# Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.4-e-x-^m-a+b-x^n-^p-c+d-x^n-^q

Nasser M. Abbasi

September 20, 2021 Compiled on September 20, 2021 at 4:41am

# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 702 ]. This is test number [ 14 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 702 ) | 0.00 ( 0 ) |
| Mathematica | 100.00 ( 702 ) | 0.00 ( 0 ) |
| Fricas | 94.44 ( 663 ) | 5.56 ( 39 ) |
| Giac | 76.21 ( 535 ) | 23.79 ( 167 ) |
| Mupad | 75.64 ( 531 ) | 24.36 ( 171 ) |
| Maple | 74.36 ( 522 ) | 25.64 ( 180 ) |
| IntegrateAlgebraic | 73.36 ( 515 ) | 26.64 ( 187 ) |
| Maxima | 55.70 ( 391 ) | 44.30 ( 311 ) |
| Sympy | 42.88 ( 301 ) | % 57.12 ( 401 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

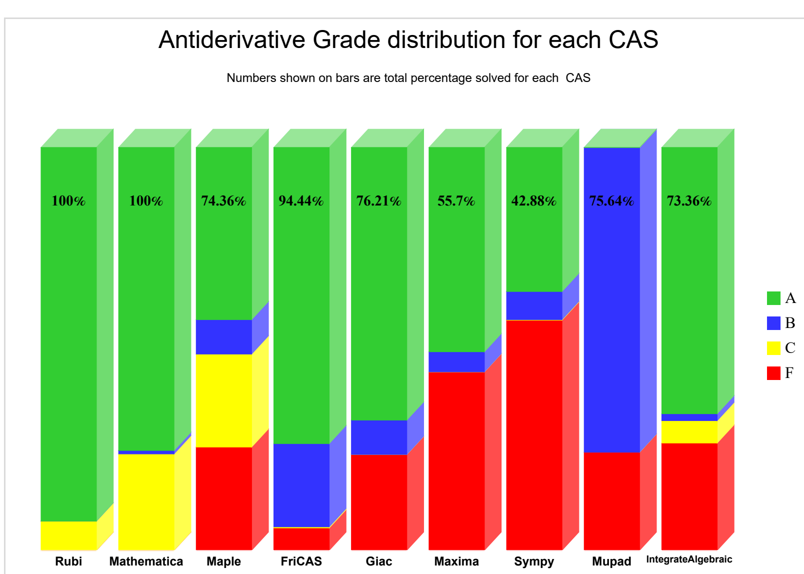
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

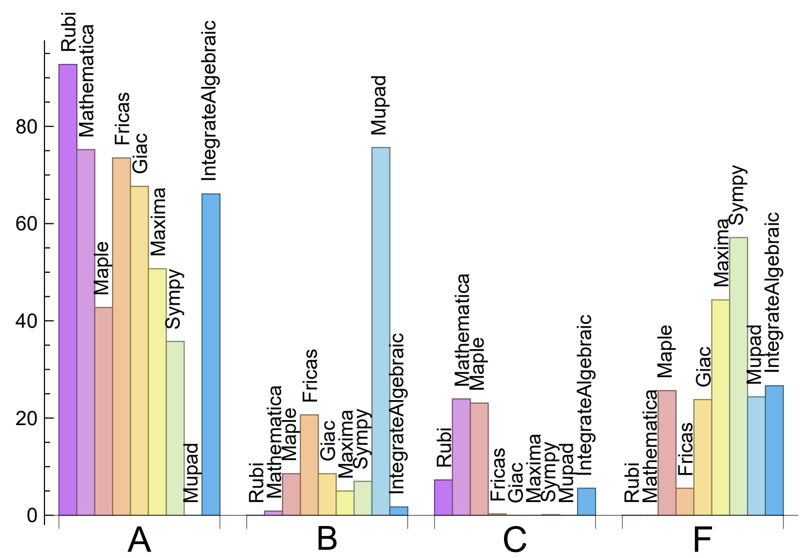
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 92.74 | 0.00 | 7.26 | 0.00 |
| Mathematica | 75.21 | 0.85 | 23.93 | 0.00 |
| Fricas | 73.50 | 20.66 | 0.28 | 5.56 |
| Giac | 67.66 | 8.55 | 0.00 | 23.79 |
| IntegrateAlgebraic | 66.10 | 1.71 | 5.56 | 26.64 |
| Maxima | 50.71 | 4.99 | 0.00 | 44.30 |
| Maple | 42.74 | 8.55 | 23.08 | 25.64 |
| Sympy | 35.75 | 6.98 | 0.14 | 57.12 |
| Mupad | N/A | 75.64 | 0.00 | 24.36 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 0 | 0.00 % | 0.00 % | 0.00 % |
| Maple | 180 | 100.00 % | 0.00 % | 0.00 % |
| Fricas | 39 | 0.00 % | 89.74 % | 10.26 % |
| IntegrateAlgebraic | 187 | 100.00 % | 0.00 % | 0.00 % |
| Giac | 167 | 69.46 % | 3.59 % | 26.95 % |
| Maxima | 311 | 76.21 % | 0.00 % | 23.79 % |
| Sympy | 401 | 62.59 % | 35.41 % | 2.00 % |
| Mupad | 171 | 100.00 % | 0.00 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

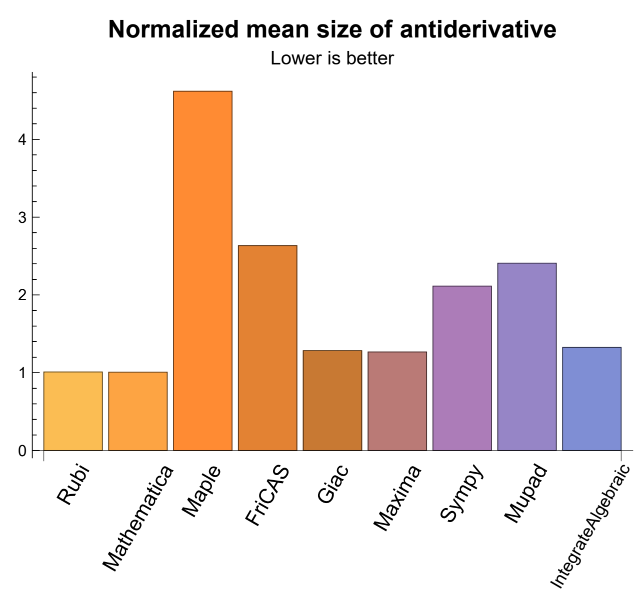
## 1.3 Performance

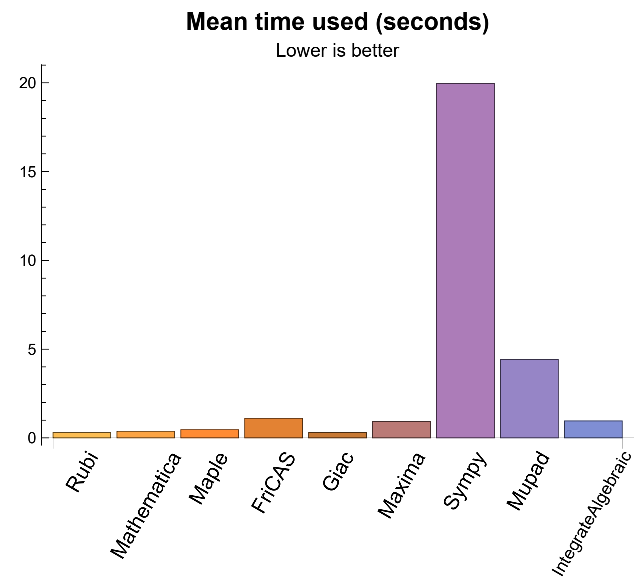
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.30 | 139.40 | 1.01 | 104.00 | 1.00 |
| Mathematica | 0.38 | 122.25 | 1.01 | 94.00 | 0.91 |
| Maple | 0.46 | 519.93 | 4.62 | 216.50 | 1.31 |
| Maxima | 0.92 | 119.81 | 1.27 | 97.00 | 1.02 |
| Fricas | 1.11 | 391.97 | 2.63 | 213.00 | 1.98 |
| Sympy | 19.97 | 194.23 | 2.11 | 114.00 | 1.17 |
| Giac | 0.30 | 151.79 | 1.28 | 114.00 | 1.04 |
| Mupad | 4.42 | 439.68 | 2.41 | 117.00 | 1.12 |
| IntegrateAlgebraic | 0.96 | 172.46 | 1.33 | 126.00 | 1.11 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

**IntegrateAlgebraic** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 370, 371, 372, 386, 398, 399, 407, 408, 409, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 486, 487, 488, 490, 491, 492}

**Mathematica** {214, 215, 237, 245, 246, 247, 248, 249, 250, 251, 252, 351, 352, 353, 355, 356, 357, 358, 366, 367, 368, 370, 371, 372, 380, 381, 383, 384, 385, 386, 393, 394, 395, 396, 397, 398, 399, 407, 408, 409, 410, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492, 520, 529, 530, 540, 541, 542, 551, 552, 560, 561, 562, 571, 572, 580, 581, 582, 649, 650}

**IntegrateAlgebraic** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

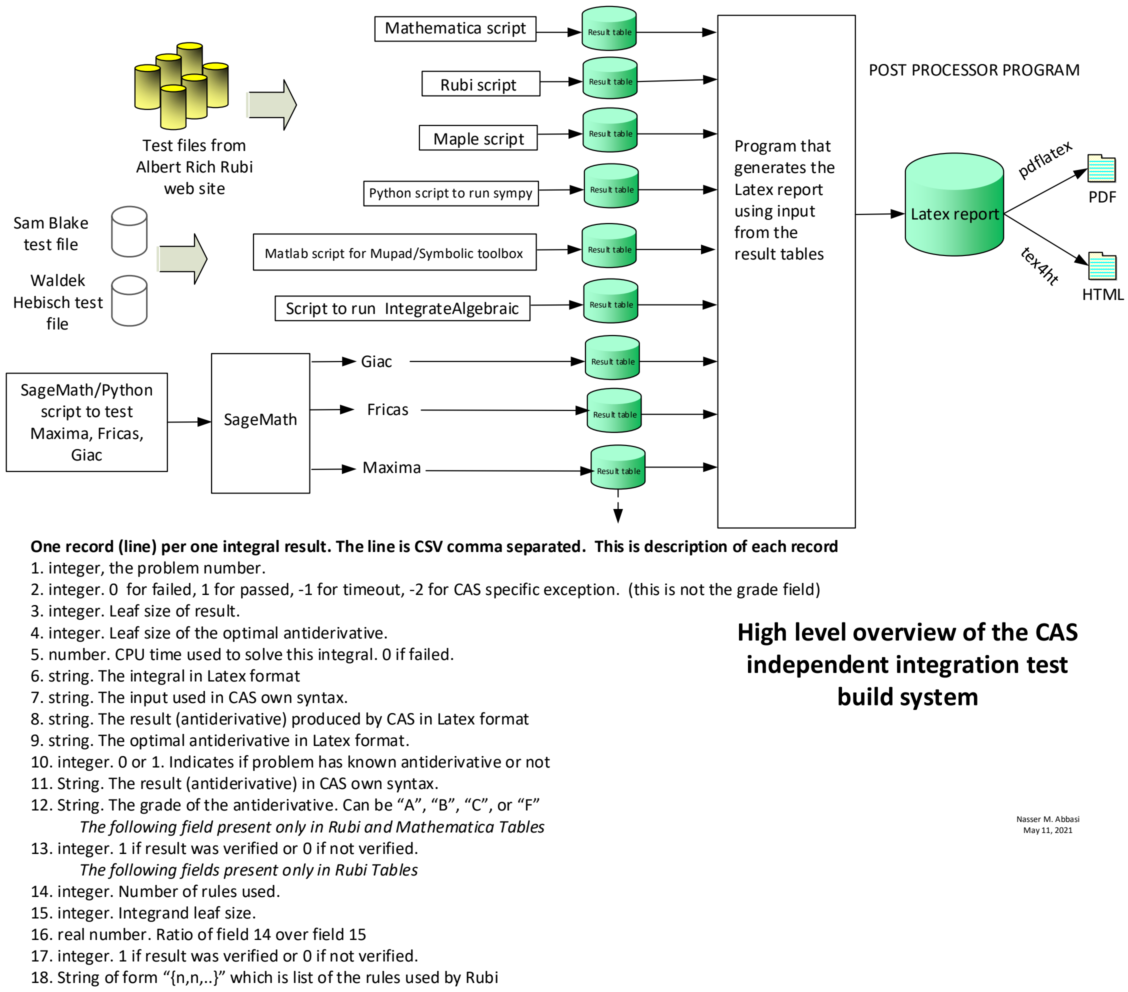
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

**Local contents**

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 359, 360, 361, 362, 363, 364, 365, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400, 401, 402, 403, 404, 405, 406, 410, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 489, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702 }

B grade: { }

C grade: { 351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 370, 371, 372, 386, 398, 399, 407, 408, 409, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 486, 487, 488, 490, 491, 492 }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 154, 157, 159, 162, 165, 167, 170, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 240, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 359, 360, 361, 362, 363, 364, 365, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 382, 384, 385, 387, 388, 389, 390, 391, 392, 403, 412, 413, 414, 415, 416, 417, 418, 419, 420, 442, 443, 444, 445, 446, 447, 464, 468, 469, 470, 471, 472, 473, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 569, 570, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 614, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 675, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699 }

B grade: { 30, 54, 652, 672, 673, 674 }

C grade: { 152, 153, 155, 156, 158, 160, 161, 163, 164, 166, 168, 169, 171, 172, 174, 186, 197, 198, 202, 203, 214, 215, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 268, 270, 271, 272, 283, 294, 296, 297, 298, 299, 300, 308, 317, 318, 319, 320, 351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 380, 381, 383, 386, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 465, 466, 467, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 518, 529, 540, 551, 552, 560, 561, 571, 572, 580, 581, 598, 601, 602, 613, 615, 638, 639, 657, 676, 700, 701, 702 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 340, 342, 343, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 616, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 658, 659, 661, 662, 663, 664, 665, 668, 669, 670, 671, 677, 678, 679, 680, 699, 700, 701 }

B grade: { 27, 30, 54, 124, 125, 126, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 586, 597, 598, 601, 602, 613, 614, 615, 625, 643, 652, 656, 660, 666, 667, 672, 673, 674, 675, 681, 682, 683, 684, 686 }

C grade: { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 373, 374, 375, 376, 377, 382, 383, 384, 385, 387, 388, 389, 390, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 409, 410, 411, 412, 413, 685 }

F grade: { 321, 322, 323, 324, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 378, 379, 380, 381, 386, 391, 392, 393, 394, 405, 406, 407, 408, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 657, 676, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 702 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 209, 210, 211, 216, 217, 218, 219, 223, 224, 225, 226, 230, 231, 232, 233, 238, 239, 240, 241, 273, 274, 275, 276, 280, 281, 282, 283, 287, 288, 289, 290, 294, 295, 296, 297, 328, 344, 345, 346, 347, 359, 360, 361, 362, 373, 374, 375, 376, 377, 387, 388, 389, 390, 400, 401, 402, 403, 404, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 629, 630, 631, 632, 633, 634, 635, 636, 637, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 676, 677, 678, 679, 685, 699, 700, 701, 702 }

B grade: { 30, 54, 180, 191, 192, 197, 198, 583, 584, 597, 598, 599, 613, 614, 615, 616, 617, 625, 626, 627, 628, 638, 639, 652, 665, 671, 672, 673, 674, 675, 680, 681, 682, 683, 684 }

C grade: { }

F grade: { 207, 208, 212, 213, 214, 215, 220, 221, 222, 227, 228, 229, 234, 235, 236, 237, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 277, 278, 279, 284, 285, 286, 291, 292, 293, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 385, 386, 391, 392, 393, 394, 395, 396, 397, 398, 399, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 154, 157, 159, 162, 165, 167, 170, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 312, 314, 315, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 373, 374, 375, 376, 377, 379, 380, 387, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 404, 405, 406, 414, 415, 416, 417, 418, 419, 420, 421, 423, 433, 434, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 460, 461, 462, 463, 483, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 533, 535, 536, 537, 538, 542, 543, 544, 545, 546, 547, 548, 549, 552, 554, 556, 557, 558, 562, 563, 564, 565, 566, 567, 568, 569, 572, 574, 576, 577, 578, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 677, 678, 679, 680, 685, 687, 688, 689, 690, 692, 693, 694, 695, 696, 699, 700, 701, 702 }

B grade: { 27, 30, 54, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 124, 125, 126, 152, 153, 155, 156, 158, 160, 161, 163, 164, 166, 168, 169, 171, 172, 174, 214, 215, 237, 251, 252, 268, 269, 271, 272, 304, 311, 313, 316, 317, 318, 319, 320, 322, 323, 348, 353, 369, 382, 383, 384, 385, 386, 395, 396, 397, 403, 407, 408, 409, 410, 411, 412, 413, 422, 428, 429, 430, 431, 432, 435, 436, 437, 457, 458, 459, 468, 469, 470, 471, 472, 473, 474, 475, 479, 480, 481, 482, 484, 485, 486, 487, 505, 506, 507, 508, 509, 510, 511, 512, 528, 529, 532, 534, 539, 540, 541, 550, 551, 553, 555, 559, 560, 561, 570, 571, 573, 575, 579, 580, 581, 603, 629, 652, 659, 665, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 686, 691, 697, 698 }

C grade: { 378, 381 }

F grade: { 245, 246, 247, 248, 249, 250, 354, 355, 356, 357, 358, 370, 371, 372, 424, 425, 426, 427, 438, 439, 440, 441, 450, 451, 452, 453, 454, 464, 465, 466, 467, 476, 477, 478, 488, 489, 490, 491, 492 }

### 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 114, 115, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 210, 211, 212, 216, 217, 218, 219, 220, 224, 225, 226, 227, 232, 233, 234, 240, 241, 242, 253, 254, 255, 256, 258, 259, 260, 261, 265, 266, 270, 271, 326, 327, 328, 335, 336, 337, 339, 513, 515, 517, 523, 524, 545, 546, 565, 566, 584, 585, 586, 587, 588, 589, 590, 595, 596, 597, 601, 602, 603, 612, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 629, 630, 631, 632, 633, 636, 639, 656, 658, 660, 661, 662, 668, 677, 678, 679, 680, 699 }

B grade: { 27, 30, 110, 111, 113, 180, 186, 192, 197, 325, 330, 331, 333, 334, 495, 496, 500, 503, 583, 591, 592, 593, 594, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 613, 614, 615, 622, 628, 634, 635, 637, 638, 657, 672, 673, 674, 681, 682, 683 }

C grade: { 651 }

F grade: { 50, 51, 52, 53, 54, 55, 107, 108, 109, 116, 117, 118, 119, 120, 121, 122, 123, 151, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 203, 208, 209, 213, 214, 215, 221, 222, 223, 228, 229, 230, 231, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 257, 262, 263, 264, 267, 268, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 332, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 497, 498, 499, 501, 502, 504, 505, 506, 507, 508, 509, 510, 511, 512, 514, 516, 518, 519, 520, 521, 522, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 600, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 659, 663, 664, 665, 666, 667, 669, 670, 671, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 325, 326, 328, 335, 336, 337, 338, 339, 341, 342, 373, 374, 375, 376, 377, 378, 379, 387, 388, 389, 390, 391, 392, 400, 401, 402, 403, 404, 405, 406, 414, 415, 416, 417, 418, 419, 420, 428, 429, 432, 433, 434, 442, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 479, 480, 481, 484, 485, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 517, 519, 521, 522, 523, 524, 525, 528, 529, 531, 532, 533, 534, 535, 536, 543, 544, 545, 546, 547, 550, 553, 554, 555, 556, 557, 563, 564, 565, 566, 567, 570, 571, 573, 574, 575, 576, 577, 583, 584, 585, 591, 592, 593, 594, 596, 597, 598, 599, 600, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 630, 640, 641, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 672, 677, 678, 679, 681, 682, 683, 685, 699 }

B grade: { 27, 30, 54, 124, 125, 126, 323, 324, 329, 332, 430, 431, 443, 482, 483, 518, 520, 530, 537, 538, 539, 540, 541, 542, 549, 558, 560, 572, 578, 579, 580, 581, 582, 586, 587, 588, 589, 590, 595, 601, 602, 603, 604, 605, 606, 619, 620, 621, 642, 643, 645, 646, 652, 673, 674, 675, 676, 700, 701, 702 }

C grade: { }

F grade: { 214, 215, 237, 245, 246, 247, 248, 249, 250, 251, 252, 327, 330, 331, 333, 334, 340, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 380, 381, 382, 383, 384, 385, 386, 393, 394, 395, 396, 397, 398, 399, 407, 408, 409, 410, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492, 514, 516, 526, 527, 548, 551, 552, 559, 561, 562, 568, 569, 618, 622, 623, 624, 625, 626, 627, 628, 629, 631, 632, 633, 634, 635, 636, 637, 638, 639, 644, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 680, 684, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

### 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 359, 360, 361, 362, 363, 364, 365, 373, 374, 375, 376, 377, 378, 379, 387, 388, 389, 390, 391, 392, 400, 401, 402, 403, 404, 405, 406, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 517, 519, 521, 522, 523, 524, 525, 531, 532, 533, 534, 535, 536, 543, 544, 545, 546, 547, 553, 554, 555, 556, 557, 563, 564, 565, 566, 567, 573, 574, 575, 576, 577, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 613, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 640, 641, 643, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 655, 656, 657, 658, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 699, 700, 701, 702 }

C grade: { }

F grade: { 245, 246, 247, 248, 249, 250, 251, 252, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 351, 352, 353, 354, 355, 356, 357, 358, 366, 367, 368, 369, 370, 371, 372, 380, 381, 382, 383, 384, 385, 386, 393, 394, 395, 396, 397, 398, 399, 407, 408, 409, 410, 411, 412, 413, 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492, 514, 516, 518, 520, 526, 527, 528, 529, 530, 537, 538, 539, 540, 541, 542, 548, 549, 550, 551, 552, 558, 559, 560, 561, 562, 568, 569, 570, 571, 572, 578, 579, 580, 581, 582, 597, 598, 611, 612, 614, 615, 626, 638, 639, 642, 651, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

### 2.1.9 IntegrateAlgebraic

A grade: { 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 455, 456, 457, 458, 459, 460, 461, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 649, 650, 655, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 675, 680, 684, 685 }

B grade: { 642, 643, 644, 645, 646, 647, 648, 651, 652, 653, 654, 686 }

C grade: { 421, 422, 423, 424, 425, 426, 427, 435, 436, 437, 438, 439, 440, 441, 448, 449, 450, 451, 452, 453, 454, 462, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 486, 487, 488, 489, 490, 491, 492 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 214, 215, 237, 245, 246, 247, 248, 249, 250, 251, 252, 398, 399, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 656, 657, 672, 673, 674, 676, 677, 678, 679, 681, 682, 683, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 28 | 27 | 29 | 29 | 29 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.82 | 0.88 | 0.88 | 0.88 | 0.85 | 0.00 |
| time (sec) | N/A | 0.033 | 0.009 | 0.037 | 0.565 | 0.652 | 0.066 | 0.179 | 0.200 | 0.000 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 28 | 27 | 29 | 29 | 29 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.82 | 0.88 | 0.88 | 0.88 | 0.85 | 0.00 |
| time (sec) | N/A | 0.017 | 0.007 | 0.045 | 0.630 | 0.699 | 0.067 | 0.148 | 2.483 | 0.000 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 25 | 24 | 26 | 26 | 26 | 25 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.86 | 0.93 | 0.93 | 0.93 | 0.89 | 0.00 |
| time (sec) | N/A | 0.013 | 0.007 | 0.036 | 0.563 | 1.078 | 0.064 | 0.150 | 0.034 | 0.000 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 28 | 28 | 25 | 27 | 28 | 26 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.97 | 0.97 | 0.86 | 0.93 | 0.97 | 0.90 | 0.00 |
| time (sec) | N/A | 0.021 | 0.009 | 0.038 | 0.571 | 0.763 | 0.113 | 0.150 | 0.035 | 0.001 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 30 | 27 | 29 | 26 | 29 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.97 | 0.87 | 0.94 | 0.84 | 0.94 | 0.90 | 0.00 |
| time (sec) | N/A | 0.015 | 0.011 | 0.049 | 0.504 | 0.727 | 0.108 | 0.147 | 0.039 | 0.000 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 28 | 24 | 24 | 28 | 24 | 23 | 24 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.86 | 0.86 | 1.00 | 0.86 | 0.82 | 0.86 | 0.00 |
| time (sec) | N/A | 0.016 | 0.009 | 0.044 | 0.478 | 0.794 | 0.115 | 0.159 | 2.337 | 0.001 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 26 | 28 | 30 | 26 | 40 | 25 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 0.97 | 1.03 | 0.90 | 1.38 | 0.86 | 0.00 |
| time (sec) | N/A | 0.022 | 0.013 | 0.050 | 0.475 | 0.810 | 0.210 | 0.164 | 0.040 | 0.001 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 32 | 28 | 29 | 29 | 31 | 31 | 29 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.90 | 0.94 | 0.94 | 1.00 | 1.00 | 0.94 | 0.00 |
| time (sec) | N/A | 0.017 | 0.013 | 0.046 | 0.632 | 1.291 | 0.250 | 0.184 | 0.033 | 0.001 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 28 | 28 | 30 | 25 | 27 | 29 | 29 | 29 | 28 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 0.89 | 0.96 | 1.04 | 1.04 | 1.04 | 1.00 | 0.00 |
| time (sec) | N/A | 0.016 | 0.012 | 0.041 | 0.488 | 0.742 | 0.287 | 0.184 | 2.316 | 0.001 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 31 | 28 | 30 | 31 | 29 | 37 | 29 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 0.97 | 1.03 | 1.07 | 1.00 | 1.28 | 1.00 | 0.00 |
| time (sec) | N/A | 0.021 | 0.018 | 0.044 | 0.509 | 0.927 | 0.542 | 0.164 | 0.052 | 0.000 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 51 | 52 | 51 | 53 | 54 | 53 | 51 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 1.24 | 1.21 | 1.26 | 1.29 | 1.26 | 1.21 | 0.00 |
| time (sec) | N/A | 0.072 | 0.016 | 0.039 | 0.491 | 1.028 | 0.076 | 0.153 | 2.375 | 0.000 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 55 | 55 | 55 | 52 | 51 | 53 | 54 | 53 | 51 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.93 | 0.96 | 0.98 | 0.96 | 0.93 | 0.00 |
| time (sec) | N/A | 0.034 | 0.008 | 0.046 | 0.608 | 1.061 | 0.076 | 0.149 | 0.043 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 50 | 49 | 48 | 50 | 51 | 50 | 48 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.98 | 0.96 | 1.00 | 1.02 | 1.00 | 0.96 | 0.00 |
| time (sec) | N/A | 0.025 | 0.039 | 0.035 | 0.663 | 0.642 | 0.076 | 0.168 | 0.043 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 51 | 52 | 52 | 49 | 53 | 52 | 49 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.13 | 1.13 | 1.07 | 1.15 | 1.13 | 1.07 | 0.00 |
| time (sec) | N/A | 0.033 | 0.038 | 0.042 | 0.538 | 0.772 | 0.145 | 0.149 | 0.038 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 53 | 53 | 51 | 53 | 49 | 52 | 50 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.96 | 1.00 | 0.92 | 0.98 | 0.94 | 0.00 |
| time (sec) | N/A | 0.028 | 0.022 | 0.043 | 0.591 | 0.736 | 0.153 | 0.151 | 0.050 | 0.001 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 50 | 49 | 48 | 53 | 49 | 48 | 48 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.98 | 0.96 | 1.06 | 0.98 | 0.96 | 0.96 | 0.00 |
| time (sec) | N/A | 0.027 | 0.016 | 0.045 | 0.487 | 0.826 | 0.152 | 0.169 | 0.046 | 0.001 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 49 | 51 | 52 | 54 | 51 | 69 | 49 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 1.00 | 1.02 | 1.06 | 1.00 | 1.35 | 0.96 | 0.00 |
| time (sec) | N/A | 0.050 | 0.026 | 0.052 | 0.483 | 0.639 | 0.273 | 0.148 | 0.043 | 0.001 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 51 | 50 | 53 | 53 | 53 | 54 | 52 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.94 | 1.00 | 1.00 | 1.00 | 1.02 | 0.98 | 0.00 |
| time (sec) | N/A | 0.031 | 0.017 | 0.049 | 0.724 | 0.717 | 0.310 | 0.149 | 0.049 | 0.001 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 50 | 46 | 51 | 53 | 53 | 51 | 50 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 1.02 | 1.06 | 1.06 | 1.02 | 1.00 | 0.00 |
| time (sec) | N/A | 0.028 | 0.020 | 0.058 | 0.574 | 1.371 | 0.354 | 0.150 | 2.371 | 0.001 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 51 | 51 | 54 | 55 | 51 | 70 | 52 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 1.06 | 1.08 | 1.00 | 1.37 | 1.02 | 0.00 |
| time (sec) | N/A | 0.038 | 0.022 | 0.067 | 0.666 | 0.747 | 0.756 | 0.207 | 2.359 | 0.001 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 54 | 48 | 54 | 53 | 58 | 56 | 53 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.91 | 1.02 | 1.00 | 1.09 | 1.06 | 1.00 | 0.00 |
| time (sec) | N/A | 0.029 | 0.017 | 0.045 | 0.567 | 0.802 | 0.874 | 0.152 | 0.045 | 0.001 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 50 | 45 | 51 | 53 | 54 | 53 | 50 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.90 | 1.02 | 1.06 | 1.08 | 1.06 | 1.00 | 0.00 |
| time (sec) | N/A | 0.029 | 0.031 | 0.046 | 0.493 | 0.826 | 0.986 | 0.148 | 2.342 | 0.001 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 124 | 119 | 125 | 136 | 125 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.02 | 1.07 | 1.16 | 1.07 | 0.91 | 0.00 |
| time (sec) | N/A | 0.098 | 0.022 | 0.038 | 0.460 | 0.714 | 0.096 | 0.160 | 0.050 | 0.000 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 107 | 124 | 119 | 125 | 136 | 125 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.13 | 1.31 | 1.25 | 1.32 | 1.43 | 1.32 | 1.13 | 0.00 |
| time (sec) | N/A | 0.237 | 0.029 | 0.038 | 0.473 | 0.734 | 0.096 | 0.173 | 2.338 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 124 | 119 | 125 | 134 | 125 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.02 | 1.07 | 1.15 | 1.07 | 0.91 | 0.00 |
| time (sec) | N/A | 0.075 | 0.020 | 0.042 | 0.475 | 0.689 | 0.096 | 0.153 | 0.041 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 124 | 119 | 125 | 136 | 125 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.02 | 1.07 | 1.16 | 1.07 | 0.91 | 0.00 |
| time (sec) | N/A | 0.068 | 0.020 | 0.033 | 0.588 | 0.517 | 0.096 | 0.152 | 0.041 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 107 | 124 | 119 | 125 | 138 | 125 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.60 | 1.85 | 1.78 | 1.87 | 2.06 | 1.87 | 1.60 | 0.00 |
| time (sec) | N/A | 0.148 | 0.025 | 0.040 | 0.447 | 1.285 | 0.100 | 0.156 | 0.040 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 124 | 119 | 125 | 136 | 125 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.02 | 1.07 | 1.16 | 1.07 | 0.91 | 0.00 |
| time (sec) | N/A | 0.070 | 0.021 | 0.042 | 0.644 | 0.820 | 0.098 | 0.227 | 0.041 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 124 | 119 | 124 | 133 | 124 | 107 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.02 | 1.06 | 1.14 | 1.06 | 0.91 | 0.00 |
| time (sec) | N/A | 0.065 | 0.018 | 0.039 | 0.443 | 0.754 | 0.096 | 0.156 | 0.040 | 0.000 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 107 | 124 | 119 | 125 | 136 | 125 | 107 | 0 |
| N.S. | 1 | 1.00 | 2.55 | 2.95 | 2.83 | 2.98 | 3.24 | 2.98 | 2.55 | 0.00 |
| time (sec) | N/A | 0.069 | 0.030 | 0.042 | 0.443 | 0.716 | 0.095 | 0.173 | 0.041 | 0.000 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 124 | 119 | 124 | 134 | 124 | 106 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.02 | 1.06 | 1.15 | 1.06 | 0.91 | 0.00 |
| time (sec) | N/A | 0.063 | 0.018 | 0.043 | 0.646 | 0.703 | 0.094 | 0.153 | 0.040 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 109 | 121 | 115 | 120 | 128 | 120 | 103 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.11 | 1.06 | 1.10 | 1.17 | 1.10 | 0.94 | 0.00 |
| time (sec) | N/A | 0.055 | 0.017 | 0.039 | 0.599 | 0.704 | 0.093 | 0.169 | 0.041 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 113 | 124 | 120 | 117 | 134 | 124 | 105 | 0 |
| N.S. | 1 | 1.00 | 1.28 | 1.41 | 1.36 | 1.33 | 1.52 | 1.41 | 1.19 | 0.00 |
| time (sec) | N/A | 0.067 | 0.029 | 0.042 | 0.449 | 0.816 | 0.242 | 0.155 | 0.046 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 112 | 125 | 118 | 121 | 129 | 124 | 106 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.12 | 1.05 | 1.08 | 1.15 | 1.11 | 0.95 | 0.00 |
| time (sec) | N/A | 0.061 | 0.035 | 0.049 | 0.539 | 0.762 | 0.248 | 0.151 | 0.042 | 0.001 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 112 | 120 | 116 | 121 | 128 | 119 | 104 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.04 | 1.08 | 1.14 | 1.06 | 0.93 | 0.00 |
| time (sec) | N/A | 0.061 | 0.032 | 0.040 | 0.607 | 1.031 | 0.244 | 0.155 | 0.043 | 0.001 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 115 | 123 | 120 | 123 | 133 | 143 | 105 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.09 | 1.06 | 1.09 | 1.18 | 1.27 | 0.93 | 0.00 |
| time (sec) | N/A | 0.117 | 0.045 | 0.054 | 0.553 | 0.750 | 0.368 | 0.158 | 2.348 | 0.001 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 115 | 123 | 121 | 121 | 133 | 127 | 109 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.09 | 1.07 | 1.07 | 1.18 | 1.12 | 0.96 | 0.00 |
| time (sec) | N/A | 0.067 | 0.063 | 0.047 | 0.489 | 0.732 | 0.422 | 0.157 | 0.042 | 0.001 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 113 | 119 | 120 | 121 | 133 | 124 | 108 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 1.06 | 1.07 | 1.18 | 1.10 | 0.96 | 0.00 |
| time (sec) | N/A | 0.063 | 0.057 | 0.043 | 0.733 | 0.527 | 0.478 | 0.179 | 0.043 | 0.001 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 106 | 124 | 122 | 123 | 131 | 148 | 113 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.09 | 1.07 | 1.08 | 1.15 | 1.30 | 0.99 | 0.00 |
| time (sec) | N/A | 0.106 | 0.057 | 0.049 | 0.649 | 0.774 | 0.941 | 0.156 | 0.049 | 0.001 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 110 | 117 | 121 | 121 | 129 | 127 | 113 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.06 | 1.10 | 1.10 | 1.17 | 1.15 | 1.03 | 0.00 |
| time (sec) | N/A | 0.067 | 0.038 | 0.047 | 0.513 | 1.109 | 1.118 | 0.173 | 2.341 | 0.001 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 113 | 114 | 120 | 121 | 133 | 124 | 111 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.01 | 1.06 | 1.07 | 1.18 | 1.10 | 0.98 | 0.00 |
| time (sec) | N/A | 0.062 | 0.038 | 0.044 | 0.593 | 0.717 | 1.211 | 0.162 | 0.044 | 0.001 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 106 | 124 | 123 | 123 | 129 | 150 | 118 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.09 | 1.08 | 1.08 | 1.13 | 1.32 | 1.04 | 0.00 |
| time (sec) | N/A | 0.104 | 0.055 | 0.049 | 0.469 | 0.645 | 2.244 | 0.154 | 0.050 | 0.001 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 118 | 111 | 122 | 121 | 131 | 127 | 118 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.97 | 1.06 | 1.05 | 1.14 | 1.10 | 1.03 | 0.00 |
| time (sec) | N/A | 0.063 | 0.024 | 0.053 | 0.465 | 1.052 | 2.931 | 0.154 | 2.364 | 0.001 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 109 | 108 | 120 | 121 | 131 | 124 | 116 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.99 | 1.10 | 1.11 | 1.20 | 1.14 | 1.06 | 0.00 |
| time (sec) | N/A | 0.064 | 0.042 | 0.054 | 0.534 | 0.793 | 3.727 | 0.183 | 0.067 | 0.001 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 118 | 124 | 123 | 123 | 129 | 149 | 122 | 0 |
| N.S. | 1 | 1.00 | 1.04 | 1.09 | 1.08 | 1.08 | 1.13 | 1.31 | 1.07 | 0.00 |
| time (sec) | N/A | 0.099 | 0.048 | 0.051 | 0.634 | 0.622 | 6.644 | 0.157 | 0.064 | 0.001 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 117 | 107 | 122 | 121 | 134 | 128 | 123 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.93 | 1.06 | 1.05 | 1.17 | 1.11 | 1.07 | 0.00 |
| time (sec) | N/A | 0.062 | 0.051 | 0.058 | 0.558 | 0.635 | 10.765 | 0.159 | 2.375 | 0.001 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 110 | 102 | 119 | 121 | 129 | 123 | 120 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 1.08 | 1.10 | 1.17 | 1.12 | 1.09 | 0.00 |
| time (sec) | N/A | 0.068 | 0.071 | 0.052 | 0.614 | 0.560 | 22.983 | 0.163 | 2.392 | 0.001 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 116 | 123 | 123 | 123 | 129 | 145 | 121 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 1.09 | 1.09 | 1.09 | 1.14 | 1.28 | 1.07 | 0.00 |
| time (sec) | N/A | 0.091 | 0.068 | 0.049 | 0.586 | 0.669 | 29.301 | 0.155 | 0.079 | 0.001 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 118 | 104 | 122 | 121 | 134 | 128 | 121 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.90 | 1.06 | 1.05 | 1.17 | 1.11 | 1.05 | 0.00 |
| time (sec) | N/A | 0.062 | 0.033 | 0.043 | 0.498 | 0.559 | 66.092 | 0.160 | 2.363 | 0.001 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 110 | 101 | 119 | 121 | 0 | 125 | 119 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.92 | 1.08 | 1.10 | 0.00 | 1.14 | 1.08 | 0.00 |
| time (sec) | N/A | 0.061 | 0.053 | 0.048 | 0.518 | 0.675 | 0.000 | 0.171 | 0.077 | 0.001 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 121 | 124 | 123 | 123 | 0 | 136 | 121 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 1.36 | 1.35 | 1.35 | 0.00 | 1.49 | 1.33 | 0.00 |
| time (sec) | N/A | 0.054 | 0.045 | 0.047 | 0.508 | 0.539 | 0.000 | 0.158 | 0.093 | 0.001 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 119 | 104 | 121 | 121 | 0 | 127 | 119 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.92 | 1.07 | 1.07 | 0.00 | 1.12 | 1.05 | 0.00 |
| time (sec) | N/A | 0.064 | 0.032 | 0.043 | 0.460 | 0.495 | 0.000 | 0.153 | 2.369 | 0.001 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 121 | 104 | 121 | 121 | 0 | 127 | 121 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 0.89 | 1.03 | 1.03 | 0.00 | 1.09 | 1.03 | 0.00 |
| time (sec) | N/A | 0.059 | 0.037 | 0.048 | 0.552 | 0.529 | 0.000 | 0.153 | 2.347 | 0.001 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 118 | 104 | 121 | 121 | 0 | 127 | 122 | 0 |
| N.S. | 1 | 1.00 | 2.46 | 2.17 | 2.52 | 2.52 | 0.00 | 2.65 | 2.54 | 0.00 |
| time (sec) | N/A | 0.032 | 0.036 | 0.046 | 0.612 | 0.614 | 0.000 | 0.183 | 2.370 | 0.001 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 117 | 104 | 121 | 121 | 0 | 127 | 121 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 1.03 | 1.03 | 0.00 | 1.09 | 1.03 | 0.00 |
| time (sec) | N/A | 0.065 | 0.050 | 0.049 | 0.697 | 0.511 | 0.000 | 0.165 | 0.064 | 0.001 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 183 | 183 | 171 | 249 | 182 | 167 | 114 | 217 | 164 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.36 | 0.99 | 0.91 | 0.62 | 1.19 | 0.90 | 0.00 |
| time (sec) | N/A | 0.149 | 0.135 | 0.044 | 1.431 | 0.681 | 1.372 | 0.237 | 0.268 | 0.001 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 47 | 62 | 50 | 51 | 46 | 52 | 52 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.15 | 0.93 | 0.94 | 0.85 | 0.96 | 0.96 | 0.00 |
| time (sec) | N/A | 0.057 | 0.022 | 0.041 | 0.499 | 0.560 | 0.960 | 0.170 | 0.079 | 0.001 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 167 | 167 | 154 | 226 | 157 | 162 | 114 | 207 | 144 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.35 | 0.94 | 0.97 | 0.68 | 1.24 | 0.86 | 0.00 |
| time (sec) | N/A | 0.120 | 0.093 | 0.045 | 1.096 | 0.528 | 0.872 | 0.186 | 2.545 | 0.001 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 162 | 162 | 152 | 221 | 154 | 145 | 87 | 186 | 162 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.36 | 0.95 | 0.90 | 0.54 | 1.15 | 1.00 | 0.00 |
| time (sec) | N/A | 0.116 | 0.093 | 0.043 | 1.084 | 0.490 | 1.147 | 0.170 | 2.610 | 0.001 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 31 | 40 | 31 | 30 | 27 | 32 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.14 | 0.89 | 0.86 | 0.77 | 0.91 | 0.89 | 0.00 |
| time (sec) | N/A | 0.033 | 0.029 | 0.040 | 0.540 | 0.580 | 0.883 | 0.217 | 0.063 | 0.001 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 150 | 152 | 198 | 131 | 382 | 92 | 161 | 126 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.32 | 0.87 | 2.55 | 0.61 | 1.07 | 0.84 | 0.00 |
| time (sec) | N/A | 0.090 | 0.057 | 0.041 | 1.144 | 0.752 | 1.106 | 0.189 | 2.566 | 0.001 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 145 | 145 | 129 | 195 | 128 | 369 | 71 | 133 | 123 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.34 | 0.88 | 2.54 | 0.49 | 0.92 | 0.85 | 0.00 |
| time (sec) | N/A | 0.077 | 0.089 | 0.045 | 1.157 | 0.694 | 1.016 | 0.176 | 2.541 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 34 | 34 | 34 | 37 | 35 | 32 | 26 | 34 | 36 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.09 | 1.03 | 0.94 | 0.76 | 1.00 | 1.06 | 0.00 |
| time (sec) | N/A | 0.033 | 0.014 | 0.050 | 0.590 | 0.481 | 2.116 | 0.193 | 0.105 | 0.001 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 134 | 195 | 140 | 372 | 90 | 155 | 126 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.33 | 0.95 | 2.53 | 0.61 | 1.05 | 0.86 | 0.00 |
| time (sec) | N/A | 0.086 | 0.094 | 0.057 | 1.157 | 0.680 | 0.940 | 0.223 | 2.545 | 0.001 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 149 | 149 | 135 | 195 | 140 | 411 | 73 | 161 | 126 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.31 | 0.94 | 2.76 | 0.49 | 1.08 | 0.85 | 0.00 |
| time (sec) | N/A | 0.093 | 0.103 | 0.048 | 1.223 | 0.499 | 1.232 | 0.176 | 0.244 | 0.001 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 49 | 56 | 48 | 47 | 41 | 69 | 46 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 1.12 | 0.96 | 0.94 | 0.82 | 1.38 | 0.92 | 0.00 |
| time (sec) | N/A | 0.049 | 0.022 | 0.048 | 0.483 | 0.505 | 2.360 | 0.165 | 2.405 | 0.001 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 165 | 165 | 154 | 216 | 147 | 158 | 112 | 197 | 178 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.31 | 0.89 | 0.96 | 0.68 | 1.19 | 1.08 | 0.00 |
| time (sec) | N/A | 0.115 | 0.141 | 0.046 | 1.137 | 0.631 | 0.961 | 0.225 | 2.591 | 0.001 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 168 | 168 | 154 | 217 | 148 | 176 | 99 | 176 | 145 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.29 | 0.88 | 1.05 | 0.59 | 1.05 | 0.86 | 0.00 |
| time (sec) | N/A | 0.119 | 0.134 | 0.048 | 1.281 | 0.601 | 0.932 | 0.176 | 2.564 | 0.001 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 70 | 81 | 70 | 73 | 61 | 99 | 70 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.17 | 1.01 | 1.06 | 0.88 | 1.43 | 1.01 | 0.00 |
| time (sec) | N/A | 0.064 | 0.033 | 0.050 | 0.492 | 0.526 | 2.688 | 0.186 | 0.127 | 0.001 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 184 | 184 | 173 | 247 | 178 | 180 | 139 | 216 | 161 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.34 | 0.97 | 0.98 | 0.76 | 1.17 | 0.88 | 0.00 |
| time (sec) | N/A | 0.134 | 0.144 | 0.057 | 1.289 | 0.851 | 1.135 | 0.183 | 2.581 | 0.001 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 233 | 233 | 203 | 288 | 218 | 271 | 156 | 244 | 209 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.24 | 0.94 | 1.16 | 0.67 | 1.05 | 0.90 | 0.00 |
| time (sec) | N/A | 0.138 | 0.169 | 0.057 | 1.233 | 0.717 | 2.116 | 0.201 | 2.620 | 0.001 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 72 | 97 | 82 | 121 | 82 | 106 | 86 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.18 | 1.00 | 1.48 | 1.00 | 1.29 | 1.05 | 0.00 |
| time (sec) | N/A | 0.091 | 0.101 | 0.053 | 0.492 | 0.571 | 2.143 | 0.183 | 0.083 | 0.001 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 215 | 215 | 185 | 266 | 192 | 257 | 151 | 236 | 179 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.24 | 0.89 | 1.20 | 0.70 | 1.10 | 0.83 | 0.00 |
| time (sec) | N/A | 0.140 | 0.169 | 0.048 | 1.102 | 0.669 | 2.120 | 0.183 | 0.268 | 0.001 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 213 | 213 | 181 | 257 | 187 | 240 | 126 | 211 | 193 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.21 | 0.88 | 1.13 | 0.59 | 0.99 | 0.91 | 0.00 |
| time (sec) | N/A | 0.130 | 0.150 | 0.086 | 1.012 | 0.847 | 2.536 | 0.180 | 2.621 | 0.001 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 50 | 74 | 60 | 81 | 56 | 91 | 62 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.23 | 1.00 | 1.35 | 0.93 | 1.52 | 1.03 | 0.00 |
| time (sec) | N/A | 0.058 | 0.063 | 0.052 | 0.458 | 0.687 | 1.608 | 0.182 | 0.081 | 0.001 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 196 | 196 | 165 | 235 | 162 | 578 | 126 | 189 | 158 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.20 | 0.83 | 2.95 | 0.64 | 0.96 | 0.81 | 0.00 |
| time (sec) | N/A | 0.110 | 0.140 | 0.080 | 1.171 | 0.789 | 2.093 | 0.208 | 2.578 | 0.001 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 190 | 190 | 160 | 228 | 157 | 573 | 102 | 166 | 150 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.20 | 0.83 | 3.02 | 0.54 | 0.87 | 0.79 | 0.00 |
| time (sec) | N/A | 0.106 | 0.170 | 0.044 | 1.216 | 0.554 | 1.668 | 0.185 | 2.577 | 0.001 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 41 | 41 | 41 | 47 | 40 | 44 | 36 | 65 | 37 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.15 | 0.98 | 1.07 | 0.88 | 1.59 | 0.90 | 0.00 |
| time (sec) | N/A | 0.038 | 0.032 | 0.055 | 0.582 | 0.564 | 1.225 | 0.184 | 2.348 | 0.001 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 171 | 171 | 146 | 223 | 160 | 548 | 117 | 186 | 145 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.30 | 0.94 | 3.20 | 0.68 | 1.09 | 0.85 | 0.00 |
| time (sec) | N/A | 0.088 | 0.134 | 0.049 | 1.210 | 0.548 | 1.482 | 0.181 | 0.250 | 0.001 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 169 | 169 | 145 | 221 | 158 | 537 | 97 | 160 | 143 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.31 | 0.93 | 3.18 | 0.57 | 0.95 | 0.85 | 0.00 |
| time (sec) | N/A | 0.086 | 0.139 | 0.057 | 1.102 | 0.705 | 1.434 | 0.177 | 2.550 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 46 | 53 | 51 | 70 | 46 | 61 | 47 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.04 | 1.00 | 1.37 | 0.90 | 1.20 | 0.92 | 0.00 |
| time (sec) | N/A | 0.046 | 0.034 | 0.054 | 0.483 | 0.699 | 1.162 | 0.214 | 0.141 | 0.001 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 195 | 196 | 164 | 241 | 166 | 570 | 122 | 180 | 156 | 0 |
| N.S. | 1 | 1.01 | 0.84 | 1.24 | 0.85 | 2.92 | 0.63 | 0.92 | 0.80 | 0.00 |
| time (sec) | N/A | 0.106 | 0.144 | 0.052 | 1.078 | 0.697 | 1.417 | 0.183 | 2.569 | 0.001 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 196 | 196 | 163 | 237 | 172 | 618 | 109 | 188 | 159 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.21 | 0.88 | 3.15 | 0.56 | 0.96 | 0.81 | 0.00 |
| time (sec) | N/A | 0.103 | 0.156 | 0.048 | 1.232 | 0.823 | 1.927 | 0.174 | 2.565 | 0.001 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 64 | 87 | 76 | 118 | 70 | 80 | 78 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.14 | 1.00 | 1.55 | 0.92 | 1.05 | 1.03 | 0.00 |
| time (sec) | N/A | 0.076 | 0.072 | 0.060 | 0.504 | 0.490 | 1.444 | 0.189 | 2.432 | 0.001 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 215 | 215 | 185 | 257 | 186 | 259 | 153 | 231 | 209 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.20 | 0.87 | 1.20 | 0.71 | 1.07 | 0.97 | 0.00 |
| time (sec) | N/A | 0.127 | 0.238 | 0.057 | 1.421 | 0.601 | 2.324 | 0.221 | 2.615 | 0.001 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 215 | 215 | 183 | 252 | 186 | 277 | 138 | 206 | 176 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.17 | 0.87 | 1.29 | 0.64 | 0.96 | 0.82 | 0.00 |
| time (sec) | N/A | 0.127 | 0.180 | 0.053 | 1.319 | 0.532 | 1.843 | 0.197 | 2.574 | 0.001 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 85 | 116 | 106 | 154 | 100 | 149 | 100 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.20 | 1.09 | 1.59 | 1.03 | 1.54 | 1.03 | 0.00 |
| time (sec) | N/A | 0.099 | 0.115 | 0.075 | 0.652 | 0.463 | 2.397 | 0.165 | 0.144 | 0.001 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 94 | 134 | 115 | 179 | 112 | 131 | 117 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.25 | 1.07 | 1.67 | 1.05 | 1.22 | 1.09 | 0.00 |
| time (sec) | N/A | 0.140 | 0.072 | 0.054 | 0.538 | 0.639 | 4.941 | 0.183 | 0.097 | 0.001 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 92 | 110 | 94 | 142 | 94 | 93 | 94 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 1.25 | 1.07 | 1.61 | 1.07 | 1.06 | 1.07 | 0.00 |
| time (sec) | N/A | 0.088 | 0.040 | 0.048 | 0.524 | 0.634 | 3.945 | 0.180 | 2.402 | 0.001 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 64 | 81 | 72 | 89 | 70 | 61 | 70 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 1.23 | 1.09 | 1.35 | 1.06 | 0.92 | 1.06 | 0.00 |
| time (sec) | N/A | 0.068 | 0.025 | 0.051 | 0.474 | 0.749 | 3.692 | 0.188 | 2.385 | 0.001 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 32 | 32 | 30 | 39 | 42 | 42 | 42 | 28 | 44 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 1.22 | 1.31 | 1.31 | 1.31 | 0.88 | 1.38 | 0.00 |
| time (sec) | N/A | 0.022 | 0.015 | 0.053 | 0.457 | 0.687 | 1.815 | 0.183 | 2.332 | 0.001 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 59 | 68 | 77 | 119 | 75 | 74 | 71 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.00 | 1.13 | 1.75 | 1.10 | 1.09 | 1.04 | 0.00 |
| time (sec) | N/A | 0.058 | 0.051 | 0.059 | 0.454 | 0.746 | 1.481 | 0.203 | 0.161 | 0.001 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 87 | 117 | 109 | 197 | 107 | 136 | 107 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.16 | 1.08 | 1.95 | 1.06 | 1.35 | 1.06 | 0.00 |
| time (sec) | N/A | 0.100 | 0.072 | 0.060 | 0.504 | 0.616 | 3.272 | 0.195 | 2.458 | 0.001 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 122 | 122 | 108 | 147 | 136 | 229 | 133 | 131 | 130 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.20 | 1.11 | 1.88 | 1.09 | 1.07 | 1.07 | 0.00 |
| time (sec) | N/A | 0.130 | 0.110 | 0.058 | 0.519 | 0.644 | 3.553 | 0.200 | 0.153 | 0.001 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 246 | 246 | 216 | 308 | 228 | 364 | 192 | 259 | 213 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.25 | 0.93 | 1.48 | 0.78 | 1.05 | 0.87 | 0.00 |
| time (sec) | N/A | 0.156 | 0.323 | 0.061 | 1.366 | 0.649 | 6.239 | 0.330 | 2.582 | 0.001 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 244 | 244 | 210 | 299 | 223 | 347 | 163 | 234 | 227 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.23 | 0.91 | 1.42 | 0.67 | 0.96 | 0.93 | 0.00 |
| time (sec) | N/A | 0.165 | 0.244 | 0.056 | 1.237 | 0.604 | 4.009 | 0.238 | 0.320 | 0.001 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 222 | 222 | 194 | 275 | 196 | 792 | 162 | 210 | 187 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.24 | 0.88 | 3.57 | 0.73 | 0.95 | 0.84 | 0.00 |
| time (sec) | N/A | 0.140 | 0.237 | 0.062 | 1.322 | 0.917 | 5.854 | 0.211 | 2.558 | 0.001 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 220 | 220 | 188 | 268 | 191 | 789 | 141 | 187 | 183 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.22 | 0.87 | 3.59 | 0.64 | 0.85 | 0.83 | 0.00 |
| time (sec) | N/A | 0.126 | 0.273 | 0.054 | 1.361 | 0.651 | 3.392 | 0.196 | 2.603 | 0.001 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 201 | 201 | 181 | 241 | 195 | 756 | 155 | 206 | 175 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.20 | 0.97 | 3.76 | 0.77 | 1.02 | 0.87 | 0.00 |
| time (sec) | N/A | 0.107 | 0.290 | 0.052 | 1.266 | 0.541 | 4.749 | 0.199 | 0.267 | 0.001 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 199 | 199 | 178 | 239 | 193 | 743 | 136 | 187 | 173 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.20 | 0.97 | 3.73 | 0.68 | 0.94 | 0.87 | 0.00 |
| time (sec) | N/A | 0.113 | 0.244 | 0.049 | 1.253 | 0.719 | 3.253 | 0.192 | 2.559 | 0.001 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 201 | 201 | 178 | 251 | 195 | 752 | 153 | 207 | 175 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.25 | 0.97 | 3.74 | 0.76 | 1.03 | 0.87 | 0.00 |
| time (sec) | N/A | 0.115 | 0.241 | 0.053 | 1.399 | 0.786 | 2.032 | 0.215 | 0.268 | 0.001 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 197 | 197 | 175 | 249 | 192 | 743 | 133 | 180 | 173 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.26 | 0.97 | 3.77 | 0.68 | 0.91 | 0.88 | 0.00 |
| time (sec) | N/A | 0.103 | 0.185 | 0.056 | 1.370 | 0.637 | 1.513 | 0.195 | 0.256 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 227 | 227 | 193 | 281 | 199 | 776 | 162 | 204 | 185 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.24 | 0.88 | 3.42 | 0.71 | 0.90 | 0.81 | 0.00 |
| time (sec) | N/A | 0.123 | 0.280 | 0.063 | 1.409 | 0.694 | 1.902 | 0.196 | 2.598 | 0.001 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 227 | 227 | 189 | 277 | 201 | 812 | 143 | 209 | 188 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.22 | 0.89 | 3.58 | 0.63 | 0.92 | 0.83 | 0.00 |
| time (sec) | N/A | 0.131 | 0.261 | 0.054 | 1.198 | 0.520 | 1.731 | 0.191 | 2.583 | 0.001 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 246 | 246 | 214 | 299 | 221 | 366 | 189 | 254 | 240 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.22 | 0.90 | 1.49 | 0.77 | 1.03 | 0.98 | 0.00 |
| time (sec) | N/A | 0.158 | 0.274 | 0.056 | 1.419 | 0.920 | 1.905 | 0.198 | 2.637 | 0.001 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 246 | 246 | 210 | 295 | 221 | 384 | 173 | 229 | 207 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.20 | 0.90 | 1.56 | 0.70 | 0.93 | 0.84 | 0.00 |
| time (sec) | N/A | 0.142 | 0.317 | 0.062 | 1.465 | 0.664 | 2.176 | 0.193 | 2.584 | 0.001 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 66 | 65 | 68 | 72 | 0 | 70 | 68 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.93 | 0.97 | 1.03 | 0.00 | 1.00 | 0.97 | 0.00 |
| time (sec) | N/A | 0.066 | 0.067 | 0.047 | 0.585 | 1.164 | 0.000 | 0.184 | 2.844 | 0.001 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 301 | 301 | 242 | 269 | 324 | 273 | 0 | 311 | 1751 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.89 | 1.08 | 0.91 | 0.00 | 1.03 | 5.82 | 0.00 |
| time (sec) | N/A | 0.310 | 0.240 | 0.050 | 1.271 | 1.359 | 0.000 | 0.214 | 11.360 | 0.001 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 296 | 296 | 238 | 266 | 349 | 228 | 0 | 308 | 873 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 0.90 | 1.18 | 0.77 | 0.00 | 1.04 | 2.95 | 0.00 |
| time (sec) | N/A | 0.267 | 0.165 | 0.052 | 1.355 | 0.937 | 0.000 | 0.212 | 1.829 | 0.001 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 43 | 50 | 49 | 42 | 144 | 51 | 51 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.94 | 0.92 | 0.79 | 2.72 | 0.96 | 0.96 | 0.00 |
| time (sec) | N/A | 0.050 | 0.061 | 0.051 | 0.463 | 0.986 | 6.748 | 0.186 | 0.313 | 0.001 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 288 | 288 | 224 | 246 | 289 | 244 | 573 | 286 | 1364 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.85 | 1.00 | 0.85 | 1.99 | 0.99 | 4.74 | 0.00 |
| time (sec) | N/A | 0.154 | 0.127 | 0.049 | 1.272 | 1.052 | 123.979 | 0.246 | 9.046 | 0.001 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 288 | 288 | 224 | 246 | 317 | 199 | 342 | 278 | 1265 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.85 | 1.10 | 0.69 | 1.19 | 0.97 | 4.39 | 0.00 |
| time (sec) | N/A | 0.148 | 0.124 | 0.054 | 1.206 | 0.797 | 20.201 | 0.204 | 8.117 | 0.001 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 31 | 42 | 41 | 31 | 138 | 51 | 602 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.93 | 0.91 | 0.69 | 3.07 | 1.13 | 13.38 | 0.00 |
| time (sec) | N/A | 0.031 | 0.020 | 0.046 | 0.483 | 0.845 | 2.474 | 0.183 | 0.257 | 0.001 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 288 | 288 | 224 | 222 | 265 | 201 | 515 | 290 | 982 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.77 | 0.92 | 0.70 | 1.79 | 1.01 | 3.41 | 0.00 |
| time (sec) | N/A | 0.143 | 0.144 | 0.047 | 1.287 | 0.841 | 14.506 | 0.247 | 5.415 | 0.001 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 288 | 288 | 224 | 222 | 293 | 254 | 447 | 278 | 1364 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 0.77 | 1.02 | 0.88 | 1.55 | 0.97 | 4.74 | 0.00 |
| time (sec) | N/A | 0.142 | 0.182 | 0.050 | 1.343 | 0.982 | 133.334 | 0.213 | 9.007 | 0.001 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 54 | 59 | 61 | 54 | 0 | 71 | 58 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.95 | 0.98 | 0.87 | 0.00 | 1.15 | 0.94 | 0.00 |
| time (sec) | N/A | 0.060 | 0.050 | 0.049 | 0.543 | 1.558 | 0.000 | 0.184 | 2.840 | 0.001 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 299 | 299 | 244 | 257 | 300 | 238 | 0 | 305 | 716 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.86 | 1.00 | 0.80 | 0.00 | 1.02 | 2.39 | 0.00 |
| time (sec) | N/A | 0.272 | 0.213 | 0.059 | 1.200 | 0.893 | 0.000 | 0.252 | 3.854 | 0.001 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 301 | 301 | 259 | 257 | 328 | 301 | 0 | 309 | 1829 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.85 | 1.09 | 1.00 | 0.00 | 1.03 | 6.08 | 0.00 |
| time (sec) | N/A | 0.255 | 0.224 | 0.052 | 1.136 | 2.623 | 0.000 | 0.206 | 11.834 | 0.001 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 88 | 87 | 87 | 99 | 0 | 111 | 87 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.00 | 1.00 | 1.14 | 0.00 | 1.28 | 1.00 | 0.00 |
| time (sec) | N/A | 0.092 | 0.106 | 0.060 | 0.485 | 4.787 | 0.000 | 0.193 | 3.223 | 0.001 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 318 | 318 | 282 | 291 | 341 | 305 | 0 | 328 | 1734 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.92 | 1.07 | 0.96 | 0.00 | 1.03 | 5.45 | 0.00 |
| time (sec) | N/A | 0.380 | 0.265 | 0.052 | 1.126 | 3.409 | 0.000 | 0.243 | 11.370 | 0.001 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 321 | 321 | 282 | 293 | 369 | 356 | 0 | 336 | 1860 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.91 | 1.15 | 1.11 | 0.00 | 1.05 | 5.79 | 0.00 |
| time (sec) | N/A | 0.456 | 0.221 | 0.054 | 1.231 | 1.042 | 0.000 | 0.206 | 11.573 | 0.001 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 119 | 124 | 117 | 127 | 0 | 165 | 118 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 0.98 | 1.07 | 0.00 | 1.39 | 0.99 | 0.00 |
| time (sec) | N/A | 0.129 | 0.063 | 0.056 | 0.520 | 11.767 | 0.000 | 0.261 | 3.212 | 0.001 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 352 | 352 | 304 | 334 | 376 | 332 | 0 | 377 | 1814 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.95 | 1.07 | 0.94 | 0.00 | 1.07 | 5.15 | 0.00 |
| time (sec) | N/A | 0.501 | 0.245 | 0.053 | 1.160 | 1.204 | 0.000 | 0.215 | 11.909 | 0.001 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 148 | 148 | 137 | 1078 | 205 | 851 | 5418 | 1331 | 559 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 7.28 | 1.39 | 5.75 | 36.61 | 8.99 | 3.78 | 0.00 |
| time (sec) | N/A | 0.099 | 0.264 | 0.052 | 0.518 | 0.915 | 23.936 | 0.285 | 3.213 | 0.355 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 66 | 262 | 91 | 215 | 1057 | 332 | 177 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 3.69 | 1.28 | 3.03 | 14.89 | 4.68 | 2.49 | 0.00 |
| time (sec) | N/A | 0.040 | 0.058 | 0.049 | 0.495 | 0.892 | 6.312 | 0.201 | 2.718 | 0.071 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 42 | 110 | 53 | 92 | 410 | 143 | 95 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 2.44 | 1.18 | 2.04 | 9.11 | 3.18 | 2.11 | 0.00 |
| time (sec) | N/A | 0.021 | 0.047 | 0.043 | 0.613 | 0.848 | 2.891 | 0.166 | 2.655 | 0.030 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 33 | 32 | 27 | 32 | 46 | 29 | 31 | 35 |
| N.S. | 1 | 1.00 | 0.85 | 0.82 | 0.69 | 0.82 | 1.18 | 0.74 | 0.79 | 0.90 |
| time (sec) | N/A | 0.015 | 0.017 | 0.046 | 0.448 | 0.879 | 20.862 | 0.168 | 0.053 | 0.022 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 33 | 32 | 27 | 32 | 46 | 29 | 31 | 41 |
| N.S. | 1 | 1.00 | 0.85 | 0.82 | 0.69 | 0.82 | 1.18 | 0.74 | 0.79 | 1.05 |
| time (sec) | N/A | 0.016 | 0.016 | 0.045 | 0.465 | 1.318 | 12.714 | 0.155 | 2.560 | 0.021 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 33 | 32 | 27 | 32 | 46 | 29 | 31 | 35 |
| N.S. | 1 | 1.00 | 0.85 | 0.82 | 0.69 | 0.82 | 1.18 | 0.74 | 0.79 | 0.90 |
| time (sec) | N/A | 0.015 | 0.015 | 0.046 | 0.449 | 0.930 | 6.742 | 0.178 | 0.044 | 0.018 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 33 | 32 | 27 | 30 | 46 | 29 | 31 | 41 |
| N.S. | 1 | 1.00 | 0.85 | 0.82 | 0.69 | 0.77 | 1.18 | 0.74 | 0.79 | 1.05 |
| time (sec) | N/A | 0.016 | 0.023 | 0.045 | 0.467 | 0.889 | 3.370 | 0.150 | 0.041 | 0.020 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 33 | 32 | 27 | 29 | 44 | 29 | 31 | 41 |
| N.S. | 1 | 1.00 | 0.89 | 0.86 | 0.73 | 0.78 | 1.19 | 0.78 | 0.84 | 1.11 |
| time (sec) | N/A | 0.015 | 0.070 | 0.045 | 0.505 | 0.745 | 2.299 | 0.153 | 2.590 | 0.019 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 35 | 32 | 27 | 29 | 44 | 29 | 31 | 35 |
| N.S. | 1 | 1.00 | 0.95 | 0.86 | 0.73 | 0.78 | 1.19 | 0.78 | 0.84 | 0.95 |
| time (sec) | N/A | 0.015 | 0.011 | 0.044 | 0.461 | 0.783 | 2.772 | 0.149 | 2.599 | 0.024 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 39 | 39 | 34 | 32 | 27 | 28 | 46 | 29 | 31 | 34 |
| N.S. | 1 | 1.00 | 0.87 | 0.82 | 0.69 | 0.72 | 1.18 | 0.74 | 0.79 | 0.87 |
| time (sec) | N/A | 0.015 | 0.013 | 0.042 | 0.482 | 0.894 | 3.736 | 0.157 | 0.040 | 0.023 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 36 | 32 | 27 | 29 | 42 | 29 | 30 | 35 |
| N.S. | 1 | 1.00 | 0.97 | 0.86 | 0.73 | 0.78 | 1.14 | 0.78 | 0.81 | 0.95 |
| time (sec) | N/A | 0.016 | 0.012 | 0.041 | 0.453 | 0.749 | 4.205 | 0.261 | 0.042 | 0.025 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 53 | 56 | 51 | 56 | 80 | 53 | 51 | 59 |
| N.S. | 1 | 1.00 | 0.84 | 0.89 | 0.81 | 0.89 | 1.27 | 0.84 | 0.81 | 0.94 |
| time (sec) | N/A | 0.031 | 0.064 | 0.049 | 0.558 | 0.760 | 47.810 | 0.153 | 2.567 | 0.032 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 63 | 56 | 51 | 56 | 80 | 53 | 51 | 69 |
| N.S. | 1 | 1.00 | 1.00 | 0.89 | 0.81 | 0.89 | 1.27 | 0.84 | 0.81 | 1.10 |
| time (sec) | N/A | 0.031 | 0.046 | 0.043 | 0.544 | 0.761 | 29.392 | 0.175 | 0.047 | 0.035 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 53 | 56 | 51 | 56 | 80 | 53 | 51 | 69 |
| N.S. | 1 | 1.00 | 0.84 | 0.89 | 0.81 | 0.89 | 1.27 | 0.84 | 0.81 | 1.10 |
| time (sec) | N/A | 0.030 | 0.060 | 0.045 | 0.499 | 0.777 | 21.960 | 0.163 | 0.047 | 0.033 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 53 | 56 | 51 | 54 | 80 | 53 | 51 | 69 |
| N.S. | 1 | 1.00 | 0.84 | 0.89 | 0.81 | 0.86 | 1.27 | 0.84 | 0.81 | 1.10 |
| time (sec) | N/A | 0.029 | 0.110 | 0.043 | 0.537 | 0.625 | 5.314 | 0.155 | 0.049 | 0.037 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 53 | 56 | 51 | 53 | 78 | 53 | 51 | 69 |
| N.S. | 1 | 1.00 | 0.87 | 0.92 | 0.84 | 0.87 | 1.28 | 0.87 | 0.84 | 1.13 |
| time (sec) | N/A | 0.029 | 0.067 | 0.046 | 0.512 | 0.823 | 9.219 | 0.227 | 0.045 | 0.033 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 60 | 56 | 51 | 53 | 78 | 53 | 51 | 59 |
| N.S. | 1 | 1.00 | 0.98 | 0.92 | 0.84 | 0.87 | 1.28 | 0.87 | 0.84 | 0.97 |
| time (sec) | N/A | 0.030 | 0.070 | 0.049 | 0.539 | 0.714 | 7.796 | 0.154 | 0.050 | 0.042 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 57 | 56 | 51 | 53 | 80 | 53 | 51 | 59 |
| N.S. | 1 | 1.00 | 0.90 | 0.89 | 0.81 | 0.84 | 1.27 | 0.84 | 0.81 | 0.94 |
| time (sec) | N/A | 0.030 | 0.067 | 0.054 | 0.510 | 0.769 | 11.737 | 0.160 | 0.048 | 0.042 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 57 | 56 | 51 | 53 | 76 | 53 | 51 | 59 |
| N.S. | 1 | 1.00 | 0.93 | 0.92 | 0.84 | 0.87 | 1.25 | 0.87 | 0.84 | 0.97 |
| time (sec) | N/A | 0.031 | 0.018 | 0.047 | 0.458 | 0.884 | 18.414 | 0.155 | 0.048 | 0.039 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 71 | 80 | 73 | 78 | 114 | 77 | 69 | 97 |
| N.S. | 1 | 1.00 | 0.84 | 0.94 | 0.86 | 0.92 | 1.34 | 0.91 | 0.81 | 1.14 |
| time (sec) | N/A | 0.044 | 0.062 | 0.043 | 0.568 | 0.725 | 92.986 | 0.164 | 2.519 | 0.041 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 85 | 80 | 73 | 78 | 114 | 77 | 69 | 97 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.86 | 0.92 | 1.34 | 0.91 | 0.81 | 1.14 |
| time (sec) | N/A | 0.041 | 0.100 | 0.043 | 0.504 | 0.815 | 58.699 | 0.154 | 0.030 | 0.042 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 85 | 80 | 73 | 78 | 114 | 77 | 69 | 97 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.86 | 0.92 | 1.34 | 0.91 | 0.81 | 1.14 |
| time (sec) | N/A | 0.042 | 0.083 | 0.049 | 0.481 | 0.829 | 39.599 | 0.170 | 0.032 | 0.043 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 71 | 80 | 73 | 76 | 114 | 77 | 69 | 97 |
| N.S. | 1 | 1.00 | 0.84 | 0.94 | 0.86 | 0.89 | 1.34 | 0.91 | 0.81 | 1.14 |
| time (sec) | N/A | 0.041 | 0.066 | 0.046 | 0.673 | 0.798 | 6.341 | 0.155 | 0.033 | 0.047 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 83 | 80 | 73 | 75 | 112 | 77 | 69 | 97 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.88 | 0.90 | 1.35 | 0.93 | 0.83 | 1.17 |
| time (sec) | N/A | 0.043 | 0.061 | 0.041 | 0.557 | 0.801 | 23.640 | 0.161 | 0.030 | 0.041 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 81 | 80 | 73 | 75 | 112 | 77 | 69 | 83 |
| N.S. | 1 | 1.00 | 0.98 | 0.96 | 0.88 | 0.90 | 1.35 | 0.93 | 0.83 | 1.00 |
| time (sec) | N/A | 0.041 | 0.044 | 0.045 | 0.561 | 1.000 | 19.488 | 0.156 | 0.033 | 0.053 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 77 | 80 | 73 | 75 | 112 | 77 | 69 | 83 |
| N.S. | 1 | 1.00 | 0.91 | 0.94 | 0.86 | 0.88 | 1.32 | 0.91 | 0.81 | 0.98 |
| time (sec) | N/A | 0.045 | 0.040 | 0.043 | 0.641 | 0.794 | 28.368 | 0.161 | 0.034 | 0.044 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 78 | 80 | 73 | 75 | 110 | 77 | 69 | 83 |
| N.S. | 1 | 1.00 | 0.94 | 0.96 | 0.88 | 0.90 | 1.33 | 0.93 | 0.83 | 1.00 |
| time (sec) | N/A | 0.041 | 0.085 | 0.041 | 0.549 | 0.676 | 29.278 | 0.163 | 0.032 | 0.047 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 67 | 78 | 58 | 143 | 0 | 64 | 111 | 71 |
| N.S. | 1 | 1.00 | 0.92 | 1.07 | 0.79 | 1.96 | 0.00 | 0.88 | 1.52 | 0.97 |
| time (sec) | N/A | 0.048 | 0.124 | 0.047 | 1.316 | 0.682 | 0.000 | 0.171 | 2.607 | 0.066 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 288 | 288 | 54 | 377 | 295 | 2433 | 881 | 289 | 1933 | 193 |
| N.S. | 1 | 1.00 | 0.19 | 1.31 | 1.02 | 8.45 | 3.06 | 1.00 | 6.71 | 0.67 |
| time (sec) | N/A | 0.519 | 0.093 | 0.178 | 1.173 | 1.075 | 177.800 | 0.211 | 2.889 | 0.210 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 270 | 270 | 45 | 356 | 212 | 3635 | 857 | 280 | 1640 | 167 |
| N.S. | 1 | 1.00 | 0.17 | 1.32 | 0.79 | 13.46 | 3.17 | 1.04 | 6.07 | 0.62 |
| time (sec) | N/A | 0.551 | 0.071 | 0.164 | 1.139 | 1.081 | 64.299 | 1.351 | 2.853 | 0.231 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 52 | 53 | 39 | 108 | 537 | 39 | 93 | 53 |
| N.S. | 1 | 1.00 | 0.98 | 1.00 | 0.74 | 2.04 | 10.13 | 0.74 | 1.75 | 1.00 |
| time (sec) | N/A | 0.036 | 0.080 | 0.054 | 1.206 | 0.807 | 22.390 | 0.166 | 2.600 | 0.046 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 268 | 268 | 43 | 353 | 278 | 2424 | 833 | 280 | 1915 | 166 |
| N.S. | 1 | 1.00 | 0.16 | 1.32 | 1.04 | 9.04 | 3.11 | 1.04 | 7.15 | 0.62 |
| time (sec) | N/A | 0.473 | 0.029 | 0.151 | 1.424 | 0.995 | 23.158 | 0.213 | 2.885 | 0.255 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 268 | 268 | 46 | 355 | 212 | 3663 | 836 | 280 | 1700 | 166 |
| N.S. | 1 | 1.00 | 0.17 | 1.32 | 0.79 | 13.67 | 3.12 | 1.04 | 6.34 | 0.62 |
| time (sec) | N/A | 0.552 | 0.014 | 0.157 | 1.343 | 1.149 | 59.226 | 0.408 | 2.856 | 0.236 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 53 | 53 | 39 | 120 | 527 | 39 | 102 | 53 |
| N.S. | 1 | 1.00 | 1.00 | 1.00 | 0.74 | 2.26 | 9.94 | 0.74 | 1.92 | 1.00 |
| time (sec) | N/A | 0.037 | 0.050 | 0.056 | 1.400 | 0.776 | 156.584 | 0.166 | 0.101 | 0.051 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 270 | 270 | 47 | 358 | 278 | 2424 | 0 | 280 | 2023 | 167 |
| N.S. | 1 | 1.00 | 0.17 | 1.33 | 1.03 | 8.98 | 0.00 | 1.04 | 7.49 | 0.62 |
| time (sec) | N/A | 0.485 | 0.019 | 0.175 | 1.424 | 0.953 | 0.000 | 0.215 | 2.913 | 0.194 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 77 | 93 | 68 | 222 | 0 | 68 | 116 | 77 |
| N.S. | 1 | 1.00 | 0.81 | 0.98 | 0.72 | 2.34 | 0.00 | 0.72 | 1.22 | 0.81 |
| time (sec) | N/A | 0.055 | 0.179 | 0.064 | 1.239 | 0.721 | 0.000 | 0.181 | 2.650 | 0.114 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 312 | 312 | 76 | 405 | 311 | 2566 | 0 | 313 | 1884 | 196 |
| N.S. | 1 | 1.00 | 0.24 | 1.30 | 1.00 | 8.22 | 0.00 | 1.00 | 6.04 | 0.63 |
| time (sec) | N/A | 0.502 | 0.087 | 0.166 | 1.355 | 0.840 | 0.000 | 0.330 | 2.888 | 0.653 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 289 | 289 | 62 | 387 | 235 | 3787 | 0 | 302 | 1578 | 190 |
| N.S. | 1 | 1.00 | 0.21 | 1.34 | 0.81 | 13.10 | 0.00 | 1.04 | 5.46 | 0.66 |
| time (sec) | N/A | 0.549 | 0.098 | 0.164 | 1.470 | 1.133 | 0.000 | 1.377 | 2.869 | 0.572 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 71 | 74 | 61 | 190 | 0 | 63 | 115 | 71 |
| N.S. | 1 | 1.00 | 1.00 | 1.04 | 0.86 | 2.68 | 0.00 | 0.89 | 1.62 | 1.00 |
| time (sec) | N/A | 0.042 | 0.055 | 0.056 | 1.399 | 0.803 | 0.000 | 0.284 | 0.141 | 0.099 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 289 | 289 | 68 | 387 | 301 | 2555 | 0 | 302 | 1922 | 190 |
| N.S. | 1 | 1.00 | 0.24 | 1.34 | 1.04 | 8.84 | 0.00 | 1.04 | 6.65 | 0.66 |
| time (sec) | N/A | 0.468 | 0.032 | 0.163 | 1.203 | 1.065 | 0.000 | 0.311 | 2.917 | 0.569 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 318 | 318 | 70 | 401 | 240 | 3798 | 0 | 307 | 1757 | 197 |
| N.S. | 1 | 1.00 | 0.22 | 1.26 | 0.75 | 11.94 | 0.00 | 0.97 | 5.53 | 0.62 |
| time (sec) | N/A | 0.686 | 0.137 | 0.162 | 1.159 | 0.966 | 0.000 | 0.468 | 2.912 | 0.610 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 97 | 79 | 93 | 67 | 232 | 0 | 66 | 139 | 79 |
| N.S. | 1 | 1.01 | 0.82 | 0.97 | 0.70 | 2.42 | 0.00 | 0.69 | 1.45 | 0.82 |
| time (sec) | N/A | 0.057 | 0.130 | 0.066 | 1.187 | 0.815 | 0.000 | 0.161 | 0.152 | 0.111 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 318 | 318 | 74 | 395 | 312 | 2584 | 0 | 313 | 2080 | 200 |
| N.S. | 1 | 1.00 | 0.23 | 1.24 | 0.98 | 8.13 | 0.00 | 0.98 | 6.54 | 0.63 |
| time (sec) | N/A | 0.504 | 0.111 | 0.159 | 1.193 | 1.085 | 0.000 | 0.219 | 2.957 | 0.598 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 93 | 96 | 96 | 314 | 0 | 84 | 133 | 92 |
| N.S. | 1 | 1.00 | 0.89 | 0.92 | 0.92 | 3.02 | 0.00 | 0.81 | 1.28 | 0.88 |
| time (sec) | N/A | 0.057 | 0.194 | 0.066 | 1.430 | 0.770 | 0.000 | 0.206 | 2.761 | 0.166 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 327 | 327 | 92 | 416 | 341 | 2714 | 0 | 328 | 1944 | 213 |
| N.S. | 1 | 1.00 | 0.28 | 1.27 | 1.04 | 8.30 | 0.00 | 1.00 | 5.94 | 0.65 |
| time (sec) | N/A | 0.493 | 0.110 | 0.161 | 1.396 | 0.786 | 0.000 | 0.232 | 2.984 | 0.780 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 327 | 327 | 62 | 411 | 271 | 3951 | 0 | 328 | 1672 | 212 |
| N.S. | 1 | 1.00 | 0.19 | 1.26 | 0.83 | 12.08 | 0.00 | 1.00 | 5.11 | 0.65 |
| time (sec) | N/A | 0.597 | 0.065 | 0.161 | 1.226 | 0.815 | 0.000 | 0.368 | 2.893 | 0.738 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 94 | 97 | 96 | 313 | 0 | 84 | 136 | 92 |
| N.S. | 1 | 1.00 | 0.90 | 0.93 | 0.92 | 3.01 | 0.00 | 0.81 | 1.31 | 0.88 |
| time (sec) | N/A | 0.060 | 0.108 | 0.067 | 1.273 | 0.568 | 0.000 | 0.180 | 2.706 | 0.163 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 321 | 321 | 91 | 407 | 336 | 2674 | 0 | 322 | 1952 | 210 |
| N.S. | 1 | 1.00 | 0.28 | 1.27 | 1.05 | 8.33 | 0.00 | 1.00 | 6.08 | 0.65 |
| time (sec) | N/A | 0.522 | 0.048 | 0.169 | 1.215 | 1.002 | 0.000 | 0.235 | 2.950 | 0.793 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 351 | 351 | 113 | 441 | 273 | 3904 | 0 | 329 | 1786 | 219 |
| N.S. | 1 | 1.00 | 0.32 | 1.26 | 0.78 | 11.12 | 0.00 | 0.94 | 5.09 | 0.62 |
| time (sec) | N/A | 0.616 | 0.133 | 0.170 | 1.289 | 0.971 | 0.000 | 0.482 | 2.911 | 0.627 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 129 | 130 | 102 | 133 | 100 | 347 | 0 | 88 | 163 | 102 |
| N.S. | 1 | 1.01 | 0.79 | 1.03 | 0.78 | 2.69 | 0.00 | 0.68 | 1.26 | 0.79 |
| time (sec) | N/A | 0.072 | 0.185 | 0.072 | 1.346 | 0.832 | 0.000 | 0.182 | 2.728 | 0.171 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 351 | 351 | 96 | 435 | 346 | 2690 | 0 | 334 | 2109 | 222 |
| N.S. | 1 | 1.00 | 0.27 | 1.24 | 0.99 | 7.66 | 0.00 | 0.95 | 6.01 | 0.63 |
| time (sec) | N/A | 0.541 | 0.122 | 0.171 | 1.342 | 1.332 | 0.000 | 0.241 | 2.959 | 0.643 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 75 | 77 | 118 | 99 | 219 | 104 | 154 | 80 |
| N.S. | 1 | 1.00 | 0.73 | 0.75 | 1.15 | 0.96 | 2.13 | 1.01 | 1.50 | 0.78 |
| time (sec) | N/A | 0.084 | 0.062 | 0.047 | 0.618 | 0.791 | 4.166 | 0.159 | 2.717 | 0.052 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 57 | 53 | 84 | 75 | 168 | 73 | 114 | 56 |
| N.S. | 1 | 1.00 | 0.78 | 0.73 | 1.15 | 1.03 | 2.30 | 1.00 | 1.56 | 0.77 |
| time (sec) | N/A | 0.060 | 0.044 | 0.041 | 0.507 | 0.835 | 1.818 | 0.175 | 2.662 | 0.040 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 34 | 31 | 49 | 50 | 117 | 44 | 44 | 34 |
| N.S. | 1 | 1.00 | 0.74 | 0.67 | 1.07 | 1.09 | 2.54 | 0.96 | 0.96 | 0.74 |
| time (sec) | N/A | 0.040 | 0.031 | 0.041 | 0.591 | 0.710 | 0.738 | 0.153 | 2.598 | 0.027 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 60 | 50 | 67 | 125 | 76 | 61 | 80 | 61 |
| N.S. | 1 | 1.00 | 0.94 | 0.78 | 1.05 | 1.95 | 1.19 | 0.95 | 1.25 | 0.95 |
| time (sec) | N/A | 0.043 | 0.051 | 0.046 | 1.204 | 0.767 | 25.832 | 0.153 | 2.714 | 0.049 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 63 | 72 | 107 | 143 | 134 | 68 | 76 | 65 |
| N.S. | 1 | 1.00 | 0.75 | 0.86 | 1.27 | 1.70 | 1.60 | 0.81 | 0.90 | 0.77 |
| time (sec) | N/A | 0.063 | 0.046 | 0.048 | 1.264 | 0.902 | 43.661 | 0.168 | 2.933 | 0.106 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 93 | 96 | 158 | 172 | 160 | 120 | 93 | 79 |
| N.S. | 1 | 1.00 | 1.06 | 1.09 | 1.80 | 1.95 | 1.82 | 1.36 | 1.06 | 0.90 |
| time (sec) | N/A | 0.070 | 0.075 | 0.049 | 1.317 | 0.758 | 129.138 | 0.200 | 3.119 | 0.118 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 78 | 77 | 118 | 124 | 267 | 104 | 206 | 80 |
| N.S. | 1 | 1.00 | 0.76 | 0.75 | 1.15 | 1.20 | 2.59 | 1.01 | 2.00 | 0.78 |
| time (sec) | N/A | 0.079 | 0.092 | 0.051 | 0.615 | 1.115 | 8.440 | 0.162 | 2.655 | 0.059 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 57 | 53 | 84 | 99 | 216 | 73 | 211 | 56 |
| N.S. | 1 | 1.00 | 0.78 | 0.73 | 1.15 | 1.36 | 2.96 | 1.00 | 2.89 | 0.77 |
| time (sec) | N/A | 0.058 | 0.055 | 0.047 | 0.610 | 0.818 | 4.755 | 0.182 | 2.715 | 0.044 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 34 | 31 | 49 | 73 | 165 | 44 | 150 | 34 |
| N.S. | 1 | 1.00 | 0.74 | 0.67 | 1.07 | 1.59 | 3.59 | 0.96 | 3.26 | 0.74 |
| time (sec) | N/A | 0.040 | 0.028 | 0.047 | 0.493 | 1.020 | 2.520 | 0.166 | 3.346 | 0.031 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 80 | 66 | 80 | 172 | 82 | 80 | 131 | 85 |
| N.S. | 1 | 1.00 | 0.99 | 0.81 | 0.99 | 2.12 | 1.01 | 0.99 | 1.62 | 1.05 |
| time (sec) | N/A | 0.057 | 0.082 | 0.049 | 1.257 | 0.823 | 66.499 | 0.165 | 2.793 | 0.071 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 80 | 101 | 134 | 169 | 223 | 103 | 111 | 85 |
| N.S. | 1 | 1.00 | 0.73 | 0.92 | 1.22 | 1.54 | 2.03 | 0.94 | 1.01 | 0.77 |
| time (sec) | N/A | 0.084 | 0.090 | 0.049 | 1.438 | 0.566 | 58.401 | 0.202 | 3.379 | 0.113 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 59 | 107 | 171 | 191 | 243 | 131 | 110 | 84 |
| N.S. | 1 | 1.00 | 0.51 | 0.93 | 1.49 | 1.66 | 2.11 | 1.14 | 0.96 | 0.73 |
| time (sec) | N/A | 0.087 | 0.036 | 0.052 | 1.306 | 0.894 | 153.107 | 0.185 | 3.472 | 0.151 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 78 | 77 | 118 | 76 | 175 | 101 | 104 | 80 |
| N.S. | 1 | 1.00 | 0.76 | 0.75 | 1.15 | 0.74 | 1.70 | 0.98 | 1.01 | 0.78 |
| time (sec) | N/A | 0.074 | 0.079 | 0.043 | 0.588 | 0.924 | 3.441 | 0.160 | 2.676 | 0.052 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 56 | 53 | 83 | 52 | 124 | 70 | 52 | 56 |
| N.S. | 1 | 1.00 | 0.77 | 0.73 | 1.14 | 0.71 | 1.70 | 0.96 | 0.71 | 0.77 |
| time (sec) | N/A | 0.055 | 0.058 | 0.047 | 0.450 | 0.810 | 1.825 | 0.160 | 2.649 | 0.042 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 33 | 30 | 48 | 29 | 75 | 38 | 29 | 33 |
| N.S. | 1 | 1.00 | 0.72 | 0.65 | 1.04 | 0.63 | 1.63 | 0.83 | 0.63 | 0.72 |
| time (sec) | N/A | 0.038 | 0.035 | 0.043 | 0.462 | 1.114 | 0.976 | 0.157 | 2.604 | 0.028 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 48 | 37 | 54 | 105 | 65 | 40 | 57 | 48 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 1.12 | 2.19 | 1.35 | 0.83 | 1.19 | 1.00 |
| time (sec) | N/A | 0.032 | 0.047 | 0.044 | 1.080 | 1.009 | 11.270 | 0.187 | 2.715 | 0.045 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 57 | 62 | 109 | 126 | 80 | 62 | 67 | 58 |
| N.S. | 1 | 1.00 | 0.98 | 1.07 | 1.88 | 2.17 | 1.38 | 1.07 | 1.16 | 1.00 |
| time (sec) | N/A | 0.046 | 0.074 | 0.053 | 1.171 | 1.506 | 31.833 | 0.161 | 2.894 | 0.066 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 81 | 102 | 178 | 173 | 163 | 121 | 95 | 80 |
| N.S. | 1 | 1.00 | 0.90 | 1.13 | 1.98 | 1.92 | 1.81 | 1.34 | 1.06 | 0.89 |
| time (sec) | N/A | 0.070 | 0.286 | 0.075 | 1.199 | 1.083 | 71.181 | 0.194 | 2.995 | 0.115 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 77 | 77 | 116 | 88 | 175 | 114 | 152 | 80 |
| N.S. | 1 | 1.00 | 0.75 | 0.75 | 1.13 | 0.85 | 1.70 | 1.11 | 1.48 | 0.78 |
| time (sec) | N/A | 0.077 | 0.053 | 0.045 | 0.590 | 1.036 | 3.902 | 0.175 | 2.771 | 0.055 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 55 | 52 | 81 | 63 | 124 | 77 | 60 | 56 |
| N.S. | 1 | 1.00 | 0.75 | 0.71 | 1.11 | 0.86 | 1.70 | 1.05 | 0.82 | 0.77 |
| time (sec) | N/A | 0.055 | 0.044 | 0.049 | 0.648 | 0.979 | 1.844 | 0.176 | 2.680 | 0.048 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 33 | 30 | 47 | 41 | 75 | 38 | 33 | 33 |
| N.S. | 1 | 1.00 | 0.72 | 0.65 | 1.02 | 0.89 | 1.63 | 0.83 | 0.72 | 0.72 |
| time (sec) | N/A | 0.037 | 0.027 | 0.046 | 0.556 | 0.552 | 1.082 | 0.177 | 2.615 | 0.035 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 58 | 57 | 70 | 170 | 56 | 53 | 65 | 58 |
| N.S. | 1 | 1.00 | 1.00 | 0.98 | 1.21 | 2.93 | 0.97 | 0.91 | 1.12 | 1.00 |
| time (sec) | N/A | 0.040 | 0.066 | 0.047 | 1.352 | 1.065 | 20.823 | 0.162 | 2.769 | 0.073 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 57 | 100 | 144 | 233 | 264 | 99 | 131 | 77 |
| N.S. | 1 | 1.00 | 0.66 | 1.16 | 1.67 | 2.71 | 3.07 | 1.15 | 1.52 | 0.90 |
| time (sec) | N/A | 0.065 | 0.020 | 0.061 | 1.388 | 0.961 | 79.790 | 0.168 | 2.929 | 0.129 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 118 | 120 | 60 | 141 | 215 | 289 | 192 | 137 | 167 | 102 |
| N.S. | 1 | 1.02 | 0.51 | 1.19 | 1.82 | 2.45 | 1.63 | 1.16 | 1.42 | 0.86 |
| time (sec) | N/A | 0.090 | 0.023 | 0.048 | 1.343 | 0.928 | 179.010 | 0.178 | 3.180 | 0.163 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 103 | 103 | 73 | 76 | 116 | 98 | 338 | 104 | 145 | 80 |
| N.S. | 1 | 1.00 | 0.71 | 0.74 | 1.13 | 0.95 | 3.28 | 1.01 | 1.41 | 0.78 |
| time (sec) | N/A | 0.078 | 0.067 | 0.048 | 0.592 | 0.842 | 5.183 | 0.166 | 2.797 | 0.059 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 54 | 53 | 84 | 75 | 240 | 63 | 60 | 56 |
| N.S. | 1 | 1.00 | 0.74 | 0.73 | 1.15 | 1.03 | 3.29 | 0.86 | 0.82 | 0.77 |
| time (sec) | N/A | 0.056 | 0.047 | 0.050 | 0.488 | 0.714 | 2.424 | 0.199 | 2.761 | 0.049 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 33 | 30 | 49 | 52 | 144 | 32 | 33 | 33 |
| N.S. | 1 | 1.00 | 0.72 | 0.65 | 1.07 | 1.13 | 3.13 | 0.70 | 0.72 | 0.72 |
| time (sec) | N/A | 0.036 | 0.029 | 0.046 | 0.499 | 0.732 | 1.335 | 0.162 | 2.678 | 0.038 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 62 | 85 | 81 | 243 | 76 | 67 | 80 | 70 |
| N.S. | 1 | 1.00 | 0.81 | 1.10 | 1.05 | 3.16 | 0.99 | 0.87 | 1.04 | 0.91 |
| time (sec) | N/A | 0.050 | 0.047 | 0.086 | 1.169 | 0.559 | 38.427 | 0.168 | 2.782 | 0.087 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 113 | 113 | 57 | 157 | 170 | 351 | 0 | 101 | 198 | 99 |
| N.S. | 1 | 1.00 | 0.50 | 1.39 | 1.50 | 3.11 | 0.00 | 0.89 | 1.75 | 0.88 |
| time (sec) | N/A | 0.087 | 0.029 | 0.091 | 1.254 | 0.747 | 0.000 | 0.186 | 2.971 | 0.135 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 77 | 506 | 69 | 156 | 85 | 82 | 109 | 78 |
| N.S. | 1 | 1.00 | 0.79 | 5.22 | 0.71 | 1.61 | 0.88 | 0.85 | 1.12 | 0.80 |
| time (sec) | N/A | 0.094 | 0.087 | 4.458 | 1.153 | 1.155 | 38.268 | 0.169 | 4.539 | 0.069 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 65 | 446 | 53 | 129 | 68 | 64 | 88 | 67 |
| N.S. | 1 | 1.00 | 0.86 | 5.87 | 0.70 | 1.70 | 0.89 | 0.84 | 1.16 | 0.88 |
| time (sec) | N/A | 0.062 | 0.041 | 0.170 | 1.299 | 0.930 | 16.958 | 0.192 | 4.278 | 0.054 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 57 | 57 | 54 | 425 | 42 | 110 | 51 | 44 | 71 | 57 |
| N.S. | 1 | 1.00 | 0.95 | 7.46 | 0.74 | 1.93 | 0.89 | 0.77 | 1.25 | 1.00 |
| time (sec) | N/A | 0.046 | 0.016 | 0.152 | 1.208 | 0.777 | 6.097 | 0.159 | 3.866 | 0.047 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 59 | 468 | 0 | 147 | 66 | 50 | 93 | 65 |
| N.S. | 1 | 1.00 | 0.91 | 7.20 | 0.00 | 2.26 | 1.02 | 0.77 | 1.43 | 1.00 |
| time (sec) | N/A | 0.056 | 0.018 | 0.187 | 0.000 | 0.815 | 13.281 | 0.165 | 4.658 | 0.052 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 88 | 511 | 0 | 194 | 0 | 72 | 113 | 88 |
| N.S. | 1 | 1.00 | 1.00 | 5.81 | 0.00 | 2.20 | 0.00 | 0.82 | 1.28 | 1.00 |
| time (sec) | N/A | 0.077 | 0.043 | 0.191 | 0.000 | 0.950 | 0.000 | 0.172 | 4.858 | 0.109 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 65 | 467 | 53 | 129 | 0 | 64 | 88 | 69 |
| N.S. | 1 | 1.00 | 0.83 | 5.99 | 0.68 | 1.65 | 0.00 | 0.82 | 1.13 | 0.88 |
| time (sec) | N/A | 0.074 | 0.070 | 0.245 | 1.345 | 0.814 | 0.000 | 0.158 | 5.383 | 0.056 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 56 | 425 | 43 | 112 | 65 | 49 | 71 | 59 |
| N.S. | 1 | 1.00 | 0.95 | 7.20 | 0.73 | 1.90 | 1.10 | 0.83 | 1.20 | 1.00 |
| time (sec) | N/A | 0.048 | 0.022 | 0.180 | 1.455 | 0.846 | 15.581 | 0.156 | 4.859 | 0.049 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 40 | 40 | 40 | 413 | 29 | 87 | 37 | 29 | 56 | 40 |
| N.S. | 1 | 1.00 | 1.00 | 10.32 | 0.72 | 2.18 | 0.92 | 0.72 | 1.40 | 1.00 |
| time (sec) | N/A | 0.036 | 0.010 | 0.175 | 1.244 | 0.722 | 10.089 | 0.183 | 5.210 | 0.040 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 65 | 65 | 59 | 433 | 0 | 148 | 63 | 53 | 94 | 65 |
| N.S. | 1 | 1.00 | 0.91 | 6.66 | 0.00 | 2.28 | 0.97 | 0.82 | 1.45 | 1.00 |
| time (sec) | N/A | 0.055 | 0.021 | 0.192 | 0.000 | 0.838 | 12.247 | 0.167 | 5.507 | 0.053 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 88 | 477 | 0 | 194 | 0 | 72 | 112 | 88 |
| N.S. | 1 | 1.00 | 1.00 | 5.42 | 0.00 | 2.20 | 0.00 | 0.82 | 1.27 | 1.00 |
| time (sec) | N/A | 0.077 | 0.035 | 0.227 | 0.000 | 0.648 | 0.000 | 0.158 | 5.719 | 0.087 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | B | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 206 | 206 | 67 | 416 | 0 | 2274 | 0 | 0 | 453 | 0 |
| N.S. | 1 | 1.00 | 0.33 | 2.02 | 0.00 | 11.04 | 0.00 | 0.00 | 2.20 | 0.00 |
| time (sec) | N/A | 0.032 | 0.026 | 0.178 | 0.000 | 2.366 | 0.000 | 0.000 | 25.804 | 15.359 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | B | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 127 | 127 | 28 | 164 | 0 | 1191 | 0 | 0 | 653 | 0 |
| N.S. | 1 | 1.00 | 0.22 | 1.29 | 0.00 | 9.38 | 0.00 | 0.00 | 5.14 | 0.00 |
| time (sec) | N/A | 0.019 | 0.029 | 0.796 | 0.000 | 1.260 | 0.000 | 0.000 | 3.419 | 20.881 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 111 | 111 | 81 | 582 | 96 | 169 | 99 | 100 | 118 | 82 |
| N.S. | 1 | 1.00 | 0.73 | 5.24 | 0.86 | 1.52 | 0.89 | 0.90 | 1.06 | 0.74 |
| time (sec) | N/A | 0.097 | 0.111 | 0.354 | 1.306 | 0.904 | 60.266 | 0.163 | 3.510 | 0.074 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 70 | 507 | 82 | 147 | 82 | 83 | 98 | 71 |
| N.S. | 1 | 1.00 | 0.78 | 5.63 | 0.91 | 1.63 | 0.91 | 0.92 | 1.09 | 0.79 |
| time (sec) | N/A | 0.085 | 0.064 | 0.161 | 1.209 | 0.752 | 30.233 | 0.171 | 3.404 | 0.063 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 69 | 69 | 58 | 446 | 66 | 121 | 65 | 65 | 78 | 59 |
| N.S. | 1 | 1.00 | 0.84 | 6.46 | 0.96 | 1.75 | 0.94 | 0.94 | 1.13 | 0.86 |
| time (sec) | N/A | 0.057 | 0.040 | 0.180 | 1.304 | 0.995 | 14.908 | 0.211 | 3.508 | 0.056 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 50 | 50 | 47 | 425 | 56 | 101 | 46 | 43 | 59 | 50 |
| N.S. | 1 | 1.00 | 0.94 | 8.50 | 1.12 | 2.02 | 0.92 | 0.86 | 1.18 | 1.00 |
| time (sec) | N/A | 0.043 | 0.027 | 0.152 | 1.154 | 0.816 | 5.109 | 0.160 | 3.496 | 0.043 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 53 | 468 | 0 | 138 | 60 | 48 | 125 | 58 |
| N.S. | 1 | 1.00 | 0.91 | 8.07 | 0.00 | 2.38 | 1.03 | 0.83 | 2.16 | 1.00 |
| time (sec) | N/A | 0.054 | 0.018 | 0.158 | 0.000 | 0.909 | 8.219 | 0.164 | 4.687 | 0.043 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 81 | 511 | 0 | 186 | 0 | 73 | 69 | 81 |
| N.S. | 1 | 1.00 | 1.00 | 6.31 | 0.00 | 2.30 | 0.00 | 0.90 | 0.85 | 1.00 |
| time (sec) | N/A | 0.072 | 0.029 | 0.176 | 0.000 | 0.760 | 0.000 | 0.170 | 3.748 | 0.081 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 96 | 574 | 0 | 188 | 0 | 100 | 83 | 95 |
| N.S. | 1 | 1.00 | 0.90 | 5.36 | 0.00 | 1.76 | 0.00 | 0.93 | 0.78 | 0.89 |
| time (sec) | N/A | 0.094 | 0.060 | 0.186 | 0.000 | 0.795 | 0.000 | 0.176 | 3.910 | 0.136 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 93 | 634 | 110 | 191 | 0 | 117 | 135 | 93 |
| N.S. | 1 | 1.00 | 0.72 | 4.88 | 0.85 | 1.47 | 0.00 | 0.90 | 1.04 | 0.72 |
| time (sec) | N/A | 0.113 | 0.133 | 0.252 | 1.285 | 0.553 | 0.000 | 0.165 | 3.525 | 0.075 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 109 | 109 | 81 | 541 | 96 | 169 | 110 | 100 | 115 | 82 |
| N.S. | 1 | 1.00 | 0.74 | 4.96 | 0.88 | 1.55 | 1.01 | 0.92 | 1.06 | 0.75 |
| time (sec) | N/A | 0.102 | 0.087 | 0.157 | 1.335 | 0.962 | 108.786 | 0.163 | 3.495 | 0.076 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 70 | 462 | 82 | 147 | 90 | 83 | 95 | 71 |
| N.S. | 1 | 1.00 | 0.80 | 5.25 | 0.93 | 1.67 | 1.02 | 0.94 | 1.08 | 0.81 |
| time (sec) | N/A | 0.072 | 0.039 | 0.176 | 1.440 | 0.627 | 62.737 | 0.184 | 3.521 | 0.066 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 58 | 441 | 68 | 121 | 65 | 65 | 75 | 59 |
| N.S. | 1 | 1.00 | 0.87 | 6.58 | 1.01 | 1.81 | 0.97 | 0.97 | 1.12 | 0.88 |
| time (sec) | N/A | 0.057 | 0.038 | 0.158 | 1.388 | 0.577 | 29.253 | 0.163 | 3.449 | 0.057 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 73 | 500 | 0 | 152 | 73 | 61 | 89 | 73 |
| N.S. | 1 | 1.00 | 1.00 | 6.85 | 0.00 | 2.08 | 1.00 | 0.84 | 1.22 | 1.00 |
| time (sec) | N/A | 0.070 | 0.048 | 0.189 | 0.000 | 0.849 | 24.743 | 0.177 | 5.893 | 0.052 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 78 | 78 | 78 | 556 | 0 | 186 | 0 | 64 | 56 | 78 |
| N.S. | 1 | 1.00 | 1.00 | 7.13 | 0.00 | 2.38 | 0.00 | 0.82 | 0.72 | 1.00 |
| time (sec) | N/A | 0.072 | 0.046 | 0.262 | 0.000 | 0.864 | 0.000 | 0.166 | 3.524 | 0.084 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 96 | 617 | 0 | 218 | 0 | 101 | 87 | 95 |
| N.S. | 1 | 1.00 | 0.92 | 5.93 | 0.00 | 2.10 | 0.00 | 0.97 | 0.84 | 0.91 |
| time (sec) | N/A | 0.095 | 0.106 | 0.235 | 0.000 | 0.874 | 0.000 | 0.190 | 3.737 | 0.108 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 69 | 528 | 82 | 146 | 0 | 82 | 98 | 72 |
| N.S. | 1 | 1.00 | 0.77 | 5.87 | 0.91 | 1.62 | 0.00 | 0.91 | 1.09 | 0.80 |
| time (sec) | N/A | 0.081 | 0.106 | 0.281 | 1.264 | 0.918 | 0.000 | 0.158 | 3.221 | 0.065 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 58 | 468 | 66 | 121 | 0 | 65 | 78 | 61 |
| N.S. | 1 | 1.00 | 0.82 | 6.59 | 0.93 | 1.70 | 0.00 | 0.92 | 1.10 | 0.86 |
| time (sec) | N/A | 0.069 | 0.069 | 0.158 | 1.312 | 1.098 | 0.000 | 0.187 | 3.393 | 0.059 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 49 | 425 | 56 | 103 | 61 | 48 | 60 | 52 |
| N.S. | 1 | 1.00 | 0.94 | 8.17 | 1.08 | 1.98 | 1.17 | 0.92 | 1.15 | 1.00 |
| time (sec) | N/A | 0.044 | 0.020 | 0.168 | 1.400 | 0.646 | 16.347 | 0.155 | 3.271 | 0.046 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 33 | 33 | 33 | 413 | 42 | 78 | 32 | 27 | 45 | 33 |
| N.S. | 1 | 1.00 | 1.00 | 12.52 | 1.27 | 2.36 | 0.97 | 0.82 | 1.36 | 1.00 |
| time (sec) | N/A | 0.033 | 0.011 | 0.195 | 1.213 | 0.836 | 11.202 | 0.160 | 3.233 | 0.034 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 58 | 58 | 51 | 433 | 0 | 139 | 58 | 54 | 47 | 58 |
| N.S. | 1 | 1.00 | 0.88 | 7.47 | 0.00 | 2.40 | 1.00 | 0.93 | 0.81 | 1.00 |
| time (sec) | N/A | 0.052 | 0.029 | 0.174 | 0.000 | 0.887 | 12.219 | 0.171 | 3.283 | 0.046 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 81 | 81 | 81 | 477 | 0 | 184 | 0 | 73 | 73 | 81 |
| N.S. | 1 | 1.00 | 1.00 | 5.89 | 0.00 | 2.27 | 0.00 | 0.90 | 0.90 | 1.00 |
| time (sec) | N/A | 0.073 | 0.052 | 0.186 | 0.000 | 0.801 | 0.000 | 0.164 | 3.417 | 0.080 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 95 | 540 | 0 | 217 | 0 | 101 | 94 | 95 |
| N.S. | 1 | 1.00 | 0.89 | 5.05 | 0.00 | 2.03 | 0.00 | 0.94 | 0.88 | 0.89 |
| time (sec) | N/A | 0.098 | 0.071 | 0.207 | 0.000 | 0.808 | 0.000 | 0.180 | 3.501 | 0.139 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | B | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 141 | 141 | 67 | 416 | 0 | 2459 | 0 | 0 | 272 | 0 |
| N.S. | 1 | 1.00 | 0.48 | 2.95 | 0.00 | 17.44 | 0.00 | 0.00 | 1.93 | 0.00 |
| time (sec) | N/A | 0.413 | 0.061 | 0.165 | 0.000 | 2.471 | 0.000 | 0.000 | 40.219 | 15.449 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 66 | 560 | 82 | 189 | 0 | 82 | 95 | 73 |
| N.S. | 1 | 1.00 | 0.73 | 6.22 | 0.91 | 2.10 | 0.00 | 0.91 | 1.06 | 0.81 |
| time (sec) | N/A | 0.104 | 0.057 | 0.297 | 1.251 | 0.725 | 0.000 | 0.197 | 3.782 | 0.077 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 53 | 501 | 68 | 161 | 0 | 58 | 75 | 62 |
| N.S. | 1 | 1.00 | 0.75 | 7.06 | 0.96 | 2.27 | 0.00 | 0.82 | 1.06 | 0.87 |
| time (sec) | N/A | 0.073 | 0.055 | 0.170 | 1.122 | 0.624 | 0.000 | 0.168 | 3.712 | 0.065 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 49 | 456 | 56 | 149 | 58 | 47 | 60 | 52 |
| N.S. | 1 | 1.00 | 0.94 | 8.77 | 1.08 | 2.87 | 1.12 | 0.90 | 1.15 | 1.00 |
| time (sec) | N/A | 0.046 | 0.035 | 0.184 | 1.209 | 0.508 | 27.571 | 0.165 | 3.677 | 0.053 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 55 | 55 | 43 | 435 | 58 | 147 | 51 | 48 | 63 | 55 |
| N.S. | 1 | 1.00 | 0.78 | 7.91 | 1.05 | 2.67 | 0.93 | 0.87 | 1.15 | 1.00 |
| time (sec) | N/A | 0.049 | 0.012 | 0.214 | 1.228 | 0.725 | 25.526 | 0.180 | 3.630 | 0.055 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 63 | 485 | 0 | 213 | 78 | 68 | 68 | 76 |
| N.S. | 1 | 1.00 | 0.83 | 6.38 | 0.00 | 2.80 | 1.03 | 0.89 | 0.89 | 1.00 |
| time (sec) | N/A | 0.070 | 0.028 | 0.189 | 0.000 | 0.657 | 17.016 | 0.162 | 3.662 | 0.064 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 77 | 549 | 0 | 272 | 0 | 100 | 88 | 91 |
| N.S. | 1 | 1.00 | 0.77 | 5.49 | 0.00 | 2.72 | 0.00 | 1.00 | 0.88 | 0.91 |
| time (sec) | N/A | 0.095 | 0.036 | 0.188 | 0.000 | 0.658 | 0.000 | 0.179 | 3.801 | 0.101 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 128 | 128 | 91 | 636 | 0 | 303 | 0 | 118 | 112 | 106 |
| N.S. | 1 | 1.00 | 0.71 | 4.97 | 0.00 | 2.37 | 0.00 | 0.92 | 0.88 | 0.83 |
| time (sec) | N/A | 0.124 | 0.079 | 0.217 | 0.000 | 0.791 | 0.000 | 0.165 | 4.025 | 0.140 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-2) | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 318 | 318 | 83 | 538 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.26 | 1.69 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.064 | 0.104 | 0.388 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 31.267 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-2) | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 324 | 324 | 83 | 509 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.26 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.070 | 0.076 | 0.410 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 31.263 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 328 | 328 | 85 | 510 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.26 | 1.55 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.067 | 0.123 | 0.413 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 31.288 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-1) | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 330 | 330 | 87 | 541 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.26 | 1.64 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.067 | 0.110 | 0.365 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 31.172 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-2) | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 310 | 310 | 83 | 538 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.27 | 1.74 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.051 | 0.149 | 0.425 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 31.157 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F(-2) | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 316 | 316 | 83 | 509 | 0 | 0 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.26 | 1.61 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.054 | 0.099 | 0.372 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 31.171 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 320 | 320 | 84 | 510 | 0 | 5060 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.26 | 1.59 | 0.00 | 15.81 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.053 | 0.106 | 0.364 | 0.000 | 22.764 | 0.000 | 0.000 | 0.000 | 31.228 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 322 | 322 | 86 | 541 | 0 | 5060 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.27 | 1.68 | 0.00 | 15.71 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.054 | 0.132 | 0.392 | 0.000 | 21.049 | 0.000 | 0.000 | 0.000 | 31.154 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 125 | 125 | 121 | 514 | 0 | 280 | 128 | 139 | 176 | 139 |
| N.S. | 1 | 1.00 | 0.97 | 4.11 | 0.00 | 2.24 | 1.02 | 1.11 | 1.41 | 1.11 |
| time (sec) | N/A | 0.129 | 0.307 | 0.425 | 0.000 | 0.700 | 30.272 | 0.174 | 6.171 | 0.161 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 88 | 458 | 0 | 195 | 95 | 96 | 136 | 100 |
| N.S. | 1 | 1.00 | 0.95 | 4.92 | 0.00 | 2.10 | 1.02 | 1.03 | 1.46 | 1.08 |
| time (sec) | N/A | 0.078 | 0.093 | 0.244 | 0.000 | 0.552 | 14.674 | 0.162 | 6.058 | 0.119 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 70 | 434 | 0 | 156 | 68 | 66 | 82 | 80 |
| N.S. | 1 | 1.00 | 1.00 | 6.20 | 0.00 | 2.23 | 0.97 | 0.94 | 1.17 | 1.14 |
| time (sec) | N/A | 0.059 | 0.037 | 0.229 | 0.000 | 0.640 | 6.349 | 0.157 | 6.156 | 0.072 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 81 | 476 | 0 | 383 | 85 | 79 | 114 | 95 |
| N.S. | 1 | 1.00 | 0.95 | 5.60 | 0.00 | 4.51 | 1.00 | 0.93 | 1.34 | 1.12 |
| time (sec) | N/A | 0.074 | 0.052 | 0.262 | 0.000 | 0.539 | 12.248 | 0.172 | 7.938 | 0.089 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 107 | 518 | 0 | 513 | 0 | 107 | 137 | 125 |
| N.S. | 1 | 1.00 | 0.93 | 4.50 | 0.00 | 4.46 | 0.00 | 0.93 | 1.19 | 1.09 |
| time (sec) | N/A | 0.125 | 0.178 | 0.249 | 0.000 | 0.518 | 0.000 | 0.192 | 5.130 | 0.264 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 154 | 154 | 145 | 605 | 0 | 410 | 153 | 193 | 330 | 195 |
| N.S. | 1 | 1.00 | 0.94 | 3.93 | 0.00 | 2.66 | 0.99 | 1.25 | 2.14 | 1.27 |
| time (sec) | N/A | 0.155 | 0.236 | 0.359 | 0.000 | 0.918 | 129.923 | 0.174 | 6.122 | 0.287 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 120 | 120 | 111 | 531 | 0 | 297 | 116 | 151 | 215 | 138 |
| N.S. | 1 | 1.00 | 0.92 | 4.42 | 0.00 | 2.48 | 0.97 | 1.26 | 1.79 | 1.15 |
| time (sec) | N/A | 0.104 | 0.121 | 0.324 | 0.000 | 0.777 | 70.599 | 0.189 | 6.134 | 0.175 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 96 | 96 | 85 | 507 | 0 | 204 | 90 | 113 | 143 | 95 |
| N.S. | 1 | 1.00 | 0.89 | 5.28 | 0.00 | 2.12 | 0.94 | 1.18 | 1.49 | 0.99 |
| time (sec) | N/A | 0.081 | 0.086 | 0.246 | 0.000 | 0.690 | 35.293 | 0.170 | 5.908 | 0.168 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 105 | 565 | 0 | 486 | 102 | 112 | 155 | 114 |
| N.S. | 1 | 1.00 | 1.01 | 5.43 | 0.00 | 4.67 | 0.98 | 1.08 | 1.49 | 1.10 |
| time (sec) | N/A | 0.112 | 0.097 | 0.261 | 0.000 | 0.999 | 39.290 | 0.177 | 7.879 | 0.232 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 116 | 116 | 108 | 620 | 0 | 538 | 0 | 121 | 167 | 130 |
| N.S. | 1 | 1.00 | 0.93 | 5.34 | 0.00 | 4.64 | 0.00 | 1.04 | 1.44 | 1.12 |
| time (sec) | N/A | 0.145 | 0.133 | 0.280 | 0.000 | 0.875 | 0.000 | 0.174 | 9.517 | 0.250 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 91 | 488 | 0 | 289 | 0 | 106 | 121 | 102 |
| N.S. | 1 | 1.00 | 0.88 | 4.69 | 0.00 | 2.78 | 0.00 | 1.02 | 1.16 | 0.98 |
| time (sec) | N/A | 0.105 | 0.216 | 0.339 | 0.000 | 0.868 | 0.000 | 0.172 | 5.426 | 0.185 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 74 | 448 | 0 | 205 | 0 | 64 | 86 | 84 |
| N.S. | 1 | 1.00 | 1.00 | 6.05 | 0.00 | 2.77 | 0.00 | 0.86 | 1.16 | 1.14 |
| time (sec) | N/A | 0.070 | 0.116 | 0.227 | 0.000 | 0.876 | 0.000 | 0.172 | 5.098 | 0.084 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 51 | 426 | 0 | 130 | 39 | 40 | 70 | 61 |
| N.S. | 1 | 1.00 | 1.00 | 8.35 | 0.00 | 2.55 | 0.76 | 0.78 | 1.37 | 1.20 |
| time (sec) | N/A | 0.050 | 0.017 | 0.275 | 0.000 | 0.851 | 10.261 | 0.157 | 5.888 | 0.053 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 81 | 453 | 0 | 431 | 70 | 71 | 114 | 95 |
| N.S. | 1 | 1.00 | 0.95 | 5.33 | 0.00 | 5.07 | 0.82 | 0.84 | 1.34 | 1.12 |
| time (sec) | N/A | 0.076 | 0.100 | 0.245 | 0.000 | 0.457 | 19.391 | 0.166 | 7.313 | 0.118 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 151 | 498 | 0 | 565 | 0 | 104 | 142 | 127 |
| N.S. | 1 | 1.00 | 1.29 | 4.26 | 0.00 | 4.83 | 0.00 | 0.89 | 1.21 | 1.09 |
| time (sec) | N/A | 0.118 | 0.131 | 0.253 | 0.000 | 0.485 | 0.000 | 0.192 | 8.424 | 0.230 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 107 | 107 | 100 | 527 | 0 | 440 | 0 | 103 | 115 | 124 |
| N.S. | 1 | 1.00 | 0.93 | 4.93 | 0.00 | 4.11 | 0.00 | 0.96 | 1.07 | 1.16 |
| time (sec) | N/A | 0.121 | 0.080 | 0.341 | 0.000 | 0.802 | 0.000 | 0.183 | 6.457 | 0.209 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 88 | 487 | 0 | 326 | 0 | 78 | 94 | 92 |
| N.S. | 1 | 1.00 | 1.07 | 5.94 | 0.00 | 3.98 | 0.00 | 0.95 | 1.15 | 1.12 |
| time (sec) | N/A | 0.074 | 0.121 | 0.271 | 0.000 | 0.812 | 0.000 | 0.191 | 5.990 | 0.150 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 52 | 463 | 0 | 236 | 66 | 73 | 89 | 109 |
| N.S. | 1 | 1.00 | 0.68 | 6.01 | 0.00 | 3.06 | 0.86 | 0.95 | 1.16 | 1.42 |
| time (sec) | N/A | 0.066 | 0.026 | 0.229 | 0.000 | 0.699 | 32.782 | 0.189 | 5.848 | 0.095 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 89 | 512 | 0 | 790 | 104 | 111 | 139 | 124 |
| N.S. | 1 | 1.00 | 0.78 | 4.49 | 0.00 | 6.93 | 0.91 | 0.97 | 1.22 | 1.09 |
| time (sec) | N/A | 0.113 | 0.048 | 0.286 | 0.000 | 0.575 | 24.626 | 0.166 | 8.444 | 0.290 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 117 | 575 | 0 | 1120 | 0 | 173 | 597 | 165 |
| N.S. | 1 | 1.00 | 0.74 | 3.64 | 0.00 | 7.09 | 0.00 | 1.09 | 3.78 | 1.04 |
| time (sec) | N/A | 0.222 | 0.058 | 0.408 | 0.000 | 0.865 | 0.000 | 0.174 | 10.472 | 0.394 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 101 | 952 | 107 | 219 | 0 | 110 | 127 | 95 |
| N.S. | 1 | 1.00 | 0.86 | 8.14 | 0.91 | 1.87 | 0.00 | 0.94 | 1.09 | 0.81 |
| time (sec) | N/A | 0.094 | 0.106 | 0.276 | 1.238 | 0.640 | 0.000 | 0.164 | 4.087 | 0.122 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 102 | 102 | 90 | 892 | 91 | 191 | 0 | 93 | 107 | 84 |
| N.S. | 1 | 1.00 | 0.88 | 8.75 | 0.89 | 1.87 | 0.00 | 0.91 | 1.05 | 0.82 |
| time (sec) | N/A | 0.076 | 0.086 | 0.207 | 1.220 | 0.680 | 0.000 | 0.184 | 4.013 | 0.079 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 79 | 874 | 79 | 165 | 0 | 69 | 87 | 73 |
| N.S. | 1 | 1.00 | 0.96 | 10.66 | 0.96 | 2.01 | 0.00 | 0.84 | 1.06 | 0.89 |
| time (sec) | N/A | 0.059 | 0.042 | 0.166 | 1.255 | 0.465 | 0.000 | 0.159 | 3.985 | 0.073 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 61 | 439 | 66 | 149 | 0 | 53 | 72 | 63 |
| N.S. | 1 | 1.00 | 0.95 | 6.86 | 1.03 | 2.33 | 0.00 | 0.83 | 1.12 | 0.98 |
| time (sec) | N/A | 0.049 | 0.062 | 0.179 | 1.392 | 0.487 | 0.000 | 0.159 | 3.926 | 0.061 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 102 | 912 | 0 | 226 | 0 | 79 | 76 | 88 |
| N.S. | 1 | 1.00 | 1.16 | 10.36 | 0.00 | 2.57 | 0.00 | 0.90 | 0.86 | 1.00 |
| time (sec) | N/A | 0.072 | 0.049 | 0.179 | 0.000 | 0.534 | 0.000 | 0.160 | 3.981 | 0.080 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 124 | 124 | 97 | 957 | 0 | 278 | 0 | 113 | 117 | 102 |
| N.S. | 1 | 1.00 | 0.78 | 7.72 | 0.00 | 2.24 | 0.00 | 0.91 | 0.94 | 0.82 |
| time (sec) | N/A | 0.102 | 0.151 | 0.211 | 0.000 | 0.522 | 0.000 | 0.172 | 4.212 | 0.116 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 164 | 164 | 112 | 1020 | 0 | 310 | 0 | 105 | 154 | 118 |
| N.S. | 1 | 1.00 | 0.68 | 6.22 | 0.00 | 1.89 | 0.00 | 0.64 | 0.94 | 0.72 |
| time (sec) | N/A | 0.131 | 0.211 | 0.188 | 0.000 | 0.473 | 0.000 | 0.181 | 4.453 | 0.170 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 111 | 998 | 119 | 239 | 0 | 127 | 147 | 104 |
| N.S. | 1 | 1.00 | 0.83 | 7.45 | 0.89 | 1.78 | 0.00 | 0.95 | 1.10 | 0.78 |
| time (sec) | N/A | 0.110 | 0.082 | 0.275 | 1.183 | 0.442 | 0.000 | 0.173 | 4.104 | 0.101 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 119 | 119 | 102 | 920 | 107 | 219 | 0 | 111 | 127 | 93 |
| N.S. | 1 | 1.00 | 0.86 | 7.73 | 0.90 | 1.84 | 0.00 | 0.93 | 1.07 | 0.78 |
| time (sec) | N/A | 0.094 | 0.126 | 0.189 | 1.391 | 0.444 | 0.000 | 0.197 | 4.052 | 0.089 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 90 | 902 | 93 | 192 | 0 | 93 | 107 | 82 |
| N.S. | 1 | 1.00 | 0.93 | 9.30 | 0.96 | 1.98 | 0.00 | 0.96 | 1.10 | 0.85 |
| time (sec) | N/A | 0.075 | 0.061 | 0.166 | 1.333 | 0.445 | 0.000 | 0.169 | 4.044 | 0.081 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 77 | 77 | 43 | 451 | 79 | 162 | 0 | 69 | 87 | 71 |
| N.S. | 1 | 1.00 | 0.56 | 5.86 | 1.03 | 2.10 | 0.00 | 0.90 | 1.13 | 0.92 |
| time (sec) | N/A | 0.060 | 0.015 | 0.177 | 1.311 | 0.439 | 0.000 | 0.169 | 3.987 | 0.068 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 100 | 956 | 0 | 220 | 0 | 70 | 101 | 85 |
| N.S. | 1 | 1.00 | 1.18 | 11.25 | 0.00 | 2.59 | 0.00 | 0.82 | 1.19 | 1.00 |
| time (sec) | N/A | 0.079 | 0.055 | 0.174 | 0.000 | 0.458 | 0.000 | 0.169 | 4.722 | 0.077 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 97 | 1014 | 0 | 280 | 0 | 114 | 110 | 103 |
| N.S. | 1 | 1.00 | 0.80 | 8.38 | 0.00 | 2.31 | 0.00 | 0.94 | 0.91 | 0.85 |
| time (sec) | N/A | 0.103 | 0.156 | 0.189 | 0.000 | 0.454 | 0.000 | 0.188 | 4.235 | 0.119 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 112 | 1075 | 0 | 310 | 0 | 129 | 151 | 118 |
| N.S. | 1 | 1.00 | 0.70 | 6.68 | 0.00 | 1.93 | 0.00 | 0.80 | 0.94 | 0.73 |
| time (sec) | N/A | 0.133 | 0.240 | 0.189 | 0.000 | 0.474 | 0.000 | 0.178 | 4.618 | 0.144 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 91 | 916 | 93 | 195 | 0 | 93 | 107 | 84 |
| N.S. | 1 | 1.00 | 0.96 | 9.64 | 0.98 | 2.05 | 0.00 | 0.98 | 1.13 | 0.88 |
| time (sec) | N/A | 0.073 | 0.048 | 0.296 | 1.344 | 0.440 | 0.000 | 0.194 | 4.061 | 0.089 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 82 | 874 | 79 | 167 | 0 | 69 | 87 | 73 |
| N.S. | 1 | 1.00 | 0.99 | 10.53 | 0.95 | 2.01 | 0.00 | 0.83 | 1.05 | 0.88 |
| time (sec) | N/A | 0.064 | 0.053 | 0.199 | 1.561 | 0.431 | 0.000 | 0.174 | 3.997 | 0.074 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 64 | 64 | 63 | 861 | 67 | 155 | 0 | 58 | 72 | 63 |
| N.S. | 1 | 1.00 | 0.98 | 13.45 | 1.05 | 2.42 | 0.00 | 0.91 | 1.12 | 0.98 |
| time (sec) | N/A | 0.051 | 0.052 | 0.174 | 1.322 | 0.470 | 0.000 | 0.162 | 4.009 | 0.063 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 64 | 442 | 72 | 153 | 0 | 59 | 75 | 67 |
| N.S. | 1 | 1.00 | 0.96 | 6.60 | 1.07 | 2.28 | 0.00 | 0.88 | 1.12 | 1.00 |
| time (sec) | N/A | 0.049 | 0.037 | 0.190 | 1.389 | 0.438 | 0.000 | 0.157 | 3.977 | 0.061 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 83 | 880 | 0 | 226 | 0 | 79 | 80 | 88 |
| N.S. | 1 | 1.00 | 0.94 | 10.00 | 0.00 | 2.57 | 0.00 | 0.90 | 0.91 | 1.00 |
| time (sec) | N/A | 0.075 | 0.096 | 0.186 | 0.000 | 0.468 | 0.000 | 0.190 | 4.008 | 0.087 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 124 | 124 | 97 | 926 | 0 | 280 | 0 | 114 | 117 | 103 |
| N.S. | 1 | 1.00 | 0.78 | 7.47 | 0.00 | 2.26 | 0.00 | 0.92 | 0.94 | 0.83 |
| time (sec) | N/A | 0.101 | 0.149 | 0.197 | 0.000 | 0.450 | 0.000 | 0.163 | 4.111 | 0.128 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 164 | 164 | 112 | 989 | 0 | 310 | 0 | 128 | 155 | 118 |
| N.S. | 1 | 1.00 | 0.68 | 6.03 | 0.00 | 1.89 | 0.00 | 0.78 | 0.95 | 0.72 |
| time (sec) | N/A | 0.129 | 0.208 | 0.197 | 0.000 | 0.650 | 0.000 | 0.181 | 4.368 | 0.151 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 88 | 970 | 98 | 233 | 0 | 88 | 111 | 127 |
| N.S. | 1 | 1.00 | 0.93 | 10.21 | 1.03 | 2.45 | 0.00 | 0.93 | 1.17 | 1.34 |
| time (sec) | N/A | 0.076 | 0.046 | 0.288 | 1.212 | 0.758 | 0.000 | 0.189 | 4.383 | 0.089 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 71 | 926 | 81 | 223 | 0 | 67 | 94 | 115 |
| N.S. | 1 | 1.00 | 0.86 | 11.16 | 0.98 | 2.69 | 0.00 | 0.81 | 1.13 | 1.39 |
| time (sec) | N/A | 0.065 | 0.105 | 0.178 | 1.412 | 0.712 | 0.000 | 0.175 | 4.300 | 0.080 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 68 | 908 | 83 | 223 | 0 | 76 | 96 | 117 |
| N.S. | 1 | 1.00 | 0.80 | 10.68 | 0.98 | 2.62 | 0.00 | 0.89 | 1.13 | 1.38 |
| time (sec) | N/A | 0.062 | 0.025 | 0.183 | 1.234 | 0.712 | 0.000 | 0.189 | 4.264 | 0.082 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 43 | 463 | 85 | 219 | 0 | 72 | 97 | 76 |
| N.S. | 1 | 1.00 | 0.49 | 5.26 | 0.97 | 2.49 | 0.00 | 0.82 | 1.10 | 0.86 |
| time (sec) | N/A | 0.064 | 0.013 | 0.190 | 1.270 | 0.595 | 0.000 | 0.161 | 4.261 | 0.095 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 106 | 106 | 97 | 953 | 0 | 316 | 0 | 93 | 101 | 163 |
| N.S. | 1 | 1.00 | 0.92 | 8.99 | 0.00 | 2.98 | 0.00 | 0.88 | 0.95 | 1.54 |
| time (sec) | N/A | 0.095 | 0.045 | 0.171 | 0.000 | 0.664 | 0.000 | 0.172 | 4.332 | 0.100 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 143 | 143 | 117 | 1019 | 0 | 368 | 0 | 129 | 133 | 114 |
| N.S. | 1 | 1.00 | 0.82 | 7.13 | 0.00 | 2.57 | 0.00 | 0.90 | 0.93 | 0.80 |
| time (sec) | N/A | 0.125 | 0.064 | 0.195 | 0.000 | 0.779 | 0.000 | 0.165 | 4.562 | 0.148 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 135 | 1106 | 0 | 398 | 0 | 149 | 171 | 208 |
| N.S. | 1 | 1.00 | 0.73 | 5.98 | 0.00 | 2.15 | 0.00 | 0.81 | 0.92 | 1.12 |
| time (sec) | N/A | 0.163 | 0.064 | 0.210 | 0.000 | 0.896 | 0.000 | 0.181 | 4.763 | 0.194 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 147 | 917 | 0 | 469 | 0 | 136 | 202 | 142 |
| N.S. | 1 | 1.00 | 0.91 | 5.70 | 0.00 | 2.91 | 0.00 | 0.84 | 1.25 | 0.88 |
| time (sec) | N/A | 0.192 | 0.291 | 0.359 | 0.000 | 0.917 | 0.000 | 0.179 | 6.842 | 0.271 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 136 | 136 | 117 | 897 | 0 | 334 | 0 | 102 | 152 | 108 |
| N.S. | 1 | 1.00 | 0.86 | 6.60 | 0.00 | 2.46 | 0.00 | 0.75 | 1.12 | 0.79 |
| time (sec) | N/A | 0.109 | 0.154 | 0.263 | 0.000 | 0.626 | 0.000 | 0.172 | 6.087 | 0.199 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 80 | 80 | 80 | 453 | 0 | 255 | 0 | 79 | 125 | 90 |
| N.S. | 1 | 1.00 | 1.00 | 5.66 | 0.00 | 3.19 | 0.00 | 0.99 | 1.56 | 1.12 |
| time (sec) | N/A | 0.065 | 0.114 | 0.270 | 0.000 | 0.809 | 0.000 | 0.202 | 5.686 | 0.176 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 112 | 934 | 0 | 856 | 0 | 114 | 182 | 131 |
| N.S. | 1 | 1.00 | 0.93 | 7.72 | 0.00 | 7.07 | 0.00 | 0.94 | 1.50 | 1.08 |
| time (sec) | N/A | 0.118 | 0.294 | 0.285 | 0.000 | 0.731 | 0.000 | 0.171 | 8.281 | 0.511 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 190 | 978 | 0 | 870 | 0 | 183 | 438 | 157 |
| N.S. | 1 | 1.00 | 1.18 | 6.07 | 0.00 | 5.40 | 0.00 | 1.14 | 2.72 | 0.98 |
| time (sec) | N/A | 0.218 | 0.283 | 0.262 | 0.000 | 1.106 | 0.000 | 0.215 | 9.692 | 0.537 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 189 | 189 | 162 | 1003 | 0 | 443 | 0 | 211 | 331 | 211 |
| N.S. | 1 | 1.00 | 0.86 | 5.31 | 0.00 | 2.34 | 0.00 | 1.12 | 1.75 | 1.12 |
| time (sec) | N/A | 0.237 | 0.185 | 0.367 | 0.000 | 0.688 | 0.000 | 0.177 | 7.752 | 0.292 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 163 | 163 | 125 | 983 | 0 | 314 | 0 | 173 | 229 | 150 |
| N.S. | 1 | 1.00 | 0.77 | 6.03 | 0.00 | 1.93 | 0.00 | 1.06 | 1.40 | 0.92 |
| time (sec) | N/A | 0.137 | 0.136 | 0.269 | 0.000 | 1.104 | 0.000 | 0.187 | 7.379 | 0.291 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 54 | 466 | 0 | 234 | 0 | 122 | 170 | 103 |
| N.S. | 1 | 1.00 | 0.57 | 4.96 | 0.00 | 2.49 | 0.00 | 1.30 | 1.81 | 1.10 |
| time (sec) | N/A | 0.081 | 0.024 | 0.266 | 0.000 | 0.904 | 0.000 | 0.169 | 7.354 | 0.193 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 131 | 131 | 122 | 1036 | 0 | 686 | 0 | 155 | 214 | 156 |
| N.S. | 1 | 1.00 | 0.93 | 7.91 | 0.00 | 5.24 | 0.00 | 1.18 | 1.63 | 1.19 |
| time (sec) | N/A | 0.140 | 0.186 | 0.267 | 0.000 | 1.070 | 0.000 | 0.174 | 9.143 | 0.308 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 170 | 170 | 142 | 1093 | 0 | 838 | 0 | 216 | 531 | 179 |
| N.S. | 1 | 1.00 | 0.84 | 6.43 | 0.00 | 4.93 | 0.00 | 1.27 | 3.12 | 1.05 |
| time (sec) | N/A | 0.258 | 0.333 | 0.248 | 0.000 | 1.182 | 0.000 | 0.217 | 10.816 | 0.678 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 107 | 911 | 0 | 475 | 0 | 134 | 160 | 143 |
| N.S. | 1 | 1.00 | 0.87 | 7.41 | 0.00 | 3.86 | 0.00 | 1.09 | 1.30 | 1.16 |
| time (sec) | N/A | 0.144 | 0.276 | 0.424 | 0.000 | 1.143 | 0.000 | 0.226 | 7.290 | 0.315 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 98 | 892 | 0 | 348 | 0 | 116 | 111 | 109 |
| N.S. | 1 | 1.00 | 0.99 | 9.01 | 0.00 | 3.52 | 0.00 | 1.17 | 1.12 | 1.10 |
| time (sec) | N/A | 0.084 | 0.097 | 0.272 | 0.000 | 1.109 | 0.000 | 0.166 | 6.846 | 0.161 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 85 | 457 | 0 | 302 | 0 | 93 | 104 | 97 |
| N.S. | 1 | 1.00 | 0.98 | 5.25 | 0.00 | 3.47 | 0.00 | 1.07 | 1.20 | 1.11 |
| time (sec) | N/A | 0.073 | 0.104 | 0.256 | 0.000 | 0.894 | 0.000 | 0.164 | 6.373 | 0.114 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 123 | 915 | 0 | 862 | 0 | 139 | 162 | 146 |
| N.S. | 1 | 1.00 | 0.93 | 6.93 | 0.00 | 6.53 | 0.00 | 1.05 | 1.23 | 1.11 |
| time (sec) | N/A | 0.139 | 0.265 | 0.276 | 0.000 | 0.927 | 0.000 | 0.163 | 9.647 | 0.307 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 163 | 961 | 0 | 1236 | 0 | 257 | 355 | 187 |
| N.S. | 1 | 1.00 | 0.88 | 5.19 | 0.00 | 6.68 | 0.00 | 1.39 | 1.92 | 1.01 |
| time (sec) | N/A | 0.243 | 0.611 | 0.287 | 0.000 | 0.905 | 0.000 | 0.172 | 11.551 | 0.642 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 149 | 134 | 978 | 0 | 746 | 0 | 195 | 367 | 151 |
| N.S. | 1 | 0.99 | 0.89 | 6.52 | 0.00 | 4.97 | 0.00 | 1.30 | 2.45 | 1.01 |
| time (sec) | N/A | 0.195 | 0.298 | 0.364 | 0.000 | 0.922 | 0.000 | 0.185 | 7.898 | 0.249 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 91 | 958 | 0 | 630 | 0 | 181 | 247 | 123 |
| N.S. | 1 | 1.00 | 0.68 | 7.15 | 0.00 | 4.70 | 0.00 | 1.35 | 1.84 | 0.92 |
| time (sec) | N/A | 0.114 | 0.033 | 0.274 | 0.000 | 1.420 | 0.000 | 0.196 | 7.704 | 0.261 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 108 | 108 | 54 | 485 | 0 | 450 | 0 | 153 | 199 | 110 |
| N.S. | 1 | 1.00 | 0.50 | 4.49 | 0.00 | 4.17 | 0.00 | 1.42 | 1.84 | 1.02 |
| time (sec) | N/A | 0.091 | 0.018 | 0.249 | 0.000 | 1.465 | 0.000 | 0.207 | 7.429 | 0.236 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 172 | 172 | 123 | 1002 | 0 | 1819 | 0 | 226 | 288 | 183 |
| N.S. | 1 | 1.00 | 0.72 | 5.83 | 0.00 | 10.58 | 0.00 | 1.31 | 1.67 | 1.06 |
| time (sec) | N/A | 0.223 | 0.127 | 0.273 | 0.000 | 1.662 | 0.000 | 0.185 | 12.182 | 0.724 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 241 | 241 | 189 | 1067 | 0 | 2384 | 0 | 367 | 18847 | 272 |
| N.S. | 1 | 1.00 | 0.78 | 4.43 | 0.00 | 9.89 | 0.00 | 1.52 | 78.20 | 1.13 |
| time (sec) | N/A | 0.357 | 0.156 | 0.246 | 0.000 | 2.004 | 0.000 | 0.222 | 19.626 | 0.949 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 88 | 123 | 0 | 0 | 256 | 0 | 104 | 283 | 122 |
| N.S. | 1 | 1.00 | 1.40 | 0.00 | 0.00 | 2.91 | 0.00 | 1.18 | 3.22 | 1.39 |
| time (sec) | N/A | 0.093 | 0.266 | 0.177 | 0.000 | 0.577 | 0.000 | 0.211 | 9.251 | 1.304 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 85 | 0 | 0 | 194 | 0 | 54 | 49 | 48 |
| N.S. | 1 | 1.00 | 1.77 | 0.00 | 0.00 | 4.04 | 0.00 | 1.12 | 1.02 | 1.00 |
| time (sec) | N/A | 0.059 | 0.096 | 0.652 | 0.000 | 0.639 | 0.000 | 0.184 | 5.043 | 0.850 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 48 | 48 | 48 | 0 | 0 | 204 | 0 | 89 | 136 | 48 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 4.25 | 0.00 | 1.85 | 2.83 | 1.00 |
| time (sec) | N/A | 0.048 | 0.019 | 0.585 | 0.000 | 0.649 | 0.000 | 0.190 | 7.572 | 0.885 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 91 | 0 | 0 | 278 | 0 | 413 | 481 | 119 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 3.05 | 0.00 | 4.54 | 5.29 | 1.31 |
| time (sec) | N/A | 0.080 | 0.065 | 0.122 | 0.000 | 0.787 | 0.000 | 0.235 | 10.775 | 1.649 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 145 | 7293 | 0 | 295 | 292 | 251 | -1 | 162 |
| N.S. | 1 | 1.00 | 0.90 | 45.30 | 0.00 | 1.83 | 1.81 | 1.56 | -0.01 | 1.01 |
| time (sec) | N/A | 0.113 | 0.242 | 8.015 | 0.000 | 1.260 | 130.857 | 0.754 | 0.000 | 0.603 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 119 | 6858 | 0 | 221 | 201 | 137 | -1 | 123 |
| N.S. | 1 | 1.00 | 0.98 | 56.68 | 0.00 | 1.83 | 1.66 | 1.13 | -0.01 | 1.02 |
| time (sec) | N/A | 0.089 | 0.145 | 1.092 | 0.000 | 1.158 | 9.670 | 0.419 | 0.000 | 0.515 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 118 | 118 | 87 | 6668 | 0 | 207 | 160 | 0 | -1 | 99 |
| N.S. | 1 | 1.00 | 0.74 | 56.51 | 0.00 | 1.75 | 1.36 | 0.00 | -0.01 | 0.84 |
| time (sec) | N/A | 0.084 | 0.209 | 1.040 | 0.000 | 1.547 | 9.715 | 0.000 | 0.000 | 0.562 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 87 | 3759 | 81 | 180 | 131 | 109 | -1 | 75 |
| N.S. | 1 | 1.00 | 1.10 | 47.58 | 1.03 | 2.28 | 1.66 | 1.38 | -0.01 | 0.95 |
| time (sec) | N/A | 0.046 | 0.181 | 0.999 | 1.217 | 0.948 | 61.862 | 0.246 | 0.000 | 0.310 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 201 | 201 | 167 | 7705 | 0 | 355 | 0 | 387 | -1 | 200 |
| N.S. | 1 | 1.00 | 0.83 | 38.33 | 0.00 | 1.77 | 0.00 | 1.93 | -0.00 | 1.00 |
| time (sec) | N/A | 0.139 | 0.335 | 1.142 | 0.000 | 1.203 | 0.000 | 1.455 | 0.000 | 0.836 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 161 | 161 | 143 | 7290 | 0 | 273 | 335 | 0 | -1 | 162 |
| N.S. | 1 | 1.00 | 0.89 | 45.28 | 0.00 | 1.70 | 2.08 | 0.00 | -0.01 | 1.01 |
| time (sec) | N/A | 0.113 | 0.182 | 1.064 | 0.000 | 1.559 | 25.464 | 0.000 | 0.000 | 0.712 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 152 | 152 | 126 | 7108 | 0 | 255 | 289 | 0 | -1 | 125 |
| N.S. | 1 | 1.00 | 0.83 | 46.76 | 0.00 | 1.68 | 1.90 | 0.00 | -0.01 | 0.82 |
| time (sec) | N/A | 0.104 | 0.154 | 1.183 | 0.000 | 1.450 | 24.317 | 0.000 | 0.000 | 0.762 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 241 | 241 | 188 | 8117 | 0 | 409 | 0 | 563 | -1 | 238 |
| N.S. | 1 | 1.00 | 0.78 | 33.68 | 0.00 | 1.70 | 0.00 | 2.34 | -0.00 | 0.99 |
| time (sec) | N/A | 0.159 | 0.314 | 1.065 | 0.000 | 1.332 | 0.000 | 1.488 | 0.000 | 0.946 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 201 | 201 | 146 | 7702 | 0 | 323 | 413 | 0 | -1 | 200 |
| N.S. | 1 | 1.00 | 0.73 | 38.32 | 0.00 | 1.61 | 2.05 | 0.00 | -0.00 | 1.00 |
| time (sec) | N/A | 0.127 | 0.426 | 1.104 | 0.000 | 1.386 | 56.284 | 0.000 | 0.000 | 0.769 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | B | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 188 | 188 | 150 | 7544 | 0 | 309 | 403 | 0 | -1 | 157 |
| N.S. | 1 | 1.00 | 0.80 | 40.13 | 0.00 | 1.64 | 2.14 | 0.00 | -0.01 | 0.84 |
| time (sec) | N/A | 0.128 | 0.213 | 1.157 | 0.000 | 1.281 | 63.945 | 0.000 | 0.000 | 0.848 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 97 | 6861 | 0 | 245 | 194 | 114 | -1 | 124 |
| N.S. | 1 | 1.00 | 0.80 | 56.70 | 0.00 | 2.02 | 1.60 | 0.94 | -0.01 | 1.02 |
| time (sec) | N/A | 0.086 | 0.169 | 1.010 | 0.000 | 1.441 | 95.303 | 0.365 | 0.000 | 1.055 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 78 | 6424 | 0 | 184 | 107 | 72 | -1 | 89 |
| N.S. | 1 | 1.00 | 0.94 | 77.40 | 0.00 | 2.22 | 1.29 | 0.87 | -0.01 | 1.07 |
| time (sec) | N/A | 0.067 | 0.059 | 1.148 | 0.000 | 1.280 | 6.806 | 0.363 | 0.000 | 0.532 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 65 | 3397 | 0 | 183 | 60 | 110 | -1 | 81 |
| N.S. | 1 | 1.00 | 0.87 | 45.29 | 0.00 | 2.44 | 0.80 | 1.47 | -0.01 | 1.08 |
| time (sec) | N/A | 0.057 | 0.064 | 1.003 | 0.000 | 1.192 | 10.112 | 0.260 | 0.000 | 0.789 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 120 | 120 | 109 | 7016 | 0 | 307 | 0 | 107 | -1 | 136 |
| N.S. | 1 | 1.00 | 0.91 | 58.47 | 0.00 | 2.56 | 0.00 | 0.89 | -0.01 | 1.13 |
| time (sec) | N/A | 0.086 | 0.176 | 1.132 | 0.000 | 1.925 | 0.000 | 0.366 | 0.000 | 1.761 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 93 | 3654 | 0 | 234 | 95 | 75 | -1 | 107 |
| N.S. | 1 | 1.00 | 1.09 | 42.99 | 0.00 | 2.75 | 1.12 | 0.88 | -0.01 | 1.26 |
| time (sec) | N/A | 0.062 | 0.099 | 1.000 | 0.000 | 0.861 | 33.620 | 0.619 | 0.000 | 1.007 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 45 | 39 | 0 | 57 | 0 | 0 | 70 | 71 |
| N.S. | 1 | 1.00 | 0.67 | 0.58 | 0.00 | 0.85 | 0.00 | 0.00 | 1.04 | 1.06 |
| time (sec) | N/A | 0.029 | 0.023 | 0.047 | 0.000 | 0.653 | 0.000 | 0.000 | 4.747 | 1.037 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 114 | 114 | 119 | 7081 | 0 | 345 | 0 | 102 | -1 | 137 |
| N.S. | 1 | 1.00 | 1.04 | 62.11 | 0.00 | 3.03 | 0.00 | 0.89 | -0.01 | 1.20 |
| time (sec) | N/A | 0.075 | 0.303 | 1.053 | 0.000 | 0.835 | 0.000 | 0.476 | 0.000 | 1.909 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 44 | 39 | 0 | 59 | 0 | 64 | 73 | 76 |
| N.S. | 1 | 1.00 | 0.56 | 0.49 | 0.00 | 0.75 | 0.00 | 0.81 | 0.92 | 0.96 |
| time (sec) | N/A | 0.031 | 0.039 | 0.049 | 0.000 | 0.615 | 0.000 | 0.324 | 4.628 | 1.263 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 65 | 62 | 0 | 93 | 0 | 0 | 115 | 100 |
| N.S. | 1 | 1.00 | 0.62 | 0.60 | 0.00 | 0.89 | 0.00 | 0.00 | 1.11 | 0.96 |
| time (sec) | N/A | 0.046 | 0.036 | 0.049 | 0.000 | 0.629 | 0.000 | 0.000 | 4.804 | 1.112 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 220 | 220 | 230 | 0 | 183 | 186 | 0 | 0 | 240 | 224 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.83 | 0.85 | 0.00 | 0.00 | 1.09 | 1.02 |
| time (sec) | N/A | 0.236 | 0.259 | 0.654 | 1.554 | 0.743 | 0.000 | 0.000 | 4.809 | 0.250 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 174 | 174 | 207 | 0 | 155 | 174 | 0 | 0 | 219 | 213 |
| N.S. | 1 | 1.00 | 1.19 | 0.00 | 0.89 | 1.00 | 0.00 | 0.00 | 1.26 | 1.22 |
| time (sec) | N/A | 0.178 | 0.139 | 0.631 | 1.406 | 0.491 | 0.000 | 0.000 | 4.650 | 0.186 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 172 | 172 | 186 | 0 | 153 | 157 | 0 | 0 | 200 | 202 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.89 | 0.91 | 0.00 | 0.00 | 1.16 | 1.17 |
| time (sec) | N/A | 0.149 | 0.096 | 0.636 | 1.434 | 0.781 | 0.000 | 0.000 | 4.661 | 0.179 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 150 | 167 | 0 | 139 | 144 | 0 | 0 | 194 | 190 |
| N.S. | 1 | 1.00 | 1.11 | 0.00 | 0.93 | 0.96 | 0.00 | 0.00 | 1.29 | 1.27 |
| time (sec) | N/A | 0.119 | 0.053 | 0.636 | 1.271 | 0.696 | 0.000 | 0.000 | 4.638 | 0.168 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 214 | 214 | 233 | 0 | 0 | 1046 | 0 | 0 | 345 | 286 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 0.00 | 4.89 | 0.00 | 0.00 | 1.61 | 1.34 |
| time (sec) | N/A | 0.174 | 0.107 | 0.598 | 0.000 | 0.720 | 0.000 | 0.000 | 5.097 | 0.351 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 268 | 268 | 280 | 0 | 0 | 321 | 0 | 0 | 455 | 318 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 1.20 | 0.00 | 0.00 | 1.70 | 1.19 |
| time (sec) | N/A | 0.247 | 0.117 | 0.615 | 0.000 | 0.738 | 0.000 | 0.000 | 5.345 | 0.569 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 283 | 283 | 314 | 0 | 0 | 345 | 0 | 0 | 490 | 340 |
| N.S. | 1 | 1.00 | 1.11 | 0.00 | 0.00 | 1.22 | 0.00 | 0.00 | 1.73 | 1.20 |
| time (sec) | N/A | 0.264 | 0.138 | 0.652 | 0.000 | 0.637 | 0.000 | 0.000 | 5.442 | 0.718 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 268 | 66 | 177 | 0 | 0 | 362 | 0 | 0 | -1 | 364 |
| N.S. | 1 | 0.25 | 0.66 | 0.00 | 0.00 | 1.35 | 0.00 | 0.00 | -0.00 | 1.36 |
| time (sec) | N/A | 0.060 | 0.210 | 0.649 | 0.000 | 0.637 | 0.000 | 0.000 | 0.000 | 1.213 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 233 | 66 | 160 | 0 | 0 | 338 | 0 | 0 | -1 | 342 |
| N.S. | 1 | 0.28 | 0.69 | 0.00 | 0.00 | 1.45 | 0.00 | 0.00 | -0.00 | 1.47 |
| time (sec) | N/A | 0.063 | 0.166 | 0.737 | 0.000 | 0.635 | 0.000 | 0.000 | 0.000 | 0.870 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 201 | 66 | 63 | 0 | 0 | 313 | 0 | 0 | -1 | 309 |
| N.S. | 1 | 0.33 | 0.31 | 0.00 | 0.00 | 1.56 | 0.00 | 0.00 | -0.00 | 1.54 |
| time (sec) | N/A | 0.043 | 0.041 | 0.710 | 0.000 | 0.615 | 0.000 | 0.000 | 0.000 | 0.597 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | A |
| verified | N/A | NO | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 156 | 77 | 45 | 0 | 0 | 0 | 0 | 0 | -1 | 205 |
| N.S. | 1 | 0.49 | 0.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 1.31 |
| time (sec) | N/A | 0.070 | 0.018 | 0.842 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.425 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 183 | 117 | 125 | 0 | 0 | 0 | 0 | 0 | -1 | 217 |
| N.S. | 1 | 0.64 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 1.19 |
| time (sec) | N/A | 0.422 | 5.104 | 0.622 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.475 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 210 | 244 | 135 | 0 | 0 | 0 | 0 | 0 | -1 | 228 |
| N.S. | 1 | 1.16 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.09 |
| time (sec) | N/A | 19.780 | 5.122 | 0.612 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.522 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F(-1) | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 237 | 423 | 130 | 0 | 0 | 0 | 0 | 0 | -1 | 239 |
| N.S. | 1 | 1.78 | 0.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.01 |
| time (sec) | N/A | 29.209 | 5.201 | 0.625 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.571 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 416 | 61 | 154 | 0 | 0 | 0 | 0 | 0 | -1 | 533 |
| N.S. | 1 | 0.15 | 0.37 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.28 |
| time (sec) | N/A | 0.031 | 0.154 | 0.631 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3.089 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F(-1) | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 223 | 223 | 163 | 0 | 183 | 209 | 0 | 0 | 261 | 225 |
| N.S. | 1 | 1.00 | 0.73 | 0.00 | 0.82 | 0.94 | 0.00 | 0.00 | 1.17 | 1.01 |
| time (sec) | N/A | 0.250 | 0.276 | 0.576 | 1.252 | 0.648 | 0.000 | 0.000 | 4.846 | 0.269 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 177 | 177 | 153 | 0 | 155 | 197 | 0 | 0 | 206 | 214 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.88 | 1.11 | 0.00 | 0.00 | 1.16 | 1.21 |
| time (sec) | N/A | 0.200 | 0.172 | 0.575 | 1.169 | 0.725 | 0.000 | 0.000 | 4.915 | 0.222 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 175 | 175 | 143 | 0 | 155 | 181 | 0 | 0 | 221 | 203 |
| N.S. | 1 | 1.00 | 0.82 | 0.00 | 0.89 | 1.03 | 0.00 | 0.00 | 1.26 | 1.16 |
| time (sec) | N/A | 0.156 | 0.147 | 0.605 | 1.120 | 0.940 | 0.000 | 0.000 | 4.839 | 0.207 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 153 | 153 | 130 | 0 | 140 | 167 | 0 | 0 | 186 | 193 |
| N.S. | 1 | 1.00 | 0.85 | 0.00 | 0.92 | 1.09 | 0.00 | 0.00 | 1.22 | 1.26 |
| time (sec) | N/A | 0.126 | 0.092 | 0.622 | 1.314 | 1.169 | 0.000 | 0.000 | 4.830 | 0.195 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 214 | 214 | 164 | 0 | 0 | 530 | 0 | 0 | 369 | 286 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 2.48 | 0.00 | 0.00 | 1.72 | 1.34 |
| time (sec) | N/A | 0.182 | 0.080 | 0.682 | 0.000 | 0.739 | 0.000 | 0.000 | 5.855 | 0.350 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 269 | 269 | 213 | 0 | 0 | 612 | 0 | 0 | 490 | 319 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 2.28 | 0.00 | 0.00 | 1.82 | 1.19 |
| time (sec) | N/A | 0.263 | 0.093 | 0.624 | 0.000 | 0.513 | 0.000 | 0.000 | 5.561 | 0.550 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 284 | 284 | 247 | 0 | 0 | 660 | 0 | 0 | 513 | 341 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 2.32 | 0.00 | 0.00 | 1.81 | 1.20 |
| time (sec) | N/A | 0.279 | 0.156 | 0.574 | 0.000 | 0.499 | 0.000 | 0.000 | 5.455 | 0.677 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 264 | 66 | 244 | 0 | 0 | 701 | 0 | 0 | -1 | 361 |
| N.S. | 1 | 0.25 | 0.92 | 0.00 | 0.00 | 2.66 | 0.00 | 0.00 | -0.00 | 1.37 |
| time (sec) | N/A | 0.063 | 0.305 | 0.595 | 0.000 | 0.518 | 0.000 | 0.000 | 0.000 | 1.136 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 229 | 66 | 216 | 0 | 0 | 653 | 0 | 0 | -1 | 339 |
| N.S. | 1 | 0.29 | 0.94 | 0.00 | 0.00 | 2.85 | 0.00 | 0.00 | -0.00 | 1.48 |
| time (sec) | N/A | 0.063 | 0.217 | 0.583 | 0.000 | 0.509 | 0.000 | 0.000 | 0.000 | 0.838 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 200 | 61 | 156 | 0 | 0 | 611 | 0 | 0 | -1 | 309 |
| N.S. | 1 | 0.30 | 0.78 | 0.00 | 0.00 | 3.06 | 0.00 | 0.00 | -0.00 | 1.54 |
| time (sec) | N/A | 0.033 | 0.163 | 0.630 | 0.000 | 0.541 | 0.000 | 0.000 | 0.000 | 0.604 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | A |
| verified | N/A | NO | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 157 | 79 | 47 | 0 | 0 | 434 | 0 | 0 | -1 | 206 |
| N.S. | 1 | 0.50 | 0.30 | 0.00 | 0.00 | 2.76 | 0.00 | 0.00 | -0.01 | 1.31 |
| time (sec) | N/A | 0.071 | 0.040 | 0.635 | 0.000 | 144.749 | 0.000 | 0.000 | 0.000 | 0.455 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | F | F | F(-1) | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 182 | 121 | 170 | 0 | 0 | 0 | 0 | 0 | -1 | 216 |
| N.S. | 1 | 0.66 | 0.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 1.19 |
| time (sec) | N/A | 0.429 | 5.203 | 0.635 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.495 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | F | F | F(-1) | F(-1) | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 209 | 244 | 179 | 0 | 0 | 0 | 0 | 0 | -1 | 227 |
| N.S. | 1 | 1.17 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.09 |
| time (sec) | N/A | 10.676 | 5.268 | 0.640 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.548 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | F | F | F(-1) | F(-1) | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 236 | 391 | 196 | 0 | 0 | 0 | 0 | 0 | -1 | 238 |
| N.S. | 1 | 1.66 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.01 |
| time (sec) | N/A | 17.601 | 5.350 | 0.580 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.615 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 127 | 127 | 113 | 792 | 119 | 118 | 0 | 134 | 133 | 148 |
| N.S. | 1 | 1.00 | 0.89 | 6.24 | 0.94 | 0.93 | 0.00 | 1.06 | 1.05 | 1.17 |
| time (sec) | N/A | 0.093 | 0.104 | 9.325 | 1.241 | 0.934 | 0.000 | 0.173 | 5.034 | 0.153 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 128 | 128 | 128 | 683 | 119 | 137 | 0 | 127 | 133 | 144 |
| N.S. | 1 | 1.00 | 1.00 | 5.34 | 0.93 | 1.07 | 0.00 | 0.99 | 1.04 | 1.12 |
| time (sec) | N/A | 0.088 | 0.057 | 8.409 | 1.248 | 0.870 | 0.000 | 0.176 | 4.730 | 0.140 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 97 | 97 | 96 | 673 | 97 | 106 | 0 | 98 | 111 | 131 |
| N.S. | 1 | 1.00 | 0.99 | 6.94 | 1.00 | 1.09 | 0.00 | 1.01 | 1.14 | 1.35 |
| time (sec) | N/A | 0.077 | 0.066 | 6.727 | 1.190 | 1.165 | 0.000 | 0.201 | 4.649 | 0.108 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 95 | 671 | 97 | 125 | 0 | 98 | 111 | 132 |
| N.S. | 1 | 1.00 | 0.97 | 6.85 | 0.99 | 1.28 | 0.00 | 1.00 | 1.13 | 1.35 |
| time (sec) | N/A | 0.064 | 0.033 | 3.820 | 1.214 | 1.008 | 0.000 | 0.176 | 4.685 | 0.122 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 73 | 476 | 86 | 90 | 0 | 87 | 100 | 116 |
| N.S. | 1 | 1.00 | 0.89 | 5.80 | 1.05 | 1.10 | 0.00 | 1.06 | 1.22 | 1.41 |
| time (sec) | N/A | 0.059 | 0.025 | 5.594 | 1.300 | 0.575 | 0.000 | 0.181 | 4.894 | 0.092 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | C | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 137 | 137 | 133 | 0 | 0 | 410 | 0 | 149 | 256 | 195 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 2.99 | 0.00 | 1.09 | 1.87 | 1.42 |
| time (sec) | N/A | 0.091 | 0.045 | 2.438 | 0.000 | 1.482 | 0.000 | 0.197 | 4.798 | 0.201 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 157 | 157 | 153 | 0 | 0 | 187 | 0 | 163 | 382 | 215 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 1.19 | 0.00 | 1.04 | 2.43 | 1.37 |
| time (sec) | N/A | 0.103 | 0.218 | 2.822 | 0.000 | 0.476 | 0.000 | 0.188 | 4.860 | 0.266 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 154 | 226 | 144 | 0 | 0 | 201 | 0 | 0 | -1 | 230 |
| N.S. | 1 | 1.47 | 0.94 | 0.00 | 0.00 | 1.31 | 0.00 | 0.00 | -0.01 | 1.49 |
| time (sec) | N/A | 0.149 | 0.158 | 1.916 | 0.000 | 0.472 | 0.000 | 0.000 | 0.000 | 0.544 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | C | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 135 | 207 | 26 | 0 | 0 | 452 | 0 | 0 | -1 | 212 |
| N.S. | 1 | 1.53 | 0.19 | 0.00 | 0.00 | 3.35 | 0.00 | 0.00 | -0.01 | 1.57 |
| time (sec) | N/A | 0.105 | 0.020 | 1.737 | 0.000 | 1.430 | 0.000 | 0.000 | 0.000 | 0.370 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 88 | 122 | 112 | 931 | 0 | 253 | 0 | 0 | -1 | 126 |
| N.S. | 1 | 1.39 | 1.27 | 10.58 | 0.00 | 2.88 | 0.00 | 0.00 | -0.01 | 1.43 |
| time (sec) | N/A | 0.057 | 0.080 | 4.935 | 0.000 | 2.602 | 0.000 | 0.000 | 0.000 | 0.271 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 105 | 139 | 82 | 929 | 0 | 307 | 0 | 0 | -1 | 145 |
| N.S. | 1 | 1.32 | 0.78 | 8.85 | 0.00 | 2.92 | 0.00 | 0.00 | -0.01 | 1.38 |
| time (sec) | N/A | 0.072 | 0.597 | 4.490 | 0.000 | 2.578 | 0.000 | 0.000 | 0.000 | 0.290 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 124 | 140 | 123 | 955 | 0 | 283 | 0 | 0 | -1 | 149 |
| N.S. | 1 | 1.13 | 0.99 | 7.70 | 0.00 | 2.28 | 0.00 | 0.00 | -0.01 | 1.20 |
| time (sec) | N/A | 0.088 | 5.246 | 2.747 | 0.000 | 2.551 | 0.000 | 0.000 | 0.000 | 0.309 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 141 | 175 | 133 | 963 | 0 | 320 | 0 | 0 | -1 | 157 |
| N.S. | 1 | 1.24 | 0.94 | 6.83 | 0.00 | 2.27 | 0.00 | 0.00 | -0.01 | 1.11 |
| time (sec) | N/A | 0.092 | 5.115 | 4.067 | 0.000 | 2.607 | 0.000 | 0.000 | 0.000 | 0.305 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 233 | 26 | 26 | 0 | 0 | 373 | 0 | 0 | -1 | 340 |
| N.S. | 1 | 0.11 | 0.11 | 0.00 | 0.00 | 1.60 | 0.00 | 0.00 | -0.00 | 1.46 |
| time (sec) | N/A | 0.013 | 0.017 | 3.084 | 0.000 | 2.318 | 0.000 | 0.000 | 0.000 | 1.075 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 125 | 125 | 151 | 1579 | 119 | 142 | 0 | 127 | 135 | 141 |
| N.S. | 1 | 1.00 | 1.21 | 12.63 | 0.95 | 1.14 | 0.00 | 1.02 | 1.08 | 1.13 |
| time (sec) | N/A | 0.095 | 0.107 | 7.448 | 1.522 | 0.451 | 0.000 | 0.179 | 5.185 | 0.139 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 98 | 98 | 135 | 1584 | 97 | 114 | 0 | 98 | 113 | 132 |
| N.S. | 1 | 1.00 | 1.38 | 16.16 | 0.99 | 1.16 | 0.00 | 1.00 | 1.15 | 1.35 |
| time (sec) | N/A | 0.075 | 0.089 | 7.461 | 1.192 | 0.479 | 0.000 | 0.178 | 4.976 | 0.125 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 118 | 1582 | 97 | 130 | 0 | 98 | 113 | 129 |
| N.S. | 1 | 1.00 | 1.24 | 16.65 | 1.02 | 1.37 | 0.00 | 1.03 | 1.19 | 1.36 |
| time (sec) | N/A | 0.062 | 0.050 | 5.099 | 1.384 | 0.465 | 0.000 | 0.234 | 4.892 | 0.104 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 83 | 83 | 94 | 529 | 86 | 98 | 0 | 87 | 102 | 117 |
| N.S. | 1 | 1.00 | 1.13 | 6.37 | 1.04 | 1.18 | 0.00 | 1.05 | 1.23 | 1.41 |
| time (sec) | N/A | 0.056 | 0.035 | 6.280 | 1.176 | 0.492 | 0.000 | 0.197 | 5.052 | 0.098 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 137 | 137 | 179 | 0 | 0 | 182 | 0 | 149 | 344 | 195 |
| N.S. | 1 | 1.00 | 1.31 | 0.00 | 0.00 | 1.33 | 0.00 | 1.09 | 2.51 | 1.42 |
| time (sec) | N/A | 0.090 | 0.052 | 2.376 | 0.000 | 0.485 | 0.000 | 0.240 | 4.921 | 0.187 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 196 | 0 | 0 | 195 | 0 | 163 | 368 | 216 |
| N.S. | 1 | 1.00 | 1.24 | 0.00 | 0.00 | 1.23 | 0.00 | 1.03 | 2.33 | 1.37 |
| time (sec) | N/A | 0.103 | 0.100 | 3.079 | 0.000 | 0.486 | 0.000 | 0.215 | 5.073 | 0.253 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 160 | 228 | 78 | 0 | 0 | 232 | 0 | 0 | -1 | 232 |
| N.S. | 1 | 1.42 | 0.49 | 0.00 | 0.00 | 1.45 | 0.00 | 0.00 | -0.01 | 1.45 |
| time (sec) | N/A | 0.163 | 0.086 | 2.045 | 0.000 | 0.462 | 0.000 | 0.000 | 0.000 | 0.540 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 139 | 207 | 26 | 0 | 0 | 197 | 0 | 0 | -1 | 212 |
| N.S. | 1 | 1.49 | 0.19 | 0.00 | 0.00 | 1.42 | 0.00 | 0.00 | -0.01 | 1.53 |
| time (sec) | N/A | 0.103 | 0.051 | 1.759 | 0.000 | 0.459 | 0.000 | 0.000 | 0.000 | 0.383 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 88 | 122 | 38 | 938 | 0 | 283 | 0 | 0 | -1 | 126 |
| N.S. | 1 | 1.39 | 0.43 | 10.66 | 0.00 | 3.22 | 0.00 | 0.00 | -0.01 | 1.43 |
| time (sec) | N/A | 0.061 | 0.007 | 4.243 | 0.000 | 1.924 | 0.000 | 0.000 | 0.000 | 0.295 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 103 | 137 | 81 | 1387 | 0 | 272 | 0 | 0 | -1 | 143 |
| N.S. | 1 | 1.33 | 0.79 | 13.47 | 0.00 | 2.64 | 0.00 | 0.00 | -0.01 | 1.39 |
| time (sec) | N/A | 0.072 | 0.555 | 4.057 | 0.000 | 1.939 | 0.000 | 0.000 | 0.000 | 0.304 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 124 | 140 | 145 | 1386 | 0 | 312 | 0 | 0 | -1 | 149 |
| N.S. | 1 | 1.13 | 1.17 | 11.18 | 0.00 | 2.52 | 0.00 | 0.00 | -0.01 | 1.20 |
| time (sec) | N/A | 0.084 | 13.899 | 3.514 | 0.000 | 1.856 | 0.000 | 0.000 | 0.000 | 0.332 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | A | F | F | F | F |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 291 | 26 | 115 | 696 | 0 | 356 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 0.09 | 0.40 | 2.39 | 0.00 | 1.22 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.019 | 0.163 | 14.511 | 0.000 | 2.518 | 0.000 | 0.000 | 0.000 | 11.003 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | A | F | F | F | F |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 294 | 26 | 120 | 991 | 0 | 396 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 0.09 | 0.41 | 3.37 | 0.00 | 1.35 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.019 | 0.100 | 14.426 | 0.000 | 2.416 | 0.000 | 0.000 | 0.000 | 34.015 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 53 | 682 | 128 | 140 | 0 | 136 | 148 | 156 |
| N.S. | 1 | 1.00 | 0.38 | 4.84 | 0.91 | 0.99 | 0.00 | 0.96 | 1.05 | 1.11 |
| time (sec) | N/A | 0.114 | 0.037 | 3.784 | 1.066 | 0.468 | 0.000 | 0.195 | 4.861 | 0.178 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 48 | 497 | 119 | 159 | 0 | 120 | 139 | 151 |
| N.S. | 1 | 1.00 | 0.37 | 3.82 | 0.92 | 1.22 | 0.00 | 0.92 | 1.07 | 1.16 |
| time (sec) | N/A | 0.101 | 0.019 | 3.791 | 1.274 | 0.454 | 0.000 | 0.180 | 5.150 | 0.159 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 41 | 672 | 108 | 130 | 0 | 109 | 128 | 146 |
| N.S. | 1 | 1.00 | 0.36 | 5.84 | 0.94 | 1.13 | 0.00 | 0.95 | 1.11 | 1.27 |
| time (sec) | N/A | 0.090 | 0.014 | 3.782 | 1.271 | 0.472 | 0.000 | 0.175 | 4.890 | 0.147 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 95 | 667 | 97 | 148 | 0 | 98 | 117 | 141 |
| N.S. | 1 | 1.00 | 0.95 | 6.67 | 0.97 | 1.48 | 0.00 | 0.98 | 1.17 | 1.41 |
| time (sec) | N/A | 0.068 | 0.093 | 3.763 | 1.526 | 0.465 | 0.000 | 0.184 | 4.846 | 0.137 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 34 | 667 | 97 | 125 | 0 | 98 | 117 | 141 |
| N.S. | 1 | 1.00 | 0.34 | 6.67 | 0.97 | 1.25 | 0.00 | 0.98 | 1.17 | 1.41 |
| time (sec) | N/A | 0.068 | 0.010 | 3.867 | 1.338 | 0.467 | 0.000 | 0.193 | 4.903 | 0.122 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 154 | 154 | 54 | 0 | 0 | 226 | 0 | 160 | 253 | 219 |
| N.S. | 1 | 1.00 | 0.35 | 0.00 | 0.00 | 1.47 | 0.00 | 1.04 | 1.64 | 1.42 |
| time (sec) | N/A | 0.111 | 0.023 | 2.719 | 0.000 | 0.490 | 0.000 | 0.187 | 5.399 | 0.248 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 175 | 175 | 64 | 0 | 0 | 238 | 0 | 181 | 399 | 232 |
| N.S. | 1 | 1.00 | 0.37 | 0.00 | 0.00 | 1.36 | 0.00 | 1.03 | 2.28 | 1.33 |
| time (sec) | N/A | 0.116 | 0.032 | 2.293 | 0.000 | 0.465 | 0.000 | 0.215 | 5.024 | 0.305 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 174 | 26 | 152 | 0 | 0 | 271 | 0 | 0 | -1 | 248 |
| N.S. | 1 | 0.15 | 0.87 | 0.00 | 0.00 | 1.56 | 0.00 | 0.00 | -0.01 | 1.43 |
| time (sec) | N/A | 0.018 | 0.248 | 1.457 | 0.000 | 0.494 | 0.000 | 0.000 | 0.000 | 0.574 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 153 | 26 | 142 | 0 | 0 | 239 | 0 | 0 | -1 | 238 |
| N.S. | 1 | 0.17 | 0.93 | 0.00 | 0.00 | 1.56 | 0.00 | 0.00 | -0.01 | 1.56 |
| time (sec) | N/A | 0.018 | 0.164 | 1.685 | 0.000 | 0.456 | 0.000 | 0.000 | 0.000 | 0.456 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 106 | 38 | 38 | 627 | 0 | 318 | 0 | 0 | -1 | 152 |
| N.S. | 1 | 0.36 | 0.36 | 5.92 | 0.00 | 3.00 | 0.00 | 0.00 | -0.01 | 1.43 |
| time (sec) | N/A | 0.018 | 0.009 | 4.228 | 0.000 | 2.584 | 0.000 | 0.000 | 0.000 | 0.355 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 106 | 140 | 100 | 944 | 0 | 288 | 0 | 0 | -1 | 152 |
| N.S. | 1 | 1.32 | 0.94 | 8.91 | 0.00 | 2.72 | 0.00 | 0.00 | -0.01 | 1.43 |
| time (sec) | N/A | 0.076 | 0.048 | 3.009 | 0.000 | 2.599 | 0.000 | 0.000 | 0.000 | 0.343 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 124 | 204 | 192 | 570 | 0 | 340 | 0 | 0 | -1 | 161 |
| N.S. | 1 | 1.65 | 1.55 | 4.60 | 0.00 | 2.74 | 0.00 | 0.00 | -0.01 | 1.30 |
| time (sec) | N/A | 8.121 | 3.108 | 2.916 | 0.000 | 2.599 | 0.000 | 0.000 | 0.000 | 0.369 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | C | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 144 | 397 | 133 | 851 | 0 | 316 | 0 | 0 | -1 | 164 |
| N.S. | 1 | 2.76 | 0.92 | 5.91 | 0.00 | 2.19 | 0.00 | 0.00 | -0.01 | 1.14 |
| time (sec) | N/A | 8.318 | 5.158 | 4.018 | 0.000 | 2.708 | 0.000 | 0.000 | 0.000 | 0.386 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | C | F | B | F | F | F | A |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 162 | 643 | 136 | 649 | 0 | 351 | 0 | 0 | -1 | 169 |
| N.S. | 1 | 3.97 | 0.84 | 4.01 | 0.00 | 2.17 | 0.00 | 0.00 | -0.01 | 1.04 |
| time (sec) | N/A | 10.595 | 5.205 | 2.684 | 0.000 | 2.587 | 0.000 | 0.000 | 0.000 | 0.386 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 264 | 264 | 270 | 0 | 0 | 325 | 0 | 379 | 442 | 340 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 1.23 | 0.00 | 1.44 | 1.67 | 1.29 |
| time (sec) | N/A | 0.386 | 0.683 | 0.652 | 0.000 | 0.448 | 0.000 | 0.292 | 4.984 | 0.502 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 220 | 220 | 230 | 0 | 0 | 282 | 0 | 320 | 336 | 283 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 1.28 | 0.00 | 1.45 | 1.53 | 1.29 |
| time (sec) | N/A | 0.259 | 0.408 | 0.629 | 0.000 | 0.456 | 0.000 | 0.253 | 4.933 | 0.368 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 186 | 186 | 204 | 0 | 0 | 222 | 0 | 276 | 298 | 239 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 0.00 | 1.19 | 0.00 | 1.48 | 1.60 | 1.28 |
| time (sec) | N/A | 0.202 | 0.466 | 0.620 | 0.000 | 0.447 | 0.000 | 0.246 | 4.617 | 0.222 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 159 | 159 | 205 | 0 | 0 | 206 | 0 | 223 | 249 | 215 |
| N.S. | 1 | 1.00 | 1.29 | 0.00 | 0.00 | 1.30 | 0.00 | 1.40 | 1.57 | 1.35 |
| time (sec) | N/A | 0.155 | 0.287 | 0.607 | 0.000 | 0.438 | 0.000 | 0.271 | 4.621 | 0.235 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 246 | 246 | 268 | 0 | 0 | 276 | 0 | 311 | 1607 | 334 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 0.00 | 1.12 | 0.00 | 1.26 | 6.53 | 1.36 |
| time (sec) | N/A | 0.234 | 0.465 | 0.566 | 0.000 | 0.446 | 0.000 | 0.805 | 4.744 | 0.445 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 340 | 340 | 366 | 0 | 0 | 429 | 0 | 351 | 1917 | 383 |
| N.S. | 1 | 1.00 | 1.08 | 0.00 | 0.00 | 1.26 | 0.00 | 1.03 | 5.64 | 1.13 |
| time (sec) | N/A | 0.384 | 1.242 | 0.617 | 0.000 | 0.558 | 0.000 | 0.776 | 9.990 | 0.674 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 370 | 370 | 411 | 0 | 0 | 472 | 0 | 465 | 2767 | 448 |
| N.S. | 1 | 1.00 | 1.11 | 0.00 | 0.00 | 1.28 | 0.00 | 1.26 | 7.48 | 1.21 |
| time (sec) | N/A | 0.489 | 1.837 | 0.633 | 0.000 | 2.003 | 0.000 | 0.810 | 12.554 | 0.811 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 336 | 64 | 226 | 0 | 0 | 494 | 0 | 0 | -1 | 613 |
| N.S. | 1 | 0.19 | 0.67 | 0.00 | 0.00 | 1.47 | 0.00 | 0.00 | -0.00 | 1.82 |
| time (sec) | N/A | 0.054 | 0.373 | 0.628 | 0.000 | 1.949 | 0.000 | 0.000 | 0.000 | 5.500 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 276 | 64 | 185 | 0 | 0 | 452 | 0 | 0 | -1 | 549 |
| N.S. | 1 | 0.23 | 0.67 | 0.00 | 0.00 | 1.64 | 0.00 | 0.00 | -0.00 | 1.99 |
| time (sec) | N/A | 0.054 | 0.194 | 0.595 | 0.000 | 0.548 | 0.000 | 0.000 | 0.000 | 4.323 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 234 | 64 | 65 | 0 | 0 | 330 | 0 | 0 | -1 | 490 |
| N.S. | 1 | 0.27 | 0.28 | 0.00 | 0.00 | 1.41 | 0.00 | 0.00 | -0.00 | 2.09 |
| time (sec) | N/A | 0.038 | 0.035 | 0.579 | 0.000 | 0.484 | 0.000 | 0.000 | 0.000 | 3.787 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 168 | 87 | 81 | 0 | 0 | 0 | 0 | 0 | -1 | 363 |
| N.S. | 1 | 0.52 | 0.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.16 |
| time (sec) | N/A | 0.064 | 0.043 | 0.602 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.275 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 204 | 145 | 146 | 0 | 0 | 0 | 0 | 0 | -1 | 394 |
| N.S. | 1 | 0.71 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.93 |
| time (sec) | N/A | 0.133 | 0.776 | 0.621 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.297 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 258 | 451 | 451 | 0 | 0 | 0 | 0 | 0 | -1 | 444 |
| N.S. | 1 | 1.75 | 1.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.72 |
| time (sec) | N/A | 0.940 | 2.753 | 0.602 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.618 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 318 | 905 | 214 | 0 | 0 | 0 | 0 | 0 | -1 | 503 |
| N.S. | 1 | 2.85 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.58 |
| time (sec) | N/A | 2.642 | 5.237 | 0.634 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.987 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 266 | 266 | 148 | 0 | 0 | 455 | 0 | 409 | 490 | 340 |
| N.S. | 1 | 1.00 | 0.56 | 0.00 | 0.00 | 1.71 | 0.00 | 1.54 | 1.84 | 1.28 |
| time (sec) | N/A | 0.322 | 0.124 | 0.605 | 0.000 | 0.687 | 0.000 | 0.287 | 5.127 | 0.648 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 223 | 223 | 104 | 0 | 0 | 398 | 0 | 350 | 385 | 284 |
| N.S. | 1 | 1.00 | 0.47 | 0.00 | 0.00 | 1.78 | 0.00 | 1.57 | 1.73 | 1.27 |
| time (sec) | N/A | 0.259 | 0.098 | 0.714 | 0.000 | 0.607 | 0.000 | 0.267 | 5.105 | 0.398 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 188 | 188 | 68 | 0 | 0 | 353 | 0 | 306 | 302 | 241 |
| N.S. | 1 | 1.00 | 0.36 | 0.00 | 0.00 | 1.88 | 0.00 | 1.63 | 1.61 | 1.28 |
| time (sec) | N/A | 0.199 | 0.046 | 0.682 | 0.000 | 0.414 | 0.000 | 0.549 | 5.058 | 0.307 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 162 | 162 | 47 | 0 | 0 | 323 | 0 | 259 | 238 | 218 |
| N.S. | 1 | 1.00 | 0.29 | 0.00 | 0.00 | 1.99 | 0.00 | 1.60 | 1.47 | 1.35 |
| time (sec) | N/A | 0.170 | 0.026 | 0.578 | 0.000 | 0.436 | 0.000 | 0.271 | 5.049 | 0.250 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 245 | 245 | 115 | 0 | 0 | 425 | 0 | 341 | 1963 | 333 |
| N.S. | 1 | 1.00 | 0.47 | 0.00 | 0.00 | 1.73 | 0.00 | 1.39 | 8.01 | 1.36 |
| time (sec) | N/A | 0.229 | 0.121 | 0.622 | 0.000 | 0.461 | 0.000 | 0.802 | 4.770 | 0.470 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 347 | 347 | 202 | 0 | 0 | 1030 | 0 | 400 | 1908 | 384 |
| N.S. | 1 | 1.00 | 0.58 | 0.00 | 0.00 | 2.97 | 0.00 | 1.15 | 5.50 | 1.11 |
| time (sec) | N/A | 0.389 | 0.158 | 0.608 | 0.000 | 0.547 | 0.000 | 0.807 | 10.571 | 0.715 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 370 | 370 | 240 | 0 | 0 | 1151 | 0 | 493 | 2788 | 448 |
| N.S. | 1 | 1.00 | 0.65 | 0.00 | 0.00 | 3.11 | 0.00 | 1.33 | 7.54 | 1.21 |
| time (sec) | N/A | 0.496 | 0.377 | 0.669 | 0.000 | 1.616 | 0.000 | 0.782 | 15.187 | 0.878 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 334 | 64 | 525 | 0 | 0 | 1164 | 0 | 0 | -1 | 608 |
| N.S. | 1 | 0.19 | 1.57 | 0.00 | 0.00 | 3.49 | 0.00 | 0.00 | -0.00 | 1.82 |
| time (sec) | N/A | 0.056 | 0.757 | 0.584 | 0.000 | 1.600 | 0.000 | 0.000 | 0.000 | 6.446 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 272 | 64 | 286 | 0 | 0 | 1091 | 0 | 0 | -1 | 549 |
| N.S. | 1 | 0.24 | 1.05 | 0.00 | 0.00 | 4.01 | 0.00 | 0.00 | -0.00 | 2.02 |
| time (sec) | N/A | 0.059 | 0.594 | 0.620 | 0.000 | 0.568 | 0.000 | 0.000 | 0.000 | 4.957 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 233 | 59 | 161 | 0 | 0 | 469 | 0 | 0 | -1 | 487 |
| N.S. | 1 | 0.25 | 0.69 | 0.00 | 0.00 | 2.01 | 0.00 | 0.00 | -0.00 | 2.09 |
| time (sec) | N/A | 0.027 | 0.059 | 0.046 | 0.000 | 0.450 | 0.000 | 0.000 | 0.000 | 0.001 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 169 | 89 | 83 | 0 | 0 | 0 | 0 | 0 | -1 | 365 |
| N.S. | 1 | 0.53 | 0.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.16 |
| time (sec) | N/A | 0.063 | 0.045 | 0.609 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.384 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 206 | 148 | 148 | 0 | 0 | 0 | 0 | 0 | -1 | 392 |
| N.S. | 1 | 0.72 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.90 |
| time (sec) | N/A | 0.135 | 0.642 | 0.598 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.503 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 257 | 451 | 451 | 0 | 0 | 0 | 0 | 0 | -1 | 444 |
| N.S. | 1 | 1.75 | 1.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.73 |
| time (sec) | N/A | 0.971 | 2.196 | 0.598 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.911 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F(-1) | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 320 | 819 | 819 | 0 | 0 | 0 | 0 | 0 | -1 | 501 |
| N.S. | 1 | 2.56 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.57 |
| time (sec) | N/A | 2.626 | 4.995 | 0.608 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3.354 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 251 | 251 | 258 | 0 | 0 | 369 | 0 | 394 | 477 | 340 |
| N.S. | 1 | 1.00 | 1.03 | 0.00 | 0.00 | 1.47 | 0.00 | 1.57 | 1.90 | 1.35 |
| time (sec) | N/A | 0.363 | 0.538 | 0.622 | 0.000 | 0.563 | 0.000 | 0.303 | 5.123 | 0.459 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 211 | 211 | 255 | 0 | 0 | 298 | 0 | 348 | 348 | 278 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 1.41 | 0.00 | 1.65 | 1.65 | 1.32 |
| time (sec) | N/A | 0.245 | 0.337 | 0.745 | 0.000 | 0.430 | 0.000 | 0.291 | 5.057 | 0.271 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 187 | 187 | 232 | 0 | 0 | 246 | 0 | 297 | 304 | 234 |
| N.S. | 1 | 1.00 | 1.24 | 0.00 | 0.00 | 1.32 | 0.00 | 1.59 | 1.63 | 1.25 |
| time (sec) | N/A | 0.208 | 0.325 | 0.647 | 0.000 | 0.440 | 0.000 | 0.297 | 4.725 | 0.236 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 261 | 261 | 331 | 0 | 0 | 320 | 0 | 357 | 796 | 349 |
| N.S. | 1 | 1.00 | 1.27 | 0.00 | 0.00 | 1.23 | 0.00 | 1.37 | 3.05 | 1.34 |
| time (sec) | N/A | 0.304 | 0.669 | 0.779 | 0.000 | 0.450 | 0.000 | 0.840 | 6.080 | 0.655 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 399 | 399 | 389 | 0 | 0 | 383 | 0 | 394 | 2047 | 398 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.96 | 0.00 | 0.99 | 5.13 | 1.00 |
| time (sec) | N/A | 0.484 | 1.548 | 0.651 | 0.000 | 0.701 | 0.000 | 0.859 | 10.881 | 0.766 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 440 | 440 | 429 | 0 | 0 | 503 | 0 | 481 | 2841 | 445 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 1.14 | 0.00 | 1.09 | 6.46 | 1.01 |
| time (sec) | N/A | 0.623 | 1.254 | 0.678 | 0.000 | 2.020 | 0.000 | 0.859 | 13.254 | 1.074 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 334 | 65 | 225 | 0 | 0 | 550 | 0 | 0 | -1 | 611 |
| N.S. | 1 | 0.19 | 0.67 | 0.00 | 0.00 | 1.65 | 0.00 | 0.00 | -0.00 | 1.83 |
| time (sec) | N/A | 0.059 | 0.312 | 0.590 | 0.000 | 2.052 | 0.000 | 0.000 | 0.000 | 9.965 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | A | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 277 | 65 | 198 | 0 | 0 | 396 | 0 | 0 | -1 | 552 |
| N.S. | 1 | 0.23 | 0.71 | 0.00 | 0.00 | 1.43 | 0.00 | 0.00 | -0.00 | 1.99 |
| time (sec) | N/A | 0.044 | 0.316 | 0.598 | 0.000 | 0.708 | 0.000 | 0.000 | 0.000 | 7.067 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 254 | 63 | 161 | 0 | 0 | 0 | 0 | 0 | -1 | 509 |
| N.S. | 1 | 0.25 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 2.00 |
| time (sec) | N/A | 0.059 | 0.547 | 0.635 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 5.538 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 201 | 90 | 84 | 0 | 0 | 0 | 0 | 0 | -1 | 383 |
| N.S. | 1 | 0.45 | 0.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.91 |
| time (sec) | N/A | 0.064 | 0.040 | 0.626 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.902 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F(-1) | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 250 | 169 | 179 | 0 | 0 | 0 | 0 | 0 | -1 | 434 |
| N.S. | 1 | 0.68 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.74 |
| time (sec) | N/A | 0.507 | 0.603 | 0.644 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3.202 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F(-1) | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 318 | 260 | 270 | 0 | 0 | 0 | 0 | 0 | -1 | 501 |
| N.S. | 1 | 0.82 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.58 |
| time (sec) | N/A | 1.801 | 2.701 | 0.638 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3.689 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F(-1) | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 392 | 1446 | 1446 | 0 | 0 | 0 | 0 | 0 | -1 | 577 |
| N.S. | 1 | 3.69 | 3.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.47 |
| time (sec) | N/A | 4.391 | 5.442 | 0.621 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 4.753 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 290 | 290 | 157 | 0 | 0 | 1004 | 0 | 454 | 438 | 340 |
| N.S. | 1 | 1.00 | 0.54 | 0.00 | 0.00 | 3.46 | 0.00 | 1.57 | 1.51 | 1.17 |
| time (sec) | N/A | 0.317 | 0.258 | 0.601 | 0.000 | 0.760 | 0.000 | 0.326 | 5.115 | 0.625 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 244 | 244 | 145 | 0 | 0 | 873 | 0 | 371 | 339 | 284 |
| N.S. | 1 | 1.00 | 0.59 | 0.00 | 0.00 | 3.58 | 0.00 | 1.52 | 1.39 | 1.16 |
| time (sec) | N/A | 0.243 | 0.138 | 0.596 | 0.000 | 0.736 | 0.000 | 0.279 | 5.090 | 0.415 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 203 | 203 | 103 | 0 | 0 | 768 | 0 | 313 | 267 | 247 |
| N.S. | 1 | 1.00 | 0.51 | 0.00 | 0.00 | 3.78 | 0.00 | 1.54 | 1.32 | 1.22 |
| time (sec) | N/A | 0.214 | 0.095 | 0.592 | 0.000 | 0.786 | 0.000 | 0.261 | 5.113 | 0.401 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 168 | 168 | 69 | 0 | 0 | 667 | 0 | 257 | 219 | 224 |
| N.S. | 1 | 1.00 | 0.41 | 0.00 | 0.00 | 3.97 | 0.00 | 1.53 | 1.30 | 1.33 |
| time (sec) | N/A | 0.161 | 0.031 | 0.612 | 0.000 | 0.732 | 0.000 | 0.279 | 5.097 | 0.206 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 145 | 145 | 50 | 0 | 0 | 592 | 0 | 226 | 208 | 201 |
| N.S. | 1 | 1.00 | 0.34 | 0.00 | 0.00 | 4.08 | 0.00 | 1.56 | 1.43 | 1.39 |
| time (sec) | N/A | 0.125 | 0.019 | 0.605 | 0.000 | 0.817 | 0.000 | 0.287 | 4.934 | 0.166 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 244 | 244 | 140 | 0 | 0 | 628 | 0 | 326 | 702 | 332 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 2.57 | 0.00 | 1.34 | 2.88 | 1.36 |
| time (sec) | N/A | 0.212 | 0.178 | 0.613 | 0.000 | 0.447 | 0.000 | 0.744 | 6.436 | 0.491 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 296 | 296 | 156 | 0 | 0 | 837 | 0 | 383 | 1929 | 386 |
| N.S. | 1 | 1.00 | 0.53 | 0.00 | 0.00 | 2.83 | 0.00 | 1.29 | 6.52 | 1.30 |
| time (sec) | N/A | 0.314 | 0.612 | 0.593 | 0.000 | 0.624 | 0.000 | 0.885 | 11.355 | 0.713 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 273 | 394 | 288 | 0 | 0 | 826 | 0 | 0 | -1 | 525 |
| N.S. | 1 | 1.44 | 1.05 | 0.00 | 0.00 | 3.03 | 0.00 | 0.00 | -0.00 | 1.92 |
| time (sec) | N/A | 0.509 | 0.818 | 0.709 | 0.000 | 0.633 | 0.000 | 0.000 | 0.000 | 4.248 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 233 | 346 | 65 | 0 | 0 | 761 | 0 | 0 | -1 | 474 |
| N.S. | 1 | 1.48 | 0.28 | 0.00 | 0.00 | 3.27 | 0.00 | 0.00 | -0.00 | 2.03 |
| time (sec) | N/A | 0.291 | 0.041 | 0.668 | 0.000 | 0.474 | 0.000 | 0.000 | 0.000 | 3.131 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 148 | 207 | 168 | 0 | 0 | 0 | 0 | 0 | -1 | 320 |
| N.S. | 1 | 1.40 | 1.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.16 |
| time (sec) | N/A | 0.127 | 0.240 | 0.040 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 176 | 235 | 124 | 0 | 0 | 0 | 0 | 0 | -1 | 349 |
| N.S. | 1 | 1.34 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 1.98 |
| time (sec) | N/A | 0.226 | 0.066 | 0.621 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.955 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 214 | 271 | 207 | 0 | 0 | 0 | 0 | 0 | -1 | 379 |
| N.S. | 1 | 1.27 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.77 |
| time (sec) | N/A | 0.286 | 1.025 | 0.586 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.306 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 262 | 317 | 821 | 0 | 0 | 0 | 0 | 0 | -1 | 419 |
| N.S. | 1 | 1.21 | 3.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.60 |
| time (sec) | N/A | 0.325 | 2.546 | 0.585 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3.109 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 241 | 241 | 251 | 0 | 0 | 1322 | 0 | 372 | 331 | 284 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 5.49 | 0.00 | 1.54 | 1.37 | 1.18 |
| time (sec) | N/A | 0.263 | 0.664 | 0.612 | 0.000 | 0.833 | 0.000 | 0.263 | 5.012 | 0.496 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 201 | 201 | 211 | 0 | 0 | 1156 | 0 | 312 | 292 | 246 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 5.75 | 0.00 | 1.55 | 1.45 | 1.22 |
| time (sec) | N/A | 0.212 | 0.425 | 0.610 | 0.000 | 0.472 | 0.000 | 0.297 | 4.688 | 0.421 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 165 | 165 | 202 | 0 | 0 | 1060 | 0 | 253 | 232 | 221 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 6.42 | 0.00 | 1.53 | 1.41 | 1.34 |
| time (sec) | N/A | 0.155 | 0.180 | 0.643 | 0.000 | 0.607 | 0.000 | 0.232 | 4.684 | 0.215 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 145 | 145 | 164 | 0 | 0 | 927 | 0 | 221 | 213 | 201 |
| N.S. | 1 | 1.00 | 1.13 | 0.00 | 0.00 | 6.39 | 0.00 | 1.52 | 1.47 | 1.39 |
| time (sec) | N/A | 0.124 | 0.062 | 0.599 | 0.000 | 0.556 | 0.000 | 0.288 | 4.847 | 0.159 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 245 | 245 | 308 | 0 | 0 | 472 | 0 | 321 | 1413 | 333 |
| N.S. | 1 | 1.00 | 1.26 | 0.00 | 0.00 | 1.93 | 0.00 | 1.31 | 5.77 | 1.36 |
| time (sec) | N/A | 0.211 | 0.446 | 0.611 | 0.000 | 0.501 | 0.000 | 0.756 | 4.942 | 0.591 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 299 | 299 | 303 | 0 | 0 | 562 | 0 | 377 | 1959 | 388 |
| N.S. | 1 | 1.00 | 1.01 | 0.00 | 0.00 | 1.88 | 0.00 | 1.26 | 6.55 | 1.30 |
| time (sec) | N/A | 0.317 | 0.591 | 0.635 | 0.000 | 1.247 | 0.000 | 0.758 | 11.141 | 0.726 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 279 | 400 | 190 | 0 | 0 | 558 | 0 | 0 | -1 | 527 |
| N.S. | 1 | 1.43 | 0.68 | 0.00 | 0.00 | 2.00 | 0.00 | 0.00 | -0.00 | 1.89 |
| time (sec) | N/A | 0.502 | 0.291 | 0.639 | 0.000 | 1.284 | 0.000 | 0.000 | 0.000 | 4.567 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 234 | 346 | 65 | 0 | 0 | 530 | 0 | 0 | -1 | 475 |
| N.S. | 1 | 1.48 | 0.28 | 0.00 | 0.00 | 2.26 | 0.00 | 0.00 | -0.00 | 2.03 |
| time (sec) | N/A | 0.308 | 0.035 | 0.636 | 0.000 | 0.552 | 0.000 | 0.000 | 0.000 | 3.425 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 149 | 208 | 83 | 0 | 0 | 0 | 0 | 0 | -1 | 319 |
| N.S. | 1 | 1.40 | 0.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.14 |
| time (sec) | N/A | 0.140 | 0.044 | 0.584 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.963 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 173 | 232 | 128 | 0 | 0 | 0 | 0 | 0 | -1 | 347 |
| N.S. | 1 | 1.34 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.01 |
| time (sec) | N/A | 0.221 | 0.065 | 0.653 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.022 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 215 | 269 | 267 | 0 | 0 | 0 | 0 | 0 | -1 | 377 |
| N.S. | 1 | 1.25 | 1.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.75 |
| time (sec) | N/A | 0.297 | 1.888 | 0.612 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.411 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 347 | 347 | 157 | 0 | 0 | 1300 | 0 | 431 | 564 | 393 |
| N.S. | 1 | 1.00 | 0.45 | 0.00 | 0.00 | 3.75 | 0.00 | 1.24 | 1.63 | 1.13 |
| time (sec) | N/A | 0.441 | 0.265 | 0.458 | 0.000 | 0.839 | 0.000 | 0.257 | 5.189 | 0.712 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 253 | 253 | 147 | 0 | 0 | 1141 | 0 | 372 | 493 | 322 |
| N.S. | 1 | 1.00 | 0.58 | 0.00 | 0.00 | 4.51 | 0.00 | 1.47 | 1.95 | 1.27 |
| time (sec) | N/A | 0.340 | 0.111 | 0.464 | 0.000 | 0.726 | 0.000 | 0.259 | 5.160 | 0.504 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 203 | 203 | 101 | 0 | 0 | 1004 | 0 | 325 | 449 | 268 |
| N.S. | 1 | 1.00 | 0.50 | 0.00 | 0.00 | 4.95 | 0.00 | 1.60 | 2.21 | 1.32 |
| time (sec) | N/A | 0.242 | 0.068 | 0.428 | 0.000 | 0.705 | 0.000 | 0.248 | 4.986 | 0.462 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | B | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 174 | 174 | 77 | 0 | 0 | 872 | 0 | 301 | 412 | 230 |
| N.S. | 1 | 1.00 | 0.44 | 0.00 | 0.00 | 5.01 | 0.00 | 1.73 | 2.37 | 1.32 |
| time (sec) | N/A | 0.171 | 0.034 | 0.674 | 0.000 | 0.821 | 0.000 | 0.254 | 4.978 | 0.256 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-2) | A | F | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 167 | 167 | 50 | 0 | 0 | 262 | 0 | 285 | 389 | 223 |
| N.S. | 1 | 1.00 | 0.30 | 0.00 | 0.00 | 1.57 | 0.00 | 1.71 | 2.33 | 1.34 |
| time (sec) | N/A | 0.165 | 0.021 | 0.612 | 0.000 | 0.670 | 0.000 | 0.256 | 4.923 | 0.241 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 271 | 271 | 86 | 0 | 0 | 975 | 0 | 389 | 3804 | 360 |
| N.S. | 1 | 1.00 | 0.32 | 0.00 | 0.00 | 3.60 | 0.00 | 1.44 | 14.04 | 1.33 |
| time (sec) | N/A | 0.316 | 0.045 | 0.708 | 0.000 | 0.798 | 0.000 | 0.793 | 5.342 | 0.851 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 357 | 357 | 117 | 0 | 0 | 1386 | 0 | 486 | 5875 | 425 |
| N.S. | 1 | 1.00 | 0.33 | 0.00 | 0.00 | 3.88 | 0.00 | 1.36 | 16.46 | 1.19 |
| time (sec) | N/A | 0.394 | 0.054 | 0.477 | 0.000 | 1.386 | 0.000 | 0.788 | 6.390 | 0.993 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 322 | 67 | 504 | 0 | 0 | 1329 | 0 | 0 | -1 | 565 |
| N.S. | 1 | 0.21 | 1.57 | 0.00 | 0.00 | 4.13 | 0.00 | 0.00 | -0.00 | 1.75 |
| time (sec) | N/A | 0.060 | 1.079 | 0.447 | 0.000 | 1.365 | 0.000 | 0.000 | 0.000 | 9.271 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | B | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 260 | 67 | 309 | 0 | 0 | 1127 | 0 | 0 | -1 | 500 |
| N.S. | 1 | 0.26 | 1.19 | 0.00 | 0.00 | 4.33 | 0.00 | 0.00 | -0.00 | 1.92 |
| time (sec) | N/A | 0.059 | 0.446 | 0.656 | 0.000 | 0.484 | 0.000 | 0.000 | 0.000 | 5.717 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 172 | 92 | 86 | 0 | 0 | 0 | 0 | 0 | -1 | 355 |
| N.S. | 1 | 0.53 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 2.06 |
| time (sec) | N/A | 0.065 | 0.061 | 0.609 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 2.565 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 179 | 238 | 256 | 0 | 0 | 0 | 0 | 0 | -1 | 352 |
| N.S. | 1 | 1.33 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 1.97 |
| time (sec) | N/A | 0.196 | 0.118 | 0.046 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 229 | 542 | 542 | 0 | 0 | 0 | 0 | 0 | -1 | 397 |
| N.S. | 1 | 2.37 | 2.37 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.73 |
| time (sec) | N/A | 1.287 | 0.832 | 0.444 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 3.193 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 287 | 950 | 950 | 0 | 0 | 0 | 0 | 0 | -1 | 452 |
| N.S. | 1 | 3.31 | 3.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.57 |
| time (sec) | N/A | 3.857 | 2.312 | 0.452 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 4.823 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | F | F | F(-1) | F | F | F | C |
| verified | N/A | NO | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 351 | 1486 | 278 | 0 | 0 | 0 | 0 | 0 | -1 | 528 |
| N.S. | 1 | 4.23 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 | 1.50 |
| time (sec) | N/A | 8.295 | 5.856 | 0.460 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 11.241 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 92 | 89 | 84 | 100 | 0 | 88 | 88 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.99 | 0.93 | 1.11 | 0.00 | 0.98 | 0.98 | 0.00 |
| time (sec) | N/A | 0.096 | 0.066 | 0.059 | 0.467 | 6.244 | 0.000 | 0.174 | 5.880 | 0.001 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 66 | 65 | 68 | 72 | 0 | 70 | 68 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.93 | 0.97 | 1.03 | 0.00 | 1.00 | 0.97 | 0.00 |
| time (sec) | N/A | 0.064 | 0.049 | 0.053 | 0.570 | 1.683 | 0.000 | 0.202 | 5.633 | 0.001 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 43 | 50 | 49 | 42 | 144 | 51 | 51 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.94 | 0.92 | 0.79 | 2.72 | 0.96 | 0.96 | 0.00 |
| time (sec) | N/A | 0.050 | 0.026 | 0.052 | 0.621 | 0.746 | 60.853 | 0.188 | 5.096 | 0.001 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 31 | 42 | 41 | 31 | 138 | 51 | 1012 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.93 | 0.91 | 0.69 | 3.07 | 1.13 | 22.49 | 0.00 |
| time (sec) | N/A | 0.031 | 0.020 | 0.051 | 0.519 | 0.422 | 1.914 | 0.337 | 4.987 | 0.001 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 62 | 62 | 54 | 59 | 61 | 54 | 0 | 73 | 58 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.95 | 0.98 | 0.87 | 0.00 | 1.18 | 0.94 | 0.00 |
| time (sec) | N/A | 0.064 | 0.033 | 0.057 | 0.710 | 1.997 | 0.000 | 0.181 | 5.492 | 0.001 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 88 | 87 | 87 | 99 | 0 | 112 | 87 | 0 |
| N.S. | 1 | 1.00 | 1.01 | 1.00 | 1.00 | 1.14 | 0.00 | 1.29 | 1.00 | 0.00 |
| time (sec) | N/A | 0.090 | 0.045 | 0.059 | 0.554 | 8.265 | 0.000 | 0.188 | 6.214 | 0.001 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 104 | 105 | 100 | 576 | 0 | 112 | 532 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.94 | 0.89 | 5.14 | 0.00 | 1.00 | 4.75 | 0.00 |
| time (sec) | N/A | 0.273 | 0.170 | 0.061 | 1.325 | 2.635 | 0.000 | 0.190 | 5.705 | 0.001 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 82 | 81 | 80 | 416 | 932 | 80 | 518 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.88 | 0.87 | 4.52 | 10.13 | 0.87 | 5.63 | 0.00 |
| time (sec) | N/A | 0.118 | 0.141 | 0.056 | 1.189 | 0.793 | 29.695 | 0.175 | 5.724 | 0.001 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 66 | 60 | 59 | 325 | 0 | 59 | 379 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.76 | 0.75 | 4.11 | 0.00 | 0.75 | 4.80 | 0.00 |
| time (sec) | N/A | 0.062 | 0.041 | 0.056 | 1.165 | 0.494 | 0.000 | 0.175 | 5.345 | 0.001 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 66 | 60 | 59 | 325 | 0 | 59 | 399 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.76 | 0.75 | 4.11 | 0.00 | 0.75 | 5.05 | 0.00 |
| time (sec) | N/A | 0.048 | 0.050 | 0.058 | 1.277 | 0.500 | 0.000 | 0.180 | 5.295 | 0.001 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 169 | 81 | 80 | 432 | 1103 | 80 | 354 | 0 |
| N.S. | 1 | 1.00 | 1.84 | 0.88 | 0.87 | 4.70 | 11.99 | 0.87 | 3.85 | 0.00 |
| time (sec) | N/A | 0.122 | 0.230 | 0.056 | 1.202 | 0.882 | 84.889 | 0.207 | 5.348 | 0.001 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 193 | 105 | 101 | 592 | 0 | 103 | 535 | 0 |
| N.S. | 1 | 1.00 | 1.72 | 0.94 | 0.90 | 5.29 | 0.00 | 0.92 | 4.78 | 0.00 |
| time (sec) | N/A | 0.218 | 0.262 | 0.062 | 1.513 | 3.447 | 0.000 | 0.184 | 5.530 | 0.001 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 457 | 457 | 377 | 328 | 375 | 1378 | 0 | 469 | 6361 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.72 | 0.82 | 3.02 | 0.00 | 1.03 | 13.92 | 0.00 |
| time (sec) | N/A | 0.443 | 0.245 | 0.059 | 1.216 | 0.722 | 0.000 | 0.223 | 5.633 | 0.001 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 449 | 449 | 340 | 320 | 363 | 1358 | 0 | 453 | 2553 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.81 | 3.02 | 0.00 | 1.01 | 5.69 | 0.00 |
| time (sec) | N/A | 0.285 | 0.121 | 0.053 | 1.421 | 0.538 | 0.000 | 0.242 | 5.725 | 0.001 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 449 | 449 | 340 | 296 | 361 | 1238 | 0 | 437 | 5889 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.66 | 0.80 | 2.76 | 0.00 | 0.97 | 13.12 | 0.00 |
| time (sec) | N/A | 0.274 | 0.113 | 0.059 | 1.393 | 0.470 | 0.000 | 0.236 | 5.859 | 0.001 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 449 | 449 | 340 | 296 | 363 | 1258 | 0 | 477 | 6633 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.66 | 0.81 | 2.80 | 0.00 | 1.06 | 14.77 | 0.00 |
| time (sec) | N/A | 0.270 | 0.139 | 0.057 | 1.207 | 0.502 | 0.000 | 0.228 | 5.497 | 0.001 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 449 | 449 | 340 | 320 | 365 | 1356 | 0 | 437 | 6153 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 0.71 | 0.81 | 3.02 | 0.00 | 0.97 | 13.70 | 0.00 |
| time (sec) | N/A | 0.259 | 0.196 | 0.056 | 1.384 | 0.627 | 0.000 | 0.215 | 5.853 | 0.001 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 460 | 460 | 385 | 331 | 384 | 1395 | 0 | 488 | 5962 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.72 | 0.83 | 3.03 | 0.00 | 1.06 | 12.96 | 0.00 |
| time (sec) | N/A | 0.445 | 0.220 | 0.058 | 1.266 | 0.894 | 0.000 | 0.207 | 6.084 | 0.001 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 462 | 462 | 406 | 343 | 390 | 1415 | 0 | 472 | 7459 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.74 | 0.84 | 3.06 | 0.00 | 1.02 | 16.15 | 0.00 |
| time (sec) | N/A | 0.432 | 0.263 | 0.059 | 1.360 | 5.399 | 0.000 | 0.194 | 6.190 | 0.001 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 479 | 479 | 428 | 365 | 405 | 1456 | 0 | 483 | 4547 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.76 | 0.85 | 3.04 | 0.00 | 1.01 | 9.49 | 0.00 |
| time (sec) | N/A | 0.597 | 0.378 | 0.066 | 1.235 | 9.197 | 0.000 | 0.235 | 6.012 | 0.001 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 88 | 1015 | 0 | 195 | 90 | 96 | 87 | 98 |
| N.S. | 1 | 1.00 | 0.95 | 10.91 | 0.00 | 2.10 | 0.97 | 1.03 | 0.94 | 1.05 |
| time (sec) | N/A | 0.082 | 0.109 | 0.272 | 0.000 | 0.433 | 17.444 | 0.165 | 4.688 | 0.109 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 120 | 120 | 114 | 1066 | 0 | 714 | 0 | 0 | -1 | 173 |
| N.S. | 1 | 1.00 | 0.95 | 8.88 | 0.00 | 5.95 | 0.00 | 0.00 | -0.01 | 1.44 |
| time (sec) | N/A | 0.154 | 0.192 | 0.278 | 0.000 | 0.557 | 0.000 | 0.000 | 0.000 | 0.474 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 70 | 70 | 69 | 988 | 0 | 156 | 65 | 66 | 54 | 80 |
| N.S. | 1 | 1.00 | 0.99 | 14.11 | 0.00 | 2.23 | 0.93 | 0.94 | 0.77 | 1.14 |
| time (sec) | N/A | 0.061 | 0.051 | 0.204 | 0.000 | 0.446 | 8.185 | 0.183 | 4.660 | 0.090 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 89 | 1000 | 0 | 612 | 0 | 0 | -1 | 144 |
| N.S. | 1 | 1.00 | 0.98 | 10.99 | 0.00 | 6.73 | 0.00 | 0.00 | -0.01 | 1.58 |
| time (sec) | N/A | 0.076 | 0.065 | 0.196 | 0.000 | 0.494 | 0.000 | 0.000 | 0.000 | 0.348 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 81 | 1037 | 0 | 383 | 82 | 79 | 199 | 95 |
| N.S. | 1 | 1.00 | 0.95 | 12.20 | 0.00 | 4.51 | 0.96 | 0.93 | 2.34 | 1.12 |
| time (sec) | N/A | 0.075 | 0.094 | 0.262 | 0.000 | 0.464 | 12.052 | 0.179 | 4.869 | 0.124 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 53 | 1075 | 0 | 281 | 0 | 121 | -1 | 128 |
| N.S. | 1 | 1.00 | 0.70 | 14.14 | 0.00 | 3.70 | 0.00 | 1.59 | -0.01 | 1.68 |
| time (sec) | N/A | 0.086 | 0.064 | 0.259 | 0.000 | 0.455 | 0.000 | 1.285 | 0.000 | 0.388 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 107 | 1107 | 0 | 513 | 0 | 107 | 269 | 125 |
| N.S. | 1 | 1.00 | 0.93 | 9.63 | 0.00 | 4.46 | 0.00 | 0.93 | 2.34 | 1.09 |
| time (sec) | N/A | 0.124 | 0.134 | 0.270 | 0.000 | 0.471 | 0.000 | 0.169 | 5.387 | 0.265 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 110 | 110 | 130 | 1116 | 0 | 329 | 0 | 225 | -1 | 151 |
| N.S. | 1 | 1.00 | 1.18 | 10.15 | 0.00 | 2.99 | 0.00 | 2.05 | -0.01 | 1.37 |
| time (sec) | N/A | 0.160 | 5.330 | 0.263 | 0.000 | 0.521 | 0.000 | 1.749 | 0.000 | 0.535 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 91 | 378 | 0 | 289 | 0 | 106 | 102 | 101 |
| N.S. | 1 | 1.00 | 0.88 | 3.63 | 0.00 | 2.78 | 0.00 | 1.02 | 0.98 | 0.97 |
| time (sec) | N/A | 0.112 | 0.257 | 0.283 | 0.000 | 0.717 | 0.000 | 0.162 | 4.825 | 0.177 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 72 | 335 | 0 | 205 | 0 | 64 | 58 | 84 |
| N.S. | 1 | 1.00 | 0.97 | 4.53 | 0.00 | 2.77 | 0.00 | 0.86 | 0.78 | 1.14 |
| time (sec) | N/A | 0.065 | 0.079 | 0.209 | 0.000 | 0.740 | 0.000 | 0.194 | 4.729 | 0.098 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 51 | 316 | 0 | 130 | 37 | 40 | 40 | 61 |
| N.S. | 1 | 1.00 | 1.00 | 6.20 | 0.00 | 2.55 | 0.73 | 0.78 | 0.78 | 1.20 |
| time (sec) | N/A | 0.049 | 0.018 | 0.213 | 0.000 | 0.694 | 14.270 | 0.156 | 4.797 | 0.052 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 81 | 347 | 0 | 431 | 66 | 71 | 652 | 95 |
| N.S. | 1 | 1.00 | 0.95 | 4.08 | 0.00 | 5.07 | 0.78 | 0.84 | 7.67 | 1.12 |
| time (sec) | N/A | 0.073 | 0.086 | 0.287 | 0.000 | 0.727 | 20.207 | 0.170 | 5.028 | 0.111 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 151 | 402 | 0 | 565 | 0 | 104 | 396 | 127 |
| N.S. | 1 | 1.00 | 1.29 | 3.44 | 0.00 | 4.83 | 0.00 | 0.89 | 3.38 | 1.09 |
| time (sec) | N/A | 0.120 | 0.170 | 0.292 | 0.000 | 0.702 | 0.000 | 0.167 | 5.351 | 0.252 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 118 | 408 | 0 | 739 | 0 | 0 | -1 | 177 |
| N.S. | 1 | 1.00 | 0.96 | 3.32 | 0.00 | 6.01 | 0.00 | 0.00 | -0.01 | 1.44 |
| time (sec) | N/A | 0.149 | 0.210 | 0.316 | 0.000 | 0.752 | 0.000 | 0.000 | 0.000 | 0.699 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 90 | 356 | 0 | 632 | 0 | 0 | -1 | 144 |
| N.S. | 1 | 1.00 | 0.99 | 3.91 | 0.00 | 6.95 | 0.00 | 0.00 | -0.01 | 1.58 |
| time (sec) | N/A | 0.087 | 0.069 | 0.333 | 0.000 | 0.674 | 0.000 | 0.000 | 0.000 | 0.455 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 95 | 322 | 0 | 245 | 0 | 72 | -1 | 106 |
| N.S. | 1 | 1.00 | 1.76 | 5.96 | 0.00 | 4.54 | 0.00 | 1.33 | -0.02 | 1.96 |
| time (sec) | N/A | 0.044 | 0.063 | 0.324 | 0.000 | 0.505 | 0.000 | 0.179 | 0.000 | 0.316 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | A | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 80 | 80 | 179 | 350 | 0 | 332 | 0 | 116 | -1 | 142 |
| N.S. | 1 | 1.00 | 2.24 | 4.38 | 0.00 | 4.15 | 0.00 | 1.45 | -0.01 | 1.78 |
| time (sec) | N/A | 0.088 | 4.830 | 0.297 | 0.000 | 0.530 | 0.000 | 0.219 | 0.000 | 0.481 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 115 | 115 | 137 | 383 | 0 | 418 | 0 | 205 | -1 | 163 |
| N.S. | 1 | 1.00 | 1.19 | 3.33 | 0.00 | 3.63 | 0.00 | 1.78 | -0.01 | 1.42 |
| time (sec) | N/A | 0.164 | 5.549 | 0.270 | 0.000 | 0.544 | 0.000 | 1.675 | 0.000 | 0.888 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 175 | 175 | 175 | 923 | 0 | 622 | 0 | 180 | 186 | 198 |
| N.S. | 1 | 1.00 | 1.00 | 5.27 | 0.00 | 3.55 | 0.00 | 1.03 | 1.06 | 1.13 |
| time (sec) | N/A | 0.173 | 0.306 | 0.247 | 0.000 | 0.551 | 0.000 | 0.175 | 5.191 | 0.486 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 107 | 876 | 0 | 475 | 0 | 134 | 144 | 143 |
| N.S. | 1 | 1.00 | 0.87 | 7.12 | 0.00 | 3.86 | 0.00 | 1.09 | 1.17 | 1.16 |
| time (sec) | N/A | 0.146 | 0.355 | 0.190 | 0.000 | 0.616 | 0.000 | 0.168 | 5.117 | 0.329 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 98 | 851 | 0 | 348 | 0 | 116 | 95 | 109 |
| N.S. | 1 | 1.00 | 0.99 | 8.60 | 0.00 | 3.52 | 0.00 | 1.17 | 0.96 | 1.10 |
| time (sec) | N/A | 0.087 | 0.113 | 0.197 | 0.000 | 0.708 | 0.000 | 0.170 | 4.932 | 0.167 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 85 | 541 | 0 | 302 | 0 | 93 | 84 | 97 |
| N.S. | 1 | 1.00 | 0.98 | 6.22 | 0.00 | 3.47 | 0.00 | 1.07 | 0.97 | 1.11 |
| time (sec) | N/A | 0.074 | 0.102 | 0.206 | 0.000 | 0.721 | 0.000 | 0.164 | 4.852 | 0.108 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 123 | 880 | 0 | 862 | 0 | 139 | 3017 | 146 |
| N.S. | 1 | 1.00 | 0.93 | 6.67 | 0.00 | 6.53 | 0.00 | 1.05 | 22.86 | 1.11 |
| time (sec) | N/A | 0.142 | 0.334 | 0.272 | 0.000 | 0.682 | 0.000 | 0.168 | 5.869 | 0.321 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 163 | 938 | 0 | 1236 | 0 | 257 | 3822 | 187 |
| N.S. | 1 | 1.00 | 0.88 | 5.07 | 0.00 | 6.68 | 0.00 | 1.39 | 20.66 | 1.01 |
| time (sec) | N/A | 0.251 | 0.695 | 0.277 | 0.000 | 0.915 | 0.000 | 0.169 | 7.047 | 0.656 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 191 | 191 | 150 | 953 | 0 | 1386 | 0 | 337 | -1 | 208 |
| N.S. | 1 | 1.00 | 0.79 | 4.99 | 0.00 | 7.26 | 0.00 | 1.76 | -0.01 | 1.09 |
| time (sec) | N/A | 0.365 | 0.566 | 0.273 | 0.000 | 1.938 | 0.000 | 0.590 | 0.000 | 2.288 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 135 | 893 | 0 | 1077 | 0 | 298 | -1 | 166 |
| N.S. | 1 | 1.00 | 0.96 | 6.33 | 0.00 | 7.64 | 0.00 | 2.11 | -0.01 | 1.18 |
| time (sec) | N/A | 0.162 | 0.224 | 0.238 | 0.000 | 0.838 | 0.000 | 0.545 | 0.000 | 1.594 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 124 | 861 | 0 | 426 | 0 | 244 | -1 | 145 |
| N.S. | 1 | 1.00 | 1.33 | 9.26 | 0.00 | 4.58 | 0.00 | 2.62 | -0.01 | 1.56 |
| time (sec) | N/A | 0.092 | 0.583 | 0.240 | 0.000 | 0.537 | 0.000 | 1.630 | 0.000 | 1.043 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 104 | 104 | 407 | 867 | 0 | 467 | 0 | 237 | -1 | 124 |
| N.S. | 1 | 1.00 | 3.91 | 8.34 | 0.00 | 4.49 | 0.00 | 2.28 | -0.01 | 1.19 |
| time (sec) | N/A | 0.094 | 6.013 | 0.194 | 0.000 | 0.599 | 0.000 | 0.376 | 0.000 | 0.643 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 149 | 149 | 155 | 885 | 0 | 612 | 0 | 418 | -1 | 159 |
| N.S. | 1 | 1.00 | 1.04 | 5.94 | 0.00 | 4.11 | 0.00 | 2.81 | -0.01 | 1.07 |
| time (sec) | N/A | 0.199 | 5.633 | 0.269 | 0.000 | 0.617 | 0.000 | 1.681 | 0.000 | 1.373 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 208 | 208 | 175 | 923 | 0 | 760 | 0 | 395 | -1 | 217 |
| N.S. | 1 | 1.00 | 0.84 | 4.44 | 0.00 | 3.65 | 0.00 | 1.90 | -0.00 | 1.04 |
| time (sec) | N/A | 0.328 | 5.871 | 0.246 | 0.000 | 0.776 | 0.000 | 2.038 | 0.000 | 2.282 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 91 | 0 | 0 | 288 | 0 | 106 | 103 | 101 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 2.77 | 0.00 | 1.02 | 0.99 | 0.97 |
| time (sec) | N/A | 0.107 | 0.277 | 0.634 | 0.000 | 0.433 | 0.000 | 0.170 | 4.866 | 0.183 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 72 | 0 | 0 | 205 | 0 | 64 | 58 | 84 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 2.77 | 0.00 | 0.86 | 0.78 | 1.14 |
| time (sec) | N/A | 0.063 | 0.130 | 0.601 | 0.000 | 0.418 | 0.000 | 0.192 | 4.745 | 0.098 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 51 | 0 | 0 | 130 | 37 | 40 | 40 | 61 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 2.55 | 0.73 | 0.78 | 0.78 | 1.20 |
| time (sec) | N/A | 0.046 | 0.020 | 0.572 | 0.000 | 0.412 | 23.762 | 0.167 | 4.725 | 0.054 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 81 | 0 | 0 | 431 | 66 | 71 | 652 | 95 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 5.07 | 0.78 | 0.84 | 7.67 | 1.12 |
| time (sec) | N/A | 0.074 | 0.126 | 0.631 | 0.000 | 0.450 | 26.110 | 0.176 | 5.191 | 0.117 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 151 | 0 | 0 | 565 | 0 | 104 | 396 | 127 |
| N.S. | 1 | 1.00 | 1.29 | 0.00 | 0.00 | 4.83 | 0.00 | 0.89 | 3.38 | 1.09 |
| time (sec) | N/A | 0.119 | 0.181 | 0.725 | 0.000 | 0.467 | 0.000 | 0.177 | 5.615 | 0.265 |

| Problem 548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 118 | 0 | 0 | 739 | 0 | 0 | -1 | 177 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 6.01 | 0.00 | 0.00 | -0.01 | 1.44 |
| time (sec) | N/A | 0.153 | 0.236 | 0.921 | 0.000 | 0.546 | 0.000 | 0.000 | 0.000 | 1.107 |

| Problem 549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 90 | 0 | 0 | 632 | 0 | 156 | -1 | 144 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 6.95 | 0.00 | 1.71 | -0.01 | 1.58 |
| time (sec) | N/A | 0.087 | 0.091 | 0.610 | 0.000 | 0.525 | 0.000 | 0.234 | 0.000 | 0.622 |

| Problem 550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 95 | 0 | 0 | 245 | 0 | 72 | -1 | 106 |
| N.S. | 1 | 1.00 | 1.76 | 0.00 | 0.00 | 4.54 | 0.00 | 1.33 | -0.02 | 1.96 |
| time (sec) | N/A | 0.049 | 0.090 | 0.605 | 0.000 | 0.495 | 0.000 | 0.247 | 0.000 | 0.399 |

| Problem 551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 80 | 80 | 179 | 0 | 0 | 332 | 0 | 0 | -1 | 142 |
| N.S. | 1 | 1.00 | 2.24 | 0.00 | 0.00 | 4.15 | 0.00 | 0.00 | -0.01 | 1.78 |
| time (sec) | N/A | 0.089 | 1.169 | 0.644 | 0.000 | 0.511 | 0.000 | 0.000 | 0.000 | 0.657 |

| Problem 552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 115 | 115 | 253 | 0 | 0 | 416 | 0 | 0 | -1 | 163 |
| N.S. | 1 | 1.00 | 2.20 | 0.00 | 0.00 | 3.62 | 0.00 | 0.00 | -0.01 | 1.42 |
| time (sec) | N/A | 0.163 | 4.650 | 0.658 | 0.000 | 0.530 | 0.000 | 0.000 | 0.000 | 1.296 |

| Problem 553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 107 | 0 | 0 | 475 | 0 | 134 | 144 | 143 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 3.86 | 0.00 | 1.09 | 1.17 | 1.16 |
| time (sec) | N/A | 0.140 | 0.357 | 0.454 | 0.000 | 0.646 | 0.000 | 0.441 | 5.093 | 0.327 |

| Problem 554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 98 | 0 | 0 | 348 | 0 | 116 | 95 | 109 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 3.52 | 0.00 | 1.17 | 0.96 | 1.10 |
| time (sec) | N/A | 0.080 | 0.096 | 0.632 | 0.000 | 0.688 | 0.000 | 0.289 | 4.989 | 0.172 |

| Problem 555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 85 | 0 | 0 | 302 | 0 | 93 | 84 | 97 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 3.47 | 0.00 | 1.07 | 0.97 | 1.11 |
| time (sec) | N/A | 0.069 | 0.101 | 0.619 | 0.000 | 0.754 | 0.000 | 0.302 | 4.929 | 0.110 |

| Problem 556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 123 | 0 | 0 | 862 | 0 | 139 | 3025 | 146 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 6.53 | 0.00 | 1.05 | 22.92 | 1.11 |
| time (sec) | N/A | 0.127 | 0.354 | 0.775 | 0.000 | 0.925 | 0.000 | 0.407 | 6.226 | 0.319 |

| Problem 557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 163 | 0 | 0 | 1236 | 0 | 257 | 3860 | 187 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 6.68 | 0.00 | 1.39 | 20.86 | 1.01 |
| time (sec) | N/A | 0.228 | 0.683 | 0.559 | 0.000 | 0.919 | 0.000 | 0.410 | 7.290 | 0.656 |

| Problem 558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 135 | 0 | 0 | 1077 | 0 | 343 | -1 | 166 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 7.64 | 0.00 | 2.43 | -0.01 | 1.18 |
| time (sec) | N/A | 0.158 | 0.276 | 0.627 | 0.000 | 1.408 | 0.000 | 0.511 | 0.000 | 2.625 |

| Problem 559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 124 | 0 | 0 | 426 | 0 | 0 | -1 | 145 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 4.58 | 0.00 | 0.00 | -0.01 | 1.56 |
| time (sec) | N/A | 0.090 | 0.581 | 0.656 | 0.000 | 0.790 | 0.000 | 0.000 | 0.000 | 1.328 |

| Problem 560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 104 | 104 | 407 | 0 | 0 | 467 | 0 | 237 | -1 | 124 |
| N.S. | 1 | 1.00 | 3.91 | 0.00 | 0.00 | 4.49 | 0.00 | 2.28 | -0.01 | 1.19 |
| time (sec) | N/A | 0.098 | 0.998 | 0.605 | 0.000 | 0.592 | 0.000 | 0.284 | 0.000 | 0.805 |

| Problem 561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 149 | 149 | 869 | 0 | 0 | 612 | 0 | 0 | -1 | 159 |
| N.S. | 1 | 1.00 | 5.83 | 0.00 | 0.00 | 4.11 | 0.00 | 0.00 | -0.01 | 1.07 |
| time (sec) | N/A | 0.194 | 5.401 | 0.483 | 0.000 | 0.608 | 0.000 | 0.000 | 0.000 | 1.634 |

| Problem 562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F(-1) | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 208 | 208 | 175 | 0 | 0 | 760 | 0 | 0 | -1 | 217 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 3.65 | 0.00 | 0.00 | -0.00 | 1.04 |
| time (sec) | N/A | 0.308 | 5.858 | 0.448 | 0.000 | 0.775 | 0.000 | 0.000 | 0.000 | 2.883 |

| Problem 563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 91 | 0 | 0 | 288 | 0 | 106 | 103 | 101 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 2.77 | 0.00 | 1.02 | 0.99 | 0.97 |
| time (sec) | N/A | 0.099 | 0.216 | 0.593 | 0.000 | 0.428 | 0.000 | 0.160 | 4.678 | 0.159 |

| Problem 564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 72 | 0 | 0 | 205 | 0 | 64 | 58 | 84 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 2.77 | 0.00 | 0.86 | 0.78 | 1.14 |
| time (sec) | N/A | 0.061 | 0.078 | 0.642 | 0.000 | 0.459 | 0.000 | 0.157 | 4.730 | 0.096 |

| Problem 565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 51 | 0 | 0 | 130 | 37 | 40 | 40 | 61 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 2.55 | 0.73 | 0.78 | 0.78 | 1.20 |
| time (sec) | N/A | 0.047 | 0.017 | 0.625 | 0.000 | 0.421 | 44.568 | 0.166 | 4.586 | 0.053 |

| Problem 566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 81 | 0 | 0 | 431 | 66 | 71 | 652 | 95 |
| N.S. | 1 | 1.00 | 0.95 | 0.00 | 0.00 | 5.07 | 0.78 | 0.84 | 7.67 | 1.12 |
| time (sec) | N/A | 0.079 | 0.099 | 0.683 | 0.000 | 0.478 | 37.858 | 0.159 | 4.813 | 0.115 |

| Problem 567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 151 | 0 | 0 | 565 | 0 | 104 | 396 | 127 |
| N.S. | 1 | 1.00 | 1.29 | 0.00 | 0.00 | 4.83 | 0.00 | 0.89 | 3.38 | 1.09 |
| time (sec) | N/A | 0.113 | 0.139 | 0.637 | 0.000 | 0.469 | 0.000 | 0.176 | 5.507 | 0.262 |

| Problem 568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 118 | 0 | 0 | 739 | 0 | 0 | -1 | 177 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 6.01 | 0.00 | 0.00 | -0.01 | 1.44 |
| time (sec) | N/A | 0.131 | 0.240 | 0.581 | 0.000 | 0.558 | 0.000 | 0.000 | 0.000 | 1.902 |

| Problem 569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 90 | 0 | 0 | 632 | 0 | 0 | -1 | 144 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 6.95 | 0.00 | 0.00 | -0.01 | 1.58 |
| time (sec) | N/A | 0.110 | 0.068 | 0.596 | 0.000 | 0.522 | 0.000 | 0.000 | 0.000 | 0.944 |

| Problem 570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 95 | 0 | 0 | 245 | 0 | 72 | -1 | 106 |
| N.S. | 1 | 1.00 | 1.76 | 0.00 | 0.00 | 4.54 | 0.00 | 1.33 | -0.02 | 1.96 |
| time (sec) | N/A | 0.055 | 0.068 | 0.616 | 0.000 | 0.486 | 0.000 | 0.203 | 0.000 | 0.490 |

| Problem 571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | A | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 80 | 80 | 179 | 0 | 0 | 332 | 0 | 116 | -1 | 142 |
| N.S. | 1 | 1.00 | 2.24 | 0.00 | 0.00 | 4.15 | 0.00 | 1.45 | -0.01 | 1.78 |
| time (sec) | N/A | 0.087 | 0.832 | 0.664 | 0.000 | 0.503 | 0.000 | 0.229 | 0.000 | 0.799 |

| Problem 572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | A | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 115 | 115 | 253 | 0 | 0 | 416 | 0 | 205 | -1 | 163 |
| N.S. | 1 | 1.00 | 2.20 | 0.00 | 0.00 | 3.62 | 0.00 | 1.78 | -0.01 | 1.42 |
| time (sec) | N/A | 0.161 | 2.273 | 0.662 | 0.000 | 0.524 | 0.000 | 1.686 | 0.000 | 2.183 |

| Problem 573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 107 | 0 | 0 | 475 | 0 | 134 | 144 | 143 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 3.86 | 0.00 | 1.09 | 1.17 | 1.16 |
| time (sec) | N/A | 0.142 | 0.257 | 0.469 | 0.000 | 0.474 | 0.000 | 0.177 | 5.021 | 0.326 |

| Problem 574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 99 | 99 | 98 | 0 | 0 | 348 | 0 | 116 | 95 | 109 |
| N.S. | 1 | 1.00 | 0.99 | 0.00 | 0.00 | 3.52 | 0.00 | 1.17 | 0.96 | 1.10 |
| time (sec) | N/A | 0.083 | 0.092 | 0.594 | 0.000 | 0.434 | 0.000 | 0.169 | 4.837 | 0.169 |

| Problem 575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F(-2) | B | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 87 | 87 | 85 | 0 | 0 | 302 | 0 | 93 | 84 | 97 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 3.47 | 0.00 | 1.07 | 0.97 | 1.11 |
| time (sec) | N/A | 0.073 | 0.096 | 0.603 | 0.000 | 0.438 | 0.000 | 0.160 | 4.797 | 0.107 |

| Problem 576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 132 | 132 | 123 | 0 | 0 | 862 | 0 | 139 | 3017 | 146 |
| N.S. | 1 | 1.00 | 0.93 | 0.00 | 0.00 | 6.53 | 0.00 | 1.05 | 22.86 | 1.11 |
| time (sec) | N/A | 0.137 | 0.355 | 0.799 | 0.000 | 0.490 | 0.000 | 0.209 | 5.820 | 0.304 |

| Problem 577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 185 | 185 | 163 | 0 | 0 | 1236 | 0 | 257 | 3832 | 187 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 6.68 | 0.00 | 1.39 | 20.71 | 1.01 |
| time (sec) | N/A | 0.229 | 0.602 | 0.658 | 0.000 | 0.556 | 0.000 | 0.172 | 7.851 | 0.647 |

| Problem 578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 141 | 141 | 135 | 0 | 0 | 1077 | 0 | 298 | -1 | 166 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 7.64 | 0.00 | 2.11 | -0.01 | 1.18 |
| time (sec) | N/A | 0.160 | 0.264 | 0.647 | 0.000 | 0.819 | 0.000 | 0.652 | 0.000 | 3.667 |

| Problem 579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-1) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 124 | 0 | 0 | 426 | 0 | 244 | -1 | 145 |
| N.S. | 1 | 1.00 | 1.33 | 0.00 | 0.00 | 4.58 | 0.00 | 2.62 | -0.01 | 1.56 |
| time (sec) | N/A | 0.086 | 0.549 | 0.642 | 0.000 | 0.539 | 0.000 | 1.595 | 0.000 | 1.815 |

| Problem 580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 104 | 104 | 407 | 0 | 0 | 467 | 0 | 237 | -1 | 124 |
| N.S. | 1 | 1.00 | 3.91 | 0.00 | 0.00 | 4.49 | 0.00 | 2.28 | -0.01 | 1.19 |
| time (sec) | N/A | 0.090 | 0.941 | 0.601 | 0.000 | 0.577 | 0.000 | 0.389 | 0.000 | 0.985 |

| Problem 581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F(-1) | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 149 | 149 | 869 | 0 | 0 | 612 | 0 | 418 | -1 | 159 |
| N.S. | 1 | 1.00 | 5.83 | 0.00 | 0.00 | 4.11 | 0.00 | 2.81 | -0.01 | 1.07 |
| time (sec) | N/A | 0.185 | 2.034 | 0.439 | 0.000 | 0.610 | 0.000 | 1.764 | 0.000 | 1.972 |

| Problem 582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | B | F | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 208 | 208 | 175 | 0 | 0 | 760 | 0 | 395 | -1 | 217 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 3.65 | 0.00 | 1.90 | -0.00 | 1.04 |
| time (sec) | N/A | 0.297 | 5.847 | 0.455 | 0.000 | 0.763 | 0.000 | 2.277 | 0.000 | 4.942 |

| Problem 583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 121 | 162 | 243 | 242 | 226 | 143 | 134 | 111 |
| N.S. | 1 | 1.00 | 0.98 | 1.32 | 1.98 | 1.97 | 1.84 | 1.16 | 1.09 | 0.90 |
| time (sec) | N/A | 0.094 | 0.257 | 0.057 | 1.361 | 0.437 | 72.291 | 0.193 | 5.713 | 0.153 |

| Problem 584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 100 | 122 | 159 | 191 | 144 | 105 | 93 | 87 |
| N.S. | 1 | 1.00 | 1.11 | 1.36 | 1.77 | 2.12 | 1.60 | 1.17 | 1.03 | 0.97 |
| time (sec) | N/A | 0.068 | 0.136 | 0.055 | 1.240 | 0.438 | 59.054 | 0.257 | 5.305 | 0.154 |

| Problem 585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 71 | 129 | 108 | 155 | 107 | 92 | 68 | 68 |
| N.S. | 1 | 1.00 | 0.85 | 1.54 | 1.29 | 1.85 | 1.27 | 1.10 | 0.81 | 0.81 |
| time (sec) | N/A | 0.058 | 0.188 | 0.056 | 1.206 | 0.433 | 60.432 | 0.289 | 5.104 | 0.109 |

| Problem 586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 82 | 109 | 67 | 166 | 75 | 163 | 57 | 74 |
| N.S. | 1 | 1.00 | 1.39 | 1.85 | 1.14 | 2.81 | 1.27 | 2.76 | 0.97 | 1.25 |
| time (sec) | N/A | 0.043 | 0.155 | 0.055 | 1.226 | 0.441 | 23.441 | 0.488 | 5.206 | 0.085 |

| Problem 587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 47 | 48 | 49 | 60 | 58 | 250 | 91 | 66 |
| N.S. | 1 | 1.00 | 1.02 | 1.04 | 1.07 | 1.30 | 1.26 | 5.43 | 1.98 | 1.43 |
| time (sec) | N/A | 0.037 | 0.017 | 0.046 | 0.724 | 0.432 | 4.022 | 0.918 | 4.817 | 0.056 |

| Problem 588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 69 | 70 | 84 | 85 | 78 | 310 | 126 | 90 |
| N.S. | 1 | 1.00 | 0.93 | 0.95 | 1.14 | 1.15 | 1.05 | 4.19 | 1.70 | 1.22 |
| time (sec) | N/A | 0.059 | 0.025 | 0.046 | 0.599 | 0.432 | 4.564 | 1.382 | 4.924 | 0.064 |

| Problem 589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 79 | 94 | 118 | 109 | 112 | 370 | 168 | 114 |
| N.S. | 1 | 1.00 | 0.76 | 0.90 | 1.13 | 1.05 | 1.08 | 3.56 | 1.62 | 1.10 |
| time (sec) | N/A | 0.079 | 0.064 | 0.048 | 0.565 | 0.482 | 5.210 | 2.061 | 5.224 | 0.080 |

| Problem 590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 90 | 118 | 152 | 133 | 146 | 430 | 210 | 138 |
| N.S. | 1 | 1.00 | 0.67 | 0.88 | 1.13 | 0.99 | 1.09 | 3.21 | 1.57 | 1.03 |
| time (sec) | N/A | 0.102 | 0.078 | 0.048 | 0.466 | 0.526 | 5.863 | 2.982 | 5.607 | 0.081 |

| Problem 591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 150 | 108 | 113 | 158 | 131 | 1386 | 175 | 117 | 112 |
| N.S. | 1 | 1.00 | 0.72 | 0.75 | 1.05 | 0.87 | 9.24 | 1.17 | 0.78 | 0.75 |
| time (sec) | N/A | 0.073 | 0.084 | 0.054 | 0.643 | 0.432 | 6.441 | 0.220 | 4.591 | 0.105 |

| Problem 592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 86 | 89 | 124 | 107 | 910 | 140 | 97 | 88 |
| N.S. | 1 | 1.00 | 0.74 | 0.76 | 1.06 | 0.91 | 7.78 | 1.20 | 0.83 | 0.75 |
| time (sec) | N/A | 0.062 | 0.065 | 0.047 | 0.564 | 0.418 | 4.920 | 0.168 | 4.518 | 0.090 |

| Problem 593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 64 | 65 | 90 | 82 | 422 | 105 | 77 | 64 |
| N.S. | 1 | 1.00 | 0.76 | 0.77 | 1.07 | 0.98 | 5.02 | 1.25 | 0.92 | 0.76 |
| time (sec) | N/A | 0.040 | 0.046 | 0.058 | 0.528 | 0.417 | 3.992 | 0.166 | 4.488 | 0.086 |

| Problem 594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 42 | 43 | 55 | 57 | 119 | 72 | 54 | 42 |
| N.S. | 1 | 1.00 | 0.79 | 0.81 | 1.04 | 1.08 | 2.25 | 1.36 | 1.02 | 0.79 |
| time (sec) | N/A | 0.025 | 0.027 | 0.050 | 0.661 | 0.417 | 3.040 | 0.213 | 4.442 | 0.067 |

| Problem 595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 66 | 66 | 84 | 83 | 75 | 156 | 107 | 116 | 80 | 83 |
| N.S. | 1 | 1.00 | 1.27 | 1.26 | 1.14 | 2.36 | 1.62 | 1.76 | 1.21 | 1.26 |
| time (sec) | N/A | 0.037 | 0.057 | 0.056 | 1.376 | 0.439 | 3.266 | 0.190 | 4.720 | 0.094 |

| Problem 596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 85 | 85 | 75 | 135 | 133 | 164 | 107 | 76 | 97 | 89 |
| N.S. | 1 | 1.00 | 0.88 | 1.59 | 1.56 | 1.93 | 1.26 | 0.89 | 1.14 | 1.05 |
| time (sec) | N/A | 0.046 | 0.090 | 0.053 | 1.224 | 0.446 | 4.484 | 0.249 | 5.108 | 0.152 |

| Problem 597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 100 | 175 | 193 | 194 | 144 | 130 | -1 | 103 |
| N.S. | 1 | 1.00 | 1.10 | 1.92 | 2.12 | 2.13 | 1.58 | 1.43 | -0.01 | 1.13 |
| time (sec) | N/A | 0.051 | 0.185 | 0.059 | 1.355 | 0.437 | 6.972 | 0.240 | 0.000 | 0.179 |

| Problem 598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | B | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 68 | 220 | 277 | 244 | 226 | 153 | -1 | 128 |
| N.S. | 1 | 1.00 | 0.55 | 1.79 | 2.25 | 1.98 | 1.84 | 1.24 | -0.01 | 1.04 |
| time (sec) | N/A | 0.081 | 0.030 | 0.059 | 1.273 | 0.455 | 11.421 | 0.301 | 0.000 | 0.238 |

| Problem 599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 123 | 162 | 240 | 243 | 253 | 144 | 130 | 112 |
| N.S. | 1 | 1.00 | 1.00 | 1.32 | 1.95 | 1.98 | 2.06 | 1.17 | 1.06 | 0.91 |
| time (sec) | N/A | 0.090 | 0.142 | 0.060 | 1.298 | 0.448 | 100.225 | 0.233 | 5.783 | 0.190 |

| Problem 600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 115 | 115 | 89 | 174 | 171 | 203 | 0 | 126 | 105 | 88 |
| N.S. | 1 | 1.00 | 0.77 | 1.51 | 1.49 | 1.77 | 0.00 | 1.10 | 0.91 | 0.77 |
| time (sec) | N/A | 0.084 | 0.245 | 0.055 | 1.327 | 0.455 | 0.000 | 0.293 | 5.703 | 0.143 |

| Problem 601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 110 | 110 | 78 | 216 | 134 | 195 | 187 | 225 | 95 | 93 |
| N.S. | 1 | 1.00 | 0.71 | 1.96 | 1.22 | 1.77 | 1.70 | 2.05 | 0.86 | 0.85 |
| time (sec) | N/A | 0.073 | 0.095 | 0.063 | 1.233 | 0.442 | 56.156 | 0.541 | 5.651 | 0.167 |

| Problem 602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 76 | 76 | 90 | 153 | 80 | 213 | 73 | 254 | 72 | 96 |
| N.S. | 1 | 1.00 | 1.18 | 2.01 | 1.05 | 2.80 | 0.96 | 3.34 | 0.95 | 1.26 |
| time (sec) | N/A | 0.054 | 0.071 | 0.066 | 1.310 | 0.435 | 54.144 | 1.068 | 5.831 | 0.113 |

| Problem 603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 46 | 46 | 49 | 48 | 49 | 84 | 138 | 370 | 122 | 90 |
| N.S. | 1 | 1.00 | 1.07 | 1.04 | 1.07 | 1.83 | 3.00 | 8.04 | 2.65 | 1.96 |
| time (sec) | N/A | 0.036 | 0.022 | 0.049 | 0.593 | 0.424 | 14.156 | 1.866 | 5.333 | 0.065 |

| Problem 604 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 74 | 74 | 71 | 70 | 84 | 109 | 194 | 430 | 164 | 114 |
| N.S. | 1 | 1.00 | 0.96 | 0.95 | 1.14 | 1.47 | 2.62 | 5.81 | 2.22 | 1.54 |
| time (sec) | N/A | 0.056 | 0.033 | 0.049 | 0.642 | 0.481 | 15.608 | 2.691 | 5.759 | 0.074 |

| Problem 605 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 94 | 94 | 118 | 134 | 262 | 490 | 206 | 138 |
| N.S. | 1 | 1.00 | 0.90 | 0.90 | 1.13 | 1.29 | 2.52 | 4.71 | 1.98 | 1.33 |
| time (sec) | N/A | 0.072 | 0.040 | 0.053 | 0.580 | 0.541 | 17.961 | 3.285 | 6.315 | 0.083 |

| Problem 606 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 134 | 134 | 115 | 118 | 152 | 157 | 326 | 550 | 248 | 162 |
| N.S. | 1 | 1.00 | 0.86 | 0.88 | 1.13 | 1.17 | 2.43 | 4.10 | 1.85 | 1.21 |
| time (sec) | N/A | 0.096 | 0.061 | 0.052 | 0.630 | 0.608 | 19.447 | 4.630 | 6.812 | 0.089 |

| Problem 607 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 150 | 110 | 115 | 158 | 155 | 3351 | 175 | 137 | 114 |
| N.S. | 1 | 1.00 | 0.73 | 0.77 | 1.05 | 1.03 | 22.34 | 1.17 | 0.91 | 0.76 |
| time (sec) | N/A | 0.072 | 0.110 | 0.049 | 0.635 | 0.418 | 12.596 | 0.183 | 4.662 | 0.130 |

| Problem 608 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 117 | 117 | 89 | 91 | 124 | 132 | 2304 | 140 | 118 | 90 |
| N.S. | 1 | 1.00 | 0.76 | 0.78 | 1.06 | 1.13 | 19.69 | 1.20 | 1.01 | 0.77 |
| time (sec) | N/A | 0.057 | 0.090 | 0.051 | 0.490 | 0.436 | 9.646 | 0.179 | 4.569 | 0.102 |

| Problem 609 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 84 | 84 | 66 | 67 | 90 | 106 | 1340 | 105 | 97 | 66 |
| N.S. | 1 | 1.00 | 0.79 | 0.80 | 1.07 | 1.26 | 15.95 | 1.25 | 1.15 | 0.79 |
| time (sec) | N/A | 0.042 | 0.055 | 0.055 | 0.755 | 0.425 | 7.638 | 0.171 | 4.551 | 0.093 |

| Problem 610 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 53 | 53 | 44 | 45 | 55 | 80 | 498 | 72 | 77 | 44 |
| N.S. | 1 | 1.00 | 0.83 | 0.85 | 1.04 | 1.51 | 9.40 | 1.36 | 1.45 | 0.83 |
| time (sec) | N/A | 0.025 | 0.036 | 0.044 | 0.647 | 0.410 | 5.948 | 0.183 | 4.626 | 0.080 |

| Problem 611 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 81 | 99 | 91 | 203 | 184 | 140 | -1 | 107 |
| N.S. | 1 | 1.00 | 0.94 | 1.15 | 1.06 | 2.36 | 2.14 | 1.63 | -0.01 | 1.24 |
| time (sec) | N/A | 0.055 | 0.147 | 0.059 | 1.350 | 0.452 | 5.249 | 0.200 | 0.000 | 0.122 |

| Problem 612 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 121 | 121 | 105 | 170 | 163 | 190 | 202 | 115 | -1 | 109 |
| N.S. | 1 | 1.00 | 0.87 | 1.40 | 1.35 | 1.57 | 1.67 | 0.95 | -0.01 | 0.90 |
| time (sec) | N/A | 0.058 | 0.088 | 0.061 | 1.359 | 0.439 | 7.646 | 0.265 | 0.000 | 0.183 |

| Problem 613 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | B | A | B | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 112 | 112 | 68 | 213 | 207 | 216 | 216 | 145 | 78 | 107 |
| N.S. | 1 | 1.00 | 0.61 | 1.90 | 1.85 | 1.93 | 1.93 | 1.29 | 0.70 | 0.96 |
| time (sec) | N/A | 0.062 | 0.034 | 0.058 | 1.424 | 0.439 | 11.776 | 0.257 | 5.858 | 0.217 |

| Problem 614 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 126 | 259 | 275 | 246 | 253 | 173 | -1 | 127 |
| N.S. | 1 | 1.00 | 1.02 | 2.11 | 2.24 | 2.00 | 2.06 | 1.41 | -0.01 | 1.03 |
| time (sec) | N/A | 0.064 | 0.244 | 0.066 | 1.383 | 0.463 | 18.949 | 0.299 | 0.000 | 0.254 |

| Problem 615 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | B | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 159 | 159 | 71 | 302 | 354 | 298 | 287 | 214 | -1 | 152 |
| N.S. | 1 | 1.00 | 0.45 | 1.90 | 2.23 | 1.87 | 1.81 | 1.35 | -0.01 | 0.96 |
| time (sec) | N/A | 0.089 | 0.049 | 0.073 | 1.573 | 0.474 | 29.163 | 0.374 | 0.000 | 0.306 |

| Problem 616 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 95 | 129 | 178 | 192 | 150 | 113 | 99 | 88 |
| N.S. | 1 | 1.00 | 1.06 | 1.43 | 1.98 | 2.13 | 1.67 | 1.26 | 1.10 | 0.98 |
| time (sec) | N/A | 0.067 | 0.112 | 0.059 | 1.171 | 0.436 | 71.064 | 0.256 | 5.348 | 0.140 |

| Problem 617 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 79 | 90 | 109 | 146 | 66 | 88 | 59 | 67 |
| N.S. | 1 | 1.00 | 1.34 | 1.53 | 1.85 | 2.47 | 1.12 | 1.49 | 1.00 | 1.14 |
| time (sec) | N/A | 0.040 | 0.055 | 0.052 | 1.195 | 0.440 | 84.411 | 0.259 | 5.077 | 0.098 |

| Problem 618 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 73 | 70 | 54 | 130 | 63 | 0 | 35 | 51 |
| N.S. | 1 | 1.00 | 1.70 | 1.63 | 1.26 | 3.02 | 1.47 | 0.00 | 0.81 | 1.19 |
| time (sec) | N/A | 0.033 | 0.049 | 0.054 | 1.105 | 0.439 | 22.339 | 0.000 | 4.851 | 0.068 |

| Problem 619 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 43 | 43 | 39 | 47 | 48 | 39 | 138 | 88 | 35 | 44 |
| N.S. | 1 | 1.00 | 0.91 | 1.09 | 1.12 | 0.91 | 3.21 | 2.05 | 0.81 | 1.02 |
| time (sec) | N/A | 0.036 | 0.048 | 0.049 | 0.534 | 0.409 | 7.054 | 0.410 | 4.565 | 0.049 |

| Problem 620 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 72 | 72 | 60 | 70 | 83 | 62 | 204 | 153 | 58 | 66 |
| N.S. | 1 | 1.00 | 0.83 | 0.97 | 1.15 | 0.86 | 2.83 | 2.12 | 0.81 | 0.92 |
| time (sec) | N/A | 0.052 | 0.039 | 0.047 | 0.489 | 0.430 | 13.172 | 0.498 | 4.676 | 0.065 |

| Problem 621 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 91 | 94 | 118 | 86 | 269 | 219 | 102 | 90 |
| N.S. | 1 | 1.00 | 0.90 | 0.93 | 1.17 | 0.85 | 2.66 | 2.17 | 1.01 | 0.89 |
| time (sec) | N/A | 0.069 | 0.048 | 0.047 | 0.576 | 0.430 | 18.001 | 0.623 | 4.716 | 0.073 |

| Problem 622 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 82 | 82 | 56 | 67 | 85 | 59 | 338 | 0 | 53 | 57 |
| N.S. | 1 | 1.00 | 0.68 | 0.82 | 1.04 | 0.72 | 4.12 | 0.00 | 0.65 | 0.70 |
| time (sec) | N/A | 0.033 | 0.075 | 0.052 | 0.491 | 0.413 | 3.553 | 0.000 | 5.314 | 0.083 |

| Problem 623 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 51 | 51 | 34 | 44 | 49 | 36 | 70 | 0 | 67 | 34 |
| N.S. | 1 | 1.00 | 0.67 | 0.86 | 0.96 | 0.71 | 1.37 | 0.00 | 1.31 | 0.67 |
| time (sec) | N/A | 0.020 | 0.056 | 0.044 | 0.592 | 0.424 | 2.755 | 0.000 | 4.917 | 0.062 |

| Problem 624 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 47 | 47 | 71 | 73 | 58 | 131 | 39 | 0 | 65 | 67 |
| N.S. | 1 | 1.00 | 1.51 | 1.55 | 1.23 | 2.79 | 0.83 | 0.00 | 1.38 | 1.43 |
| time (sec) | N/A | 0.029 | 0.041 | 0.051 | 1.276 | 0.444 | 2.856 | 0.000 | 4.998 | 0.081 |

| Problem 625 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 61 | 61 | 80 | 105 | 121 | 144 | 66 | 0 | 94 | 82 |
| N.S. | 1 | 1.00 | 1.31 | 1.72 | 1.98 | 2.36 | 1.08 | 0.00 | 1.54 | 1.34 |
| time (sec) | N/A | 0.038 | 0.064 | 0.057 | 1.177 | 0.423 | 4.921 | 0.000 | 5.529 | 0.116 |

| Problem 626 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 107 | 146 | 200 | 201 | 150 | 0 | -1 | 104 |
| N.S. | 1 | 1.00 | 1.15 | 1.57 | 2.15 | 2.16 | 1.61 | 0.00 | -0.01 | 1.12 |
| time (sec) | N/A | 0.050 | 0.179 | 0.063 | 1.234 | 0.454 | 8.461 | 0.000 | 0.000 | 0.201 |

| Problem 627 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 118 | 120 | 111 | 140 | 215 | 304 | 177 | 0 | 134 | 119 |
| N.S. | 1 | 1.02 | 0.94 | 1.19 | 1.82 | 2.58 | 1.50 | 0.00 | 1.14 | 1.01 |
| time (sec) | N/A | 0.085 | 0.192 | 0.060 | 1.385 | 0.462 | 103.904 | 0.000 | 6.339 | 0.183 |

| Problem 628 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | B | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 89 | 115 | 144 | 249 | 264 | 0 | 90 | 93 |
| N.S. | 1 | 1.00 | 1.03 | 1.34 | 1.67 | 2.90 | 3.07 | 0.00 | 1.05 | 1.08 |
| time (sec) | N/A | 0.058 | 0.217 | 0.058 | 1.204 | 0.438 | 52.419 | 0.000 | 5.609 | 0.178 |

| Problem 629 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 73 | 75 | 69 | 200 | 49 | 0 | 54 | 73 |
| N.S. | 1 | 1.00 | 1.40 | 1.44 | 1.33 | 3.85 | 0.94 | 0.00 | 1.04 | 1.40 |
| time (sec) | N/A | 0.040 | 0.094 | 0.058 | 1.260 | 0.461 | 20.333 | 0.000 | 5.058 | 0.110 |

| Problem 630 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 42 | 42 | 36 | 46 | 46 | 46 | 68 | 37 | 46 | 46 |
| N.S. | 1 | 1.00 | 0.86 | 1.10 | 1.10 | 1.10 | 1.62 | 0.88 | 1.10 | 1.10 |
| time (sec) | N/A | 0.035 | 0.021 | 0.052 | 0.461 | 0.422 | 3.421 | 0.264 | 4.519 | 0.061 |

| Problem 631 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 68 | 68 | 60 | 69 | 81 | 73 | 61 | 0 | 66 | 75 |
| N.S. | 1 | 1.00 | 0.88 | 1.01 | 1.19 | 1.07 | 0.90 | 0.00 | 0.97 | 1.10 |
| time (sec) | N/A | 0.052 | 0.027 | 0.050 | 0.611 | 0.422 | 11.071 | 0.000 | 4.642 | 0.080 |

| Problem 632 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 100 | 100 | 81 | 94 | 116 | 98 | 90 | 0 | 91 | 99 |
| N.S. | 1 | 1.00 | 0.81 | 0.94 | 1.16 | 0.98 | 0.90 | 0.00 | 0.91 | 0.99 |
| time (sec) | N/A | 0.071 | 0.030 | 0.053 | 0.615 | 0.429 | 13.279 | 0.000 | 4.837 | 0.099 |

| Problem 633 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 126 | 126 | 104 | 118 | 151 | 121 | 122 | 0 | 154 | 123 |
| N.S. | 1 | 1.00 | 0.83 | 0.94 | 1.20 | 0.96 | 0.97 | 0.00 | 1.22 | 0.98 |
| time (sec) | N/A | 0.089 | 0.035 | 0.054 | 0.549 | 0.454 | 16.143 | 0.000 | 4.920 | 0.108 |

| Problem 634 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 111 | 111 | 80 | 91 | 128 | 95 | 561 | 0 | 79 | 90 |
| N.S. | 1 | 1.00 | 0.72 | 0.82 | 1.15 | 0.86 | 5.05 | 0.00 | 0.71 | 0.81 |
| time (sec) | N/A | 0.049 | 0.076 | 0.055 | 0.558 | 0.425 | 7.831 | 0.000 | 5.768 | 0.104 |

| Problem 635 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 79 | 79 | 57 | 66 | 90 | 70 | 267 | 0 | 81 | 65 |
| N.S. | 1 | 1.00 | 0.72 | 0.84 | 1.14 | 0.89 | 3.38 | 0.00 | 1.03 | 0.82 |
| time (sec) | N/A | 0.030 | 0.050 | 0.061 | 0.533 | 0.429 | 7.306 | 0.000 | 5.195 | 0.090 |

| Problem 636 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 45 | 45 | 33 | 43 | 53 | 47 | 65 | 0 | 38 | 40 |
| N.S. | 1 | 1.00 | 0.73 | 0.96 | 1.18 | 1.04 | 1.44 | 0.00 | 0.84 | 0.89 |
| time (sec) | N/A | 0.030 | 0.026 | 0.044 | 0.681 | 0.479 | 7.513 | 0.000 | 4.898 | 0.079 |

| Problem 637 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F(-2) | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 59 | 59 | 71 | 79 | 80 | 195 | 206 | 0 | 60 | 77 |
| N.S. | 1 | 1.00 | 1.20 | 1.34 | 1.36 | 3.31 | 3.49 | 0.00 | 1.02 | 1.31 |
| time (sec) | N/A | 0.034 | 0.039 | 0.059 | 1.215 | 0.463 | 11.714 | 0.000 | 5.106 | 0.117 |

| Problem 638 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | B | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 92 | 92 | 57 | 132 | 162 | 248 | 262 | 0 | -1 | 101 |
| N.S. | 1 | 1.00 | 0.62 | 1.43 | 1.76 | 2.70 | 2.85 | 0.00 | -0.01 | 1.10 |
| time (sec) | N/A | 0.050 | 0.029 | 0.059 | 1.218 | 0.439 | 18.437 | 0.000 | 0.000 | 0.242 |

| Problem 639 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | B | A | A | F(-2) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 123 | 123 | 60 | 157 | 243 | 314 | 180 | 0 | -1 | 126 |
| N.S. | 1 | 1.00 | 0.49 | 1.28 | 1.98 | 2.55 | 1.46 | 0.00 | -0.01 | 1.02 |
| time (sec) | N/A | 0.069 | 0.030 | 0.063 | 1.284 | 0.446 | 29.807 | 0.000 | 0.000 | 0.249 |

| Problem 640 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 135 | 135 | 111 | 75 | 57 | 62 | 0 | 162 | 831 | 216 |
| N.S. | 1 | 1.00 | 0.82 | 0.56 | 0.42 | 0.46 | 0.00 | 1.20 | 6.16 | 1.60 |
| time (sec) | N/A | 0.061 | 0.069 | 0.054 | 0.546 | 0.394 | 0.000 | 0.290 | 52.028 | 2.437 |

| Problem 641 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 104 | 104 | 99 | 65 | 47 | 57 | 0 | 127 | 632 | 176 |
| N.S. | 1 | 1.00 | 0.95 | 0.62 | 0.45 | 0.55 | 0.00 | 1.22 | 6.08 | 1.69 |
| time (sec) | N/A | 0.049 | 0.056 | 0.049 | 0.489 | 0.410 | 0.000 | 0.217 | 31.390 | 1.303 |

| Problem 642 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 73 | 73 | 87 | 52 | 37 | 52 | 0 | 92 | -1 | 404 |
| N.S. | 1 | 1.00 | 1.19 | 0.71 | 0.51 | 0.71 | 0.00 | 1.26 | -0.01 | 5.53 |
| time (sec) | N/A | 0.032 | 0.075 | 0.047 | 0.581 | 0.405 | 0.000 | 0.189 | 0.000 | 1.475 |

| Problem 643 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 37 | 37 | 72 | 72 | 26 | 46 | 0 | 57 | 41 | 268 |
| N.S. | 1 | 1.00 | 1.95 | 1.95 | 0.70 | 1.24 | 0.00 | 1.54 | 1.11 | 7.24 |
| time (sec) | N/A | 0.021 | 0.035 | 0.050 | 0.494 | 0.412 | 0.000 | 0.175 | 5.069 | 1.175 |

| Problem 644 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 67 | 67 | 74 | 47 | 27 | 55 | 0 | 0 | 129 | 184 |
| N.S. | 1 | 1.00 | 1.10 | 0.70 | 0.40 | 0.82 | 0.00 | 0.00 | 1.93 | 2.75 |
| time (sec) | N/A | 0.031 | 0.079 | 0.054 | 1.280 | 0.396 | 0.000 | 0.000 | 6.249 | 1.133 |

| Problem 645 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 31 | 31 | 31 | 23 | 10 | 30 | 0 | 48 | 31 | 512 |
| N.S. | 1 | 1.00 | 1.00 | 0.74 | 0.32 | 0.97 | 0.00 | 1.55 | 1.00 | 16.52 |
| time (sec) | N/A | 0.009 | 0.016 | 0.052 | 1.320 | 0.421 | 0.000 | 0.229 | 5.257 | 2.382 |

| Problem 646 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 36 | 28 | 21 | 37 | 0 | 90 | 43 | 836 |
| N.S. | 1 | 1.00 | 0.57 | 0.44 | 0.33 | 0.59 | 0.00 | 1.43 | 0.68 | 13.27 |
| time (sec) | N/A | 0.020 | 0.021 | 0.052 | 1.312 | 0.408 | 0.000 | 0.280 | 5.075 | 10.092 |

| Problem 647 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 41 | 33 | 31 | 44 | 0 | 111 | 55 | 1160 |
| N.S. | 1 | 1.00 | 0.44 | 0.35 | 0.33 | 0.47 | 0.00 | 1.18 | 0.59 | 12.34 |
| time (sec) | N/A | 0.031 | 0.031 | 0.054 | 1.550 | 0.397 | 0.000 | 0.290 | 5.038 | 36.638 |

| Problem 648 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 125 | 125 | 46 | 38 | 41 | 49 | 0 | 132 | 67 | 1484 |
| N.S. | 1 | 1.00 | 0.37 | 0.30 | 0.33 | 0.39 | 0.00 | 1.06 | 0.54 | 11.87 |
| time (sec) | N/A | 0.042 | 0.033 | 0.061 | 1.217 | 0.405 | 0.000 | 0.426 | 5.023 | 111.252 |

| Problem 649 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 104 | 104 | 67 | 65 | 47 | 57 | 0 | 76 | 632 | 176 |
| N.S. | 1 | 1.00 | 0.64 | 0.62 | 0.45 | 0.55 | 0.00 | 0.73 | 6.08 | 1.69 |
| time (sec) | N/A | 0.045 | 0.054 | 0.058 | 0.612 | 0.406 | 0.000 | 0.231 | 27.092 | 1.421 |

| Problem 650 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD | NO |
| size | 73 | 73 | 62 | 55 | 37 | 52 | 0 | 59 | 429 | 136 |
| N.S. | 1 | 1.00 | 0.85 | 0.75 | 0.51 | 0.71 | 0.00 | 0.81 | 5.88 | 1.86 |
| time (sec) | N/A | 0.032 | 0.036 | 0.063 | 0.460 | 0.405 | 0.000 | 0.227 | 18.765 | 1.323 |

| Problem 651 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 35 | 35 | 55 | 41 | 24 | 46 | 83 | 39 | -1 | 268 |
| N.S. | 1 | 1.00 | 1.57 | 1.17 | 0.69 | 1.31 | 2.37 | 1.11 | -0.03 | 7.66 |
| time (sec) | N/A | 0.021 | 0.012 | 0.061 | 0.453 | 0.413 | 27.907 | 0.223 | 0.000 | 1.268 |

| Problem 652 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | F | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 8 | 26 | 40 | 16 | 27 | 0 | 20 | 6 | 38 |
| N.S. | 1 | 1.00 | 3.25 | 5.00 | 2.00 | 3.38 | 0.00 | 2.50 | 0.75 | 4.75 |
| time (sec) | N/A | 0.011 | 0.007 | 0.049 | 0.647 | 0.407 | 0.000 | 0.195 | 5.287 | 1.016 |

| Problem 653 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 29 | 29 | 29 | 20 | 10 | 25 | 0 | 25 | 19 | 188 |
| N.S. | 1 | 1.00 | 1.00 | 0.69 | 0.34 | 0.86 | 0.00 | 0.86 | 0.66 | 6.48 |
| time (sec) | N/A | 0.010 | 0.010 | 0.056 | 1.208 | 0.416 | 0.000 | 0.222 | 5.561 | 1.137 |

| Problem 654 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 36 | 25 | 21 | 34 | 0 | 48 | 33 | 512 |
| N.S. | 1 | 1.00 | 0.57 | 0.40 | 0.33 | 0.54 | 0.00 | 0.76 | 0.52 | 8.13 |
| time (sec) | N/A | 0.020 | 0.014 | 0.058 | 1.200 | 0.400 | 0.000 | 0.222 | 5.504 | 2.746 |

| Problem 655 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 94 | 94 | 41 | 30 | 31 | 39 | 0 | 69 | 43 | 127 |
| N.S. | 1 | 1.00 | 0.44 | 0.32 | 0.33 | 0.41 | 0.00 | 0.73 | 0.46 | 1.35 |
| time (sec) | N/A | 0.033 | 0.029 | 0.064 | 1.297 | 0.407 | 0.000 | 0.257 | 5.658 | 1.371 |

| Problem 656 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 36 | 15 | 11 | 10 | 16 | 11 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 2.40 | 1.00 | 0.73 | 0.67 | 1.07 | 0.73 | 0.00 |
| time (sec) | N/A | 0.014 | 0.006 | 0.052 | 0.455 | 0.399 | 0.122 | 0.185 | 0.087 | 0.001 |

| Problem 657 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | A | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 22 | 22 | 110 | 0 | 36 | 40 | 39 | 38 | 31 | 0 |
| N.S. | 1 | 1.00 | 5.00 | 0.00 | 1.64 | 1.82 | 1.77 | 1.73 | 1.41 | 0.00 |
| time (sec) | N/A | 0.015 | 0.166 | 0.674 | 0.954 | 0.411 | 8.426 | 0.292 | 4.942 | 0.258 |

| Problem 658 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 63 | 63 | 56 | 69 | 69 | 58 | 332 | 0 | 162 | 67 |
| N.S. | 1 | 1.00 | 0.89 | 1.10 | 1.10 | 0.92 | 5.27 | 0.00 | 2.57 | 1.06 |
| time (sec) | N/A | 0.068 | 0.065 | 0.099 | 0.565 | 0.450 | 4.025 | 0.000 | 5.724 | 0.069 |

| Problem 659 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 101 | 101 | 97 | 131 | 151 | 224 | 0 | 0 | -1 | 109 |
| N.S. | 1 | 1.00 | 0.96 | 1.30 | 1.50 | 2.22 | 0.00 | 0.00 | -0.01 | 1.08 |
| time (sec) | N/A | 0.105 | 0.329 | 0.105 | 0.665 | 0.466 | 0.000 | 0.000 | 0.000 | 0.128 |

| Problem 660 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 130 | 130 | 115 | 284 | 231 | 177 | 320 | 0 | -1 | 155 |
| N.S. | 1 | 1.00 | 0.88 | 2.18 | 1.78 | 1.36 | 2.46 | 0.00 | -0.01 | 1.19 |
| time (sec) | N/A | 0.138 | 0.354 | 0.082 | 0.531 | 0.454 | 135.872 | 0.000 | 0.000 | 0.106 |

| Problem 661 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 90 | 90 | 82 | 173 | 150 | 108 | 202 | 0 | -1 | 97 |
| N.S. | 1 | 1.00 | 0.91 | 1.92 | 1.67 | 1.20 | 2.24 | 0.00 | -0.01 | 1.08 |
| time (sec) | N/A | 0.089 | 0.138 | 0.077 | 0.698 | 0.441 | 63.508 | 0.000 | 0.000 | 0.079 |

| Problem 662 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 60 | 60 | 50 | 87 | 83 | 56 | 105 | 0 | -1 | 55 |
| N.S. | 1 | 1.00 | 0.83 | 1.45 | 1.38 | 0.93 | 1.75 | 0.00 | -0.02 | 0.92 |
| time (sec) | N/A | 0.054 | 0.058 | 0.071 | 0.699 | 0.431 | 26.972 | 0.000 | 0.000 | 0.052 |

| Problem 663 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 54 | 54 | 44 | 59 | 60 | 45 | 0 | 0 | -1 | 55 |
| N.S. | 1 | 1.00 | 0.81 | 1.09 | 1.11 | 0.83 | 0.00 | 0.00 | -0.02 | 1.02 |
| time (sec) | N/A | 0.054 | 0.045 | 0.070 | 0.586 | 0.444 | 0.000 | 0.000 | 0.000 | 0.060 |

| Problem 664 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 75 | 75 | 58 | 109 | 121 | 120 | 0 | 0 | -1 | 75 |
| N.S. | 1 | 1.00 | 0.77 | 1.45 | 1.61 | 1.60 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.062 | 0.135 | 0.092 | 0.515 | 0.447 | 0.000 | 0.000 | 0.000 | 0.090 |

| Problem 665 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 105 | 105 | 97 | 203 | 243 | 267 | 0 | 0 | -1 | 100 |
| N.S. | 1 | 1.00 | 0.92 | 1.93 | 2.31 | 2.54 | 0.00 | 0.00 | -0.01 | 0.95 |
| time (sec) | N/A | 0.090 | 0.168 | 0.112 | 0.663 | 0.462 | 0.000 | 0.000 | 0.000 | 0.125 |

| Problem 666 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 158 | 158 | 138 | 342 | 286 | 230 | 0 | 0 | -1 | 220 |
| N.S. | 1 | 1.00 | 0.87 | 2.16 | 1.81 | 1.46 | 0.00 | 0.00 | -0.01 | 1.39 |
| time (sec) | N/A | 0.151 | 0.314 | 0.070 | 0.663 | 0.486 | 0.000 | 0.000 | 0.000 | 0.134 |

| Problem 667 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 118 | 118 | 103 | 236 | 192 | 146 | 0 | 0 | -1 | 141 |
| N.S. | 1 | 1.00 | 0.87 | 2.00 | 1.63 | 1.24 | 0.00 | 0.00 | -0.01 | 1.19 |
| time (sec) | N/A | 0.115 | 0.269 | 0.072 | 0.541 | 0.477 | 0.000 | 0.000 | 0.000 | 0.102 |

| Problem 668 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 86 | 86 | 78 | 125 | 112 | 82 | 139 | 0 | -1 | 82 |
| N.S. | 1 | 1.00 | 0.91 | 1.45 | 1.30 | 0.95 | 1.62 | 0.00 | -0.01 | 0.95 |
| time (sec) | N/A | 0.076 | 0.099 | 0.072 | 0.534 | 0.435 | 57.945 | 0.000 | 0.000 | 0.069 |

| Problem 669 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 71 | 71 | 66 | 78 | 81 | 74 | 0 | 0 | -1 | 70 |
| N.S. | 1 | 1.00 | 0.93 | 1.10 | 1.14 | 1.04 | 0.00 | 0.00 | -0.01 | 0.99 |
| time (sec) | N/A | 0.069 | 0.105 | 0.096 | 0.626 | 0.463 | 0.000 | 0.000 | 0.000 | 0.073 |

| Problem 670 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 90 | 163 | 147 | 166 | 0 | 0 | -1 | 95 |
| N.S. | 1 | 1.00 | 0.95 | 1.72 | 1.55 | 1.75 | 0.00 | 0.00 | -0.01 | 1.00 |
| time (sec) | N/A | 0.090 | 0.110 | 0.080 | 0.582 | 0.451 | 0.000 | 0.000 | 0.000 | 0.114 |

| Problem 671 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F(-2) | F | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 120 | 120 | 112 | 214 | 262 | 301 | 0 | 0 | -1 | 110 |
| N.S. | 1 | 1.00 | 0.93 | 1.78 | 2.18 | 2.51 | 0.00 | 0.00 | -0.01 | 0.92 |
| time (sec) | N/A | 0.105 | 0.275 | 0.121 | 0.677 | 0.454 | 0.000 | 0.000 | 0.000 | 0.146 |

| Problem 672 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 172 | 155 | 154 | 154 | 175 | 13 | 154 | 0 |
| N.S. | 1 | 1.00 | 12.29 | 11.07 | 11.00 | 11.00 | 12.50 | 0.93 | 11.00 | 0.00 |
| time (sec) | N/A | 0.002 | 0.006 | 0.040 | 0.566 | 0.354 | 0.115 | 0.148 | 0.156 | 0.000 |

| Problem 673 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 182 | 157 | 156 | 156 | 182 | 156 | 156 | 0 |
| N.S. | 1 | 1.00 | 11.38 | 9.81 | 9.75 | 9.75 | 11.38 | 9.75 | 9.75 | 0.00 |
| time (sec) | N/A | 0.041 | 0.006 | 0.038 | 0.731 | 0.359 | 0.122 | 0.188 | 4.925 | 0.000 |

| Problem 674 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 186 | 157 | 156 | 156 | 185 | 156 | 156 | 0 |
| N.S. | 1 | 1.00 | 11.62 | 9.81 | 9.75 | 9.75 | 11.56 | 9.75 | 9.75 | 0.00 |
| time (sec) | N/A | 0.036 | 0.021 | 0.038 | 0.624 | 0.360 | 0.121 | 0.163 | 4.772 | 0.000 |

| Problem 675 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 230 | 229 | 189 | 0 | 189 | 229 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 10.95 | 10.90 | 9.00 | 0.00 | 9.00 | 10.90 | 1.00 |
| time (sec) | N/A | 0.016 | 0.190 | 0.079 | 0.644 | 0.457 | 0.000 | 0.572 | 5.209 | 0.041 |

| Problem 676 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | B | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 13 | 13 | 107 | 0 | 16 | 32 | 0 | 70 | 25 | 0 |
| N.S. | 1 | 1.00 | 8.23 | 0.00 | 1.23 | 2.46 | 0.00 | 5.38 | 1.92 | 0.00 |
| time (sec) | N/A | 0.014 | 0.184 | 0.675 | 0.842 | 0.433 | 0.000 | 0.279 | 4.810 | 0.096 |

| Problem 677 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 8 | 9 | 9 | 9 | 9 | 10 | 8 | 11 | 8 | 0 |
| N.S. | 1 | 1.12 | 1.12 | 1.12 | 1.12 | 1.25 | 1.00 | 1.38 | 1.00 | 0.00 |
| time (sec) | N/A | 0.006 | 0.018 | 0.042 | 0.478 | 0.400 | 0.124 | 0.152 | 4.662 | 0.001 |

| Problem 678 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 14 | 17 | 13 | 12 | 18 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 1.13 | 0.87 | 0.80 | 1.20 | 0.87 | 0.00 |
| time (sec) | N/A | 0.015 | 0.009 | 0.053 | 0.488 | 0.397 | 0.178 | 0.151 | 0.060 | 0.001 |

| Problem 679 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 14 | 17 | 13 | 12 | 15 | 13 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 1.13 | 0.87 | 0.80 | 1.00 | 0.87 | 0.00 |
| time (sec) | N/A | 0.014 | 0.020 | 0.048 | 0.554 | 0.403 | 0.193 | 0.162 | 4.628 | 0.001 |

| Problem 680 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | A | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 15 | 15 | 15 | 17 | 47 | 17 | 29 | 0 | 15 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 1.13 | 3.13 | 1.13 | 1.93 | 0.00 | 1.00 | 1.60 |
| time (sec) | N/A | 0.015 | 0.016 | 0.047 | 0.489 | 0.435 | 0.619 | 0.000 | 4.723 | 0.032 |

| Problem 681 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 14 | 14 | 14 | 177 | 81 | 81 | 87 | 13 | 12 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 12.64 | 5.79 | 5.79 | 6.21 | 0.93 | 0.86 | 0.00 |
| time (sec) | N/A | 0.002 | 0.024 | 0.061 | 0.612 | 0.423 | 0.886 | 0.170 | 7.087 | 0.001 |

| Problem 682 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 197 | 81 | 81 | 87 | 15 | 14 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 12.31 | 5.06 | 5.06 | 5.44 | 0.94 | 0.88 | 0.00 |
| time (sec) | N/A | 0.014 | 0.047 | 0.063 | 0.638 | 0.408 | 1.356 | 0.155 | 2.315 | 0.001 |

| Problem 683 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | B | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 16 | 16 | 16 | 197 | 81 | 81 | 87 | 15 | 14 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 12.31 | 5.06 | 5.06 | 5.44 | 0.94 | 0.88 | 0.00 |
| time (sec) | N/A | 0.013 | 0.054 | 0.062 | 0.614 | 0.394 | 1.908 | 0.169 | 9.981 | 0.001 |

| Problem 684 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | F | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 21 | 21 | 21 | 203 | 612 | 105 | 0 | 0 | 105 | 21 |
| N.S. | 1 | 1.00 | 1.00 | 9.67 | 29.14 | 5.00 | 0.00 | 0.00 | 5.00 | 1.00 |
| time (sec) | N/A | 0.013 | 0.251 | 0.098 | 0.787 | 0.476 | 0.000 | 0.000 | 4.987 | 0.096 |

| Problem 685 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F(-1) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 52 | 52 | 44 | 86 | 53 | 46 | 0 | 56 | 37 | 46 |
| N.S. | 1 | 1.00 | 0.85 | 1.65 | 1.02 | 0.88 | 0.00 | 1.08 | 0.71 | 0.88 |
| time (sec) | N/A | 0.028 | 0.043 | 1.147 | 1.252 | 0.423 | 0.000 | 0.155 | 4.817 | 0.071 |

| Problem 686 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 93 | 93 | 142 | 143 | 0 | 757 | 0 | 0 | -1 | 442 |
| N.S. | 1 | 1.00 | 1.53 | 1.54 | 0.00 | 8.14 | 0.00 | 0.00 | -0.01 | 4.75 |
| time (sec) | N/A | 0.086 | 0.682 | 0.102 | 0.000 | 0.706 | 0.000 | 0.000 | 0.000 | 2.670 |

| Problem 687 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 252 | 252 | 223 | 0 | 0 | 607 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.00 | 0.00 | 2.41 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.234 | 1.093 | 1.016 | 0.000 | 0.542 | 0.000 | 0.000 | 0.000 | 0.952 |

| Problem 688 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 199 | 199 | 178 | 0 | 0 | 469 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 2.36 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.164 | 0.672 | 0.961 | 0.000 | 0.496 | 0.000 | 0.000 | 0.000 | 0.428 |

| Problem 689 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 146 | 146 | 141 | 0 | 0 | 359 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 2.46 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.121 | 0.513 | 1.012 | 0.000 | 0.461 | 0.000 | 0.000 | 0.000 | 0.141 |

| Problem 690 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 89 | 89 | 123 | 0 | 0 | 281 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.38 | 0.00 | 0.00 | 3.16 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.088 | 0.321 | 0.974 | 0.000 | 0.524 | 0.000 | 0.000 | 0.000 | 0.144 |

| Problem 691 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 91 | 91 | 122 | 0 | 0 | 408 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.34 | 0.00 | 0.00 | 4.48 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.089 | 0.901 | 0.947 | 0.000 | 0.597 | 0.000 | 0.000 | 0.000 | 0.315 |

| Problem 692 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 95 | 95 | 57 | 0 | 0 | 135 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.60 | 0.00 | 0.00 | 1.42 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.065 | 0.075 | 0.918 | 0.000 | 0.739 | 0.000 | 0.000 | 0.000 | 0.859 |

| Problem 693 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 358 | 358 | 274 | 0 | 0 | 771 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 2.15 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.405 | 2.024 | 0.965 | 0.000 | 0.584 | 0.000 | 0.000 | 0.000 | 0.959 |

| Problem 694 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 291 | 291 | 241 | 0 | 0 | 607 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.00 | 0.00 | 2.09 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.316 | 1.007 | 0.934 | 0.000 | 0.542 | 0.000 | 0.000 | 0.000 | 0.423 |

| Problem 695 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 221 | 221 | 191 | 0 | 0 | 471 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 2.13 | 0.00 | 0.00 | -0.00 | 0.00 |
| time (sec) | N/A | 0.245 | 0.624 | 0.993 | 0.000 | 0.513 | 0.000 | 0.000 | 0.000 | 0.143 |

| Problem 696 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 150 | 150 | 157 | 0 | 0 | 361 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 0.00 | 0.00 | 2.41 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.152 | 0.464 | 0.997 | 0.000 | 0.500 | 0.000 | 0.000 | 0.000 | 0.145 |

| Problem 697 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 133 | 133 | 185 | 0 | 0 | 540 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 0.00 | 0.00 | 4.06 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.161 | 0.522 | 1.003 | 0.000 | 0.574 | 0.000 | 0.000 | 0.000 | 0.312 |

| Problem 698 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 147 | 147 | 217 | 0 | 0 | 769 | 0 | 0 | -1 | 0 |
| N.S. | 1 | 1.00 | 1.48 | 0.00 | 0.00 | 5.23 | 0.00 | 0.00 | -0.01 | 0.00 |
| time (sec) | N/A | 0.142 | 0.812 | 1.091 | 0.000 | 0.794 | 0.000 | 0.000 | 0.000 | 0.851 |

| Problem 699 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 20 | 20 | 20 | 21 | 29 | 25 | 46 | 35 | 22 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 1.45 | 1.25 | 2.30 | 1.75 | 1.10 | 0.00 |
| time (sec) | N/A | 0.004 | 0.018 | 0.046 | 0.824 | 0.420 | 3.098 | 0.164 | 4.795 | 0.036 |

| Problem 700 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 97 | 26 | 35 | 32 | 0 | 52 | 47 | 0 |
| N.S. | 1 | 1.00 | 3.59 | 0.96 | 1.30 | 1.19 | 0.00 | 1.93 | 1.74 | 0.00 |
| time (sec) | N/A | 0.009 | 0.132 | 0.044 | 0.889 | 0.433 | 0.000 | 0.191 | 4.882 | 0.040 |

| Problem 701 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 97 | 26 | 35 | 32 | 0 | 56 | 47 | 0 |
| N.S. | 1 | 1.00 | 3.59 | 0.96 | 1.30 | 1.19 | 0.00 | 2.07 | 1.74 | 0.00 |
| time (sec) | N/A | 0.009 | 0.096 | 0.045 | 0.692 | 0.434 | 0.000 | 0.262 | 4.902 | 0.045 |

| Problem 702 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad | I.A. |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | A | F(-1) | B | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD | Yes |
| size | 27 | 27 | 101 | 0 | 39 | 35 | 0 | 66 | 54 | 0 |
| N.S. | 1 | 1.00 | 3.74 | 0.00 | 1.44 | 1.30 | 0.00 | 2.44 | 2.00 | 0.00 |
| time (sec) | N/A | 0.012 | 0.162 | 0.688 | 0.957 | 0.451 | 0.000 | 0.253 | 4.863 | 0.109 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [102] had the largest ratio of [.4706]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 2 | A | 2 | 1 | 1.00 | 16 | 0.062 |
| 3 | A | 2 | 1 | 1.00 | 15 | 0.067 |
| 4 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 5 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 6 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 7 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 8 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 9 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 10 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 11 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 12 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 13 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 14 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 15 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 16 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 17 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 18 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 19 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 20 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 21 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 22 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 23 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 24 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 25 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 26 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 27 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 28 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 29 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 30 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 31 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 32 | A | 2 | 1 | 1.00 | 17 | 0.059 |
| 33 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 34 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 35 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 36 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 37 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 38 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 39 | A | 3 | 2 | 1.00 | 20 | 0.100 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 40 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 41 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 42 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 43 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 44 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 45 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 46 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 47 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 48 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 49 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 50 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 51 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 52 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 53 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 54 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 55 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 56 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 57 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 58 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 59 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 60 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 61 | A | 7 | 7 | 1.00 | 18 | 0.389 |
| 62 | A | 7 | 7 | 1.00 | 17 | 0.412 |
| 63 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 64 | A | 7 | 7 | 1.00 | 20 | 0.350 |
| 65 | A | 7 | 7 | 1.00 | 20 | 0.350 |
| 66 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 67 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 68 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 69 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 70 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 71 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 72 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 73 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 74 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 75 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 76 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 77 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 78 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 79 | A | 7 | 7 | 1.00 | 18 | 0.389 |
| 80 | A | 7 | 7 | 1.00 | 17 | 0.412 |
| 81 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 82 | A | 8 | 8 | 1.01 | 20 | 0.400 |
| 83 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 84 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 85 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 86 | A | 9 | 8 | 1.00 | 20 | 0.400 |
| 87 | A | 3 | 2 | 1.00 | 20 | 0.100 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 88 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 89 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 90 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 91 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 92 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 93 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 94 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 95 | A | 10 | 9 | 1.00 | 20 | 0.450 |
| 96 | A | 10 | 9 | 1.00 | 20 | 0.450 |
| 97 | A | 9 | 9 | 1.00 | 20 | 0.450 |
| 98 | A | 9 | 9 | 1.00 | 20 | 0.450 |
| 99 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 100 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 101 | A | 8 | 8 | 1.00 | 18 | 0.444 |
| 102 | A | 8 | 8 | 1.00 | 17 | 0.471 |
| 103 | A | 9 | 9 | 1.00 | 20 | 0.450 |
| 104 | A | 9 | 9 | 1.00 | 20 | 0.450 |
| 105 | A | 10 | 9 | 1.00 | 20 | 0.450 |
| 106 | A | 10 | 9 | 1.00 | 20 | 0.450 |
| 107 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 108 | A | 15 | 8 | 1.00 | 22 | 0.364 |
| 109 | A | 14 | 8 | 1.00 | 22 | 0.364 |
| 110 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 111 | A | 13 | 7 | 1.00 | 22 | 0.318 |
| 112 | A | 13 | 7 | 1.00 | 22 | 0.318 |
| 113 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 114 | A | 13 | 7 | 1.00 | 20 | 0.350 |
| 115 | A | 13 | 7 | 1.00 | 19 | 0.368 |
| 116 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 117 | A | 15 | 8 | 1.00 | 22 | 0.364 |
| 118 | A | 14 | 8 | 1.00 | 22 | 0.364 |
| 119 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 120 | A | 16 | 9 | 1.00 | 22 | 0.409 |
| 121 | A | 15 | 9 | 1.00 | 22 | 0.409 |
| 122 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 123 | A | 17 | 9 | 1.00 | 22 | 0.409 |
| 124 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 125 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 126 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 127 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 128 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 129 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 130 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 131 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 132 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 133 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 134 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 135 | A | 2 | 1 | 1.00 | 22 | 0.045 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 136 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 137 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 138 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 139 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 140 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 141 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 142 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 143 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 144 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 145 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 146 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 147 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 148 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 149 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 150 | A | 2 | 1 | 1.00 | 22 | 0.045 |
| 151 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 152 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 153 | A | 12 | 8 | 1.00 | 22 | 0.364 |
| 154 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 155 | A | 12 | 8 | 1.00 | 22 | 0.364 |
| 156 | A | 12 | 8 | 1.00 | 22 | 0.364 |
| 157 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 158 | A | 12 | 8 | 1.00 | 22 | 0.364 |
| 159 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 160 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 161 | A | 12 | 8 | 1.00 | 22 | 0.364 |
| 162 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 163 | A | 12 | 8 | 1.00 | 22 | 0.364 |
| 164 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 165 | A | 5 | 5 | 1.01 | 22 | 0.227 |
| 166 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 167 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 168 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 169 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 170 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 171 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 172 | A | 14 | 10 | 1.00 | 22 | 0.454 |
| 173 | A | 6 | 6 | 1.01 | 22 | 0.273 |
| 174 | A | 14 | 10 | 1.00 | 22 | 0.454 |
| 175 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 176 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 177 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 178 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 179 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 180 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 181 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 182 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 183 | A | 3 | 2 | 1.00 | 22 | 0.091 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 184 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 185 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 186 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 187 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 188 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 189 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 190 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 191 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 192 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 193 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 194 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 195 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 196 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 197 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 198 | A | 6 | 5 | 1.02 | 22 | 0.227 |
| 199 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 200 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 201 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 202 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 203 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 204 | A | 6 | 5 | 1.00 | 26 | 0.192 |
| 205 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 206 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 207 | A | 6 | 5 | 1.00 | 26 | 0.192 |
| 208 | A | 7 | 6 | 1.00 | 26 | 0.231 |
| 209 | A | 5 | 4 | 1.00 | 26 | 0.154 |
| 210 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 211 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 212 | A | 6 | 5 | 1.00 | 26 | 0.192 |
| 213 | A | 7 | 6 | 1.00 | 26 | 0.231 |
| 214 | A | 1 | 1 | 1.00 | 24 | 0.042 |
| 215 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 216 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 217 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 218 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 219 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 220 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 221 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 222 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 223 | A | 7 | 5 | 1.00 | 27 | 0.185 |
| 224 | A | 7 | 5 | 1.00 | 27 | 0.185 |
| 225 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 226 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 227 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 228 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 229 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 230 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 231 | A | 5 | 4 | 1.00 | 27 | 0.148 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 232 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 233 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 234 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 235 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 236 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 237 | A | 8 | 7 | 1.00 | 25 | 0.280 |
| 238 | A | 7 | 5 | 1.00 | 27 | 0.185 |
| 239 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 240 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 241 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 242 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 243 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 244 | A | 9 | 8 | 1.00 | 27 | 0.296 |
| 245 | A | 1 | 1 | 1.00 | 33 | 0.030 |
| 246 | A | 1 | 1 | 1.00 | 35 | 0.029 |
| 247 | A | 1 | 1 | 1.00 | 35 | 0.029 |
| 248 | A | 1 | 1 | 1.00 | 37 | 0.027 |
| 249 | A | 1 | 1 | 1.00 | 33 | 0.030 |
| 250 | A | 1 | 1 | 1.00 | 35 | 0.029 |
| 251 | A | 1 | 1 | 1.00 | 36 | 0.028 |
| 252 | A | 1 | 1 | 1.00 | 36 | 0.028 |
| 253 | A | 6 | 5 | 1.00 | 24 | 0.208 |
| 254 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 255 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 256 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 257 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 258 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 259 | A | 6 | 5 | 1.00 | 24 | 0.208 |
| 260 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 261 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 262 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 263 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 264 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 265 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 266 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 267 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 268 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 269 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 270 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 271 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 272 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 273 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 274 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 275 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 276 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 277 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 278 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 279 | A | 9 | 7 | 1.00 | 27 | 0.259 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 280 | A | 7 | 7 | 1.00 | 27 | 0.259 |
| 281 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 282 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 283 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 284 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 285 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 286 | A | 9 | 7 | 1.00 | 27 | 0.259 |
| 287 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 288 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 289 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 290 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 291 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 292 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 293 | A | 9 | 7 | 1.00 | 27 | 0.259 |
| 294 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 295 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 296 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 297 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 298 | A | 8 | 7 | 1.00 | 27 | 0.259 |
| 299 | A | 9 | 8 | 1.00 | 27 | 0.296 |
| 300 | A | 10 | 8 | 1.00 | 27 | 0.296 |
| 301 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 302 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 303 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 304 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 305 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 306 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 307 | A | 6 | 5 | 1.00 | 24 | 0.208 |
| 308 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 309 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 310 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 311 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 312 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 313 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 314 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 315 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 316 | A | 5 | 5 | 0.99 | 24 | 0.208 |
| 317 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 318 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 319 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 320 | A | 9 | 7 | 1.00 | 24 | 0.292 |
| 321 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 322 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 323 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 324 | A | 4 | 4 | 1.00 | 26 | 0.154 |
| 325 | A | 7 | 7 | 1.00 | 26 | 0.269 |
| 326 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 327 | A | 6 | 6 | 1.00 | 26 | 0.231 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 328 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 329 | A | 8 | 7 | 1.00 | 26 | 0.269 |
| 330 | A | 7 | 6 | 1.00 | 26 | 0.231 |
| 331 | A | 7 | 6 | 1.00 | 26 | 0.231 |
| 332 | A | 9 | 7 | 1.00 | 26 | 0.269 |
| 333 | A | 8 | 6 | 1.00 | 26 | 0.231 |
| 334 | A | 8 | 6 | 1.00 | 26 | 0.231 |
| 335 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 336 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 337 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 338 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 339 | A | 5 | 5 | 1.00 | 26 | 0.192 |
| 340 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 341 | A | 6 | 6 | 1.00 | 26 | 0.231 |
| 342 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 343 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 344 | A | 8 | 7 | 1.00 | 28 | 0.250 |
| 345 | A | 8 | 7 | 1.00 | 28 | 0.250 |
| 346 | A | 7 | 7 | 1.00 | 28 | 0.250 |
| 347 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 348 | A | 10 | 6 | 1.00 | 28 | 0.214 |
| 349 | A | 13 | 8 | 1.00 | 28 | 0.286 |
| 350 | A | 12 | 8 | 1.00 | 28 | 0.286 |
| 351 | C | 2 | 2 | 0.25 | 28 | 0.071 |
| 352 | C | 2 | 2 | 0.28 | 28 | 0.071 |
| 353 | C | 2 | 2 | 0.33 | 26 | 0.077 |
| 354 | C | 2 | 2 | 0.49 | 28 | 0.071 |
| 355 | C | 2 | 2 | 0.64 | 28 | 0.071 |
| 356 | C | 2 | 2 | 1.16 | 28 | 0.071 |
| 357 | C | 2 | 2 | 1.78 | 28 | 0.071 |
| 358 | C | 2 | 2 | 0.15 | 25 | 0.080 |
| 359 | A | 8 | 7 | 1.00 | 28 | 0.250 |
| 360 | A | 8 | 7 | 1.00 | 28 | 0.250 |
| 361 | A | 7 | 7 | 1.00 | 28 | 0.250 |
| 362 | A | 6 | 6 | 1.00 | 28 | 0.214 |
| 363 | A | 10 | 6 | 1.00 | 28 | 0.214 |
| 364 | A | 13 | 8 | 1.00 | 28 | 0.286 |
| 365 | A | 12 | 8 | 1.00 | 28 | 0.286 |
| 366 | C | 2 | 2 | 0.25 | 28 | 0.071 |
| 367 | C | 2 | 2 | 0.29 | 28 | 0.071 |
| 368 | C | 2 | 2 | 0.30 | 25 | 0.080 |
| 369 | C | 2 | 2 | 0.50 | 28 | 0.071 |
| 370 | C | 2 | 2 | 0.66 | 28 | 0.071 |
| 371 | C | 2 | 2 | 1.17 | 28 | 0.071 |
| 372 | C | 2 | 2 | 1.66 | 28 | 0.071 |
| 373 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 374 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 375 | A | 7 | 6 | 1.00 | 22 | 0.273 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 376 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 377 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 378 | A | 10 | 7 | 1.00 | 22 | 0.318 |
| 379 | A | 11 | 8 | 1.00 | 22 | 0.364 |
| 380 | A | 15 | 10 | 1.47 | 22 | 0.454 |
| 381 | A | 14 | 9 | 1.53 | 22 | 0.409 |
| 382 | A | 7 | 7 | 1.39 | 19 | 0.368 |
| 383 | A | 8 | 8 | 1.32 | 22 | 0.364 |
| 384 | A | 9 | 8 | 1.13 | 22 | 0.364 |
| 385 | A | 9 | 8 | 1.24 | 22 | 0.364 |
| 386 | C | 1 | 1 | 0.11 | 20 | 0.050 |
| 387 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 388 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 389 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 390 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 391 | A | 10 | 7 | 1.00 | 22 | 0.318 |
| 392 | A | 11 | 8 | 1.00 | 22 | 0.364 |
| 393 | A | 14 | 10 | 1.42 | 22 | 0.454 |
| 394 | A | 14 | 9 | 1.49 | 22 | 0.409 |
| 395 | A | 7 | 7 | 1.39 | 20 | 0.350 |
| 396 | A | 8 | 8 | 1.33 | 22 | 0.364 |
| 397 | A | 9 | 8 | 1.13 | 22 | 0.364 |
| 398 | C | 1 | 1 | 0.09 | 22 | 0.045 |
| 399 | C | 1 | 1 | 0.09 | 22 | 0.045 |
| 400 | A | 11 | 9 | 1.00 | 22 | 0.409 |
| 401 | A | 11 | 9 | 1.00 | 22 | 0.409 |
| 402 | A | 9 | 8 | 1.00 | 22 | 0.364 |
| 403 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 404 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 405 | A | 11 | 8 | 1.00 | 22 | 0.364 |
| 406 | A | 13 | 9 | 1.00 | 22 | 0.409 |
| 407 | C | 1 | 1 | 0.15 | 22 | 0.045 |
| 408 | C | 1 | 1 | 0.17 | 22 | 0.045 |
| 409 | C | 1 | 1 | 0.36 | 22 | 0.045 |
| 410 | A | 8 | 8 | 1.32 | 19 | 0.421 |
| 411 | C | 1 | 1 | 1.65 | 22 | 0.045 |
| 412 | C | 1 | 1 | 2.76 | 22 | 0.045 |
| 413 | C | 1 | 1 | 3.97 | 22 | 0.045 |
| 414 | A | 8 | 7 | 1.00 | 24 | 0.292 |
| 415 | A | 8 | 7 | 1.00 | 24 | 0.292 |
| 416 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 417 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 418 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 419 | A | 13 | 9 | 1.00 | 24 | 0.375 |
| 420 | A | 12 | 9 | 1.00 | 24 | 0.375 |
| 421 | C | 2 | 2 | 0.19 | 24 | 0.083 |
| 422 | C | 2 | 2 | 0.23 | 24 | 0.083 |
| 423 | C | 2 | 2 | 0.27 | 22 | 0.091 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 424 | C | 2 | 2 | 0.52 | 24 | 0.083 |
| 425 | C | 2 | 2 | 0.71 | 24 | 0.083 |
| 426 | C | 2 | 2 | 1.75 | 24 | 0.083 |
| 427 | C | 2 | 2 | 2.85 | 24 | 0.083 |
| 428 | A | 8 | 7 | 1.00 | 24 | 0.292 |
| 429 | A | 8 | 7 | 1.00 | 24 | 0.292 |
| 430 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 431 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 432 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 433 | A | 13 | 9 | 1.00 | 24 | 0.375 |
| 434 | A | 12 | 9 | 1.00 | 24 | 0.375 |
| 435 | C | 2 | 2 | 0.19 | 24 | 0.083 |
| 436 | C | 2 | 2 | 0.24 | 24 | 0.083 |
| 437 | C | 2 | 2 | 0.25 | 21 | 0.095 |
| 438 | C | 2 | 2 | 0.53 | 24 | 0.083 |
| 439 | C | 2 | 2 | 0.72 | 24 | 0.083 |
| 440 | C | 2 | 2 | 1.75 | 24 | 0.083 |
| 441 | C | 2 | 2 | 2.56 | 24 | 0.083 |
| 442 | A | 9 | 7 | 1.00 | 24 | 0.292 |
| 443 | A | 8 | 7 | 1.00 | 24 | 0.292 |
| 444 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 445 | A | 11 | 8 | 1.00 | 24 | 0.333 |
| 446 | A | 15 | 9 | 1.00 | 24 | 0.375 |
| 447 | A | 14 | 10 | 1.00 | 24 | 0.417 |
| 448 | C | 2 | 2 | 0.19 | 24 | 0.083 |
| 449 | C | 2 | 2 | 0.23 | 22 | 0.091 |
| 450 | C | 2 | 2 | 0.25 | 24 | 0.083 |
| 451 | C | 2 | 2 | 0.45 | 24 | 0.083 |
| 452 | C | 2 | 2 | 0.68 | 24 | 0.083 |
| 453 | C | 2 | 2 | 0.82 | 24 | 0.083 |
| 454 | C | 2 | 2 | 3.69 | 24 | 0.083 |
| 455 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 456 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 457 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 458 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 459 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 460 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 461 | A | 11 | 8 | 1.00 | 24 | 0.333 |
| 462 | A | 15 | 9 | 1.44 | 24 | 0.375 |
| 463 | A | 14 | 8 | 1.48 | 24 | 0.333 |
| 464 | A | 7 | 7 | 1.40 | 21 | 0.333 |
| 465 | A | 8 | 8 | 1.34 | 24 | 0.333 |
| 466 | A | 9 | 8 | 1.27 | 24 | 0.333 |
| 467 | A | 9 | 8 | 1.21 | 24 | 0.333 |
| 468 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 469 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 470 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 471 | A | 5 | 5 | 1.00 | 24 | 0.208 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 472 | A | 10 | 7 | 1.00 | 24 | 0.292 |
| 473 | A | 11 | 8 | 1.00 | 24 | 0.333 |
| 474 | A | 16 | 9 | 1.43 | 24 | 0.375 |
| 475 | A | 14 | 8 | 1.48 | 24 | 0.333 |
| 476 | A | 7 | 7 | 1.40 | 22 | 0.318 |
| 477 | A | 8 | 8 | 1.34 | 24 | 0.333 |
| 478 | A | 9 | 8 | 1.25 | 24 | 0.333 |
| 479 | A | 11 | 7 | 1.00 | 24 | 0.292 |
| 480 | A | 9 | 7 | 1.00 | 24 | 0.292 |
| 481 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 482 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 483 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 484 | A | 11 | 8 | 1.00 | 24 | 0.333 |
| 485 | A | 13 | 9 | 1.00 | 24 | 0.375 |
| 486 | C | 2 | 2 | 0.21 | 24 | 0.083 |
| 487 | C | 2 | 2 | 0.26 | 24 | 0.083 |
| 488 | C | 2 | 2 | 0.53 | 24 | 0.083 |
| 489 | A | 8 | 8 | 1.33 | 21 | 0.381 |
| 490 | C | 2 | 2 | 2.37 | 24 | 0.083 |
| 491 | C | 2 | 2 | 3.31 | 24 | 0.083 |
| 492 | C | 2 | 2 | 4.23 | 24 | 0.083 |
| 493 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 494 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 495 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 496 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 497 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 498 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 499 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 500 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 501 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 502 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 503 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 504 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 505 | A | 20 | 8 | 1.00 | 22 | 0.364 |
| 506 | A | 19 | 7 | 1.00 | 22 | 0.318 |
| 507 | A | 19 | 7 | 1.00 | 22 | 0.318 |
| 508 | A | 19 | 7 | 1.00 | 22 | 0.318 |
| 509 | A | 19 | 7 | 1.00 | 19 | 0.368 |
| 510 | A | 21 | 8 | 1.00 | 22 | 0.364 |
| 511 | A | 20 | 8 | 1.00 | 22 | 0.364 |
| 512 | A | 22 | 9 | 1.00 | 22 | 0.409 |
| 513 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 514 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 515 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 516 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 517 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 518 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 519 | A | 7 | 5 | 1.00 | 24 | 0.208 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 520 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 521 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 522 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 523 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 524 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 525 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 526 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 527 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 528 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 529 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 530 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 531 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 532 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 533 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 534 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 535 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 536 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 537 | A | 8 | 8 | 1.00 | 24 | 0.333 |
| 538 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 539 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 540 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 541 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 542 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 543 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 544 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 545 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 546 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 547 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 548 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 549 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 550 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 551 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 552 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 553 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 554 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 555 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 556 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 557 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 558 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 559 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 560 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 561 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 562 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 563 | A | 5 | 4 | 1.00 | 24 | 0.167 |
| 564 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 565 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 566 | A | 6 | 4 | 1.00 | 24 | 0.167 |
| 567 | A | 7 | 5 | 1.00 | 24 | 0.208 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 568 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 569 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 570 | A | 3 | 3 | 1.00 | 24 | 0.125 |
| 571 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 572 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 573 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 574 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 575 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 576 | A | 7 | 5 | 1.00 | 24 | 0.208 |
| 577 | A | 8 | 6 | 1.00 | 24 | 0.250 |
| 578 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 579 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 580 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 581 | A | 6 | 6 | 1.00 | 24 | 0.250 |
| 582 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 583 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 584 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 585 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 586 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 587 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 588 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 589 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 590 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 591 | A | 5 | 3 | 1.00 | 22 | 0.136 |
| 592 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 593 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 594 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 595 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 596 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 597 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 598 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 599 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 600 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 601 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 602 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 603 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 604 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 605 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 606 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 607 | A | 5 | 3 | 1.00 | 22 | 0.136 |
| 608 | A | 4 | 3 | 1.00 | 22 | 0.136 |
| 609 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 610 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 611 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 612 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 613 | A | 6 | 5 | 1.00 | 19 | 0.263 |
| 614 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 615 | A | 7 | 6 | 1.00 | 22 | 0.273 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 616 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 617 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 618 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 619 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 620 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 621 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 622 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 623 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 624 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 625 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 626 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 627 | A | 6 | 5 | 1.02 | 22 | 0.227 |
| 628 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 629 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 630 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 631 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 632 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 633 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 634 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 635 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 636 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 637 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 638 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 639 | A | 6 | 6 | 1.00 | 22 | 0.273 |
| 640 | A | 6 | 4 | 1.00 | 28 | 0.143 |
| 641 | A | 5 | 4 | 1.00 | 28 | 0.143 |
| 642 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 643 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 644 | A | 4 | 4 | 1.00 | 28 | 0.143 |
| 645 | A | 1 | 1 | 1.00 | 28 | 0.036 |
| 646 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 647 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 648 | A | 4 | 2 | 1.00 | 28 | 0.071 |
| 649 | A | 5 | 3 | 1.00 | 28 | 0.107 |
| 650 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 651 | A | 3 | 3 | 1.00 | 28 | 0.107 |
| 652 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 653 | A | 1 | 1 | 1.00 | 28 | 0.036 |
| 654 | A | 2 | 2 | 1.00 | 28 | 0.071 |
| 655 | A | 3 | 2 | 1.00 | 28 | 0.071 |
| 656 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 657 | A | 1 | 1 | 1.00 | 33 | 0.030 |
| 658 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 659 | A | 3 | 2 | 1.00 | 22 | 0.091 |
| 660 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 661 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 662 | A | 3 | 2 | 1.00 | 24 | 0.083 |
| 663 | A | 3 | 2 | 1.00 | 26 | 0.077 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 664 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 665 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 666 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 667 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 668 | A | 3 | 2 | 1.00 | 24 | 0.083 |
| 669 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 670 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 671 | A | 3 | 2 | 1.00 | 26 | 0.077 |
| 672 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 673 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 674 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 675 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 676 | A | 1 | 1 | 1.00 | 31 | 0.032 |
| 677 | A | 2 | 1 | 1.12 | 17 | 0.059 |
| 678 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 679 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 680 | A | 3 | 2 | 1.00 | 21 | 0.095 |
| 681 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 682 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 683 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 684 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 685 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 686 | A | 7 | 7 | 1.00 | 26 | 0.269 |
| 687 | A | 8 | 6 | 1.00 | 30 | 0.200 |
| 688 | A | 7 | 6 | 1.00 | 30 | 0.200 |
| 689 | A | 6 | 6 | 1.00 | 30 | 0.200 |
| 690 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 691 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 692 | A | 3 | 3 | 1.00 | 30 | 0.100 |
| 693 | A | 9 | 7 | 1.00 | 30 | 0.233 |
| 694 | A | 8 | 7 | 1.00 | 30 | 0.233 |
| 695 | A | 7 | 7 | 1.00 | 30 | 0.233 |
| 696 | A | 6 | 6 | 1.00 | 30 | 0.200 |
| 697 | A | 6 | 6 | 1.00 | 30 | 0.200 |
| 698 | A | 6 | 6 | 1.00 | 30 | 0.200 |
| 699 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 700 | A | 1 | 1 | 1.00 | 27 | 0.037 |
| 701 | A | 1 | 1 | 1.00 | 27 | 0.037 |
| 702 | A | 1 | 1 | 1.00 | 27 | 0.037 |

# Chapter 3

# Listing of integrals

## Local contents

## 3.1 $\int x^2 \left(a+b x^3\right) \left(A+B x^3\right) \, dx$

**Optimal**. Leaf size=33

$$\frac{1}{6} x^6 (a B+A b)+\frac{1}{3} a A x^3+\frac{1}{9} b B x^9$$

**Rubi** [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{1}{6} x^6 (a B+A b)+\frac{1}{3} a A x^3+\frac{1}{9} b B x^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(a+b x^3\right) \left(A+B x^3\right) \, dx &= \frac{1}{3} \text{Subst}\left(\int (a+b x)(A+B x) \, dx,x,x^3\right)\\ &= \frac{1}{3} \text{Subst}\left(\int \left(a A+(A b+a B) x+b B x^2\right) \, dx,x,x^3\right)\\ &= \frac{1}{3} a A x^3+\frac{1}{6} (A b+a B) x^6+\frac{1}{9} b B x^9 \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{6} x^6 (a B+A b)+\frac{1}{3} a A x^3+\frac{1}{9} b B x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+b x^3\right) \left(A+B x^3\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3)*(A + B*x^3),x]
[Out] IntegrateAlgebraic[x^2*(a + b*x^3)*(A + B*x^3), x]
```

**fricas** [A] time = 0.65, size = 29, normalized size = 0.88

$$\frac{1}{9}x^9bB + \frac{1}{6}x^6aB + \frac{1}{6}x^6bA + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/9*x^9*b*B + 1/6*x^6*a*B + 1/6*x^6*b*A + 1/3*x^3*a*A
```

**giac** [A] time = 0.18, size = 29, normalized size = 0.88

$$\frac{1}{9}Bbx^9 + \frac{1}{6}Bax^6 + \frac{1}{6}Abx^6 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3
```

**maple** [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bb\,x^9}{9} + \frac{(Ab + Ba)\,x^6}{6} + \frac{Aa\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*(B*x^3+A),x)
[Out] 1/3*A*a*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

**maxima** [A] time = 0.57, size = 27, normalized size = 0.82

$$\frac{1}{9}Bbx^9 + \frac{1}{6}(Ba + Ab)x^6 + \frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

**mupad** [B] time = 0.20, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^9}{9} + \left(\frac{A\,b}{6} + \frac{B\,a}{6}\right)x^6 + \frac{A\,a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3),x)
[Out] x^6*((A*b)/6 + (B*a)/6) + (A*a*x^3)/3 + (B*b*x^9)/9
```

**sympy** [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Aax^3}{3} + \frac{Bbx^9}{9} + x^6\left(\frac{Ab}{6} + \frac{Ba}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*(B*x**3+A),x)
[Out] A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)
```

## 3.2 $\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=33

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

**Rubi** [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {448}

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx+(Ab+aB)x^4+bBx^7\right)\,dx\\&=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^3)*(A + B*x^3), x]
```

**fricas** [A] time = 0.70, size = 29, normalized size = 0.88

$$\frac{1}{8}x^8bB+\frac{1}{5}x^5aB+\frac{1}{5}x^5bA+\frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/8*x^8*b*B + 1/5*x^5*a*B + 1/5*x^5*b*A + 1/2*x^2*a*A
```

**giac [A]** time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{8}Bbx^8 + \frac{1}{5}Bax^5 + \frac{1}{5}Abx^5 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2
```

**maple [A]** time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bb\,x^8}{8} + \frac{(Ab + Ba)\,x^5}{5} + \frac{Aa\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*(B*x^3+A),x)
[Out] 1/2*A*a*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

**maxima [A]** time = 0.63, size = 27, normalized size = 0.82

$$\frac{1}{8}Bbx^8 + \frac{1}{5}(Ba + Ab)x^5 + \frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

**mupad [B]** time = 2.48, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^8}{8} + \left(\frac{A\,b}{5} + \frac{B\,a}{5}\right)\,x^5 + \frac{A\,a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3),x)
[Out] x^5*((A*b)/5 + (B*a)/5) + (A*a*x^2)/2 + (B*b*x^8)/8
```

**sympy [A]** time = 0.07, size = 29, normalized size = 0.88

$$\frac{Aax^2}{2} + \frac{Bbx^8}{8} + x^5\left(\frac{Ab}{5} + \frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*(B*x**3+A),x)
[Out] A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)
```

## 3.3 $\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=28

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

**Rubi** [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(A + B*x^3),x]
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aA+(Ab+aB)x^3+bBx^6\right)\,dx\\ &= aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(A + B*x^3),x]
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)*(A + B*x^3),x]
[Out] IntegrateAlgebraic[(a + b*x^3)*(A + B*x^3), x]
```

**fricas** [A] time = 1.08, size = 26, normalized size = 0.93

$$\frac{1}{7}x^7bB+\frac{1}{4}x^4aB+\frac{1}{4}x^4bA+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*B + 1/4*x^4*a*B + 1/4*x^4*b*A + x*a*A
```

**giac** [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{7}Bbx^7 + \frac{1}{4}Bax^4 + \frac{1}{4}Abx^4 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x
```

**maple** [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{Bb\,x^7}{7} + \frac{(Ab+Ba)\,x^4}{4} + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A),x)
```

```
[Out] A*a*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

**maxima** [A] time = 0.56, size = 24, normalized size = 0.86

$$\frac{1}{7}Bbx^7 + \frac{1}{4}(Ba+Ab)x^4 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

**mupad** [B] time = 0.03, size = 25, normalized size = 0.89

$$\frac{B\,b\,x^7}{7} + \left(\frac{A\,b}{4} + \frac{B\,a}{4}\right)\,x^4 + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^4*((A*b)/4 + (B*a)/4) + A*a*x + (B*b*x^7)/7
```

**sympy** [A] time = 0.06, size = 26, normalized size = 0.93

$$Aax + \frac{Bbx^7}{7} + x^4\left(\frac{Ab}{4} + \frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)
```

## 3.4 $\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx$

**Optimal**. Leaf size=29

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

**Rubi** [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(Ab+aB+\frac{aA}{x}+bBx\right)dx,x,x^3\right)\\
&= \frac{1}{3}(Ab+aB)x^3+\frac{1}{6}bBx^6+aA\log(x)
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)(A + Bx^3)}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x,x]
[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x, x]
```

**fricas [A]** time = 0.76, size = 25, normalized size = 0.86

$$\frac{1}{6} Bbx^6 + \frac{1}{3} (Ba + Ab)x^3 + Aa \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)
```

**giac [A]** time = 0.15, size = 28, normalized size = 0.97

$$\frac{1}{6} Bbx^6 + \frac{1}{3} Bax^3 + \frac{1}{3} Abx^3 + Aa \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")
[Out] 1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))
```

**maple [A]** time = 0.04, size = 28, normalized size = 0.97

$$\frac{Bb\,x^6}{6} + \frac{Ab\,x^3}{3} + \frac{Ba\,x^3}{3} + Aa\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x,x)
[Out] 1/6*b*B*x^6+1/3*A*x^3*b+1/3*B*a*x^3+A*a*ln(x)
```

**maxima [A]** time = 0.57, size = 28, normalized size = 0.97

$$\frac{1}{6} Bbx^6 + \frac{1}{3} (Ba + Ab)x^3 + \frac{1}{3} Aa \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)
```

**mupad [B]** time = 0.03, size = 26, normalized size = 0.90

$$x^3 \left(\frac{A\,b}{3} + \frac{B\,a}{3}\right) + \frac{B\,b\,x^6}{6} + A\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x,x)
[Out] x^3*((A*b)/3 + (B*a)/3) + (B*b*x^6)/6 + A*a*log(x)
```

**sympy [A]** time = 0.11, size = 27, normalized size = 0.93

$$Aa\log(x) + \frac{Bbx^6}{6} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x,x)
[Out] A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)
```

## 3.5 $\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx$

**Optimal**. Leaf size=31

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx &= \int \left(\frac{aA}{x^2}+(Ab+aB)x+bBx^4\right) dx \\
&= -\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^2, x]
```

**fricas [A]** time = 0.73, size = 29, normalized size = 0.94

$$\frac{2\,Bbx^6+5\,(Ba+Ab)x^3-10\,Aa}{10\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")
[Out] 1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x
```

**giac [A]** time = 0.15, size = 29, normalized size = 0.94

$$\frac{1}{5}Bbx^5 + \frac{1}{2}Bax^2 + \frac{1}{2}Abx^2 - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")
[Out] 1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x
```

**maple [A]** time = 0.05, size = 30, normalized size = 0.97

$$\frac{Bb\,x^5}{5} + \frac{Ab\,x^2}{2} + \frac{Ba\,x^2}{2} - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^2,x)
[Out] 1/5*b*B*x^5+1/2*A*b*x^2+1/2*B*a*x^2-A*a/x
```

**maxima [A]** time = 0.50, size = 27, normalized size = 0.87

$$\frac{1}{5}Bbx^5 + \frac{1}{2}(Ba + Ab)x^2 - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")
[Out] 1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x
```

**mupad [B]** time = 0.04, size = 28, normalized size = 0.90

$$x^2\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right) - \frac{A\,a}{x} + \frac{B\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^2,x)
[Out] x^2*((A*b)/2 + (B*a)/2) - (A*a)/x + (B*b*x^5)/5
```

**sympy [A]** time = 0.11, size = 26, normalized size = 0.84

$$-\frac{Aa}{x} + \frac{Bbx^5}{5} + x^2\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**2,x)
[Out] -A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)
```

## 3.6 $\int \frac{(a+bx^3)(A+Bx^3)}{x^3} dx$

**Optimal**. Leaf size=28

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) + (A*b + a*B)*x + (b*B*x^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^3} dx &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^3}+bBx^3\right) dx\\ &= -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 1.00

$$x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^3, x]
```

**fricas [A]** time = 0.79, size = 28, normalized size = 1.00

$$\frac{Bbx^6+4(Ba+Ab)x^3-2Aa}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")
[Out] 1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2
```

**giac [A]** time = 0.16, size = 23, normalized size = 0.82

$$\frac{1}{4}\,Bbx^4 + Bax + Abx - \frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")
[Out] 1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2
```

**maple [A]** time = 0.04, size = 24, normalized size = 0.86

$$\frac{Bb\,x^4}{4} + Abx + Bax - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^3,x)
[Out] 1/4*b*B*x^4+A*b*x+B*a*x-1/2*A*a/x^2
```

**maxima [A]** time = 0.48, size = 24, normalized size = 0.86

$$\frac{1}{4}\,Bbx^4 + (Ba + Ab)x - \frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")
[Out] 1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2
```

**mupad [B]** time = 2.34, size = 24, normalized size = 0.86

$$x\,\left(A\,b+B\,a\right)-\frac{A\,a}{2\,x^2}+\frac{B\,b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^3,x)
[Out] x*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^4)/4
```

**sympy [A]** time = 0.12, size = 24, normalized size = 0.86

$$-\frac{Aa}{2x^2} + \frac{Bbx^4}{4} + x\left(Ab + Ba\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**3,x)
[Out] -A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)
```

## 3.7 $\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx$

**Optimal**. Leaf size=29

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 76}

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2} dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(bB + \frac{aA}{x^2} + \frac{Ab+aB}{x}\right) dx, x, x^3\right) \\
&= -\frac{aA}{3x^3} + \frac{1}{3}bBx^3 + (Ab+aB)\log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$\log(x)(aB + Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^4,x]
[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^4, x]
```

**fricas [A]** time = 0.81, size = 30, normalized size = 1.03

$$\frac{Bbx^6 + 3\,(Ba + Ab)x^3\log(x) - Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")
[Out] 1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3
```

**giac [A]** time = 0.16, size = 40, normalized size = 1.38

$$\frac{1}{3}\,Bbx^3 + (Ba + Ab)\log\left(|x|\right) - \frac{Bax^3 + Abx^3 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")
[Out] 1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3
```

**maple [A]** time = 0.05, size = 26, normalized size = 0.90

$$\frac{Bb\,x^3}{3} + Ab\ln(x) + Ba\ln(x) - \frac{Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^4,x)
[Out] 1/3*B*b*x^3-1/3*A*a/x^3+A*b*ln(x)+B*a*ln(x)
```

**maxima [A]** time = 0.48, size = 28, normalized size = 0.97

$$\frac{1}{3}\,Bbx^3 + \frac{1}{3}\,(Ba + Ab)\log\left(x^3\right) - \frac{Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")
[Out] 1/3*B*b*x^3 + 1/3*(B*a + A*b)*log(x^3) - 1/3*A*a/x^3
```

**mupad [B]** time = 0.04, size = 25, normalized size = 0.86

$$\ln(x)\,\left(A\,b + B\,a\right) - \frac{A\,a}{3\,x^3} + \frac{B\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^4,x)
[Out] log(x)*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^3)/3
```

**sympy [A]** time = 0.21, size = 26, normalized size = 0.90

$$-\frac{Aa}{3x^3} + \frac{Bbx^3}{3} + (Ab + Ba)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**4,x)
[Out] -A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*log(x)
```

## 3.8 $\int \frac{(a+bx^3)(A+Bx^3)}{x^5} \, dx$

**Optimal**. Leaf size=31

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/x + (b*B*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^5} \, dx &= \int \left(\frac{aA}{x^5}+\frac{Ab+aB}{x^2}+bBx\right) dx\\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 32, normalized size = 1.03

$$\frac{-aB-Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 + (-(A*b) - a*B)/x + (b*B*x^2)/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^5, x]
```

**fricas [A]** time = 1.29, size = 29, normalized size = 0.94

$$\frac{2\,Bbx^6-4\,(Ba+Ab)x^3-Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")
[Out] 1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4
```

**giac [A]** time = 0.18, size = 31, normalized size = 1.00

$$\frac{1}{2}\,Bbx^2 - \frac{4\,Bax^3 + 4\,Abx^3 + Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")
[Out] 1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4
```

**maple [A]** time = 0.05, size = 28, normalized size = 0.90

$$\frac{Bb\,x^2}{2} - \frac{Ab + Ba}{x} - \frac{Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^5,x)
[Out] 1/2*b*B*x^2-1/4*a*A/x^4-(A*b+B*a)/x
```

**maxima [A]** time = 0.63, size = 29, normalized size = 0.94

$$\frac{1}{2}\,Bbx^2 - \frac{4\,(Ba + Ab)x^3 + Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")
[Out] 1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4
```

**mupad [B]** time = 0.03, size = 29, normalized size = 0.94

$$\frac{B\,b\,x^2}{2} - \frac{(A\,b + B\,a)\;x^3 + \frac{A\,a}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^5,x)
[Out] (B*b*x^2)/2 - ((A*a)/4 + x^3*(A*b + B*a))/x^4
```

**sympy [A]** time = 0.25, size = 31, normalized size = 1.00

$$\frac{Bbx^2}{2} + \frac{-Aa + x^3\,(-4Ab - 4Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**5,x)
[Out] B*b*x**2/2 + (-A*a + x**3*(-4*A*b - 4*B*a))/(4*x**4)
```

## 3.9 $\int \frac{(a+bx^3)(A+Bx^3)}{x^6} \, dx$

**Optimal**. Leaf size=28

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(2*x^2) + b*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx &= \int \left(bB+\frac{aA}{x^6}+\frac{Ab+aB}{x^3}\right)dx\\ &= -\frac{aA}{5x^5}-\frac{Ab+aB}{2x^2}+bBx\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.07

$$\frac{-aB-Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 + (-(A*b) - a*B)/(2*x^2) + b*B*x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^6, x]
```

**fricas [A]** time = 0.74, size = 29, normalized size = 1.04

$$\frac{10\,Bbx^6-5\,(Ba+Ab)x^3-2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")
[Out] 1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5
```

**giac [A]** time = 0.18, size = 29, normalized size = 1.04

$$Bbx - \frac{5\,Bax^3 + 5\,Abx^3 + 2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")
[Out] B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5
```

**maple [A]** time = 0.04, size = 25, normalized size = 0.89

$$Bbx - \frac{Ab + Ba}{2x^2} - \frac{Aa}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^6,x)
[Out] B*b*x-1/5*a*A/x^5-1/2*(A*b+B*a)/x^2
```

**maxima [A]** time = 0.49, size = 27, normalized size = 0.96

$$Bbx - \frac{5\,(Ba + Ab)x^3 + 2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")
[Out] B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5
```

**mupad [B]** time = 2.32, size = 28, normalized size = 1.00

$$B\,b\,x - \frac{\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right)\,x^3 + \frac{A\,a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^6,x)
[Out] B*b*x - ((A*a)/5 + x^3*((A*b)/2 + (B*a)/2))/x^5
```

**sympy [A]** time = 0.29, size = 29, normalized size = 1.04

$$Bbx + \frac{-2Aa + x^3\,(-5Ab - 5Ba)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**6,x)
[Out] B*b*x + (-2*A*a + x**3*(-5*A*b - 5*B*a))/(10*x**5)
```

## 3.10 $\int \frac{(a+bx^3)(A+Bx^3)}{x^7} dx$

**Optimal**. Leaf size=29

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(3*x^3) + b*B*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{aA}{x^3}+\frac{Ab+aB}{x^2}+\frac{bB}{x}\right)dx,x,x^3\right)\\ &= -\frac{aA}{6x^6}-\frac{Ab+aB}{3x^3}+bB\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 31, normalized size = 1.07

$$\frac{-aB-Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^7,x]
[Out] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^7, x]
```

**fricas [A]** time = 0.93, size = 31, normalized size = 1.07

$$\frac{6\,Bbx^6\log(x) - 2\,(Ba + Ab)x^3 - Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="fricas")
[Out] 1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6
```

**giac [A]** time = 0.16, size = 37, normalized size = 1.28

$$Bb\log\left(|x|\right) - \frac{3\,Bbx^6 + 2\,Bax^3 + 2\,Abx^3 + Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="giac")
[Out] B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6
```

**maple [A]** time = 0.04, size = 28, normalized size = 0.97

$$Bb\ln(x) - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} - \frac{Aa}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^7,x)
[Out] -1/3/x^3*A*b-1/3/x^3*B*a-1/6*a*A/x^6+B*b*ln(x)
```

**maxima [A]** time = 0.51, size = 30, normalized size = 1.03

$$\frac{1}{3}\,Bb\log\left(x^3\right) - \frac{2\,(Ba + Ab)x^3 + Aa}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")
[Out] 1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6
```

**mupad [B]** time = 0.05, size = 29, normalized size = 1.00

$$B\,b\,\ln(x) - \frac{\left(\frac{A\,b}{3} + \frac{B\,a}{3}\right)\,x^3 + \frac{A\,a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^7,x)
[Out] B*b*log(x) - ((A*a)/6 + x^3*((A*b)/3 + (B*a)/3))/x^6
```

**sympy [A]** time = 0.54, size = 29, normalized size = 1.00

$$Bb\log\left(x\right) + \frac{-Aa + x^3\left(-2Ab - 2Ba\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**7,x)
```

```
[Out] B*b*log(x) + (-A*a + x**3*(-2*A*b - 2*B*a))/(6*x**6)
```

## 3.11 $\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=42

$$\frac{\left(a+bx^3\right)^3 (Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{\left(a+bx^3\right)^3 (Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^3)/(9*b^2) + (B*(a + b*x^3)^4)/(12*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^2(A+Bx)\, dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right) dx, x, x^3\right)\\ &= \frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 1.21

$$\frac{1}{36}x^3\left(12a^2A+4bx^6(2aB+Ab)+6ax^3(aB+2Ab)+3b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x^
9))/36
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^3)^2*(A + B*x^3), x]
```

**fricas [A]** time = 1.03, size = 53, normalized size = 1.26

$$\frac{1}{12}x^{12}b^2B + \frac{2}{9}x^9baB + \frac{1}{9}x^9b^2A + \frac{1}{6}x^6a^2B + \frac{1}{3}x^6baA + \frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^2*B + 2/9*x^9*b*a*B + 1/9*x^9*b^2*A + 1/6*x^6*a^2*B + 1/3*x^6*b
*a*A + 1/3*x^3*a^2*A
```

**giac [A]** time = 0.15, size = 53, normalized size = 1.26

$$\frac{1}{12}\,Bb^2x^{12} + \frac{2}{9}\,Babx^9 + \frac{1}{9}\,Ab^2x^9 + \frac{1}{6}\,Ba^2x^6 + \frac{1}{3}\,Aabx^6 + \frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*b
*x^6 + 1/3*A*a^2*x^3
```

**maple [A]** time = 0.04, size = 52, normalized size = 1.24

$$\frac{B\,b^2x^{12}}{12} + \frac{\left(b^2A+2abB\right)x^9}{9} + \frac{A\,a^2x^3}{3} + \frac{\left(2abA+a^2B\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/12*b^2*B*x^12+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*A*a^2*x
^3
```

**maxima [A]** time = 0.49, size = 51, normalized size = 1.21

$$\frac{1}{12}\,Bb^2x^{12} + \frac{1}{9}\left(2\,Bab+Ab^2\right)x^9 + \frac{1}{6}\left(Ba^2+2\,Aab\right)x^6 + \frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1
/3*A*a^2*x^3
```

**mupad [B]** time = 2.38, size = 51, normalized size = 1.21

$$x^6\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^9\left(\frac{A\,b^2}{9}+\frac{2\,B\,a\,b}{9}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^6*((B*a^2)/6 + (A*a*b)/3) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (A*a^2*x^3)/3
 + (B*b^2*x^12)/12
```

**sympy [A]** time = 0.08, size = 54, normalized size = 1.29

$$\frac{Aa^2x^3}{3} + \frac{Bb^2x^{12}}{12} + x^9\left(\frac{Ab^2}{9} + \frac{2Bab}{9}\right) + x^6\left(\frac{Aab}{3} + \frac{Ba^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*b
/3 + B*a**2/6)
```

## 3.12 $\int x \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=55

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

**Rubi [A]** time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx &= \int \left(a^2Ax+a(2Ab+aB)x^4+b(Ab+2aB)x^7+b^2Bx^{10}\right) \, dx\\ &= \frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^3)^2*(A + B*x^3), x]
```

**fricas [A]** time = 1.06, size = 53, normalized size = 0.96

$$\frac{1}{11}x^{11}b^2B+\frac{1}{4}x^8baB+\frac{1}{8}x^8b^2A+\frac{1}{5}x^5a^2B+\frac{2}{5}x^5baA+\frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^2*B + 1/4*x^8*b*a*B + 1/8*x^8*b^2*A + 1/5*x^5*a^2*B + 2/5*x^5*b
*a*A + 1/2*x^2*a^2*A
```

**giac [A]** time = 0.15, size = 53, normalized size = 0.96

$$\frac{1}{11}Bb^2x^{11} + \frac{1}{4}Babx^8 + \frac{1}{8}Ab^2x^8 + \frac{1}{5}Ba^2x^5 + \frac{2}{5}Aabx^5 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/11*B*b^2*x^11 + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*b
*x^5 + 1/2*A*a^2*x^2
```

**maple [A]** time = 0.05, size = 52, normalized size = 0.95

$$\frac{B\,b^2x^{11}}{11} + \frac{\left(b^2A + 2abB\right)x^8}{8} + \frac{A\,a^2x^2}{2} + \frac{\left(2abA + a^2B\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/11*b^2*B*x^11+1/8*(A*b^2+2*B*a*b)*x^8+1/5*(2*A*a*b+B*a^2)*x^5+1/2*a^2*A*x
^2
```

**maxima [A]** time = 0.61, size = 51, normalized size = 0.93

$$\frac{1}{11}Bb^2x^{11} + \frac{1}{8}\left(2\,Bab + Ab^2\right)x^8 + \frac{1}{5}\left(Ba^2 + 2\,Aab\right)x^5 + \frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1
/2*A*a^2*x^2
```

**mupad [B]** time = 0.04, size = 51, normalized size = 0.93

$$x^5\left(\frac{B\,a^2}{5} + \frac{2\,A\,b\,a}{5}\right) + x^8\left(\frac{A\,b^2}{8} + \frac{B\,a\,b}{4}\right) + \frac{A\,a^2\,x^2}{2} + \frac{B\,b^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^8*((A*b^2)/8 + (B*a*b)/4) + (A*a^2*x^2)/2
 + (B*b^2*x^11)/11
```

**sympy [A]** time = 0.08, size = 54, normalized size = 0.98

$$\frac{Aa^2x^2}{2} + \frac{Bb^2x^{11}}{11} + x^8\left(\frac{Ab^2}{8} + \frac{Bab}{4}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ba^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

## 3.13 $\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=50

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx &= \int \left(a^2A+a(2Ab+aB)x^3+b(Ab+2aB)x^6+b^2Bx^9\right) dx\\ &= a^2Ax+\frac{1}{4}a(2Ab+aB)x^4+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{10}b^2Bx^{10}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 50, normalized size = 1.00

$$a^2Ax+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{4}ax^4(aB+2Ab)+\frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2*(A + B*x^3), x]
```

**fricas [A]** time = 0.64, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}b^2B+\frac{2}{7}x^7baB+\frac{1}{7}x^7b^2A+\frac{1}{4}x^4a^2B+\frac{1}{2}x^4baA+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2*B + 2/7*x^7*b*a*B + 1/7*x^7*b^2*A + 1/4*x^4*a^2*B + 1/2*x^4*b
*a*A + x*a^2*A
```

**giac [A]** time = 0.17, size = 50, normalized size = 1.00

$$\frac{1}{10}\,Bb^2x^{10}+\frac{2}{7}\,Babx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{2}\,Aabx^4+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/10*B*b^2*x^10 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*b
*x^4 + A*a^2*x
```

**maple [A]** time = 0.04, size = 49, normalized size = 0.98

$$\frac{B\,b^2x^{10}}{10}+\frac{\left(b^2A+2abB\right)x^7}{7}+A\,a^2x+\frac{\left(2abA+a^2B\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/10*b^2*B*x^10+1/7*(A*b^2+2*B*a*b)*x^7+1/4*(2*A*a*b+B*a^2)*x^4+A*a^2*x
```

**maxima [A]** time = 0.66, size = 48, normalized size = 0.96

$$\frac{1}{10}\,Bb^2x^{10}+\frac{1}{7}\left(2\,Bab+Ab^2\right)x^7+\frac{1}{4}\left(Ba^2+2\,Aab\right)x^4+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A
*a^2*x
```

**mupad [B]** time = 0.04, size = 48, normalized size = 0.96

$$x^4\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{B\,b^2\,x^{10}}{10}+A\,a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (B*b^2*x^10)/
10 + A*a^2*x
```

**sympy [A]** time = 0.08, size = 51, normalized size = 1.02

$$Aa^2x+\frac{Bb^2x^{10}}{10}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 +
B*a**2/4)
```

## 3.14 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}\,dx$

Optimal. Leaf size=46

$$a^2A\log(x)+\frac{2}{3}aAbx^3+\frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{6}Ab^2x^6$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 80, 43}

$$a^2A\log(x)+\frac{2}{3}aAbx^3+\frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{6}Ab^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*b*x^3)/3 + (A*b^2*x^6)/6 + (B*(a + b*x^3)^3)/(9*b) + a^2*A*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x}\,dx,x,x^3\right)\\
&= \frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{3}A\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx,x,x^3\right)\\
&= \frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{3}A\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx,x,x^3\right)\\
&= \frac{2}{3}aAbx^3+\frac{1}{6}Ab^2x^6+\frac{B\left(a+bx^3\right)^3}{9b}+a^2A\log(x)
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 51, normalized size = 1.11

$$a^2A\log(x)+\frac{1}{6}bx^6(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]
[Out] (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*L
og[x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x,x]
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x, x]
```

**fricas** [A] time = 0.77, size = 49, normalized size = 1.07

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3+Aa^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + A*a
^2*log(x)
```

**giac** [A] time = 0.15, size = 52, normalized size = 1.13

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{3}\,Babx^6+\frac{1}{6}\,Ab^2x^6+\frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+Aa^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")
[Out] 1/9*B*b^2*x^9 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*log(abs(x))
```

**maple** [A] time = 0.04, size = 52, normalized size = 1.13

$$\frac{B\,b^2x^9}{9}+\frac{A\,b^2x^6}{6}+\frac{Bab\,x^6}{3}+\frac{2Aab\,x^3}{3}+\frac{B\,a^2x^3}{3}+A\,a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x,x)
[Out] 1/9*B*b^2*x^9+1/6*A*b^2*x^6+1/3*B*x^6*a*b+2/3*a*A*b*x^3+1/3*B*a^2*x^3+A*a^2
*ln(x)
```

**maxima** [A] time = 0.54, size = 52, normalized size = 1.13

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3+\frac{1}{3}\,Aa^2\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + 1/3
*A*a^2*log(x^3)
```

**mupad [B]** time = 0.04, size = 49, normalized size = 1.07

$$x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right)+\frac{B\,b^2\,x^9}{9}+A\,a^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x,x)
```

```
[Out] x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^6*((A*b^2)/6 + (B*a*b)/3) + (B*b^2*x^9)/9
 + A*a^2*log(x)
```

**sympy [A]** time = 0.15, size = 53, normalized size = 1.15

$$Aa^2\log(x)+\frac{Bb^2x^9}{9}+x^6\left(\frac{Ab^2}{6}+\frac{Bab}{3}\right)+x^3\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**9/9 + x**6*(A*b**2/6 + B*a*b/3) + x**3*(2*A*a*b/3
 + B*a**2/3)
```

## 3.15 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2}\,dx$

**Optimal**. Leaf size=53

$$-\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2}\,dx &= \int \left(\frac{a^2A}{x^2}+a(2Ab+aB)x+b(Ab+2aB)x^4+b^2Bx^7\right)dx\\ &= -\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 53, normalized size = 1.00

$$-\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^2, x]
```

**fricas** [A] time = 0.74, size = 53, normalized size = 1.00

$$\frac{5 Bb^2x^9 + 8\left(2 Bab + Ab^2\right)x^6 + 20\left(Ba^2 + 2 Aab\right)x^3 - 40 Aa^2}{40 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 40
*A*a^2)/x
```

**giac** [A] time = 0.15, size = 52, normalized size = 0.98

$$\frac{1}{8} Bb^2x^8 + \frac{2}{5} Babx^5 + \frac{1}{5} Ab^2x^5 + \frac{1}{2} Ba^2x^2 + Aabx^2 - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2 -
 A*a^2/x
```

**maple** [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{B b^2x^8}{8} + \frac{A b^2x^5}{5} + \frac{2Bab x^5}{5} + Aab x^2 + \frac{B a^2x^2}{2} - \frac{A a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^2,x)
```

```
[Out] 1/8*b^2*B*x^8+1/5*A*x^5*b^2+2/5*B*x^5*a*b+A*a*b*x^2+1/2*B*a^2*x^2-A*a^2/x
```

**maxima** [A] time = 0.59, size = 51, normalized size = 0.96

$$\frac{1}{8} Bb^2x^8 + \frac{1}{5}\left(2 Bab + Ab^2\right)x^5 + \frac{1}{2}\left(Ba^2 + 2 Aab\right)x^2 - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/2*(B*a^2 + 2*A*a*b)*x^2 - A*a
^2/x
```

**mupad** [B] time = 0.05, size = 50, normalized size = 0.94

$$x^2\left(\frac{B a^2}{2} + A b a\right) + x^5\left(\frac{A b^2}{5} + \frac{2 B a b}{5}\right) - \frac{A a^2}{x} + \frac{B b^2 x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^2,x)
```

```
[Out] x^2*((B*a^2)/2 + A*a*b) + x^5*((A*b^2)/5 + (2*B*a*b)/5) - (A*a^2)/x + (B*b^
2*x^8)/8
```

**sympy** [A] time = 0.15, size = 49, normalized size = 0.92

$$-\frac{Aa^2}{x} + \frac{Bb^2x^8}{8} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*a
**2/2)
```

## 3.16 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$

**Optimal**. Leaf size=50

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx &= \int \left(a(2Ab+aB)+\frac{a^2A}{x^3}+b(Ab+2aB)x^3+b^2Bx^6\right) dx\\ &= -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 1.00

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^3, x]
```

**fricas** [A] time = 0.83, size = 53, normalized size = 1.06

$$\frac{4\,Bb^2x^9 + 7\left(2\,Bab + Ab^2\right)x^6 + 28\left(Ba^2 + 2\,Aab\right)x^3 - 14\,Aa^2}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 14
*A*a^2)/x^2
```

**giac** [A] time = 0.17, size = 48, normalized size = 0.96

$$\frac{1}{7}\,Bb^2x^7 + \frac{1}{2}\,Babx^4 + \frac{1}{4}\,Ab^2x^4 + Ba^2x + 2\,Aabx - \frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*A
*a^2/x^2
```

**maple** [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{B\,b^2x^7}{7} + \frac{A\,b^2x^4}{4} + \frac{Bab\,x^4}{2} + 2Aabx + B\,a^2x - \frac{A\,a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^3,x)
```

```
[Out] 1/7*b^2*B*x^7+1/4*A*b^2*x^4+1/2*B*x^4*a*b+2*a*b*A*x+B*a^2*x-1/2*A*a^2/x^2
```

**maxima** [A] time = 0.49, size = 48, normalized size = 0.96

$$\frac{1}{7}\,Bb^2x^7 + \frac{1}{4}\left(2\,Bab + Ab^2\right)x^4 + \left(Ba^2 + 2\,Aab\right)x - \frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^2
/x^2
```

**mupad** [B] time = 0.05, size = 48, normalized size = 0.96

$$x^4\left(\frac{A\,b^2}{4} + \frac{B\,a\,b}{2}\right) + x\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{2\,x^2} + \frac{B\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^3,x)
```

```
[Out] x^4*((A*b^2)/4 + (B*a*b)/2) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b^
2*x^7)/7
```

**sympy** [A] time = 0.15, size = 49, normalized size = 0.98

$$-\frac{Aa^2}{2x^2} + \frac{Bb^2x^7}{7} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x\left(2Aab + Ba^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b +
 B*a**2)
```

## 3.17 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4} dx$

**Optimal**. Leaf size=51

$$-\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^6)/6 + a*(2*A*b + a*B
)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2}dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(b(Ab+2aB)+\frac{a^2A}{x^2}+\frac{a(2Ab+aB)}{x}+b^2Bx\right)dx,x,x^3\right)\\ &= -\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 49, normalized size = 0.96

$$\frac{1}{6}\left(-\frac{2a^2A}{x^3}+2bx^3(2aB+Ab)+6a\log(x)(aB+2Ab)+b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] ((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Log
[x])/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^4, x]
```

**fricas [A]** time = 0.64, size = 54, normalized size = 1.06

$$\frac{Bb^2x^9+2\left(2\,Bab+Ab^2\right)x^6+6\left(Ba^2+2\,Aab\right)x^3\log(x)-2\,Aa^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x) -
 2*A*a^2)/x^3
```

**giac [A]** time = 0.15, size = 69, normalized size = 1.35

$$\frac{1}{6}\,Bb^2x^6+\frac{2}{3}\,Babx^3+\frac{1}{3}\,Ab^2x^3+\left(Ba^2+2\,Aab\right)\log\left(|x|\right)-\frac{Ba^2x^3+2\,Aabx^3+Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(x
)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3
```

**maple [A]** time = 0.05, size = 51, normalized size = 1.00

$$\frac{B\,b^2x^6}{6}+\frac{A\,b^2x^3}{3}+\frac{2Bab\,x^3}{3}+2Aab\ln(x)+B\,a^2\ln(x)-\frac{A\,a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^4,x)
```

```
[Out] 1/6*b^2*B*x^6+1/3*A*x^3*b^2+2/3*B*x^3*a*b-1/3*A*a^2/x^3+2*A*ln(x)*a*b+B*a^2
*ln(x)
```

**maxima [A]** time = 0.48, size = 52, normalized size = 1.02

$$\frac{1}{6}\,Bb^2x^6+\frac{1}{3}\left(2\,Bab+Ab^2\right)x^3+\frac{1}{3}\left(Ba^2+2\,Aab\right)\log\left(x^3\right)-\frac{Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3)
- 1/3*A*a^2/x^3
```

**mupad [B]** time = 0.04, size = 49, normalized size = 0.96

$$x^3\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\ln(x)\,\left(B\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{3\,x^3}+\frac{B\,b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^4,x)
```

```
[Out] x^3*((A*b^2)/3 + (2*B*a*b)/3) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3)
+ (B*b^2*x^6)/6
```

**sympy [A]** time = 0.27, size = 51, normalized size = 1.00

$$-\frac{Aa^2}{3x^3}+\frac{Bb^2x^6}{6}+a\left(2Ab+Ba\right)\log\left(x\right)+x^3\left(\frac{Ab^2}{3}+\frac{2Bab}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3
+ 2*B*a*b/3)
```

## 3.18 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx$

**Optimal**. Leaf size=53

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x
^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx&=\int \left(\frac{a^2A}{x^5}+\frac{a(2Ab+aB)}{x^2}+b(Ab+2aB)x+b^2Bx^4\right)dx\\&=-\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{x}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{5}b^2Bx^5\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 0.96

$$\frac{-5a^2A+10bx^6(2aB+Ab)-20ax^3(aB+2Ab)+4b^2Bx^9}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] (-5*a^2*A - 20*a*(2*A*b + a*B)*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)/
(20*x^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^5, x]
```

**fricas** [A] time = 0.72, size = 53, normalized size = 1.00

$$\frac{4\,Bb^2x^9+10\left(2\,Bab+Ab^2\right)x^6-20\left(Ba^2+2\,Aab\right)x^3-5\,Aa^2}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^4
```

**giac** [A] time = 0.15, size = 54, normalized size = 1.02

$$\frac{1}{5}\,Bb^2x^5+Babx^2+\frac{1}{2}\,Ab^2x^2-\frac{4\,Ba^2x^3+8\,Aabx^3+Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3
+ A*a^2)/x^4
```

**maple** [A] time = 0.05, size = 50, normalized size = 0.94

$$\frac{B\,b^2x^5}{5}+\frac{A\,b^2x^2}{2}+Bab\,x^2-\frac{(2Ab+Ba)\,a}{x}-\frac{A\,a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^5,x)
```

```
[Out] 1/5*b^2*B*x^5+1/2*A*x^2*b^2+B*a*b*x^2-1/4*a^2*A/x^4-a*(2*A*b+B*a)/x
```

**maxima** [A] time = 0.72, size = 53, normalized size = 1.00

$$\frac{1}{5}\,Bb^2x^5+\frac{1}{2}\left(2\,Bab+Ab^2\right)x^2-\frac{4\left(Ba^2+2\,Aab\right)x^3+Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/2*(2*B*a*b + A*b^2)*x^2 - 1/4*(4*(B*a^2 + 2*A*a*b)*x^3 +
A*a^2)/x^4
```

**mupad** [B] time = 0.05, size = 52, normalized size = 0.98

$$x^2\left(\frac{A\,b^2}{2}+B\,a\,b\right)-\frac{x^3\,\left(B\,a^2+2\,A\,b\,a\right)+\frac{A\,a^2}{4}}{x^4}+\frac{B\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^5,x)
```

```
[Out] x^2*((A*b^2)/2 + B*a*b) - (x^3*(B*a^2 + 2*A*a*b) + (A*a^2)/4)/x^4 + (B*b^2*
x^5)/5
```

**sympy** [A] time = 0.31, size = 53, normalized size = 1.00

$$\frac{Bb^2x^5}{5}+x^2\left(\frac{Ab^2}{2}+Bab\right)+\frac{-Aa^2+x^3\left(-8Aab-4Ba^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**2*x**5/5 + x**2*(A*b**2/2 + B*a*b) + (-A*a**2 + x**3*(-8*A*a*b - 4*B*a
**2))/(4*x**4)
```

## 3.19 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx$

**Optimal**. Leaf size=50

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx &= \int \left(b(Ab+2aB)+\frac{a^2A}{x^6}+\frac{a(2Ab+aB)}{x^3}+b^2Bx^3\right)dx\\ &= -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{2x^2}+b(Ab+2aB)x+\frac{1}{4}b^2Bx^4\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 1.00

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{2x^2}+bx(2aB+Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^6, x]
```

**fricas [A]** time = 1.37, size = 53, normalized size = 1.06

$$\frac{5\,Bb^2x^9+20\left(2\,Bab+Ab^2\right)x^6-10\left(Ba^2+2\,Aab\right)x^3-4\,Aa^2}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^5
```

**giac [A]** time = 0.15, size = 51, normalized size = 1.02

$$\frac{1}{4}\,Bb^2x^4+2\,Babx+Ab^2x-\frac{5\,Ba^2x^3+10\,Aabx^3+2\,Aa^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2*
A*a^2)/x^5
```

**maple [A]** time = 0.06, size = 46, normalized size = 0.92

$$\frac{B\,b^2x^4}{4}+A\,b^2x+2Babx-\frac{(2Ab+Ba)\,a}{2x^2}-\frac{A\,a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^6,x)
```

```
[Out] 1/4*B*b^2*x^4+b^2*A*x+2*a*b*B*x-1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2
```

**maxima [A]** time = 0.57, size = 51, normalized size = 1.02

$$\frac{1}{4}\,Bb^2x^4+\left(2\,Bab+Ab^2\right)x-\frac{5\left(Ba^2+2\,Aab\right)x^3+2\,Aa^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*a
^2)/x^5
```

**mupad [B]** time = 2.37, size = 50, normalized size = 1.00

$$x\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{x^3\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{5}}{x^5}+\frac{B\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^6,x)
```

```
[Out] x*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/2 + A*a*b) + (A*a^2)/5)/x^5 + (B*b^2*x^
4)/4
```

**sympy [A]** time = 0.35, size = 53, normalized size = 1.06

$$\frac{Bb^2x^4}{4}+x\left(Ab^2+2Bab\right)+\frac{-2Aa^2+x^3\left(-10Aab-5Ba^2\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) + (-2*A*a**2 + x**3*(-10*A*a*b - 5*B*a
**2))/(10*x**5)
```

## 3.20 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx$

**Optimal**. Leaf size=51

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{3x^3}+b\log(x)(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

**Rubi [A]** time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{3x^3}+b\log(x)(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(3*x^3) + (b^2*B*x^3)/3 + b*(A*b + 2*a
*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(b^2B+\frac{a^2A}{x^3}+\frac{a(2Ab+aB)}{x^2}+\frac{b(Ab+2aB)}{x}\right)dx,x,x^3\right)\\ &= -\frac{a^2A}{6x^6}-\frac{a(2Ab+aB)}{3x^3}+\frac{1}{3}b^2Bx^3+b(Ab+2aB)\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 1.00

$$\frac{1}{6}\left(-\frac{a^2(A+2Bx^3)}{x^6}+6b\log(x)(2aB+Ab)-\frac{4aAb}{x^3}+2b^2Bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] ((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B)
*Log[x])/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^7, x]
```

**fricas [A]** time = 0.75, size = 55, normalized size = 1.08

$$\frac{2\,Bb^2x^9+6\left(2\,Bab+Ab^2\right)x^6\log(x)-2\left(Ba^2+2\,Aab\right)x^3-Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^3
 - A*a^2)/x^6
```

**giac [A]** time = 0.21, size = 70, normalized size = 1.37

$$\frac{1}{3}\,Bb^2x^3+\left(2\,Bab+Ab^2\right)\log\left(|x|\right)-\frac{6\,Babx^6+3\,Ab^2x^6+2\,Ba^2x^3+4\,Aabx^3+Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2*
x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6
```

**maple [A]** time = 0.07, size = 51, normalized size = 1.00

$$\frac{B\,b^2x^3}{3}+A\,b^2\ln(x)+2Bab\ln(x)-\frac{2Aab}{3x^3}-\frac{B\,a^2}{3x^3}-\frac{A\,a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^7,x)
```

```
[Out] 1/3*b^2*B*x^3-2/3*a/x^3*A*b-1/3*a^2/x^3*B-1/6*a^2*A/x^6+A*ln(x)*b^2+2*B*ln(
x)*a*b
```

**maxima [A]** time = 0.67, size = 54, normalized size = 1.06

$$\frac{1}{3}\,Bb^2x^3+\frac{1}{3}\left(2\,Bab+Ab^2\right)\log\left(x^3\right)-\frac{2\left(Ba^2+2\,Aab\right)x^3+Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*x
^3 + A*a^2)/x^6
```

**mupad [B]** time = 2.36, size = 52, normalized size = 1.02

$$\ln(x)\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{x^3\,\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+\frac{A\,a^2}{6}}{x^6}+\frac{B\,b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^7,x)
```

```
[Out] log(x)*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/6)/x^6
+ (B*b^2*x^3)/3
```

**sympy [A]** time = 0.76, size = 51, normalized size = 1.00

$$\frac{Bb^2x^3}{3}+b\,(Ab+2Ba)\log{(x)}+\frac{-Aa^2+x^3\left(-4Aab-2Ba^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**3*(-4*A*a*b - 2*B*a*
*2))/(6*x**6)
```

## 3.21 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8} \, dx$

**Optimal**. Leaf size=53

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{x}+\frac{1}{2}b^2Bx^2$$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{x}+\frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*B
*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx &= \int \left(\frac{a^2A}{x^8}+\frac{a(2Ab+aB)}{x^5}+\frac{b(Ab+2aB)}{x^2}+b^2Bx\right)dx\\ &= -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{x}+\frac{1}{2}b^2Bx^2\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 54, normalized size = 1.02

$$-\frac{a^2(4A+7Bx^3)+14abx^3(A+4Bx^3)-14b^2x^6(Bx^3-2A)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -1/28*(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A + 7
*B*x^3))/x^7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^8, x]
```

**fricas** [A] time = 0.80, size = 53, normalized size = 1.00

$$\frac{14\,Bb^2x^9 - 28\left(2\,Bab + Ab^2\right)x^6 - 7\left(Ba^2 + 2\,Aab\right)x^3 - 4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^7
```

**giac** [A] time = 0.15, size = 56, normalized size = 1.06

$$\frac{1}{2}\,Bb^2x^2 - \frac{56\,Babx^6 + 28\,Ab^2x^6 + 7\,Ba^2x^3 + 14\,Aabx^3 + 4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b*
x^3 + 4*A*a^2)/x^7
```

**maple** [A] time = 0.04, size = 48, normalized size = 0.91

$$\frac{B\,b^2x^2}{2} - \frac{(Ab + 2Ba)\,b}{x} - \frac{(2Ab + Ba)\,a}{4x^4} - \frac{A\,a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^8,x)
```

```
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

**maxima** [A] time = 0.57, size = 54, normalized size = 1.02

$$\frac{1}{2}\,Bb^2x^2 - \frac{28\left(2\,Bab + Ab^2\right)x^6 + 7\left(Ba^2 + 2\,Aab\right)x^3 + 4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 +
4*A*a^2)/x^7
```

**mupad** [B] time = 0.05, size = 53, normalized size = 1.00

$$\frac{B\,b^2\,x^2}{2} - \frac{x^3\,\left(\frac{B\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^6\,\left(A\,b^2 + 2\,B\,a\,b\right) + \frac{A\,a^2}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^8,x)
```

```
[Out] (B*b^2*x^2)/2 - (x^3*((B*a^2)/4 + (A*a*b)/2) + x^6*(A*b^2 + 2*B*a*b) + (A*a
^2)/7)/x^7
```

**sympy** [A] time = 0.87, size = 58, normalized size = 1.09

$$\frac{Bb^2x^2}{2} + \frac{-4Aa^2 + x^6\left(-28Ab^2 - 56Bab\right) + x^3\left(-14Aab - 7Ba^2\right)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**2*x**2/2 + (-4*A*a**2 + x**6*(-28*A*b**2 - 56*B*a*b) + x**3*(-14*A*a*b
 - 7*B*a**2))/(28*x**7)
```

## 3.22 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx$

**Optimal**. Leaf size=50

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^2*A)/(8*x^8) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx &= \int \left(b^2B+\frac{a^2A}{x^9}+\frac{a(2Ab+aB)}{x^6}+\frac{b(Ab+2aB)}{x^3}\right)dx\\ &= -\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 50, normalized size = 1.00

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^9, x]
```

**fricas** [A] time = 0.83, size = 53, normalized size = 1.06

$$\frac{40\,Bb^2x^9 - 20\left(2\,Bab + Ab^2\right)x^6 - 8\left(Ba^2 + 2\,Aab\right)x^3 - 5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^8
```

**giac** [A] time = 0.15, size = 53, normalized size = 1.06

$$Bb^2x - \frac{40\,Babx^6 + 20\,Ab^2x^6 + 8\,Ba^2x^3 + 16\,Aabx^3 + 5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 +
5*A*a^2)/x^8
```

**maple** [A] time = 0.05, size = 45, normalized size = 0.90

$$B\,b^2x - \frac{(Ab + 2Ba)\,b}{2x^2} - \frac{(2Ab + Ba)\,a}{5x^5} - \frac{A\,a^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^9,x)
```

```
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

**maxima** [A] time = 0.49, size = 51, normalized size = 1.02

$$Bb^2x - \frac{20\left(2\,Bab + Ab^2\right)x^6 + 8\left(Ba^2 + 2\,Aab\right)x^3 + 5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a^
2)/x^8
```

**mupad** [B] time = 2.34, size = 50, normalized size = 1.00

$$B\,b^2\,x - \frac{x^3\left(\frac{B\,a^2}{5} + \frac{2\,A\,b\,a}{5}\right) + x^6\left(\frac{A\,b^2}{2} + B\,a\,b\right) + \frac{A\,a^2}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^9,x)
```

```
[Out] B*b^2*x - (x^3*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/2 + B*a*b) + (A*a^2
)/8)/x^8
```

**sympy** [A] time = 0.99, size = 54, normalized size = 1.08

$$Bb^2x + \frac{-5Aa^2 + x^6\left(-20Ab^2 - 40Bab\right) + x^3\left(-16Aab - 8Ba^2\right)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**2*x + (-5*A*a**2 + x**6*(-20*A*b**2 - 40*B*a*b) + x**3*(-16*A*a*b - 8*
B*a**2))/(40*x**8)
```

## 3.23 $\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=117

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB-$$

**Rubi [A]** time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+Ab)+\frac{1}{28}b^5Bx^{28}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \int \left(a^5Ax^9+a^4(5Ab+aB)x^{12}+5a^3b(2Ab+aB)x^{15}+10a^2b^2(Ab+aB)x^{18}+\right.\\ &= \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+aB)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+Ab)+\frac{1}{28}b^5Bx^{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^9*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [A]** time = 0.71, size = 125, normalized size = 1.07

$$\frac{1}{28}x^{28}b^5B+\frac{1}{5}x^{25}b^4aB+\frac{1}{25}x^{25}b^5A+\frac{5}{11}x^{22}b^3a^2B+\frac{5}{22}x^{22}b^4aA+\frac{10}{19}x^{19}b^2a^3B+\frac{10}{19}x^{19}b^3a^2A+\frac{5}{16}x^{16}ba^4B+\frac{5}{8}x^{16}b^2a^3A+\frac{1}{13}x^{13}a^5B+\frac{5}{13}x^{13}ba^4A+\frac{1}{10}x^{10}a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/28*x^28*b^5*B + 1/5*x^25*b^4*a*B + 1/25*x^25*b^5*A + 5/11*x^22*b^3*a^2*B
+ 5/22*x^22*b^4*a*A + 10/19*x^19*b^2*a^3*B + 10/19*x^19*b^3*a^2*A + 5/16*x^
16*b*a^4*B + 5/8*x^16*b^2*a^3*A + 1/13*x^13*a^5*B + 5/13*x^13*b*a^4*A + 1/1
0*x^10*a^5*A
```

**giac [A]** time = 0.16, size = 125, normalized size = 1.07

$$\frac{1}{28}Bb^5x^{28}+\frac{1}{5}Bab^4x^{25}+\frac{1}{25}Ab^5x^{25}+\frac{5}{11}Ba^2b^3x^{22}+\frac{5}{22}Aab^4x^{22}+\frac{10}{19}Ba^3b^2x^{19}+\frac{10}{19}Aa^2b^3x^{19}+\frac{5}{16}Ba^4bx^{16}+\frac{5}{8}Aa^3b^2x^{16}+\frac{1}{13}Ba^5x^{13}+\frac{5}{13}Aa^4bx^{13}+\frac{1}{10}Aa^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/28*B*b^5*x^28 + 1/5*B*a*b^4*x^25 + 1/25*A*b^5*x^25 + 5/11*B*a^2*b^3*x^22
+ 5/22*A*a*b^4*x^22 + 10/19*B*a^3*b^2*x^19 + 10/19*A*a^2*b^3*x^19 + 5/16*B*
a^4*b*x^16 + 5/8*A*a^3*b^2*x^16 + 1/13*B*a^5*x^13 + 5/13*A*a^4*b*x^13 + 1/1
0*A*a^5*x^10
```

**maple [A]** time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{28}}{28}+\frac{\left(b^5A+5a\,b^4B\right)x^{25}}{25}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{22}}{22}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{19}}{19}+\frac{A\,a^5x^{10}}{10}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{16}}{16}+\frac{\left(5a^4bA+a^5B\right)x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/28*b^5*B*x^28+1/25*(A*b^5+5*B*a*b^4)*x^25+1/22*(5*A*a*b^4+10*B*a^2*b^3)*x
^22+1/19*(10*A*a^2*b^3+10*B*a^3*b^2)*x^19+1/16*(10*A*a^3*b^2+5*B*a^4*b)*x^1
6+1/13*(5*A*a^4*b+B*a^5)*x^13+1/10*a^5*A*x^10
```

**maxima [A]** time = 0.46, size = 119, normalized size = 1.02

$$\frac{1}{28}Bb^5x^{28}+\frac{1}{25}\left(5\,Bab^4+Ab^5\right)x^{25}+\frac{5}{22}\left(2\,Ba^2b^3+Aab^4\right)x^{22}+\frac{10}{19}\left(Ba^3b^2+Aa^2b^3\right)x^{19}+\frac{5}{16}\left(Ba^4b+2\,Aa^3b^2\right)x^{16}+\frac{1}{10}Aa^5x^{10}+\frac{1}{13}\left(Ba^5+5\,Aa^4b\right)x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*b
^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b^2
)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

**mupad [B]** time = 0.05, size = 107, normalized size = 0.91

$$x^{13}\left(\frac{B\,a^5}{13}+\frac{5\,A\,b\,a^4}{13}\right)+x^{25}\left(\frac{A\,b^5}{25}+\frac{B\,a\,b^4}{5}\right)+\frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{28}}{28}+\frac{10\,a^2\,b^2\,x^{19}\,\left(A\,b+B\,a\right)}{19}+\frac{5\,a^3\,b\,x^{16}\,\left(2\,A\,b+B\,a\right)}{16}+\frac{5\,a\,b^3\,x^{22}\,\left(A\,b+2\,B\,a\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^13*((B*a^5)/13 + (5*A*a^4*b)/13) + x^25*((A*b^5)/25 + (B*a*b^4)/5) + (A*a
^5*x^10)/10 + (B*b^5*x^28)/28 + (10*a^2*b^2*x^19*(A*b + B*a))/19 + (5*a^3*b
*x^16*(2*A*b + B*a))/16 + (5*a*b^3*x^22*(A*b + 2*B*a))/22
```

<u>**sympy**</u> **[A]** time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{28}}{28}+x^{25}\left(\frac{Ab^5}{25}+\frac{Bab^4}{5}\right)+x^{22}\left(\frac{5Aab^4}{22}+\frac{5Ba^2b^3}{11}\right)+x^{19}\left(\frac{10Aa^2b^3}{19}+\frac{10Ba^3b^2}{19}\right)+x^{16}\left(\frac{5Aa^3b^2}{8}+\frac{5Ba^4b}{16}\right)+x^{13}\left(\frac{5Aa^4b}{13}+\frac{Ba^5}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22*
(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3*b
**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13 +
B*a**5/13)
```

## 3.24 $\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=95

$$\frac{a^2 \left(a+bx^3\right)^6 (Ab-aB)}{18b^4}+\frac{\left(a+bx^3\right)^8 (Ab-3aB)}{24b^4}-\frac{a \left(a+bx^3\right)^7 (2Ab-3aB)}{21b^4}+\frac{B \left(a+bx^3\right)^9}{27b^4}$$

**Rubi [A]** time = 0.24, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{a^2 \left(a+bx^3\right)^6 (Ab-aB)}{18b^4}+\frac{\left(a+bx^3\right)^8 (Ab-3aB)}{24b^4}-\frac{a \left(a+bx^3\right)^7 (2Ab-3aB)}{21b^4}+\frac{B \left(a+bx^3\right)^9}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^3)^6)/(18*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^7
)/(21*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(24*b^4) + (B*(a + b*x^3)^9)/(27
*b^4)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^6}{b^3}+\frac{(Ab-3aB}{b}\right.\right.\\ &= \frac{a^2(Ab-aB)\left(a+bx^3\right)^6}{18b^4}-\frac{a(2Ab-3aB)\left(a+bx^3\right)^7}{21b^4}+\frac{(Ab-3aB)\left(a+bx^3\right)^8}{24b^4}+\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 107, normalized size = 1.13

$$\frac{x^9 \left(168a^5A+126a^4x^3(aB+5Ab)+504a^3bx^6(aB+2Ab)+840a^2b^2x^9(aB+Ab)+63b^4x^{15}(5aB+Ab)+360ab^3x^{12}(2aB+Ab)+56b^5Bx^{18}\right)}{1512}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6 +
 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b +
 5*a*B)*x^15 + 56*b^5*B*x^18))/1512
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [A]** time = 0.73, size = 125, normalized size = 1.32

$$\frac{1}{27}x^{27}b^5B+\frac{5}{24}x^{24}b^4aB+\frac{1}{24}x^{24}b^5A+\frac{10}{21}x^{21}b^3a^2B+\frac{5}{21}x^{21}b^4aA+\frac{5}{9}x^{18}b^2a^3B+\frac{5}{9}x^{18}b^3a^2A+\frac{1}{3}x^{15}ba^4B+\frac{2}{3}x^{15}b^2a^3A+\frac{1}{12}x^{12}a^5B+\frac{5}{12}x^{12}ba^4A+\frac{1}{9}x^9a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/27*x^27*b^5*B + 5/24*x^24*b^4*a*B + 1/24*x^24*b^5*A + 10/21*x^21*b^3*a^2*
B + 5/21*x^21*b^4*a*A + 5/9*x^18*b^2*a^3*B + 5/9*x^18*b^3*a^2*A + 1/3*x^15*
b*a^4*B + 2/3*x^15*b^2*a^3*A + 1/12*x^12*a^5*B + 5/12*x^12*b*a^4*A + 1/9*x^
9*a^5*A
```

**giac [A]** time = 0.17, size = 125, normalized size = 1.32

$$\frac{1}{27}Bb^5x^{27}+\frac{5}{24}Bab^4x^{24}+\frac{1}{24}Ab^5x^{24}+\frac{10}{21}Ba^2b^3x^{21}+\frac{5}{21}Aab^4x^{21}+\frac{5}{9}Ba^3b^2x^{18}+\frac{5}{9}Aa^2b^3x^{18}+\frac{1}{3}Ba^4bx^{15}+\frac{2}{3}Aa^3b^2x^{15}+\frac{1}{12}Ba^5x^{12}+\frac{5}{12}Aa^4bx^{12}+\frac{1}{9}Aa^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/27*B*b^5*x^27 + 5/24*B*a*b^4*x^24 + 1/24*A*b^5*x^24 + 10/21*B*a^2*b^3*x^2
1 + 5/21*A*a*b^4*x^21 + 5/9*B*a^3*b^2*x^18 + 5/9*A*a^2*b^3*x^18 + 1/3*B*a^4
*b*x^15 + 2/3*A*a^3*b^2*x^15 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/9*A*
a^5*x^9
```

**maple [A]** time = 0.04, size = 124, normalized size = 1.31

$$\frac{B\,b^5x^{27}}{27}+\frac{\left(b^5A+5a\,b^4B\right)x^{24}}{24}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{21}}{21}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{18}}{18}+\frac{A\,a^5x^9}{9}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{15}}{15}+\frac{\left(5a^4bA+a^5B\right)x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/27*b^5*B*x^27+1/24*(A*b^5+5*B*a*b^4)*x^24+1/21*(5*A*a*b^4+10*B*a^2*b^3)*x
^21+1/18*(10*A*a^2*b^3+10*B*a^3*b^2)*x^18+1/15*(10*A*a^3*b^2+5*B*a^4*b)*x^1
5+1/12*(5*A*a^4*b+B*a^5)*x^12+1/9*a^5*A*x^9
```

**maxima [A]** time = 0.47, size = 119, normalized size = 1.25

$$\frac{1}{27}Bb^5x^{27}+\frac{1}{24}\left(5Bab^4+Ab^5\right)x^{24}+\frac{5}{21}\left(2Ba^2b^3+Aab^4\right)x^{21}+\frac{5}{9}\left(Ba^3b^2+Aa^2b^3\right)x^{18}+\frac{1}{3}\left(Ba^4b+2Aa^3b^2\right)x^{15}+\frac{1}{9}Aa^5x^9+\frac{1}{12}\left(Ba^5+5Aa^4b\right)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*b
^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)*x
^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

**mupad [B]** time = 2.34, size = 107, normalized size = 1.13

$$x^{12}\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{24}\left(\frac{A\,b^5}{24}+\frac{5\,B\,a\,b^4}{24}\right)+\frac{A\,a^5\,x^9}{9}+\frac{B\,b^5\,x^{27}}{27}+\frac{5\,a^2\,b^2\,x^{18}\,\left(A\,b+B\,a\right)}{9}+\frac{a^3\,b\,x^{15}\,\left(2\,A\,b+B\,a\right)}{3}+\frac{5\,a\,b^3\,x^{21}\,\left(A\,b+2\,B\,a\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^24*((A*b^5)/24 + (5*B*a*b^4)/24) + (
A*a^5*x^9)/9 + (B*b^5*x^27)/27 + (5*a^2*b^2*x^18*(A*b + B*a))/9 + (a^3*b*x^
15*(2*A*b + B*a))/3 + (5*a*b^3*x^21*(A*b + 2*B*a))/21
```

**sympy [A]** time = 0.10, size = 136, normalized size = 1.43

$$\frac{Aa^5x^9}{9}+\frac{Bb^5x^{27}}{27}+x^{24}\left(\frac{Ab^5}{24}+\frac{5Bab^4}{24}\right)+x^{21}\left(\frac{5Aab^4}{21}+\frac{10Ba^2b^3}{21}\right)+x^{18}\left(\frac{5Aa^2b^3}{9}+\frac{5Ba^3b^2}{9}\right)+x^{15}\left(\frac{2Aa^3b^2}{3}+\frac{Ba^4b}{3}\right)+x^{12}\left(\frac{5Aa^4b}{12}+\frac{Ba^5}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**21
*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*b*
*2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*a**
5/12)
```

## 3.25 $\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=117

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab$$

**Rubi [A]** time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{8}a^5Ax^8+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+\frac{1}{26}b^5Bx^{26}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5Ax^7+a^4(5Ab+aB)x^{10}+5a^3b(2Ab+aB)x^{13}+10a^2b^2(Ab+aB)x^{16}+\right.\\ &= \frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{10}{17}a^2b^2(Ab+aB)x^{1}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+\frac{1}{26}b^5Bx^{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [A]** time = 0.69, size = 125, normalized size = 1.07

$$\frac{1}{26}x^{26}b^5B+\frac{5}{23}x^{23}b^4aB+\frac{1}{23}x^{23}b^5A+\frac{1}{2}x^{20}b^3a^2B+\frac{1}{4}x^{20}b^4aA+\frac{10}{17}x^{17}b^2a^3B+\frac{10}{17}x^{17}b^3a^2A+\frac{5}{14}x^{14}ba^4B+\frac{5}{7}x^{14}b^2a^3A+\frac{1}{11}x^{11}a^5B+\frac{5}{11}x^{11}ba^4A+\frac{1}{8}x^8a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/26*x^26*b^5*B + 5/23*x^23*b^4*a*B + 1/23*x^23*b^5*A + 1/2*x^20*b^3*a^2*B
+ 1/4*x^20*b^4*a*A + 10/17*x^17*b^2*a^3*B + 10/17*x^17*b^3*a^2*A + 5/14*x^1
4*b*a^4*B + 5/7*x^14*b^2*a^3*A + 1/11*x^11*a^5*B + 5/11*x^11*b*a^4*A + 1/8*
x^8*a^5*A
```

**giac [A]** time = 0.15, size = 125, normalized size = 1.07

$$\frac{1}{26}Bb^5x^{26}+\frac{5}{23}Bab^4x^{23}+\frac{1}{23}Ab^5x^{23}+\frac{1}{2}Ba^2b^3x^{20}+\frac{1}{4}Aab^4x^{20}+\frac{10}{17}Ba^3b^2x^{17}+\frac{10}{17}Aa^2b^3x^{17}+\frac{5}{14}Ba^4bx^{14}+\frac{5}{7}Aa^3b^2x^{14}+\frac{1}{11}Ba^5x^{11}+\frac{5}{11}Aa^4bx^{11}+\frac{1}{8}Aa^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/26*B*b^5*x^26 + 5/23*B*a*b^4*x^23 + 1/23*A*b^5*x^23 + 1/2*B*a^2*b^3*x^20
+ 1/4*A*a*b^4*x^20 + 10/17*B*a^3*b^2*x^17 + 10/17*A*a^2*b^3*x^17 + 5/14*B*a
^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/8*
A*a^5*x^8
```

**maple [A]** time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{26}}{26}+\frac{\left(b^5A+5a\,b^4B\right)x^{23}}{23}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{20}}{20}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{17}}{17}+\frac{A\,a^5x^8}{8}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{14}}{14}+\frac{\left(5a^4bA+a^5B\right)x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/26*b^5*B*x^26+1/23*(A*b^5+5*B*a*b^4)*x^23+1/20*(5*A*a*b^4+10*B*a^2*b^3)*x
^20+1/17*(10*A*a^2*b^3+10*B*a^3*b^2)*x^17+1/14*(10*A*a^3*b^2+5*B*a^4*b)*x^1
4+1/11*(5*A*a^4*b+B*a^5)*x^11+1/8*a^5*A*x^8
```

**maxima [A]** time = 0.47, size = 119, normalized size = 1.02

$$\frac{1}{26}Bb^5x^{26}+\frac{1}{23}\left(5\,Bab^4+Ab^5\right)x^{23}+\frac{1}{4}\left(2\,Ba^2b^3+Aab^4\right)x^{20}+\frac{10}{17}\left(Ba^3b^2+Aa^2b^3\right)x^{17}+\frac{5}{14}\left(Ba^4b+2\,Aa^3b^2\right)x^{14}+\frac{1}{8}Aa^5x^8+\frac{1}{11}\left(Ba^5+5\,Aa^4b\right)x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b^
4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^2)
*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

**mupad [B]** time = 0.04, size = 107, normalized size = 0.91

$$x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{23}\left(\frac{A\,b^5}{23}+\frac{5\,B\,a\,b^4}{23}\right)+\frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{26}}{26}+\frac{10\,a^2\,b^2\,x^{17}\,\left(A\,b+B\,a\right)}{17}+\frac{5\,a^3\,b\,x^{14}\,\left(2\,A\,b+B\,a\right)}{14}+\frac{a\,b^3\,x^{20}\,\left(A\,b+2\,B\,a\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^23*((A*b^5)/23 + (5*B*a*b^4)/23) + (
A*a^5*x^8)/8 + (B*b^5*x^26)/26 + (10*a^2*b^2*x^17*(A*b + B*a))/17 + (5*a^3*
b*x^14*(2*A*b + B*a))/14 + (a*b^3*x^20*(A*b + 2*B*a))/4
```

**sympy [A]** time = 0.10, size = 134, normalized size = 1.15

$$\frac{Aa^5x^8}{8}+\frac{Bb^5x^{26}}{26}+x^{23}\left(\frac{Ab^5}{23}+\frac{5Bab^4}{23}\right)+x^{20}\left(\frac{Aab^4}{4}+\frac{Ba^2b^3}{2}\right)+x^{17}\left(\frac{10Aa^2b^3}{17}+\frac{10Ba^3b^2}{17}\right)+x^{14}\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4b}{14}\right)+x^{11}\left(\frac{5Aa^4b}{11}+\frac{Ba^5}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**26/26 + x**23*(A*b**5/23 + 5*B*a*b**4/23) + x**20
*(A*a*b**4/4 + B*a**2*b**3/2) + x**17*(10*A*a**2*b**3/17 + 10*B*a**3*b**2/1
7) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**11*(5*A*a**4*b/11 + B*a**
5/11)
```

## 3.26 $\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=117

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab)+$$

**Rubi [A]** time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab)+\frac{1}{25}b^5Bx^{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^6\left(a+bx^3\right)^5\left(A+Bx^3\right)dx=\int\left(a^5Ax^6+a^4(5Ab+aB)x^9+5a^3b(2Ab+aB)x^{12}+10a^2b^2(Ab+aB)x^{15}+5a\right.$$
$$=\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+$$

**Mathematica [A]** time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab)+\frac{1}{25}b^5Bx^{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(a+bx^3\right)^5\left(A+Bx^3\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [A]** time = 0.52, size = 125, normalized size = 1.07

$$\frac{1}{25}x^{25}b^5B+\frac{5}{22}x^{22}b^4aB+\frac{1}{22}x^{22}b^5A+\frac{10}{19}x^{19}b^3a^2B+\frac{5}{19}x^{19}b^4aA+\frac{5}{8}x^{16}b^2a^3B+\frac{5}{8}x^{16}b^3a^2A+\frac{5}{13}x^{13}ba^4B+\frac{10}{13}x^{13}b^2a^3A+\frac{1}{10}x^{10}a^5B+\frac{1}{2}x^{10}ba^4A+\frac{1}{7}x^7a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/25*x^25*b^5*B + 5/22*x^22*b^4*a*B + 1/22*x^22*b^5*A + 10/19*x^19*b^3*a^2*
B + 5/19*x^19*b^4*a*A + 5/8*x^16*b^2*a^3*B + 5/8*x^16*b^3*a^2*A + 5/13*x^13
*b*a^4*B + 10/13*x^13*b^2*a^3*A + 1/10*x^10*a^5*B + 1/2*x^10*b*a^4*A + 1/7*
x^7*a^5*A
```

**giac [A]** time = 0.15, size = 125, normalized size = 1.07

$$\frac{1}{25}Bb^5x^{25}+\frac{5}{22}Bab^4x^{22}+\frac{1}{22}Ab^5x^{22}+\frac{10}{19}Ba^2b^3x^{19}+\frac{5}{19}Aab^4x^{19}+\frac{5}{8}Ba^3b^2x^{16}+\frac{5}{8}Aa^2b^3x^{16}+\frac{5}{13}Ba^4bx^{13}+\frac{10}{13}Aa^3b^2x^{13}+\frac{1}{10}Ba^5x^{10}+\frac{1}{2}Aa^4bx^{10}+\frac{1}{7}Aa^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/25*B*b^5*x^25 + 5/22*B*a*b^4*x^22 + 1/22*A*b^5*x^22 + 10/19*B*a^2*b^3*x^1
9 + 5/19*A*a*b^4*x^19 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/13*B*a^
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/7*
A*a^5*x^7
```

**maple [A]** time = 0.03, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{25}}{25}+\frac{\left(b^5A+5a\,b^4B\right)x^{22}}{22}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{19}}{19}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{16}}{16}+\frac{A\,a^5x^7}{7}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{13}}{13}+\frac{\left(5a^4bA+a^5B\right)x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/25*b^5*B*x^25+1/22*(A*b^5+5*B*a*b^4)*x^22+1/19*(5*A*a*b^4+10*B*a^2*b^3)*x
^19+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/13*(10*A*a^3*b^2+5*B*a^4*b)*x^1
3+1/10*(5*A*a^4*b+B*a^5)*x^10+1/7*a^5*A*x^7
```

**maxima [A]** time = 0.59, size = 119, normalized size = 1.02

$$\frac{1}{25}Bb^5x^{25}+\frac{1}{22}\left(5\,Bab^4+Ab^5\right)x^{22}+\frac{5}{19}\left(2\,Ba^2b^3+Aab^4\right)x^{19}+\frac{5}{8}\left(Ba^3b^2+Aa^2b^3\right)x^{16}+\frac{5}{13}\left(Ba^4b+2\,Aa^3b^2\right)x^{13}+\frac{1}{7}Aa^5x^7+\frac{1}{10}\left(Ba^5+5\,Aa^4b\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*b
^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*
x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

**mupad [B]** time = 0.04, size = 107, normalized size = 0.91

$$x^{10}\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{22}\left(\frac{A\,b^5}{22}+\frac{5\,B\,a\,b^4}{22}\right)+\frac{A\,a^5\,x^7}{7}+\frac{B\,b^5\,x^{25}}{25}+\frac{5\,a^2\,b^2\,x^{16}\,\left(A\,b+B\,a\right)}{8}+\frac{5\,a^3\,b\,x^{13}\,\left(2\,A\,b+B\,a\right)}{13}+\frac{5\,a\,b^3\,x^{19}\,\left(A\,b+2\,B\,a\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^22*((A*b^5)/22 + (5*B*a*b^4)/22) + (A*a
^5*x^7)/7 + (B*b^5*x^25)/25 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x^1
3*(2*A*b + B*a))/13 + (5*a*b^3*x^19*(A*b + 2*B*a))/19
```

**sympy [A]** time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7} + \frac{Bb^5x^{25}}{25} + x^{22}\left(\frac{Ab^5}{22} + \frac{5Bab^4}{22}\right) + x^{19}\left(\frac{5Aab^4}{19} + \frac{10Ba^2b^3}{19}\right) + x^{16}\left(\frac{5Aa^2b^3}{8} + \frac{5Ba^3b^2}{8}\right) + x^{13}\left(\frac{10Aa^3b^2}{13} + \frac{5Ba^4b}{13}\right) + x^{10}\left(\frac{Aa^4b}{2} + \frac{Ba^5}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**25/25 + x**22*(A*b**5/22 + 5*B*a*b**4/22) + x**19
*(5*A*a*b**4/19 + 10*B*a**2*b**3/19) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b*
*2/8) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**10*(A*a**4*b/2 + B*a
**5/10)
```

## 3.27 $\int x^5 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=67

$$\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{21b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{18b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}$$

**Rubi [A]** time = 0.15, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{21b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{18b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^3)^6)/(18*b^3) + ((A*b - 2*a*B)*(a + b*x^3)^7)/(21
*b^3) + (B*(a + b*x^3)^8)/(24*b^3)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^5}{b^2}+\frac{(Ab-2aB)(a+bx)^6}{b^2}+\frac{B(a+bx)^7}{b^2}\right) d\\ &= -\frac{a(Ab-aB)\left(a+bx^3\right)^6}{18b^3}+\frac{(Ab-2aB)\left(a+bx^3\right)^7}{21b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 107, normalized size = 1.60

$$\frac{1}{504}x^6\left(84a^5A+56a^4x^3(aB+5Ab)+210a^3bx^6(aB+2Ab)+336a^2b^2x^9(aB+Ab)+24b^4x^{15}(5aB+Ab)+140ab^3x^{12}(2aB+Ab)+21b^5Bx^{18}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B)*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 + 3
36*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b + 5
*a*B)*x^15 + 21*b^5*B*x^18))/504
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [B]** time = 1.28, size = 125, normalized size = 1.87

$$\frac{1}{24}x^{24}b^5B+\frac{5}{21}x^{21}b^4aB+\frac{1}{21}x^{21}b^5A+\frac{5}{9}x^{18}b^3a^2B+\frac{5}{18}x^{18}b^4aA+\frac{2}{3}x^{15}b^2a^3B+\frac{2}{3}x^{15}b^3a^2A+\frac{5}{12}x^{12}ba^4B+\frac{5}{6}x^{12}b^2a^3A+\frac{1}{9}x^9a^5B+\frac{5}{9}x^9ba^4A+\frac{1}{6}x^6a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^5*B + 5/21*x^21*b^4*a*B + 1/21*x^21*b^5*A + 5/9*x^18*b^3*a^2*B
+ 5/18*x^18*b^4*a*A + 2/3*x^15*b^2*a^3*B + 2/3*x^15*b^3*a^2*A + 5/12*x^12*b
*a^4*B + 5/6*x^12*b^2*a^3*A + 1/9*x^9*a^5*B + 5/9*x^9*b*a^4*A + 1/6*x^6*a^5
*A
```

**giac [B]** time = 0.16, size = 125, normalized size = 1.87

$$\frac{1}{24}Bb^5x^{24}+\frac{5}{21}Bab^4x^{21}+\frac{1}{21}Ab^5x^{21}+\frac{5}{9}Ba^2b^3x^{18}+\frac{5}{18}Aab^4x^{18}+\frac{2}{3}Ba^3b^2x^{15}+\frac{2}{3}Aa^2b^3x^{15}+\frac{5}{12}Ba^4bx^{12}+\frac{5}{6}Aa^3b^2x^{12}+\frac{1}{9}Ba^5x^9+\frac{5}{9}Aa^4bx^9+\frac{1}{6}Aa^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/24*B*b^5*x^24 + 5/21*B*a*b^4*x^21 + 1/21*A*b^5*x^21 + 5/9*B*a^2*b^3*x^18
+ 5/18*A*a*b^4*x^18 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/6*A*a^5*x
^6
```

**maple [B]** time = 0.04, size = 124, normalized size = 1.85

$$\frac{B\,b^5x^{24}}{24}+\frac{\left(b^5A+5a\,b^4B\right)x^{21}}{21}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{18}}{18}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{15}}{15}+\frac{A\,a^5x^6}{6}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{12}}{12}+\frac{\left(5a^4bA+a^5B\right)x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/24*b^5*B*x^24+1/21*(A*b^5+5*B*a*b^4)*x^21+1/18*(5*A*a*b^4+10*B*a^2*b^3)*x
^18+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/12*(10*A*a^3*b^2+5*B*a^4*b)*x^1
2+1/9*(5*A*a^4*b+B*a^5)*x^9+1/6*a^5*A*x^6
```

**maxima [A]** time = 0.45, size = 119, normalized size = 1.78

$$\frac{1}{24}Bb^5x^{24}+\frac{1}{21}\left(5\,Bab^4+Ab^5\right)x^{21}+\frac{5}{18}\left(2\,Ba^2b^3+Aab^4\right)x^{18}+\frac{2}{3}\left(Ba^3b^2+Aa^2b^3\right)x^{15}+\frac{5}{12}\left(Ba^4b+2\,Aa^3b^2\right)x^{12}+\frac{1}{6}Aa^5x^6+\frac{1}{9}\left(Ba^5+5\,Aa^4b\right)x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*
x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

**mupad [B]** time = 0.04, size = 107, normalized size = 1.60

$$x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^{21}\left(\frac{A\,b^5}{21}+\frac{5\,B\,a\,b^4}{21}\right)+\frac{A\,a^5\,x^6}{6}+\frac{B\,b^5\,x^{24}}{24}+\frac{2\,a^2\,b^2\,x^{15}\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^3\,b\,x^{12}\,\left(2\,A\,b+B\,a\right)}{12}+\frac{5\,a\,b^3\,x^{18}\,\left(A\,b+2\,B\,a\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^21*((A*b^5)/21 + (5*B*a*b^4)/21) + (A*a
^5*x^6)/6 + (B*b^5*x^24)/24 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*x^1
2*(2*A*b + B*a))/12 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

**sympy [B]** time = 0.10, size = 138, normalized size = 2.06

$$\frac{Aa^5x^6}{6} + \frac{Bb^5x^{24}}{24} + x^{21}\left(\frac{Ab^5}{21} + \frac{5Bab^4}{21}\right) + x^{18}\left(\frac{5Aab^4}{18} + \frac{5Ba^2b^3}{9}\right) + x^{15}\left(\frac{2Aa^2b^3}{3} + \frac{2Ba^3b^2}{3}\right) + x^{12}\left(\frac{5Aa^3b^2}{6} + \frac{5Ba^4b}{12}\right) + x^9\left(\frac{5Aa^4b}{9} + \frac{Ba^5}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**18
*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b**2
/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a**5
/9)
```

## 3.28 $\int x^4 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=117

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{23}$$

**Rubi [A]** time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{23}b^5Bx^{23}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \int \left(a^5Ax^4+a^4(5Ab+aB)x^7+5a^3b(2Ab+aB)x^{10}+10a^2b^2(Ab+aB)x^{13}+5ab\right.\\ &= \frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{23}b^5Bx^{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas** [A] time = 0.82, size = 125, normalized size = 1.07

$$\frac{1}{23}x^{23}b^5B+\frac{1}{4}x^{20}b^4aB+\frac{1}{20}x^{20}b^5A+\frac{10}{17}x^{17}b^3a^2B+\frac{5}{17}x^{17}b^4aA+\frac{5}{7}x^{14}b^2a^3B+\frac{5}{7}x^{14}b^3a^2A+\frac{5}{11}x^{11}ba^4B+\frac{10}{11}x^{11}b^2a^3A+\frac{1}{8}x^8a^5B+\frac{5}{8}x^8ba^4A+\frac{1}{5}x^5a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/23*x^23*b^5*B + 1/4*x^20*b^4*a*B + 1/20*x^20*b^5*A + 10/17*x^17*b^3*a^2*B
 + 5/17*x^17*b^4*a*A + 5/7*x^14*b^2*a^3*B + 5/7*x^14*b^3*a^2*A + 5/11*x^11*
b*a^4*B + 10/11*x^11*b^2*a^3*A + 1/8*x^8*a^5*B + 5/8*x^8*b*a^4*A + 1/5*x^5*
a^5*A
```

**giac** [A] time = 0.23, size = 125, normalized size = 1.07

$$\frac{1}{23}Bb^5x^{23}+\frac{1}{4}Bab^4x^{20}+\frac{1}{20}Ab^5x^{20}+\frac{10}{17}Ba^2b^3x^{17}+\frac{5}{17}Aab^4x^{17}+\frac{5}{7}Ba^3b^2x^{14}+\frac{5}{7}Aa^2b^3x^{14}+\frac{5}{11}Ba^4bx^{11}+\frac{10}{11}Aa^3b^2x^{11}+\frac{1}{8}Ba^5x^8+\frac{5}{8}Aa^4bx^8+\frac{1}{5}Aa^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/23*B*b^5*x^23 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 10/17*B*a^2*b^3*x^17
 + 5/17*A*a*b^4*x^17 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/11*B*a^4
*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/5*A*a^
5*x^5
```

**maple** [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{23}}{23}+\frac{\left(b^5A+5a\,b^4B\right)x^{20}}{20}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{17}}{17}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{14}}{14}+\frac{A\,a^5x^5}{5}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{11}}{11}+\frac{\left(5a^4bA+a^5B\right)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/23*b^5*B*x^23+1/20*(A*b^5+5*B*a*b^4)*x^20+1/17*(5*A*a*b^4+10*B*a^2*b^3)*x
^17+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/11*(10*A*a^3*b^2+5*B*a^4*b)*x^1
1+1/8*(5*A*a^4*b+B*a^5)*x^8+1/5*a^5*A*x^5
```

**maxima** [A] time = 0.64, size = 119, normalized size = 1.02

$$\frac{1}{23}Bb^5x^{23}+\frac{1}{20}\left(5\,Bab^4+Ab^5\right)x^{20}+\frac{5}{17}\left(2\,Ba^2b^3+Aab^4\right)x^{17}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{5}{11}\left(Ba^4b+2\,Aa^3b^2\right)x^{11}+\frac{1}{5}Aa^5x^5+\frac{1}{8}\left(Ba^5+5\,Aa^4b\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*
x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

**mupad** [B] time = 0.04, size = 107, normalized size = 0.91

$$x^8\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right)+\frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{23}}{23}+\frac{5\,a^2\,b^2\,x^{14}\,\left(A\,b+B\,a\right)}{7}+\frac{5\,a^3\,b\,x^{11}\,\left(2\,A\,b+B\,a\right)}{11}+\frac{5\,a\,b^3\,x^{17}\,\left(A\,b+2\,B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a^5*
x^5)/5 + (B*b^5*x^23)/23 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^11*(
2*A*b + B*a))/11 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

**sympy** [A] time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5}+\frac{Bb^5x^{23}}{23}+x^{20}\left(\frac{Ab^5}{20}+\frac{Bab^4}{4}\right)+x^{17}\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{14}\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{11}\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4b}{11}\right)+x^8\left(\frac{5Aa^4b}{8}+\frac{Ba^5}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**23/23 + x**20*(A*b**5/20 + B*a*b**4/4) + x**17*(5
*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2/
7) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**8*(5*A*a**4*b/8 + B*a**
5/8)
```

## 3.29 $\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=117

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)$$

**Rubi [A]** time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{4}a^5Ax^4+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx &= \int \left(a^5Ax^3+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^9+10a^2b^2(Ab+aB)x^{12}+5\right.\\ &= \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas** [A] time = 0.75, size = 124, normalized size = 1.06

$$\frac{1}{22}x^{22}b^5B+\frac{5}{19}x^{19}b^4aB+\frac{1}{19}x^{19}b^5A+\frac{5}{8}x^{16}b^3a^2B+\frac{5}{16}x^{16}b^4aA+\frac{10}{13}x^{13}b^2a^3B+\frac{10}{13}x^{13}b^3a^2A+\frac{1}{2}x^{10}ba^4B+x^{10}b^2a^3A+\frac{1}{7}x^7a^5B+\frac{5}{7}x^7ba^4A+\frac{1}{4}x^4a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5*B + 5/19*x^19*b^4*a*B + 1/19*x^19*b^5*A + 5/8*x^16*b^3*a^2*B
+ 5/16*x^16*b^4*a*A + 10/13*x^13*b^2*a^3*B + 10/13*x^13*b^3*a^2*A + 1/2*x^1
0*b*a^4*B + x^10*b^2*a^3*A + 1/7*x^7*a^5*B + 5/7*x^7*b*a^4*A + 1/4*x^4*a^5*
A
```

**giac** [A] time = 0.16, size = 124, normalized size = 1.06

$$\frac{1}{22}Bb^5x^{22}+\frac{5}{19}Bab^4x^{19}+\frac{1}{19}Ab^5x^{19}+\frac{5}{8}Ba^2b^3x^{16}+\frac{5}{16}Aab^4x^{16}+\frac{10}{13}Ba^3b^2x^{13}+\frac{10}{13}Aa^2b^3x^{13}+\frac{1}{2}Ba^4bx^{10}+Aa^3b^2x^{10}+\frac{1}{7}Ba^5x^7+\frac{5}{7}Aa^4bx^7+\frac{1}{4}Aa^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 1/2*B*a
^4*b*x^10 + A*a^3*b^2*x^10 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/4*A*a^5*x^
4
```

**maple** [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{22}}{22}+\frac{\left(b^5A+5a\,b^4B\right)x^{19}}{19}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{16}}{16}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{13}}{13}+\frac{A\,a^5x^4}{4}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{10}}{10}+\frac{\left(5a^4bA+a^5B\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/22*b^5*B*x^22+1/19*(A*b^5+5*B*a*b^4)*x^19+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x
^16+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^1
0+1/7*(5*A*a^4*b+B*a^5)*x^7+1/4*a^5*A*x^4
```

**maxima** [A] time = 0.44, size = 119, normalized size = 1.02

$$\frac{1}{22}Bb^5x^{22}+\frac{1}{19}\left(5Bab^4+Ab^5\right)x^{19}+\frac{5}{16}\left(2Ba^2b^3+Aab^4\right)x^{16}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{2}\left(Ba^4b+2Aa^3b^2\right)x^{10}+\frac{1}{4}Aa^5x^4+\frac{1}{7}\left(Ba^5+5Aa^4b\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^2)
*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

**mupad** [B] time = 0.04, size = 107, normalized size = 0.91

$$x^7\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^{19}\left(\frac{A\,b^5}{19}+\frac{5\,B\,a\,b^4}{19}\right)+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{22}}{22}+\frac{10\,a^2\,b^2\,x^{13}\,\left(A\,b+B\,a\right)}{13}+\frac{a^3\,b\,x^{10}\,\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b^3\,x^{16}\,\left(A\,b+2\,B\,a\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (A*a
^5*x^4)/4 + (B*b^5*x^22)/22 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (a^3*b*x^1
0*(2*A*b + B*a))/2 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

**sympy [A]** time = 0.10, size = 133, normalized size = 1.14

$$\frac{Aa^5x^4}{4} + \frac{Bb^5x^{22}}{22} + x^{19}\left(\frac{Ab^5}{19} + \frac{5Bab^4}{19}\right) + x^{16}\left(\frac{5Aab^4}{16} + \frac{5Ba^2b^3}{8}\right) + x^{13}\left(\frac{10Aa^2b^3}{13} + \frac{10Ba^3b^2}{13}\right) + x^{10}\left(Aa^3b^2 + \frac{Ba^4b}{2}\right) + x^7\left(\frac{5Aa^4b}{7} + \frac{Ba^5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b
**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5/7)
```

## 3.30 $\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=42

$$\frac{\left(a+bx^3\right)^6 (Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{\left(a+bx^3\right)^6 (Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^6)/(18*b^2) + (B*(a + b*x^3)^7)/(21*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^5(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right)dx, x, x^3\right)\\ &= \frac{(Ab-aB)\left(a+bx^3\right)^6}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}\end{aligned}$$

**Mathematica [B]** time = 0.03, size = 107, normalized size = 2.55

$$\frac{1}{126}x^3\left(42a^5A+21a^4x^3(aB+5Ab)+70a^3bx^6(aB+2Ab)+105a^2b^2x^9(aB+Ab)+7b^4x^{15}(5aB+Ab)+42ab^3x^{12}(2aB+Ab)+6b^5Bx^{18}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 10
5*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*a*
B)*x^15 + 6*b^5*B*x^18))/126
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [B]** time = 0.72, size = 125, normalized size = 2.98

$$\frac{1}{21}x^{21}b^5B + \frac{5}{18}x^{18}b^4aB + \frac{1}{18}x^{18}b^5A + \frac{2}{3}x^{15}b^3a^2B + \frac{1}{3}x^{15}b^4aA + \frac{5}{6}x^{12}b^2a^3B + \frac{5}{6}x^{12}b^3a^2A + \frac{5}{9}x^9ba^4B + \frac{10}{9}x^9b^2a^3A + \frac{1}{6}x^6a^5B + \frac{5}{6}x^6ba^4A + \frac{1}{3}x^3a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^5*B + 5/18*x^18*b^4*a*B + 1/18*x^18*b^5*A + 2/3*x^15*b^3*a^2*B
+ 1/3*x^15*b^4*a*A + 5/6*x^12*b^2*a^3*B + 5/6*x^12*b^3*a^2*A + 5/9*x^9*b*a^
4*B + 10/9*x^9*b^2*a^3*A + 1/6*x^6*a^5*B + 5/6*x^6*b*a^4*A + 1/3*x^3*a^5*A
```

**giac [B]** time = 0.17, size = 125, normalized size = 2.98

$$\frac{1}{21}Bb^5x^{21} + \frac{5}{18}Bab^4x^{18} + \frac{1}{18}Ab^5x^{18} + \frac{2}{3}Ba^2b^3x^{15} + \frac{1}{3}Aab^4x^{15} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9 + \frac{1}{6}Ba^5x^6 + \frac{5}{6}Aa^4bx^6 + \frac{1}{3}Aa^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*b*
x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x^3
```

**maple [B]** time = 0.04, size = 124, normalized size = 2.95

$$\frac{B\,b^5x^{21}}{21} + \frac{\left(b^5A + 5a\,b^4B\right)x^{18}}{18} + \frac{\left(5a\,b^4A + 10a^2b^3B\right)x^{15}}{15} + \frac{\left(10a^2b^3A + 10a^3b^2B\right)x^{12}}{12} + \frac{A\,a^5x^3}{3} + \frac{\left(10a^3b^2A + 5a^4bB\right)x^9}{9} + \frac{\left(5a^4bA + a^5B\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/21*b^5*B*x^21+1/18*(A*b^5+5*B*a*b^4)*x^18+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x
^15+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+
1/6*(5*A*a^4*b+B*a^5)*x^6+1/3*a^5*A*x^3
```

**maxima [B]** time = 0.44, size = 119, normalized size = 2.83

$$\frac{1}{21}Bb^5x^{21} + \frac{1}{18}\left(5\,Bab^4 + Ab^5\right)x^{18} + \frac{1}{3}\left(2\,Ba^2b^3 + Aab^4\right)x^{15} + \frac{5}{6}\left(Ba^3b^2 + Aa^2b^3\right)x^{12} + \frac{5}{9}\left(Ba^4b + 2\,Aa^3b^2\right)x^9 + \frac{1}{3}Aa^5x^3 + \frac{1}{6}\left(Ba^5 + 5\,Aa^4b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^
9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

**mupad [B]** time = 0.04, size = 107, normalized size = 2.55

$$x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{18}\left(\frac{A\,b^5}{18}+\frac{5\,B\,a\,b^4}{18}\right)+\frac{A\,a^5\,x^3}{3}+\frac{B\,b^5\,x^{21}}{21}+\frac{5\,a^2\,b^2\,x^{12}\,\left(A\,b+B\,a\right)}{6}+\frac{5\,a^3\,b\,x^9\,\left(2\,A\,b+B\,a\right)}{9}+\frac{a\,b^3\,x^{15}\,\left(A\,b+2\,B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*a
^5*x^3)/3 + (B*b^5*x^21)/21 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (5*a^3*b*x^9
*(2*A*b + B*a))/9 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

**sympy [B]** time = 0.10, size = 136, normalized size = 3.24

$$\frac{Aa^5x^3}{3} + \frac{Bb^5x^{21}}{21} + x^{18}\left(\frac{Ab^5}{18} + \frac{5Bab^4}{18}\right) + x^{15}\left(\frac{Aab^4}{3} + \frac{2Ba^2b^3}{3}\right) + x^{12}\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^9\left(\frac{10Aa^3b^2}{9} + \frac{5Ba^4b}{9}\right) + x^6\left(\frac{5Aa^4b}{6} + \frac{Ba^5}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/6)
 + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

## 3.31 $\int x \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=117

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+$$

**Rubi [A]** time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{2}a^5Ax^2+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)dx=\int\left(a^5Ax+a^4(5Ab+aB)x^4+5a^3b(2Ab+aB)x^7+10a^2b^2(Ab+aB)x^{10}+5al\right.$$
$$=\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{8}a^3b(2Ab+aB)x^8+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{1}$$

**Mathematica [A]** time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [A]** time = 0.70, size = 124, normalized size = 1.06

$$\frac{1}{20}x^{20}b^5B+\frac{5}{17}x^{17}b^4aB+\frac{1}{17}x^{17}b^5A+\frac{5}{7}x^{14}b^3a^2B+\frac{5}{14}x^{14}b^4aA+\frac{10}{11}x^{11}b^2a^3B+\frac{10}{11}x^{11}b^3a^2A+\frac{5}{8}x^8ba^4B+\frac{5}{4}x^8b^2a^3A+\frac{1}{5}x^5a^5B+x^5ba^4A+\frac{1}{2}x^2a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5*B + 5/17*x^17*b^4*a*B + 1/17*x^17*b^5*A + 5/7*x^14*b^3*a^2*B
+ 5/14*x^14*b^4*a*A + 10/11*x^11*b^2*a^3*B + 10/11*x^11*b^3*a^2*A + 5/8*x^8
*b*a^4*B + 5/4*x^8*b^2*a^3*A + 1/5*x^5*a^5*B + x^5*b*a^4*A + 1/2*x^2*a^5*A
```

**giac [A]** time = 0.15, size = 124, normalized size = 1.06

$$\frac{1}{20}Bb^5x^{20}+\frac{5}{17}Bab^4x^{17}+\frac{1}{17}Ab^5x^{17}+\frac{5}{7}Ba^2b^3x^{14}+\frac{5}{14}Aab^4x^{14}+\frac{10}{11}Ba^3b^2x^{11}+\frac{10}{11}Aa^2b^3x^{11}+\frac{5}{8}Ba^4bx^8+\frac{5}{4}Aa^3b^2x^8+\frac{1}{5}Ba^5x^5+Aa^4bx^5+\frac{1}{2}Aa^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/8*B*a
^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x^2
```

**maple [A]** time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{20}}{20}+\frac{\left(b^5A+5a\,b^4B\right)x^{17}}{17}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{14}}{14}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{11}}{11}+\frac{A\,a^5x^2}{2}+\frac{\left(10a^3b^2A+5a^4bB\right)x^8}{8}+\frac{\left(5a^4bA+a^5B\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/20*b^5*B*x^20+1/17*(A*b^5+5*B*a*b^4)*x^17+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x
^14+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+
1/5*(5*A*a^4*b+B*a^5)*x^5+1/2*a^5*A*x^2
```

**maxima [A]** time = 0.65, size = 119, normalized size = 1.02

$$\frac{1}{20}Bb^5x^{20}+\frac{1}{17}\left(5\,Bab^4+Ab^5\right)x^{17}+\frac{5}{14}\left(2\,Ba^2b^3+Aab^4\right)x^{14}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{5}{8}\left(Ba^4b+2\,Aa^3b^2\right)x^8+\frac{1}{2}Aa^5x^2+\frac{1}{5}\left(Ba^5+5\,Aa^4b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^2)
*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

**mupad [B]** time = 0.04, size = 106, normalized size = 0.91

$$x^5\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^{17}\left(\frac{A\,b^5}{17}+\frac{5\,B\,a\,b^4}{17}\right)+\frac{A\,a^5\,x^2}{2}+\frac{B\,b^5\,x^{20}}{20}+\frac{10\,a^2\,b^2\,x^{11}\,\left(A\,b+B\,a\right)}{11}+\frac{5\,a^3\,b\,x^8\,\left(2\,A\,b+B\,a\right)}{8}+\frac{5\,a\,b^3\,x^{14}\,\left(A\,b+2\,B\,a\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^5*((B*a^5)/5 + A*a^4*b) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a^5*x^2
)/2 + (B*b^5*x^20)/20 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^8*(2*
A*b + B*a))/8 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

**sympy [A]** time = 0.09, size = 134, normalized size = 1.15

$$\frac{Aa^5x^2}{2}+\frac{Bb^5x^{20}}{20}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{14}\left(\frac{5Aab^4}{14}+\frac{5Ba^2b^3}{7}\right)+x^{11}\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^8\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^5\left(Aa^4b+\frac{Ba^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b
**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**5/5
)
```

### 3.32 $\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=109

$$a^5 Ax+\frac{1}{4}a^4x^4(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5Bx^1$$

**Rubi [A]** time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {373}

$$a^2b^2x^{10}(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{4}a^4x^4(aB+5Ab)+a^5Ax+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \int \left(a^5A+a^4(5Ab+aB)x^3+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^9+5ab^3(Ab+\right.$$
$$= a^5Ax+\frac{1}{4}a^4(5Ab+aB)x^4+\frac{5}{7}a^3b(2Ab+aB)x^7+a^2b^2(Ab+aB)x^{10}+\frac{5}{13}ab^3(Ab+$$

**Mathematica [A]** time = 0.02, size = 109, normalized size = 1.00

$$a^5Ax+\frac{1}{4}a^4x^4(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [A]** time = 0.70, size = 120, normalized size = 1.10

$$\frac{1}{19}x^{19}b^5B+\frac{5}{16}x^{16}b^4aB+\frac{1}{16}x^{16}b^5A+\frac{10}{13}x^{13}b^3a^2B+\frac{5}{13}x^{13}b^4aA+x^{10}b^2a^3B+x^{10}b^3a^2A+\frac{5}{7}x^7ba^4B+\frac{10}{7}x^7b^2a^3A+\frac{1}{4}x^4a^5B+\frac{5}{4}x^4ba^4A+xa^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5*B + 5/16*x^16*b^4*a*B + 1/16*x^16*b^5*A + 10/13*x^13*b^3*a^2*
B + 5/13*x^13*b^4*a*A + x^10*b^2*a^3*B + x^10*b^3*a^2*A + 5/7*x^7*b*a^4*B +
 10/7*x^7*b^2*a^3*A + 1/4*x^4*a^5*B + 5/4*x^4*b*a^4*A + x*a^5*A
```

**giac [A]** time = 0.17, size = 120, normalized size = 1.10

$$\frac{1}{19}Bb^5x^{19} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{7}Ba^4bx^7 + \frac{10}{7}Aa^3b^2x^7 + \frac{1}{4}Ba^5x^4 + \frac{5}{4}Aa^4bx^4 + Aa^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 10/13*B*a^2*b^3*x^1
3 + 5/13*A*a*b^4*x^13 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/7*B*a^4*b*x^7 +
 10/7*A*a^3*b^2*x^7 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + A*a^5*x
```

**maple [A]** time = 0.04, size = 121, normalized size = 1.11

$$\frac{B\,b^5x^{19}}{19} + \frac{\left(b^5A + 5a\,b^4B\right)x^{16}}{16} + \frac{\left(5a\,b^4A + 10a^2b^3B\right)x^{13}}{13} + \frac{\left(10a^2b^3A + 10a^3b^2B\right)x^{10}}{10} + A\,a^5x + \frac{\left(10a^3b^2A + 5a^4bB\right)x^7}{7} + \frac{\left(5a^4bA + a^5B\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/19*b^5*B*x^19+1/16*(A*b^5+5*B*a*b^4)*x^16+1/13*(5*A*a*b^4+10*B*a^2*b^3)*x
^13+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/7*(10*A*a^3*b^2+5*B*a^4*b)*x^7+
1/4*(5*A*a^4*b+B*a^5)*x^4+a^5*A*x
```

**maxima [A]** time = 0.60, size = 115, normalized size = 1.06

$$\frac{1}{19}Bb^5x^{19} + \frac{1}{16}\left(5\,Bab^4 + Ab^5\right)x^{16} + \frac{5}{13}\left(2\,Ba^2b^3 + Aab^4\right)x^{13} + \left(Ba^3b^2 + Aa^2b^3\right)x^{10} + \frac{5}{7}\left(Ba^4b + 2\,Aa^3b^2\right)x^7 + Aa^5x + \frac{1}{4}\left(Ba^5 + 5\,Aa^4b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 +
 A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

**mupad [B]** time = 0.04, size = 103, normalized size = 0.94

$$x^4\left(\frac{B\,a^5}{4} + \frac{5\,A\,b\,a^4}{4}\right) + x^{16}\left(\frac{A\,b^5}{16} + \frac{5\,B\,a\,b^4}{16}\right) + \frac{B\,b^5\,x^{19}}{19} + A\,a^5\,x + a^2\,b^2\,x^{10}\,\left(A\,b + B\,a\right) + \frac{5\,a^3\,b\,x^7\,\left(2\,A\,b + B\,a\right)}{7} + \frac{5\,a\,b^3\,x^{13}\,\left(A\,b + 2\,B\,a\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (B*b
^5*x^19)/19 + A*a^5*x + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^7*(2*A*b + B*
a))/7 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

**sympy [A]** time = 0.09, size = 128, normalized size = 1.17

$$Aa^5x + \frac{Bb^5x^{19}}{19} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{13}\left(\frac{5Aab^4}{13} + \frac{10Ba^2b^3}{13}\right) + x^{10}\left(Aa^2b^3 + Ba^3b^2\right) + x^7\left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^4\left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**19/19 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**13*(5*A
*a*b**4/13 + 10*B*a**2*b**3/13) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x**7*
(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

## 3.33 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x}\,dx$

**Optimal**. Leaf size=88

$$a^5A\log(x)+\frac{5}{3}a^4Abx^3+\frac{5}{3}a^3Ab^2x^6+\frac{10}{9}a^2Ab^3x^9+\frac{5}{12}aAb^4x^{12}+\frac{B\left(a+bx^3\right)^6}{18b}+\frac{1}{15}Ab^5x^{15}$$

**Rubi** [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 80, 43}

$$\frac{10}{9}a^2Ab^3x^9+\frac{5}{3}a^3Ab^2x^6+\frac{5}{3}a^4Abx^3+a^5A\log(x)+\frac{5}{12}aAb^4x^{12}+\frac{B\left(a+bx^3\right)^6}{18b}+\frac{1}{15}Ab^5x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (5*a^4*A*b*x^3)/3 + (5*a^3*A*b^2*x^6)/3 + (10*a^2*A*b^3*x^9)/9 + (5*a*A*b^4
*x^12)/12 + (A*b^5*x^15)/15 + (B*(a + b*x^3)^6)/(18*b) + a^5*A*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x}\,dx,x,x^3\right)\\
&= \frac{B\left(a+bx^3\right)^6}{18b}+\frac{1}{3}A\,\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx,x,x^3\right)\\
&= \frac{B\left(a+bx^3\right)^6}{18b}+\frac{1}{3}A\,\text{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x\right.\right.\\
&= \frac{5}{3}a^4Abx^3+\frac{5}{3}a^3Ab^2x^6+\frac{10}{9}a^2Ab^3x^9+\frac{5}{12}aAb^4x^{12}+\frac{1}{15}Ab^5x^{15}+\frac{B\left(a+bx^3\right)^6}{18b}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 113, normalized size = 1.28

$$a^5A\log(x)+\frac{1}{3}a^4x^3(aB+5Ab)+\frac{5}{6}a^3bx^6(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{12}ab^3x^{12}(2aB+Ab)+\frac{1}{18}b^5Bx^{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A*
b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^15
)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x, x]
```

**fricas [A]** time = 0.82, size = 117, normalized size = 1.33

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{15}\left(5Bab^4+Ab^5\right)x^{15}+\frac{5}{12}\left(2Ba^2b^3+Aab^4\right)x^{12}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{6}\left(Ba^4b+2Aa^3b^2\right)x^6+Aa^5\log(x)+\frac{1}{3}\left(Ba^5+5Aa^4b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + A*a^5*log(x) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

**giac [A]** time = 0.16, size = 124, normalized size = 1.41

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{3}Bab^4x^{15}+\frac{1}{15}Ab^5x^{15}+\frac{5}{6}Ba^2b^3x^{12}+\frac{5}{12}Aab^4x^{12}+\frac{10}{9}Ba^3b^2x^9+\frac{10}{9}Aa^2b^3x^9+\frac{5}{6}Ba^4bx^6+\frac{5}{3}Aa^3b^2x^6+\frac{1}{3}Ba^5x^3+\frac{5}{3}Aa^4bx^3+Aa^5\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 5/6*B*a^2*b^3*x^12 +
 5/12*A*a*b^4*x^12 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/6*B*a^4*b*
x^6 + 5/3*A*a^3*b^2*x^6 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*log(abs(x
))
```

**maple [A]** time = 0.04, size = 124, normalized size = 1.41

$$\frac{Bb^5x^{18}}{18}+\frac{Ab^5x^{15}}{15}+\frac{Bab^4x^{15}}{3}+\frac{5Aab^4x^{12}}{12}+\frac{5Ba^2b^3x^{12}}{6}+\frac{10Aa^2b^3x^9}{9}+\frac{10Ba^3b^2x^9}{9}+\frac{5Aa^3b^2x^6}{3}+\frac{5Ba^4bx^6}{6}+\frac{5Aa^4bx^3}{3}+\frac{Ba^5x^3}{3}+Aa^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x,x)
```

```
[Out] 1/18*B*b^5*x^18+1/15*A*b^5*x^15+1/3*B*x^15*a*b^4+5/12*a*A*b^4*x^12+5/6*B*x^
12*a^2*b^3+10/9*a^2*A*b^3*x^9+10/9*B*x^9*a^3*b^2+5/3*a^3*A*b^2*x^6+5/6*B*x^
6*a^4*b+5/3*a^4*A*b*x^3+1/3*B*x^3*a^5+a^5*A*ln(x)
```

**maxima [A]** time = 0.45, size = 120, normalized size = 1.36

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{15}\left(5Bab^4+Ab^5\right)x^{15}+\frac{5}{12}\left(2Ba^2b^3+Aab^4\right)x^{12}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{6}\left(Ba^4b+2Aa^3b^2\right)x^6+\frac{1}{3}Aa^5\log\left(x^3\right)+\frac{1}{3}\left(Ba^5+5Aa^4b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 1/3*A*a^5*log(x^3) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

**mupad [B]** time = 0.05, size = 105, normalized size = 1.19

$$x^3\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+x^{15}\left(\frac{A\,b^5}{15}+\frac{B\,a\,b^4}{3}\right)+\frac{B\,b^5\,x^{18}}{18}+A\,a^5\,\ln(x)+\frac{10\,a^2\,b^2\,x^9\,(A\,b+B\,a)}{9}+\frac{5\,a^3\,b\,x^6\,(2\,A\,b+B\,a)}{6}+\frac{5\,a\,b^3\,x^{12}\,(A\,b+2\,B\,a)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x,x)
```

```
[Out] x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (B*b^5*
x^18)/18 + A*a^5*log(x) + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^6*(2*
A*b + B*a))/6 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

**sympy [A]** time = 0.24, size = 134, normalized size = 1.52

$$Aa^5\log{(x)}+\frac{Bb^5x^{18}}{18}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{12}\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^9\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^6\left(\frac{5Aa^3b^2}{3}+\frac{5Ba^4b}{6}\right)+x^3\left(\frac{5Aa^4b}{3}+\frac{Ba^5}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**18/18 + x**15*(A*b**5/15 + B*a*b**4/3) + x**12*(5
*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2/9
) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

## 3.34 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx$

**Optimal**. Leaf size=112

$$-\frac{a^5A}{x}+\frac{1}{2}a^4x^2(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{5}{4}a^2b^2x^8(aB+Ab)+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5Bx^1$$

**Rubi [A]** time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{5}{4}a^2b^2x^8(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{2}a^4x^2(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx = \int \left(\frac{a^5A}{x^2}+a^4(5Ab+aB)x+5a^3b(2Ab+aB)x^4+10a^2b^2(Ab+aB)x^7+5ab^3(Ab+\right.$$

$$=-\frac{a^5A}{x}+\frac{1}{2}a^4(5Ab+aB)x^2+a^3b(2Ab+aB)x^5+\frac{5}{4}a^2b^2(Ab+aB)x^8+\frac{5}{11}ab^3(Ab+$$

**Mathematica [A]** time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5A}{x}+\frac{1}{2}a^4x^2(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{5}{4}a^2b^2x^8(aB+Ab)+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^2, x]

**fricas [A]** time = 0.76, size = 121, normalized size = 1.08

$$\frac{308\,Bb^{5}x^{18} + 374\left(5\,Bab^{4} + Ab^{5}\right)x^{15} + 2380\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{12} + 6545\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{9} + 5236\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{6} - 5236\,Aa^{5} + 2618\left(Ba^{5} + 5\,Aa^{4}b\right)x^{3}}{5236\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5236*(308*B*b^5*x^18 + 374*(5*B*a*b^4 + A*b^5)*x^15 + 2380*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5236*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 5236*A*a^5 + 2618*(B*a^5 + 5*A*a^4*b)*x^3)/x
```

**giac [A]** time = 0.15, size = 124, normalized size = 1.11

$$\frac{1}{17}\,Bb^{5}x^{17} + \frac{5}{14}\,Bab^{4}x^{14} + \frac{1}{14}\,Ab^{5}x^{14} + \frac{10}{11}\,Ba^{2}b^{3}x^{11} + \frac{5}{11}\,Aab^{4}x^{11} + \frac{5}{4}\,Ba^{3}b^{2}x^{8} + \frac{5}{4}\,Aa^{2}b^{3}x^{8} + Ba^{4}bx^{5} + 2\,Aa^{3}b^{2}x^{5} + \frac{1}{2}\,Ba^{5}x^{2} + \frac{5}{2}\,Aa^{4}bx^{2} - \frac{Aa^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + B*a^4*b*x^5
 + 2*A*a^3*b^2*x^5 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 - A*a^5/x
```

**maple [A]** time = 0.05, size = 125, normalized size = 1.12

$$\frac{B\,b^{5}x^{17}}{17} + \frac{A\,b^{5}x^{14}}{14} + \frac{5Ba\,b^{4}x^{14}}{14} + \frac{5Aa\,b^{4}x^{11}}{11} + \frac{10B\,a^{2}b^{3}x^{11}}{11} + \frac{5A\,a^{2}b^{3}x^{8}}{4} + \frac{5B\,a^{3}b^{2}x^{8}}{4} + 2A\,a^{3}b^{2}x^{5} + B\,a^{4}b\,x^{5} + \frac{5A\,a^{4}b\,x^{2}}{2} + \frac{B\,a^{5}x^{2}}{2} - \frac{A\,a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^2,x)
```

```
[Out] 1/17*b^5*B*x^17+1/14*A*x^14*b^5+5/14*B*x^14*a*b^4+5/11*A*x^11*a*b^4+10/11*B
*x^11*a^2*b^3+5/4*A*x^8*a^2*b^3+5/4*B*x^8*a^3*b^2+2*A*x^5*a^3*b^2+B*x^5*a^4
*b+5/2*A*x^2*a^4*b+1/2*B*x^2*a^5-a^5*A/x
```

**maxima [A]** time = 0.54, size = 118, normalized size = 1.05

$$\frac{1}{17}\,Bb^{5}x^{17} + \frac{1}{14}\left(5\,Bab^{4} + Ab^{5}\right)x^{14} + \frac{5}{11}\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{11} + \frac{5}{4}\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{8} + \left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{5} - \frac{Aa^{5}}{x} + \frac{1}{2}\left(Ba^{5} + 5\,Aa^{4}b\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5 -
A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

**mupad [B]** time = 0.04, size = 106, normalized size = 0.95

$$x^{2}\left(\frac{B\,a^{5}}{2} + \frac{5\,A\,b\,a^{4}}{2}\right) + x^{14}\left(\frac{A\,b^{5}}{14} + \frac{5\,B\,a\,b^{4}}{14}\right) - \frac{A\,a^{5}}{x} + \frac{B\,b^{5}\,x^{17}}{17} + \frac{5\,a^{2}\,b^{2}\,x^{8}\,\left(A\,b + B\,a\right)}{4} + a^{3}\,b\,x^{5}\,\left(2\,A\,b + B\,a\right) + \frac{5\,a\,b^{3}\,x^{11}\,\left(A\,b + 2\,B\,a\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^2,x)
```

```
[Out] x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) - (A*a
^5)/x + (B*b^5*x^17)/17 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + a^3*b*x^5*(2*A*b
+ B*a) + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

**sympy [A]** time = 0.25, size = 129, normalized size = 1.15

$$-\frac{Aa^5}{x}+\frac{Bb^5x^{17}}{17}+x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right)+x^{11}\left(\frac{5Aab^4}{11}+\frac{10Ba^2b^3}{11}\right)+x^8\left(\frac{5Aa^2b^3}{4}+\frac{5Ba^3b^2}{4}\right)+x^5\left(2Aa^3b^2+Ba^4b\right)+x^2\left(\frac{5Aa^4b}{2}+\frac{Ba^5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5*
A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4)
 + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

## 3.35 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx$

**Optimal**. Leaf size=112

$$-\frac{a^5 A}{2x^2}+a^4 x(aB+5Ab)+\frac{5}{4}a^3 bx^4(aB+2Ab)+\frac{10}{7}a^2 b^2 x^7(aB+Ab)+\frac{1}{13}b^4 x^{13}(5aB+Ab)+\frac{1}{2}ab^3 x^{10}(2aB+Ab)+\frac{1}{16}b^5 B$$

**Rubi [A]** time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{7}a^2 b^2 x^7 (aB + Ab) + \frac{5}{4}a^3 bx^4 (aB + 2Ab) + a^4 x(aB + 5Ab) - \frac{a^5 A}{2x^2} + \frac{1}{13}b^4 x^{13}(5aB + Ab) + \frac{1}{2}ab^3 x^{10}(2aB + Ab) + \frac{1}{16}b^5 Bx^{16}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^3} dx = \int \left(a^4(5Ab + aB) + \frac{a^5 A}{x^3} + 5a^3 b(2Ab + aB)x^3 + 10a^2 b^2 (Ab + aB)x^6 + 5ab^3(Ab\right.$$
$$= -\frac{a^5 A}{2x^2} + a^4(5Ab + aB)x + \frac{5}{4}a^3 b(2Ab + aB)x^4 + \frac{10}{7}a^2 b^2 (Ab + aB)x^7 + \frac{1}{2}ab^3(Ab$$

**Mathematica [A]** time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5 A}{2x^2} + a^4 x(aB + 5Ab) + \frac{5}{4}a^3 bx^4(aB + 2Ab) + \frac{10}{7}a^2 b^2 x^7(aB + Ab) + \frac{1}{13}b^4 x^{13}(5aB + Ab) + \frac{1}{2}ab^3 x^{10}(2aB + Ab) + \frac{1}{16}b^5 Bx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^3, x]
```

**fricas [A]** time = 1.03, size = 121, normalized size = 1.08

$$\frac{91\,Bb^5x^{18}+112\left(5\,Bab^4+Ab^5\right)x^{15}+728\left(2\,Ba^2b^3+Aab^4\right)x^{12}+2080\left(Ba^3b^2+Aa^2b^3\right)x^9+1820\left(Ba^4b+2\,Aa^3b^2\right)x^6-728\,Aa^5+1456\left(Ba^5+5\,Aa^4b\right)x^3}{1456\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/1456*(91*B*b^5*x^18 + 112*(5*B*a*b^4 + A*b^5)*x^15 + 728*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1820*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 728*A*a^5 + 1456*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
```

**giac [A]** time = 0.16, size = 119, normalized size = 1.06

$$\frac{1}{16}\,Bb^5x^{16}+\frac{5}{13}\,Bab^4x^{13}+\frac{1}{13}\,Ab^5x^{13}+Ba^2b^3x^{10}+\frac{1}{2}\,Aab^4x^{10}+\frac{10}{7}\,Ba^3b^2x^7+\frac{10}{7}\,Aa^2b^3x^7+\frac{5}{4}\,Ba^4bx^4+\frac{5}{2}\,Aa^3b^2x^4+Ba^5x+5\,Aa^4bx-\frac{Aa^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/4*B*a^4*b*x^4
+ 5/2*A*a^3*b^2*x^4 + B*a^5*x + 5*A*a^4*b*x - 1/2*A*a^5/x^2
```

**maple [A]** time = 0.04, size = 120, normalized size = 1.07

$$\frac{B\,b^5x^{16}}{16}+\frac{A\,b^5x^{13}}{13}+\frac{5Ba\,b^4x^{13}}{13}+\frac{Aa\,b^4x^{10}}{2}+B\,a^2b^3x^{10}+\frac{10A\,a^2b^3x^7}{7}+\frac{10B\,a^3b^2x^7}{7}+\frac{5A\,a^3b^2x^4}{2}+\frac{5B\,a^4b\,x^4}{4}+5A\,a^4bx+B\,a^5x-\frac{A\,a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^3,x)
```

```
[Out] 1/16*b^5*B*x^16+1/13*A*x^13*b^5+5/13*B*x^13*a*b^4+1/2*A*x^10*a*b^4+B*x^10*a
^2*b^3+10/7*A*x^7*a^2*b^3+10/7*B*x^7*a^3*b^2+5/2*A*x^4*a^3*b^2+5/4*B*x^4*a^
4*b+5*a^4*b*A*x+B*a^5*x-1/2*a^5*A/x^2
```

**maxima [A]** time = 0.61, size = 116, normalized size = 1.04

$$\frac{1}{16}\,Bb^5x^{16}+\frac{1}{13}\left(5\,Bab^4+Ab^5\right)x^{13}+\frac{1}{2}\left(2\,Ba^2b^3+Aab^4\right)x^{10}+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4-\frac{Aa^5}{2\,x^2}+\left(Ba^5+5\,Aa^4b\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*x^
4 - 1/2*A*a^5/x^2 + (B*a^5 + 5*A*a^4*b)*x
```

**mupad [B]** time = 0.04, size = 104, normalized size = 0.93

$$x\,\left(B\,a^5+5\,A\,b\,a^4\right)+x^{13}\,\left(\frac{A\,b^5}{13}+\frac{5\,B\,a\,b^4}{13}\right)-\frac{A\,a^5}{2\,x^2}+\frac{B\,b^5\,x^{16}}{16}+\frac{10\,a^2\,b^2\,x^7\,\left(A\,b+B\,a\right)}{7}+\frac{5\,a^3\,b\,x^4\,\left(2\,A\,b+B\,a\right)}{4}+\frac{a\,b^3\,x^{10}\,\left(A\,b+2\,B\,a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^3,x)
```

```
[Out] x*(B*a^5 + 5*A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) - (A*a^5)/(2*x^2
) + (B*b^5*x^16)/16 + (10*a^2*b^2*x^7*(A*b + B*a))/7 + (5*a^3*b*x^4*(2*A*b
+ B*a))/4 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

**sympy [A]** time = 0.24, size = 128, normalized size = 1.14

$$-\frac{Aa^5}{2x^2}+\frac{Bb^5x^{16}}{16}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)+x^{10}\left(\frac{Aab^4}{2}+Ba^2b^3\right)+x^7\left(\frac{10Aa^2b^3}{7}+\frac{10Ba^3b^2}{7}\right)+x^4\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x\left(5Aa^4b+Ba^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**16/16 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x*
*10*(A*a*b**4/2 + B*a**2*b**3) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7)
 + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x*(5*A*a**4*b + B*a**5)
```

## 3.36 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^4}\,dx$

**Optimal**. Leaf size=113

$$-\frac{a^5A}{3x^3}+a^4\log(x)(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}b^5B$$

**Rubi [A]** time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4\log(x)(aB+5Ab)-\frac{a^5A}{3x^3}+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^2}+\frac{a^4(5Ab+aB)}{x}+10a^2b^2(Ab+aB)x+5ab^3\right.\right.\\ &= -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 115, normalized size = 1.02

$$-\frac{a^5A}{3x^3}+\frac{5}{3}a^3bx^3(aB+2Ab)+\frac{5}{3}a^2b^2x^6(aB+Ab)+\log(x)\left(a^5B+5a^4Ab\right)+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^4, x]
```

**fricas** [A] time = 0.75, size = 123, normalized size = 1.09

$$\frac{12\,Bb^5x^{18}+15\left(5\,Bab^4+Ab^5\right)x^{15}+100\left(2\,Ba^2b^3+Aab^4\right)x^{12}+300\left(Ba^3b^2+Aa^2b^3\right)x^9+300\left(Ba^4b+2\,Aa^3b^2\right)x^6-60\,Aa^5+180\left(Ba^5+5\,Aa^4b\right)x^3\log(x)}{180\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2)*
x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3
```

**giac** [A] time = 0.16, size = 143, normalized size = 1.27

$$\frac{1}{15}\,Bb^5x^{15}+\frac{5}{12}\,Bab^4x^{12}+\frac{1}{12}\,Ab^5x^{12}+\frac{10}{9}\,Ba^2b^3x^9+\frac{5}{9}\,Aab^4x^9+\frac{5}{3}\,Ba^3b^2x^6+\frac{5}{3}\,Aa^2b^3x^6+\frac{5}{3}\,Ba^4bx^3+\frac{10}{3}\,Aa^3b^2x^3+\left(Ba^5+5\,Aa^4b\right)\log\left(|x|\right)-\frac{Ba^5x^3+5\,Aa^4bx^3+Aa^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/3*B*a^4*b*x^3
 + 10/3*A*a^3*b^2*x^3 + (B*a^5 + 5*A*a^4*b)*log(abs(x)) - 1/3*(B*a^5*x^3 +
5*A*a^4*b*x^3 + A*a^5)/x^3
```

**maple** [A] time = 0.05, size = 123, normalized size = 1.09

$$\frac{B\,b^5x^{15}}{15}+\frac{A\,b^5x^{12}}{12}+\frac{5Ba\,b^4x^{12}}{12}+\frac{5Aa\,b^4x^9}{9}+\frac{10B\,a^2b^3x^9}{9}+\frac{5A\,a^2b^3x^6}{3}+\frac{5B\,a^3b^2x^6}{3}+\frac{10A\,a^3b^2x^3}{3}+\frac{5B\,a^4b\,x^3}{3}+5A\,a^4b\ln(x)+B\,a^5\ln(x)-\frac{A\,a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^4,x)
```

```
[Out] 1/15*b^5*B*x^15+1/12*A*x^12*b^5+5/12*B*x^12*a*b^4+5/9*A*x^9*a*b^4+10/9*B*x^
9*a^2*b^3+5/3*A*x^6*a^2*b^3+5/3*B*x^6*a^3*b^2+10/3*A*x^3*a^3*b^2+5/3*B*x^3*
a^4*b-1/3*a^5*A/x^3+5*A*ln(x)*a^4*b+B*ln(x)*a^5
```

**maxima** [A] time = 0.55, size = 120, normalized size = 1.06

$$\frac{1}{15}\,Bb^5x^{15}+\frac{1}{12}\left(5\,Bab^4+Ab^5\right)x^{12}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^3-\frac{Aa^5}{3\,x^3}+\frac{1}{3}\left(Ba^5+5\,Aa^4b\right)\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*x^3
- 1/3*A*a^5/x^3 + 1/3*(B*a^5 + 5*A*a^4*b)*log(x^3)
```

**mupad [B]** time = 2.35, size = 105, normalized size = 0.93

$$x^{12}\left(\frac{A\,b^5}{12}+\frac{5\,B\,a\,b^4}{12}\right)+\ln(x)\,\left(B\,a^5+5\,A\,b\,a^4\right)-\frac{A\,a^5}{3\,x^3}+\frac{B\,b^5\,x^{15}}{15}+\frac{5\,a^2\,b^2\,x^6\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^3\,b\,x^3\,\left(2\,A\,b+B\,a\right)}{3}+\frac{5\,a\,b^3\,x^9\,\left(A\,b+2\,B\,a\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^4,x)
```

```
[Out] x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/(
3*x^3) + (B*b^5*x^15)/15 + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^3*(2*
A*b + B*a))/3 + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

**sympy [A]** time = 0.37, size = 133, normalized size = 1.18

$$-\frac{Aa^5}{3x^3}+\frac{Bb^5x^{15}}{15}+a^4\left(5Ab+Ba\right)\log\left(x\right)+x^{12}\left(\frac{Ab^5}{12}+\frac{5Bab^4}{12}\right)+x^9\left(\frac{5Aab^4}{9}+\frac{10Ba^2b^3}{9}\right)+x^6\left(\frac{5Aa^2b^3}{3}+\frac{5Ba^3b^2}{3}\right)+x^3\left(\frac{10Aa^3b^2}{3}+\frac{5Ba^4b}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*log(x) + x**12*(A*b
**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(5*
A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3)
```

## 3.37 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx$

**Optimal**. Leaf size=113

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{x}+\frac{5}{2}a^3bx^2(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

**Rubi [A]** time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$2a^2b^2x^5(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)-\frac{a^4(aB+5Ab)}{x}-\frac{a^5A}{4x^4}+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a
*B)*x^11)/11 + (b^5*B*x^14)/14
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = \int \left(\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^2}+5a^3b(2Ab+aB)x+10a^2b^2(Ab+aB)x^4+5ab^3(Ab\right.$$
$$= -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2+2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+$$

**Mathematica [A]** time = 0.06, size = 115, normalized size = 1.02

$$-\frac{a^5A}{4x^4}+\frac{5}{2}a^3bx^2(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{a^5(-B)-5a^4Ab}{x}+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*
a*B)*x^11)/11 + (b^5*B*x^14)/14
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^5, x]
```

**fricas [A]** time = 0.73, size = 121, normalized size = 1.07

$$\frac{44 Bb^5x^{18} + 56 \left(5 Bab^4 + Ab^5\right)x^{15} + 385 \left(2 Ba^2b^3 + Aab^4\right)x^{12} + 1232 \left(Ba^3b^2 + Aa^2b^3\right)x^9 + 1540 \left(Ba^4b + 2 Aa^3b^2\right)x^6 - 154 Aa^5 - 616 \left(Ba^5 + 5 Aa^4b\right)x^3}{616 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/616*(44*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 385*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

**giac [A]** time = 0.16, size = 127, normalized size = 1.12

$$\frac{1}{14} Bb^5x^{14} + \frac{5}{11} Bab^4x^{11} + \frac{1}{11} Ab^5x^{11} + \frac{5}{4} Ba^2b^3x^8 + \frac{5}{8} Aab^4x^8 + 2 Ba^3b^2x^5 + 2 Aa^2b^3x^5 + \frac{5}{2} Ba^4bx^2 + 5 Aa^3b^2x^2 - \frac{4 Ba^5x^3 + 20 Aa^4bx^3 + Aa^5}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 5/4*B*a^2*b^3*x^8 +
 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4
```

**maple [A]** time = 0.05, size = 123, normalized size = 1.09

$$\frac{B b^5x^{14}}{14} + \frac{A b^5x^{11}}{11} + \frac{5Ba b^4x^{11}}{11} + \frac{5Aa b^4x^8}{8} + \frac{5B a^2b^3x^8}{4} + 2A a^2b^3x^5 + 2B a^3b^2x^5 + 5A a^3b^2x^2 + \frac{5B a^4b x^2}{2} - \frac{(5Ab + Ba) a^4}{x} - \frac{A a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^5,x)
```

```
[Out] 1/14*b^5*B*x^14+1/11*A*x^11*b^5+5/11*B*x^11*a*b^4+5/8*A*x^8*a*b^4+5/4*B*x^8
*a^2*b^3+2*A*x^5*a^2*b^3+2*B*x^5*a^3*b^2+5*A*x^2*a^3*b^2+5/2*B*x^2*a^4*b-1/
4*a^5*A/x^4-a^4*(5*A*b+B*a)/x
```

**maxima [A]** time = 0.49, size = 121, normalized size = 1.07

$$\frac{1}{14} Bb^5x^{14} + \frac{1}{11} \left(5 Bab^4 + Ab^5\right)x^{11} + \frac{5}{8} \left(2 Ba^2b^3 + Aab^4\right)x^8 + 2 \left(Ba^3b^2 + Aa^2b^3\right)x^5 + \frac{5}{2} \left(Ba^4b + 2 Aa^3b^2\right)x^2 - \frac{Aa^5 + 4 \left(Ba^5 + 5 Aa^4b\right)x^3}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2 -
1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

**mupad [B]** time = 0.04, size = 109, normalized size = 0.96

$$x^{11} \left(\frac{A b^5}{11} + \frac{5 B a b^4}{11}\right) - \frac{\frac{A a^5}{4} + x^3 \left(B a^5 + 5 A b a^4\right)}{x^4} + \frac{B b^5 x^{14}}{14} + 2 a^2 b^2 x^5 \left(A b + B a\right) + \frac{5 a^3 b x^2 \left(2 A b + B a\right)}{2} + \frac{5 a b^3 x^8 \left(A b + 2 B a\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^5,x)
```

```
[Out] x^11*((A*b^5)/11 + (5*B*a*b^4)/11) - ((A*a^5)/4 + x^3*(B*a^5 + 5*A*a^4*b))/
x^4 + (B*b^5*x^14)/14 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^2*(2*A*b + B
*a))/2 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

**sympy [A]** time = 0.42, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{14}}{14}+x^{11}\left(\frac{Ab^5}{11}+\frac{5Bab^4}{11}\right)+x^8\left(\frac{5Aab^4}{8}+\frac{5Ba^2b^3}{4}\right)+x^5\left(2Aa^2b^3+2Ba^3b^2\right)+x^2\left(5Aa^3b^2+\frac{5Ba^4b}{2}\right)+\frac{-Aa^5+x^3\left(-20Aa^4b-4Ba^5\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 +
5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*b*
*2 + 5*B*a**4*b/2) + (-A*a**5 + x**3*(-20*A*a**4*b - 4*B*a**5))/(4*x**4)
```

## 3.38 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx$

**Optimal**. Leaf size=113

$$-\frac{a^5 A}{5x^5}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

**Rubi [A]** time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{5}{2}a^2b^2x^4(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)-\frac{a^5A}{5x^5}+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx = \int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^3}+10a^2b^2(Ab+aB)x^3+5ab^3(Ab+2a\right.$$
$$= -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB$$

**Mathematica [A]** time = 0.06, size = 113, normalized size = 1.00

$$-\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^6, x]
```

**fricas [A]** time = 0.53, size = 121, normalized size = 1.07

$$\frac{70\,Bb^5x^{18} + 91\,\left(5\,Bab^4 + Ab^5\right)x^{15} + 650\,\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 2275\,\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 4550\,\left(Ba^4b + 2\,Aa^3b^2\right)x^6 - 182\,Aa^5 - 455\,\left(Ba^5 + 5\,Aa^4b\right)x^3}{910\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/910*(70*B*b^5*x^18 + 91*(5*B*a*b^4 + A*b^5)*x^15 + 650*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 4550*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 182*A*a^5 - 455*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

**giac [A]** time = 0.18, size = 124, normalized size = 1.10

$$\frac{1}{13}\,Bb^5x^{13} + \frac{1}{2}\,Bab^4x^{10} + \frac{1}{10}\,Ab^5x^{10} + \frac{10}{7}\,Ba^2b^3x^7 + \frac{5}{7}\,Aab^4x^7 + \frac{5}{2}\,Ba^3b^2x^4 + \frac{5}{2}\,Aa^2b^3x^4 + 5\,Ba^4bx + 10\,Aa^3b^2x - \frac{5\,Ba^5x^3 + 25\,Aa^4bx^3 + 2\,Aa^5}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/7*B*a^2*b^3*x^7 +
 5/7*A*a*b^4*x^7 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5*B*a^4*b*x + 10
*A*a^3*b^2*x - 1/10*(5*B*a^5*x^3 + 25*A*a^4*b*x^3 + 2*A*a^5)/x^5
```

**maple [A]** time = 0.04, size = 119, normalized size = 1.05

$$\frac{B\,b^5x^{13}}{13} + \frac{A\,b^5x^{10}}{10} + \frac{Ba\,b^4x^{10}}{2} + \frac{5Aa\,b^4x^7}{7} + \frac{10B\,a^2b^3x^7}{7} + \frac{5A\,a^2b^3x^4}{2} + \frac{5B\,a^3b^2x^4}{2} + 10A\,a^3b^2x + 5B\,a^4bx - \frac{(5Ab + Ba)\,a^4}{2x^2} - \frac{A\,a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^6,x)
```

```
[Out] 1/13*b^5*B*x^13+1/10*A*x^10*b^5+1/2*B*x^10*a*b^4+5/7*A*x^7*a*b^4+10/7*B*x^7
*a^2*b^3+5/2*A*x^4*a^2*b^3+5/2*B*x^4*a^3*b^2+10*A*a^3*b^2*x+5*B*a^4*b*x-1/5
*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2
```

**maxima [A]** time = 0.73, size = 120, normalized size = 1.06

$$\frac{1}{13}\,Bb^5x^{13} + \frac{1}{10}\left(5\,Bab^4 + Ab^5\right)x^{10} + \frac{5}{7}\left(2\,Ba^2b^3 + Aab^4\right)x^7 + \frac{5}{2}\left(Ba^3b^2 + Aa^2b^3\right)x^4 + 5\left(Ba^4b + 2\,Aa^3b^2\right)x - \frac{2\,Aa^5 + 5\left(Ba^5 + 5\,Aa^4b\right)x^3}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b^
4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/
10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

**mupad [B]** time = 0.04, size = 108, normalized size = 0.96

$$x^{10}\left(\frac{A\,b^5}{10} + \frac{B\,a\,b^4}{2}\right) - \frac{\frac{A\,a^5}{5} + x^3\left(\frac{B\,a^5}{2} + \frac{5\,A\,b\,a^4}{2}\right)}{x^5} + \frac{B\,b^5\,x^{13}}{13} + \frac{5\,a^2\,b^2\,x^4\,\left(A\,b + B\,a\right)}{2} + 5\,a^3\,b\,x\,\left(2\,A\,b + B\,a\right) + \frac{5\,a\,b^3\,x^7\,\left(A\,b + 2\,B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^6,x)
```

```
[Out] x^10*((A*b^5)/10 + (B*a*b^4)/2) - ((A*a^5)/5 + x^3*((B*a^5)/2 + (5*A*a^4*b)
/2))/x^5 + (B*b^5*x^13)/13 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + 5*a^3*b*x*(2*A
*b + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7
```

**sympy [A]** time = 0.48, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{13}}{13}+x^{10}\left(\frac{Ab^5}{10}+\frac{Bab^4}{2}\right)+x^7\left(\frac{5Aab^4}{7}+\frac{10Ba^2b^3}{7}\right)+x^4\left(\frac{5Aa^2b^3}{2}+\frac{5Ba^3b^2}{2}\right)+x\left(10Aa^3b^2+5Ba^4b\right)+\frac{-2Aa^5+x^3\left(-25Aa^4b-5Ba^5\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10*
B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*b*
*2 + 5*B*a**4*b) + (-2*A*a**5 + x**3*(-25*A*a**4*b - 5*B*a**5))/(10*x**5)
```

## 3.39 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} \, dx$

**Optimal**. Leaf size=114

$$-\frac{a^5 A}{6x^6}-\frac{a^4(aB+5Ab)}{3x^3}+5a^3b\log(x)(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{12}b^5$$

**Rubi [A]** time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{10}{3}a^2b^2x^3(aB+Ab)-\frac{a^4(aB+5Ab)}{3x^3}+5a^3b\log(x)(aB+2Ab)-\frac{a^5A}{6x^6}+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{12}b^5Bx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(3*x^3) + (10*a^2*b^2*(A*b + a*B)*x^
3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x
^12)/12 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^3}+\frac{a^4(5Ab+aB)}{x^2}+\frac{5a^3b(2Ab+aB)}{x}+5a\right.\right.\\ &= -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{9}b^4(Ab+\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 106, normalized size = 0.93

$$\frac{1}{36}\left(-\frac{6a^5A}{x^6}-\frac{12a^4(aB+5Ab)}{x^3}+180a^3b\log(x)(aB+2Ab)+120a^2b^2x^3(aB+Ab)+4b^4x^9(5aB+Ab)+30ab^3x^6(2aB+Ab)+3b^5Bx^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] ((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 180
*a^3*b*(2*A*b + a*B)*Log[x])/36
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^7, x]
```

**fricas [A]** time = 0.77, size = 123, normalized size = 1.08

$$\frac{3\,Bb^5x^{18}+4\left(5\,Bab^4+Ab^5\right)x^{15}+30\left(2\,Ba^2b^3+Aab^4\right)x^{12}+120\left(Ba^3b^2+Aa^2b^3\right)x^9+180\left(Ba^4b+2\,Aa^3b^2\right)x^6\log(x)-6\,Aa^5-12\left(Ba^5+5\,Aa^4b\right)x^3}{36\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^6*
log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

**giac [A]** time = 0.16, size = 148, normalized size = 1.30

$$\frac{1}{12}\,Bb^5x^{12}+\frac{5}{9}\,Bab^4x^9+\frac{1}{9}\,Ab^5x^9+\frac{5}{3}\,Ba^2b^3x^6+\frac{5}{6}\,Aab^4x^6+\frac{10}{3}\,Ba^3b^2x^3+\frac{10}{3}\,Aa^2b^3x^3+5\left(Ba^4b+2\,Aa^3b^2\right)\log\left(|x|\right)-\frac{15\,Ba^4bx^6+30\,Aa^3b^2x^6+2\,Ba^5x^3+10\,Aa^4bx^3+Aa^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*(B*a^4*b + 2*A*a
^3*b^2)*log(abs(x)) - 1/6*(15*B*a^4*b*x^6 + 30*A*a^3*b^2*x^6 + 2*B*a^5*x^3
+ 10*A*a^4*b*x^3 + A*a^5)/x^6
```

**maple [A]** time = 0.05, size = 124, normalized size = 1.09

$$\frac{B\,b^5x^{12}}{12}+\frac{A\,b^5x^9}{9}+\frac{5Ba\,b^4x^9}{9}+\frac{5Aa\,b^4x^6}{6}+\frac{5B\,a^2b^3x^6}{3}+\frac{10A\,a^2b^3x^3}{3}+\frac{10B\,a^3b^2x^3}{3}+10A\,a^3b^2\ln(x)+5B\,a^4b\ln(x)-\frac{5A\,a^4b}{3x^3}-\frac{B\,a^5}{3x^3}-\frac{A\,a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^7,x)
```

```
[Out] 1/12*b^5*B*x^12+1/9*A*x^9*b^5+5/9*B*x^9*a*b^4+5/6*A*x^6*a*b^4+5/3*B*x^6*a^2
*b^3+10/3*A*a^2*b^3*x^3+10/3*B*a^3*b^2*x^3-5/3*a^4/x^3*A*b-1/3*a^5/x^3*B-1/
6*a^5*A/x^6+10*A*ln(x)*a^3*b^2+5*B*ln(x)*a^4*b
```

**maxima [A]** time = 0.65, size = 122, normalized size = 1.07

$$\frac{1}{12}\,Bb^5x^{12}+\frac{1}{9}\left(5\,Bab^4+Ab^5\right)x^9+\frac{5}{6}\left(2\,Ba^2b^3+Aab^4\right)x^6+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3+\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)\log\left(x^3\right)-\frac{Aa^5+2\left(Ba^5+5\,Aa^4b\right)x^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*log(x
^3) - 1/6*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

**mupad [B]** time = 0.05, size = 113, normalized size = 0.99

$$\ln(x)\,\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)-\frac{\frac{A\,a^5}{6}+x^3\,\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)}{x^6}+x^9\,\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+\frac{B\,b^5\,x^{12}}{12}+\frac{10\,a^2\,b^2\,x^3\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a\,b^3\,x^6\,\left(A\,b+2\,B\,a\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^7,x)
```

```
[Out] log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/6 + x^3*((B*a^5)/3 + (5*A*a^4*
b)/3))/x^6 + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) + (B*b^5*x^12)/12 + (10*a^2*b^
2*x^3*(A*b + B*a))/3 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

**sympy [A]** time = 0.94, size = 131, normalized size = 1.15

$$\frac{Bb^5x^{12}}{12}+5a^3b\,(2Ab+Ba)\log{(x)}+x^9\left(\frac{Ab^5}{9}+\frac{5Bab^4}{9}\right)+x^6\left(\frac{5Aab^4}{6}+\frac{5Ba^2b^3}{3}\right)+x^3\left(\frac{10Aa^2b^3}{3}+\frac{10Ba^3b^2}{3}\right)+\frac{-Aa^5+x^3\left(-10Aa^4b-2Ba^5\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**5*x**12/12 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**9*(A*b**5/9 + 5*B*a*b*
*4/9) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**3*(10*A*a**2*b**3/3 + 10
*B*a**3*b**2/3) + (-A*a**5 + x**3*(-10*A*a**4*b - 2*B*a**5))/(6*x**6)
```

## 3.40 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx$

**Optimal**. Leaf size=110

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

**Rubi [A]** time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$5a^2b^2x^2(aB+Ab)-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}-\frac{a^5A}{7x^7}+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx &= \int \left(\frac{a^5A}{x^8}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^2}+10a^2b^2(Ab+aB)x+5ab^3(Ab+2a\right.\\ &= -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 110, normalized size = 1.00

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^8, x]
```

**fricas [A]** time = 1.11, size = 121, normalized size = 1.10

$$\frac{56\,Bb^5x^{18}+77\left(5\,Bab^4+Ab^5\right)x^{15}+616\left(2\,Ba^2b^3+Aab^4\right)x^{12}+3080\left(Ba^3b^2+Aa^2b^3\right)x^9-3080\left(Ba^4b+2\,Aa^3b^2\right)x^6-88\,Aa^5-154\left(Ba^5+5\,Aa^4b\right)x^3}{616\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/616*(56*B*b^5*x^18 + 77*(5*B*a*b^4 + A*b^5)*x^15 + 616*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

**giac [A]** time = 0.17, size = 127, normalized size = 1.15

$$\frac{1}{11}\,Bb^5x^{11}+\frac{5}{8}\,Bab^4x^8+\frac{1}{8}\,Ab^5x^8+2\,Ba^2b^3x^5+Aab^4x^5+5\,Ba^3b^2x^2+5\,Aa^2b^3x^2-\frac{140\,Ba^4bx^6+280\,Aa^3b^2x^6+7\,Ba^5x^3+35\,Aa^4bx^3+4\,Aa^5}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*b
^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*A*
a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7
```

**maple [A]** time = 0.05, size = 117, normalized size = 1.06

$$\frac{B\,b^5x^{11}}{11}+\frac{A\,b^5x^8}{8}+\frac{5Ba\,b^4x^8}{8}+Aa\,b^4x^5+2B\,a^2b^3x^5+5A\,a^2b^3x^2+5B\,a^3b^2x^2-\frac{5\,(2Ab+Ba)\,a^3b}{x}-\frac{(5Ab+Ba)\,a^4}{4x^4}-\frac{A\,a^5}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^8,x)
```

```
[Out] 1/11*b^5*B*x^11+1/8*A*x^8*b^5+5/8*B*x^8*a*b^4+A*a*b^4*x^5+2*B*a^2*b^3*x^5+5
*A*x^2*a^2*b^3+5*B*x^2*a^3*b^2-1/4*a^4*(5*A*b+B*a)/x^4-1/7*a^5*A/x^7-5*a^3*
b*(2*A*b+B*a)/x
```

**maxima [A]** time = 0.51, size = 121, normalized size = 1.10

$$\frac{1}{11}\,Bb^5x^{11}+\frac{1}{8}\left(5\,Bab^4+Ab^5\right)x^8+\left(2\,Ba^2b^3+Aab^4\right)x^5+5\left(Ba^3b^2+Aa^2b^3\right)x^2-\frac{140\left(Ba^4b+2\,Aa^3b^2\right)x^6+4\,Aa^5+7\left(Ba^5+5\,Aa^4b\right)x^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^5
 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6 +
4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

**mupad [B]** time = 2.34, size = 113, normalized size = 1.03

$$x^8\left(\frac{A\,b^5}{8}+\frac{5\,B\,a\,b^4}{8}\right)-\frac{\frac{A\,a^5}{7}+x^6\,\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)+x^3\,\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)}{x^7}+\frac{B\,b^5\,x^{11}}{11}+5\,a^2\,b^2\,x^2\,\left(A\,b+B\,a\right)+a\,b^3\,x^5\,\left(A\,b+2\,B\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^8,x)
```

```
[Out] x^8*((A*b^5)/8 + (5*B*a*b^4)/8) - ((A*a^5)/7 + x^6*(10*A*a^3*b^2 + 5*B*a^4*
b) + x^3*((B*a^5)/4 + (5*A*a^4*b)/4))/x^7 + (B*b^5*x^11)/11 + 5*a^2*b^2*x^2
*(A*b + B*a) + a*b^3*x^5*(A*b + 2*B*a)
```

**sympy [A]** time = 1.12, size = 129, normalized size = 1.17

$$\frac{Bb^5x^{11}}{11} + x^8\left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^5\left(Aab^4 + 2Ba^2b^3\right) + x^2\left(5Aa^2b^3 + 5Ba^3b^2\right) + \frac{-4Aa^5 + x^6\left(-280Aa^3b^2 - 140Ba^4b\right) + x^3\left(-35Aa^4b - 7Ba^5\right)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a**
2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) + (-4*A*a**5 + x**6*(-280*A*
a**3*b**2 - 140*B*a**4*b) + x**3*(-35*A*a**4*b - 7*B*a**5))/(28*x**7)
```

## 3.41 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} \, dx$

**Optimal**. Leaf size=113

$$-\frac{a^5 A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

**Rubi [A]** time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$10a^2b^2x(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{a^5A}{8x^8}+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} \, dx &= \int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^9}+\frac{a^4(5Ab+aB)}{x^6}+\frac{5a^3b(2Ab+aB)}{x^3}+5ab^3(Ab+2\right.\\ &= -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2a\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 113, normalized size = 1.00

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^9, x]
```

**fricas** [A] time = 0.72, size = 121, normalized size = 1.07

$$\frac{28\,Bb^5x^{18} + 40\left(5\,Bab^4 + Ab^5\right)x^{15} + 350\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 2800\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 700\left(Ba^4b + 2\,Aa^3b^2\right)x^6 - 35\,Aa^5 - 56\left(Ba^5 + 5\,Aa^4b\right)x^3}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/280*(28*B*b^5*x^18 + 40*(5*B*a*b^4 + A*b^5)*x^15 + 350*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 700*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 35*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

**giac** [A] time = 0.16, size = 124, normalized size = 1.10

$$\frac{1}{10}\,Bb^5x^{10} + \frac{5}{7}\,Bab^4x^7 + \frac{1}{7}\,Ab^5x^7 + \frac{5}{2}\,Ba^2b^3x^4 + \frac{5}{4}\,Aab^4x^4 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{100\,Ba^4bx^6 + 200\,Aa^3b^2x^6 + 8\,Ba^5x^3 + 40\,Aa^4bx^3 + 5\,Aa^5}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/2*B*a^2*b^3*x^4 + 5/4
*A*a*b^4*x^4 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/40*(100*B*a^4*b*x^6 + 20
0*A*a^3*b^2*x^6 + 8*B*a^5*x^3 + 40*A*a^4*b*x^3 + 5*A*a^5)/x^8
```

**maple** [A] time = 0.04, size = 114, normalized size = 1.01

$$\frac{B\,b^5x^{10}}{10} + \frac{A\,b^5x^7}{7} + \frac{5Ba\,b^4x^7}{7} + \frac{5Aa\,b^4x^4}{4} + \frac{5B\,a^2b^3x^4}{2} + 10A\,a^2b^3x + 10B\,a^3b^2x - \frac{5\,(2Ab + Ba)\,a^3b}{2x^2} - \frac{(5Ab + Ba)\,a^4}{5x^5} - \frac{A\,a^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^9,x)
```

```
[Out] 1/10*b^5*B*x^10+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+5/4*A*x^4*a*b^4+5/2*B*x^4*a^2
*b^3+10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/5*a^4*(5*A*b+B*a)/x^5-1/8*a^5*A/x^8-5/
2*a^3*b*(2*A*b+B*a)/x^2
```

**maxima** [A] time = 0.59, size = 120, normalized size = 1.06

$$\frac{1}{10}\,Bb^5x^{10} + \frac{1}{7}\left(5\,Bab^4 + Ab^5\right)x^7 + \frac{5}{4}\left(2\,Ba^2b^3 + Aab^4\right)x^4 + 10\left(Ba^3b^2 + Aa^2b^3\right)x - \frac{100\left(Ba^4b + 2\,Aa^3b^2\right)x^6 + 5\,Aa^5 + 8\left(Ba^5 + 5\,Aa^4b\right)x^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4)
*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x^6
 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

**mupad** [B] time = 0.04, size = 111, normalized size = 0.98

$$x^7\left(\frac{A\,b^5}{7} + \frac{5\,B\,a\,b^4}{7}\right) - \frac{\frac{A\,a^5}{8} + x^6\left(\frac{5\,B\,a^4\,b}{2} + 5\,A\,a^3\,b^2\right) + x^3\left(\frac{B\,a^5}{5} + A\,b\,a^4\right)}{x^8} + \frac{B\,b^5\,x^{10}}{10} + 10\,a^2\,b^2\,x\,\left(A\,b + B\,a\right) + \frac{5\,a\,b^3\,x^4\,\left(A\,b + 2\,B\,a\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^9,x)
```

```
[Out] x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/8 + x^6*(5*A*a^3*b^2 + (5*B*a^4*
b)/2) + x^3*((B*a^5)/5 + A*a^4*b))/x^8 + (B*b^5*x^10)/10 + 10*a^2*b^2*x*(A*
b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

**sympy** [**A**] time = 1.21, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{10}}{10} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^4\left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2}\right) + x\left(10Aa^2b^3 + 10Ba^3b^2\right) + \frac{-5Aa^5 + x^6\left(-200Aa^3b^2 - 100Ba^4b\right) + x^3\left(-40Aa^4b - 8Ba^5\right)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**5*x**10/10 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**4*(5*A*a*b**4/4 + 5*B
*a**2*b**3/2) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-5*A*a**5 + x**6*(-2
00*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-40*A*a**4*b - 8*B*a**5))/(40*x**8)
```

## 3.42 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{10}}\,dx$

**Optimal**. Leaf size=114

$$-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{3x^3}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

**Rubi [A]** time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$10a^2b^2\log(x)(aB+Ab)-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{a^5A}{9x^9}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*
x^9)/9 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{10}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^2}+\frac{10a^2b\right.\right.\\ &= -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{6}b^4(Ab+5aB)x\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 106, normalized size = 0.93

$$\frac{1}{18}\left(-\frac{2a^5A}{x^9}-\frac{3a^4(aB+5Ab)}{x^6}-\frac{30a^3b(aB+2Ab)}{x^3}+180a^2b^2\log(x)(aB+Ab)+3b^4x^6(5aB+Ab)+30ab^3x^3(2aB+Ab)+2b^5Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

[Out] ((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3 + 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 180*a^2*b^2*(A*b + a*B)*Log[x])/18

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^10,x]

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^10, x]

**fricas [A]** time = 0.64, size = 123, normalized size = 1.08

$$\frac{2\,Bb^5x^{18}+3\left(5\,Bab^4+Ab^5\right)x^{15}+30\left(2\,Ba^2b^3+Aab^4\right)x^{12}+180\left(Ba^3b^2+Aa^2b^3\right)x^9\log(x)-30\left(Ba^4b+2\,Aa^3b^2\right)x^6-2\,Aa^5-3\left(Ba^5+5\,Aa^4b\right)x^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="fricas")

[Out] 1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9

**giac [A]** time = 0.15, size = 150, normalized size = 1.32

$$\frac{1}{9}\,Bb^5x^9+\frac{5}{6}\,Bab^4x^6+\frac{1}{6}\,Ab^5x^6+\frac{10}{3}\,Ba^2b^3x^3+\frac{5}{3}\,Aab^4x^3+10\left(Ba^3b^2+Aa^2b^3\right)\log\left(|x|\right)-\frac{110\,Ba^3b^2x^9+110\,Aa^2b^3x^9+30\,Ba^4bx^6+60\,Aa^3b^2x^6+3\,Ba^5x^3+15\,Aa^4bx^3+2\,Aa^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")

[Out] 1/9*B*b^5*x^9 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 10/3*B*a^2*b^3*x^3 + 5/3*A*a*b^4*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/18*(110*B*a^3*b^2*x^9 + 110*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 3*B*a^5*x^3 + 15*A*a^4*b*x^3 + 2*A*a^5)/x^9

**maple [A]** time = 0.05, size = 124, normalized size = 1.09

$$\frac{B\,b^5x^9}{9}+\frac{A\,b^5x^6}{6}+\frac{5Ba\,b^4x^6}{6}+\frac{5Aa\,b^4x^3}{3}+\frac{10B\,a^2b^3x^3}{3}+10A\,a^2b^3\ln(x)+10B\,a^3b^2\ln(x)-\frac{10A\,a^3b^2}{3x^3}-\frac{5B\,a^4b}{3x^3}-\frac{5A\,a^4b}{6x^6}-\frac{B\,a^5}{6x^6}-\frac{A\,a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^10,x)

[Out] 1/9*B*b^5*x^9+1/6*A*x^6*b^5+5/6*B*x^6*a*b^4+5/3*A*x^3*a*b^4+10/3*B*x^3*a^2*b^3-10/3*a^3*b^2/x^3*A-5/3*a^4*b/x^3*B-1/9*a^5*A/x^9-5/6*a^4/x^6*A*b-1/6*a^5/x^6*B+10*A*ln(x)*a^2*b^3+10*B*ln(x)*a^3*b^2

**maxima [A]** time = 0.47, size = 123, normalized size = 1.08

$$\frac{1}{9}\,Bb^5x^9+\frac{1}{6}\left(5\,Bab^4+Ab^5\right)x^6+\frac{5}{3}\left(2\,Ba^2b^3+Aab^4\right)x^3+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)\log\left(x^3\right)-\frac{30\left(Ba^4b+2\,Aa^3b^2\right)x^6+2\,Aa^5+3\left(Ba^5+5\,Aa^4b\right)x^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")

[Out] 1/9*B*b^5*x^9 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x^3 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*log(x^3) - 1/18*(30*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 2*A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9

**mupad [B]** time = 0.05, size = 118, normalized size = 1.04

$$x^6\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)-\frac{\frac{A\,a^5}{9}+x^6\left(\frac{5\,B\,a^4\,b}{3}+\frac{10\,A\,a^3\,b^2}{3}\right)+x^3\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)}{x^9}+\ln(x)\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)+\frac{B\,b^5\,x^9}{9}+\frac{5\,a\,b^3\,x^3\,(A\,b+2\,B\,a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^10,x)
```

```
[Out] x^6*((A*b^5)/6 + (5*B*a*b^4)/6) - ((A*a^5)/9 + x^6*((10*A*a^3*b^2)/3 + (5*B
*a^4*b)/3) + x^3*((B*a^5)/6 + (5*A*a^4*b)/6))/x^9 + log(x)*(10*A*a^2*b^3 +
10*B*a^3*b^2) + (B*b^5*x^9)/9 + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

**sympy [A]** time = 2.24, size = 129, normalized size = 1.13

$$\frac{Bb^5x^9}{9}+10a^2b^2\,(Ab+Ba)\log{(x)}+x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right)+x^3\left(\frac{5Aab^4}{3}+\frac{10Ba^2b^3}{3}\right)+\frac{-2Aa^5+x^6\left(-60Aa^3b^2-30Ba^4b\right)+x^3\left(-15Aa^4b-3Ba^5\right)}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)
```

```
[Out] B*b**5*x**9/9 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b*
*4/6) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) + (-2*A*a**5 + x**6*(-60*A*a
**3*b**2 - 30*B*a**4*b) + x**3*(-15*A*a**4*b - 3*B*a**5))/(18*x**9)
```

## 3.43 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx$

**Optimal**. Leaf size=115

$$-\frac{a^5 A}{10x^{10}}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

**Rubi [A]** time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^5 A}{10x^{10}}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))/
(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4
*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx &= \int \left(\frac{a^5 A}{x^{11}}+\frac{a^4(5Ab+aB)}{x^8}+\frac{5a^3b(2Ab+aB)}{x^5}+\frac{10a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+\right.\\ &= -\frac{a^5 A}{10x^{10}}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{10a^2b^2(Ab+aB)}{x}+\frac{5}{2}ab^3(Ab+2\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 118, normalized size = 1.03

$$\frac{-4a^5(7A+10Bx^3)-50a^4bx^3(4A+7Bx^3)-700a^3b^2x^6(A+4Bx^3)+1400a^2b^3x^9(Bx^3-2A)+140ab^4x^{12}(5A+2Bx^3)+7b^5x^{15}(8A+5Bx^3)}{280x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] (1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^3
*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A + 7
*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^11, x]
```

**fricas [A]** time = 1.05, size = 121, normalized size = 1.05

$$\frac{35 Bb^5x^{18} + 56\left(5 Bab^4 + Ab^5\right)x^{15} + 700\left(2 Ba^2b^3 + Aab^4\right)x^{12} - 2800\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 350\left(Ba^4b + 2 Aa^3b^2\right)x^6 - 28 Aa^5 - 40\left(Ba^5 + 5 Aa^4b\right)x^3}{280 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] 1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*a
*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

**giac [A]** time = 0.15, size = 127, normalized size = 1.10

$$\frac{1}{8} Bb^5x^8 + Bab^4x^5 + \frac{1}{5} Ab^5x^5 + 5 Ba^2b^3x^2 + \frac{5}{2} Aab^4x^2 - \frac{1400 Ba^3b^2x^9 + 1400 Aa^2b^3x^9 + 175 Ba^4bx^6 + 350 Aa^3b^2x^6 + 20 Ba^5x^3 + 100 Aa^4bx^3 + 14 Aa^5}{140 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^4
*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 + 3
50*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10
```

**maple [A]** time = 0.05, size = 111, normalized size = 0.97

$$\frac{B b^5x^8}{8} + \frac{A b^5x^5}{5} + Ba b^4x^5 + \frac{5Aa b^4x^2}{2} + 5B a^2b^3x^2 - \frac{10\left(Ab + Ba\right) a^2b^2}{x} - \frac{5\left(2Ab + Ba\right) a^3b}{4x^4} - \frac{\left(5Ab + Ba\right) a^4}{7x^7} - \frac{A a^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^11,x)
```

```
[Out] 1/8*b^5*B*x^8+1/5*A*x^5*b^5+B*x^5*a*b^4+5/2*A*x^2*a*b^4+5*B*x^2*a^2*b^3-5/4
*a^3*b*(2*A*b+B*a)/x^4-1/7*a^4*(5*A*b+B*a)/x^7-10*a^2*b^2*(A*b+B*a)/x-1/10*
a^5*A/x^10
```

**maxima [A]** time = 0.47, size = 122, normalized size = 1.06

$$\frac{1}{8} Bb^5x^8 + \frac{1}{5}\left(5 Bab^4 + Ab^5\right)x^5 + \frac{5}{2}\left(2 Ba^2b^3 + Aab^4\right)x^2 - \frac{1400\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 175\left(Ba^4b + 2 Aa^3b^2\right)x^6 + 14 Aa^5 + 20\left(Ba^5 + 5 Aa^4b\right)x^3}{140 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x
^2 - 1/140*(1400*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 175*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 14*A*a^5 + 20*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

**mupad [B]** time = 2.36, size = 118, normalized size = 1.03

$$x^5\left(\frac{A b^5}{5} + B a b^4\right) - \frac{\frac{A a^5}{10} + x^6\left(\frac{5 B a^4 b}{4} + \frac{5 A a^3 b^2}{2}\right) + x^3\left(\frac{B a^5}{7} + \frac{5 A b a^4}{7}\right) + x^9\left(10 B a^3 b^2 + 10 A a^2 b^3\right)}{x^{10}} + \frac{B b^5 x^8}{8} + \frac{5 a b^3 x^2\left(A b + 2 B a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^11,x)
```

```
[Out] x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/10 + x^6*((5*A*a^3*b^2)/2 + (5*B*a^4*b
)/4) + x^3*((B*a^5)/7 + (5*A*a^4*b)/7) + x^9*(10*A*a^2*b^3 + 10*B*a^3*b^2))
/x^10 + (B*b^5*x^8)/8 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

**sympy [A]** time = 2.93, size = 131, normalized size = 1.14

$$\frac{Bb^5x^8}{8}+x^5\left(\frac{Ab^5}{5}+Bab^4\right)+x^2\left(\frac{5Aab^4}{2}+5Ba^2b^3\right)+\frac{-14Aa^5+x^9\left(-1400Aa^2b^3-1400Ba^3b^2\right)+x^6\left(-350Aa^3b^2-175Ba^4b\right)+x^3\left(-100Aa^4b-20Ba^5\right)}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)
```

```
[Out] B*b**5*x**8/8 + x**5*(A*b**5/5 + B*a*b**4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*
b**3) + (-14*A*a**5 + x**9*(-1400*A*a**2*b**3 - 1400*B*a**3*b**2) + x**6*(-
350*A*a**3*b**2 - 175*B*a**4*b) + x**3*(-100*A*a**4*b - 20*B*a**5))/(140*x*
*10)
```

## 3.44 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx$

**Optimal**. Leaf size=109

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

**Rubi [A]** time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^5A}{11x^{11}}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = \int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^9}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{10a^2b^2(Ab+aB)}{x^3}\right.$$
$$=-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+2aB)x$$

**Mathematica [A]** time = 0.04, size = 109, normalized size = 1.00

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^12, x]
```

**fricas [A]** time = 0.79, size = 121, normalized size = 1.11

$$\frac{88 Bb^5x^{18} + 154\left(5 Bab^4 + Ab^5\right)x^{15} + 3080\left(2 Ba^2b^3 + Aab^4\right)x^{12} - 3080\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 616\left(Ba^4b + 2 Aa^3b^2\right)x^6 - 56 Aa^5 - 77\left(Ba^5 + 5 Aa^4b\right)x^3}{616 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")
```

```
[Out] 1/616*(88*B*b^5*x^18 + 154*(5*B*a*b^4 + A*b^5)*x^15 + 3080*(2*B*a^2*b^3 + A
*a*b^4)*x^12 - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

**giac [A]** time = 0.18, size = 124, normalized size = 1.14

$$\frac{1}{7} Bb^5x^7 + \frac{5}{4} Bab^4x^4 + \frac{1}{4} Ab^5x^4 + 10 Ba^2b^3x + 5 Aab^4x - \frac{440 Ba^3b^2x^9 + 440 Aa^2b^3x^9 + 88 Ba^4bx^6 + 176 Aa^3b^2x^6 + 11 Ba^5x^3 + 55 Aa^4bx^3 + 8 Aa^5}{88 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*A*
a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^11
```

**maple [A]** time = 0.05, size = 108, normalized size = 0.99

$$\frac{B b^5x^7}{7} + \frac{A b^5x^4}{4} + \frac{5Ba b^4x^4}{4} + 5Aa b^4x + 10B a^2b^3x - \frac{5(Ab + Ba) a^2b^2}{x^2} - \frac{(2Ab + Ba) a^3b}{x^5} - \frac{(5Ab + Ba) a^4}{8x^8} - \frac{A a^5}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^12,x)
```

```
[Out] 1/7*b^5*B*x^7+1/4*A*x^4*b^5+5/4*B*x^4*a*b^4+5*a*b^4*A*x+10*a^2*b^3*B*x-a^3*
b*(2*A*b+B*a)/x^5-1/8*a^4*(5*A*b+B*a)/x^8-5*a^2*b^2*(A*b+B*a)/x^2-1/11*a^5*
A/x^11
```

**maxima [A]** time = 0.53, size = 120, normalized size = 1.10

$$\frac{1}{7} Bb^5x^7 + \frac{1}{4}\left(5 Bab^4 + Ab^5\right)x^4 + 5\left(2 Ba^2b^3 + Aab^4\right)x - \frac{440\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 88\left(Ba^4b + 2 Aa^3b^2\right)x^6 + 8 Aa^5 + 11\left(Ba^5 + 5 Aa^4b\right)x^3}{88 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 8
*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

**mupad [B]** time = 0.07, size = 116, normalized size = 1.06

$$x^4\left(\frac{A b^5}{4} + \frac{5 B a b^4}{4}\right) - \frac{\frac{A a^5}{11} + x^6\left(B a^4 b + 2 A a^3 b^2\right) + x^3\left(\frac{B a^5}{8} + \frac{5 A b a^4}{8}\right) + x^9\left(5 B a^3 b^2 + 5 A a^2 b^3\right)}{x^{11}} + \frac{B b^5 x^7}{7} + 5 a b^3 x\,(A b + 2 B a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^12,x)
```

```
[Out] x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/11 + x^6*(2*A*a^3*b^2 + B*a^4*b)
 + x^3*((B*a^5)/8 + (5*A*a^4*b)/8) + x^9*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^11
+ (B*b^5*x^7)/7 + 5*a*b^3*x*(A*b + 2*B*a)
```

**sympy [A]** time = 3.73, size = 131, normalized size = 1.20

$$\frac{Bb^5x^7}{7} + x^4\left(\frac{Ab^5}{4} + \frac{5Bab^4}{4}\right) + x\left(5Aab^4 + 10Ba^2b^3\right) + \frac{-8Aa^5 + x^9\left(-440Aa^2b^3 - 440Ba^3b^2\right) + x^6\left(-176Aa^3b^2 - 88Ba^4b\right) + x^3\left(-55Aa^4b - 11Ba^5\right)}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)
```

```
[Out] B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) + (-8*A*a**5 + x**9*(-440*A*a**2*b**3 - 440*B*a**3*b**2) + x**6*(-176
*A*a**3*b**2 - 88*B*a**4*b) + x**3*(-55*A*a**4*b - 11*B*a**5))/(88*x**11)
```

## 3.45 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{13}}\,dx$

**Optimal**. Leaf size=114

$$-\frac{a^5A}{12x^{12}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{10a^2b^2(aB+Ab)}{3x^3}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{6}b^5Bx$$

**Rubi [A]** time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{10a^2b^2(aB+Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{a^5A}{12x^{12}}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -(a^5*A)/(12*x^12) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (10*a^2*b^2*(A*b + a*B))/(3*x^3) + (b^4*(A*b + 5*a*B)*x^3)/3 + (b
^5*B*x^6)/6 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{13}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(b^4(Ab+5aB)+\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^3}+\frac{10a^2b\right.\right.\\ &= -\frac{a^5A}{12x^{12}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{10a^2b^2(Ab+aB)}{3x^3}+\frac{1}{3}b^4(Ab+5a\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 118, normalized size = 1.04

$$-\frac{a^5\left(3A+4Bx^3\right)+10a^4bx^3\left(2A+3Bx^3\right)+60a^3b^2x^6\left(A+2Bx^3\right)+120a^2Ab^3x^9-180ab^3x^{12}\log(x)(2aB+Ab)-60ab^4Bx^{15}-6b^5x^{15}\left(2A+Bx^3\right)}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -1/36*(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60*
a^3*b^2*x^6*(A + 2*B*x^3) + 10*a^4*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B*x
^3) - 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/x^12
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^13, x]
```

**fricas [A]** time = 0.62, size = 123, normalized size = 1.08

$$\frac{6\,Bb^5x^{18}+12\left(5\,Bab^4+Ab^5\right)x^{15}+180\left(2\,Ba^2b^3+Aab^4\right)x^{12}\log(x)-120\left(Ba^3b^2+Aa^2b^3\right)x^9-30\left(Ba^4b+2\,Aa^3b^2\right)x^6-3\,Aa^5-4\left(Ba^5+5\,Aa^4b\right)x^3}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*b
^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

**giac [A]** time = 0.16, size = 149, normalized size = 1.31

$$\frac{1}{6}\,Bb^5x^6+\frac{5}{3}\,Bab^4x^3+\frac{1}{3}\,Ab^5x^3+5\left(2\,Ba^2b^3+Aab^4\right)\log\left(|x|\right)-\frac{250\,Ba^2b^3x^{12}+125\,Aab^4x^{12}+120\,Ba^3b^2x^9+120\,Aa^2b^3x^9+30\,Ba^4bx^6+60\,Aa^3b^2x^6+4\,Ba^5x^3+20\,Aa^4bx^3+3\,Aa^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)
*log(abs(x)) - 1/36*(250*B*a^2*b^3*x^12 + 125*A*a*b^4*x^12 + 120*B*a^3*b^2*
x^9 + 120*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 4*B*a^5*x^3 +
 20*A*a^4*b*x^3 + 3*A*a^5)/x^12
```

**maple [A]** time = 0.05, size = 124, normalized size = 1.09

$$\frac{B\,b^5x^6}{6}+\frac{A\,b^5x^3}{3}+\frac{5Ba\,b^4x^3}{3}+5Aa\,b^4\ln(x)+10B\,a^2b^3\ln(x)-\frac{10A\,a^2b^3}{3x^3}-\frac{10B\,a^3b^2}{3x^3}-\frac{5A\,a^3b^2}{3x^6}-\frac{5B\,a^4b}{6x^6}-\frac{5A\,a^4b}{9x^9}-\frac{B\,a^5}{9x^9}-\frac{A\,a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^13,x)
```

```
[Out] 1/6*b^5*B*x^6+1/3*A*x^3*b^5+5/3*B*x^3*a*b^4-1/12*a^5*A/x^12-10/3*b^3*a^2/x^
3*A-10/3*b^2*a^3/x^3*B-5/9*a^4/x^9*A*b-1/9*a^5/x^9*B-5/3*a^3*b^2/x^6*A-5/6*
a^4*b/x^6*B+5*A*ln(x)*a*b^4+10*B*ln(x)*a^2*b^3
```

**maxima [A]** time = 0.63, size = 123, normalized size = 1.08

$$\frac{1}{6}\,Bb^5x^6+\frac{1}{3}\left(5\,Bab^4+Ab^5\right)x^3+\frac{5}{3}\left(2\,Ba^2b^3+Aab^4\right)\log\left(x^3\right)-\frac{120\left(Ba^3b^2+Aa^2b^3\right)x^9+30\left(Ba^4b+2\,Aa^3b^2\right)x^6+3\,Aa^5+4\left(Ba^5+5\,Aa^4b\right)x^3}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^3) - 1/36*(120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 3*A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

**mupad [B]** time = 0.06, size = 122, normalized size = 1.07

$$\ln(x)\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)-\frac{\frac{A\,a^5}{12}+x^6\left(\frac{5\,B\,a^4\,b}{6}+\frac{5\,A\,a^3\,b^2}{3}\right)+x^3\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^9\left(\frac{10\,B\,a^3\,b^2}{3}+\frac{10\,A\,a^2\,b^3}{3}\right)}{x^{12}}+x^3\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+\frac{B\,b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^13,x)
```

```
[Out] log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/12 + x^6*((5*A*a^3*b^2)/3 + (5
*B*a^4*b)/6) + x^3*((B*a^5)/9 + (5*A*a^4*b)/9) + x^9*((10*A*a^2*b^3)/3 + (1
0*B*a^3*b^2)/3))/x^12 + x^3*((A*b^5)/3 + (5*B*a*b^4)/3) + (B*b^5*x^6)/6
```

**sympy [A]** time = 6.64, size = 129, normalized size = 1.13

$$\frac{Bb^5x^6}{6}+5ab^3\left(Ab+2Ba\right)\log\left(x\right)+x^3\left(\frac{Ab^5}{3}+\frac{5Bab^4}{3}\right)+\frac{-3Aa^5+x^9\left(-120Aa^2b^3-120Ba^3b^2\right)+x^6\left(-60Aa^3b^2-30Ba^4b\right)+x^3\left(-20Aa^4b-4Ba^5\right)}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)
```

```
[Out] B*b**5*x**6/6 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**4
/3) + (-3*A*a**5 + x**9*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**6*(-60*A*
a**3*b**2 - 30*B*a**4*b) + x**3*(-20*A*a**4*b - 4*B*a**5))/(36*x**12)
```

## 3.46 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx$

**Optimal**. Leaf size=115

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{5a^2b^2(aB+Ab)}{2x^4}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

**Rubi** [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{2x^4}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{a^5A}{13x^{13}}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x + (
b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx = \int \left(\frac{a^5A}{x^{14}}+\frac{a^4(5Ab+aB)}{x^{11}}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^5}+\frac{5ab^3(Ab+2a\dots}{x^2}\right.$$
$$=-\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{10x^{10}}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{x}$$

**Mathematica** [A] time = 0.05, size = 117, normalized size = 1.02

$$-\frac{a^5(70A+91Bx^3)+65a^4bx^3(7A+10Bx^3)+325a^3b^2x^6(4A+7Bx^3)+2275a^2b^3x^9(A+4Bx^3)-2275ab^4x^{12}(Bx^3-2A)-91b^5x^{15}(5A+2Bx^3)}{910x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -1/910*(-2275*a*b^4*x^12*(-2*A + B*x^3) - 91*b^5*x^15*(5*A + 2*B*x^3) + 227
5*a^2*b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*x^
3*(7*A + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/x^13
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^14, x]

**fricas [A]** time = 0.64, size = 121, normalized size = 1.05

$$\frac{182 Bb^5x^{18} + 455 \left(5 Bab^4 + Ab^5\right)x^{15} - 4550 \left(2 Ba^2b^3 + Aab^4\right)x^{12} - 2275 \left(Ba^3b^2 + Aa^2b^3\right)x^9 - 650 \left(Ba^4b + 2 Aa^3b^2\right)x^6 - 70 Aa^5 - 91 \left(Ba^5 + 5 Aa^4b\right)x^3}{910 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")

[Out] 1/910*(182*B*b^5*x^18 + 455*(5*B*a*b^4 + A*b^5)*x^15 - 4550*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13

**giac [A]** time = 0.16, size = 128, normalized size = 1.11

$$\frac{1}{5} Bb^5x^5 + \frac{5}{2} Bab^4x^2 + \frac{1}{2} Ab^5x^2 - \frac{9100 Ba^2b^3x^{12} + 4550 Aab^4x^{12} + 2275 Ba^3b^2x^9 + 2275 Aa^2b^3x^9 + 650 Ba^4bx^6 + 1300 Aa^3b^2x^6 + 91 Ba^5x^3 + 455 Aa^4bx^3 + 70 Aa^5}{910 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")

[Out] 1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^12 + 4550*A*a*b^4*x^12 + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a^4*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)/x^13

**maple [A]** time = 0.06, size = 107, normalized size = 0.93

$$\frac{B b^5x^5}{5} + \frac{A b^5x^2}{2} + \frac{5Ba b^4x^2}{2} - \frac{5 (Ab + 2Ba) a b^3}{x} - \frac{5 (Ab + Ba) a^2b^2}{2x^4} - \frac{5 (2Ab + Ba) a^3b}{7x^7} - \frac{(5Ab + Ba) a^4}{10x^{10}} - \frac{A a^5}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^14,x)

[Out] 1/5*b^5*B*x^5+1/2*A*x^2*b^5+5/2*B*x^2*a*b^4-5/2*a^2*b^2*(A*b+B*a)/x^4-1/13*a^5*A/x^13-5/7*a^3*b*(2*A*b+B*a)/x^7-5*a*b^3*(A*b+2*B*a)/x-1/10*a^4*(5*A*b+B*a)/x^10

**maxima [A]** time = 0.56, size = 122, normalized size = 1.06

$$\frac{1}{5} Bb^5x^5 + \frac{1}{2} \left(5 Bab^4 + Ab^5\right)x^2 - \frac{4550 \left(2 Ba^2b^3 + Aab^4\right)x^{12} + 2275 \left(Ba^3b^2 + Aa^2b^3\right)x^9 + 650 \left(Ba^4b + 2 Aa^3b^2\right)x^6 + 70 Aa^5 + 91 \left(Ba^5 + 5 Aa^4b\right)x^3}{910 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")

[Out] 1/5*B*b^5*x^5 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 - 1/910*(4550*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 650*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 70*A*a^5 + 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13

**mupad [B]** time = 2.37, size = 123, normalized size = 1.07

$$x^2 \left(\frac{A b^5}{2} + \frac{5 B a b^4}{2}\right) - \frac{\frac{A a^5}{13} + x^{12} \left(10 B a^2 b^3 + 5 A a b^4\right) + x^6 \left(\frac{5 B a^4 b}{7} + \frac{10 A a^3 b^2}{7}\right) + x^3 \left(\frac{B a^5}{10} + \frac{A b a^4}{2}\right) + x^9 \left(\frac{5 B a^3 b^2}{2} + \frac{5 A a^2 b^3}{2}\right)}{x^{13}} + \frac{B b^5 x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^14,x)

[Out] x^2*((A*b^5)/2 + (5*B*a*b^4)/2) - ((A*a^5)/13 + x^12*(10*B*a^2*b^3 + 5*A*a*b^4) + x^6*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^3*((B*a^5)/10 + (A*a^4*b)/2) + x^9*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^13 + (B*b^5*x^5)/5

**sympy [A]** time = 10.76, size = 134, normalized size = 1.17

$$\frac{Bb^5x^5}{5}+x^2\left(\frac{Ab^5}{2}+\frac{5Bab^4}{2}\right)+\frac{-70Aa^5+x^{12}\left(-4550Aab^4-9100Ba^2b^3\right)+x^9\left(-2275Aa^2b^3-2275Ba^3b^2\right)+x^6\left(-1300Aa^3b^2-650Ba^4b\right)+x^3\left(-455Aa^4b-91Ba^5\right)}{910x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)
```

```
[Out] B*b**5*x**5/5 + x**2*(A*b**5/2 + 5*B*a*b**4/2) + (-70*A*a**5 + x**12*(-4550
*A*a*b**4 - 9100*B*a**2*b**3) + x**9*(-2275*A*a**2*b**3 - 2275*B*a**3*b**2)
 + x**6*(-1300*A*a**3*b**2 - 650*B*a**4*b) + x**3*(-455*A*a**4*b - 91*B*a**
5))/(910*x**13)
```

## 3.47 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx$

**Optimal**. Leaf size=110

$$-\frac{a^5 A}{14x^{14}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{2a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{4}b^5Bx^4$$

**Rubi** [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{a^5 A}{14x^{14}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4x(5aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -(a^5*A)/(14*x^14) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \int \left(b^4(Ab+5aB)+\frac{a^5 A}{x^{15}}+\frac{a^4(5Ab+aB)}{x^{12}}+\frac{5a^3b(2Ab+aB)}{x^9}+\frac{10a^2b^2(Ab+a}{x^6}\right.$$

$$= -\frac{a^5 A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2a}{2x^2}$$

**Mathematica** [A] time = 0.07, size = 110, normalized size = 1.00

$$-\frac{a^5 A}{14x^{14}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{2a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^15, x]
```

**fricas [A]** time = 0.56, size = 121, normalized size = 1.10

$$\frac{154 Bb^5x^{18} + 616 \left(5 Bab^4 + Ab^5\right)x^{15} - 1540 \left(2 Ba^2b^3 + Aab^4\right)x^{12} - 1232 \left(Ba^3b^2 + Aa^2b^3\right)x^9 - 385 \left(Ba^4b + 2 Aa^3b^2\right)x^6 - 44 Aa^5 - 56 \left(Ba^5 + 5 Aa^4b\right)x^3}{616 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")
```

```
[Out] 1/616*(154*B*b^5*x^18 + 616*(5*B*a*b^4 + A*b^5)*x^15 - 1540*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 385*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 44*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

**giac [A]** time = 0.16, size = 123, normalized size = 1.12

$$\frac{1}{4} Bb^5x^4 + 5 Bab^4x + Ab^5x - \frac{3080 Ba^2b^3x^{12} + 1540 Aab^4x^{12} + 1232 Ba^3b^2x^9 + 1232 Aa^2b^3x^9 + 385 Ba^4bx^6 + 770 Aa^3b^2x^6 + 56 Ba^5x^3 + 280 Aa^4bx^3 + 44 Aa^5}{616 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5*B*a*b^4*x + A*b^5*x - 1/616*(3080*B*a^2*b^3*x^12 + 1540*A
*a*b^4*x^12 + 1232*B*a^3*b^2*x^9 + 1232*A*a^2*b^3*x^9 + 385*B*a^4*b*x^6 + 7
70*A*a^3*b^2*x^6 + 56*B*a^5*x^3 + 280*A*a^4*b*x^3 + 44*A*a^5)/x^14
```

**maple [A]** time = 0.05, size = 102, normalized size = 0.93

$$\frac{B b^5x^4}{4} + A b^5x + 5Ba b^4x - \frac{5 (Ab + 2Ba) a b^3}{2x^2} - \frac{2 (Ab + Ba) a^2b^2}{x^5} - \frac{5 (2Ab + Ba) a^3b}{8x^8} - \frac{(5Ab + Ba) a^4}{11x^{11}} - \frac{A a^5}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^15,x)
```

```
[Out] 1/4*b^5*B*x^4+b^5*A*x+5*a*b^4*B*x-2*a^2*b^2*(A*b+B*a)/x^5-5/8*a^3*b*(2*A*b+
B*a)/x^8-5/2*a*b^3*(A*b+2*B*a)/x^2-1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^1
1
```

**maxima [A]** time = 0.61, size = 119, normalized size = 1.08

$$\frac{1}{4} Bb^5x^4 + \left(5 Bab^4 + Ab^5\right)x - \frac{1540 \left(2 Ba^2b^3 + Aab^4\right)x^{12} + 1232 \left(Ba^3b^2 + Aa^2b^3\right)x^9 + 385 \left(Ba^4b + 2 Aa^3b^2\right)x^6 + 44 Aa^5 + 56 \left(Ba^5 + 5 Aa^4b\right)x^3}{616 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + (5*B*a*b^4 + A*b^5)*x - 1/616*(1540*(2*B*a^2*b^3 + A*a*b^4)
*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 385*(B*a^4*b + 2*A*a^3*b^2)*x^6
+ 44*A*a^5 + 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

**mupad [B]** time = 2.39, size = 120, normalized size = 1.09

$$x \left(A b^5 + 5 B a b^4\right) - \frac{\frac{A a^5}{14} + x^{12} \left(5 B a^2 b^3 + \frac{5 A a b^4}{2}\right) + x^6 \left(\frac{5 B a^4 b}{8} + \frac{5 A a^3 b^2}{4}\right) + x^3 \left(\frac{B a^5}{11} + \frac{5 A b a^4}{11}\right) + x^9 \left(2 B a^3 b^2 + 2 A a^2 b^3\right)}{x^{14}} + \frac{B b^5 x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^15,x)
```

```
[Out] x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/14 + x^12*(5*B*a^2*b^3 + (5*A*a*b^4)/2) +
x^6*((5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^3*((B*a^5)/11 + (5*A*a^4*b)/11) +
 x^9*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^14 + (B*b^5*x^4)/4
```

**sympy [A]** time = 22.98, size = 129, normalized size = 1.17

$$\frac{Bb^5x^4}{4}+x\left(Ab^5+5Bab^4\right)+\frac{-44Aa^5+x^{12}\left(-1540Aab^4-3080Ba^2b^3\right)+x^9\left(-1232Aa^2b^3-1232Ba^3b^2\right)+x^6\left(-770Aa^3b^2-385Ba^4b\right)+x^3\left(-280Aa^4b-56Ba^5\right)}{616x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

```
[Out] B*b**5*x**4/4 + x*(A*b**5 + 5*B*a*b**4) + (-44*A*a**5 + x**12*(-1540*A*a*b*
*4 - 3080*B*a**2*b**3) + x**9*(-1232*A*a**2*b**3 - 1232*B*a**3*b**2) + x**6
*(-770*A*a**3*b**2 - 385*B*a**4*b) + x**3*(-280*A*a**4*b - 56*B*a**5))/(616
*x**14)
```

## 3.48 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx$

**Optimal**. Leaf size=113

$$-\frac{a^5 A}{15x^{15}}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{5a^2b^2(aB+Ab)}{3x^6}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{3x^3}+\frac{1}{3}b^5Bx^3$$

**Rubi [A]** time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{3x^6}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{a^5A}{15x^{15}}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^4\log(x)(5aB+Ab)+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (5*a*b^3*(A*b + 2*a*B))/(3*x^
3) + (b^5*B*x^3)/3 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6} dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(b^5B+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+aB)}{x^3}\right.\right.\\
&= -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab+2aB)}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 116, normalized size = 1.03

$$b^4\log(x)(5aB+Ab)-\frac{3a^5(4A+5Bx^3)+25a^4bx^3(3A+4Bx^3)+100a^3b^2x^6(2A+3Bx^3)+300a^2b^3x^9(A+2Bx^3)+300aAb^4x^{12}-60b^5Bx^{18}}{180x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -1/180*(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) +
100*a^3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4*A
 + 5*B*x^3))/x^15 + b^4*(A*b + 5*a*B)*Log[x]
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^16, x]
```

**fricas [A]** time = 0.67, size = 123, normalized size = 1.09

$$\frac{60\,Bb^5x^{18}+180\left(5\,Bab^4+Ab^5\right)x^{15}\log(x)-300\left(2\,Ba^2b^3+Aab^4\right)x^{12}-300\left(Ba^3b^2+Aa^2b^3\right)x^9-100\left(Ba^4b+2\,Aa^3b^2\right)x^6-12\,Aa^5-15\left(Ba^5+5\,Aa^4b\right)x^3}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

**giac [A]** time = 0.15, size = 145, normalized size = 1.28

$$\frac{1}{3}\,Bb^5x^3+\left(5\,Bab^4+Ab^5\right)\log\left(|x|\right)-\frac{685\,Bab^4x^{15}+137\,Ab^5x^{15}+600\,Ba^2b^3x^{12}+300\,Aab^4x^{12}+300\,Ba^3b^2x^9+300\,Aa^2b^3x^9+100\,Ba^4bx^6+200\,Aa^3b^2x^6+15\,Ba^5x^3+75\,Aa^4bx^3+12\,Aa^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/180*(685*B*a*b^4*x^15 +
 137*A*b^5*x^15 + 600*B*a^2*b^3*x^12 + 300*A*a*b^4*x^12 + 300*B*a^3*b^2*x^9
 + 300*A*a^2*b^3*x^9 + 100*B*a^4*b*x^6 + 200*A*a^3*b^2*x^6 + 15*B*a^5*x^3 +
 75*A*a^4*b*x^3 + 12*A*a^5)/x^15
```

**maple [A]** time = 0.05, size = 123, normalized size = 1.09

$$\frac{B\,b^5x^3}{3}+A\,b^5\ln(x)+5Ba\,b^4\ln(x)-\frac{5Aa\,b^4}{3x^3}-\frac{10B\,a^2b^3}{3x^3}-\frac{5A\,a^2b^3}{3x^6}-\frac{5B\,a^3b^2}{3x^6}-\frac{10A\,a^3b^2}{9x^9}-\frac{5B\,a^4b}{9x^9}-\frac{5A\,a^4b}{12x^{12}}-\frac{B\,a^5}{12x^{12}}-\frac{A\,a^5}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^16,x)
```

```
[Out] 1/3*b^5*B*x^3-5/12*a^4/x^12*A*b-1/12*a^5/x^12*B-5/3*a*b^4/x^3*A-10/3*a^2*b^
3/x^3*B-1/15*a^5*A/x^15-10/9*a^3*b^2/x^9*A-5/9*a^4*b/x^9*B-5/3*b^3*a^2/x^6*
A-5/3*b^2*a^3/x^6*B+A*ln(x)*b^5+5*B*ln(x)*a*b^4
```

**maxima [A]** time = 0.59, size = 123, normalized size = 1.09

$$\frac{1}{3}\,Bb^5x^3+\frac{1}{3}\left(5\,Bab^4+Ab^5\right)\log\left(x^3\right)-\frac{300\left(2\,Ba^2b^3+Aab^4\right)x^{12}+300\left(Ba^3b^2+Aa^2b^3\right)x^9+100\left(Ba^4b+2\,Aa^3b^2\right)x^6+12\,Aa^5+15\left(Ba^5+5\,Aa^4b\right)x^3}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + 1/3*(5*B*a*b^4 + A*b^5)*log(x^3) - 1/180*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 100*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 12*A*a^5 + 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

**mupad [B]** time = 0.08, size = 121, normalized size = 1.07

$$\ln(x)\left(A\,b^5+5\,B\,a\,b^4\right)-\frac{\frac{A\,a^5}{15}+x^{12}\left(\frac{10\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{3}\right)+x^6\left(\frac{5\,B\,a^4\,b}{9}+\frac{10\,A\,a^3\,b^2}{9}\right)+x^3\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^9\left(\frac{5\,B\,a^3\,b^2}{3}+\frac{5\,A\,a^2\,b^3}{3}\right)}{x^{15}}+\frac{B\,b^5\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^16,x)
```

```
[Out] log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/15 + x^12*((10*B*a^2*b^3)/3 + (5*A*a*
b^4)/3) + x^6*((10*A*a^3*b^2)/9 + (5*B*a^4*b)/9) + x^3*((B*a^5)/12 + (5*A*a
^4*b)/12) + x^9*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^15 + (B*b^5*x^3)/3
```

**sympy [A]** time = 29.30, size = 129, normalized size = 1.14

$$\frac{Bb^5x^3}{3}+b^4\left(Ab+5Ba\right)\log\left(x\right)+\frac{-12Aa^5+x^{12}\left(-300Aab^4-600Ba^2b^3\right)+x^9\left(-300Aa^2b^3-300Ba^3b^2\right)+x^6\left(-200Aa^3b^2-100Ba^4b\right)+x^3\left(-75Aa^4b-15Ba^5\right)}{180x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

```
[Out] B*b**5*x**3/3 + b**4*(A*b + 5*B*a)*log(x) + (-12*A*a**5 + x**12*(-300*A*a*b
**4 - 600*B*a**2*b**3) + x**9*(-300*A*a**2*b**3 - 300*B*a**3*b**2) + x**6*(
-200*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-75*A*a**4*b - 15*B*a**5))/(180*x*
*15)
```

## 3.49 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} \, dx$

**Optimal**. Leaf size=115

$$-\frac{a^5A}{16x^{16}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{4x^4}+\frac{1}{2}b^5Bx^2$$

**Rubi [A]** time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{a^5A}{16x^{16}}-\frac{5ab^3(2aB+Ab)}{4x^4}-\frac{b^4(5aB+Ab)}{x}+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -(a^5*A)/(16*x^16) - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))/
(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*x^
4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx &= \int \left(\frac{a^5A}{x^{17}}+\frac{a^4(5Ab+aB)}{x^{14}}+\frac{5a^3b(2Ab+aB)}{x^{11}}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(Ab+\right.\\ &= -\frac{a^5A}{16x^{16}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{a^3b(2Ab+aB)}{2x^{10}}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{5ab^3(Ab+2a}{4x^4}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 118, normalized size = 1.03

$$-\frac{7a^5(13A+16Bx^3)+56a^4bx^3(10A+13Bx^3)+208a^3b^2x^6(7A+10Bx^3)+520a^2b^3x^9(4A+7Bx^3)+1820ab^4x^{12}(A+4Bx^3)-728b^5x^{15}(Bx^3-2A)}{1456x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -1/1456*(-728*b^5*x^15*(-2*A + B*x^3) + 1820*a*b^4*x^12*(A + 4*B*x^3) + 520
*a^2*b^3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*b*
x^3*(10*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/x^16
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^17, x]
```

**fricas [A]** time = 0.56, size = 121, normalized size = 1.05

$$\frac{728 Bb^5x^{18} - 1456 \left(5 Bab^4 + Ab^5\right)x^{15} - 1820 \left(2 Ba^2b^3 + Aab^4\right)x^{12} - 2080 \left(Ba^3b^2 + Aa^2b^3\right)x^9 - 728 \left(Ba^4b + 2 Aa^3b^2\right)x^6 - 91 Aa^5 - 112 \left(Ba^5 + 5 Aa^4b\right)x^3}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="fricas")
```

```
[Out] 1/1456*(728*B*b^5*x^18 - 1456*(5*B*a*b^4 + A*b^5)*x^15 - 1820*(2*B*a^2*b^3
+ A*a*b^4)*x^12 - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

**giac [A]** time = 0.16, size = 128, normalized size = 1.11

$$\frac{1}{2} Bb^5x^2 - \frac{7280 Bab^4x^{15} + 1456 Ab^5x^{15} + 3640 Ba^2b^3x^{12} + 1820 Aab^4x^{12} + 2080 Ba^3b^2x^9 + 2080 Aa^2b^3x^9 + 728 Ba^4bx^6 + 1456 Aa^3b^2x^6 + 112 Ba^5x^3 + 560 Aa^4bx^3 + 91 Aa^5}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^15 + 1456*A*b^5*x^15 + 3640*B*a^2*b^
3*x^12 + 1820*A*a*b^4*x^12 + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 728*
B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*A*a
^5)/x^16
```

**maple [A]** time = 0.04, size = 104, normalized size = 0.90

$$\frac{B b^5x^2}{2} - \frac{(Ab + 5Ba) b^4}{x} - \frac{5 (Ab + 2Ba) a b^3}{4x^4} - \frac{10 (Ab + Ba) a^2b^2}{7x^7} - \frac{(2Ab + Ba) a^3b}{2x^{10}} - \frac{(5Ab + Ba) a^4}{13x^{13}} - \frac{A a^5}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^17,x)
```

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

**maxima [A]** time = 0.50, size = 122, normalized size = 1.06

$$\frac{1}{2} Bb^5x^2 - \frac{1456 \left(5 Bab^4 + Ab^5\right)x^{15} + 1820 \left(2 Ba^2b^3 + Aab^4\right)x^{12} + 2080 \left(Ba^3b^2 + Aa^2b^3\right)x^9 + 728 \left(Ba^4b + 2 Aa^3b^2\right)x^6 + 91 Aa^5 + 112 \left(Ba^5 + 5 Aa^4b\right)x^3}{1456 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^15 + 1820*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

**mupad [B]** time = 2.36, size = 121, normalized size = 1.05

$$\frac{B b^5 x^2}{2} - \frac{\frac{A a^5}{16} + x^6 \left(\frac{B a^4 b}{2} + A a^3 b^2\right) + x^{12} \left(\frac{5 B a^2 b^3}{2} + \frac{5 A a b^4}{4}\right) + x^3 \left(\frac{B a^5}{13} + \frac{5 A b a^4}{13}\right) + x^{15} \left(A b^5 + 5 B a b^4\right) + x^9 \left(\frac{10 B a^3 b^2}{7} + \frac{10 A a^2 b^3}{7}\right)}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^17,x)
```

```
[Out] (B*b^5*x^2)/2 - ((A*a^5)/16 + x^6*(A*a^3*b^2 + (B*a^4*b)/2) + x^12*((5*B*a^
2*b^3)/2 + (5*A*a*b^4)/4) + x^3*((B*a^5)/13 + (5*A*a^4*b)/13) + x^15*(A*b^5
 + 5*B*a*b^4) + x^9*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7))/x^16
```

**sympy [A]** time = 66.09, size = 134, normalized size = 1.17

$$\frac{Bb^5x^2}{2}+\frac{-91Aa^5+x^{15}\left(-1456Ab^5-7280Bab^4\right)+x^{12}\left(-1820Aab^4-3640Ba^2b^3\right)+x^9\left(-2080Aa^2b^3-2080Ba^3b^2\right)+x^6\left(-1456Aa^3b^2-728Ba^4b\right)+x^3\left(-560Aa^4b-112Ba^5\right)}{1456x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)
```

```
[Out] B*b**5*x**2/2 + (-91*A*a**5 + x**15*(-1456*A*b**5 - 7280*B*a*b**4) + x**12*
(-1820*A*a*b**4 - 3640*B*a**2*b**3) + x**9*(-2080*A*a**2*b**3 - 2080*B*a**3
*b**2) + x**6*(-1456*A*a**3*b**2 - 728*B*a**4*b) + x**3*(-560*A*a**4*b - 11
2*B*a**5))/(1456*x**16)
```

## 3.50 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx$

**Optimal**. Leaf size=110

$$-\frac{a^5 A}{17x^{17}}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{2x^2}-\frac{ab^3(2aB+Ab)}{x^5}+b^5Bx$$

**Rubi** [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5 A}{17x^{17}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{2x^2}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx = \int \left(b^5B+\frac{a^5A}{x^{18}}+\frac{a^4(5Ab+aB)}{x^{15}}+\frac{5a^3b(2Ab+aB)}{x^{12}}+\frac{10a^2b^2(Ab+aB)}{x^9}+\frac{5ab^3(Ab\right.$$
$$=-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-$$

**Mathematica** [A] time = 0.05, size = 110, normalized size = 1.00

$$-\frac{a^5 A}{17x^{17}}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{2x^2}-\frac{ab^3(2aB+Ab)}{x^5}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^18, x]
```

**fricas [A]** time = 0.67, size = 121, normalized size = 1.10

$$\frac{5236\,Bb^{5}x^{18} - 2618\left(5\,Bab^{4} + Ab^{5}\right)x^{15} - 5236\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{12} - 6545\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{9} - 2380\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{6} - 308\,Aa^{5} - 374\left(Ba^{5} + 5\,Aa^{4}b\right)x^{3}}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")
```

```
[Out] 1/5236*(5236*B*b^5*x^18 - 2618*(5*B*a*b^4 + A*b^5)*x^15 - 5236*(2*B*a^2*b^3
 + A*a*b^4)*x^12 - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

**giac [A]** time = 0.17, size = 125, normalized size = 1.14

$$Bb^{5}x - \frac{13090\,Bab^{4}x^{15} + 2618\,Ab^{5}x^{15} + 10472\,Ba^{2}b^{3}x^{12} + 5236\,Aab^{4}x^{12} + 6545\,Ba^{3}b^{2}x^{9} + 6545\,Aa^{2}b^{3}x^{9} + 2380\,Ba^{4}bx^{6} + 4760\,Aa^{3}b^{2}x^{6} + 374\,Ba^{5}x^{3} + 1870\,Aa^{4}bx^{3} + 308\,Aa^{5}}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/5236*(13090*B*a*b^4*x^15 + 2618*A*b^5*x^15 + 10472*B*a^2*b^3*x^
12 + 5236*A*a*b^4*x^12 + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B*a
^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A*a^
5)/x^17
```

**maple [A]** time = 0.05, size = 101, normalized size = 0.92

$$B\,b^{5}x - \frac{(Ab+5Ba)\,b^{4}}{2x^{2}} - \frac{(Ab+2Ba)\,a\,b^{3}}{x^{5}} - \frac{5\,(Ab+Ba)\,a^{2}b^{2}}{4x^{8}} - \frac{5\,(2Ab+Ba)\,a^{3}b}{11x^{11}} - \frac{(5Ab+Ba)\,a^{4}}{14x^{14}} - \frac{A\,a^{5}}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^18,x)
```

```
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

**maxima [A]** time = 0.52, size = 119, normalized size = 1.08

$$Bb^{5}x - \frac{2618\left(5\,Bab^{4} + Ab^{5}\right)x^{15} + 5236\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{12} + 6545\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{9} + 2380\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{6} + 308\,Aa^{5} + 374\left(Ba^{5} + 5\,Aa^{4}b\right)x^{3}}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^15 + 5236*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

**mupad [B]** time = 0.08, size = 119, normalized size = 1.08

$$B\,b^{5}\,x - \frac{\frac{A\,a^{5}}{17} + x^{12}\,\left(2\,B\,a^{2}\,b^{3} + A\,a\,b^{4}\right) + x^{6}\,\left(\frac{5\,B\,a^{4}\,b}{11} + \frac{10\,A\,a^{3}\,b^{2}}{11}\right) + x^{3}\,\left(\frac{B\,a^{5}}{14} + \frac{5\,A\,b\,a^{4}}{14}\right) + x^{15}\,\left(\frac{A\,b^{5}}{2} + \frac{5\,B\,a\,b^{4}}{2}\right) + x^{9}\,\left(\frac{5\,B\,a^{3}\,b^{2}}{4} + \frac{5\,A\,a^{2}\,b^{3}}{4}\right)}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^18,x)
```

```
[Out] B*b^5*x - ((A*a^5)/17 + x^12*(2*B*a^2*b^3 + A*a*b^4) + x^6*((10*A*a^3*b^2)/
11 + (5*B*a^4*b)/11) + x^3*((B*a^5)/14 + (5*A*a^4*b)/14) + x^15*((A*b^5)/2
+ (5*B*a*b^4)/2) + x^9*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4))/x^17
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)
```

```
[Out] Timed out
```

## 3.51 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} \, dx$

**Optimal**. Leaf size=91

$$-\frac{a^5B}{15x^{15}}-\frac{5a^4bB}{12x^{12}}-\frac{10a^3b^2B}{9x^9}-\frac{5a^2b^3B}{3x^6}-\frac{A\left(a+bx^3\right)^6}{18ax^{18}}-\frac{5ab^4B}{3x^3}+b^5B\log(x)$$

**Rubi** [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 78, 43}

$$-\frac{5a^2b^3B}{3x^6}-\frac{10a^3b^2B}{9x^9}-\frac{5a^4bB}{12x^{12}}-\frac{a^5B}{15x^{15}}-\frac{A\left(a+bx^3\right)^6}{18ax^{18}}-\frac{5ab^4B}{3x^3}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -(a^5*B)/(15*x^15) - (5*a^4*b*B)/(12*x^12) - (10*a^3*b^2*B)/(9*x^9) - (5*a^
2*b^3*B)/(3*x^6) - (5*a*b^4*B)/(3*x^3) - (A*(a + b*x^3)^6)/(18*a*x^18) + b^
5*B*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{19}} \, dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7} \, dx, x, x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^6}{18ax^{18}}+\frac{1}{3}B\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^6} \, dx, x, x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^6}{18ax^{18}}+\frac{1}{3}B\operatorname{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right) d\right.\\
&= -\frac{a^5B}{15x^{15}}-\frac{5a^4bB}{12x^{12}}-\frac{10a^3b^2B}{9x^9}-\frac{5a^2b^3B}{3x^6}-\frac{5ab^4B}{3x^3}-\frac{A\left(a+bx^3\right)^6}{18ax^{18}}+b^5B\log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 121, normalized size = 1.33

$$-\frac{2a^5(5A+6Bx^3)+15a^4bx^3(4A+5Bx^3)+50a^3b^2x^6(3A+4Bx^3)+100a^2b^3x^9(2A+3Bx^3)+150ab^4x^{12}(A+2Bx^3)+60Ab^5x^{15}-180b^5Bx^{18}\log(x)}{180x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -1/180*(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*A
 + 3*B*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^3)
 + 2*a^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/x^18
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^19, x]
```

**fricas [A]** time = 0.54, size = 123, normalized size = 1.35

$$\frac{180\,Bb^5x^{18}\log(x)-60\left(5\,Bab^4+Ab^5\right)x^{15}-150\left(2\,Ba^2b^3+Aab^4\right)x^{12}-200\left(Ba^3b^2+Aa^2b^3\right)x^9-75\left(Ba^4b+2\,Aa^3b^2\right)x^6-10\,Aa^5-12\left(Ba^5+5\,Aa^4b\right)x^3}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")
```

```
[Out] 1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*a^
3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

**giac [A]** time = 0.16, size = 136, normalized size = 1.49

$$Bb^5\log(|x|)-\frac{147\,Bb^5x^{18}+300\,Bab^4x^{15}+60\,Ab^5x^{15}+300\,Ba^2b^3x^{12}+150\,Aab^4x^{12}+200\,Ba^3b^2x^9+200\,Aa^2b^3x^9+75\,Ba^4bx^6+150\,Aa^3b^2x^6+12\,Ba^5x^3+60\,Aa^4bx^3+10\,Aa^5}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")
```

```
[Out] B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^1
5 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2*b
^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*x^3
 + 10*A*a^5)/x^18
```

**maple [A]** time = 0.05, size = 124, normalized size = 1.36

$$B\,b^5\ln(x)-\frac{A\,b^5}{3x^3}-\frac{5B a\,b^4}{3x^3}-\frac{5Aa\,b^4}{6x^6}-\frac{5B\,a^2b^3}{3x^6}-\frac{10A\,a^2b^3}{9x^9}-\frac{10B\,a^3b^2}{9x^9}-\frac{5A\,a^3b^2}{6x^{12}}-\frac{5B\,a^4b}{12x^{12}}-\frac{A\,a^4b}{3x^{15}}-\frac{B\,a^5}{15x^{15}}-\frac{A\,a^5}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^19,x)
```

```
[Out] -5/6*a^3*b^2/x^12*A-5/12*a^4*b*B/x^12-1/3*b^5/x^3*A-5/3*a*b^4*B/x^3-1/18*A*
a^5/x^18-1/3*a^4/x^15*A*b-1/15*a^5*B/x^15-10/9*b^3*a^2/x^9*A-10/9*a^3*b^2*B
/x^9-5/6*a*b^4/x^6*A-5/3*a^2*b^3*B/x^6+b^5*B*ln(x)
```

**maxima [A]** time = 0.51, size = 123, normalized size = 1.35

$$\frac{1}{3}\,Bb^5\log\left(x^3\right)-\frac{60\left(5\,Bab^4+Ab^5\right)x^{15}+150\left(2\,Ba^2b^3+Aab^4\right)x^{12}+200\left(Ba^3b^2+Aa^2b^3\right)x^9+75\left(Ba^4b+2\,Aa^3b^2\right)x^6+10\,Aa^5+12\left(Ba^5+5\,Aa^4b\right)x^3}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

**mupad [B]** time = 0.09, size = 121, normalized size = 1.33

$$B\,b^5\,\ln(x)-\frac{\frac{A\,a^5}{18}+x^{12}\,\left(\frac{5\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{6}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{12}+\frac{5\,A\,a^3\,b^2}{6}\right)+x^3\,\left(\frac{B\,a^5}{15}+\frac{A\,b\,a^4}{3}\right)+x^{15}\,\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+x^9\,\left(\frac{10\,B\,a^3\,b^2}{9}+\frac{10\,A\,a^2\,b^3}{9}\right)}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^19,x)
```

```
[Out] B*b^5*log(x) - ((A*a^5)/18 + x^12*((5*B*a^2*b^3)/3 + (5*A*a*b^4)/6) + x^6*(
(5*A*a^3*b^2)/6 + (5*B*a^4*b)/12) + x^3*((B*a^5)/15 + (A*a^4*b)/3) + x^15*(
(A*b^5)/3 + (5*B*a*b^4)/3) + x^9*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9))/x^1
8
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

```
[Out] Timed out
```

## 3.52 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx$

**Optimal**. Leaf size=113

$$-\frac{a^5 A}{19x^{19}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{x}$$

**Rubi [A]** time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5 A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7)
- (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx &= \int \left(\frac{a^5 A}{x^{20}}+\frac{a^4(5Ab+aB)}{x^{17}}+\frac{5a^3b(2Ab+aB)}{x^{14}}+\frac{10a^2b^2(Ab+aB)}{x^{11}}+\frac{5ab^3(Ab+2a\ldots}{x^8}\right.\\ &= -\frac{a^5 A}{19x^{19}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{a^2b^2(Ab+aB)}{x^{10}}-\frac{5ab^3(Ab+2aB)}{7x^7}-\ldots\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 119, normalized size = 1.05

$$-\frac{91a^5(16A+19Bx^3)+665a^4bx^3(13A+16Bx^3)+2128a^3b^2x^6(10A+13Bx^3)+3952a^2b^3x^9(7A+10Bx^3)+4940ab^4x^{12}(4A+7Bx^3)+6916b^5x^{15}(A+4Bx^3)}{27664x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -1/27664*(6916*b^5*x^15*(A + 4*B*x^3) + 4940*a*b^4*x^12*(4*A + 7*B*x^3) + 3
952*a^2*b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 665
*a^4*b*x^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/x^19
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^20, x]
```

**fricas [A]** time = 0.49, size = 121, normalized size = 1.07

$$-\frac{27664 Bb^5x^{18} + 6916 \left(5 Bab^4 + Ab^5\right)x^{15} + 19760 \left(2 Ba^2b^3 + Aab^4\right)x^{12} + 27664 \left(Ba^3b^2 + Aa^2b^3\right)x^9 + 10640 \left(Ba^4b + 2 Aa^3b^2\right)x^6 + 1456 Aa^5 + 1729 \left(Ba^5 + 5 Aa^4b\right)x^3}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

**giac [A]** time = 0.15, size = 127, normalized size = 1.12

$$-\frac{27664 Bb^5x^{18} + 34580 Bab^4x^{15} + 6916 Ab^5x^{15} + 39520 Ba^2b^3x^{12} + 19760 Aab^4x^{12} + 27664 Ba^3b^2x^9 + 27664 Aa^2b^3x^9 + 10640 Ba^4bx^6 + 21280 Aa^3b^2x^6 + 1729 Ba^5x^3 + 8645 Aa^4bx^3 + 1456 Aa^5}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 34580*B*a*b^4*x^15 + 6916*A*b^5*x^15 + 39520*B
*a^2*b^3*x^12 + 19760*A*a*b^4*x^12 + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^3*
x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*a^4
*b*x^3 + 1456*A*a^5)/x^19
```

**maple [A]** time = 0.04, size = 104, normalized size = 0.92

$$-\frac{B b^5}{x} - \frac{(Ab + 5Ba) b^4}{4x^4} - \frac{5 (Ab + 2Ba) a b^3}{7x^7} - \frac{(Ab + Ba) a^2b^2}{x^{10}} - \frac{5 (2Ab + Ba) a^3b}{13x^{13}} - \frac{(5Ab + Ba) a^4}{16x^{16}} - \frac{A a^5}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^20,x)
```

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

**maxima [A]** time = 0.46, size = 121, normalized size = 1.07

$$-\frac{27664 Bb^5x^{18} + 6916 \left(5 Bab^4 + Ab^5\right)x^{15} + 19760 \left(2 Ba^2b^3 + Aab^4\right)x^{12} + 27664 \left(Ba^3b^2 + Aa^2b^3\right)x^9 + 10640 \left(Ba^4b + 2 Aa^3b^2\right)x^6 + 1456 Aa^5 + 1729 \left(Ba^5 + 5 Aa^4b\right)x^3}{27664 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

**mupad [B]** time = 2.37, size = 119, normalized size = 1.05

$$-\frac{\frac{A a^5}{19} + x^{12} \left(\frac{10 B a^2 b^3}{7} + \frac{5 A a b^4}{7}\right) + x^6 \left(\frac{5 B a^4 b}{13} + \frac{10 A a^3 b^2}{13}\right) + x^3 \left(\frac{B a^5}{16} + \frac{5 A b a^4}{16}\right) + x^{15} \left(\frac{A b^5}{4} + \frac{5 B a b^4}{4}\right) + x^9 \left(B a^3 b^2 + A a^2 b^3\right) + B b^5 x^{18}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^20,x)
```

```
[Out] -((A*a^5)/19 + x^12*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^6*((10*A*a^3*b^2
)/13 + (5*B*a^4*b)/13) + x^3*((B*a^5)/16 + (5*A*a^4*b)/16) + x^15*((A*b^5)/
4 + (5*B*a*b^4)/4) + x^9*(A*a^2*b^3 + B*a^3*b^2) + B*b^5*x^18)/x^19
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)
```

```
[Out] Timed out
```

## 3.53 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} \, dx$

**Optimal**. Leaf size=117

$$-\frac{a^5 A}{20x^{20}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^5B}{2x^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{a^5 A}{20x^{20}}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{b^5B}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} \, dx = \int \left(\frac{a^5 A}{x^{21}}+\frac{a^4(5Ab+aB)}{x^{18}}+\frac{5a^3b(2Ab+aB)}{x^{15}}+\frac{10a^2b^2(Ab+aB)}{x^{12}}+\frac{5ab^3(Ab+\right.$$
$$= -\frac{a^5 A}{20x^{20}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2\right.}{8x^8}$$

**Mathematica [A]** time = 0.04, size = 121, normalized size = 1.03

$$-\frac{154a^5\left(17A+20Bx^3\right)+1100a^4bx^3\left(14A+17Bx^3\right)+3400a^3b^2x^6\left(11A+14Bx^3\right)+5950a^2b^3x^9\left(8A+11Bx^3\right)+6545ab^4x^{12}\left(5A+8Bx^3\right)+5236b^5x^{15}\left(2A+5Bx^3\right)}{52360x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -1/52360*(5236*b^5*x^15*(2*A + 5*B*x^3) + 6545*a*b^4*x^12*(5*A + 8*B*x^3) +
 5950*a^2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) + 1
100*a^4*b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/x^20
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^21, x]
```

**fricas** [A]   time = 0.53, size = 121, normalized size = 1.03

$$-\frac{26180 \, B b^{5} x^{18} + 10472 \, {\left(5 \, B a b^{4} + A b^{5}\right)} x^{15} + 32725 \, {\left(2 \, B a^{2} b^{3} + A a b^{4}\right)} x^{12} + 47600 \, {\left(B a^{3} b^{2} + A a^{2} b^{3}\right)} x^{9} + 18700 \, {\left(B a^{4} b + 2 \, A a^{3} b^{2}\right)} x^{6} + 2618 \, A a^{5} + 3080 \, {\left(B a^{5} + 5 \, A a^{4} b\right)} x^{3}}{52360 \, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

**giac** [A]   time = 0.15, size = 127, normalized size = 1.09

$$-\frac{26180 \, B b^{5} x^{18} + 52360 \, B a b^{4} x^{15} + 10472 \, A b^{5} x^{15} + 65450 \, B a^{2} b^{3} x^{12} + 32725 \, A a b^{4} x^{12} + 47600 \, B a^{3} b^{2} x^{9} + 47600 \, A a^{2} b^{3} x^{9} + 18700 \, B a^{4} b x^{6} + 37400 \, A a^{3} b^{2} x^{6} + 3080 \, B a^{5} x^{3} + 15400 \, A a^{4} b x^{3} + 2618 \, A a^{5}}{52360 \, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 52360*B*a*b^4*x^15 + 10472*A*b^5*x^15 + 65450*
B*a^2*b^3*x^12 + 32725*A*a*b^4*x^12 + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b^3
*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*A*a
^4*b*x^3 + 2618*A*a^5)/x^20
```

**maple** [A]   time = 0.05, size = 104, normalized size = 0.89

$$-\frac{B \,b^{5}}{2 x^{2}}-\frac{\left(A b +5 B a \right) b^{4}}{5 x^{5}}-\frac{5 \left(A b +2 B a \right) a \,b^{3}}{8 x^{8}}-\frac{10 \left(A b +B a \right) a^{2} b^{2}}{11 x^{11}}-\frac{5 \left(2 A b +B a \right) a^{3} b}{14 x^{14}}-\frac{\left(5 A b +B a \right) a^{4}}{17 x^{17}}-\frac{A \,a^{5}}{20 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^21,x)
```

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

**maxima** [A]   time = 0.55, size = 121, normalized size = 1.03

$$-\frac{26180 \, B b^{5} x^{18} + 10472 \, {\left(5 \, B a b^{4} + A b^{5}\right)} x^{15} + 32725 \, {\left(2 \, B a^{2} b^{3} + A a b^{4}\right)} x^{12} + 47600 \, {\left(B a^{3} b^{2} + A a^{2} b^{3}\right)} x^{9} + 18700 \, {\left(B a^{4} b + 2 \, A a^{3} b^{2}\right)} x^{6} + 2618 \, A a^{5} + 3080 \, {\left(B a^{5} + 5 \, A a^{4} b\right)} x^{3}}{52360 \, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

**mupad** [B]   time = 2.35, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{20}+x^{12}\,\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{14}+\frac{5\,A\,a^3\,b^2}{7}\right)+x^3\,\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{15}\,\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^9\,\left(\frac{10\,B\,a^3\,b^2}{11}+\frac{10\,A\,a^2\,b^3}{11}\right)+\frac{B\,b^5\,x^{18}}{2}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^21,x)
```

```
[Out] -((A*a^5)/20 + x^12*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^6*((5*A*a^3*b^2)/
7 + (5*B*a^4*b)/14) + x^3*((B*a^5)/17 + (5*A*a^4*b)/17) + x^15*((A*b^5)/5 +
```

```
 B*a*b^4) + x^9*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^18)/2)/x
^20
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**21,x)
```

```
[Out] Timed out
```

## 3.54 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx$

**Optimal**. Leaf size=48

$$\frac{(a+bx^3)^6(Ab-7aB)}{126a^2x^{18}}-\frac{A(a+bx^3)^6}{21ax^{21}}$$

**Rubi [A]** time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 78, 37}

$$\frac{(a+bx^3)^6(Ab-7aB)}{126a^2x^{18}}-\frac{A(a+bx^3)^6}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -(A*(a + b*x^3)^6)/(21*a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(126*a^2*x^1
8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx,x,x^3\right)\\ &= -\frac{A(a+bx^3)^6}{21ax^{21}}+\frac{(-Ab+7aB)\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^3\right)}{21a}\\ &= -\frac{A(a+bx^3)^6}{21ax^{21}}+\frac{(Ab-7aB)(a+bx^3)^6}{126a^2x^{18}}\end{aligned}$$

**Mathematica [B]** time = 0.04, size = 118, normalized size = 2.46

$$-\frac{a^5\left(6A+7Bx^3\right)+7a^4bx^3\left(5A+6Bx^3\right)+21a^3b^2x^6\left(4A+5Bx^3\right)+35a^2b^3x^9\left(3A+4Bx^3\right)+35ab^4x^{12}\left(2A+3Bx^3\right)+21b^5x^{15}\left(A+2Bx^3\right)}{126x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -1/126*(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2*
b^3*x^9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5*A
 + 6*B*x^3) + a^5*(6*A + 7*B*x^3))/x^21
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{22}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^22, x]
```

**fricas [B]** time = 0.61, size = 121, normalized size = 2.52

$$-\frac{42\,Bb^5x^{18}+21\left(5\,Bab^4+Ab^5\right)x^{15}+70\left(2\,Ba^2b^3+Aab^4\right)x^{12}+105\left(Ba^3b^2+Aa^2b^3\right)x^9+42\left(Ba^4b+2\,Aa^3b^2\right)x^6+6\,Aa^5+7\left(Ba^5+5\,Aa^4b\right)x^3}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

**giac [B]** time = 0.18, size = 127, normalized size = 2.65

$$-\frac{42\,Bb^5x^{18}+105\,Bab^4x^{15}+21\,Ab^5x^{15}+140\,Ba^2b^3x^{12}+70\,Aab^4x^{12}+105\,Ba^3b^2x^9+105\,Aa^2b^3x^9+42\,Ba^4bx^6+84\,Aa^3b^2x^6+7\,Ba^5x^3+35\,Aa^4bx^3+6\,Aa^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 105*B*a*b^4*x^15 + 21*A*b^5*x^15 + 140*B*a^2*b^3*x^
12 + 70*A*a*b^4*x^12 + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b*x
^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^21
```

**maple [B]** time = 0.05, size = 104, normalized size = 2.17

$$-\frac{B\,b^5}{3x^3}-\frac{(Ab+5Ba)\,b^4}{6x^6}-\frac{5\,(Ab+2Ba)\,a\,b^3}{9x^9}-\frac{5\,(Ab+Ba)\,a^2b^2}{6x^{12}}-\frac{(2Ab+Ba)\,a^3b}{3x^{15}}-\frac{(5Ab+Ba)\,a^4}{18x^{18}}-\frac{A\,a^5}{21x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^22,x)
```

```
[Out] -5/6*b^2*a^2*(A*b+B*a)/x^12-1/3*B*b^5/x^3-1/18*a^4*(5*A*b+B*a)/x^18-1/3*a^3
*b*(2*A*b+B*a)/x^15-5/9*a*b^3*(A*b+2*B*a)/x^9-1/6*b^4*(A*b+5*B*a)/x^6-1/21*
A*a^5/x^21
```

**maxima [B]** time = 0.61, size = 121, normalized size = 2.52

$$-\frac{42\,Bb^5x^{18}+21\left(5\,Bab^4+Ab^5\right)x^{15}+70\left(2\,Ba^2b^3+Aab^4\right)x^{12}+105\left(Ba^3b^2+Aa^2b^3\right)x^9+42\left(Ba^4b+2\,Aa^3b^2\right)x^6+6\,Aa^5+7\left(Ba^5+5\,Aa^4b\right)x^3}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

**mupad [B]** time = 2.37, size = 122, normalized size = 2.54

$$-\frac{\frac{A\,a^5}{21}+x^6\,\left(\frac{B\,a^4\,b}{3}+\frac{2\,A\,a^3\,b^2}{3}\right)+x^{12}\,\left(\frac{10\,B\,a^2\,b^3}{9}+\frac{5\,A\,a\,b^4}{9}\right)+x^3\,\left(\frac{B\,a^5}{18}+\frac{5\,A\,b\,a^4}{18}\right)+x^{15}\,\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+x^9\,\left(\frac{5\,B\,a^3\,b^2}{6}+\frac{5\,A\,a^2\,b^3}{6}\right)+\frac{B\,b^5\,x^{18}}{3}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^22,x)
```

```
[Out] -((A*a^5)/21 + x^6*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^12*((10*B*a^2*b^3)/9
 + (5*A*a*b^4)/9) + x^3*((B*a^5)/18 + (5*A*a^4*b)/18) + x^15*((A*b^5)/6 + (
5*B*a*b^4)/6) + x^9*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^18)/3)/x
^21
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)
```

```
[Out] Timed out
```

## 3.55 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} \, dx$

**Optimal**. Leaf size=117

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{4x^4}$$

**Rubi [A]** time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{22x^{22}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} \, dx = \int \left(\frac{a^5A}{x^{23}}+\frac{a^4(5Ab+aB)}{x^{20}}+\frac{5a^3b(2Ab+aB)}{x^{17}}+\frac{10a^2b^2(Ab+aB)}{x^{14}}+\frac{5ab^3(Ab+\right.$$
$$=-\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab+2a}{2x^{10}}$$

**Mathematica [A]** time = 0.05, size = 117, normalized size = 1.00

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

[Out] IntegrateAlgebraic[((a + b*x^3)^5*(A + B*x^3))/x^23, x]

**fricas** [A] time = 0.51, size = 121, normalized size = 1.03

$$-\frac{76076\,Bb^5x^{18}+43472\left(5\,Bab^4+Ab^5\right)x^{15}+152152\left(2\,Ba^2b^3+Aab^4\right)x^{12}+234080\left(Ba^3b^2+Aa^2b^3\right)x^9+95095\left(Ba^4b+2\,Aa^3b^2\right)x^6+13832\,Aa^5+16016\left(Ba^5+5\,Aa^4b\right)x^3}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")

[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22

**giac** [A] time = 0.16, size = 127, normalized size = 1.09

$$-\frac{76076\,Bb^5x^{18}+217360\,Bab^4x^{15}+43472\,Ab^5x^{15}+304304\,Ba^2b^3x^{12}+152152\,Aab^4x^{12}+234080\,Ba^3b^2x^9+234080\,Aa^2b^3x^9+95095\,Ba^4bx^6+190190\,Aa^3b^2x^6+16016\,Ba^5x^3+80080\,Aa^4bx^3+13832\,Aa^5}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")

[Out] -1/304304*(76076*B*b^5*x^18 + 217360*B*a*b^4*x^15 + 43472*A*b^5*x^15 + 304304*B*a^2*b^3*x^12 + 152152*A*a*b^4*x^12 + 234080*B*a^3*b^2*x^9 + 234080*A*a^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3 + 80080*A*a^4*b*x^3 + 13832*A*a^5)/x^22

**maple** [A] time = 0.05, size = 104, normalized size = 0.89

$$-\frac{B\,b^5}{4x^4}-\frac{(Ab+5Ba)\,b^4}{7x^7}-\frac{(Ab+2Ba)\,a\,b^3}{2x^{10}}-\frac{10\,(Ab+Ba)\,a^2b^2}{13x^{13}}-\frac{5\,(2Ab+Ba)\,a^3b}{16x^{16}}-\frac{(5Ab+Ba)\,a^4}{19x^{19}}-\frac{A\,a^5}{22x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^5*(B*x^3+A)/x^23,x)

[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/13*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7-1/4*b^5*B/x^4

**maxima** [A] time = 0.70, size = 121, normalized size = 1.03

$$-\frac{76076\,Bb^5x^{18}+43472\left(5\,Bab^4+Ab^5\right)x^{15}+152152\left(2\,Ba^2b^3+Aab^4\right)x^{12}+234080\left(Ba^3b^2+Aa^2b^3\right)x^9+95095\left(Ba^4b+2\,Aa^3b^2\right)x^6+13832\,Aa^5+16016\left(Ba^5+5\,Aa^4b\right)x^3}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")

[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22

**mupad** [B] time = 0.06, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{22}+x^{12}\left(B\,a^2\,b^3+\frac{A\,a\,b^4}{2}\right)+x^6\left(\frac{5\,B\,a^4\,b}{16}+\frac{5\,A\,a^3\,b^2}{8}\right)+x^3\left(\frac{B\,a^5}{19}+\frac{5\,A\,b\,a^4}{19}\right)+x^{15}\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right)+x^9\left(\frac{10\,B\,a^3\,b^2}{13}+\frac{10\,A\,a^2\,b^3}{13}\right)+\frac{B\,b^5\,x^{18}}{4}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^23,x)

[Out] -((A*a^5)/22 + x^12*(B*a^2*b^3 + (A*a*b^4)/2) + x^6*((5*A*a^3*b^2)/8 + (5*B*a^4*b)/16) + x^3*((B*a^5)/19 + (5*A*a^4*b)/19) + x^15*((A*b^5)/7 + (5*B*a*

```
b^4)/7) + x^9*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^18)/4)/x^2
2
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)
```

```
[Out] Timed out
```

## 3.56 $\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$

**Optimal**. Leaf size=183

$$-\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}-\frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{10/3}}-$$

**Rubi [A]** time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {459, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}-\frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{10/3}}+\frac{x^4(Ab-aB)}{4b^2}-\frac{ax(Ab-aB)}{b^3}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^7)/(7*b) - (a^(
4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*b^(10/3)) + (a^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) -
 (a^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^
(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{7b}$$

$$= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{7b}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^2(Ab-aB)\right)\int \frac{1}{a+bx^3}\,dx}{b^3}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^3} + \frac{\left(a^{4/3}(Ab-aB)\right.}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{10/3}} - \frac{\left(a^{4/3}(Ab-a\right.}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{10/3}} - \frac{a^{4/3}(Ab-aB}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} - \frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{10/3}} + \frac{a^{4/3}(Ab-aB}{}$$

**Mathematica [A]** time = 0.13, size = 171, normalized size = 0.93

$$\frac{14a^{4/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-28a^{4/3}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+28\sqrt{3}\,a^{4/3}(aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+21b^{4/3}x^4(Ab-aB)+84a\sqrt[3]{b}\,x(aB-Ab)+12b^{7/3}Bx^7}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B*
x^7 + 28*Sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*
(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*b^(10/3)
)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \left(A + Bx^3\right)}{a + bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3), x]
```

**fricas [A]** time = 0.68, size = 167, normalized size = 0.91

$$\frac{12 Bb^2x^7 - 21\left(Bab - Ab^2\right)x^4 - 28\sqrt{3}\left(Ba^2 - Aab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) + 14\left(Ba^2-Aab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 28\left(Ba^2-Aab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 84\left(Ba^2-Aab\right)x}{84b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(a
/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a^2
 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2 -
A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x)/b^3
```

**giac [A]** time = 0.24, size = 217, normalized size = 1.19

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - \left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4} + \frac{\left(Ba^3b^4 - Aa^2b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7} + \frac{4Bb^6x^7 - 7Bab^5x^4 + 7Ab^6x^4 + 28Ba^2b^4x - 28Aab^5x}{28b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (-a
*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B*a^
3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(
4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^5*x)/
b^7
```

**maple [A]** time = 0.04, size = 249, normalized size = 1.36

$$\frac{Bx^7}{7b} + \frac{Ax^4}{4b} - \frac{Bax^4}{4b^2} + \frac{\sqrt{3}\,Aa^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{Aa^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{Aa^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{Aax}{b^2} - \frac{\sqrt{3}\,Ba^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} - \frac{Ba^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{Ba^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{Ba^2x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/7*B*x^7/b+1/4/b*A*x^4-1/4/b^2*B*x^4*a-1/b^2*a*A*x+1/b^3*a^2*B*x+1/3*a^2/b
^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)*B-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*a^3/b^4
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3*a^2/b^3/(a/b)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3*a^3/b^4/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

**maxima [A]** time = 1.43, size = 182, normalized size = 0.99

$$\frac{4Bb^2x^7 - 7\left(Bab - Ab^2\right)x^4 + 28\left(Ba^2 - Aab\right)x}{28b^3} - \frac{\sqrt{3}\left(Ba^3 - Aa^2b\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Ba^3 - Aa^2b\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(Ba^3 - Aa^2b\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*B*b^2*x^7 - 7*(B*a*b - A*b^2)*x^4 + 28*(B*a^2 - A*a*b)*x)/b^3 - 1/3
*sqrt(3)*(B*a^3 - A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^4*(a/b)^(2/3)) + 1/6*(B*a^3 - A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/
b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(B*a^3 - A*a^2*b)*log(x + (a/b)^(1/3))/(b
^4*(a/b)^(2/3))
```

**mupad [B]** time = 0.27, size = 164, normalized size = 0.90

$$x^4\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)+\frac{B\,x^7}{7\,b}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}}-\frac{a\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}}+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^7)/(7*b) + (a^(4/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(10/3)) - (a*x*(A/b - (B*a)/b^2))/b - (a^(4/3)*log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*b^(10/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3
))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(10/3))
```

**sympy [A]** time = 1.37, size = 114, normalized size = 0.62

$$\frac{Bx^7}{7b}+x^4\left(\frac{A}{4b}-\frac{Ba}{4b^2}\right)+x\left(-\frac{Aa}{b^2}+\frac{Ba^2}{b^3}\right)+\operatorname{RootSum}\left(27t^3b^{10}-A^3a^4b^3+3A^2Ba^5b^2-3AB^2a^6b+B^3a^7,\left(t\mapsto t\log\left(-\frac{3tb^3}{-Aab+Ba^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**7/(7*b) + x**4*(A/(4*b) - B*a/(4*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
+ RootSum(27*_t**3*b**10 - A**3*a**4*b**3 + 3*A**2*B*a**5*b**2 - 3*A*B**2*a
**6*b + B**3*a**7, Lambda(_t, _t*log(-3*_t*b**3/(-A*a*b + B*a**2) + x)))
```

## 3.57 $\int \frac{x^5\left(A+Bx^3\right)}{a+bx^3}\,dx$

**Optimal**. Leaf size=54

$$-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^6}{6b}$$

**Rubi** [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{x^3(Ab-aB)}{3b^2}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}+\frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*b^2) + (B*x^6)/(6*b) - (a*(A*b - a*B)*Log[a + b*x^3])/
(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(-Ab+aB)}{b^2(a+bx)}\right)dx,x,x^3\right)\\
&= \frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 47, normalized size = 0.87

$$\frac{bx^3\left(-2aB+2Ab+bBx^3\right)+2a(aB-Ab)\log\left(a+bx^3\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b^
3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(A + Bx^3\right)}{a + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3), x]
```

**fricas [A]** time = 0.56, size = 51, normalized size = 0.94

$$\frac{Bb^2x^6 - 2\left(Bab - Ab^2\right)x^3 + 2\left(Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))/
b^3
```

**giac [A]** time = 0.17, size = 52, normalized size = 0.96

$$\frac{Bbx^6 - 2\,Bax^3 + 2\,Abx^3}{6\,b^2} + \frac{\left(Ba^2 - Aab\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*x
^3 + a))/b^3
```

**maple [A]** time = 0.04, size = 62, normalized size = 1.15

$$\frac{B\,x^6}{6b} + \frac{A\,x^3}{3b} - \frac{Ba\,x^3}{3b^2} - \frac{Aa\ln\left(b\,x^3 + a\right)}{3b^2} + \frac{B\,a^2\ln\left(b\,x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/6*B*x^6/b+1/3/b*A*x^3-1/3/b^2*B*a*x^3-1/3*a/b^2*ln(b*x^3+a)*A+1/3*a^2/b^3
*ln(b*x^3+a)*B
```

**maxima [A]** time = 0.50, size = 50, normalized size = 0.93

$$\frac{Bbx^6 - 2\,(Ba - Ab)x^3}{6\,b^2} + \frac{\left(Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)/
b^3
```

**mupad [B]** time = 0.08, size = 52, normalized size = 0.96

$$x^3\left(\frac{A}{3\,b}-\frac{B\,a}{3\,b^2}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(B\,a^2-A\,a\,b\right)}{3\,b^3}+\frac{B\,x^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x^3)*(B*a^2 - A*a*b))/(3*b^3) +
(B*x^6)/(6*b)
```

**sympy [A]** time = 0.96, size = 46, normalized size = 0.85

$$\frac{Bx^6}{6b}+\frac{a\left(-Ab+Ba\right)\log\left(a+bx^3\right)}{3b^3}+x^3\left(\frac{A}{3b}-\frac{Ba}{3b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) + x**3*(A/(3*b) - B*
a/(3*b**2))
```

## 3.58 $\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$

**Optimal**. Leaf size=167

$$-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}+\frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {459, 321, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}+\frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^5)/(5*b) + (a^(2/3)*(A*b - a*B)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (a^(2/3)*(A*b
- a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) - (a^(2/3)*(A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{5b}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} - \frac{(a(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{b^2}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{7/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{6b^{8/3}}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{6b^{8/3}}$$
$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} -$$

**Mathematica [A]** time = 0.09, size = 154, normalized size = 0.92

$$\frac{5a^{2/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-10a^{2/3}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}\,a^{2/3}(aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+15b^{2/3}x^2(Ab-aB)+6b^{5/3}Bx^5}{30b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b)
+ a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) + a
*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(30*b^(8/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx^3\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3), x]
```

**fricas [A]** time = 0.53, size = 162, normalized size = 0.97

$$\frac{6\,Bbx^{5}-15\,(Ba-Ab)x^{2}+10\,\sqrt{3}\,(Ba-Ab)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+5\,(Ba-Ab)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax^{2}-bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)-10\,(Ba-Ab)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}\right)}{30\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1/
3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A*b
)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10
*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b^2
```

**giac [A]** time = 0.19, size = 207, normalized size = 1.24

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}}Ba-\left(-ab^{2}\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{4}}+\frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}}Ba-\left(-ab^{2}\right)^{\frac{2}{3}}Ab\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{4}}-\frac{\left(Ba^{2}b^{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Aab^{4}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{5}}+\frac{2\,Bb^{4}x^{5}-5\,Bab^{3}x^{2}+5\,Ab^{4}x^{2}}{10\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2)^
(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3*(-
a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5
```

**maple [A]** time = 0.04, size = 226, normalized size = 1.35

$$\frac{B\,x^{5}}{5b}+\frac{A\,x^{2}}{2b}-\frac{B\,a\,x^{2}}{2b^{2}}-\frac{\sqrt{3}\,A\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{A\,a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{A\,a\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{\sqrt{3}\,B\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{B\,a^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{B\,a^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/5*B*x^5/b+1/2/b*A*x^2-1/2/b^2*B*x^2*a+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))*A-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B-1/6*a/b^2/(a/b)^(1/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))*B-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*A+1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*B
```

**maxima [A]** time = 1.10, size = 157, normalized size = 0.94

$$\frac{\sqrt{3}\left(Ba^{2}-Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,Bbx^{5}-5\,(Ba-Ab)x^{2}}{10\,b^{2}}+\frac{\left(Ba^{2}-Aab\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(Ba^{2}-Aab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(b^3*(a/b)^(1/3)) + 1/10*(2*B*b*x^5 - 5*(B*a - A*b)*x^2)/b^2 + 1/6*(B*
a^2 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 1/3
*(B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

**mupad [B]** time = 2.55, size = 144, normalized size = 0.86

$$x^2\left(\frac{A}{2\,b}-\frac{B\,a}{2\,b^2}\right)+\frac{B\,x^5}{5\,b}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^2*(A/(2*b) - (B*a)/(2*b^2)) + (B*x^5)/(5*b) + (a^(2/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(8/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/
3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(8/3)) - (a^(2/3)*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
 - B*a))/(3*b^(8/3))
```

**sympy [A]** time = 0.87, size = 114, normalized size = 0.68

$$\frac{Bx^5}{5b}+x^2\left(\frac{A}{2b}-\frac{Ba}{2b^2}\right)+\operatorname{RootSum}\left(27t^3b^8-A^3a^2b^3+3A^2Ba^3b^2-3AB^2a^4b+B^3a^5,\left(t\mapsto t\log\left(\frac{9t^2b^5}{A^2ab^2-2ABa^2b+B^2a^3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**5/(5*b) + x**2*(A/(2*b) - B*a/(2*b**2)) + RootSum(27*_t**3*b**8 - A**3
*a**2*b**3 + 3*A**2*B*a**3*b**2 - 3*A*B**2*a**4*b + B**3*a**5, Lambda(_t, _
t*log(9*_t**2*b**5/(A**2*a*b**2 - 2*A*B*a**2*b + B**2*a**3) + x)))
```

## 3.59 $\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$

**Optimal**. Leaf size=162

$$\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{7/3}}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {459, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{x(Ab-aB)}{b^2}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}+\frac{Bx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^4)/(4*b) + (a^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (a^(1/3)*(A*b - a*B)
*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) + (a^(1/3)*(A*b - a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{Bx^4}{4b}-\frac{(-4Ab+4aB)\int \frac{x^3}{a+bx^3}\,dx}{4b}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{(a(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{b^2}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^2}-\frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,d
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}}{6b^{7/3}}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x\right.}{6b^{7/3}}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}+\frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+
\end{aligned}
$$

**Mathematica [A]** time = 0.09, size = 152, normalized size = 0.94

$$
\frac{-2\sqrt[3]{a}\,(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+12\sqrt[3]{b}\,x(Ab-aB)+4\sqrt[3]{a}\,(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-4\sqrt{3}\sqrt[3]{a}\,(aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+3b^{4/3}Bx^4}{12b^{7/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) + a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B)*
Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^3\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3),x]
[Out] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3), x]
```

**fricas [A]** time = 0.49, size = 145, normalized size = 0.90

$$\frac{3\,Bbx^4-4\,\sqrt{3}\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+2\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-4\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)-12\,(Ba-Ab)x}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/12*(3*B*b*x^4 - 4*sqrt(3)*(B*a - A*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*
b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 2*(B*a - A*b)*(-a/b)^(1/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3)) - 4*(B*a - A*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/
3)) - 12*(B*a - A*b)*x)/b^2
```

**giac [A]** time = 0.17, size = 186, normalized size = 1.15

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}-\frac{\left(Ba^2b^2-Aab^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4}+\frac{Bb^3x^4-4\,Bab^2x+4\,Ab^3x}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(
1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 - 1/3*(B*a^2*b^2 - A
*a*b^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(B*b^3*x^4 -
4*B*a*b^2*x + 4*A*b^3*x)/b^4
```

**maple [A]** time = 0.04, size = 221, normalized size = 1.36

$$\frac{B\,x^4}{4b}-\frac{\sqrt{3}\,Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{Aa\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{Aa\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{Ax}{b}+\frac{\sqrt{3}\,B\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{B\,a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{B\,a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{Bax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a),x)
[Out] 1/4*B*x^4/b+1/b*A*x-1/b^2*B*a*x-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1
/3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*A-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))*B-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x-1))*A+1/3*a^2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))*B
```

**maxima [A]** time = 1.08, size = 154, normalized size = 0.95

$$\frac{Bbx^4-4\,(Ba-Ab)x}{4\,b^2}+\frac{\sqrt{3}\left(Ba^2-Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(Ba^2-Aab\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Ba^2-Aab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 4*(B*a - A*b)*x)/b^2 + 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/6*(B*a^2 -
 A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(B*a
^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

**mupad [B]** time = 2.61, size = 162, normalized size = 1.00

$$x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)+\frac{B\,x^4}{4\,b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(A\,b-B\,a)}{3\,b^{7/3}}-\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,b^{7/3}}+\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x*(A/b - (B*a)/b^2) + (B*x^4)/(4*b) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3
)*x)*(A*b - B*a))/(3*b^(7/3)) - ((-a)^(1/3)*log(2*a*b^(1/3)*x - 3^(1/2)*(-a
)^(4/3)*1i - (-a)^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*b^(7/3)) +
((-a)^(1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/
2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(7/3))
```

**sympy [A]** time = 1.15, size = 87, normalized size = 0.54

$$\frac{Bx^4}{4b}+x\left(\frac{A}{b}-\frac{Ba}{b^2}\right)+\operatorname{RootSum}\left(27t^3b^7+A^3ab^3-3A^2Ba^2b^2+3AB^2a^3b-B^3a^4,\left(t\mapsto t\log\left(\frac{3tb^2}{-Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**4/(4*b) + x*(A/b - B*a/b**2) + RootSum(27*_t**3*b**7 + A**3*a*b**3 - 3
*A**2*B*a**2*b**2 + 3*A*B**2*a**3*b - B**3*a**4, Lambda(_t, _t*log(3*_t*b**
2/(-A*b + B*a) + x)))
```

## 3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx$

**Optimal**. Leaf size=35

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}+\frac{Bx^3}{3b}$$

**Rubi [A]** time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^3)/(3*b) + ((A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{a+bx}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Bx^3}{3b}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 31, normalized size = 0.89

$$\frac{(Ab-aB)\log\left(a+bx^3\right)+bBx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(A+Bx^3\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3),x]
[Out] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3), x]
```

**fricas [A]** time = 0.58, size = 30, normalized size = 0.86

$$\frac{Bbx^3 - (Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2
```

**giac [A]** time = 0.22, size = 32, normalized size = 0.91

$$\frac{Bx^3}{3\,b} - \frac{(Ba - Ab)\log\left(\left|bx^3 + a\right|\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2
```

**maple [A]** time = 0.04, size = 40, normalized size = 1.14

$$\frac{B\,x^3}{3b} + \frac{A\ln\left(b\,x^3 + a\right)}{3b} - \frac{Ba\ln\left(b\,x^3 + a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a),x)
[Out] 1/3*B*x^3/b+1/3/b*ln(b*x^3+a)*A-1/3/b^2*ln(b*x^3+a)*B*a
```

**maxima [A]** time = 0.54, size = 31, normalized size = 0.89

$$\frac{Bx^3}{3\,b} - \frac{(Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(b*x^3 + a)/b^2
```

**mupad [B]** time = 0.06, size = 31, normalized size = 0.89

$$\frac{B\,x^3}{3\,b} + \frac{\ln\left(b\,x^3 + a\right)\,(A\,b - B\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3),x)
[Out] (B*x^3)/(3*b) + (log(a + b*x^3)*(A*b - B*a))/(3*b^2)
```

**sympy [A]** time = 0.88, size = 27, normalized size = 0.77

$$\frac{Bx^3}{3b} - \frac{(-Ab + Ba)\log\left(a + bx^3\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**3/(3*b) - (-A*b + B*a)*log(a + b*x**3)/(3*b**2)
```

## 3.61 $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$

**Optimal**. Leaf size=150

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}+\frac{Bx^2}{2b}$$

**Rubi [A]** time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {459, 292, 31, 634, 617, 204, 628}

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*b^(5/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(-2Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{2b}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,b^{4/3}} + \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,b^{4/3}}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\int \ldots}{2\ldots}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-a\ldots}{\ldots}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}\ldots\right.}{6\sqrt[3]{a}\ldots}$$

**Mathematica [A]** time = 0.06, size = 152, normalized size = 1.01

$$-\frac{(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{5/3}} + \frac{(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{5/3}} - \frac{(aB-Ab)\tan^{-1}\left(\frac{2\sqrt[3]{b}\,x-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} + \frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(1/3)*b^(5/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^3\right)}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3), x]
```

**fricas [A]** time = 0.75, size = 382, normalized size = 2.55

$$\left[\frac{3\,Bab^2x^2 - 3\,\sqrt{\frac{1}{3}}\left(Ba^2b - Aab^2\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\left(-ab^2\right)^{\frac{2}{3}}x^2 + \left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}} - 3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3 + a}\right) - \left(-ab^2\right)^{\frac{2}{3}}\left(Ba - Ab\right)\log\left(b^2x^2 + \left(-ab^2\right)^{\frac{1}{3}}bx + \left(-ab^2\right)^{\frac{2}{3}}\right) + 2\left(-ab^2\right)^{\frac{2}{3}}\left(Ba - Ab\right)\log\left(bx - \left(-ab^2\right)^{\frac{1}{3}}\right)}{6\,ab^3}, \frac{3\,Bab^2x^2 - 6\,\sqrt{\frac{1}{3}}\left(Ba^2b - Aab^2\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx + \left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right) - \left(-ab^2\right)^{\frac{2}{3}}\left(Ba - Ab\right)\log\left(b^2x^2 + \left(-ab^2\right)^{\frac{1}{3}}bx + \left(-ab^2\right)^{\frac{2}{3}}\right) + 2\left(-ab^2\right)^{\frac{2}{3}}\left(Ba - Ab\right)\log\left(bx - \left(-ab^2\right)^{\frac{1}{3}}\right)}{6\,ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^
2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (-a
*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
+ 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6*(3*B
*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(
2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*(-a
*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]
```

**giac [A]** time = 0.19, size = 161, normalized size = 1.07

$$\frac{Bx^2}{2\,b} - \frac{\sqrt{3}\left(Ba - Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}b} + \frac{\left(Ba - Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}b} + \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b) + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-
a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

**maple [A]** time = 0.04, size = 198, normalized size = 1.32

$$\frac{B\,x^2}{2b} + \frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{A\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b} + \frac{A\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b} - \frac{\sqrt{3}\,Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{Ba\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{Ba\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/2*B*x^2/b-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A+1/3/b^2/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))*B*a+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/
b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B*a+1/3/b*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/b^2*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B*a
```

**maxima [A]** time = 1.14, size = 131, normalized size = 0.87

$$\frac{Bx^2}{2\,b} - \frac{\sqrt{3}\left(Ba - Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(Ba - Ab\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\left(Ba - Ab\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^
2*(a/b)^(1/3))
```

**mupad [B]** time = 2.57, size = 126, normalized size = 0.84

$$\frac{B\,x^2}{2\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{1/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{1/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{1/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (B*x^2)/(2*b) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
- (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
A*b - B*a))/(3*a^(1/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a
^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
```

**sympy [A]** time = 1.11, size = 92, normalized size = 0.61

$$\frac{Bx^2}{2b}+\operatorname{RootSum}\left(27t^3ab^5+A^3b^3-3A^2Bab^2+3AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{9t^2ab^3}{A^2b^2-2ABab+B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**2/(2*b) + RootSum(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A*
B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(9*_t**2*a*b**3/(A**2*b**2 - 2*A*
B*a*b + B**2*a**2) + x)))
```

## 3.62 $\int \frac{A+Bx^3}{a+bx^3} \, dx$

**Optimal**. Leaf size=145

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{Bx}{b}$$

**Rubi [A]** time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (B*x)/b - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{a+bx^3}\,dx &= \frac{Bx}{b}-\frac{(-Ab+aB)\int \frac{1}{a+bx^3}\,dx}{b}\\
&= \frac{Bx}{b}+\frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b}+\frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b}\\
&= \frac{Bx}{b}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\int \frac{\cdots}{a^{2/3}-\sqrt[3]{a}\cdots}}{2\sqrt[3]{a}\,b\cdots}\\
&= \frac{Bx}{b}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\,\mathrm{Su}\cdots}{\cdots}\\
&= \frac{Bx}{b}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\cdots\right.}{6a^{2/3}b^{4/3}}
\end{aligned}
$$

**Mathematica [A]** time = 0.09, size = 129, normalized size = 0.89

$$
\frac{-(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}Bx+2(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{A+Bx^3}{a+bx^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3), x]
```

**fricas [A]** time = 0.69, size = 369, normalized size = 2.54

$$
\left[\frac{6Ba^2bx-3\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+\left(a^2b\right)^{\frac{2}{3}}(Ba-Ab)\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)-2\left(a^2b\right)^{\frac{2}{3}}(Ba-Ab)\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)}{6a^2b^2},\ \frac{6Ba^2bx-6\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)+\left(a^2b\right)^{\frac{2}{3}}(Ba-Ab)\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)-2\left(a^2b\right)^{\frac{2}{3}}(Ba-Ab)\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)}{6a^2b^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*B*a^2*b*x - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(a^2*b)^(1/3)/b)*
log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (a^2*b
)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(a
^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*B*a^2
*b*x - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/
3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*
b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(
a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2)]
```

**giac [A]** time = 0.18, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}}+\frac{Bx}{b}+\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(-a*b^2)^(2/3) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/(-a*b^2)^(2/3) + B*x/b + 1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b)
```

**maple [A]** time = 0.04, size = 195, normalized size = 1.34

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\sqrt{3}\,Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{Ba\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{Ba\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a),x)
```

```
[Out] B*x/b+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^
(1/3))*B*a-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/b^2/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B*a+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B*a
```

**maxima [A]** time = 1.16, size = 128, normalized size = 0.88

$$\frac{Bx}{b}-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] B*x/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b
```

```
)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

**mupad [B]** time = 2.54, size = 123, normalized size = 0.85

$$\frac{B\,x}{b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3),x)
```

```
[Out] (B*x)/b + (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(2/3)*b^(4/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*a^(2/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(2/3)*b^(4/3))
```

**sympy [A]** time = 1.02, size = 71, normalized size = 0.49

$$\frac{Bx}{b}+\operatorname{RootSum}\left(27t^{3}a^{2}b^{4}-A^{3}b^{3}+3A^{2}Bab^{2}-3AB^{2}a^{2}b+B^{3}a^{3},\left(t\mapsto t\log\left(-\frac{3tab}{-Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x/b + RootSum(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2
*a**2*b + B**3*a**3, Lambda(_t, _t*log(-3*_t*a*b/(-A*b + B*a) + x)))
```

## 3.63 $\int \frac{A+Bx^3}{x(a+bx^3)} dx$

**Optimal**. Leaf size=34

$$\frac{A \log(x)}{a} - \frac{(Ab - aB) \log \left(a + bx^3\right)}{3ab}$$

**Rubi [A]** time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 72}

$$\frac{A \log(x)}{a} - \frac{(Ab - aB) \log \left(a + bx^3\right)}{3ab}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^3])/(3*a*b)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx^3}{x \left(a + bx^3\right)} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{A + Bx}{x(a + bx)} dx, x, x^3\right) \\
&= \frac{1}{3} \text{Subst}\left(\int \left(\frac{A}{ax} + \frac{-Ab + aB}{a(a + bx)}\right) dx, x, x^3\right) \\
&= \frac{A \log(x)}{a} - \frac{(Ab - aB) \log \left(a + bx^3\right)}{3ab}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 34, normalized size = 1.00

$$\frac{(aB - Ab) \log \left(a + bx^3\right)}{3ab} + \frac{A \log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x \left(a + bx^3\right)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)),x]
[Out] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)), x]
```

**fricas** [A] time = 0.48, size = 32, normalized size = 0.94

$$\frac{3\,Ab\log(x) + (Ba - Ab)\log\left(bx^3 + a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)
```

**giac** [A] time = 0.19, size = 34, normalized size = 1.00

$$\frac{A\log(|x|)}{a} + \frac{(Ba - Ab)\log\left(\left|bx^3 + a\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")
[Out] A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)
```

**maple** [A] time = 0.05, size = 37, normalized size = 1.09

$$\frac{A\ln(x)}{a} - \frac{A\ln\left(b\,x^3 + a\right)}{3a} + \frac{B\ln\left(b\,x^3 + a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a),x)
[Out] -1/3/a*ln(b*x^3+a)*A+1/3/b*ln(b*x^3+a)*B+A/a*ln(x)
```

**maxima** [A] time = 0.59, size = 35, normalized size = 1.03

$$\frac{A\log\left(x^3\right)}{3\,a} + \frac{(Ba - Ab)\log\left(bx^3 + a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*A*log(x^3)/a + 1/3*(B*a - A*b)*log(b*x^3 + a)/(a*b)
```

**mupad** [B] time = 0.10, size = 36, normalized size = 1.06

$$\frac{B\,\ln\left(b\,x^3 + a\right)}{3\,b} - \frac{A\,\ln\left(b\,x^3 + a\right)}{3\,a} + \frac{A\,\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)),x)
[Out] (B*log(a + b*x^3))/(3*b) - (A*log(a + b*x^3))/(3*a) + (A*log(x))/a
```

**sympy** [A] time = 2.12, size = 26, normalized size = 0.76

$$\frac{A\log(x)}{a} + \frac{(-Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a),x)
```

```
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**3)/(3*a*b)
```

## 3.64 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=147

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{A}{ax}$$

**Rubi [A]** time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {453, 292, 31, 634, 617, 204, 628}

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(A/(a*x)) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(4/3)*b^(2/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
4/3)*b^(2/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = -\frac{A}{ax} - \frac{(Ab-aB)\int \frac{x}{a+bx^3}\,dx}{a}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}\sqrt[3]{b}} - \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{2/3}} - \frac{(Ab-aB)\int \frac{\cdots}{a^{2/3}-\cdots}}{2a\sqrt[3]{\cdots}}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}} - \frac{(Ab-aB)\cdots}{\cdots}$$

$$= -\frac{A}{ax} + \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\cdots\right)}{6a^{4/3}b^{2\cdots}}$$

**Mathematica [A]** time = 0.09, size = 134, normalized size = 0.91

$$\frac{-x(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-6\sqrt[3]{a}Ab^{2/3}+2x(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}x(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{4/3}b^{2/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*x
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)), x]
```

**fricas** [A] time = 0.68, size = 372, normalized size = 2.53

$$\left[-\frac{6\,Aab^{2}+3\,\sqrt{\frac{1}{3}}\left(Ba^{2}b-Aab^{2}\right)x\sqrt{-\frac{\left(ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab-3\,\sqrt{\frac{1}{3}}\left(abx+2\left(ab^{2}\right)^{\frac{2}{3}}x^{2}-\left(ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(ab^{2}\right)^{\frac{2}{3}}x}{bx^{3}+a}\right)-\left(ab^{2}\right)^{\frac{2}{3}}(Ba-Ab)x\log\left(b^{2}x^{2}-\left(ab^{2}\right)^{\frac{1}{3}}bx+\left(ab^{2}\right)^{\frac{2}{3}}\right)+2\left(ab^{2}\right)^{\frac{2}{3}}(Ba-Ab)x\log\left(bx+\left(ab^{2}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b^{2}x},\ -\frac{6\,Aab^{2}+6\,\sqrt{\frac{1}{3}}\left(Ba^{2}b-Aab^{2}\right)x\sqrt{\frac{\left(ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,bx-\left(ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{\frac{\left(ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)-\left(ab^{2}\right)^{\frac{2}{3}}(Ba-Ab)x\log\left(b^{2}x^{2}-\left(ab^{2}\right)^{\frac{1}{3}}bx+\left(ab^{2}\right)^{\frac{2}{3}}\right)+2\left(ab^{2}\right)^{\frac{2}{3}}(Ba-Ab)x\log\left(bx+\left(ab^{2}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*A*a*b^2 + 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt(-(a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^
(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - (a*b^2)
^(2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(
a*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x), -1/6*(6*A
*a*b^2 + 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt((a*b^2)^(1/3)/a)*arctan(-sq
rt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (a*b^2)^(2/3)*(B
*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(a*b^2)^(2
/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x)]
```

**giac** [A] time = 0.22, size = 155, normalized size = 1.05

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{1}{3}}a}-\frac{(Ba-Ab)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{1}{3}}a}-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{2}}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/((-a*b^2)^(1/3)*a) - 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a) - 1/3*(B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - A/(a*x)
```

**maple** [A] time = 0.06, size = 195, normalized size = 1.33

$$-\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a),x)
```

```
[Out] 1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B
-1/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/b/(a/b)^(1/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*A+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))*B-A/a/x
```

**maxima** [A] time = 1.16, size = 140, normalized size = 0.95

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(1/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(1/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
 - A/(a*x)
```

**mupad** [B] time = 2.54, size = 126, normalized size = 0.86

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{A}{a\,x}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(4/3)*b^(2/3)) - A/(a*x) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b -
B*a))/(3*a^(4/3)*b^(2/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(4/3)*b^(2/3))
```

**sympy** [A] time = 0.94, size = 90, normalized size = 0.61

$$-\frac{A}{ax}+\operatorname{RootSum}\left(27t^3a^4b^2-A^3b^3+3A^2Bab^2-3AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{9t^2a^3b}{A^2b^2-2ABab+B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a),x)
```

```
[Out] -A/(a*x) + RootSum(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B
**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(9*_t**2*a**3*b/(A**2*b**2 - 2*A*B
*a*b + B**2*a**2) + x)))
```

## 3.65 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=149

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}-\frac{A}{2ax^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {453, 200, 31, 634, 617, 204, 628}

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] -A/(2*a*x^2) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(5/3)*b^(1/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a
^(5/3)*b^(1/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(6*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx=-\frac{A}{2ax^2}-\frac{(2Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{2a}$$

$$=-\frac{A}{2ax^2}-\frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}}-\frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}}$$

$$=-\frac{A}{2ax^2}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}}+\frac{(Ab-aB)\int \frac{-}{a^{2/3}}}{6a^{5/}}$$

$$=-\frac{A}{2ax^2}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aI}{}$$

$$=-\frac{A}{2ax^2}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\right.}{6a^{5/3}}$$

**Mathematica [A]** time = 0.10, size = 135, normalized size = 0.91

$$\frac{\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{3a^{2/3}A}{x^2}+\frac{2(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3
) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(
6*a^(5/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)), x]
```

**fricas** [A] time = 0.50, size = 411, normalized size = 2.76

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)x^2\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2abx^2+\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+\left(-a^2b\right)^{\frac{2}{3}}(Ba-Ab)x^2\log\left(abx^2-\left(-a^2b\right)^{\frac{2}{3}}x-\left(-a^2b\right)^{\frac{1}{3}}a\right)-2\left(-a^2b\right)^{\frac{2}{3}}(Ba-Ab)x^2\log\left(abx+\left(-a^2b\right)^{\frac{2}{3}}\right)+3Aa^2b}{6a^3bx^2},\ \frac{6\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)x^2\sqrt{-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(-a^2b\right)^{\frac{2}{3}}(Ba-Ab)x^2\log\left(abx^2-\left(-a^2b\right)^{\frac{2}{3}}x-\left(-a^2b\right)^{\frac{1}{3}}a\right)+2\left(-a^2b\right)^{\frac{2}{3}}(Ba-Ab)x^2\log\left(abx+\left(-a^2b\right)^{\frac{2}{3}}\right)-3Aa^2b}{6a^3bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt((-a^2*b)^(1/3)/b)*log((2*a*
b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3
)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) + (-a^2*b)^(2/
3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(
-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^(2/3)) + 3*A*a^2*b)/(a^3
*b*x^2), 1/6*(6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt(-(-a^2*b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3
)/b)/a^2) - (-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
 (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^
(2/3)) - 3*A*a^2*b)/(a^3*b*x^2)]
```

**giac** [A] time = 0.18, size = 161, normalized size = 1.08

$$-\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b}-\frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*
((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a^2*b) + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2*A/(a*x^2)
```

**maple** [A] time = 0.05, size = 195, normalized size = 1.31

$$-\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a),x)
```

```
[Out] -1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*
B+1/6/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*A+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*B-1/2*A/a/x^2
```

**maxima** [A] time = 1.22, size = 140, normalized size = 0.94

$$\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(2/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(2/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
 - 1/2*A/(a*x^2)
```

**mupad [B]** time = 0.24, size = 126, normalized size = 0.85

$$-\frac{A}{2\,a\,x^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b - B*a))/(3*a^(5/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a
^(5/3)*b^(1/3)) - A/(2*a*x^2) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(
1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(5/3)*b^(1/3))
```

**sympy [A]** time = 1.23, size = 73, normalized size = 0.49

$$-\frac{A}{2ax^2}+\operatorname{RootSum}\left(27t^3a^5b+A^3b^3-3A^2Bab^2+3AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{3ta^2}{-Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a),x)
```

```
[Out] -A/(2*a*x**2) + RootSum(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A
*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(3*_t*a**2/(-A*b + B*a) + x)))
```

## 3.66 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=50

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -A/(3*a*x^3) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3*a
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx&=\frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx,x,x^3\right)\\&=\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}-\frac{b(-Ab+aB)}{a^2(a+bx)}\right)dx,x,x^3\right)\\&=-\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 0.98

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}+\frac{\log(x)(aB-Ab)}{a^2}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*A/(a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])
/(3*a^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)), x]
```

**fricas [A]** time = 0.50, size = 47, normalized size = 0.94

$$-\frac{(Ba-Ab)x^3\log\left(bx^3+a\right)-3\,(Ba-Ab)x^3\log(x)+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^2
*x^3)
```

**giac [A]** time = 0.17, size = 69, normalized size = 1.38

$$\frac{(Ba-Ab)\log\left(|x|\right)}{a^2}-\frac{\left(Bab-Ab^2\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^2b}-\frac{Bax^3-Abx^3+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2*
b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)
```

**maple [A]** time = 0.05, size = 56, normalized size = 1.12

$$-\frac{Ab\ln(x)}{a^2}+\frac{Ab\ln\left(b\,x^3+a\right)}{3a^2}+\frac{B\ln(x)}{a}-\frac{B\ln\left(b\,x^3+a\right)}{3a}-\frac{A}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3/a^2*ln(b*x^3+a)*A*b-1/3/a*ln(b*x^3+a)*B-1/3*A/a/x^3-1/a^2*ln(x)*A*b+B/a
*ln(x)
```

**maxima [A]** time = 0.48, size = 48, normalized size = 0.96

$$-\frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,a^2}+\frac{(Ba-Ab)\log\left(x^3\right)}{3\,a^2}-\frac{A}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A/
(a*x^3)
```

**mupad** [B] time = 2.40, size = 46, normalized size = 0.92

$$\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-B\,a\right)}{3\,a^2}-\frac{A}{3\,a\,x^3}-\frac{\ln(x)\,\left(A\,b-B\,a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(A*b - B*a))/(3*a^2) - A/(3*a*x^3) - (log(x)*(A*b - B*a))/a
^2
```

**sympy** [A] time = 2.36, size = 41, normalized size = 0.82

$$-\frac{A}{3ax^3}+\frac{(-Ab+Ba)\log(x)}{a^2}-\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a),x)
```

```
[Out] -A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3*
a**2)
```

## 3.67 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=165

$$\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{A}{}$$

**Rubi [A]** time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{A}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(a^2*x) - (b^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (b^(1/3)*(A*b - a*B)*
Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)) + (b^(1/3)*(A*b - a*B)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = -\frac{A}{4ax^4}-\frac{(4Ab-4aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{4a}$$

$$= -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}+\frac{(b(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{7/3}}+\frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}}{3a^{7/3}}$$

$$= -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}}+\frac{\left(\sqrt[3]{b}\,(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}}}{6a^{7/3}}$$

$$= -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}}+\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x\right.}{6a^{7/3}}$$

$$= -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}}+$$

**Mathematica [A]** time = 0.14, size = 154, normalized size = 0.93

$$\frac{2\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-\frac{3a^{4/3}A}{x^4}+\frac{12\sqrt[3]{a}\,(Ab-aB)}{x}+4\sqrt[3]{b}\,(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-4\sqrt{3}\,\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{12a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b -
 a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a*B
)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)), x]
```

**fricas [A]** time = 0.63, size = 158, normalized size = 0.96

$$-\frac{4\sqrt{3}(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+4(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+12(Ba-Ab)x^3+3Aa}{12a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*(B*a - A*b)*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(
1/3) + 1/3*sqrt(3)) - 2*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 4*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)
```

**giac [A]** time = 0.22, size = 197, normalized size = 1.19

$$\frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3b}-\frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3b}-\frac{4Bax^3-4Abx^3+Aa}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^3 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6*((-a*b^2)^(
2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^
3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)
```

**maple [A]** time = 0.05, size = 216, normalized size = 1.31

$$\frac{\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{Ab\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{Ab}{a^2x}-\frac{B}{ax}-\frac{A}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a),x)
```

```
[Out] -1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))*B+1/6/a^2*b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/a/(a/b)
^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3/a^2*b*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/4*A/a/x^4+1/a^2/x*A*b-B/a/x
```

**maxima [A]** time = 1.14, size = 147, normalized size = 0.89

$$-\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{4(Ba-Ab)x^3+Aa}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(1/3)
) - 1/4*(4*(B*a - A*b)*x^3 + A*a)/(a^2*x^4)
```

**mupad [B]** time = 2.59, size = 178, normalized size = 1.08

$$\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(A\,b-B\,a)}{3\,a^{7/3}}-\frac{\frac{A}{4\,a}-\frac{x^3\,(A\,b-B\,a)}{a^2}}{x^4}+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{7/3}}-\frac{(-b)^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)),x)
```

```
[Out] ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(A*b - B*a))/(3*a^(7/3)) - (A/(
4*a) - (x^3*(A*b - B*a))/a^2)/x^4 + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*
b^3*x + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(7/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3
)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(7/3))
```

**sympy [A]** time = 0.96, size = 112, normalized size = 0.68

$$\operatorname{RootSum}\left(27t^3a^7+A^3b^4-3A^2Bab^3+3AB^2a^2b^2-B^3a^3b,\left(t\mapsto t\log\left(\frac{9t^2a^5}{A^2b^3-2ABab^2+B^2a^2b}+x\right)\right)\right)+\frac{-Aa+x^3\,(4Ab-4Ba)}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 -
B**3*a**3*b, Lambda(_t, _t*log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B**
2*a**2*b) + x))) + (-A*a + x**3*(4*A*b - 4*B*a))/(4*a**2*x**4)
```

## 3.68 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=168

$$-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+$$

**Rubi [A]** time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {453, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(2*a^2*x^2) - (b^(2/3)*(A*b - a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) + (b^(2/3)*(A*b - a
*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)) - (b^(2/3)*(A*b - a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{A}{5ax^5}-\frac{(5Ab-5aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{5a}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{(b(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{(b(Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{8/3}}+\frac{(b(Ab-aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{8/3}}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}-\frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}}}{6a^{8/3}}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\right.}{6a^{8/3}}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}$$

**Mathematica [A]** time = 0.13, size = 154, normalized size = 0.92

$$\frac{5b^{2/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\frac{15a^{2/3}(Ab-aB)}{x^2}-\frac{6a^{5/3}A}{x^5}+10b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-10\sqrt{3}\,b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{30a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A*
b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)), x]
```

**fricas [A]** time = 0.60, size = 176, normalized size = 1.05

$$-\frac{10\sqrt{3}(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-5(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+15(Ba-Ab)x^3+6Aa}{30a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x
*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(b^
2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(B*a - A*b)*x^5*(
b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 15*(B*a - A*b)*x^3 + 6*A*a)/(
a^2*x^5)
```

**giac [A]** time = 0.18, size = 176, normalized size = 1.05

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3}+\frac{\left(Bab-Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3}-\frac{5Bax^3-5Abx^3+2Aa}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/a^3 - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 1/10*(5*B*a*x^3 - 5*A*b*x^
3 + 2*A*a)/(a^2*x^5)
```

**maple [A]** time = 0.05, size = 217, normalized size = 1.29

$$\frac{\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{Ab\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{Ab}{2a^2x^2}-\frac{B}{2a\,x^2}-\frac{A}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a),x)
```

```
[Out] 1/3/a^2*b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*B-1/6/a^2*b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/a/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3/a^2*b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/5*A/a/x^5+1/2/a^2/x^2*A*b-1/2/a/x^2*B
```

**maxima** [A] time = 1.28, size = 148, normalized size = 0.88

$$-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,(Ba-Ab)x^3+2\,Aa}{10\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(2/3)
) - 1/10*(5*(B*a - A*b)*x^3 + 2*A*a)/(a^2*x^5)
```

**mupad** [B] time = 2.56, size = 145, normalized size = 0.86

$$\frac{b^{2/3}\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-B\,a)}{3\,a^{8/3}}-\frac{\frac{A}{5\,a}-\frac{x^3\,(A\,b-B\,a)}{2\,a^2}}{x^5}-\frac{b^{2/3}\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,a^{8/3}}+\frac{b^{2/3}\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b-B\,a)}{3\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(8/3)) - (A/(5*a) - (x^
3*(A*b - B*a))/(2*a^2))/x^5 - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(8/3)) + (b^(2/3)*log(
3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B
*a))/(3*a^(8/3))
```

**sympy** [A] time = 0.93, size = 99, normalized size = 0.59

$$\operatorname{RootSum}\left(27t^3a^8-A^3b^5+3A^2Bab^4-3AB^2a^2b^3+B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{3ta^3}{-Ab^2+Bab}+x\right)\right)\right)+\frac{-2Aa+x^3\,(5Ab-5Ba)}{10a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8 - A**3*b**5 + 3*A**2*B*a*b**4 - 3*A*B**2*a**2*b**3 +
B**3*a**3*b**2, Lambda(_t, _t*log(-3*_t*a**3/(-A*b**2 + B*a*b) + x))) + (-2
*A*a + x**3*(5*A*b - 5*B*a))/(10*a**2*x**5)
```

## 3.69 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=69

$$-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{6ax^6}$$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(3*a^2*x^3) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A
*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax^3}+\frac{-Ab+aB}{a^2x^2}-\frac{b(-Ab+aB)}{a^3x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx,x,x^3\right)\\
&= -\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 70, normalized size = 1.01

$$\frac{6bx^6\log(x)(Ab-aB)-a\left(aA+2aBx^3-2Abx^3\right)+2bx^6(aB-Ab)\log\left(a+bx^3\right)}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*Log[x] + 2*b*(-(A
*b) + a*B)*x^6*Log[a + b*x^3])/(6*a^3*x^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)), x]
```

**fricas [A]** time = 0.53, size = 73, normalized size = 1.06

$$\frac{2\left(Bab-Ab^2\right)x^6\log\left(bx^3+a\right)-6\left(Bab-Ab^2\right)x^6\log(x)-2\left(Ba^2-Aab\right)x^3-Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a*b - A*b^2)*x^6*log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*log(x) -
2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)
```

**giac [A]** time = 0.19, size = 99, normalized size = 1.43

$$-\frac{\left(Bab-Ab^2\right)\log\left(|x|\right)}{a^3}+\frac{\left(Bab^2-Ab^3\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^3b}+\frac{3\,Babx^6-3\,Ab^2x^6-2\,Ba^2x^3+2\,Aabx^3-Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*log(abs(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*log(abs(b*x^3 + a)
)/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 - A*
a^2)/(a^3*x^6)
```

**maple [A]** time = 0.05, size = 81, normalized size = 1.17

$$\frac{A\,b^2\ln(x)}{a^3}-\frac{A\,b^2\ln\left(b\,x^3+a\right)}{3a^3}-\frac{Bb\ln(x)}{a^2}+\frac{Bb\ln\left(b\,x^3+a\right)}{3a^2}+\frac{Ab}{3a^2x^3}-\frac{B}{3a\,x^3}-\frac{A}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a),x)
```

```
[Out] -1/3*b^2/a^3*ln(b*x^3+a)*A+1/3*b/a^2*ln(b*x^3+a)*B-1/6*A/a/x^6+1/3/a^2/x^3*
A*b-1/3/a/x^3*B+b^2/a^3*ln(x)*A-b/a^2*ln(x)*B
```

**maxima [A]** time = 0.49, size = 70, normalized size = 1.01

$$\frac{\left(Bab-Ab^2\right)\log\left(bx^3+a\right)}{3\,a^3}-\frac{\left(Bab-Ab^2\right)\log\left(x^3\right)}{3\,a^3}-\frac{2\,(Ba-Ab)x^3+Aa}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*a*b - A*b^2)*log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*log(x^3)/a^3 -
 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)
```

**mupad [B]** time = 0.13, size = 70, normalized size = 1.01

$$\frac{\ln(x)\,\left(A\,b^2-B\,a\,b\right)}{a^3}-\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b^2-B\,a\,b\right)}{3\,a^3}-\frac{\frac{A}{6\,a}-\frac{x^3\,\left(A\,b-B\,a\right)}{3\,a^2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)),x)
```

```
[Out] (log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^3)*(A*b^2 - B*a*b))/(3*a^3) - (
A/(6*a) - (x^3*(A*b - B*a))/(3*a^2))/x^6
```

**sympy [A]** time = 2.69, size = 61, normalized size = 0.88

$$\frac{-Aa+x^3\,(2Ab-2Ba)}{6a^2x^6}-\frac{b\,(-Ab+Ba)\log{(x)}}{a^3}+\frac{b\,(-Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a),x)
```

```
[Out] (-A*a + x**3*(2*A*b - 2*B*a))/(6*a**2*x**6) - b*(-A*b + B*a)*log(x)/a**3 +
b*(-A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

## 3.70 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=184

$$-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}}$$

**Rubi [A]** time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}-\frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(a^3*x) + (b^(4/3)
*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(10/3)) + (b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)) - (b^
(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx &= -\frac{A}{7ax^7}-\frac{(7Ab-7aB)\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{7a}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}+\frac{(b(Ab-aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{a^2}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}-\frac{\left(b^2(Ab-aB)\right)\int \frac{x}{a+bx^3}\,dx}{a^3}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}}-\frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{\cdots}{a^{2/3}-\cdots}}{3a^{10/3}}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{\left(b^{4/3}(Ab-aB)\right)\int \frac{\cdots}{a^{2/3}\cdots}}{6a^{10/3}}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/\cdots}\right.}{6a^{\cdots}}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{\cdots}\right.}{3a^{10/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 173, normalized size = 0.94

$$\frac{14b^{4/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{21a^{4/3}(Ab-aB)}{x^4}-\frac{12a^{7/3}A}{x^7}+28b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}\,b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{84\sqrt[3]{a}b(aB-Ab)}{x}}{84a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b)
 + a*B))/x + 28*Sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4/3)
*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(10/3
))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^8 \left(a + bx^3\right)}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^8*(a + b*x^3)), x]
```

**fricas [A]** time = 0.85, size = 180, normalized size = 0.98

$$\frac{28\,\sqrt{3}\left(Bab - Ab^{2}\right)x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\,\sqrt{3}\right) + 14\left(Bab - Ab^{2}\right)x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2} - ax\left(\frac{b}{a}\right)^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 28\left(Bab - Ab^{2}\right)x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx + a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right) + 84\left(Bab - Ab^{2}\right)x^{6} - 21\left(Ba^{2} - Aab\right)x^{3} - 12\,Aa^{2}}{84\,a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)
^(1/3) - 1/3*sqrt(3)) + 14*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x^2 - a*x*
(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x +
 a*(b/a)^(2/3)) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*A*a^
2)/(a^3*x^7)
```

**giac [A]** time = 0.18, size = 216, normalized size = 1.17

$$-\frac{\sqrt{3}\left(\left(-ab^{2}\right)^{\frac{2}{3}} Ba - \left(-ab^{2}\right)^{\frac{2}{3}} Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{4}} - \frac{\left(Bab^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{4}} + \frac{\left(\left(-ab^{2}\right)^{\frac{2}{3}} Ba - \left(-ab^{2}\right)^{\frac{2}{3}} Ab\right)\log\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{4}} + \frac{28\,Babx^{6} - 28\,Ab^{2}x^{6} - 7\,Ba^{2}x^{3} + 7\,Aabx^{3} - 4\,Aa^{2}}{28\,a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 - 1/3*(B*a*b^2*(-a/b)^(1/3) - A*b^3*(
-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/6*((-a*b^2)^(2
/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4
+ 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*a^2)/
(a^3*x^7)
```

**maple [A]** time = 0.06, size = 247, normalized size = 1.34

$$-\frac{\sqrt{3}\,A\,b^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}} + \frac{A\,b^{2}\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}} - \frac{A\,b^{2}\ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{3}} + \frac{\sqrt{3}\,Bb\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2}} - \frac{Bb\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2}} + \frac{Bb\ln\left(x^{2} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}} a^{2}} - \frac{A\,b^{2}}{a^{3}x} + \frac{Bb}{a^{2}x} + \frac{Ab}{4a^{2}x^{4}} - \frac{B}{4a\,x^{4}} - \frac{A}{7a\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a),x)
```

```
[Out] 1/3/a^3*b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))*B-1/6/a^3*b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/
a^2*b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a^3*b^2*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/3/a^2*b*3^(1/2)/(a/b
```

```
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/7*A/a/x^7+1/4/a^2/x^4*A
*b-1/4/a/x^4*B-b^2/a^3/x*A+b/a^2/x*B
```

**maxima [A]** time = 1.29, size = 178, normalized size = 0.97

$$\frac{\sqrt{3}\left(Bab-Ab^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(Bab-Ab^{2}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(Bab-Ab^{2}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{28\left(Bab-Ab^{2}\right)x^{6}-7\left(Ba^{2}-Aab\right)x^{3}-4Aa^{2}}{28a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a*b - A*b^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a^3*(a/b)^(1/3)) + 1/6*(B*a*b - A*b^2)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) - 1/3*(B*a*b - A*b^2)*log(x + (a/b)^(1/3))/(a^3*
(a/b)^(1/3)) + 1/28*(28*(B*a*b - A*b^2)*x^6 - 7*(B*a^2 - A*a*b)*x^3 - 4*A*a
^2)/(a^3*x^7)
```

**mupad [B]** time = 2.58, size = 161, normalized size = 0.88

$$\frac{b^{4/3}\ln\left(b^{1/3}x+a^{1/3}\right)(Ab-Ba)}{3a^{10/3}}-\frac{\frac{A}{7a}-\frac{x^{3}(Ab-Ba)}{4a^{2}}+\frac{bx^{6}(Ab-Ba)}{a^{3}}}{x^{7}}+\frac{b^{4/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab-Ba)}{3a^{10/3}}-\frac{b^{4/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab-Ba)}{3a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(10/3)) - (A/(7*a) - (x
^3*(A*b - B*a))/(4*a^2) + (b*x^6*(A*b - B*a))/a^3)/x^7 + (b^(4/3)*log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(10/3)) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(10/3))
```

**sympy [A]** time = 1.13, size = 139, normalized size = 0.76

$$\operatorname{RootSum}\left(27t^{3}a^{10}-A^{3}b^{7}+3A^{2}Bab^{6}-3AB^{2}a^{2}b^{5}+B^{3}a^{3}b^{4},\left(t\mapsto t\log\left(\frac{9t^{2}a^{7}}{A^{2}b^{5}-2ABab^{4}+B^{2}a^{2}b^{3}}+x\right)\right)\right)+\frac{-4Aa^{2}+x^{6}\left(-28Ab^{2}+28Bab\right)+x^{3}\left(7Aab-7Ba^{2}\right)}{28a^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5 +
 B**3*a**3*b**4, Lambda(_t, _t*log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4 +
 B**2*a**2*b**3) + x))) + (-4*A*a**2 + x**6*(-28*A*b**2 + 28*B*a*b) + x**3*
(7*A*a*b - 7*B*a**2))/(28*a**3*x**7)
```

## 3.71 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=233

$$-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{13/3}}+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{13/3}}-\frac{a^{4/3}(7Ab-10aB)\tan^{-}}{3\sqrt{3}\,b^{13/3}}$$

**Rubi [A]** time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{13/3}}+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{13/3}}-\frac{a^{4/3}(7Ab-10aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{13/3}}-\frac{x^7(7Ab-10aB)}{21ab^2}+\frac{x^4(7Ab-10aB)}{12b^3}-\frac{ax(7Ab-10aB)}{3b^4}+\frac{x^{10}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(7*A*b - 10*a*B)*x)/(3*b^4) + ((7*A*b - 10*a*B)*x^4)/(12*b^3) - ((7*A*b
 - 10*a*B)*x^7)/(21*a*b^2) + ((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - (a^(4
/3)*(7*A*b - 10*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*b^(13/3)) + (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*
b^(13/3)) - (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{(-7Ab+10aB)\int \frac{x^9}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{(-7Ab+10aB)\int \left(\frac{a^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{b}-\frac{a^3}{b^3\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{\left(a^2(7Ab-10a\cdots\right.}{3b\cdots}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{\left(a^{4/3}(7Ab-10\cdots\right.}{\cdots}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{a^{4/3}(7Ab-10a\cdots}{\cdots}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}+\frac{a^{4/3}(7Ab-10a\cdots}{\cdots}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}+\frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)}-\frac{a^{4/3}(7Ab-10a\cdots}{3\cdots}$$

**Mathematica** [A] time = 0.17, size = 203, normalized size = 0.87

$$\frac{14a^{4/3}(10aB-7Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-28a^{4/3}(10aB-7Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}a^{4/3}(10aB-7Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{84a^2\sqrt[3]{b}x(aB-Ab)}{a+bx^3}+63b^{4/3}x^4(Ab-2aB)+252a\sqrt[3]{b}x(3aB-2Ab)+36b^{7/3}Bx^7}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7/
3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*Sqrt[3]*a^(4/
3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 28*a^(4/
3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10*a*B
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(252*b^(13/3))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 \left(A + Bx^3\right)}{\left(a + bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas** [A] time = 0.72, size = 271, normalized size = 1.16

$$\frac{36\,Bb^3x^{10} - 9\left(10\,Bab^2 - 7\,Ab^3\right)x^7 + 63\left(10\,Ba^2b - 7\,Aab^2\right)x^4 - 28\,\sqrt{3}\left(10\,Ba^3 - 7\,Aa^2b + \left(10\,Ba^2b - 7\,Aab^2\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}\,a}{3\,a}\right) + 14\left(10\,Ba^3 - 7\,Aa^2b + \left(10\,Ba^2b - 7\,Aab^2\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) - 28\left(10\,Ba^3 - 7\,Aa^2b + \left(10\,Ba^2b - 7\,Aab^2\right)x^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) + 84\left(10\,Ba^3 - 7\,Aa^2b\right)x}{252\left(b^5x^3 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A*
a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^
3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(
10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A
*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^2*b)*x
)/(b^5*x^3 + a*b^4)
```

**giac** [A] time = 0.20, size = 244, normalized size = 1.05

$$-\frac{\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - 7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} + \frac{\left(10\,Ba^3 - 7\,Aa^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2 - 7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5} + \frac{Ba^3x - Aa^2bx}{3\left(bx^3 + a\right)b^4} + \frac{4\,Bb^{12}x^7 - 14\,Bab^{11}x^4 + 7\,Ab^{12}x^4 + 84\,Ba^2b^{10}x - 56\,Aab^{11}x}{28\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*b)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/3)*
B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 - 14*B
*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14
```

**maple** [A] time = 0.06, size = 288, normalized size = 1.24

$$\frac{B\,x^7}{7b^2} + \frac{A\,x^4}{4b^2} - \frac{Ba\,x^4}{2b^3} - \frac{A\,a^2x}{3\left(b\,x^3 + a\right)b^3} + \frac{B\,a^3x}{3\left(b\,x^3 + a\right)b^4} + \frac{7\sqrt{3}\;A\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{7A\,a^2\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} - \frac{7A\,a^2\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} - \frac{2Aax}{b^3} - \frac{10\sqrt{3}\;B\,a^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5} - \frac{10B\,a^3\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5} + \frac{5B\,a^3\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5} + \frac{3B\,a^2x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/7/b^2*B*x^7+1/4/b^2*A*x^4-1/2/b^3*B*x^4*a-2/b^3*a*A*x+3/b^4*a^2*B*x-1/3*a
^2/b^3*x/(b*x^3+a)*A+1/3*a^3/b^4*x/(b*x^3+a)*B+7/9*a^2/b^4*A/(a/b)^(2/3)*ln
```

```
(x+(a/b)^(1/3))-7/18*a^2/b^4*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+7/9*a^2/b^4*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-10/9*a^3/b^5*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/9*a^3/b^5*B/(a/b)^(2/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-10/9*a^3/b^5*B/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

**maxima [A]** time = 1.23, size = 218, normalized size = 0.94

$$\frac{\left(Ba^3 - Aa^2b\right)x}{3\left(b^5x^3 + ab^4\right)} + \frac{4\,Bb^2x^7 - 7\left(2\,Bab - Ab^2\right)x^4 + 28\left(3\,Ba^2 - 2\,Aab\right)x}{28\,b^4} - \frac{\sqrt{3}\left(10\,Ba^3 - 7\,Aa^2b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(10\,Ba^3 - 7\,Aa^2b\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(10\,Ba^3 - 7\,Aa^2b\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)*x/(b^5*x^3 + a*b^4) + 1/28*(4*B*b^2*x^7 - 7*(2*B*a*b
 - A*b^2)*x^4 + 28*(3*B*a^2 - 2*A*a*b)*x)/b^4 - 1/9*sqrt(3)*(10*B*a^3 - 7*A*
a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)
) + 1/18*(10*B*a^3 - 7*A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5
*(a/b)^(2/3)) - 1/9*(10*B*a^3 - 7*A*a^2*b)*log(x + (a/b)^(1/3))/(b^5*(a/b)^
(2/3))
```

**mupad [B]** time = 2.62, size = 209, normalized size = 0.90

$$x^4\left(\frac{A}{4\,b^2} - \frac{B\,a}{2\,b^3}\right) - x\left(\frac{2\,a\left(\frac{A}{b^2} - \frac{2\,B\,a}{b^3}\right)}{b} + \frac{B\,a^2}{b^4}\right) + \frac{B\,x^7}{7\,b^2} + \frac{x\left(\frac{B\,a^3}{3} - \frac{A\,a^2\,b}{3}\right)}{b^5\,x^3 + a\,b^4} + \frac{a^{4/3}\,\ln\left(b^{1/3}\,x + a^{1/3}\right)\,(7\,A\,b - 10\,B\,a)}{9\,b^{13/3}} - \frac{a^{4/3}\,\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(7\,A\,b - 10\,B\,a)}{9\,b^{13/3}} + \frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(7\,A\,b - 10\,B\,a)}{9\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2
)/b^4) + (B*x^7)/(7*b^2) + (x*((B*a^3)/3 - (A*a^2*b)/3))/(a*b^4 + b^5*x^3)
+ (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(7*A*b - 10*B*a))/(9*b^(13/3)) - (a^(4/
3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
7*A*b - 10*B*a))/(9*b^(13/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 10*B*a))/(9*b^(13/3))
```

**sympy [A]** time = 2.12, size = 156, normalized size = 0.67

$$\frac{Bx^7}{7b^2} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{x\left(-Aa^2b + Ba^3\right)}{3ab^4 + 3b^5x^3} + \operatorname{RootSum}\left(729t^3b^{13} - 343A^3a^4b^3 + 1470A^2Ba^5b^2 - 2100AB^2a^6b + 1000B^3a^7, \left(t \mapsto t\log\left(-\frac{9tb^4}{-7Aab + 10Ba^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x**4*(A/(4*b**2) - B*a/(2*b**3)) + x*(-2*A*a/b**3 + 3*B*a
**2/b**4) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729*_
t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2*a**6*
b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**2) + x
)))
```

## 3.72 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2} dx$

**Optimal**. Leaf size=82

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^6}{6b^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}+\frac{x^3(Ab-2aB)}{3b^3}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^6)/(6*b^2) - (a^2*(A*b - a*B))/(3*b^4*(a
 + b*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^2}+\frac{a(-2Ab+3aB)}{b^3(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^6}{6b^2}-\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 72, normalized size = 0.88

$$\frac{\frac{2a^2(aB-Ab)}{a+bx^3}+2bx^3(Ab-2aB)+2a(3aB-2Ab)\log(a+bx^3)+b^2Bx^6}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^3)}{(a + bx^3)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]** time = 0.57, size = 121, normalized size = 1.48

$$\frac{Bb^3x^9 - (3\,Bab^2 - 2\,Ab^3)x^6 + 2\,Ba^3 - 2\,Aa^2b - 2\,(2\,Ba^2b - Aab^2)x^3 + 2\,(3\,Ba^3 - 2\,Aa^2b + (3\,Ba^2b - 2\,Aab^2)x^3)\log(bx^3 + a)}{6\,(b^5x^3 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^3*x^9 - (3*B*a*b^2 - 2*A*b^3)*x^6 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^3 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^3)
*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

**giac [A]** time = 0.18, size = 106, normalized size = 1.29

$$\frac{(3\,Ba^2 - 2\,Aab)\log(|bx^3 + a|)}{3\,b^4} + \frac{Bb^2x^6 - 4\,Babx^3 + 2\,Ab^2x^3}{6\,b^4} - \frac{3\,Ba^2bx^3 - 2\,Aab^2x^3 + 2\,Ba^3 - Aa^2b}{3\,(bx^3 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(B*b^2*x^6 - 4*B*a*b*
x^3 + 2*A*b^2*x^3)/b^4 - 1/3*(3*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 2*B*a^3 - A*a
^2*b)/((b*x^3 + a)*b^4)
```

**maple [A]** time = 0.05, size = 97, normalized size = 1.18

$$\frac{B\,x^6}{6b^2} + \frac{A\,x^3}{3b^2} - \frac{2Ba\,x^3}{3b^3} - \frac{A\,a^2}{3\,(b\,x^3 + a)\,b^3} - \frac{2Aa\ln(b\,x^3 + a)}{3b^3} + \frac{B\,a^3}{3\,(b\,x^3 + a)\,b^4} + \frac{B\,a^2\ln(b\,x^3 + a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*B*x^6/b^2+1/3/b^2*A*x^3-2/3/b^3*B*a*x^3-2/3*a/b^3*ln(b*x^3+a)*A+a^2/b^4
*ln(b*x^3+a)*B-1/3*a^2/b^3/(b*x^3+a)*A+1/3*a^3/b^4/(b*x^3+a)*B
```

**maxima [A]** time = 0.49, size = 82, normalized size = 1.00

$$\frac{Ba^3 - Aa^2b}{3\,(b^5x^3 + ab^4)} + \frac{Bbx^6 - 2\,(2\,Ba - Ab)x^3}{6\,b^3} + \frac{(3\,Ba^2 - 2\,Aab)\log(bx^3 + a)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)/(b^5*x^3 + a*b^4) + 1/6*(B*b*x^6 - 2*(2*B*a - A*b)*x^
3)/b^3 + 1/3*(3*B*a^2 - 2*A*a*b)*log(b*x^3 + a)/b^4
```

**mupad [B]** time = 0.08, size = 86, normalized size = 1.05

$$x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{3\,b^4}+\frac{B\,x^6}{6\,b^2}+\frac{B\,a^3-A\,a^2\,b}{3\,b\,\left(b^4\,x^3+a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) + (log(a + b*x^3)*(3*B*a^2 - 2*A*a*b))/(3
*b^4) + (B*x^6)/(6*b^2) + (B*a^3 - A*a^2*b)/(3*b*(a*b^3 + b^4*x^3))
```

**sympy [A]** time = 2.14, size = 82, normalized size = 1.00

$$\frac{Bx^6}{6b^2}+\frac{a\left(-2Ab+3Ba\right)\log\left(a+bx^3\right)}{3b^4}+x^3\left(\frac{A}{3b^2}-\frac{2Ba}{3b^3}\right)+\frac{-Aa^2b+Ba^3}{3ab^4+3b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**3)/(3*b**4) + x**3*(A/(3*
b**2) - 2*B*a/(3*b**3)) + (-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3)
```

## 3.73 $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=215

$$-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\ldots}{\sqrt{3}\ldots}\right)}{3\sqrt{3}\,b^{11/3}}$$

**Rubi [A]** time = 0.14, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{11/3}}-\frac{x^5(5Ab-8aB)}{15ab^2}+\frac{x^2(5Ab-8aB)}{6b^3}+\frac{x^8(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((5*A*b - 8*a*B)*x^2)/(6*b^3) - ((5*A*b - 8*a*B)*x^5)/(15*a*b^2) + ((A*b -
a*B)*x^8)/(3*a*b*(a + b*x^3)) + (a^(2/3)*(5*A*b - 8*a*B)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(11/3)) + (a^(2/3)*(5*A*b - 8
*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(11/3)) - (a^(2/3)*(5*A*b - 8*a*B)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{(-5Ab+8aB)\int \frac{x^7}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{(-5Ab+8aB)\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}-\frac{(a(5Ab-8aB))\int \frac{x}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{\left(a^{2/3}(5Ab-8aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9b^{10/3}}-$$

$$= \frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{11/3}}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{11/3}}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3}-\frac{(5Ab-8aB)x^5}{15ab^2}+\frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{11/3}}$$

**Mathematica [A]** time = 0.17, size = 185, normalized size = 0.86

$$\frac{5a^{2/3}(8aB-5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-10a^{2/3}(8aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-10\sqrt{3}\,a^{2/3}(8aB-5Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+45b^{2/3}x^2(Ab-2aB)+\frac{30ab^{2/3}x^2(Ab-aB)}{a+bx^3}+18b^{5/3}Bx^5}{90b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*B
)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^(1/
3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(90*b^(11/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \left(A + Bx^3\right)}{\left(a + bx^3\right)^2}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]** time = 0.67, size = 257, normalized size = 1.20

$$\frac{18\,Bb^2x^8 - 9\left(8\,Bab - 5\,Ab^2\right)x^5 - 15\left(8\,Ba^2 - 5\,Aab\right)x^2 + 10\,\sqrt{3}\left(\left(8\,Bab - 5\,Ab^2\right)x^3 + 8\,Ba^2 - 5\,Aab\right)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}\,a}{3\,a}\right) + 5\left(\left(8\,Bab - 5\,Ab^2\right)x^3 + 8\,Ba^2 - 5\,Aab\right)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2 - bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) - 10\left(\left(8\,Bab - 5\,Ab^2\right)x^3 + 8\,Ba^2 - 5\,Aab\right)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax + b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)}{90\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^2
 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)
*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b -
5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)
^(2/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a
*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^4*x^3 + a*b^3)
```

**giac [A]** time = 0.18, size = 236, normalized size = 1.10

$$-\frac{\left(8\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3} - \frac{\sqrt{3}\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} - \frac{Ba^2x^2 - Aabx^2}{3\left(bx^3 + a\right)b^3} + \frac{\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5} + \frac{2\,Bb^8x^5 - 10\,Bab^7x^2 + 5\,Ab^8x^2}{10\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2
/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3*(B
*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a - 5*(-
a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*(2*B*
b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10
```

**maple [A]** time = 0.05, size = 266, normalized size = 1.24

$$\frac{B\,x^5}{5b^2} + \frac{Aa\,x^2}{3\left(b\,x^3 + a\right)b^2} - \frac{B\,a^2x^2}{3\left(b\,x^3 + a\right)b^3} + \frac{A\,x^2}{2b^2} - \frac{Ba\,x^2}{b^3} - \frac{5\sqrt{3}\,Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{5Aa\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} - \frac{5Aa\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{8\sqrt{3}\,B\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} - \frac{8B\,a^2\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4} + \frac{4B\,a^2\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/5/b^2*B*x^5+1/2/b^2*A*x^2-1/b^3*B*x^2*a+1/3*a/b^2*x^2/(b*x^3+a)*A-1/3*a^2
/b^3*x^2/(b*x^3+a)*B+5/9*a/b^3*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18*a/b^3*A
```

```
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9*a/b^3*A*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-8/9*a^2/b^4*B/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))+4/9*a^2/b^4*B/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+8/
9*a^2/b^4*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

**maxima [A]** time = 1.10, size = 192, normalized size = 0.89

$$-\frac{\left(Ba^2-Aab\right)x^2}{3\left(b^4x^3+ab^3\right)}+\frac{\sqrt{3}\left(8\,Ba^2-5\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,Bbx^5-5\,(2\,Ba-Ab)x^2}{10\,b^3}+\frac{\left(8\,Ba^2-5\,Aab\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(8\,Ba^2-5\,Aab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x^2/(b^4*x^3 + a*b^3) + 1/9*sqrt(3)*(8*B*a^2 - 5*A*a*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1
/10*(2*B*b*x^5 - 5*(2*B*a - A*b)*x^2)/b^3 + 1/18*(8*B*a^2 - 5*A*a*b)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) - 1/9*(8*B*a^2 - 5*A*a*b
)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

**mupad [B]** time = 0.27, size = 179, normalized size = 0.83

$$x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)+\frac{B\,x^5}{5\,b^2}-\frac{x^2\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-8\,B\,a)}{9\,b^{11/3}}+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(5\,A\,b-8\,B\,a)}{9\,b^{11/3}}-\frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(5\,A\,b-8\,B\,a)}{9\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^2*(A/(2*b^2) - (B*a)/b^3) + (B*x^5)/(5*b^2) - (x^2*((B*a^2)/3 - (A*a*b)/3
))/(a*b^3 + b^4*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 8*B*a))/(
9*b^(11/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(5*A*b - 8*B*a))/(9*b^(11/3)) - (a^(2/3)*log(3^(1/2)*a^(
1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 8*B*a))/(9
*b^(11/3))
```

**sympy [A]** time = 2.12, size = 151, normalized size = 0.70

$$\frac{Bx^5}{5b^2}+x^2\left(\frac{A}{2b^2}-\frac{Ba}{b^3}\right)+\frac{x^2\left(Aab-Ba^2\right)}{3ab^3+3b^4x^3}+\operatorname{RootSum}\left(729t^3b^{11}-125A^3a^2b^3+600A^2Ba^3b^2-960AB^2a^4b+512B^3a^5,\left(t\mapsto t\log\left(\frac{81t^2b^7}{25A^2ab^2-80ABa^2b+64B^2a^3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x**2*(A/(2*b**2) - B*a/b**3) + x**2*(A*a*b - B*a**2)/(3*a
*b**3 + 3*b**4*x**3) + RootSum(729*_t**3*b**11 - 125*A**3*a**2*b**3 + 600*A
**2*B*a**3*b**2 - 960*A*B**2*a**4*b + 512*B**3*a**5, Lambda(_t, _t*log(81*_
t**2*b**7/(25*A**2*a*b**2 - 80*A*B*a**2*b + 64*B**2*a**3) + x)))
```

## 3.74 $\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=213

$$\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{10/3}}-\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}}+\frac{\sqrt[3]{a}\,(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{10/3}}$$

**Rubi** [A] time = 0.13, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{10/3}}-\frac{x^4(4Ab-7aB)}{12ab^2}+\frac{x(4Ab-7aB)}{3b^3}-\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}}+\frac{\sqrt[3]{a}\,(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{10/3}}+\frac{x^7(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((4*A*b - 7*a*B)*x)/(3*b^3) - ((4*A*b - 7*a*B)*x^4)/(12*a*b^2) + ((A*b - a*
B)*x^7)/(3*a*b*(a + b*x^3)) + (a^(1/3)*(4*A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(10/3)) - (a^(1/3)*(4*A*b - 7*a
*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(10/3)) + (a^(1/3)*(4*A*b - 7*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{(-4Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{(-4Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{(a(4Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\left(\sqrt[3]{a}\,(4Ab-7aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9b^3} - \cdots$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}} + \cdots$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}} + \cdots$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{\sqrt[3]{a}\,(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{10/3}} - \cdots$$

**Mathematica [A]** time = 0.15, size = 181, normalized size = 0.85

$$\frac{-2\sqrt[3]{a}\,(7aB-4Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\frac{12a\sqrt[3]{b}\,x(Ab-aB)}{a+bx^3}+36\sqrt[3]{b}\,x(Ab-2aB)+4\sqrt[3]{a}\,(7aB-4Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-4\sqrt{3}\sqrt[3]{a}\,(7aB-4Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+9b^{4/3}Bx^4}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*x
)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x] -
 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(36*b^(10/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \left(A + Bx^3\right)}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]** time = 0.85, size = 240, normalized size = 1.13

$$\frac{9 \, B b^{2} x^{7} - 9 \, {\left(7 \, B a b - 4 \, A b^{2}\right)} x^{4} - 4 \, \sqrt{3} {\left({\left(7 \, B a b - 4 \, A b^{2}\right)} x^{3} + 7 \, B a^{2} - 4 \, A a b\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \, \sqrt{3} b x \left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3} a}{3 \, a}\right) + 2 \, {\left({\left(7 \, B a b - 4 \, A b^{2}\right)} x^{3} + 7 \, B a^{2} - 4 \, A a b\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 4 \, {\left({\left(7 \, B a b - 4 \, A b^{2}\right)} x^{3} + 7 \, B a^{2} - 4 \, A a b\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - 12 \, {\left(7 \, B a^{2} - 4 \, A a b\right)} x}{36 \, {\left(b^{4} x^{3} + a b^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(9*B*b^2*x^7 - 9*(7*B*a*b - 4*A*b^2)*x^4 - 4*sqrt(3)*((7*B*a*b - 4*A*b
^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^
(2/3) - sqrt(3)*a)/a) + 2*((7*B*a*b - 4*A*b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a
/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*((7*B*a*b - 4*A*b^2)
*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 12*(7*B*a^2
- 4*A*a*b)*x)/(b^4*x^3 + a*b^3)
```

**giac [A]** time = 0.18, size = 211, normalized size = 0.99

$$\frac{\sqrt{3} {\left(7 \, \left(-a b^{2}\right)^{\frac{1}{3}} B a - 4 \, \left(-a b^{2}\right)^{\frac{1}{3}} A b\right)} \arctan\left(\frac{\sqrt{3} {\left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}}{3 \, \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \, b^{4}} - \frac{{\left(7 \, B a^{2} - 4 \, A a b\right)} \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left({\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right|}\right)}{9 \, a b^{3}} + \frac{{\left(7 \, \left(-a b^{2}\right)^{\frac{1}{3}} B a - 4 \, \left(-a b^{2}\right)^{\frac{1}{3}} A b\right)} \log\left(x^{2} + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \, b^{4}} - \frac{B a^{2} x - A a b x}{3 \, {\left(b x^{3} + a\right)} b^{3}} + \frac{B b^{6} x^{4} - 8 \, B a b^{5} x + 4 \, A b^{6} x}{4 \, b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/9*(7*B*a^2 - 4*A*a*b)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(7*(-a*b^2)^(1/3)*B*a - 4*(-
a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2
*x - A*a*b*x)/((b*x^3 + a)*b^3) + 1/4*(B*b^6*x^4 - 8*B*a*b^5*x + 4*A*b^6*x)
/b^8
```

**maple [A]** time = 0.09, size = 257, normalized size = 1.21

$$\frac{B x^{4}}{4 b^{2}}+\frac{A a x}{3 \left(b \,x^{3}+a \right) b^{2}}-\frac{B \,a^{2} x}{3 \left(b \,x^{3}+a \right) b^{3}}-\frac{4 \sqrt{3}\, A a \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}-\frac{4 A a \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{2 A a \ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{3}}+\frac{A x}{b^{2}}+\frac{7 \sqrt{3}\, B \,a^{2} \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}+\frac{7 B \,a^{2} \ln \left(x +\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}-\frac{7 B \,a^{2} \ln \left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x +\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^{4}}-\frac{2 B a x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/4/b^2*B*x^4+1/b^2*A*x-2/b^3*B*a*x+1/3*a/b^2*x/(b*x^3+a)*A-1/3*a^2/b^3*x/(
b*x^3+a)*B-4/9*a/b^3*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9*a/b^3*A/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*a/b^3*A/(a/b)^(2/3)*3^(1/2)*arctan(
```

```
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+7/9*a^2/b^4*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-7/18*a^2/b^4*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+7/9*a^2/b^4*
B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

**maxima [A]** time = 1.01, size = 187, normalized size = 0.88

$$-\frac{\left(Ba^2-Aab\right)x}{3\left(b^4x^3+ab^3\right)}+\frac{Bbx^4-4\,(2\,Ba-Ab)x}{4\,b^3}+\frac{\sqrt{3}\left(7\,Ba^2-4\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(7\,Ba^2-4\,Aab\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(7\,Ba^2-4\,Aab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x/(b^4*x^3 + a*b^3) + 1/4*(B*b*x^4 - 4*(2*B*a - A*b)*x
)/b^3 + 1/9*sqrt(3)*(7*B*a^2 - 4*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/18*(7*B*a^2 - 4*A*a*b)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(7*B*a^2 - 4*A*a*b)*log(x
 + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

**mupad [B]** time = 2.62, size = 193, normalized size = 0.91

$$x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{B\,x^4}{4\,b^2}+\frac{{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3}+a\,b^{1/3}\,x\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}-\frac{{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}+\frac{{(-a)}^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-{(-a)}^{4/3}+\sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) + (
B*x^4)/(4*b^2) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(4*A*b - 7*B*a))
/(9*b^(10/3)) - ((-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^
(1/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3)) + ((-a)^(1/3)
*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 -
1/2)*(4*A*b - 7*B*a))/(9*b^(10/3))
```

**sympy [A]** time = 2.54, size = 126, normalized size = 0.59

$$\frac{Bx^4}{4b^2}+x\left(\frac{A}{b^2}-\frac{2Ba}{b^3}\right)+\frac{x\left(Aab-Ba^2\right)}{3ab^3+3b^4x^3}+\operatorname{RootSum}\left(729t^3b^{10}+64A^3ab^3-336A^2Ba^2b^2+588AB^2a^3b-343B^3a^4,\left(t\mapsto t\log\left(\frac{9tb^3}{-4Ab+7Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(3*a*b**3 +
3*b**4*x**3) + RootSum(729*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2*b
**2 + 588*A*B**2*a**3*b - 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4*A*
b + 7*B*a) + x)))
```

## 3.75 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=60

$$\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}+\frac{Bx^3}{3b^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (B*x^3)/(3*b^2) + (a*(A*b - a*B))/(3*b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[
a + b*x^3])/(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b^2}+\frac{a(-Ab+aB)}{b^2(a+bx)^2}+\frac{Ab-2aB}{b^2(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{Bx^3}{3b^2}+\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 50, normalized size = 0.83

$$\frac{\frac{a(Ab-aB)}{a+bx^3}+(Ab-2aB)\log(a+bx^3)+bBx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*b
^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(A+Bx^3\right)}{\left(a+bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]** time = 0.69, size = 81, normalized size = 1.35

$$\frac{Bb^2x^6 + Babx^3 - Ba^2 + Aab - \left(\left(2\,Bab - Ab^2\right)x^3 + 2\,Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{3\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + B*a*b*x^3 - B*a^2 + A*a*b - ((2*B*a*b - A*b^2)*x^3 + 2*B*a
^2 - A*a*b)*log(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

**giac [A]** time = 0.18, size = 91, normalized size = 1.52

$$\frac{\frac{\left(bx^3+a\right)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^3+a} - \frac{Aab^2}{bx^3+a}}{b^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^3 + a) - A*a*b^2/(b*x^3 + a))/b^3)/b
```

**maple [A]** time = 0.05, size = 74, normalized size = 1.23

$$\frac{B\,x^3}{3b^2} + \frac{Aa}{3\left(b\,x^3+a\right)b^2} + \frac{A\ln\left(b\,x^3+a\right)}{3b^2} - \frac{B\,a^2}{3\left(b\,x^3+a\right)b^3} - \frac{2Ba\ln\left(b\,x^3+a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*B*x^3/b^2+1/3/b^2*ln(b*x^3+a)*A-2/3/b^3*ln(b*x^3+a)*B*a+1/3/b^2*a/(b*x^
3+a)*A-1/3/b^3*a^2/(b*x^3+a)*B
```

**maxima [A]** time = 0.46, size = 60, normalized size = 1.00

$$\frac{Bx^3}{3\,b^2} - \frac{Ba^2 - Aab}{3\left(b^4x^3 + ab^3\right)} - \frac{(2\,Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^2 - 1/3*(B*a^2 - A*a*b)/(b^4*x^3 + a*b^3) - 1/3*(2*B*a - A*b)*l
og(b*x^3 + a)/b^3
```

**mupad** [B] time = 0.08, size = 62, normalized size = 1.03

$$\frac{B\,x^3}{3\,b^2}+\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-2\,B\,a\right)}{3\,b^3}-\frac{B\,a^2-A\,a\,b}{3\,b\,\left(b^3\,x^3+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^3)/(3*b^2) + (log(a + b*x^3)*(A*b - 2*B*a))/(3*b^3) - (B*a^2 - A*a*b)/
(3*b*(a*b^2 + b^3*x^3))
```

**sympy** [A] time = 1.61, size = 56, normalized size = 0.93

$$\frac{Bx^3}{3b^2}+\frac{Aab-Ba^2}{3ab^3+3b^4x^3}-\frac{(-Ab+2Ba)\log\left(a+bx^3\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) + (A*a*b - B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*a
)*log(a + b*x**3)/(3*b**3)
```

## 3.76 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=196

$$\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-x$$

**Rubi [A]** time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 321, 292, 31, 634, 617, 204, 628}

$$\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{x^2(2Ab-5aB)}{6ab^2}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}+\frac{x^5(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((2*A*b - 5*a*B)*x^2)/(6*a*b^2) + ((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3)) -
((2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(1/3)*b^(8/3)) - ((2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3
)*b^(8/3)) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}+\frac{(-2Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}+\frac{(2Ab-5aB)\int \frac{x}{a+bx^3}\,dx}{3b^2}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9\sqrt[3]{a}\,b^{7/3}}+\frac{(2Ab-5aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{}}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}}}{9\sqrt[3]{a}\,b^{7/3}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}}+\frac{(2Ab-5aB)\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{}}}{18\sqrt[3]{a}\,b^{8}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}}+\frac{(2Ab-5aB)\log\left(a^{2/3}\right.}{18\sqrt[3]{}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}\right.}{9\sqrt[3]{a}\,b^{8/3}}$$

**Mathematica** [A] time = 0.14, size = 165, normalized size = 0.84

$$\frac{\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{a}}-\frac{6b^{2/3}x^2(Ab-aB)}{a+bx^3}+\frac{2(5aB-2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a}}+\frac{2\sqrt{3}\,(5aB-2Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+9b^{2/3}Bx^2}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-2
*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(-2
*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(A + Bx^3\right)}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]** time = 0.79, size = 578, normalized size = 2.95

$$\left[\frac{9\,Bab^3x^5 + 3\left(5\,Ba^2b^2 - 2\,Aab^3\right)x^2 - 3\,\sqrt{\frac{1}{3}}\left(5\,Ba^3b - 2\,Aa^2b^2 + \left(5\,Ba^2b^2 - 2\,Aab^3\right)x^3\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\left(-ab^2\right)^{\frac{2}{3}}x^2 + \left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}} - 3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3 + a}\right) - \left(\left(5\,Bab - 2\,Ab^2\right)x^3 + 5\,Ba^2 - 2\,Aab\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2 + \left(-ab^2\right)^{\frac{1}{3}}bx + \left(-ab^2\right)^{\frac{2}{3}}\right) + 2\left(\left(5\,Bab - 2\,Ab^2\right)x^3 + 5\,Ba^2 - 2\,Aab\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx - \left(-ab^2\right)^{\frac{1}{3}}\right)}{18\left(ab^5x^3 + a^2b^4\right)}, \frac{9\,Bab^3x^5 + 3\left(5\,Ba^2b^2 - 2\,Aab^3\right)x^2 - 6\,\sqrt{\frac{1}{3}}\left(5\,Ba^3b - 2\,Aa^2b^2 + \left(5\,Ba^2b^2 - 2\,Aab^3\right)x^3\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx + \left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right) - \left(\left(5\,Bab - 2\,Ab^2\right)x^3 + 5\,Ba^2 - 2\,Aab\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2 + \left(-ab^2\right)^{\frac{1}{3}}bx + \left(-ab^2\right)^{\frac{2}{3}}\right) + 2\left(\left(5\,Bab - 2\,Ab^2\right)x^3 + 5\,Ba^2 - 2\,Aab\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx - \left(-ab^2\right)^{\frac{1}{3}}\right)}{18\left(ab^5x^3 + a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*a
^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*
log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)
^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B
*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b
^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*
A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4), 1/1
8*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5*B*a^3*b
 - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a
*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2
)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*
a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4)]
```

**giac [A]** time = 0.21, size = 189, normalized size = 0.96

$$\frac{Bx^2}{2\,b^2} - \frac{\sqrt{3}\,(5\,Ba - 2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,b^2} + \frac{(5\,Ba - 2\,Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,b^2} + \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax^2 - Abx^2}{3\left(bx^3 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^2) + 1/18*(5*B*a - 2*A*b)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^2) + 1/9*(5*B*a*(-a/b)^(
1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
+ 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b^2)
```

**maple [A]** time = 0.08, size = 235, normalized size = 1.20

$$-\frac{A\,x^2}{3\left(b\,x^3 + a\right)b} + \frac{Ba\,x^2}{3\left(b\,x^3 + a\right)b^2} + \frac{B\,x^2}{2b^2} + \frac{2\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{2A\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{A\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{5\sqrt{3}\,Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} + \frac{5Ba\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3} - \frac{5Ba\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/2*B*x^2/b^2-1/3/b*x^2/(b*x^3+a)*A+1/3/b^2*x^2/(b*x^3+a)*B*a+5/9/b^3*B*a/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18/b^3*B*a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))-5/9/b^3*B*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-2/9/b^2*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*A/(a/b)^(1/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/b^2*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))
```

**maxima [A]** time = 1.17, size = 162, normalized size = 0.83

$$\frac{(Ba-Ab)x^2}{3\left(b^3x^3+ab^2\right)}+\frac{Bx^2}{2\,b^2}-\frac{\sqrt{3}\,(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(5\,Ba-2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(5\,Ba-2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^2/(b^3*x^3 + a*b^2) + 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a
- 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/
3)) - 1/18*(5*B*a - 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b
)^(1/3)) + 1/9*(5*B*a - 2*A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

**mupad [B]** time = 2.58, size = 158, normalized size = 0.81

$$\frac{B\,x^2}{2\,b^2}-\frac{x^2\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^2)/(2*b^2) - (x^2*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (log(b^(1/3
)*x + a^(1/3))*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*
1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*A*b - 5*B*a))/(9*a^(1
/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 + 1/2)*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3))
```

**sympy [A]** time = 2.09, size = 126, normalized size = 0.64

$$\frac{Bx^2}{2b^2}+\frac{x^2\,(-Ab+Ba)}{3ab^2+3b^3x^3}+\operatorname{RootSum}\left(729t^3ab^8+8A^3b^3-60A^2Bab^2+150AB^2a^2b-125B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2ab^5}{4A^2b^2-20ABab+25B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*
_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B**
3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25*B*
*2*a**2) + x)))
```

## 3.77 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=190

$$-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{x(A$$

**Rubi [A]** time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{x(Ab-4aB)}{3ab^2}+\frac{x^4(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 4*a*B)*x)/(3*a*b^2) + ((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((A*
b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^
(2/3)*b^(7/3)) + ((A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(7/3
)) - ((A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(
2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx=\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(-Ab+4aB)\int\frac{x^3}{a+bx^3}\,dx}{3ab}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\int\frac{1}{a+bx^3}\,dx}{3b^2}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b^2}+\frac{(Ab-4aB)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/\cdots}}\cdots}{9a^{2/3}b^2}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}\cdots}{18a^{2/3}b^{7/3}}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\log\left(a^{2/3}-\cdots\right.}{18a^{2/3}b^{7/\cdots}}$$

$$=-\frac{(Ab-4aB)x}{3ab^2}+\frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\cdots\right.}{9a^{2/3}b^{7/3}}$$

**Mathematica** [A] time = 0.17, size = 160, normalized size = 0.84

$$\frac{\frac{(4aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{2(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{2\sqrt{3}(4aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{6\sqrt[3]{b}x(Ab-aB)}{a+bx^3}+18\sqrt[3]{b}Bx}{18b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*b
) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b -
 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(A + Bx^3\right)}{\left(a + bx^3\right)^2}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]** time = 0.55, size = 573, normalized size = 3.02

$$\left[\frac{18\,Ba^2b^2x^4 - 3\,\sqrt{\frac{1}{3}}\left(4\,Ba^3b - Aa^2b^2 + \left(4\,Ba^2b^2 - Aab^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\left(a^2b\right)^{\frac{1}{3}}ax - a^2 + 3\,\sqrt{\frac{1}{3}}\left(2\,abx^2 + \left(a^2b\right)^{\frac{2}{3}}x - \left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right) + \left(\left(4\,Bab - Ab^2\right)x^3 + 4\,Ba^2 - Aab\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2 - \left(a^2b\right)^{\frac{2}{3}}x + \left(a^2b\right)^{\frac{1}{3}}a\right) - 2\left(\left(4\,Bab - Ab^2\right)x^3 + 4\,Ba^2 - Aab\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx + \left(a^2b\right)^{\frac{2}{3}}\right) + 6\left(4\,Ba^3b - Aa^2b^2\right)x}{18\left(a^2b^4x^3 + a^3b^3\right)}, \frac{18\,Ba^2b^2x^4 - 6\,\sqrt{\frac{1}{3}}\left(4\,Ba^3b - Aa^2b^2 + \left(4\,Ba^2b^2 - Aab^3\right)x^3\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2b\right)^{\frac{2}{3}}x - \left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right) + \left(\left(4\,Bab - Ab^2\right)x^3 + 4\,Ba^2 - Aab\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2 - \left(a^2b\right)^{\frac{2}{3}}x + \left(a^2b\right)^{\frac{1}{3}}a\right) - 2\left(\left(4\,Bab - Ab^2\right)x^3 + 4\,Ba^2 - Aab\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx + \left(a^2b\right)^{\frac{2}{3}}\right) + 6\left(4\,Ba^3b - Aa^2b^2\right)x}{18\left(a^2b^4x^3 + a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2
 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*
x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a
*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*a^2*b^2
*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)*x^3)*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2
*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a*b -
A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*
(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]
```

**giac [A]** time = 0.19, size = 166, normalized size = 0.87

$$\frac{\sqrt{3}\left(4\,Ba - Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b} + \frac{\left(4\,Ba - Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b} + \frac{Bx}{b^2} + \frac{\left(4\,Ba - Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax - Abx}{3\left(bx^3 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(2/3)*b) + 1/18*(4*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/((-a*b^2)^(2/3)*b) + B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*b^2)
```

**maple [A]** time = 0.04, size = 228, normalized size = 1.20

$$-\frac{Ax}{3\left(b\,x^3 + a\right)b} + \frac{Bax}{3\left(b\,x^3 + a\right)b^2} + \frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{A\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{A\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{4\sqrt{3}\,Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{4Ba\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{2Ba\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{Bx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] B/b^2*x-1/3/b*x/(b*x^3+a)*A+1/3/b^2*x/(b*x^3+a)*B*a-4/9/b^3*B*a/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))+2/9/b^3*B*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)-4/9/b^3*B*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1
/9/b^2*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*A/(a/b)^(2/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))+1/9/b^2*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))
```

**maxima [A]** time = 1.22, size = 157, normalized size = 0.83

$$\frac{(Ba - Ab)x}{3\left(b^3x^3 + ab^2\right)} + \frac{Bx}{b^2} - \frac{\sqrt{3}\,(4\,Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4\,Ba - Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4\,Ba - Ab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x/(b^3*x^3 + a*b^2) + B*x/b^2 - 1/9*sqrt(3)*(4*B*a - A*b)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/18
*(4*B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1
/9*(4*B*a - A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

**mupad [B]** time = 2.58, size = 150, normalized size = 0.79

$$\frac{B\,x}{b^2} - \frac{x\,\left(\frac{A\,b}{3} - \frac{B\,a}{3}\right)}{b^3\,x^3 + a\,b^2} + \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,\left(A\,b - 4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}} - \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b - 4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b - 4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x)/b^2 - (x*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (log(b^(1/3)*x + a^
(1/3))*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
 + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
```

**sympy [A]** time = 1.67, size = 102, normalized size = 0.54

$$\frac{Bx}{b^2} + \frac{x\,(-Ab + Ba)}{3ab^2 + 3b^3x^3} + \operatorname{RootSum}\left(729t^3a^2b^7 - A^3b^3 + 12A^2Bab^2 - 48AB^2a^2b + 64B^3a^3, \left(t \mapsto t\log\left(-\frac{9tab^2}{-Ab + 4Ba} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**2
*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, Lam
bda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))
```

## 3.78 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=41

$$\frac{aB-Ab}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{B\log(a+bx^3)}{3b^2}-\frac{Ab-aB}{3b^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)dx,x,x^3\right)\\ &= -\frac{Ab-aB}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 41, normalized size = 1.00

$$\frac{aB-Ab}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (-(A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(A + Bx^3\right)}{\left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
[Out] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]** time = 0.56, size = 44, normalized size = 1.07

$$\frac{Ba - Ab + \left(Bbx^3 + Ba\right)\log\left(bx^3 + a\right)}{3\left(b^3x^3 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] 1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

**giac [A]** time = 0.18, size = 65, normalized size = 1.59

$$-\frac{B\left(\frac{\log\left(\frac{|bx^3+a|}{\left(bx^3+a\right)^2|b|}\right)}{b} - \frac{a}{\left(bx^3+a\right)b}\right)}{3\,b} - \frac{A}{3\left(bx^3 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
[Out] -1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b
 - 1/3*A/((b*x^3 + a)*b)
```

**maple [A]** time = 0.06, size = 47, normalized size = 1.15

$$-\frac{A}{3\left(b\,x^3 + a\right)b} + \frac{Ba}{3\left(b\,x^3 + a\right)b^2} + \frac{B\ln\left(b\,x^3 + a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^2,x)
[Out] 1/3*B*ln(b*x^3+a)/b^2-1/3/b/(b*x^3+a)*A+1/3/b^2/(b*x^3+a)*B*a
```

**maxima [A]** time = 0.58, size = 40, normalized size = 0.98

$$\frac{Ba - Ab}{3\left(b^3x^3 + ab^2\right)} + \frac{B\log\left(bx^3 + a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2
```

**mupad** [B] time = 2.35, size = 37, normalized size = 0.90

$$\frac{B\ln\left(b\,x^3+a\right)}{3\,b^2}-\frac{A\,b-B\,a}{3\,b^2\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*log(a + b*x^3))/(3*b^2) - (A*b - B*a)/(3*b^2*(a + b*x^3))
```

**sympy** [A] time = 1.23, size = 36, normalized size = 0.88

$$\frac{B\log\left(a+bx^3\right)}{3b^2}+\frac{-Ab+Ba}{3ab^2+3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**2) + (-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3)
```

## 3.79 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=171

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^2(Ab-}{3ab\,(a-}$$

**Rubi [A]** time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {457, 292, 31, 634, 617, 204, 628}

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^2(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - ((A*b + 2*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(5/3)) + ((A*b + 2*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx = \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}+\frac{(Ab+2aB)\int \frac{x}{a+bx^3}dx}{3ab}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{9a^{4/3}b^{4/3}}+\frac{(Ab+2aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{9a^{4/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}}+\frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{18a^{4/3}b^{5/3}}+ \frac{(A}{}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}}+\frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right.}{18a^{4/3}b^{5/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}-\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}}+\frac{(Ab+2}{}$$

**Mathematica [A]** time = 0.13, size = 146, normalized size = 0.85

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{6\sqrt[3]{a}b^{2/3}x^2(aB-Ab)}{a+bx^3}-2(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(2aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{4/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(1/
3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(4/3)*b^(5/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^2, x]
```

**fricas [A]**     time = 0.55, size = 548, normalized size = 3.20

$$\left[-\frac{6\left(Ba^2b^2 - Aab^3\right)x^2 - 3\sqrt{\frac{1}{3}}\left(2Ba^3b + Aa^2b^2 + \left(2Ba^2b^2 + Aab^3\right)x^3\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3 - ab + 3\sqrt{\frac{1}{3}}\left(abx + 2(-ab^2)^{\frac{2}{3}}x^2 + (-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}} - 3(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right) - \left(\left(2Bab + Ab^2\right)x^3 + 2Ba^2 + Aab\right)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^2 + (-ab^2)^{\frac{1}{3}}bx + (-ab^2)^{\frac{2}{3}}\right) + 2\left(\left(2Bab + Ab^2\right)x^3 + 2Ba^2 + Aab\right)(-ab^2)^{\frac{2}{3}}\log\left(bx - (-ab^2)^{\frac{1}{3}}\right)}{18\left(a^2b^4x^3 + a^3b^3\right)}, -\frac{6\left(Ba^2b^2 - Aab^3\right)x^2 - 6\sqrt{\frac{1}{3}}\left(2Ba^3b + Aa^2b^2 + \left(2Ba^2b^2 + Aab^3\right)x^3\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx + (-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right) - \left(\left(2Bab + Ab^2\right)x^3 + 2Ba^2 + Aab\right)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^2 + (-ab^2)^{\frac{1}{3}}bx + (-ab^2)^{\frac{2}{3}}\right) + 2\left(\left(2Bab + Ab^2\right)x^3 + 2Ba^2 + Aab\right)(-ab^2)^{\frac{2}{3}}\log\left(bx - (-ab^2)^{\frac{1}{3}}\right)}{18\left(a^2b^4x^3 + a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 +
(2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b +
3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)
^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 + 2*B
*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x -
 (-a*b^2)^(1/3)))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a*b^3)*x
^2 - 6*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt
(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2
)^(1/3)/a)/b) - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((2*B*a*b + A*b^2)*x^3
 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x^3
+ a^3*b^3)]
```

**giac [A]**     time = 0.18, size = 186, normalized size = 1.09

$$\frac{\sqrt{3}\,(2\,Ba + Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}ab} - \frac{(2\,Ba + Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}ab} - \frac{\left(2\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} + Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{Bax^2 - Abx^2}{3\left(bx^3 + a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a*b) - 1/18*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b) - 1/9*(2*B*a*(-a/b)^(1/3) + A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x^2 - A*b*x
^2)/((b*x^3 + a)*a*b)
```

**maple [A]**     time = 0.05, size = 223, normalized size = 1.30

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab} - \frac{A\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab} + \frac{A\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab} + \frac{2\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{2B\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{B\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{(Ab - Ba)\,x^2}{3\left(b\,x^3 + a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9/b/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-2/9
/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1/18/b/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))*A+1/9/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+
1/9/b/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+2/9/b
^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

**maxima** [A] time = 1.21, size = 160, normalized size = 0.94

$$-\frac{(Ba-Ab)x^2}{3\left(ab^2x^3+a^2b\right)}+\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^2/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*B*a + A*b)*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/18*(2
*B*a + A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/
9*(2*B*a + A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

**mupad** [B] time = 0.25, size = 145, normalized size = 0.85

$$\frac{x^2\,(A\,b-B\,a)}{3\,a\,b\,\left(b\,x^3+a\right)}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(b^
(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) + (x^2*(A*b - B*a))/(
3*a*b*(a + b*x^3))
```

**sympy** [A] time = 1.48, size = 117, normalized size = 0.68

$$\frac{x^2\,(Ab-Ba)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^4b^5+A^3b^3+6A^2Bab^2+12AB^2a^2b+8B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a^3b^3}{A^2b^2+4ABab+4B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5 +
 A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t, _
t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

## 3.80 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^2} dx$

**Optimal**. Leaf size=169

$$-\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(Ab}{3ab\,(a}$$

**Rubi [A]** time = 0.09, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*A*b + a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*A*b + a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b} + \frac{(2Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}} + \frac{(2A\ldots}{\ldots}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+a\ldots}{\ldots}$$

**Mathematica [A]** time = 0.14, size = 145, normalized size = 0.86

$$\frac{-(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{6a^{2/3}\sqrt[3]{b}x(aB-Ab)}{a+bx^3}+2(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(aB+2Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^2, x]
```

**fricas** [A] time = 0.70, size = 537, normalized size = 3.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B
*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a
^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^
2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b + 2
*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

**giac** [A] time = 0.18, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{(Ba+2\,Ab)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^{2}\right)^{\frac{2}{3}}a}-\frac{(Ba+2\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b}-\frac{Bax-Abx}{3\left(bx^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a*b)
```

**maple** [A] time = 0.06, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{(Ab-Ba)\,x}{3\left(b\,x^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/9/b
^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))*A-1/18/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+2/
9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/9/b^2
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

**maxima** [A] time = 1.10, size = 158, normalized size = 0.93

$$-\frac{(Ba-Ab)x}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba+2\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(B*a
 + 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*
(B*a + 2*A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

**mupad [B]** time = 2.55, size = 143, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,A\,b+B\,a\right)}{9\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,A\,b+B\,a\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,A\,b+B\,a\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{x\,\left(A\,b-B\,a\right)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) + (x*(A*b - B*a))/(3*
a*b*(a + b*x^3))
```

**sympy [A]** time = 1.43, size = 97, normalized size = 0.57

$$\frac{x\left(Ab-Ba\right)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^5b^4-8A^3b^3-12A^2Bab^2-6AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{2Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - 8*
A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*l
og(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

## 3.81 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=51

$$-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

**Rubi** [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^3)) + (A*Log[x])/a^2 - (A*Log[a + b*x^3])/(3*a^
2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x}+\frac{-Ab+aB}{a(a+bx)^2}-\frac{Ab}{a^2(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Ab-aB}{3ab\left(a+bx^3\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 46, normalized size = 0.90

$$\frac{\frac{a(Ab-aB)}{b\left(a+bx^3\right)}-A\log\left(a+bx^3\right)+3A\log(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^2), x]
```

**fricas** [A] time = 0.70, size = 70, normalized size = 1.37

$$-\frac{Ba^2-Aab+\left(Ab^2x^3+Aab\right)\log\left(bx^3+a\right)-3\left(Ab^2x^3+Aab\right)\log(x)}{3\left(a^2b^2x^3+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 + A
*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)
```

**giac** [A] time = 0.21, size = 61, normalized size = 1.20

$$-\frac{A\log\left(\left|bx^3+a\right|\right)}{3\,a^2}+\frac{A\log\left(|x|\right)}{a^2}+\frac{Ab^2x^3-Ba^2+2\,Aab}{3\left(bx^3+a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^2
 + 2*A*a*b)/((b*x^3 + a)*a^2*b)
```

**maple** [A] time = 0.05, size = 53, normalized size = 1.04

$$\frac{A}{3\left(b\,x^3+a\right)a}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)}{3a^2}-\frac{B}{3\left(b\,x^3+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/3*A*ln(b*x^3+a)/a^2+1/3/a/(b*x^3+a)*A-1/3/b/(b*x^3+a)*B+A/a^2*ln(x)
```

**maxima** [A] time = 0.48, size = 51, normalized size = 1.00

$$-\frac{Ba-Ab}{3\left(ab^2x^3+a^2b\right)}-\frac{A\log\left(bx^3+a\right)}{3\,a^2}+\frac{A\log\left(x^3\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*log
(x^3)/a^2
```

**mupad [B]** time = 0.14, size = 47, normalized size = 0.92

$$\frac{A\ \ln(x)}{a^2}-\frac{A\ \ln\left(b\,x^3+a\right)}{3\,a^2}+\frac{A\,b-B\,a}{3\,a\,b\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^2),x)
```

```
[Out] (A*log(x))/a^2 - (A*log(a + b*x^3))/(3*a^2) + (A*b - B*a)/(3*a*b*(a + b*x^3
))
```

**sympy [A]** time = 1.16, size = 46, normalized size = 0.90

$$\frac{A\log{(x)}}{a^2}-\frac{A\log{\left(\frac{a}{b}+x^3\right)}}{3a^2}+\frac{Ab-Ba}{3a^2b+3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**3)/(3*a**2) + (A*b - B*a)/(3*a**2*b + 3*a*b*
*2*x**3)
```

## 3.82 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=195

$$-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}+\frac{aB}{3}$$

**Rubi** [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}-\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(4*A*b - a*B)/(3*a^2*b*x) + (A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b -
a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*
b^(2/3)) + ((4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(2/3)) - (
(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b
^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} - \frac{(4Ab-aB)\int \frac{x}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{7/3}\sqrt[3]{b}} - \frac{(4Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^{2}}}{9a^{7/3}\sqrt[3]{b}}$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2}}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+}}{18a^{7/3}b^{2/3}}$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{18a^{7/3}b^{2/3}}$$

$$= -\frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)} + \frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{9a^{7/3}b^{2/3}}$$

**Mathematica** [A] time = 0.14, size = 164, normalized size = 0.84

$$\frac{\frac{(aB-4Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}} + \frac{2\sqrt{3}\,(4Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{6\sqrt[3]{a}\,x^2(aB-Ab)}{a+bx^3} - \frac{18\sqrt[3]{a}\,A}{x}}{18a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3
]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(
4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^2), x]
```

**fricas [A]** time = 0.70, size = 570, normalized size = 2.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*sqrt(1/3)*((B*a^2*
b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*lo
g((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/
3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((B*a*b -
4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/
3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(
a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x), -1/18*(18
*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*sqrt(1/3)*((B*a^2*b^2 - 4*A*
a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((B*a*b - 4*A*b^2)*
x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x +
(a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2
/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x)]
```

**giac [A]** time = 0.18, size = 180, normalized size = 0.92

$$\frac{\sqrt{3}\,(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^2}-\frac{(Ba-4\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^2}-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}+\frac{Bax^3-4\,Abx^3-3\,Aa}{3\left(bx^4+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a^2) - 1/18*(B*a - 4*A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2) - 1/9*(B*a*(-a/b)^(1/3) - 4*A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/3*(B*a*x^3 - 4*A*b*x^3
 - 3*A*a)/((b*x^4 + a*x)*a^2)
```

**maple [A]** time = 0.05, size = 241, normalized size = 1.24

$$-\frac{Ab\,x^2}{3\left(b\,x^3+a\right)a^2}+\frac{B\,x^2}{3\left(b\,x^3+a\right)a}-\frac{4\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{A}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^2/(b*x^3+a)*A*b+1/3/a*x^2/(b*x^3+a)*B+4/9/a^2*A/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))-2/9/a^2*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/a
^2*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a*B/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+1/9/a*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-A/a^2/x
```

**maxima [A]** time = 1.08, size = 166, normalized size = 0.85

$$\frac{(Ba-4\,Ab)x^3-3\,Aa}{3\left(a^2bx^4+a^3x\right)}+\frac{\sqrt{3}\,(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 4*A*b)*x^3 - 3*A*a)/(a^2*b*x^4 + a^3*x) + 1/9*sqrt(3)*(B*a - 4*
A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)
) + 1/18*(B*a - 4*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(1/3)) - 1/9*(B*a - 4*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

**mupad [B]** time = 2.57, size = 156, normalized size = 0.80

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}}-\frac{\frac{A}{a}+\frac{x^3\,\left(4\,A\,b-B\,a\right)}{3\,a^2}}{b\,x^4+a\,x}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (A/a + (x^3*
(4*A*b - B*a))/(3*a^2))/(a*x + b*x^4) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*A
*b - B*a))/(9*a^(7/3)*b^(2/3))
```

**sympy [A]** time = 1.42, size = 122, normalized size = 0.63

$$\frac{-3Aa+x^3\left(-4Ab+Ba\right)}{3a^3x+3a^2bx^4}+\operatorname{RootSum}\left(729t^3a^7b^2-64A^3b^3+48A^2Bab^2-12AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a^5b}{16A^2b^2-8ABab+B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t*
*3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*a*
*3, Lambda(_t, _t*log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a**2
) + x)))
```

## 3.83 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=196

$$\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{2}{}$$

**Rubi** [A] time = 0.10, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{5Ab-2aB}{6a^2bx^2}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*b*x^2) + (A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((5*
A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(8/3)*b^(1/3)) - ((5*A*b - 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^
(1/3)) + ((5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx &= \frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{3ab}\\
&= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{3a^2}\\
&= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{8/3}}-\frac{(5Ab-2aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\ldots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+\ldots}}{9a^{8/3}}\ldots\\
&= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\int \frac{\ldots}{a^{2/3}-\sqrt[3]{a}\ldots}}{6a^{7/3}}\ldots\\
&= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\log\left(a^{2/3}\ldots\right.}{18a^{8/3}\ldots}\ldots\\
&= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}\ldots\right.}{9a^{8/3}\sqrt[3]{b}}\ldots
\end{aligned}
$$

**Mathematica [A]** time = 0.16, size = 163, normalized size = 0.83

$$
\frac{\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{6a^{2/3}x(aB-Ab)}{a+bx^3}-\frac{9a^{2/3}A}{x^2}+\frac{2(2aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}(5Ab-2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{18a^{8/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*Sqrt[3]
*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*
(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^2), x]
```

**fricas [A]** time = 0.82, size = 618, normalized size = 3.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*sqrt(1/3)*((2*B*a^2
*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (
-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) +
((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*
x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (
2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^
2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 - 6*
sqrt(1/3)*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*s
qrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3
)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5
*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*
log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^2*x^5 + a^5*b*x^2)]
```

**giac [A]** time = 0.17, size = 188, normalized size = 0.96

$$-\frac{(2Ba-5Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^3}+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba-5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^3b}+\frac{Bax-Abx}{3\left(bx^3+a\right)a^2}+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba-5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^3b}-\frac{A}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*B*a - 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/9*sqrt
(3)*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a^2
) + 1/18*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^3*b) - 1/2*A/(a^2*x^2)
```

**maple [A]** time = 0.05, size = 237, normalized size = 1.21

$$-\frac{Abx}{3\left(bx^3+a\right)a^2}+\frac{Bx}{3\left(bx^3+a\right)a}-\frac{5\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{5A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{5A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{2\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{A}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x/(b*x^3+a)*A*b+1/3/a*x/(b*x^3+a)*B-5/9/a^2*A/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))+5/18/a^2*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9/a^2*
A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a*B/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a*B/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+2/9/a*B/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-1/2*A/a^2/x^2
```

**maxima [A]** time = 1.23, size = 172, normalized size = 0.88

$$\frac{(2\,Ba-5\,Ab)x^3-3\,Aa}{6\left(a^2bx^5+a^3x^2\right)}+\frac{\sqrt{3}\,(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((2*B*a - 5*A*b)*x^3 - 3*A*a)/(a^2*b*x^5 + a^3*x^2) + 1/9*sqrt(3)*(2*B*
a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)
^(2/3)) - 1/18*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*
b*(a/b)^(2/3)) + 1/9*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3
))
```

**mupad [B]** time = 2.57, size = 159, normalized size = 0.81

$$-\frac{\frac{A}{2\,a}+\frac{x^3\,(5\,A\,b-2\,B\,a)}{6\,a^2}}{b\,x^5+a\,x^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*
A*b - 2*B*a))/(9*a^(8/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(5*A*b - 2*B*
a))/(9*a^(8/3)*b^(1/3)) - (A/(2*a) + (x^3*(5*A*b - 2*B*a))/(6*a^2))/(a*x^2
+ b*x^5) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(5*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3))
```

**sympy [A]** time = 1.93, size = 109, normalized size = 0.56

$$\frac{-3Aa+x^3\,(-5Ab+2Ba)}{6a^3x^2+6a^2bx^5}+\operatorname{RootSum}\left(729t^3a^8b+125A^3b^3-150A^2Bab^2+60AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^3}{-5Ab+2Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(72
9*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8*B
**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))
```

## 3.84 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=76

$$\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{A}{3a^2x^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -A/(3*a^2*x^3) - (A*b - a*B)/(3*a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x])/a
^3 + ((2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^2}+\frac{-2Ab+aB}{a^3x}-\frac{b(-Ab+aB)}{a^2(a+bx)^2}-\frac{b(-2Ab+aB)}{a^3(a+bx)}\right)dx,x,x^3\right)\\
&= -\frac{A}{3a^2x^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 64, normalized size = 0.84

$$\frac{\frac{a(aB-Ab)}{a+bx^3}+(2Ab-aB)\log\left(a+bx^3\right)+3\log(x)(aB-2Ab)-\frac{aA}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^2), x]
```

**fricas [A]** time = 0.49, size = 118, normalized size = 1.55

$$\frac{\left(Ba^2-2\,Aab\right)x^3-Aa^2-\left(\left(Bab-2\,Ab^2\right)x^6+\left(Ba^2-2\,Aab\right)x^3\right)\log\left(bx^3+a\right)+3\left(\left(Bab-2\,Ab^2\right)x^6+\left(Ba^2-2\,Aab\right)x^3\right)\log(x)}{3\left(a^3bx^6+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*
a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x^3
)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

**giac [A]** time = 0.19, size = 80, normalized size = 1.05

$$\frac{(Ba-2\,Ab)\log\left(|x|\right)}{a^3}+\frac{Bax^3-2\,Abx^3-Aa}{3\left(bx^6+ax^3\right)a^2}-\frac{\left(Bab-2\,Ab^2\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 + a
*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)
```

**maple [A]** time = 0.06, size = 87, normalized size = 1.14

$$-\frac{Ab}{3\left(b\,x^3+a\right)a^2}-\frac{2Ab\ln(x)}{a^3}+\frac{2Ab\ln\left(b\,x^3+a\right)}{3a^3}+\frac{B}{3\left(b\,x^3+a\right)a}+\frac{B\ln(x)}{a^2}-\frac{B\ln\left(b\,x^3+a\right)}{3a^2}-\frac{A}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 2/3*b/a^3*ln(b*x^3+a)*A-1/3/a^2*ln(b*x^3+a)*B-1/3*b/a^2/(b*x^3+a)*A+1/3/a/(
b*x^3+a)*B-1/3*A/a^2/x^3-2/a^3*ln(x)*A*b+B/a^2*ln(x)
```

**maxima [A]** time = 0.50, size = 76, normalized size = 1.00

$$\frac{(Ba-2\,Ab)x^3-Aa}{3\left(a^2bx^6+a^3x^3\right)}-\frac{(Ba-2\,Ab)\log\left(bx^3+a\right)}{3\,a^3}+\frac{(Ba-2\,Ab)\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*log
(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3
```

**mupad [B]** time = 2.43, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b-B\,a\right)}{3\,a^3}-\frac{\frac{A}{3\,a}+\frac{x^3\,\left(2\,A\,b-B\,a\right)}{3\,a^2}}{b\,x^6+a\,x^3}-\frac{\ln(x)\,\left(2\,A\,b-B\,a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (log(a + b*x^3)*(2*A*b - B*a))/(3*a^3) - (A/(3*a) + (x^3*(2*A*b - B*a))/(3*
a^2))/(a*x^3 + b*x^6) - (log(x)*(2*A*b - B*a))/a^3
```

**sympy [A]** time = 1.44, size = 70, normalized size = 0.92

$$\frac{-Aa+x^3\left(-2Ab+Ba\right)}{3a^3x^3+3a^2bx^6}+\frac{\left(-2Ab+Ba\right)\log\left(x\right)}{a^3}-\frac{\left(-2Ab+Ba\right)\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)
```

```
[Out] (-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

## 3.85 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=215

$$\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{10/3}}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{10/3}}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}$$

**Rubi [A]** time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{10/3}}-\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{10/3}}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] -(7*A*b - 4*a*B)/(12*a^2*b*x^4) + (7*A*b - 4*a*B)/(3*a^3*x) + (A*b - a*B)/(
3*a*b*x^4*(a + b*x^3)) - (b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (b^(1/3)*(7*A*b - 4*a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)) + (b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} + \frac{(7Ab-4aB)\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{(7Ab-4aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3a^2}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} + \frac{(b(7Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\left(b^{2/3}(7Ab-4aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{10/3}} + \cdots$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}} + \cdots$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}} + \cdots$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\sqrt[3]{b}(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} - \cdots$$

**Mathematica [A]** time = 0.24, size = 185, normalized size = 0.86

$$\frac{2\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{9a^{4/3}A}{x^4}-\frac{12\sqrt[3]{a}bx^2(aB-Ab)}{a+bx^3}-\frac{36\sqrt[3]{a}(aB-2Ab)}{x}+4\sqrt[3]{b}(4aB-7Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-4\sqrt{3}\sqrt[3]{b}(7Ab-4aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b)
 + a*B)*x^2)/(a + b*x^3) - 4*Sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) + b^
(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(36*a^(10/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^2), x]
```

**fricas [A]** time = 0.60, size = 259, normalized size = 1.20

$$-\frac{12\left(4Bab-7Ab^2\right)x^6+9\left(4Ba^2-7Aab\right)x^3+9Aa^2+4\sqrt{3}\left(\left(4Bab-7Ab^2\right)x^7+\left(4Ba^2-7Aab\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\left(\left(4Bab-7Ab^2\right)x^7+\left(4Ba^2-7Aab\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+4\left(\left(4Bab-7Ab^2\right)x^7+\left(4Ba^2-7Aab\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{36\left(a^3bx^7+a^4x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 + 4
*sqrt(3)*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 2*((4*B*a*b - 7*A*b^2)*x^
7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*
(-b/a)^(1/3)) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a
)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^3*b*x^7 + a^4*x^4)
```

**giac [A]** time = 0.22, size = 231, normalized size = 1.07

$$\frac{\left(4Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4}+\frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba-7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^4b}-\frac{Babx^2-Ab^2x^2}{3\left(bx^3+a\right)a^3}-\frac{\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba-7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^4b}-\frac{4Bax^3-8Abx^3+Aa}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(4*B*a*b*(-a/b)^(1/3) - 7*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/a^4 + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)*A
*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/3*(B*
a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^(2/3)*B*a - 7*(-a
*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/4*(4*
B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)
```

**maple [A]** time = 0.06, size = 257, normalized size = 1.20

$$\frac{Ab^2x^2}{3\left(bx^3+a\right)a^3}-\frac{Bbx^2}{3\left(bx^3+a\right)a^2}+\frac{7\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{7Ab\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{4\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{2Ab}{a^3x}-\frac{B}{a^2x}-\frac{A}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^2,x)
```

```
[Out] 1/3*b^2/a^3*x^2/(b*x^3+a)*A-1/3*b/a^2*x^2/(b*x^3+a)*B-7/9*b/a^3*A/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))+7/18*b/a^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
```

```
/3))+7/9*b/a^3*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)+4/9/a^2*B/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*B/(a/b)^(1/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))-4/9/a^2*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))-1/4/a^2*A/x^4+2*A/a^3*b/x-B/a^2/x
```

**maxima [A]** time = 1.42, size = 186, normalized size = 0.87

$$-\frac{4\left(4\,Bab-7\,Ab^2\right)x^6+3\left(4\,Ba^2-7\,Aab\right)x^3+3\,Aa^2}{12\left(a^3bx^7+a^4x^4\right)}-\frac{\sqrt{3}\,(4\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(4\,Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(4\,Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(4*B*a*b - 7*A*b^2)*x^6 + 3*(4*B*a^2 - 7*A*a*b)*x^3 + 3*A*a^2)/(a^
3*b*x^7 + a^4*x^4) - 1/9*sqrt(3)*(4*B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(1/3)) - 1/18*(4*B*a - 7*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)) + 1/9*(4*B*a - 7*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

**mupad [B]** time = 2.62, size = 209, normalized size = 0.97

$$\frac{\frac{x^3\,(7\,A\,b-4\,B\,a)}{4\,a^2}-\frac{A}{4\,a}+\frac{b\,x^6\,(7\,A\,b-4\,B\,a)}{3\,a^3}}{b\,x^7+a\,x^4}+\frac{(-b)^{1/3}\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}+\frac{(-b)^{1/3}\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}-\frac{(-b)^{1/3}\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(7*A*b - 4*B*a))/(4*a^2) - A/(4*a) + (b*x^6*(7*A*b - 4*B*a))/(3*a^3))
/(a*x^4 + b*x^7) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(7*A*b - 4*B
*a))/(9*a^(10/3)) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*
a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
 - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3
)*1i)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
```

**sympy [A]** time = 2.32, size = 153, normalized size = 0.71

$$\operatorname{RootSum}\left(729t^3a^{10}+343A^3b^4-588A^2Bab^3+336AB^2a^2b^2-64B^3a^3b,\left(t\mapsto t\log\left(\frac{81t^2a^7}{49A^2b^3-56ABab^2+16B^2a^2b}+x\right)\right)\right)+\frac{-3Aa^2+x^6\left(28Ab^2-16Bab\right)+x^3\left(21Aab-12Ba^2\right)}{12a^4x^4+12a^3bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a*
*2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 - 5
6*A*B*a*b**2 + 16*B**2*a**2*b) + x))) + (-3*A*a**2 + x**6*(28*A*b**2 - 16*B
*a*b) + x**3*(21*A*a*b - 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)
```

## 3.86 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=215

$$-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}\,a^{11/3}}$$

**Rubi** [A] time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}+\frac{8Ab-5aB}{6a^3x^2}-\frac{8Ab-5aB}{15a^2bx^5}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -(8*A*b - 5*a*B)/(15*a^2*b*x^5) + (8*A*b - 5*a*B)/(6*a^3*x^2) + (A*b - a*B)
/(3*a*b*x^5*(a + b*x^3)) - (b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)) + (b^(2/3)*(8*A*b - 5*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)) - (b^(2/3)*(8*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(8Ab-5aB)\int \frac{1}{x^6\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}-\frac{(8Ab-5aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{3a^2}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(b(8Ab-5aB))\int \frac{1}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(b(8Ab-5aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{11/3}}+\frac{(b(8A\ldots}{\ldots}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}}-\frac{(b\ldots}{\ldots}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}}-\frac{b^{2\ldots}}{\ldots}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}+\frac{b^{2\ldots}}{\ldots}$$

**Mathematica [A]** time = 0.18, size = 183, normalized size = 0.85

$$\frac{5b^{2/3}(5aB-8Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{30a^{2/3}bx(aB-Ab)}{a+bx^3}-\frac{45a^{2/3}(aB-2Ab)}{x^2}-\frac{18a^{5/3}A}{x^5}+10b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(A
*b) + a*B)*x)/(a + b*x^3) - 10*Sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(90*a^(11/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^6 \left(a + bx^3\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^2), x]
```

**fricas [A]** time = 0.53, size = 277, normalized size = 1.29

$$-\frac{15 \left(5 Bab - 8 Ab^2\right)x^6 + 9 \left(5 Ba^2 - 8 Aab\right)x^3 + 18 Aa^2 + 10 \sqrt{3} \left(\left(5 Bab - 8 Ab^2\right)x^8 + \left(5 Ba^2 - 8 Aab\right)x^5\right) \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2 \sqrt{3} ax \left(\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3} b}{3b}\right) - 5 \left(\left(5 Bab - 8 Ab^2\right)x^8 + \left(5 Ba^2 - 8 Aab\right)x^5\right) \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \log\left(b^2x^2 - abx \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} + a^2 \left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right) + 10 \left(\left(5 Bab - 8 Ab^2\right)x^8 + \left(5 Ba^2 - 8 Aab\right)x^5\right) \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \log\left(bx + a \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)}{90 \left(a^3bx^8 + a^4x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 +
10*sqrt(3)*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*((5*B*a*b
 - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*((5*B*a*b - 8*A*b^2)*x^8 +
(5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)))/(a^3
*b*x^8 + a^4*x^5)
```

**giac [A]** time = 0.20, size = 206, normalized size = 0.96

$$-\frac{\sqrt{3} \left(5 \left(-ab^2\right)^{\frac{1}{3}} Ba - 8 \left(-ab^2\right)^{\frac{1}{3}} Ab\right) \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a^4} + \frac{\left(5 Bab - 8 Ab^2\right) \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 a^4} - \frac{\left(5 \left(-ab^2\right)^{\frac{1}{3}} Ba - 8 \left(-ab^2\right)^{\frac{1}{3}} Ab\right) \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 a^4} - \frac{Babx - Ab^2x}{3 \left(bx^3 + a\right)a^3} - \frac{5 Bax^3 - 10 Abx^3 + 2 Aa}{10 a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/18*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b
^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4 - 1/3*(B*a*b*x
- A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(a^3*x
^5)
```

**maple [A]** time = 0.05, size = 252, normalized size = 1.17

$$\frac{A b^2 x}{3 \left(b x^3 + a\right) a^3} - \frac{Bbx}{3 \left(b x^3 + a\right) a^2} + \frac{8 \sqrt{3}\, Ab \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^3} + \frac{8Ab \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^3} - \frac{4Ab \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^3} - \frac{5 \sqrt{3}\, B \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^2} - \frac{5B \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^2} + \frac{5B \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 \left(\frac{a}{b}\right)^{\frac{2}{3}} a^2} + \frac{Ab}{a^3x^2} - \frac{B}{2a^2x^2} - \frac{A}{5a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^2,x)
```

```
[Out] 1/3*b^2/a^3*x/(b*x^3+a)*A-1/3*b/a^2*x/(b*x^3+a)*B+8/9*b/a^3*A/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))-4/9*b/a^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
8/9*b/a^3*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/9
/a^2*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))-5/9/a^2*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))-1/5/a^2*A/x^5+1/a^3/x^2*A*b-1/2/a^2/x^2*B
```

**maxima [A]** time = 1.32, size = 186, normalized size = 0.87

$$-\frac{5\left(5\,Bab-8\,Ab^{2}\right)x^{6}+3\left(5\,Ba^{2}-8\,Aab\right)x^{3}+6\,Aa^{2}}{30\left(a^{3}bx^{8}+a^{4}x^{5}\right)}-\frac{\sqrt{3}\left(5\,Ba-8\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(5\,Ba-8\,Ab\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(5\,Ba-8\,Ab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(5*(5*B*a*b - 8*A*b^2)*x^6 + 3*(5*B*a^2 - 8*A*a*b)*x^3 + 6*A*a^2)/(a^
3*b*x^8 + a^4*x^5) - 1/9*sqrt(3)*(5*B*a - 8*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(2/3)) + 1/18*(5*B*a - 8*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) - 1/9*(5*B*a - 8*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

**mupad [B]** time = 2.57, size = 176, normalized size = 0.82

$$\frac{\frac{x^{3}\,\left(8\,A\,b-5\,B\,a\right)}{10\,a^{2}}-\frac{A}{5\,a}+\frac{b\,x^{6}\,\left(8\,A\,b-5\,B\,a\right)}{6\,a^{3}}}{b\,x^{8}+a\,x^{5}}+\frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(8\,A\,b-5\,B\,a\right)}{9\,a^{11/3}}-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(8\,A\,b-5\,B\,a\right)}{9\,a^{11/3}}+\frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(8\,A\,b-5\,B\,a\right)}{9\,a^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(8*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (b*x^6*(8*A*b - 5*B*a))/(6*a^3)
)/(a*x^5 + b*x^8) + (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(8*A*b - 5*B*a))/(9*a
^(11/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/
2)*1i)/2 + 1/2)*(8*A*b - 5*B*a))/(9*a^(11/3)) + (b^(2/3)*log(3^(1/2)*a^(1/3
)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*A*b - 5*B*a))/(9*a^
(11/3))
```

**sympy [A]** time = 1.84, size = 138, normalized size = 0.64

$$\operatorname{RootSum}\left(729t^{3}a^{11}-512A^{3}b^{5}+960A^{2}Bab^{4}-600AB^{2}a^{2}b^{3}+125B^{3}a^{3}b^{2},\left(t\mapsto t\log\left(-\frac{9ta^{4}}{-8Ab^{2}+5Bab}+x\right)\right)\right)+\frac{-6Aa^{2}+x^{6}\left(40Ab^{2}-25Bab\right)+x^{3}\left(24Aab-15Ba^{2}\right)}{30a^{4}x^{5}+30a^{3}bx^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a*
*2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5*B
*a*b) + x))) + (-6*A*a**2 + x**6*(40*A*b**2 - 25*B*a*b) + x**3*(24*A*a*b -
15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)
```

## 3.87 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=97

$$-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{6a^2x^6}$$

**Rubi [A]** time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(A*b - a*B))/(3*a^3*(a + b*
x^3)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^3])
/(3*a^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)}\right)\right.\\ &= -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^6}+\frac{2ab(aB-Ab)}{a+bx^3}+\frac{2a(aB-2Ab)}{x^3}+2b(3Ab-2aB)\log\left(a+bx^3\right)-6b\log(x)(3Ab-2aB)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a +
b*x^3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])/a
^4
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^2), x]
```

**fricas [A]** time = 0.46, size = 154, normalized size = 1.59

$$-\frac{2\left(2\,Ba^2b-3\,Aab^2\right)x^6+Aa^3+\left(2\,Ba^3-3\,Aa^2b\right)x^3-2\left(\left(2\,Bab^2-3\,Ab^3\right)x^9+\left(2\,Ba^2b-3\,Aab^2\right)x^6\right)\log\left(bx^3+a\right)+6\left(\left(2\,Bab^2-3\,Ab^3\right)x^9+\left(2\,Ba^2b-3\,Aab^2\right)x^6\right)\log(x)}{6\left(a^4bx^9+a^5x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a) +
 6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(x))/(a^4*b
*x^9 + a^5*x^6)
```

**giac [A]** time = 0.17, size = 149, normalized size = 1.54

$$-\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(|x|\right)}{a^4}+\frac{\left(2\,Bab^2-3\,Ab^3\right)\log\left(|bx^3+a|\right)}{3\,a^4b}-\frac{2\,Bab^2x^3-3\,Ab^3x^3+3\,Ba^2b-4\,Aab^2}{3\left(bx^3+a\right)a^4}+\frac{6\,Babx^6-9\,Ab^2x^6-2\,Ba^2x^3+4\,Aabx^3-Aa^2}{6\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*log(abs(b*
x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*b^
2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*A*a
*b*x^3 - A*a^2)/(a^4*x^6)
```

**maple [A]** time = 0.08, size = 116, normalized size = 1.20

$$\frac{A\,b^2}{3\left(b\,x^3+a\right)a^3}+\frac{3A\,b^2\ln(x)}{a^4}-\frac{A\,b^2\ln\left(b\,x^3+a\right)}{a^4}-\frac{Bb}{3\left(b\,x^3+a\right)a^2}-\frac{2Bb\ln(x)}{a^3}+\frac{2Bb\ln\left(b\,x^3+a\right)}{3a^3}+\frac{2Ab}{3a^3x^3}-\frac{B}{3a^2x^3}-\frac{A}{6a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^2,x)
```

```
[Out] -1/a^4*b^2*ln(b*x^3+a)*A+2/3/a^3*b*ln(b*x^3+a)*B+1/3/a^3*b^2/(b*x^3+a)*A-1/
3/a^2*b/(b*x^3+a)*B-1/6*A/a^2/x^6+2/3/a^3/x^3*A*b-1/3/a^2/x^3*B+3*b^2/a^4*l
n(x)*A-2*b/a^3*ln(x)*B
```

**maxima [A]** time = 0.65, size = 106, normalized size = 1.09

$$-\frac{2\left(2\,Bab-3\,Ab^2\right)x^6+\left(2\,Ba^2-3\,Aab\right)x^3+Aa^2}{6\left(a^3bx^9+a^4x^6\right)}+\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(bx^3+a\right)}{3\,a^4}-\frac{\left(2\,Bab-3\,Ab^2\right)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*x
^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b -
 3*A*b^2)*log(x^3)/a^4
```

**mupad** [B] time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^3\,(3\,A\,b-2\,B\,a)}{6\,a^2}-\frac{A}{6\,a}+\frac{b\,x^6\,(3\,A\,b-2\,B\,a)}{3\,a^3}}{b\,x^9+a\,x^6}-\frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{3\,a^4}+\frac{\ln(x)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(3*A*b - 2*B*a))/(6*a^2) - A/(6*a) + (b*x^6*(3*A*b - 2*B*a))/(3*a^3))
/(a*x^6 + b*x^9) - (log(a + b*x^3)*(3*A*b^2 - 2*B*a*b))/(3*a^4) + (log(x)*(
3*A*b^2 - 2*B*a*b))/a^4
```

**sympy** [A] time = 2.40, size = 100, normalized size = 1.03

$$\frac{-Aa^2+x^6\left(6Ab^2-4Bab\right)+x^3\left(3Aab-2Ba^2\right)}{6a^4x^6+6a^3bx^9}-\frac{b\,(-3Ab+2Ba)\log{(x)}}{a^4}+\frac{b\,(-3Ab+2Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**2,x)
```

```
[Out] (-A*a**2 + x**6*(6*A*b**2 - 4*B*a*b) + x**3*(3*A*a*b - 2*B*a**2))/(6*a**4*x
**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a)*
log(a/b + x**3)/(3*a**4)
```

## 3.88 $\int \frac{x^{11}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=107

$$\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^6}{6b^3}$$

**Rubi [A]** time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}+\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}+\frac{x^3(Ab-3aB)}{3b^4}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^6)/(6*b^3) + (a^3*(A*b - a*B))/(6*b^5*(a
 + b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(3*b^5*(a + b*x^3)) - (a*(A*b - 2*a*B)
*Log[a + b*x^3])/b^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^3}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2}+\frac{3a(-Ab+2aB)}{b^4(a+bx)}\right)\right.\\&=\frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^6}{6b^3}+\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 94, normalized size = 0.88

$$\frac{\frac{a^3(Ab-aB)}{\left(a+bx^3\right)^2}+\frac{2a^2(4aB-3Ab)}{a+bx^3}+2bx^3(Ab-3aB)+6a(2aB-Ab)\log\left(a+bx^3\right)+b^2Bx^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(6*
b^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^11*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [A]** time = 0.64, size = 179, normalized size = 1.67

$$\frac{Bb^4x^{12}-2\left(2\,Bab^3-Ab^4\right)x^9-\left(11\,Ba^2b^2-4\,Aab^3\right)x^6+7\,Ba^4-5\,Aa^3b+2\left(Ba^3b-2\,Aa^2b^2\right)x^3+6\left(\left(2\,Ba^2b^2-Aab^3\right)x^6+2\,Ba^4-Aa^3b+2\left(2\,Ba^3b-Aa^2b^2\right)x^3\right)\log\left(bx^3+a\right)}{6\left(b^7x^6+2\,ab^6x^3+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^4*x^12 - 2*(2*B*a*b^3 - A*b^4)*x^9 - (11*B*a^2*b^2 - 4*A*a*b^3)*x^
6 + 7*B*a^4 - 5*A*a^3*b + 2*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 6*((2*B*a^2*b^2 -
 A*a*b^3)*x^6 + 2*B*a^4 - A*a^3*b + 2*(2*B*a^3*b - A*a^2*b^2)*x^3)*log(b*x^
3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

**giac [A]** time = 0.18, size = 131, normalized size = 1.22

$$\frac{\left(2\,Ba^2-Aab\right)\log\left(\left|bx^3+a\right|\right)}{b^5}+\frac{Bb^3x^6-6\,Bab^2x^3+2\,Ab^3x^3}{6\,b^6}-\frac{18\,Ba^2b^2x^6-9\,Aab^3x^6+28\,Ba^3bx^3-12\,Aa^2b^2x^3+11\,Ba^4-4\,Aa^3b}{6\left(bx^3+a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (2*B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^5 + 1/6*(B*b^3*x^6 - 6*B*a*b^2*x^3
+ 2*A*b^3*x^3)/b^6 - 1/6*(18*B*a^2*b^2*x^6 - 9*A*a*b^3*x^6 + 28*B*a^3*b*x^3
 - 12*A*a^2*b^2*x^3 + 11*B*a^4 - 4*A*a^3*b)/((b*x^3 + a)^2*b^5)
```

**maple [A]** time = 0.05, size = 134, normalized size = 1.25

$$\frac{B\,x^6}{6b^3}+\frac{A\,x^3}{3b^3}-\frac{Ba\,x^3}{b^4}+\frac{A\,a^3}{6\left(b\,x^3+a\right)^2b^4}-\frac{B\,a^4}{6\left(b\,x^3+a\right)^2b^5}-\frac{A\,a^2}{\left(b\,x^3+a\right)b^4}-\frac{Aa\ln\left(b\,x^3+a\right)}{b^4}+\frac{4B\,a^3}{3\left(b\,x^3+a\right)b^5}+\frac{2B\,a^2\ln\left(b\,x^3+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*B*x^6/b^3+1/3/b^3*A*x^3-1/b^4*B*a*x^3+1/6*a^3/b^4/(b*x^3+a)^2*A-1/6*a^4
/b^5/(b*x^3+a)^2*B-a/b^4*ln(b*x^3+a)*A+2*a^2/b^5*ln(b*x^3+a)*B-a^2/b^4/(b*x
^3+a)*A+4/3*a^3/b^5/(b*x^3+a)*B
```

**maxima [A]** time = 0.54, size = 115, normalized size = 1.07

$$\frac{7\,Ba^4-5\,Aa^3b+2\left(4\,Ba^3b-3\,Aa^2b^2\right)x^3}{6\left(b^7x^6+2\,ab^6x^3+a^2b^5\right)}+\frac{Bbx^6-2\,(3\,Ba-Ab)x^3}{6\,b^4}+\frac{\left(2\,Ba^2-Aab\right)\log\left(bx^3+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^3)/(b^7*x^6 + 2*a*
b^6*x^3 + a^2*b^5) + 1/6*(B*b*x^6 - 2*(3*B*a - A*b)*x^3)/b^4 + (2*B*a^2 - A
*a*b)*log(b*x^3 + a)/b^5
```

**mupad [B]** time = 0.10, size = 117, normalized size = 1.09

$$\frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{6\,b}+x^3\,\left(\frac{4\,B\,a^3}{3}-A\,a^2\,b\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^3\,\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(2\,B\,a^2-A\,a\,b\right)}{b^5}+\frac{B\,x^6}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((7*B*a^4 - 5*A*a^3*b)/(6*b) + x^3*((4*B*a^3)/3 - A*a^2*b))/(a^2*b^4 + b^6*
x^6 + 2*a*b^5*x^3) + x^3*(A/(3*b^3) - (B*a)/b^4) + (log(a + b*x^3)*(2*B*a^2
 - A*a*b))/b^5 + (B*x^6)/(6*b^3)
```

**sympy [A]** time = 4.94, size = 112, normalized size = 1.05

$$\frac{Bx^6}{6b^3}+\frac{a\left(-Ab+2Ba\right)\log\left(a+bx^3\right)}{b^5}+x^3\left(\frac{A}{3b^3}-\frac{Ba}{b^4}\right)+\frac{-5Aa^3b+7Ba^4+x^3\left(-6Aa^2b^2+8Ba^3b\right)}{6a^2b^5+12ab^6x^3+6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*log(a + b*x**3)/b**5 + x**3*(A/(3*b**3)
- B*a/b**4) + (-5*A*a**3*b + 7*B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b))
/(6*a**2*b**5 + 12*a*b**6*x**3 + 6*b**7*x**6)
```

## 3.89 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=88

$$-\frac{a^2(Ab-aB)}{6b^4\left(a+bx^3\right)^2}+\frac{a(2Ab-3aB)}{3b^4\left(a+bx^3\right)}+\frac{(Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{Bx^3}{3b^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{6b^4\left(a+bx^3\right)^2}+\frac{a(2Ab-3aB)}{3b^4\left(a+bx^3\right)}+\frac{(Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) - (a^2*(A*b - a*B))/(6*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a
*B))/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b^3}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^3}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^2}+\frac{Ab-3aB}{b^3(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)}{6b^4\left(a+bx^3\right)^2}+\frac{a(2Ab-3aB)}{3b^4\left(a+bx^3\right)}+\frac{(Ab-3aB)\log\left(a+bx^3\right)}{3b^4}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 92, normalized size = 1.05

$$\frac{2aAb-3a^2B}{3b^4\left(a+bx^3\right)}+\frac{a^3B-a^2Ab}{6b^4\left(a+bx^3\right)^2}+\frac{(Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) + (-(a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b - 3
*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [A]** time = 0.63, size = 142, normalized size = 1.61

$$\frac{2 Bb^3x^9 + 4 Bab^2x^6 - 5 Ba^3 + 3 Aa^2b - 4 (Ba^2b - Aab^2)x^3 - 2 ((3 Bab^2 - Ab^3)x^6 + 3 Ba^3 - Aa^2b + 2 (3 Ba^2b - Aab^2)x^3) \log (bx^3 + a)}{6 (b^6x^6 + 2 ab^5x^3 + a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b
^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A
*a*b^2)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

**giac [A]** time = 0.18, size = 93, normalized size = 1.06

$$\frac{Bx^3}{3 b^3} - \frac{(3 Ba - Ab) \log (|bx^3 + a|)}{3 b^4} + \frac{9 Bab^2x^6 - 3 Ab^3x^6 + 12 Ba^2bx^3 - 2 Aab^2x^3 + 4 Ba^3}{6 (bx^3 + a)^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*x^3/b^3 - 1/3*(3*B*a - A*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(9*B*a*b^2*
x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a)^
2*b^4)
```

**maple [A]** time = 0.05, size = 110, normalized size = 1.25

$$\frac{B x^3}{3b^3} - \frac{A a^2}{6 (b x^3 + a)^2 b^3} + \frac{B a^3}{6 (b x^3 + a)^2 b^4} + \frac{2Aa}{3 (b x^3 + a) b^3} + \frac{A \ln (b x^3 + a)}{3b^3} - \frac{B a^2}{(b x^3 + a) b^4} - \frac{Ba \ln (b x^3 + a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*B*x^3/b^3-1/6/b^3*a^2/(b*x^3+a)^2*A+1/6/b^4*a^3/(b*x^3+a)^2*B+1/3/b^3*l
n(b*x^3+a)*A-1/b^4*ln(b*x^3+a)*B*a+2/3/b^3*a/(b*x^3+a)*A-1/b^4*a^2/(b*x^3+a
)*B
```

**maxima [A]** time = 0.52, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3 b^3} - \frac{5 Ba^3 - 3 Aa^2b + 2 (3 Ba^2b - 2 Aab^2)x^3}{6 (b^6x^6 + 2 ab^5x^3 + a^2b^4)} - \frac{(3 Ba - Ab) \log (bx^3 + a)}{3 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^3 - 1/6*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/(
b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*B*a - A*b)*log(b*x^3 + a)/b^4
```

**mupad [B]** time = 2.40, size = 94, normalized size = 1.07

$$\frac{B\,x^3}{3\,b^3}-\frac{x^3\,\left(B\,a^2-\frac{2\,A\,a\,b}{3}\right)+\frac{5\,B\,a^3-3\,A\,a^2\,b}{6\,b}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-3\,B\,a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x^3)/(3*b^3) - (x^3*(B*a^2 - (2*A*a*b)/3) + (5*B*a^3 - 3*A*a^2*b)/(6*b))
/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (log(a + b*x^3)*(A*b - 3*B*a))/(3*b^4)
```

**sympy [A]** time = 3.94, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3b^3}+\frac{3Aa^2b-5Ba^3+x^3\left(4Aab^2-6Ba^2b\right)}{6a^2b^4+12ab^5x^3+6b^6x^6}-\frac{(-Ab+3Ba)\log\left(a+bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) + (3*A*a**2*b - 5*B*a**3 + x**3*(4*A*a*b**2 - 6*B*a**2*b))/
(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*log(a + b*x**
3)/(3*b**4)
```

## 3.90 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=66

$$-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}+\frac{B\log\left(a+bx^3\right)}{3b^3}$$

**Rubi [A]** time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}+\frac{B\log\left(a+bx^3\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (a*(A*b - a*B))/(6*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(3*b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3}+\frac{Ab-2aB}{b^2(a+bx)^2}+\frac{B}{b^2(a+bx)}\right)dx,x,x^3\right)\\
&= \frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 64, normalized size = 0.97

$$\frac{3a^2B-ab\left(A-4Bx^3\right)+2B\left(a+bx^3\right)^2\log\left(a+bx^3\right)-2Ab^2x^3}{6b^3\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x^
3])/(6*b^3*(a + b*x^3)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (A + Bx^3)}{(a + bx^3)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [A]** time = 0.75, size = 89, normalized size = 1.35

$$\frac{2\left(2\,Bab - Ab^2\right)x^3 + 3\,Ba^2 - Aab + 2\left(Bb^2x^6 + 2\,Babx^3 + Ba^2\right)\log\left(bx^3 + a\right)}{6\left(b^5x^6 + 2\,ab^4x^3 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^3
 + B*a^2)*log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

**giac [A]** time = 0.19, size = 61, normalized size = 0.92

$$\frac{B\log\left(\left|bx^3 + a\right|\right)}{3\,b^3} + \frac{2\,(2\,Ba - Ab)x^3 + \frac{3\,Ba^2 - Aab}{b}}{6\left(bx^3 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*log(abs(b*x^3 + a))/b^3 + 1/6*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*b
)/b)/((b*x^3 + a)^2*b^2)
```

**maple [A]** time = 0.05, size = 81, normalized size = 1.23

$$\frac{Aa}{6\left(b\,x^3 + a\right)^2 b^2} - \frac{B\,a^2}{6\left(b\,x^3 + a\right)^2 b^3} - \frac{A}{3\left(b\,x^3 + a\right)b^2} + \frac{2Ba}{3\left(b\,x^3 + a\right)b^3} + \frac{B\ln\left(b\,x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*a/b^2/(b*x^3+a)^2*A-1/6*a^2/b^3/(b*x^3+a)^2*B+1/3*B*ln(b*x^3+a)/b^3-1/3
/b^2/(b*x^3+a)*A+2/3/b^3/(b*x^3+a)*B*a
```

**maxima [A]** time = 0.47, size = 72, normalized size = 1.09

$$\frac{2\left(2\,Bab - Ab^2\right)x^3 + 3\,Ba^2 - Aab}{6\left(b^5x^6 + 2\,ab^4x^3 + a^2b^3\right)} + \frac{B\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a^
2*b^3) + 1/3*B*log(b*x^3 + a)/b^3
```

**mupad** [B] time = 2.38, size = 70, normalized size = 1.06

$$\frac{\frac{3\,B\,a^2-A\,a\,b}{6\,b^3}-\frac{x^3\,(A\,b-2\,B\,a)}{3\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((3*B*a^2 - A*a*b)/(6*b^3) - (x^3*(A*b - 2*B*a))/(3*b^2))/(a^2 + b^2*x^6 +
2*a*b*x^3) + (B*log(a + b*x^3))/(3*b^3)
```

**sympy** [A] time = 3.69, size = 70, normalized size = 1.06

$$\frac{B\log\left(a+bx^3\right)}{3b^3}+\frac{-Aab+3Ba^2+x^3\left(-2Ab^2+4Bab\right)}{6a^2b^3+12ab^4x^3+6b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**3) + (-A*a*b + 3*B*a**2 + x**3*(-2*A*b**2 + 4*B*a*b
))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
```

## 3.91 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx$

**Optimal**. Leaf size=32

$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

**Rubi [A]** time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 37}

$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -(A + B*x^3)^2/(6*(A*b - a*B)*(a + b*x^3)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^3} dx, x, x^3\right)\\ &= -\frac{(A+Bx^3)^2}{6(Ab-aB)(a+bx^3)^2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 30, normalized size = 0.94

$$-\frac{B(a+2bx^3)+Ab}{6b^2(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(A*b + B*(a + 2*b*x^3))/(b^2*(a + b*x^3)^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(A + Bx^3\right)}{\left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
[Out] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [A]** time = 0.69, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

**giac [A]** time = 0.18, size = 28, normalized size = 0.88

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\left(bx^3 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)
```

**maple [A]** time = 0.05, size = 39, normalized size = 1.22

$$-\frac{B}{3\left(b\,x^3 + a\right)b^2} - \frac{Ab - Ba}{6\left(b\,x^3 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^3,x)
[Out] -1/6*(A*b-B*a)/b^2/(b*x^3+a)^2-1/3*B/b^2/(b*x^3+a)
```

**maxima [A]** time = 0.46, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

**mupad [B]** time = 2.33, size = 44, normalized size = 1.38

$$-\frac{\frac{A\,b+B\,a}{6\,b^2} + \frac{B\,x^3}{3\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] -((A*b + B*a)/(6*b^2) + (B*x^3)/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

**sympy** [**A**] time = 1.82, size = 42, normalized size = 1.31

$$\frac{-Ab - Ba - 2Bbx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (-A*b - B*a - 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

## 3.92 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=68

$$-\frac{A\log\left(a+bx^3\right)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{A}{3a^2\left(a+bx^3\right)}+\frac{Ab-aB}{6ab\left(a+bx^3\right)^2}$$

**Rubi** [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{A}{3a^2\left(a+bx^3\right)}-\frac{A\log\left(a+bx^3\right)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{Ab-aB}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] (A*b - a*B)/(6*a*b*(a + b*x^3)^2) + A/(3*a^2*(a + b*x^3)) + (A*Log[x])/a^3
- (A*Log[a + b*x^3])/(3*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x}+\frac{-Ab+aB}{a(a+bx)^3}-\frac{Ab}{a^2(a+bx)^2}-\frac{Ab}{a^3(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Ab-aB}{6ab\left(a+bx^3\right)^2}+\frac{A}{3a^2\left(a+bx^3\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 59, normalized size = 0.87

$$\frac{\frac{a\left(-a^2B+3aAb+2Ab^2x^3\right)}{b\left(a+bx^3\right)^2}-2A\log\left(a+bx^3\right)+6A\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*L
og[a + b*x^3])/(6*a^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^3), x]
```

**fricas [A]** time = 0.75, size = 119, normalized size = 1.75

$$\frac{2\,Aab^2x^3-Ba^3+3\,Aa^2b-2\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\log\left(bx^3+a\right)+6\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\log(x)}{6\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a
^2*b)*log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*log(x))/(a^3
*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

**giac [A]** time = 0.20, size = 74, normalized size = 1.09

$$-\frac{A\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{A\log\left(|x|\right)}{a^3}+\frac{3\,Ab^3x^6+8\,Aab^2x^3-Ba^3+6\,Aa^2b}{6\left(bx^3+a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^3 + A*log(abs(x))/a^3 + 1/6*(3*A*b^3*x^6 + 8*A
*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)
```

**maple [A]** time = 0.06, size = 68, normalized size = 1.00

$$\frac{A}{6\left(b\,x^3+a\right)^2a}-\frac{B}{6\left(b\,x^3+a\right)^2b}+\frac{A}{3\left(b\,x^3+a\right)a^2}+\frac{A\ln(x)}{a^3}-\frac{A\ln\left(b\,x^3+a\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^3,x)
```

```
[Out] 1/6/a/(b*x^3+a)^2*A-1/6/b/(b*x^3+a)^2*B-1/3*A*ln(b*x^3+a)/a^3+1/3*A/a^2/(b*
x^3+a)+A/a^3*ln(x)
```

**maxima [A]** time = 0.45, size = 77, normalized size = 1.13

$$\frac{2\,Ab^2x^3-Ba^2+3\,Aab}{6\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}-\frac{A\log\left(bx^3+a\right)}{3\,a^3}+\frac{A\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) -
 1/3*A*log(b*x^3 + a)/a^3 + 1/3*A*log(x^3)/a^3
```

**mupad [B]** time = 0.16, size = 71, normalized size = 1.04

$$\frac{\frac{3\,A\,b-B\,a}{6\,a\,b}+\frac{A\,b\,x^3}{3\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a^3}+\frac{A\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*A*b - B*a)/(6*a*b) + (A*b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) - (
A*log(a + b*x^3))/(3*a^3) + (A*log(x))/a^3
```

**sympy [A]** time = 1.48, size = 75, normalized size = 1.10

$$\frac{A\log{(x)}}{a^3}-\frac{A\log{\left(\frac{a}{b}+x^3\right)}}{3a^3}+\frac{3Aab+2Ab^2x^3-Ba^2}{6a^4b+12a^3b^2x^3+6a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**3)/(3*a**3) + (3*A*a*b + 2*A*b**2*x**3 - B*a
**2)/(6*a**4*b + 12*a**3*b**2*x**3 + 6*a**2*b**3*x**6)
```

## 3.93 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=101

$$\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}$$

**Rubi [A]** time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -A/(3*a^3*x^3) - (A*b - a*B)/(6*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(3*a^3*(
a + b*x^3)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3])/(
3*a^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^2}+\frac{-3Ab+aB}{a^4x}-\frac{b(-Ab+aB)}{a^2(a+bx)^3}-\frac{b(-2Ab+aB)}{a^3(a+bx)^2}-\frac{b(-3Ab+aB)}{a^4(a+bx)}\right)dx,x\right.\\ &= -\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 87, normalized size = 0.86

$$\frac{\frac{a^2(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{2a(aB-2Ab)}{a+bx^3}+2(3Ab-aB)\log\left(a+bx^3\right)+6\log(x)(aB-3Ab)-\frac{2aA}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6*a
^4)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{x^4 \left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^3), x]
```

**fricas [B]** time = 0.62, size = 197, normalized size = 1.95

$$\frac{2\left(Ba^2b - 3\,Aab^2\right)x^6 - 2\,Aa^3 + 3\left(Ba^3 - 3\,Aa^2b\right)x^3 - 2\left(\left(Bab^2 - 3\,Ab^3\right)x^9 + 2\left(Ba^2b - 3\,Aab^2\right)x^6 + \left(Ba^3 - 3\,Aa^2b\right)x^3\right)\log\left(bx^3 + a\right) + 6\left(\left(Bab^2 - 3\,Ab^3\right)x^9 + 2\left(Ba^2b - 3\,Aab^2\right)x^6 + \left(Ba^3 - 3\,Aa^2b\right)x^3\right)\log(x)}{6\left(a^4b^2x^9 + 2\,a^5bx^6 + a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2*
((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2*b
)*x^3)*log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2
)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x
^3)
```

**giac [A]** time = 0.20, size = 136, normalized size = 1.35

$$\frac{(Ba - 3\,Ab)\log\left(|x|\right)}{a^4} - \frac{\left(Bab - 3\,Ab^2\right)\log\left(|bx^3 + a|\right)}{3\,a^4b} + \frac{3\,Bab^2x^6 - 9\,Ab^3x^6 + 8\,Ba^2bx^3 - 22\,Aab^2x^3 + 6\,Ba^3 - 14\,Aa^2b}{6\left(bx^3 + a\right)^2a^4} - \frac{Bax^3 - 3\,Abx^3 + Aa}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (B*a - 3*A*b)*log(abs(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*log(abs(b*x^3 + a))/(
a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^3
+ 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 + A*
a)/(a^4*x^3)
```

**maple [A]** time = 0.06, size = 117, normalized size = 1.16

$$-\frac{Ab}{6\left(bx^3 + a\right)^2a^2} + \frac{B}{6\left(bx^3 + a\right)^2a} - \frac{2Ab}{3\left(bx^3 + a\right)a^3} - \frac{3Ab\ln(x)}{a^4} + \frac{Ab\ln\left(bx^3 + a\right)}{a^4} + \frac{B}{3\left(bx^3 + a\right)a^2} + \frac{B\ln(x)}{a^3} - \frac{B\ln\left(bx^3 + a\right)}{3a^3} - \frac{A}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -1/6/a^2*b/(b*x^3+a)^2*A+1/6/a/(b*x^3+a)^2*B+1/a^4*b*ln(b*x^3+a)*A-1/3/a^3*
ln(b*x^3+a)*B-2/3/a^3*b/(b*x^3+a)*A+1/3/a^2/(b*x^3+a)*B-1/3*A/a^3/x^3-3*A/a
^4*b*ln(x)+B/a^3*ln(x)
```

**maxima [A]** time = 0.50, size = 109, normalized size = 1.08

$$\frac{2\left(Bab - 3\,Ab^2\right)x^6 + 3\left(Ba^2 - 3\,Aab\right)x^3 - 2\,Aa^2}{6\left(a^3b^2x^9 + 2\,a^4bx^6 + a^5x^3\right)} - \frac{(Ba - 3\,Ab)\log\left(bx^3 + a\right)}{3\,a^4} + \frac{(Ba - 3\,Ab)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2*
x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*log(b*x^3 + a)/a^4 + 1/3*(
B*a - 3*A*b)*log(x^3)/a^4
```

**mupad [B]** time = 2.46, size = 107, normalized size = 1.06

$$\frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b-B\,a\right)}{3\,a^4}-\frac{\frac{A}{3\,a}+\frac{x^3\,\left(3\,A\,b-B\,a\right)}{2\,a^2}+\frac{b\,x^6\,\left(3\,A\,b-B\,a\right)}{3\,a^3}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}-\frac{\ln(x)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (log(a + b*x^3)*(3*A*b - B*a))/(3*a^4) - (A/(3*a) + (x^3*(3*A*b - B*a))/(2*
a^2) + (b*x^6*(3*A*b - B*a))/(3*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (lo
g(x)*(3*A*b - B*a))/a^4
```

**sympy [A]** time = 3.27, size = 107, normalized size = 1.06

$$\frac{-2Aa^2+x^6\left(-6Ab^2+2Bab\right)+x^3\left(-9Aab+3Ba^2\right)}{6a^5x^3+12a^4bx^6+6a^3b^2x^9}+\frac{(-3Ab+Ba)\log(x)}{a^4}-\frac{(-3Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a*
*5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4 -
 (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)
```

## 3.94 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=122

$$-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}-\frac{A}{6a^3x^6}$$

**Rubi** [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(3*a^4*x^3) + (b*(A*b - a*B))/(6*a^3*(a + b*
x^3)^2) + (b*(3*A*b - 2*a*B))/(3*a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Log[
x])/a^5 - (b*(2*A*b - a*B)*Log[a + b*x^3])/a^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^3}+\frac{-3Ab+aB}{a^4x^2}-\frac{3b(-2Ab+aB)}{a^5x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^3}+\frac{b^2(-3Ab+2aB}{a^4(a+bx)^2}\right.\right.\\ &= -\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Al}{}\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 108, normalized size = 0.89

$$\frac{\frac{a^2b(Ab-aB)}{\left(a+bx^3\right)^2}-\frac{a^2A}{x^6}+\frac{2ab(3Ab-2aB)}{a+bx^3}-\frac{2a(aB-3Ab)}{x^3}+6b(aB-2Ab)\log\left(a+bx^3\right)+18b\log(x)(2Ab-aB)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*A)/x^6) - (2*a*(-3*A*b + a*B))/x^3 + (a^2*b*(A*b - a*B))/(a + b*x^3
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^3) + 18*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^3])/(6*a^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^3), x]
```

**fricas [B]** time = 0.64, size = 229, normalized size = 1.88

$$-\frac{6\left(Ba^2b^2-2\,Aab^3\right)x^9+9\left(Ba^3b-2\,Aa^2b^2\right)x^6+Aa^4+2\left(Ba^4-2\,Aa^3b\right)x^3-6\left(\left(Bab^3-2\,Ab^4\right)x^{12}+2\left(Ba^2b^2-2\,Aab^3\right)x^9+\left(Ba^3b-2\,Aa^2b^2\right)x^6\right)\log\left(bx^3+a\right)+18\left(\left(Bab^3-2\,Ab^4\right)x^{12}+2\left(Ba^2b^2-2\,Aab^3\right)x^9+\left(Ba^3b-2\,Aa^2b^2\right)x^6\right)\log(x)}{6\left(a^5b^2x^{12}+2\,a^6bx^9+a^7x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^4
 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 -
 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(b*x^3 + a) + 18*((B*a*b^
3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)
*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)
```

**giac [A]** time = 0.20, size = 131, normalized size = 1.07

$$-\frac{3\left(Bab-2\,Ab^2\right)\log\left(|x|\right)}{a^5}+\frac{\left(Bab^2-2\,Ab^3\right)\log\left(\left|bx^3+a\right|\right)}{a^5b}-\frac{6\,Bab^2x^9-12\,Ab^3x^9+9\,Ba^2bx^6-18\,Aab^2x^6+2\,Ba^3x^3-4\,Aa^2bx^3+Aa^3}{6\left(bx^6+ax^3\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + (B*a*b^2 - 2*A*b^3)*log(abs(b*x^3 +
a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*b^
2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)
```

**maple [A]** time = 0.06, size = 147, normalized size = 1.20

$$\frac{A\,b^2}{6\left(b\,x^3+a\right)^2a^3}-\frac{Bb}{6\left(b\,x^3+a\right)^2a^2}+\frac{A\,b^2}{\left(b\,x^3+a\right)a^4}+\frac{6A\,b^2\ln(x)}{a^5}-\frac{2A\,b^2\ln\left(b\,x^3+a\right)}{a^5}-\frac{2Bb}{3\left(b\,x^3+a\right)a^3}-\frac{3Bb\ln(x)}{a^4}+\frac{Bb\ln\left(b\,x^3+a\right)}{a^4}+\frac{Ab}{a^4x^3}-\frac{B}{3a^3x^3}-\frac{A}{6a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^3,x)
```

```
[Out] 1/6/a^3*b^2/(b*x^3+a)^2*A-1/6/a^2*b/(b*x^3+a)^2*B-2/a^5*b^2*ln(b*x^3+a)*A+1
/a^4*b*ln(b*x^3+a)*B+1/a^4*b^2/(b*x^3+a)*A-2/3/a^3*b/(b*x^3+a)*B-1/6*A/a^3/
x^6+1/a^4/x^3*A*b-1/3/a^3/x^3*B+6*b^2/a^5*ln(x)*A-3*b/a^4*ln(x)*B
```

**maxima [A]** time = 0.52, size = 136, normalized size = 1.11

$$-\frac{6\left(Bab^2-2\,Ab^3\right)x^9+9\left(Ba^2b-2\,Aab^2\right)x^6+Aa^3+2\left(Ba^3-2\,Aa^2b\right)x^3}{6\left(a^4b^2x^{12}+2\,a^5bx^9+a^6x^6\right)}+\frac{\left(Bab-2\,Ab^2\right)\log\left(bx^3+a\right)}{a^5}-\frac{\left(Bab-2\,Ab^2\right)\log\left(x^3\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^12 + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b - 2
*A*b^2)*log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*log(x^3)/a^5
```

**mupad [B]** time = 0.15, size = 130, normalized size = 1.07

$$\frac{\frac{x^3\,(2\,A\,b-B\,a)}{3\,a^2}-\frac{A}{6\,a}+\frac{b^2\,x^9\,(2\,A\,b-B\,a)}{a^4}+\frac{3\,b\,x^6\,(2\,A\,b-B\,a)}{2\,a^3}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}}-\frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b^2-B\,a\,b\right)}{a^5}+\frac{\ln(x)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(2*A*b - B*a))/(3*a^2) - A/(6*a) + (b^2*x^9*(2*A*b - B*a))/a^4 + (3*b
*x^6*(2*A*b - B*a))/(2*a^3))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9) - (log(a + b*
x^3)*(2*A*b^2 - B*a*b))/a^5 + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

**sympy [A]** time = 3.55, size = 133, normalized size = 1.09

$$\frac{-Aa^3+x^9\left(12Ab^3-6Bab^2\right)+x^6\left(18Aab^2-9Ba^2b\right)+x^3\left(4Aa^2b-2Ba^3\right)}{6a^6x^6+12a^5bx^9+6a^4b^2x^{12}}-\frac{3b\,(-2Ab+Ba)\log{(x)}}{a^5}+\frac{b\,(-2Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**3,x)
```

```
[Out] (-A*a**3 + x**9*(12*A*b**3 - 6*B*a*b**2) + x**6*(18*A*a*b**2 - 9*B*a**2*b)
+ x**3*(4*A*a**2*b - 2*B*a**3))/(6*a**6*x**6 + 12*a**5*b*x**9 + 6*a**4*b**2
*x**12) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + b*(-2*A*b + B*a)*log(a/b + x**3)
/a**5
```

## 3.95 $\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=246

$$-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\tan^{-}}{9\sqrt{3}\,b^{14/3}}$$

**Rubi [A]** time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 288, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,b^{14/3}}+\frac{x^8(5Ab-11aB)}{18ab^2\left(a+bx^3\right)}-\frac{4x^5(5Ab-11aB)}{45ab^3}+\frac{2x^2(5Ab-11aB)}{9b^4}+\frac{x^{11}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*(5*A*b - 11*a*B)*x^2)/(9*b^4) - (4*(5*A*b - 11*a*B)*x^5)/(45*a*b^3) + ((
A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) + ((5*A*b - 11*a*B)*x^8)/(18*a*b^2*(
a + b*x^3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(14/3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*Log[a^
(1/3) + b^(1/3)*x])/(27*b^(14/3)) - (2*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(-5Ab+11aB)\int \frac{x^{10}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \frac{x^7}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx}{9ab^2}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4a(5Ab-1\cdots}{\cdots}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{\left(4a^{2/3}(5Ab-\cdots\right.}{\cdots}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-\cdots}{\cdots}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-\cdots}{\cdots}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-\cdots}{\cdots}$$

**Mathematica [A]** time = 0.32, size = 216, normalized size = 0.88

$$\frac{20a^{2/3}(11aB-5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-40a^{2/3}(11aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-40\sqrt{3}a^{2/3}(11aB-5Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{45a^2b^{2/3}x^2(aB-Ab)}{\left(a+bx^3\right)^2}+135b^{2/3}x^2(Ab-3aB)+\frac{30ab^{2/3}x^2(7Ab-10aB)}{a+bx^3}+54b^{5/3}Bx^5}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(A*b - 3*a*B)*x^2 + 54*b^(5/3)*B*x^5 + (45*a^2*b^(2/3)*(-(A*b)
 + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^(2/3)*(7*A*b - 10*a*B)*x^2)/(a + b*x^3
) - 40*Sqrt[3]*a^(2/3)*(-5*A*b + 11*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] - 40*a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*a^(2
/3)*(-5*A*b + 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
b^(14/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^10*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas** [**A**] time = 0.65, size = 364, normalized size = 1.48

$$\frac{54\,Bb^{3}x^{11} - 27\,\left(11\,Bab^{2} - 5\,Ab^{3}\right)x^{8} - 96\,\left(11\,Ba^{2}b - 5\,Aab^{2}\right)x^{5} - 60\,\left(11\,Ba^{3} - 5\,Aa^{2}b\right)x^{2} + 40\,\sqrt{3}\left(\left(11\,Bab^{2} - 5\,Ab^{3}\right)x^{6} + 11\,Ba^{3} - 5\,Aa^{2}b + 2\,\left(11\,Ba^{2}b - 5\,Aab^{2}\right)x^{3}\right)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}} - \sqrt{3}a}{3\,a}\right) + 20\,\left(\left(11\,Bab^{2} - 5\,Ab^{3}\right)x^{6} + 11\,Ba^{3} - 5\,Aa^{2}b + 2\,\left(11\,Ba^{2}b - 5\,Aab^{2}\right)x^{3}\right)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax^{2} - bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}} + a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right) - 40\,\left(\left(11\,Bab^{2} - 5\,Ab^{3}\right)x^{6} + 11\,Ba^{3} - 5\,Aa^{2}b + 2\,\left(11\,Ba^{2}b - 5\,Aab^{2}\right)x^{3}\right)\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax + b\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}\right)}{270\,\left(b^{6}x^{6} + 2\,ab^{5}x^{3} + a^{2}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270*(54*B*b^3*x^11 - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*A
*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*sqrt(3)*((11*B*a*b^2 - 5*A
*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 20*((11
*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)
*x^3)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3))
- 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5
*A*a*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^6*x^6 + 2*a
*b^5*x^3 + a^2*b^4)
```

**giac** [**A**] time = 0.33, size = 259, normalized size = 1.05

$$-\frac{4\,\left(11\,Ba^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^{4}} - \frac{4\,\sqrt{3}\left(11\,\left(-ab^{2}\right)^{\frac{2}{3}}Ba - 5\,\left(-ab^{2}\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{6}} + \frac{2\,\left(11\,\left(-ab^{2}\right)^{\frac{2}{3}}Ba - 5\,\left(-ab^{2}\right)^{\frac{2}{3}}Ab\right)\log\left(x^{2} + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^{6}} - \frac{20\,Ba^{2}bx^{5} - 14\,Aab^{2}x^{5} + 17\,Ba^{3}x^{2} - 11\,Aa^{2}bx^{2}}{18\,\left(bx^{3} + a\right)^{2}b^{4}} + \frac{2\,Bb^{12}x^{5} - 15\,Bab^{11}x^{2} + 5\,Ab^{12}x^{2}}{10\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*(11*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^4) - 4/27*sqrt(3)*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2
)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 + 2/
27*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/b^6 - 1/18*(20*B*a^2*b*x^5 - 14*A*a*b^2*x^5 + 17*B*a^3*x^2
- 11*A*a^2*b*x^2)/((b*x^3 + a)^2*b^4) + 1/10*(2*B*b^12*x^5 - 15*B*a*b^11*x^
2 + 5*A*b^12*x^2)/b^15
```

**maple** [**A**] time = 0.06, size = 308, normalized size = 1.25

$$\frac{7Aa\,x^{5}}{9\left(b\,x^{3}+a\right)^{2}b^{2}} - \frac{10B\,a^{2}x^{5}}{9\left(b\,x^{3}+a\right)^{2}b^{3}} + \frac{B\,x^{5}}{5b^{3}} + \frac{11A\,a^{2}x^{2}}{18\left(b\,x^{3}+a\right)^{2}b^{3}} - \frac{17B\,a^{3}x^{2}}{18\left(b\,x^{3}+a\right)^{2}b^{4}} + \frac{A\,x^{2}}{2b^{3}} - \frac{3Ba\,x^{2}}{2b^{4}} - \frac{20\sqrt{3}\,Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{4}} + \frac{20Aa\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{4}} - \frac{10Aa\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{4}} + \frac{44\sqrt{3}\,B\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{5}} - \frac{44B\,a^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{5}} + \frac{22B\,a^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/5/b^3*B*x^5+1/2/b^3*A*x^2-3/2/b^4*B*x^2*a+7/9*a/b^2/(b*x^3+a)^2*A*x^5-10/
9*a^2/b^3/(b*x^3+a)^2*B*x^5+11/18*a^2/b^3/(b*x^3+a)^2*A*x^2-17/18*a^3/b^4/(
b*x^3+a)^2*B*x^2+20/27*a/b^4*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-10/27*a/b^4*A/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27*a/b^4*A*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-44/27*a^2/b^5*B/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+22/27*a^2/b^5*B/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+44/27*a^2/b^5*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))
```

**maxima** [**A**] time = 1.37, size = 228, normalized size = 0.93

$$-\frac{2\,\left(10\,Ba^{2}b - 7\,Aab^{2}\right)x^{5} + \left(17\,Ba^{3} - 11\,Aa^{2}b\right)x^{2}}{18\,\left(b^{6}x^{6} + 2\,ab^{5}x^{3} + a^{2}b^{4}\right)} + \frac{4\,\sqrt{3}\left(11\,Ba^{2} - 5\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^{5} - 5\,(3\,Ba - Ab)x^{2}}{10\,b^{4}} + \frac{2\,\left(11\,Ba^{2} - 5\,Aab\right)\log\left(x^{2} - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\,\left(11\,Ba^{2} - 5\,Aab\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^{5}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(10*B*a^2*b - 7*A*a*b^2)*x^5 + (17*B*a^3 - 11*A*a^2*b)*x^2)/(b^6*x
^6 + 2*a*b^5*x^3 + a^2*b^4) + 4/27*sqrt(3)*(11*B*a^2 - 5*A*a*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/10*(2*B*b*x^
5 - 5*(3*B*a - A*b)*x^2)/b^4 + 2/27*(11*B*a^2 - 5*A*a*b)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) - 4/27*(11*B*a^2 - 5*A*a*b)*log(x +
(a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

**mupad [B]** time = 2.58, size = 213, normalized size = 0.87

$$\frac{x^5\left(\frac{7\,A\,a\,b^2}{9}-\frac{10\,B\,a^2\,b}{9}\right)-x^2\left(\frac{17\,B\,a^3}{18}-\frac{11\,A\,a^2\,b}{18}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^2\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right)+\frac{B\,x^5}{5\,b^3}+\frac{4\,a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,A\,b-11\,B\,a\right)}{27\,b^{14/3}}+\frac{4\,a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-11\,B\,a\right)}{27\,b^{14/3}}-\frac{4\,a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,A\,b-11\,B\,a\right)}{27\,b^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^5*((7*A*a*b^2)/9 - (10*B*a^2*b)/9) - x^2*((17*B*a^3)/18 - (11*A*a^2*b)/1
8))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) +
 (B*x^5)/(5*b^3) + (4*a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 11*B*a))/(2
7*b^(14/3)) + (4*a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(5*A*b - 11*B*a))/(27*b^(14/3)) - (4*a^(2/3)*log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 11*B
*a))/(27*b^(14/3))
```

**sympy [A]** time = 6.24, size = 192, normalized size = 0.78

$$\frac{Bx^5}{5b^3}+x^2\left(\frac{A}{2b^3}-\frac{3Ba}{2b^4}\right)+\frac{x^5\left(14Aab^2-20Ba^2b\right)+x^2\left(11Aa^2b-17Ba^3\right)}{18a^2b^4+36ab^5x^3+18b^6x^6}+\operatorname{RootSum}\left(19683t^3b^{14}-8000A^3a^2b^3+52800A^2Ba^3b^2-116160AB^2a^4b+85184B^3a^5,\left(t\mapsto t\log\left(\frac{729t^2b^9}{400A^2ab^2-1760ABa^2b+1936B^2a^3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + x**2*(A/(2*b**3) - 3*B*a/(2*b**4)) + (x**5*(14*A*a*b**2 -
 20*B*a**2*b) + x**2*(11*A*a**2*b - 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x
**3 + 18*b**6*x**6) + RootSum(19683*_t**3*b**14 - 8000*A**3*a**2*b**3 + 528
00*A**2*B*a**3*b**2 - 116160*A*B**2*a**4*b + 85184*B**3*a**5, Lambda(_t, _t
*log(729*_t**2*b**9/(400*A**2*a*b**2 - 1760*A*B*a**2*b + 1936*B**2*a**3) +
x)))
```

## 3.96 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3} \, dx$

**Optimal**. Leaf size=244

$$\frac{7\sqrt[3]{a}\,(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54b^{13/3}}-\frac{7\sqrt[3]{a}\,(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27b^{13/3}}+\frac{7\sqrt[3]{a}\,(2Ab-5aB)\tan^{-1}}{9\sqrt{3}\,b^{13/3}}$$

**Rubi [A]** time = 0.17, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 288, 302, 200, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{a}\,(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54b^{13/3}}+\frac{x^7(2Ab-5aB)}{9ab^2\left(a+bx^3\right)}-\frac{7x^4(2Ab-5aB)}{36ab^3}+\frac{7x(2Ab-5aB)}{9b^4}-\frac{7\sqrt[3]{a}\,(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27b^{13/3}}+\frac{7\sqrt[3]{a}\,(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,b^{13/3}}+\frac{x^{10}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (7*(2*A*b - 5*a*B)*x)/(9*b^4) - (7*(2*A*b - 5*a*B)*x^4)/(36*a*b^3) + ((A*b
- a*B)*x^10)/(6*a*b*(a + b*x^3)^2) + ((2*A*b - 5*a*B)*x^7)/(9*a*b^2*(a + b*
x^3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(13/3)) - (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x])/(27*b^(13/3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^9 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(-4Ab+10aB)\int \frac{x^9}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)} - \frac{(7(2Ab-5aB))\int \frac{x^6}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)} - \frac{(7(2Ab-5aB))\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{9ab^2}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)} - \frac{(7a(2Ab-5a\ldots}{9b\ldots}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)} - \frac{\left(7\sqrt[3]{a}\,(2Ab-5\ldots\right.}{\ldots}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)} - \frac{7\sqrt[3]{a}\,(2Ab-5\ldots}{\ldots}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)} - \frac{7\sqrt[3]{a}\,(2Ab-5\ldots}{\ldots}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)} + \frac{7\sqrt[3]{a}\,(2Ab-5\ldots}{9\ldots}$$

**Mathematica [A]** time = 0.24, size = 210, normalized size = 0.86

$$\frac{-14\sqrt[3]{a}\,(5aB-2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\frac{18a^2\sqrt[3]{b}\,x(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{6a\sqrt[3]{b}\,x(13Ab-19aB)}{a+bx^3}+108\sqrt[3]{b}\,x(Ab-3aB)+28\sqrt[3]{a}\,(5aB-2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-28\sqrt{3}\,\sqrt[3]{a}\,(5aB-2Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+27b^{4/3}Bx^4}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(A*b - 3*a*B)*x + 27*b^(4/3)*B*x^4 + (18*a^2*b^(1/3)*(-(A*b) +
 a*B)*x)/(a + b*x^3)^2 + (6*a*b^(1/3)*(13*A*b - 19*a*B)*x)/(a + b*x^3) - 28
*Sqrt[3]*a^(1/3)*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 28*a^(1/3)*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x] - 14*a^(1/3)*(-2*
A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*b^(13/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^9*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [A]** time = 0.60, size = 347, normalized size = 1.42

$$\frac{27 Bb^3x^{10} - 54\left(5Bab^2-2Ab^3\right)x^7 - 147\left(5Ba^2b-2Aab^2\right)x^4 - 28\sqrt{3}\left(\left(5Bab^2-2Ab^3\right)x^6+5Ba^3-2Aa^2b+2\left(5Ba^2b-2Aab^2\right)x^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right) + 14\left(\left(5Bab^2-2Ab^3\right)x^6+5Ba^3-2Aa^2b+2\left(5Ba^2b-2Aab^2\right)x^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) - 28\left(\left(5Bab^2-2Ab^3\right)x^6+5Ba^3-2Aa^2b+2\left(5Ba^2b-2Aab^2\right)x^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) - 84\left(5Ba^3-2Aa^2b\right)x}{108\left(b^6x^6+2ab^5x^3+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(27*B*b^3*x^10 - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A*
a*b^2)*x^4 - 28*sqrt(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b +
2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b
)^(2/3) - sqrt(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2
*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3)) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5
*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 84*(5*B*a^3
 - 2*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

**giac [A]** time = 0.24, size = 234, normalized size = 0.96

$$\frac{7\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^5} - \frac{7\left(5Ba^2-2Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^4} + \frac{7\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^5} - \frac{19Ba^2bx^4-13Aab^2x^4+16Ba^3x-10Aa^2bx}{18\left(bx^3+a\right)^2b^4} + \frac{Bb^9x^4-12Bab^8x+4Ab^9x}{4b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 7/54*(5*(-a*b^2)^(1/3)*B*a - 2*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 - 1/18*(19
*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a)^2*b
^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^12
```

**maple [A]** time = 0.06, size = 299, normalized size = 1.23

$$\frac{13Aax^4}{18\left(bx^3+a\right)^2b^2} - \frac{19Ba^2x^4}{18\left(bx^3+a\right)^2b^3} + \frac{Bx^4}{4b^3} + \frac{5Aa^2x}{9\left(bx^3+a\right)^2b^3} - \frac{8Ba^3x}{9\left(bx^3+a\right)^2b^4} - \frac{14\sqrt{3}\,Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} - \frac{14Aa\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{7Aa\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{Ax}{b^3} + \frac{35\sqrt{3}\,Ba^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5} + \frac{35Ba^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5} - \frac{35Ba^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5} - \frac{3Bax}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/4/b^3*B*x^4+1/b^3*A*x-3/b^4*B*a*x+13/18*a/b^2/(b*x^3+a)^2*A*x^4-19/18*a^2
/b^3/(b*x^3+a)^2*B*x^4+5/9*a^2/b^3/(b*x^3+a)^2*A*x-8/9*a^3/b^4/(b*x^3+a)^2*
B*x-14/27*a/b^4*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27*a/b^4*A/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27*a/b^4*A/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+35/27*a^2/b^5*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-35/54*a^2/b^5*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+35/27*a^2/b
^5*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

**maxima [A]** time = 1.24, size = 223, normalized size = 0.91

$$-\frac{\left(19Ba^2b-13Aab^2\right)x^4+2\left(8Ba^3-5Aa^2b\right)x}{18\left(b^6x^6+2ab^5x^3+a^2b^4\right)} + \frac{Bbx^4-4\left(3Ba-Ab\right)x}{4b^4} + \frac{7\sqrt{3}\left(5Ba^2-2Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{7\left(5Ba^2-2Aab\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{7\left(5Ba^2-2Aab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((19*B*a^2*b - 13*A*a*b^2)*x^4 + 2*(8*B*a^3 - 5*A*a^2*b)*x)/(b^6*x^6
+ 2*a*b^5*x^3 + a^2*b^4) + 1/4*(B*b*x^4 - 4*(3*B*a - A*b)*x)/b^4 + 7/27*sqr
```

```
t(3)*(5*B*a^2 - 2*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(b^5*(a/b)^(2/3)) - 7/54*(5*B*a^2 - 2*A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^5*(a/b)^(2/3)) + 7/27*(5*B*a^2 - 2*A*a*b)*log(x + (a/b)^(1/3)
)/(b^5*(a/b)^(2/3))
```

**mupad [B]** time = 0.32, size = 227, normalized size = 0.93

$$\frac{x^4\left(\frac{13\,A\,a\,b^2}{18}-\frac{19\,B\,a^2\,b}{18}\right)-x\left(\frac{8\,B\,a^3}{9}-\frac{5\,A\,a^2\,b}{9}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)+\frac{B\,x^4}{4\,b^3}+\frac{7\,{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3}+a\,b^{1/3}\,x\right)\,\left(2\,A\,b-5\,B\,a\right)}{27\,b^{13/3}}-\frac{7\,{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{27\,b^{13/3}}+\frac{7\,{(-a)}^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-{(-a)}^{4/3}+\sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{27\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^4*((13*A*a*b^2)/18 - (19*B*a^2*b)/18) - x*((8*B*a^3)/9 - (5*A*a^2*b)/9))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^4)/(4*b
^3) + (7*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(2*A*b - 5*B*a))/(27*b^(1
3/3)) - (7*(-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^(1/3)*
x)*((3^(1/2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3)) + (7*(-a)^(1/3)*lo
g(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b - 5*B*a))/(27*b^(13/3))
```

**sympy [A]** time = 4.01, size = 163, normalized size = 0.67

$$\frac{Bx^4}{4b^3}+x\left(\frac{A}{b^3}-\frac{3Ba}{b^4}\right)+\frac{x^4\left(13Aab^2-19Ba^2b\right)+x\left(10Aa^2b-16Ba^3\right)}{18a^2b^4+36ab^5x^3+18b^6x^6}+\operatorname{RootSum}\left(19683t^3b^{13}+2744A^3ab^3-20580A^2Ba^2b^2+51450AB^2a^3b-42875B^3a^4,\left(t\mapsto t\log\left(\frac{27tb^4}{-14Ab+35Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + x*(A/b**3 - 3*B*a/b**4) + (x**4*(13*A*a*b**2 - 19*B*a**2*
b) + x*(10*A*a**2*b - 16*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*
x**6) + RootSum(19683*_t**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2*b*
*2 + 51450*A*B**2*a**3*b - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**4/(-
14*A*b + 35*B*a) + x)))
```

## 3.97 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=222

$$\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}\,b^{11/3}}-\frac{5x^2(}{}$$

**Rubi [A]** time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 288, 321, 292, 31, 634, 617, 204, 628}

$$\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{11/3}}+\frac{x^5(Ab-4aB)}{9ab^2\left(a+bx^3\right)}-\frac{5x^2(Ab-4aB)}{18ab^3}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}\,b^{11/3}}+\frac{x^8(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-5*(A*b - 4*a*B)*x^2)/(18*a*b^3) + ((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2)
 + ((A*b - 4*a*B)*x^5)/(9*a*b^2*(a + b*x^3)) - (5*(A*b - 4*a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(11/3)) - (5*(
A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(11/3)) + (5*(A*b - 4*
a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(-2Ab+8aB)\int \frac{x^7}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{(5(Ab-4aB))\int \frac{x^4}{a+bx^3}\,dx}{9ab^2}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} + \frac{(5(Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{9b^3}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{(5(Ab-4aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27\sqrt[3]{a}\,b^{10/3}} + \frac{(5(A\ldots}{\ldots}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}\,b^{11/3}} + \frac{(5\ldots}{\ldots}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}\,b^{11/3}} + \frac{5(\ldots}{\ldots}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}\,b^{11/3}} - \frac{5(\ldots}{\ldots}$$

**Mathematica [A]** time = 0.24, size = 194, normalized size = 0.87

$$\frac{\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}} - \frac{6b^{2/3}x^2(4Ab-7aB)}{a+bx^3} + \frac{9ab^{2/3}x^2(Ab-aB)}{\left(a+bx^3\right)^2} + \frac{10(4aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{10\sqrt{3}(4aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + 27b^{2/3}Bx^2}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/3
)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (10*(-(A*b) + 4*a*B)*Log[a^(1
/3) + b^(1/3)*x])/a^(1/3) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/a^(1/3))/(54*b^(11/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^7*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [B]** time = 0.92, size = 792, normalized size = 3.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
A*a^2*b^3)*x^2 - 15*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*
a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^
2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)
*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*((4*B*a*b^2
- A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A
*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5), 1/54*(27
*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 - A*a^2*b^3
)*x^2 - 30*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*a^3*b^2 +
 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*((4*B*a*b^2 - A*b^3
)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log
(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A*b^3)*x
^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)]
```

**giac [A]**     time = 0.21, size = 210, normalized size = 0.95

$$\frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}\,(4\,Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\,(4\,Ba - Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\left(4\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{14\,Babx^5 - 8\,Ab^2x^5 + 11\,Ba^2x^2 - 5\,Aabx^2}{18\,\left(bx^3 + a\right)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) + 5/54*(4*B*a - A*b)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3) + 5/27*(4*B*a*(-a/b)^(1/
3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/
18*(14*B*a*b*x^5 - 8*A*b^2*x^5 + 11*B*a^2*x^2 - 5*A*a*b*x^2)/((b*x^3 + a)^2
*b^3)
```

**maple [A]**     time = 0.06, size = 275, normalized size = 1.24

$$-\frac{4A\,x^5}{9\left(b\,x^3 + a\right)^2 b} + \frac{7Ba\,x^5}{9\left(b\,x^3 + a\right)^2 b^2} - \frac{5Aa\,x^2}{18\left(b\,x^3 + a\right)^2 b^2} + \frac{11B\,a^2x^2}{18\left(b\,x^3 + a\right)^2 b^3} + \frac{B\,x^2}{2b^3} + \frac{5\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} - \frac{5A\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} + \frac{5A\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} - \frac{20\sqrt{3}\;Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^4} + \frac{20Ba\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^4} - \frac{10Ba\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/2*B/b^3*x^2-4/9/b/(b*x^3+a)^2*A*x^5+7/9/b^2/(b*x^3+a)^2*B*x^5*a-5/18/b^2/
(b*x^3+a)^2*A*x^2*a+11/18/b^3/(b*x^3+a)^2*B*x^2*a^2-5/27/b^3*A/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+5/54/b^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
5/27/b^3*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+20/2
7/b^4*B*a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-10/27/b^4*B*a/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))-20/27/b^4*B*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))
```

**maxima [A]**     time = 1.32, size = 196, normalized size = 0.88

$$\frac{2\left(7\,Bab - 4\,Ab^2\right)x^5 + \left(11\,Ba^2 - 5\,Aab\right)x^2}{18\left(b^5x^6 + 2\,ab^4x^3 + a^2b^3\right)} + \frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}\,(4\,Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{5\,(4\,Ba - Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,(4\,Ba - Ab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(7*B*a*b - 4*A*b^2)*x^5 + (11*B*a^2 - 5*A*a*b)*x^2)/(b^5*x^6 + 2*a*
b^4*x^3 + a^2*b^3) + 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) - 5/54*(4*B*a -
A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) + 5/27*(4*B*a
 - A*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

**mupad [B]** time = 2.56, size = 187, normalized size = 0.84

$$\frac{x^2\left(\frac{11\,B\,a^2}{18}-\frac{5\,A\,a\,b}{18}\right)-x^5\left(\frac{4\,A\,b^2}{9}-\frac{7\,B\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{B\,x^2}{2\,b^3}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}-\frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}+\frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^2*((11*B*a^2)/18 - (5*A*a*b)/18) - x^5*((4*A*b^2)/9 - (7*B*a*b)/9))/(a^2
*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (B*x^2)/(2*b^3) - (5*log(b^(1/3)*x + a^(1/3
))*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3)) - (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3
)) + (5*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1
/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3))
```

**sympy [A]** time = 5.85, size = 162, normalized size = 0.73

$$\frac{Bx^2}{2b^3}+\frac{x^5\left(-8Ab^2+14Bab\right)+x^2\left(-5Aab+11Ba^2\right)}{18a^2b^3+36ab^4x^3+18b^5x^6}+\operatorname{RootSum}\left(19683t^3ab^{11}+125A^3b^3-1500A^2Bab^2+6000AB^2a^2b-8000B^3a^3,\left(t\mapsto t\log\left(\frac{729t^2ab^7}{25A^2b^2-200ABab+400B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**2
))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a*b
**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B**3*
a**3, Lambda(_t, _t*log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b + 400*
B**2*a**2) + x)))
```

## 3.98 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3} dx$

**Optimal**. Leaf size=220

$$-\frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}-2$$

**Rubi [A]** time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 288, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}+\frac{x^4(Ab-7aB)}{18ab^2\left(a+bx^3\right)}-\frac{2x(Ab-7aB)}{9ab^3}+\frac{x^7(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-2*(A*b - 7*a*B)*x)/(9*a*b^3) + ((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) +
((A*b - 7*a*B)*x^4)/(18*a*b^2*(a + b*x^3)) - (2*(A*b - 7*a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(10/3)) + (2*(A*
b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(10/3)) - ((A*b - 7*a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(2/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6 (A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(-Ab+7aB)\int \frac{x^6}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} - \frac{(2(Ab-7aB))\int \frac{x^3}{a+bx^3}\,dx}{9ab^2}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} + \frac{(2(Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{9b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} + \frac{(2(Ab-7aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{2/3}b^3} + \cdots$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \cdots$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \cdots$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} - \frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}} + \cdots$$

**Mathematica [A]** time = 0.27, size = 188, normalized size = 0.85

$$\frac{\frac{2(7aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} + \frac{4(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{4\sqrt{3}(7aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{3\sqrt[3]{b}x(7Ab-13aB)}{a+bx^3} + \frac{9a\sqrt[3]{b}x(Ab-aB)}{(a+bx^3)^2} + 54\sqrt[3]{b}Bx}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(7
*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b^(1
/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 (A+Bx^3)}{(a+bx^3)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [B]** time = 0.65, size = 789, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*((
7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2
*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(7*
B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*B*
a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/54*(54*B*a
^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*B*a^2*b^3
- A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*b^3)*x^3)*s
qrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*
b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*
x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(
7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*
B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```

**giac [A]** time = 0.20, size = 187, normalized size = 0.85

$$\frac{2\sqrt{3}(7Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}b^2}+\frac{(7Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}b^2}+\frac{Bx}{b^3}+\frac{2(7Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^3}+\frac{13Babx^4-7Ab^2x^4+10Ba^2x-4Aabx}{18\left(bx^3+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*b^2) + 1/27*(7*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + B*x/b^3 + 2/27*(7*B*a - A*b)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(13*B*a*b*x^4 - 7*A*b^2*x^4 +
 10*B*a^2*x - 4*A*a*b*x)/((b*x^3 + a)^2*b^3)
```

**maple [A]** time = 0.05, size = 268, normalized size = 1.22

$$-\frac{7Ax^4}{18\left(bx^3+a\right)^2b}+\frac{13Bax^4}{18\left(bx^3+a\right)^2b^2}-\frac{2Aax}{9\left(bx^3+a\right)^2b^2}+\frac{5Ba^2x}{9\left(bx^3+a\right)^2b^3}+\frac{2\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{14\sqrt{3}\,Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{14Ba\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{7Ba\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{Bx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] B/b^3*x-7/18/b/(b*x^3+a)^2*A*x^4+13/18/b^2/(b*x^3+a)^2*B*x^4*a-2/9/b^2/(b*x
^3+a)^2*a*A*x+5/9/b^3/(b*x^3+a)^2*a^2*B*x+2/27/b^3*A/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))-1/27/b^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/b^3*A
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/27/b^4*B*a/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27/b^4*B*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))-14/27/b^4*B*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))
```

**maxima [A]** time = 1.36, size = 191, normalized size = 0.87

$$\frac{\left(13Bab-7Ab^2\right)x^4+2\left(5Ba^2-2Aab\right)x}{18\left(b^5x^6+2ab^4x^3+a^2b^3\right)}+\frac{Bx}{b^3}-\frac{2\sqrt{3}(7Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(7Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2(7Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((13*B*a*b - 7*A*b^2)*x^4 + 2*(5*B*a^2 - 2*A*a*b)*x)/(b^5*x^6 + 2*a*b^
4*x^3 + a^2*b^3) + B*x/b^3 - 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) + 1/27*(7*B*a - A*b)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 2/27*(7*B*a - A*b)*
log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

**mupad [B]** time = 2.60, size = 183, normalized size = 0.83

$$\frac{B\,x}{b^3}-\frac{x^4\,\left(\frac{7\,A\,b^2}{18}-\frac{13\,B\,a\,b}{18}\right)-x\,\left(\frac{5\,B\,a^2}{9}-\frac{2\,A\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}-\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}+\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x)/b^3 - (x^4*((7*A*b^2)/18 - (13*B*a*b)/18) - x*((5*B*a^2)/9 - (2*A*a*b
)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*log(b^(1/3)*x + a^(1/3))*(A*b
- 7*B*a))/(27*a^(2/3)*b^(10/3)) - (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x +
 a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3)) + (2*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - 7*B*a))/(27*a^(2/3)*b^(10/3))
```

**sympy [A]** time = 3.39, size = 141, normalized size = 0.64

$$\frac{Bx}{b^3}+\frac{x^4\left(-7Ab^2+13Bab\right)+x\left(-4Aab+10Ba^2\right)}{18a^2b^3+36ab^4x^3+18b^5x^6}+\operatorname{RootSum}\left(19683t^3a^2b^{10}-8A^3b^3+168A^2Bab^2-1176AB^2a^2b+2744B^3a^3,\left(t\mapsto t\log\left(-\frac{27tab^3}{-2Ab+14Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a**
2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a**2*b**10 -
8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, Lamb
da(_t, _t*log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))
```

## 3.99 $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=201

$$\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}}-\frac{x^2(5aB$$

**Rubi [A]** time = 0.11, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 288, 292, 31, 634, 617, 204, 628}

$$\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}}-\frac{x^2(5aB+Ab)}{18ab^2\left(a+bx^3\right)}+\frac{x^5(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) - ((A*b + 5*a*B)*x^2)/(18*a*b^2*(a
+ b*x^3)) - ((A*b + 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(9*Sqrt[3]*a^(4/3)*b^(8/3)) - ((A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(
27*a^(4/3)*b^(8/3)) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(4/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^5}{6ab(a+bx^3)^2} + \frac{(Ab+5aB)\int \frac{x^4}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^5}{6ab(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2(a+bx^3)} + \frac{(Ab+5aB)\int \frac{x}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^5}{6ab(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2(a+bx^3)} - \frac{(Ab+5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{4/3}b^{7/3}} + \frac{(Ab+5aB)\int \frac{\sqrt[3]{a}+\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{\cdots}}}{27a^{4/3}b^{7/3}}$$

$$= \frac{(Ab-aB)x^5}{6ab(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2(a+bx^3)} - \frac{(Ab+5aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\int \frac{-\sqrt[3]{\cdots}}{a^{2/3}-\cdots}}{54a^{4/3}b^{\cdots}}$$

$$= \frac{(Ab-aB)x^5}{6ab(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2(a+bx^3)} - \frac{(Ab+5aB)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\log(a^{2}\cdots}{54a\cdots}$$

$$= \frac{(Ab-aB)x^5}{6ab(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2(a+bx^3)} - \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}} - \frac{(Ab+5aB)\log(\sqrt[3]{\cdots}}{27a^{4/3}b^{8/3}}$$

**Mathematica [A]** time = 0.29, size = 181, normalized size = 0.90

$$\frac{\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}} - \frac{2(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}} - \frac{2\sqrt{3}(5aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{6b^{2/3}x^2(Ab-4aB)}{a(a+bx^3)} - \frac{9b^{2/3}x^2(Ab-aB)}{(a+bx^3)^2}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2)
/(a*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3)
+ ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(
54*b^(8/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(A + Bx^3\right)}{\left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [B]** time = 0.54, size = 756, normalized size = 3.76

$$\left[-\frac{6 \left(4 Ba^2b^3 - Aab^4\right)x^5 + 3\left(5Ba^3b^2 + Aa^2b^3\right)x^2 - 3\sqrt{\frac{1}{3}}\left(\left(5Ba^2b^3 + Aab^4\right)x^6 + 5Ba^4b + Aa^3b^2 + 2\left(5Ba^3b^2 + Aa^2b^3\right)x^3\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x^3 - ab + 3\sqrt{\frac{1}{3}}\left(abx + 2\left(-ab^2\right)^{\frac{2}{3}}x^2 + \left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}} - 3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3 + a}\right) - \left(\left(5Bab^2 + Ab^3\right)x^6 + 5Ba^3 + Aa^2b + 2\left(5Ba^2b + Aab^2\right)x^3\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2 + \left(-ab^2\right)^{\frac{1}{3}}bx + \left(-ab^2\right)^{\frac{2}{3}}\right) + 2\left(\left(5Bab^2 + Ab^3\right)x^6 + 5Ba^3 + Aa^2b + 2\left(5Ba^2b + Aab^2\right)x^3\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx - \left(-ab^2\right)^{\frac{1}{3}}\right)}{54\left(a^2b^6x^6 + 2a^3b^5x^3 + a^4b^4\right)}, -\frac{6\left(4Ba^2b^3 - Aab^4\right)x^5 + 3\left(5Ba^3b^2 + Aa^2b^3\right)x^2 - 6\sqrt{\frac{1}{3}}\left(\left(5Ba^2b^3 + Aab^4\right)x^6 + 5Ba^4b + Aa^3b^2 + 2\left(5Ba^3b^2 + Aa^2b^3\right)x^3\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx + \left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right) - \left(\left(5Bab^2 + Ab^3\right)x^6 + 5Ba^3 + Aa^2b + 2\left(5Ba^2b + Aab^2\right)x^3\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2 + \left(-ab^2\right)^{\frac{1}{3}}bx + \left(-ab^2\right)^{\frac{2}{3}}\right) + 2\left(\left(5Bab^2 + Ab^3\right)x^6 + 5Ba^3 + Aa^2b + 2\left(5Ba^2b + Aab^2\right)x^3\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx - \left(-ab^2\right)^{\frac{1}{3}}\right)}{54\left(a^2b^6x^6 + 2a^3b^5x^3 + a^4b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 3
*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^
3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^
3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 +
A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b^3 - A*a*
b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^2*b^3 + A*
a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(
-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)
^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b
+ A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*
a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*
b^5*x^3 + a^4*b^4)]
```

**giac [A]** time = 0.20, size = 206, normalized size = 1.02

$$\frac{\sqrt{3}\left(5Ba + Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{\left(5Ba + Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{\left(5Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} + Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b^2} - \frac{8Babx^5 - 2Ab^2x^5 + 5Ba^2x^2 + Aabx^2}{18\left(bx^3 + a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a*b^2) - 1/54*(5*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b^2) - 1/27*(5*B*a*(-a/b)^(1/3) + A*b*(-a
/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(8*B*a*
b*x^5 - 2*A*b^2*x^5 + 5*B*a^2*x^2 + A*a*b*x^2)/((b*x^3 + a)^2*a*b^2)
```

**maple [A]** time = 0.05, size = 241, normalized size = 1.20

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{5\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{5B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{\frac{(Ab-4Ba)x^{5}}{9ab}-\frac{(Ab+5Ba)x^{2}}{18b^{2}}}{\left(b\,x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(A*b-4*B*a)/a/b*x^5-1/18*(A*b+5*B*a)/b^2*x^2)/(b*x^3+a)^2-1/27/b^2/a/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1/5
4/b^2/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+5/54/b^3/(a/b)^(1/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

**maxima [A]** time = 1.27, size = 195, normalized size = 0.97

$$-\frac{2\left(4\,Bab-Ab^{2}\right)x^{5}+\left(5\,Ba^{2}+Aab\right)x^{2}}{18\left(ab^{4}x^{6}+2\,a^{2}b^{3}x^{3}+a^{3}b^{2}\right)}+\frac{\sqrt{3}\,(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(5\,Ba+Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(5\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(4*B*a*b - A*b^2)*x^5 + (5*B*a^2 + A*a*b)*x^2)/(a*b^4*x^6 + 2*a^2*
b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(1/3)) + 1/54*(5*B*a + A*b)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) - 1/27*(5*B*a + A*b)*log(
x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

**mupad [B]** time = 0.27, size = 175, normalized size = 0.87

$$-\frac{\frac{x^{2}\,(A\,b+5\,B\,a)}{18\,b^{2}}-\frac{x^{5}\,(A\,b-4\,B\,a)}{9\,a\,b}}{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b + 5*B*a))
/(27*a^(4/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - ((x^2*(A*b + 5*B
*a))/(18*b^2) - (x^5*(A*b - 4*B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

**sympy [A]** time = 4.75, size = 155, normalized size = 0.77

$$\frac{x^{5}\left(2Ab^{2}-8Bab\right)+x^{2}\left(-Aab-5Ba^{2}\right)}{18a^{3}b^{2}+36a^{2}b^{3}x^{3}+18ab^{4}x^{6}}+\operatorname{RootSum}\left(19683t^{3}a^{4}b^{8}+A^{3}b^{3}+15A^{2}Bab^{2}+75AB^{2}a^{2}b+125B^{3}a^{3},\left(t\mapsto t\log\left(\frac{729t^{2}a^{3}b^{5}}{A^{2}b^{2}+10ABab+25B^{2}a^{2}}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(2*A*b**2 - 8*B*a*b) + x**2*(-A*a*b - 5*B*a**2))/(18*a**3*b**2 + 36*a
**2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**4*b**8 + A**3*b**3
 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, Lambda(_t, _t*log(7
29*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))
```

## 3.100 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3} dx$

**Optimal**. Leaf size=199

$$-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}+\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}-\frac{x(2a}{9ab^2(}$$

**Rubi** [A] time = 0.11, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 288, 200, 31, 634, 617, 204, 628}

$$-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}+\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}-\frac{x(2aB+Ab)}{9ab^2\left(a+bx^3\right)}+\frac{x^4(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) - ((A*b + 2*a*B)*x)/(9*a*b^2*(a + b
*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(5/3)*b^(7/3)) + ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*
a^(5/3)*b^(7/3)) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+4aB)\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$=\frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}+\frac{(Ab+2aB)\int \frac{1}{a+bx^3}\,dx}{9ab^2}$$

$$=\frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}+\frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b^2}+\frac{(Ab+2aB)\int \frac{2\sqrt[3]{a}-\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots}}{27a^{5/3}b^2}$$

$$=\frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}+\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\cdots}{a^{2/3}-\sqrt[3]{\cdots}}}{54a^{5/3}b^{7\cdots}}$$

$$=\frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}+\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(Ab+2aB)\log\left(a^{2/3}\cdots\right.}{54a^{5\cdots}}$$

$$=\frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}-\frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}+\frac{(Ab+2aB)\log\left(\sqrt[3]{a}\cdots\right.}{27a^{5/3}b^{7/3}}$$

**Mathematica [A]** time = 0.24, size = 178, normalized size = 0.89

$$\frac{-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{2(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{2\sqrt{3}(2aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{3\sqrt[3]{b}x(Ab-7aB)}{a\left(a+bx^3\right)}-\frac{9\sqrt[3]{b}x(Ab-aB)}{\left(a+bx^3\right)^2}}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a*
(a + b*x^3)) - (2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - ((
A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b
^(7/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^3))/(a + b*x^3)^3, x]
```

**fricas [B]** time = 0.72, size = 743, normalized size = 3.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b
^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-(a
^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*
a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
 + a)) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*
b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^
3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/
(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)
*x^4 - 6*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2
*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(
a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b^2
+ A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((2*B*a*b^2 + A*b^3
)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(
a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/(a^3*b^5*x^6 + 2*a^4*
b^4*x^3 + a^5*b^3)]
```

**giac [A]** time = 0.19, size = 187, normalized size = 0.94

$$-\frac{\sqrt{3}(2Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{(2Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{(2Ba+Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b^2} - \frac{7Babx^4-Ab^2x^4+4Ba^2x+2Aabx}{18\left(bx^3+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(2*B*a + A*b)*(-a/b)^(1/3)*log(a
```

```
bs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(7*B*a*b*x^4 - A*b^2*x^4 + 4*B*a^2*x
 + 2*A*a*b*x)/((b*x^3 + a)^2*a*b^2)
```

**maple** [A] time = 0.05, size = 239, normalized size = 1.20

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{2\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/18*(A*b-7*B*a)/a/b*x^4-1/9*(A*b+2*B*a)/b^2*x)/(b*x^3+a)^2+1/27/b^2/a/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*A+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-1/54/
b^2/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/27/b^3/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/27/b^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

**maxima** [A] time = 1.25, size = 193, normalized size = 0.97

$$-\frac{\left(7\,Bab-Ab^2\right)x^4+2\left(2\,Ba^2+Aab\right)x}{18\left(ab^4x^6+2\,a^2b^3x^3+a^3b^2\right)}+\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*B*a*b - A*b^2)*x^4 + 2*(2*B*a^2 + A*a*b)*x)/(a*b^4*x^6 + 2*a^2*b^
3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/54*(2*B*a + A*b)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(2*B*a + A*b)*log(x
+ (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

**mupad** [B] time = 2.56, size = 173, normalized size = 0.87

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}-\frac{\frac{x\,\left(A\,b+2\,B\,a\right)}{9\,b^2}-\frac{x^4\,\left(A\,b-7\,B\,a\right)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) - ((x*(A*b +
2*B*a))/(9*b^2) - (x^4*(A*b - 7*B*a))/(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3))
```

**sympy** [A] time = 3.25, size = 136, normalized size = 0.68

$$\frac{x^4\left(Ab^2-7Bab\right)+x\left(-2Aab-4Ba^2\right)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}+\operatorname{RootSum}\left(19683t^3a^5b^7-A^3b^3-6A^2Bab^2-12AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^2b^2}{Ab+2Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(A*b**2 - 7*B*a*b) + x*(-2*A*a*b - 4*B*a**2))/(18*a**3*b**2 + 36*a**2
*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**5*b**7 - A**3*b**3 -
6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(27*_t*a
**2*b**2/(A*b + 2*B*a) + x)))
```

## 3.101 $\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=201

$$\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}}+\frac{x^2(aB+}{9a^2b(a}$$

**Rubi [A]** time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {457, 290, 292, 31, 634, 617, 204, 628}

$$\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}}+\frac{x^2(aB+2Ab)}{9a^2b\left(a+bx^3\right)}+\frac{x^2(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*x^2)/(9*a^2*b*(a +
 b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(9*Sqrt[3]*a^(7/3)*b^(5/3)) - ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(7/3)*b^(5/3)) + ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(7/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2}+\frac{(4Ab+2aB)\int \frac{x}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)}+\frac{(2Ab+aB)\int \frac{x}{a+bx^3}\,dx}{9a^2b}\\
&= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)}-\frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}b^{4/3}}+\frac{(2Ab+aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b} \cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x \cdots}}{27a^{7/3}b^{4/3}}\\
&= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)}-\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}+\frac{(2Ab+aB)\int \frac{-\sqrt[3]{a} \cdots}{a^{2/3}-\sqrt[3]{a} \cdots}}{54a^{7/3}b^{5/ \cdots}}\\
&= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)}-\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}}+\frac{(2Ab+aB)\log\left(a^{2/3} \cdots\right.}{54a^{7/ \cdots}}\\
&= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)}-\frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}}-\frac{(2Ab+aB)\log\left(\sqrt[3]{a} \cdots\right.}{27a^{7/3}b^{5/3}}
\end{aligned}
$$

**Mathematica [A]** time = 0.24, size = 178, normalized size = 0.89

$$
\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{9a^{4/3}b^{2/3}x^2(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{6\sqrt[3]{a}b^{2/3}x^2(aB+2Ab)}{a+bx^3}-2(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(aB+2Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{7/3}b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

[Out] ((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3)*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] + (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(5/3))

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^3,x]

[Out] IntegrateAlgebraic[(x*(A + B*x^3))/(a + b*x^3)^3, x]

**fricas [B]** time = 0.79, size = 752, normalized size = 3.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] [1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3)]

**giac [A]** time = 0.21, size = 207, normalized size = 1.03

$$\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^2b}-\frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}a^2b}-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}+\frac{2\,Babx^5+4\,Ab^2x^5-Ba^2x^2+7\,Aabx^2}{18\left(bx^3+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")

[Out] 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/54*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b) - 1/27*(B*a*(-a/b)^(1/3) + 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(2*B*a*b*x^5 + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b)

**maple** [A] time = 0.05, size = 251, normalized size = 1.25

$$\frac{2\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{\frac{(2Ab+Ba)x^{5}}{9a^{2}}+\frac{(7Ab-Ba)x^{2}}{18ab}}{\left(b\,x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2-2/27/a^2/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1
/27/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/54/a/b^2/(a/b)^
(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

**maxima** [A] time = 1.40, size = 195, normalized size = 0.97

$$\frac{2\left(Bab+2\,Ab^{2}\right)x^{5}-\left(Ba^{2}-7\,Aab\right)x^{2}}{18\left(a^{2}b^{3}x^{6}+2\,a^{3}b^{2}x^{3}+a^{4}b\right)}+\frac{\sqrt{3}\left(Ba+2\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(Ba+2\,Ab\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(Ba+2\,Ab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(B*a*b + 2*A*b^2)*x^5 - (B*a^2 - 7*A*a*b)*x^2)/(a^2*b^3*x^6 + 2*a^3
*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/54*(B*a + 2*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(1/3)) - 1/27*(B*a + 2*A*b)*l
og(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3))
```

**mupad** [B] time = 0.27, size = 175, normalized size = 0.87

$$\frac{\frac{x^{5}\,(2\,A\,b+B\,a)}{9\,a^{2}}+\frac{x^{2}\,(7\,A\,b-B\,a)}{18\,a\,b}}{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^5*(2*A*b + B*a))/(9*a^2) + (x^2*(7*A*b - B*a))/(18*a*b))/(a^2 + b^2*x^6
 + 2*a*b*x^3) - (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3
)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3))
```

**sympy** [A] time = 2.03, size = 153, normalized size = 0.76

$$\frac{x^{5}\left(4Ab^{2}+2Bab\right)+x^{2}\left(7Aab-Ba^{2}\right)}{18a^{4}b+36a^{3}b^{2}x^{3}+18a^{2}b^{3}x^{6}}+\operatorname{RootSum}\left(19683t^{3}a^{7}b^{5}+8A^{3}b^{3}+12A^{2}Bab^{2}+6AB^{2}a^{2}b+B^{3}a^{3},\left(t\mapsto t\log\left(\frac{729t^{2}a^{5}b^{3}}{4A^{2}b^{2}+4ABab+B^{2}a^{2}}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3*
b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b**
3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(729*_
t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))
```

## 3.102 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=197

$$-\frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}+\frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{x(aB}{18a^2b}$$

**Rubi [A]** time = 0.10, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}+\frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{x(aB+5Ab)}{18a^2b\left(a+bx^3\right)}+\frac{x(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*x)/(18*a^2*b*(a + b*
x^3)) - ((5*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(8/3)*b^(4/3)) + ((5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(8/3)*b^(4/3)) - ((5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(54*a^(8/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)\int \frac{1}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)}+\frac{(5Ab+aB)\int \frac{1}{a+bx^3}\,dx}{9a^2b}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)}+\frac{(5Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}b}+\frac{(5Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\ldots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\ldots}}{27a^{8/3}b}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)}+\frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(5Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{\ldots}}{a^{2/3}-\sqrt[3]{a}\ldots}}{54a^{8/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)}+\frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(5Ab+aB)\log\left(a^{2/3}\ldots\right.}{54a^{8/3}\ldots}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)}-\frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}b^{4/3}}+\frac{(5Ab+aB)\log\left(\sqrt[3]{a}\ldots\right.}{27a^{8/3}b^{4/3}}$$

**Mathematica [A]** time = 0.19, size = 175, normalized size = 0.89

$$\frac{-(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{9a^{5/3}\sqrt[3]{b}x(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{3a^{2/3}\sqrt[3]{b}x(aB+5Ab)}{a+bx^3}+2(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(aB+5Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{54a^{8/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*(
5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A*b
+ a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^3}{\left(a + bx^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(a + b*x^3)^3, x]
```

**fricas [B]** time = 0.64, size = 743, normalized size = 3.77

$$\left[\frac{3\left(Ba^3b^2+5Aa^2b^3\right)x^4+3\sqrt{\frac{1}{3}}\left(\left(Ba^2b^3+5Aab^4\right)x^6+Ba^4b+5Aa^3b^2+2\left(Ba^3b^2+5Aa^2b^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(\left(Bab^2+5Ab^3\right)x^6+Ba^3+5Aa^2b+2\left(Ba^2b+5Aab^2\right)x^3\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(\left(Bab^2+5Ab^3\right)x^6+Ba^3+5Aa^2b+2\left(Ba^2b+5Aab^2\right)x^3\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)-6\left(Ba^4b-4Aa^3b^2\right)x}{54\left(a^4b^4x^6+2a^5b^3x^3+a^6b^2\right)}, \frac{3\left(Ba^3b^2+5Aa^2b^3\right)x^4+6\sqrt{\frac{1}{3}}\left(\left(Ba^2b^3+5Aab^4\right)x^6+Ba^4b+5Aa^3b^2+2\left(Ba^3b^2+5Aa^2b^3\right)x^3\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(\left(Bab^2+5Ab^3\right)x^6+Ba^3+5Aa^2b+2\left(Ba^2b+5Aab^2\right)x^3\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(\left(Bab^2+5Ab^3\right)x^6+Ba^3+5Aa^2b+2\left(Ba^2b+5Aab^2\right)x^3\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)-6\left(Ba^4b-4Aa^3b^2\right)x}{54\left(a^4b^4x^6+2a^5b^3x^3+a^6b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^
4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(a^
2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a
*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
+ a)) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b
^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2
*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3
)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(
a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x
^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(
B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b^2 + 5*
A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b^2 + 5*A*b^3)*
x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*
b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(a^4*b^4*x^6 + 2*a^5*b^
3*x^3 + a^6*b^2)]
```

**giac [A]** time = 0.20, size = 180, normalized size = 0.91

$$-\frac{\sqrt{3}\,(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2}-\frac{(Ba+5\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2}-\frac{(Ba+5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b}+\frac{Babx^4+5\,Ab^2x^4-2\,Ba^2x+8\,Aabx}{18\left(bx^3+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(B*a + 5*A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2*x +
 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)
```

**maple [A]** time = 0.06, size = 249, normalized size = 1.26

$$\frac{5\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{\frac{(5Ab+Ba)x^{4}}{18a^{2}}+\frac{(4Ab-Ba)x}{9ab}}{\left(b\,x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+5/27/a^2/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-5/5
4/a^2/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/54/a/b^2/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

**maxima [A]** time = 1.37, size = 192, normalized size = 0.97

$$\frac{\left(Bab+5\,Ab^{2}\right)x^{4}-2\left(Ba^{2}-4\,Aab\right)x}{18\left(a^{2}b^{3}x^{6}+2\,a^{3}b^{2}x^{3}+a^{4}b\right)}+\frac{\sqrt{3}\,(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba+5\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba+5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((B*a*b + 5*A*b^2)*x^4 - 2*(B*a^2 - 4*A*a*b)*x)/(a^2*b^3*x^6 + 2*a^3*b
^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b
)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/54*(B*a + 5*A*b)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) + 1/27*(B*a + 5*A*b)*log
(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

**mupad [B]** time = 0.26, size = 173, normalized size = 0.88

$$\frac{\frac{x^{4}\,(5\,A\,b+B\,a)}{18\,a^{2}}+\frac{x\,(4\,A\,b-B\,a)}{9\,a\,b}}{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^3,x)
```

```
[Out] ((x^4*(5*A*b + B*a))/(18*a^2) + (x*(4*A*b - B*a))/(9*a*b))/(a^2 + b^2*x^6 +
 2*a*b*x^3) + (log(b^(1/3)*x + a^(1/3))*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(5*A*b + B*a))/(27*a^(8/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
```

**sympy [A]** time = 1.51, size = 133, normalized size = 0.68

$$\frac{x^{4}\left(5Ab^{2}+Bab\right)+x\left(8Aab-2Ba^{2}\right)}{18a^{4}b+36a^{3}b^{2}x^{3}+18a^{2}b^{3}x^{6}}+\operatorname{RootSum}\left(19683t^{3}a^{8}b^{4}-125A^{3}b^{3}-75A^{2}Bab^{2}-15AB^{2}a^{2}b-B^{3}a^{3},\left(t\mapsto t\log\left(\frac{27ta^{3}b}{5Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b**
2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b**3
 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27*_t
*a**3*b/(5*A*b + B*a) + x)))
```

## 3.103 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=227

$$-\frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}}-2(7\ldots$$

**Rubi [A]** time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}-\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] (-2*(7*A*b - a*B))/(9*a^3*b*x) + (A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + (7*A
*b - a*B)/(18*a^2*b*x*(a + b*x^3)) + (2*(7*A*b - a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(2/3)) + (2*(7*A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(2/3)) - ((7*A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{(7Ab-aB)\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{(2(7Ab-aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} - \frac{(2(7Ab-aB))\int \frac{x}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{(2(7Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{10/3}\sqrt[3]{b}} - \cdots$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \cdots$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \cdots$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}} + \cdots$$

**Mathematica [A]** time = 0.28, size = 193, normalized size = 0.85

$$\frac{\frac{2(aB-7Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{9a^{4/3}x^2(aB-Ab)}{\left(a+bx^3\right)^2} + \frac{4(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{4\sqrt{3}(7Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{6\sqrt[3]{a}x^2(2aB-5Ab)}{a+bx^3} - \frac{54\sqrt[3]{a}A}{x}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1
/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*Sqrt[3]*(7*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(7*A*b - a*B)*Log[a^(1/3) +
 b^(1/3)*x])/b^(2/3) + (2*(-7*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(2/3))/(54*a^(10/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^2*(a + b*x^3)^3), x]
```

**fricas [B]** time = 0.69, size = 776, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*
a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*
A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b
^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*
sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2 - 7*
A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x), 1/54
*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*a^2*b
^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*A*a^
2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b^3)
*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x)]
```

**giac** [A] time = 0.20, size = 204, normalized size = 0.90

$$\frac{2\sqrt{3}\,(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}}-\frac{(Ba-7\,Ab)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}}-\frac{2\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{4}}-\frac{A}{a^{3}x}+\frac{4\,Babx^{5}-10\,Ab^{2}x^{5}+7\,Ba^{2}x^{2}-13\,Aabx^{2}}{18\left(bx^{3}+a\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^3) - 1/27*(B*a - 7*A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3) - 2/27*(B*a*(-a/b)^(1/3) - 7*A*b*(-a/b)^
(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - A/(a^3*x) + 1/18*(4*B*
a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 + a)^2*a^3)
```

**maple** [A] time = 0.06, size = 281, normalized size = 1.24

$$-\frac{5Ab^{2}x^{5}}{9\left(bx^{3}+a\right)^{2}a^{3}}+\frac{2Bbx^{5}}{9\left(bx^{3}+a\right)^{2}a^{2}}-\frac{13Abx^{2}}{18\left(bx^{3}+a\right)^{2}a^{2}}+\frac{7Bx^{2}}{18\left(bx^{3}+a\right)^{2}a}-\frac{14\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{14A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{7A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{2\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{A}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -5/9/a^3/(b*x^3+a)^2*A*x^5*b^2+2/9/a^2/(b*x^3+a)^2*B*x^5*b-13/18/a^2/(b*x^3
+a)^2*A*x^2*b+7/18/a/(b*x^3+a)^2*B*x^2+14/27/a^3*A/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))-7/27/a^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27/a^3*A*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/27/a^2*B/b/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27/a^2*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+2/27/a^2*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-A/a^3/x
```

**maxima** [A] time = 1.41, size = 199, normalized size = 0.88

$$\frac{4\left(Bab-7\,Ab^{2}\right)x^{6}+7\left(Ba^{2}-7\,Aab\right)x^{3}-18\,Aa^{2}}{18\left(a^{3}b^{2}x^{7}+2\,a^{4}bx^{4}+a^{5}x\right)}+\frac{2\sqrt{3}\,(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-7\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{3}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,(Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{3}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*(B*a*b - 7*A*b^2)*x^6 + 7*(B*a^2 - 7*A*a*b)*x^3 - 18*A*a^2)/(a^3*b^
2*x^7 + 2*a^4*b*x^4 + a^5*x) + 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) + 1/27*(B*a - 7*A*b)
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) - 2/27*(B*a - 7
*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1/3))
```

**mupad [B]** time = 2.60, size = 185, normalized size = 0.81

$$\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}-\frac{\frac{A}{a}+\frac{7\,x^3\,\left(7\,A\,b-B\,a\right)}{18\,a^2}+\frac{2\,b\,x^6\,\left(7\,A\,b-B\,a\right)}{9\,a^3}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7}+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] (2*log(b^(1/3)*x + a^(1/3))*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (A/a + (
7*x^3*(7*A*b - B*a))/(18*a^2) + (2*b*x^6*(7*A*b - B*a))/(9*a^3))/(a^2*x + b
^2*x^7 + 2*a*b*x^4) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (2*log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a))/(
27*a^(10/3)*b^(2/3))
```

**sympy [A]** time = 1.90, size = 162, normalized size = 0.71

$$\frac{-18Aa^{2}+x^{6}\left(-28Ab^{2}+4Bab\right)+x^{3}\left(-49Aab+7Ba^{2}\right)}{18a^{5}x+36a^{4}bx^{4}+18a^{3}b^{2}x^{7}}+\operatorname{RootSum}{\left(19683t^{3}a^{10}b^{2}-2744A^{3}b^{3}+1176A^{2}Bab^{2}-168AB^{2}a^{2}b+8B^{3}a^{3},\left(t\mapsto t\log{\left(\frac{729t^{2}a^{7}b}{196A^{2}b^{2}-56ABab+4B^{2}a^{2}}+x\right)}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**3,x)
```

```
[Out] (-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(1
8*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + RootSum(19683*_t**3*a**10*
b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*a**
3, Lambda(_t, _t*log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B**2*
a**2) + x)))
```

## 3.104 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=227

$$\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}}+\frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}}-\frac{5}{}$$

**Rubi [A]** time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}-\frac{5(4Ab-aB)}{18a^3bx^2}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}}+\frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}}+\frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] (-5*(4*A*b - a*B))/(18*a^3*b*x^2) + (A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) +
 (4*A*b - a*B)/(9*a^2*b*x^2*(a + b*x^3)) + (5*(4*A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(1/3)) - (5*(4*A*
b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(1/3)) + (5*(4*A*b - a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{(8Ab-2aB)\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{(5(4Ab-aB))\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{11/3}} -$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}\sqrt[3]{b}}$$

**Mathematica [A]** time = 0.26, size = 189, normalized size = 0.83

$$\frac{\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}} + \frac{9a^{5/3}x(aB-Ab)}{\left(a+bx^3\right)^2} + \frac{3a^{2/3}x(5aB-11Ab)}{a+bx^3} - \frac{27a^{2/3}A}{x^2} + \frac{10(aB-4Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{10\sqrt{3}(4Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2
/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^3*(a + b*x^3)^3), x]
```

**fricas [B]** time = 0.52, size = 812, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a^
3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4*A
*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*
a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((B*a*b^
2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(
-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a
*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2
)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5
+ a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a
^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*
a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(-(-a^2*b)^(1
/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2
*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^
5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
- (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)
*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))
/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

**giac** [A] time = 0.19, size = 209, normalized size = 0.92

$$-\frac{5\,(Ba-4\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4}+\frac{5\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\,\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}+\frac{5\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\,\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b}+\frac{5\,Babx^6-20\,Ab^2x^6+8\,Ba^2x^3-32\,Aabx^3-9\,Aa^2}{18\,\left(bx^4+ax\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -5/27*(B*a - 4*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 5/27*sqrt
(3)*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) + 5/54*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^
(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 1/18*(5*B*a*b
*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 + a*x)^
2*a^3)
```

**maple** [A] time = 0.05, size = 277, normalized size = 1.22

$$-\frac{11A\,b^2x^4}{18\left(b\,x^3+a\right)^2a^3}+\frac{5Bb\,x^4}{18\left(b\,x^3+a\right)^2a^2}-\frac{7Abx}{9\left(b\,x^3+a\right)^2a^2}+\frac{4Bx}{9\left(b\,x^3+a\right)^2a}-\frac{20\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{20A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{10A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{5\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{5B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{5B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{A}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -11/18/a^3/(b*x^3+a)^2*A*x^4*b^2+5/18/a^2/(b*x^3+a)^2*B*x^4*b-7/9/a^2/(b*x^
3+a)^2*b*A*x+4/9/a/(b*x^3+a)^2*B*x-20/27/a^3*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)+10/27/a^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27/a^3*A/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/27/a^2*B/b/(a/b)
^(2/3)*ln(x+(a/b)^(1/3))-5/54/a^2*B/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))+5/27/a^2*B/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-1/2*A/a^3/x^2
```

**maxima** [A] time = 1.20, size = 201, normalized size = 0.89

$$\frac{5\left(Bab-4\,Ab^2\right)x^6+8\left(Ba^2-4\,Aab\right)x^3-9\,Aa^2}{18\left(a^3b^2x^8+2\,a^4bx^5+a^5x^2\right)}+\frac{5\,\sqrt{3}\,(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(5*(B*a*b - 4*A*b^2)*x^6 + 8*(B*a^2 - 4*A*a*b)*x^3 - 9*A*a^2)/(a^3*b^2
*x^8 + 2*a^4*b*x^5 + a^5*x^2) + 5/27*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(
3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) - 5/54*(B*a - 4*A*b
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) + 5/27*(B*a -
4*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

**mupad [B]** time = 2.58, size = 188, normalized size = 0.83

$$-\frac{\frac{A}{2\,a}+\frac{4\,x^3\,\left(4\,A\,b-B\,a\right)}{9\,a^2}+\frac{5\,b\,x^6\,\left(4\,A\,b-B\,a\right)}{18\,a^3}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}+\frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}-\frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
4*A*b - B*a))/(27*a^(11/3)*b^(1/3)) - (5*log(b^(1/3)*x + a^(1/3))*(4*A*b -
B*a))/(27*a^(11/3)*b^(1/3)) - (A/(2*a) + (4*x^3*(4*A*b - B*a))/(9*a^2) + (5
*b*x^6*(4*A*b - B*a))/(18*a^3))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) - (5*log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B
*a))/(27*a^(11/3)*b^(1/3))
```

**sympy [A]** time = 1.73, size = 143, normalized size = 0.63

$$\frac{-9Aa^2+x^6\left(-20Ab^2+5Bab\right)+x^3\left(-32Aab+8Ba^2\right)}{18a^5x^2+36a^4bx^5+18a^3b^2x^8}+\operatorname{RootSum}\left(19683t^3a^{11}b+8000A^3b^3-6000A^2Bab^2+1500AB^2a^2b-125B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^4}{-20Ab+5Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**3,x)
```

```
[Out] (-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(18
*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a**1
1*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**3*a
**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))
```

## 3.105 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=246

$$\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{13/3}}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{13/3}}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\tan^{-1}\left(\sqrt[3]{\ldots}\right.}{9\sqrt{3}\,a^{13/3}}$$

**Rubi [A]** time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{13/3}}+\frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)}-\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{7(5Ab-2aB)}{9a^4x}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{13/3}}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{13/3}}+\frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] (-7*(5*A*b - 2*a*B))/(36*a^3*b*x^4) + (7*(5*A*b - 2*a*B))/(9*a^4*x) + (A*b
 - a*B)/(6*a*b*x^4*(a + b*x^3)^2) + (5*A*b - 2*a*B)/(9*a^2*b*x^4*(a + b*x^3)
) - (7*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(13/3)) - (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(1/3) + b^(1
/3)*x])/(27*a^(13/3)) + (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{(10Ab-4aB)\int \frac{1}{x^5\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7(5Ab-2aB))\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{(7(5Ab-2aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{9a^3}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7b(5Ab-2aB \cdots}{9a \cdots}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{(7b^{2/3}(5Ab-2 \cdots}{27 \cdots}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab-2a \cdots}{2 \cdots}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab-2a \cdots}{2 \cdots}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab-2a \cdots}{9 \cdots}$$

**Mathematica** [A] time = 0.27, size = 214, normalized size = 0.87

$$\frac{14\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-\frac{18a^{4/3}bx^2(aB-Ab)}{\left(a+bx^3\right)^2}-\frac{27a^{4/3}A}{x^4}-\frac{12\sqrt[3]{a}\,bx^2(5aB-8Ab)}{a+bx^3}-\frac{108\sqrt[3]{a}\,(aB-3Ab)}{x}+28\sqrt[3]{b}\,(2aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-28\sqrt{3}\sqrt[3]{b}\,(5Ab-2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*A)/x^4 - (108*a^(1/3)*(-3*A*b + a*B))/x - (18*a^(4/3)*b*(-(A*
b) + a*B)*x^2)/(a + b*x^3)^2 - (12*a^(1/3)*b*(-8*A*b + 5*a*B)*x^2)/(a + b*x
^3) - 28*Sqrt[3]*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] + 28*b^(1/3)*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(1/
3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*a^(
13/3))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^5*(a + b*x^3)^3), x]
```

**fricas** [A] time = 0.92, size = 366, normalized size = 1.49

$$-\frac{84\left(2 Bab^2-5 Ab^3\right)x^9+147\left(2 Ba^2b-5 Aab^2\right)x^6+27 Aa^3+54\left(2 Ba^3-5 Aa^2b\right)x^3+28\sqrt{3}\left(\left(2 Bab^2-5 Ab^3\right)x^{10}+2\left(2 Ba^2b-5 Aab^2\right)x^7+\left(2 Ba^3-5 Aa^2b\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-14\left(\left(2 Bab^2-5 Ab^3\right)x^{10}+2\left(2 Ba^2b-5 Aab^2\right)x^7+\left(2 Ba^3-5 Aa^2b\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+28\left(\left(2 Bab^2-5 Ab^3\right)x^{10}+2\left(2 Ba^2b-5 Aab^2\right)x^7+\left(2 Ba^3-5 Aa^2b\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{108\left(a^4b^2x^{10}+2 a^5bx^7+a^6x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(84*(2*B*a*b^2 - 5*A*b^3)*x^9 + 147*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 27
*A*a^3 + 54*(2*B*a^3 - 5*A*a^2*b)*x^3 + 28*sqrt(3)*((2*B*a*b^2 - 5*A*b^3)*x
^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/a)^(1/
3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 14*((2*B*a*b^2 - 5*A*
b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/
a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 28*((2*B*a*b^2 -
5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*
(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x
^4)
```

**giac** [A] time = 0.20, size = 254, normalized size = 1.03

$$\frac{7\left(2 Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5 Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27 a^5}+\frac{7\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^5b}-\frac{7\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 a^5b}-\frac{10 Bab^2x^5-16 Ab^3x^5+13 Ba^2bx^2-19 Aab^2x^2}{18\left(bx^3+a\right)^2a^4}-\frac{4 Bax^3-12 Abx^3+Aa}{4 a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*(2*B*a*b*(-a/b)^(1/3) - 5*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/a^5 + 7/27*sqrt(3)*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)
*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 7/54*
(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^5*b) - 1/18*(10*B*a*b^2*x^5 - 16*A*b^3*x^5 + 13*B*a^2*b*x^2
- 19*A*a*b^2*x^2)/((b*x^3 + a)^2*a^4) - 1/4*(4*B*a*x^3 - 12*A*b*x^3 + A*a)/
(a^4*x^4)
```

**maple** [A] time = 0.06, size = 299, normalized size = 1.22

$$\frac{8Ab^3x^5}{9\left(bx^3+a\right)^2a^4}-\frac{5Bb^2x^5}{9\left(bx^3+a\right)^2a^3}+\frac{19Ab^2x^2}{18\left(bx^3+a\right)^2a^3}-\frac{13Bbx^2}{18\left(bx^3+a\right)^2a^2}+\frac{35\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}-\frac{35Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}+\frac{35Ab\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a^4}-\frac{14\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7B\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{3Ab}{a^4x}-\frac{B}{a^3x}-\frac{A}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^3,x)
```

```
[Out] 8/9/a^4*b^3/(b*x^3+a)^2*x^5*A-5/9/a^3*b^2/(b*x^3+a)^2*x^5*B+19/18/a^3*b^2/(
b*x^3+a)^2*A*x^2-13/18/a^2*b/(b*x^3+a)^2*B*x^2-35/27/a^4*b*A/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+35/54/a^4*b*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+35/27/a^4*b*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
14/27/a^3*B/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-7/27/a^3*B/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))-14/27/a^3*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))-1/4/a^3*A/x^4+3/a^4/x*A*b-B/a^3/x
```

**maxima** [A] time = 1.42, size = 221, normalized size = 0.90

$$-\frac{28\left(2 Bab^2-5 Ab^3\right)x^9+49\left(2 Ba^2b-5 Aab^2\right)x^6+9 Aa^3+18\left(2 Ba^3-5 Aa^2b\right)x^3}{36\left(a^4b^2x^{10}+2 a^5bx^7+a^6x^4\right)}-\frac{7\sqrt{3}\left(2 Ba-5 Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27 a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\left(2 Ba-5 Ab\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54 a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\left(2 Ba-5 Ab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27 a^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*(28*(2*B*a*b^2 - 5*A*b^3)*x^9 + 49*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 9*A*
a^3 + 18*(2*B*a^3 - 5*A*a^2*b)*x^3)/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)
- 7/27*sqrt(3)*(2*B*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a^4*(a/b)^(1/3)) - 7/54*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(a^4*(a/b)^(1/3)) + 7/27*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/
(a^4*(a/b)^(1/3))
```

**mupad [B]** time = 2.64, size = 240, normalized size = 0.98

$$\frac{\frac{x^3\,(5\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{4\,a}+\frac{7\,b^2\,x^9\,(5\,A\,b-2\,B\,a)}{9\,a^4}+\frac{49\,b\,x^6\,(5\,A\,b-2\,B\,a)}{36\,a^3}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}+\frac{7\,(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}+\frac{7\,(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}-\frac{7\,(-b)^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(5*A*b - 2*B*a))/(2*a^2) - A/(4*a) + (7*b^2*x^9*(5*A*b - 2*B*a))/(9*a
^4) + (49*b*x^6*(5*A*b - 2*B*a))/(36*a^3))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7)
 + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(5*A*b - 2*B*a))/(27*a^(13
/3)) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*a^(1/3)*(-b
)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3)) - (7*(-b)
^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((
3^(1/2)*1i)/2 + 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3))
```

**sympy [A]** time = 1.91, size = 189, normalized size = 0.77

$$\operatorname{RootSum}\left(19683t^3a^{13}+42875A^3b^4-51450A^2Bab^3+20580AB^2a^2b^2-2744B^3a^3b,\left(t\mapsto t\log\left(\frac{729t^2a^9}{1225A^2b^3-980ABab^2+196B^2a^2b}+x\right)\right)\right)+\frac{-9Aa^3+x^9\left(140Ab^3-56Bab^2\right)+x^6\left(245Aab^2-98Ba^2b\right)+x^3\left(90Aa^2b-36Ba^3\right)}{36a^6x^4+72a^5bx^7+36a^4b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*A
*B**2*a**2*b**2 - 2744*B**3*a**3*b, Lambda(_t, _t*log(729*_t**2*a**9/(1225*
A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) + (-9*A*a**3 + x**9*(1
40*A*b**3 - 56*B*a*b**2) + x**6*(245*A*a*b**2 - 98*B*a**2*b) + x**3*(90*A*a
**2*b - 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**10)
```

## 3.106 $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=246

$$-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}}-\frac{4b^{2/3}(11Ab-5aB)\,\mathrm{ta}}{9\sqrt{3}\,a^{1}}$$

**Rubi** [A] time = 0.14, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}}-\frac{4b^{2/3}(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{14/3}}+\frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)}+\frac{2(11Ab-5aB)}{9a^4x^2}-\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] (-4*(11*A*b - 5*a*B))/(45*a^3*b*x^5) + (2*(11*A*b - 5*a*B))/(9*a^4*x^2) + (
A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + (11*A*b - 5*a*B)/(18*a^2*b*x^5*(a +
b*x^3)) - (4*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)) + (4*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(14/3)) - (2*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{(11Ab-5aB)\int \frac{1}{x^6\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4(11Ab-5aB))\int \frac{1}{x^6\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{(4(11Ab-5aB))\int \frac{1}{x^3\left(a+bx^3\right)}}{9a^3}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11Ab}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11Ab}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(11A}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(11A}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{4b^{2/3}(11A}{}$$

**Mathematica [A]** time = 0.32, size = 210, normalized size = 0.85

$$\frac{20b^{2/3}(5aB-11Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{45a^{5/3}bx(aB-Ab)}{\left(a+bx^3\right)^2} - \frac{15a^{2/3}bx(11aB-17Ab)}{a+bx^3} - \frac{135a^{2/3}(aB-3Ab)}{x^2} - \frac{54a^{5/3}A}{x^5} + 40b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - 40\sqrt{3}b^{2/3}(11Ab-5aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*A)/x^5 - (135*a^(2/3)*(-3*A*b + a*B))/x^2 - (45*a^(5/3)*b*(-(
A*b) + a*B)*x)/(a + b*x^3)^2 - (15*a^(2/3)*b*(-17*A*b + 11*a*B)*x)/(a + b*x
^3) - 40*Sqrt[3]*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]] + 40*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*b^(2
/3)*(-11*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
a^(14/3))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^3)/(x^6*(a + b*x^3)^3), x]
```

**fricas** [A] time = 0.66, size = 384, normalized size = 1.56

$$-\frac{60\left(5\,Bab^{2}-11\,Ab^{3}\right)x^{9}+96\left(5\,Ba^{2}b-11\,Aab^{2}\right)x^{6}+54\,Aa^{3}+27\left(5\,Ba^{3}-11\,Aa^{2}b\right)x^{3}+40\,\sqrt{3}\left(\left(5\,Bab^{2}-11\,Ab^{3}\right)x^{11}+2\left(5\,Ba^{2}b-11\,Aab^{2}\right)x^{8}+\left(5\,Ba^{3}-11\,Aa^{2}b\right)x^{5}\right)\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-20\left(\left(5\,Bab^{2}-11\,Ab^{3}\right)x^{11}+2\left(5\,Ba^{2}b-11\,Aab^{2}\right)x^{8}+\left(5\,Ba^{3}-11\,Aa^{2}b\right)x^{5}\right)\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+40\left(\left(5\,Bab^{2}-11\,Ab^{3}\right)x^{11}+2\left(5\,Ba^{2}b-11\,Aab^{2}\right)x^{8}+\left(5\,Ba^{3}-11\,Aa^{2}b\right)x^{5}\right)\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)}{270\left(a^{4}b^{2}x^{11}+2\,a^{5}bx^{8}+a^{6}x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/270*(60*(5*B*a*b^2 - 11*A*b^3)*x^9 + 96*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 5
4*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*sqrt(3)*((5*B*a*b^2 - 11*A*b^3
)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/
a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(
(5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 1
1*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(
b^2/a^2)^(2/3)) + 40*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b
^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)))/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5)
```

**giac** [A] time = 0.19, size = 229, normalized size = 0.93

$$-\frac{4\,\sqrt{3}\left(5\left(-ab^{2}\right)^{\frac{1}{3}}Ba-11\left(-ab^{2}\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{5}}+\frac{4\left(5\,Bab-11\,Ab^{2}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{5}}-\frac{2\left(5\left(-ab^{2}\right)^{\frac{1}{3}}Ba-11\left(-ab^{2}\right)^{\frac{1}{3}}Ab\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{5}}-\frac{11\,Bab^{2}x^{4}-17\,Ab^{3}x^{4}+14\,Ba^{2}bx-20\,Aab^{2}x}{18\left(bx^{3}+a\right)^{2}a^{4}}-\frac{5\,Bax^{3}-15\,Abx^{3}+2\,Aa}{10\,a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 11*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 2/27*(5*(-a*b^2)^(1/3)*B*a - 11*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 - 1/18*(11
*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/((b*x^3 + a)^2*a
^4) - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)
```

**maple** [A] time = 0.06, size = 295, normalized size = 1.20

$$\frac{17A\,b^{3}x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{4}}-\frac{11B\,b^{2}x^{4}}{18\left(b\,x^{3}+a\right)^{2}a^{3}}+\frac{10A\,b^{2}x}{9\left(b\,x^{3}+a\right)^{2}a^{3}}-\frac{7Bbx}{9\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{44\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}+\frac{44Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{22Ab\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{4}}-\frac{20\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{20B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{10B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{3Ab}{2a^{4}x^{2}}-\frac{B}{2a^{3}x^{2}}-\frac{A}{5a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^3,x)
```

```
[Out] 17/18/a^4*b^3/(b*x^3+a)^2*A*x^4-11/18/a^3*b^2/(b*x^3+a)^2*B*x^4+10/9/a^3*b^
2/(b*x^3+a)^2*A*x-7/9/a^2*b/(b*x^3+a)^2*B*x+44/27/a^4*b*A/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-22/27/a^4*b*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+44
/27/a^4*b*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-20/
27/a^3*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+10/27/a^3*B/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))-20/27/a^3*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))-1/5/a^3*A/x^5+3/2/a^4/x^2*A*b-1/2/a^3/x^2*B
```

**maxima** [A] time = 1.46, size = 221, normalized size = 0.90

$$-\frac{20\left(5\,Bab^{2}-11\,Ab^{3}\right)x^{9}+32\left(5\,Ba^{2}b-11\,Aab^{2}\right)x^{6}+18\,Aa^{3}+9\left(5\,Ba^{3}-11\,Aa^{2}b\right)x^{3}}{90\left(a^{4}b^{2}x^{11}+2\,a^{5}bx^{8}+a^{6}x^{5}\right)}-\frac{4\,\sqrt{3}\left(5\,Ba-11\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\left(5\,Ba-11\,Ab\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{4\left(5\,Ba-11\,Ab\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/90*(20*(5*B*a*b^2 - 11*A*b^3)*x^9 + 32*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 18
*A*a^3 + 9*(5*B*a^3 - 11*A*a^2*b)*x^3)/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^
5) - 4/27*sqrt(3)*(5*B*a - 11*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(
a/b)^(1/3))/(a^4*(a/b)^(2/3)) + 2/27*(5*B*a - 11*A*b)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 4/27*(5*B*a - 11*A*b)*log(x + (a/b)^(
1/3))/(a^4*(a/b)^(2/3))
```

**mupad [B]** time = 2.58, size = 207, normalized size = 0.84

$$\frac{\frac{x^3\,(11\,A\,b-5\,B\,a)}{10\,a^2}-\frac{A}{5\,a}+\frac{2\,b^2\,x^9\,(11\,A\,b-5\,B\,a)}{9\,a^4}+\frac{16\,b\,x^6\,(11\,A\,b-5\,B\,a)}{45\,a^3}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}+\frac{4\,b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(11\,A\,b-5\,B\,a)}{27\,a^{14/3}}-\frac{4\,b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(11\,A\,b-5\,B\,a)}{27\,a^{14/3}}+\frac{4\,b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(11\,A\,b-5\,B\,a)}{27\,a^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(11*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (2*b^2*x^9*(11*A*b - 5*B*a))/(
9*a^4) + (16*b*x^6*(11*A*b - 5*B*a))/(45*a^3))/(a^2*x^5 + b^2*x^11 + 2*a*b*
x^8) + (4*b^(2/3)*log(b^(1/3)*x + a^(1/3))*(11*A*b - 5*B*a))/(27*a^(14/3))
- (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/
2 + 1/2)*(11*A*b - 5*B*a))/(27*a^(14/3)) + (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1
i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a))/(27*a^(
14/3))
```

**sympy [A]** time = 2.18, size = 173, normalized size = 0.70

$$\operatorname{RootSum}\left(19683t^3a^{14}-85184A^3b^5+116160A^2Bab^4-52800AB^2a^2b^3+8000B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{27ta^5}{-44Ab^2+20Bab}+x\right)\right)\right)+\frac{-18Aa^3+x^9\left(220Ab^3-100Bab^2\right)+x^6\left(352Aab^2-160Ba^2b\right)+x^3\left(99Aa^2b-45Ba^3\right)}{90a^6x^5+180a^5bx^8+90a^4b^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800*
A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, Lambda(_t, _t*log(-27*_t*a**5/(-44*
A*b**2 + 20*B*a*b) + x))) + (-18*A*a**3 + x**9*(220*A*b**3 - 100*B*a*b**2)
+ x**6*(352*A*a*b**2 - 160*B*a**2*b) + x**3*(99*A*a**2*b - 45*B*a**3))/(90*
a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11)
```

## 3.107 $\int \frac{x^8}{(a+bx^3)(c+dx^3)} \, dx$

**Optimal**. Leaf size=70

$$\frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)} + \frac{x^3}{3bd}$$

**Rubi** [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$\frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)} + \frac{x^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^3/(3*b*d) + (a^2*Log[a + b*x^3])/(3*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3
])/(3*d^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^3)(c+dx^3)} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)} \, dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right) dx, x, x^3\right) \\
&= \frac{x^3}{3bd} + \frac{a^2 \log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2 \log(c+dx^3)}{3d^2(bc-ad)}
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 66, normalized size = 0.94

$$\frac{a^2 d^2 \log(a+bx^3) - b\left(dx^3(ad-bc) + bc^2 \log(c+dx^3)\right)}{3b^2 d^2 (bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))/
(3*b^2*d^2*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[x^8/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 1.16, size = 72, normalized size = 1.03

$$\frac{a^2d^2\log\left(bx^3+a\right)-b^2c^2\log\left(dx^3+c\right)+\left(b^2cd-abd^2\right)x^3}{3\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)*
x^3)/(b^3*c*d^2 - a*b^2*d^3)
```

**giac [A]** time = 0.18, size = 70, normalized size = 1.00

$$\frac{a^2\log\left(\left|bx^3+a\right|\right)}{3\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(\left|dx^3+c\right|\right)}{3\left(bcd^2-ad^3\right)}+\frac{x^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a^2*log(abs(b*x^3 + a))/(b^3*c - a*b^2*d) - 1/3*c^2*log(abs(d*x^3 + c))
/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)
```

**maple [A]** time = 0.05, size = 65, normalized size = 0.93

$$-\frac{a^2\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,b^2}+\frac{x^3}{3bd}+\frac{c^2\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*x^3/b/d-1/3*a^2/b^2/(a*d-b*c)*ln(b*x^3+a)+1/3*c^2/d^2/(a*d-b*c)*ln(d*x^
3+c)
```

**maxima [A]** time = 0.59, size = 68, normalized size = 0.97

$$\frac{a^2\log\left(bx^3+a\right)}{3\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(dx^3+c\right)}{3\left(bcd^2-ad^3\right)}+\frac{x^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2
- a*d^3) + 1/3*x^3/(b*d)
```

**mupad [B]** time = 2.84, size = 68, normalized size = 0.97

$$\frac{a^2 \ln\left(b x^3 + a\right)}{3 b^3 c - 3 a b^2 d} + \frac{c^2 \ln\left(d x^3 + c\right)}{3 a d^3 - 3 b c d^2} + \frac{x^3}{3 b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (a^2*log(a + b*x^3))/(3*b^3*c - 3*a*b^2*d) + (c^2*log(c + d*x^3))/(3*a*d^3
- 3*b*c*d^2) + x^3/(3*b*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.108 $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=301

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2}\right.}{6d^{5/3}(bc-ad)}$$

**Rubi [A]** time = 0.31, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 584, 292, 31, 634, 617, 204, 628}

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}+\frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)}+\frac{c^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^2/(2*b*d) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(5/3)*(b*c - a*d)) + (c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)) - (a^(5/3)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)*(b*c - a*d)) + (c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(
5/3)*(b*c - a*d)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)*(b*c - a*d)) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*d^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{x^2}{2bd}-\frac{\int \frac{x\left(2ac+2(bc+ad)x^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{2bd}$$

$$=\frac{x^2}{2bd}-\frac{\int \left(\frac{2a^2dx}{(-bc+ad)\left(a+bx^3\right)}+\frac{2bc^2x}{(bc-ad)\left(c+dx^3\right)}\right)dx}{2bd}$$

$$=\frac{x^2}{2bd}+\frac{a^2\int \frac{x}{a+bx^3}\,dx}{b(bc-ad)}-\frac{c^2\int \frac{x}{c+dx^3}\,dx}{d(bc-ad)}$$

$$=\frac{x^2}{2bd}-\frac{a^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}(bc-ad)}+\frac{a^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{4/3}(bc-ad)}+\frac{c^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d^{4/3}(bc-ad)}-\frac{c^{5/3}\int \cdots}{3\cdots}$$

$$=\frac{x^2}{2bd}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}+\frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)}+\frac{a^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{5/3}(bc-ad)}+\cdots$$

$$=\frac{x^2}{2bd}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}+\frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/\cdots}\right.}{6b^{5/3}(bc-ad)}$$

$$=\frac{x^2}{2bd}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)}+\frac{c^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}+\frac{c^{5/3}\cdots}{3\cdots}$$

**Mathematica [A]**   time = 0.24, size = 242, normalized size = 0.80

$$\frac{\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{5/3}}-\frac{2a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{5/3}}-\frac{2\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}}-\frac{3ax^2}{b}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{5/3}}+\frac{2c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{5/3}}+\frac{2\sqrt{3}c^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{5/3}}+\frac{3cx^2}{d}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-3*a*x^2)/b + (3*c*x^2)/d - (2*Sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/b^(5/3) + (2*Sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) +
(2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2])/d^(5/3))/(6*b*c - 6*a*d)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[x^7/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas** [A] time = 1.36, size = 273, normalized size = 0.91

$$\frac{2\sqrt{3}\,ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3a}\right)-2\sqrt{3}\,bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,dx\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\,c}{3c}\right)+ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx^2-dx\left(-\frac{c^2}{d^2}\right)^{\frac{2}{3}}-c\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\right)-2\,ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)-2\,bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx+d\left(-\frac{c^2}{d^2}\right)^{\frac{2}{3}}\right)+3\,(bc-ad)x^2}{6\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*d*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3
) - sqrt(3)*a)/a) - 2*sqrt(3)*b*c*(-c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*
x*(-c^2/d^2)^(1/3) + sqrt(3)*c)/c) + a*d*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a
^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + b*c*(-c^2/d^2)^(1/3)*log(c*x^2 - d*x*(
-c^2/d^2)^(2/3) - c*(-c^2/d^2)^(1/3)) - 2*a*d*(a^2/b^2)^(1/3)*log(a*x + b*(
a^2/b^2)^(2/3)) - 2*b*c*(-c^2/d^2)^(1/3)*log(c*x + d*(-c^2/d^2)^(2/3)) + 3*
(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)
```

**giac** [A] time = 0.21, size = 311, normalized size = 1.03

$$-\frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{c^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2d-acd^2\right)}-\frac{\left(-ab^2\right)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^4c-\sqrt{3}\,ab^3d}+\frac{\left(-cd^2\right)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^3-\sqrt{3}\,ad^4}+\frac{\left(-ab^2\right)^{\frac{2}{3}}a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^4c-ab^3d\right)}-\frac{\left(-cd^2\right)^{\frac{2}{3}}c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^3-ad^4\right)}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) - (-a*b^2)^
(2/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^4*
c - sqrt(3)*a*b^3*d) + (-c*d^2)^(2/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-a*b^2)^(2/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^4*c - a*b^3*d) - 1/6*(-c*d^2)^
(2/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^3 - a*d^4) + 1/2*x^
2/(b*d)
```

**maple** [A] time = 0.05, size = 269, normalized size = 0.89

$$-\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{\sqrt{3}\,c^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2}-\frac{c^2\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2}+\frac{c^2\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d^2}+\frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/2*x^2/b/d+1/3*a^2/b^2/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*a^2/b^2
/(a*d-b*c)/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*a^2/b^2/(a*d-b
*c)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*c^2/d^2
/(a*d-b*c)/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6*c^2/d^2/(a*d-b*c)/(c/d)^(1/3)*
ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*c^2/d^2/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)
*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

**maxima** [A] time = 1.27, size = 324, normalized size = 1.08

$$\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c-ab^2d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,c^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2-ad^3\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
 - a*b^2*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/((b*c*d^2 - a*d^3)*(c/d)^(1/3)) + 1/6*a^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3)) - 1/6*c
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d
)^(1/3)) - 1/3*a^2*log(x + (a/b)^(1/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^
(1/3)) + 1/3*c^2*log(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d)^(1
/3)) + 1/2*x^2/(b*d)
```

**mupad** [B] time = 11.36, size = 1751, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((((27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))*(a^5/(b^5*(a*d
- b*c)^3))^(1/3))/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2
*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2
+ b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^
2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3) + log(((((27*a^2*b*c^2*d*x*(a^2*d^2 +
b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^
5*(a*d - b*c)^3))^(2/3))*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/3 - (9*(a*b^7*c^
8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d
- b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*
(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1
/3) - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*c^2*d*x*(a^2*d^
2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)
*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3)
)^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b
^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x*(a^2*d^2 + b^2*c^
2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*
d^2 - 81*a*b^7*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)^2
*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (
27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d
- b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3))^(1/3))/6 - (9*(a*b^7*c^8 + a
^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)
^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5
/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3)*(
3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*
```

```
c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1
)^2*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d
^5*(a*d - b*c)^3))^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a
^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x
*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d
^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)
*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)
^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/6
 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(
-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c
*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*
a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i - 1))/2 + x^2/(2*b*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

## 3.109 $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=296

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}(bc-ad)}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}\right.}{6d^{4/3}(bc-ad)}$$

**Rubi [A]** time = 0.27, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}(bc-ad)}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}-\frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{4/3}(bc-ad)}+\frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x/(b*d) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*b^(4/3)*(b*c - a*d)) + (c^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]
*c^(1/3))])/(Sqrt[3]*d^(4/3)*(b*c - a*d)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(4/3)*(b*c - a*d)) - (c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(4/3)
*(b*c - a*d)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*b^(4/3)*(b*c - a*d)) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(6*d^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{bd}\\
&= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^3}\,dx}{d(bc-ad)}\\
&= \frac{x}{bd} + \frac{a^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b(bc-ad)} + \frac{a^{4/3}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b(bc-ad)} - \frac{c^{4/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d(bc-ad)} - \frac{c^{4/3}\int
\\
&= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}(bc-ad)} -
\\
&= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\right.}{6b^{4/3}(bc-ad)}
\\
&= \frac{x}{bd} - \frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)} + \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/}}{}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 238, normalized size = 0.80

$$\frac{-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{4/3}}+\frac{2a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}}-\frac{2\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}-\frac{6ax}{b}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{4/3}}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{4/3}}+\frac{2\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{4/3}}+\frac{6cx}{d}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/
```

```
3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (2*c
^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2])/d^(4/3))/(6*b*c - 6*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[x^6/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 0.94, size = 228, normalized size = 0.77

$$-\frac{2\sqrt{3}\,ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+2\sqrt{3}\,bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,dx\left(\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,c}{3\,c}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)+2\,ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+2\,bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)-6\,(bc-ad)x}{6\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a*d*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - s
qrt(3)*a)/a) + 2*sqrt(3)*b*c*(c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c/d)^(2
/3) - sqrt(3)*c)/c) - a*d*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3)) - b*c*(c/d)^(1/3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)) + 2*a*d*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 2*b*c*(c/d)^(1/3)*log(x + (c/d)^(1/3)) - 6*
(b*c - a*d)*x)/(b^2*c*d - a*b*d^2)
```

**giac [A]** time = 0.21, size = 308, normalized size = 1.04

$$-\frac{a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^3c-\sqrt{3}\,ab^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^2-\sqrt{3}\,ad^3}+\frac{\left(-ab^2\right)^{\frac{1}{3}}a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c-ab^2d\right)}-\frac{\left(-cd^2\right)^{\frac{1}{3}}c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2-ad^3\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (-a*b^2)^
(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*
c - sqrt(3)*a*b^2*d) - (-c*d^2)^(1/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) + 1/6*(-a*b^2)^(1/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) - 1/6*(-c*d^2)^
(1/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3) + x/(b*d
)
```

**maple [A]** time = 0.05, size = 266, normalized size = 0.90

$$-\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{a^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{a^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{\sqrt{3}\,c^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{c^2\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}-\frac{c^2\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] x/b/d-1/3/b^2*a^2/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/b^2*a^2/(a*d-
b*c)/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2*a^2/(a*d-b*c)/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/d^2*c^2/(a*d-
b*c)/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d^2*c^2/(a*d-b*c)/(c/d)^(2/3)*ln(x^2
-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d^2*c^2/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

**maxima [A]** time = 1.35, size = 349, normalized size = 1.18

$$\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,c^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d^2*(c/d)^(1/3) - a*d^3*(
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*a^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) + 1/6*c^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + 1/3*a^2*log(x +
(a/b)^(1/3))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) - 1/3*c^2*log(x + (c
/d)^(1/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + x/(b*d)
```

**mupad [B]** time = 1.83, size = 873, normalized size = 2.95

$$\ln\left(a\,x+b^2\,c\,{\left(-\frac{a^4}{b^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}-a\,b\,d\,{\left(-\frac{a^4}{b^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\right)\,{\left(\frac{a^4}{-27\,a^3\,b^4\,d^3+81\,a^2\,b^5\,c\,d^2-81\,a\,b^6\,c^2\,d+27\,b^7\,c^3}\right)}^{1/3}+\ln\left(c\,x+a\,d^2\,{\left(\frac{c^4}{d^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}-b\,c\,d\,{\left(\frac{c^4}{d^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\right)\,{\left(\frac{c^4}{27\,a^3\,d^7-81\,a^2\,b\,c\,d^6+81\,a\,b^2\,c^2\,d^5-27\,b^3\,c^3\,d^4}\right)}^{1/3}+\frac{x}{b\,d}+\frac{\ln\left(\frac{3\,x\,\left(a^6\,c^2\,d^4+a^2\,b^4\,c^6\right)}{b\,d}-\frac{3\,a\,c^2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-\frac{a^4}{b^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\,\left(a^5\,d^5-a^4\,b\,c\,d^4+a\,b^4\,c^4\,d-b^5\,c^5\right)}{2\,d}\right)\,{\left(\frac{a^4}{-27\,a^3\,b^4\,d^3+81\,a^2\,b^5\,c\,d^2-81\,a\,b^6\,c^2\,d+27\,b^7\,c^3}\right)}^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{\ln\left(\frac{3\,x\,\left(a^6\,c^2\,d^4+a^2\,b^4\,c^6\right)}{b\,d}+\frac{3\,a\,c^2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(-\frac{a^4}{b^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\,\left(a^5\,d^5-a^4\,b\,c\,d^4+a\,b^4\,c^4\,d-b^5\,c^5\right)}{2\,d}\right)\,{\left(\frac{a^4}{-27\,a^3\,b^4\,d^3+81\,a^2\,b^5\,c\,d^2-81\,a\,b^6\,c^2\,d+27\,b^7\,c^3}\right)}^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+\frac{\ln\left(\frac{3\,x\,\left(a^6\,c^2\,d^4+a^2\,b^4\,c^6\right)}{b\,d}+\frac{3\,a^2\,c\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{c^4}{d^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\,\left(a^5\,d^5-a^4\,b\,c\,d^4+a\,b^4\,c^4\,d-b^5\,c^5\right)}{2\,b}\right)\,{\left(\frac{c^4}{27\,a^3\,d^7-81\,a^2\,b\,c\,d^6+81\,a\,b^2\,c^2\,d^5-27\,b^3\,c^3\,d^4}\right)}^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{\ln\left(\frac{3\,x\,\left(a^6\,c^2\,d^4+a^2\,b^4\,c^6\right)}{b\,d}-\frac{3\,a^2\,c\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{c^4}{d^4\,{\left(a\,d-b\,c\right)}^3}\right)}^{1/3}\,\left(a^5\,d^5-a^4\,b\,c\,d^4+a\,b^4\,c^4\,d-b^5\,c^5\right)}{2\,b}\right)\,{\left(\frac{c^4}{27\,a^3\,d^7-81\,a^2\,b\,c\,d^6+81\,a\,b^2\,c^2\,d^5-27\,b^3\,c^3\,d^4}\right)}^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^2*c*(-a^4/(b^4*(a*d - b*c)^3))^(1/3) - a*b*d*(-a^4/(b^4*(a*d -
b*c)^3))^(1/3))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a
*b^6*c^2*d))^(1/3) + log(c*x + a*d^2*(c^4/(d^4*(a*d - b*c)^3))^(1/3) - b*c*
d*(c^4/(d^4*(a*d - b*c)^3))^(1/3))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a
*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3) + x/(b*d) + (log((3*x*(a^2*b^4*c^6 +
a^6*c^2*d^4))/(b*d) - (3*a*c^2*(3^(1/2)*1i - 1)*(-a^4/(b^4*(a*d - b*c)^3))^
(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*
c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1
i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a*c^2*(3^(1/2
)*1i + 1)*(-a^4/(b^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d
 - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^
2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((3*x*(a^2*b^4*c^6 + a
^6*c^2*d^4))/(b*d) + (3*a^2*c*(3^(1/2)*1i - 1)*(c^4/(d^4*(a*d - b*c)^3))^(1
/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^
7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i
- 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) - (3*a^2*c*(3^(1/2)*
1i + 1)*(c^4/(d^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d -
a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 -
 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i + 1))/2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.110 $\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx$

**Optimal**. Leaf size=53

$$\frac{c\log(c+dx^3)}{3d(bc-ad)}-\frac{a\log(a+bx^3)}{3b(bc-ad)}$$

**Rubi** [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c\log(c+dx^3)}{3d(bc-ad)}-\frac{a\log(a+bx^3)}{3b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(a*Log[a + b*x^3])/(3*b*(b*c - a*d)) + (c*Log[c + d*x^3])/(3*d*(b*c - a*d)
)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx,x,x^3\right)\\ &= -\frac{a\log(a+bx^3)}{3b(bc-ad)}+\frac{c\log(c+dx^3)}{3d(bc-ad)}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 43, normalized size = 0.81

$$-\frac{ad\log(a+bx^3)-bc\log(c+dx^3)}{3b^2cd-3abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[x^5/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 0.99, size = 42, normalized size = 0.79

$$-\frac{ad\log\left(bx^3+a\right)-bc\log\left(dx^3+c\right)}{3\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)
```

**giac [A]** time = 0.19, size = 51, normalized size = 0.96

$$-\frac{a\log\left(\left|bx^3+a\right|\right)}{3\left(b^2c-abd\right)}+\frac{c\log\left(\left|dx^3+c\right|\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a*log(abs(b*x^3 + a))/(b^2*c - a*b*d) + 1/3*c*log(abs(d*x^3 + c))/(b*c
*d - a*d^2)
```

**maple [A]** time = 0.05, size = 50, normalized size = 0.94

$$\frac{a\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,b}-\frac{c\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*a/(a*d-b*c)/b*ln(b*x^3+a)-1/3*c/(a*d-b*c)/d*ln(d*x^3+c)
```

**maxima [A]** time = 0.46, size = 49, normalized size = 0.92

$$-\frac{a\log\left(bx^3+a\right)}{3\left(b^2c-abd\right)}+\frac{c\log\left(dx^3+c\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^2
)
```

**mupad [B]** time = 0.31, size = 51, normalized size = 0.96

$$-\frac{a\,\ln\left(b\,x^3+a\right)}{3\,b^2\,c-3\,a\,b\,d}-\frac{c\,\ln\left(d\,x^3+c\right)}{3\,a\,d^2-3\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] - (a*log(a + b*x^3))/(3*b^2*c - 3*a*b*d) - (c*log(c + d*x^3))/(3*a*d^2 - 3*
b*c*d)
```

**sympy** [B] time = 6.75, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^3+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{3b\,(ad-bc)}-\frac{c\log\left(x^3+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(3*d*(a*d - b*c))
```

## 3.111 $\int \frac{x^4}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=288

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}(bc-ad)}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+\right.}{6d^{2/3}(bc-ad)}$$

**Rubi [A]** time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}(bc-ad)}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}-\frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{2/3}(bc-ad)}-\frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3
)*(b*c - a*d)) - (c^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
2/3)*(b*c - a*d)) - (c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(2/3)*(b*c - a*
d)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(
b*c - a*d)) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{a\int \frac{x}{a+bx^3}\,dx}{bc-ad}+\frac{c\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{b}\,(bc-ad)}-\frac{a^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3\sqrt[3]{b}\,(bc-ad)}-\frac{c^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3\sqrt[3]{d}\,(bc-ad)}+\frac{c^{2/3}\int \frac{\sqrt[3]{c}+\ldots}{c^{2/3}-\sqrt[3]{c}\,\ldots}}{3\sqrt[3]{d}\,(bc\ldots}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)}-\frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{2/3}(bc-ad)}-\frac{a^{2/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6b^{2/3}(bc-ad)}-\frac{a\int \ldots}{2\ldots}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)}-\frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3d^{2/3}(bc-ad)}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\ldots$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}(bc-ad)}-\frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}(bc-ad)}-\frac{c^{2/3}\log\left(\sqrt[3]{c}\ldots\right.}{3d^{2/3}(bc\ldots}$$

**Mathematica [A]** time = 0.13, size = 224, normalized size = 0.78

$$\frac{-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{d^{2/3}}-\frac{2c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{d^{2/3}}-\frac{2\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{2/3}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) -
(2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) + (
2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(2/3))
/(6*b*c - 6*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas** [A] time = 1.05, size = 244, normalized size = 0.85

$$-\frac{2\sqrt{3}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-2\sqrt{3}\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}c}{3c}\right)-\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx^2-dx\left(\frac{c^2}{d^2}\right)^{\frac{2}{3}}+c\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\right)+2\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)+2\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\log\left(cx+d\left(\frac{c^2}{d^2}\right)^{\frac{2}{3}}\right)}{6(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3)
 + sqrt(3)*a)/a) - 2*sqrt(3)*(c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c^2
/d^2)^(1/3) - sqrt(3)*c)/c) - (-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(
2/3) - a*(-a^2/b^2)^(1/3)) - (c^2/d^2)^(1/3)*log(c*x^2 - d*x*(c^2/d^2)^(2/3
) + c*(c^2/d^2)^(1/3)) + 2*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)) +
 2*(c^2/d^2)^(1/3)*log(c*x + d*(c^2/d^2)^(2/3)))/(b*c - a*d)
```

**giac** [A] time = 0.25, size = 286, normalized size = 0.99

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c-ab^2d\right)}+\frac{\left(-cd^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2-ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(2/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b
^2*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3)
)/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^(2/3)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3)
```

**maple** [A] time = 0.05, size = 246, normalized size = 0.85

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{c\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d}-\frac{c\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*a/(a*d-b*c)/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*a/(a*d-b*c)/b/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*a/(a*d-b*c)*3^(1/2)/b/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*c/(a*d-b*c)/d/(c/d)^(1/3)*ln(x
+(c/d)^(1/3))-1/6*c/(a*d-b*c)/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3
))-1/3*c/(a*d-b*c)*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*
x-1))
```

**maxima** [A] time = 1.27, size = 289, normalized size = 1.00

$$-\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c-abd\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd-ad^2\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
- a*b*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c*d - a*d^2)*(c/d)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) + 1/6*c*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3)) + 1/3*
a*log(x + (a/b)^(1/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) - 1/3*c*log(
x + (c/d)^(1/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3))
```

**mupad [B]** time = 9.05, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^3*c^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3) + a^2*b*d^2*(-a^2/(b^2*(
a*d - b*c)^3))^(2/3) - 2*a*b^2*c*d*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))*(a^2/(
27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3) + l
og(c*x + a^2*d^3*(c^2/(d^2*(a*d - b*c)^3))^(2/3) + b^2*c^2*d*(c^2/(d^2*(a*d
 - b*c)^3))^(2/3) - 2*a*b*c*d^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3))*(c^2/(27*a
^3*d^5 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3) + (log(
((3^(1/2)*1i - 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(54
*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5*c^5*d
+ 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 27*a^3
*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1
)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*
(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(
a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b^5*c
^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 2
7*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i + 1))
/2 + (log(((3^(1/2)*1i - 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1
)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d^2
*(a*d - b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5
*c^5*d + 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5 -
 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i - 1
))/2 - (log(((3^(1/2)*1i + 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i +
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d
^2*(a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b
^5*c^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5
 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i +
 1))/2
```

**sympy [B]** time = 123.98, size = 573, normalized size = 1.99

$$\operatorname{RootSum}{\left(t^{3} \left(27 a^{3} d^{5} - 81 a^{2} b c d^{4} + 81 a b^{2} c^{2} d^{3} - 27 b^{3} c^{3} d^{2}\right) - c^{2}, \left( t \mapsto t \log{\left(x + \frac{243 t^{5} a^{6} b^{2} d^{8} - 1458 t^{5} a^{5} b^{3} c d^{7} + 3645 t^{5} a^{4} b^{4} c^{2} d^{6} - 4860 t^{5} a^{3} b^{5} c^{3} d^{5} + 3645 t^{5} a^{2} b^{6} c^{4} d^{4} - 1458 t^{5} a b^{7} c^{5} d^{3} + 243 t^{5} b^{8} c^{6} d^{2} + 9 t^{2} a^{5} d^{5} - 18 t^{2} a^{4} b c d^{4} + 9 t^{2} a^{3} b^{2} c^{2} d^{3} + 9 t^{2} a^{2} b^{3} c^{3} d^{2} - 18 t^{2} a b^{4} c^{4} d + 9 t^{2} b^{5} c^{5}}{a^{3} c d^{2} + a b^{2} c^{3}} \right)} \right)\right)} + \operatorname{RootSum}{\left(t^{3} \left(27 a^{3} b^{2} d^{3} - 81 a^{2} b^{3} c d^{2} + 81 a b^{4} c^{2} d - 27 b^{5} c^{3}\right) + a^{2}, \left( t \mapsto t \log{\left(x + \frac{243 t^{5} a^{6} b^{2} d^{8} - 1458 t^{5} a^{5} b^{3} c d^{7} + 3645 t^{5} a^{4} b^{4} c^{2} d^{6} - 4860 t^{5} a^{3} b^{5} c^{3} d^{5} + 3645 t^{5} a^{2} b^{6} c^{4} d^{4} - 1458 t^{5} a b^{7} c^{5} d^{3} + 243 t^{5} b^{8} c^{6} d^{2} + 9 t^{2} a^{5} d^{5} - 18 t^{2} a^{4} b c d^{4} + 9 t^{2} a^{3} b^{2} c^{2} d^{3} + 9 t^{2} a^{2} b^{3} c^{3} d^{2} - 18 t^{2} a b^{4} c^{4} d + 9 t^{2} b^{5} c^{5}}{a^{3} c d^{2} + a b^{2} c^{3}} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**5 - 81*a**2*b*c*d**4 + 81*a*b**2*c**2*d**3 - 27*b
**3*c**3*d**2) - c**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 14
58*_t**5*a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**
```

```
3*b**5*c**3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*
d**3 + 243*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**
4 + 9*_t**2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*
b**4*c**4*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3)))) + RootSum(_
t**3*(27*a**3*b**2*d**3 - 81*a**2*b**3*c*d**2 + 81*a*b**4*c**2*d - 27*b**5*
c**3) + a**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 1458*_t**5*
a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**3*b**5*c*
*3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*d**3 + 24
3*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**4 + 9*_t*
*2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*b**4*c**4
*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3))))
```

## 3.112 $\int \frac{x^3}{(a+bx^3)(c+dx^3)} \, dx$

**Optimal**. Leaf size=288

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{3\sqrt[3]{d}(bc-ad)}$$

**Rubi [A]** time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}+\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3
)*(b*c - a*d)) - (c^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(1/3)*(b*c - a*d)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
1/3)*(b*c - a*d)) + (c^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(1/3)*(b*c - a*
d)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)*(
b*c - a*d)) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{a\int \frac{1}{a+bx^3}\,dx}{bc-ad}+\frac{c\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3(bc-ad)}-\frac{\sqrt[3]{a}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3(bc-ad)}+\frac{\sqrt[3]{c}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3(bc-ad)}+\frac{\sqrt[3]{c}\int \frac{\cdots}{c^{2/3}-\cdots}}{3(\cdots}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}-\frac{a^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2(bc-ad)}+\cdots$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\cdots\right)}{6\sqrt[3]{b}(bc-ad)}$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log(\cdots}{3\sqrt[3]{d}\cdots}$$

**Mathematica [A]** time = 0.12, size = 224, normalized size = 0.78

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{\sqrt[3]{d}}+\frac{2\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{d}}-\frac{2\sqrt{3}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) -
(2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) - (
2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(1/3))
/(6*b*c - 6*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas** [A] time = 0.80, size = 199, normalized size = 0.69

$$-\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)+2\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{6\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)
*a)/a) + 2*sqrt(3)*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(-c/d)^(2/3) - sq
rt(3)*c)/c) - (a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - (-c/d)^(
1/3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)) + 2*(a/b)^(1/3)*log(x + (a/b)
^(1/3)) + 2*(-c/d)^(1/3)*log(x - (-c/d)^(1/3)))/(b*c - a*d)
```

**giac** [A] time = 0.20, size = 278, normalized size = 0.97

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd}+\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c-abd\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(1/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b
*d) + (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/
(sqrt(3)*b*c*d - sqrt(3)*a*d^2) - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(b^2*c - a*b*d) + 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)^
(1/3) + (-c/d)^(2/3))/(b*c*d - a*d^2)
```

**maple** [A] time = 0.05, size = 246, normalized size = 0.85

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{c\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{c\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*a/(a*d-b*c)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*a/(a*d-b*c)/b/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*a/(a*d-b*c)/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*c/(a*d-b*c)/d/(c/d)^(2/3)*ln(x+
(c/d)^(1/3))+1/6*c/(a*d-b*c)/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3)
)-1/3*c/(a*d-b*c)/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x
-1))
```

**maxima** [A] time = 1.21, size = 317, normalized size = 1.10

$$-\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c*
(a/b)^(1/3) - a*b*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c*(a/
b)^(2/3) - a*b*d*(a/b)^(2/3)) - 1/6*c*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2
*c*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) + 1/3*c*log(x + (c/d)^(1/3))/(b*c*d*(c/
d)^(2/3) - a*d^2*(c/d)^(2/3))
```

**mupad [B]** time = 8.12, size = 1265, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(x + a*d*(a/(b*(a*d - b*c)^3))^(1/3) - b*c*(a/(b*(a*d - b*c)^3))^(1/3))*
(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3)
+ log(x - a*d*(-c/(d*(a*d - b*c)^3))^(1/3) + b*c*(-c/(d*(a*d - b*c)^3))^(1/
3))*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1
/3) + (log(((3^(1/2)*1i - 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)
^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d
+ b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^
(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b
^3*c^2*d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-a/
(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(8
1*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c
)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^(2/3)
)/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^
2*d^4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-a/(27*b
^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 + (log
(((3^(1/2)*1i - 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a*
b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a
*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/
36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*
d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-c/(27*a^3
*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2 - (log((
(3^(1/2)*1i + 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a*b^
3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/36
 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^
4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-c/(27*a^3*d
^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2
```

**sympy [A]** time = 20.20, size = 342, normalized size = 1.19

$$\operatorname{RootSum}{\left(t^{3} \left(27 a^{3} d^{4} - 81 a^{2} b c d^{3} + 81 a b^{2} c^{2} d^{2} - 27 b^{3} c^{3} d\right) + c, \left( t \mapsto t \log{\left(x + \frac{162 t^{4} a^{4} b d^{5} - 648 t^{4} a^{3} b^{2} c d^{4} + 972 t^{4} a^{2} b^{3} c^{2} d^{3} - 648 t^{4} a b^{4} c^{3} d^{2} + 162 t^{4} b^{5} c^{4} d - 3 t a^{2} d^{2} + 6 t a b c d - 3 t b^{2} c^{2}}{a d + b c} \right)} \right)\right)} + \operatorname{RootSum}{\left(t^{3} \left(27 a^{3} b d^{3} - 81 a^{2} b^{2} c d^{2} + 81 a b^{3} c^{2} d - 27 b^{4} c^{3}\right) - a, \left( t \mapsto t \log{\left(x + \frac{162 t^{4} a^{4} b d^{5} - 648 t^{4} a^{3} b^{2} c d^{4} + 972 t^{4} a^{2} b^{3} c^{2} d^{3} - 648 t^{4} a b^{4} c^{3} d^{2} + 162 t^{4} b^{5} c^{4} d - 3 t a^{2} d^{2} + 6 t a b c d - 3 t b^{2} c^{2}}{a d + b c} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*b
**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4*a
**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*d**
2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c**2)
/(a*d + b*c)))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2 + 81*
a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*
d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t*
```

```
*4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d
 - 3*_t*b**2*c**2)/(a*d + b*c))))
```

## 3.113 $\int \frac{x^2}{(a+bx^3)(c+dx^3)} \, dx$

**Optimal**. Leaf size=45

$$\frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[a + b*x^3]/(3*(b*c - a*d)) - Log[c + d*x^3]/(3*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^3)(c+dx^3)} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)} \, dx, x, x^3\right) \\
&= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx} \, dx, x, x^3\right)}{3(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{1}{c+dx} \, dx, x, x^3\right)}{3(bc-ad)} \\
&= \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^3) - \log(c+dx^3)}{3bc-3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)*(c + d*x^3)),x]
[Out] IntegrateAlgebraic[x^2/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 0.84, size = 31, normalized size = 0.69

$$\frac{\log\left(bx^3+a\right)-\log\left(dx^3+c\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
[Out] 1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)
```

**giac [A]** time = 0.18, size = 51, normalized size = 1.13

$$\frac{b\log\left(\left|bx^3+a\right|\right)}{3\left(b^2c-abd\right)}-\frac{d\log\left(\left|dx^3+c\right|\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
[Out] 1/3*b*log(abs(b*x^3 + a))/(b^2*c - a*b*d) - 1/3*d*log(abs(d*x^3 + c))/(b*c*
d - a*d^2)
```

**maple [A]** time = 0.05, size = 42, normalized size = 0.93

$$-\frac{\ln\left(b\,x^3+a\right)}{3\,(ad-bc)}+\frac{\ln\left(d\,x^3+c\right)}{3ad-3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c),x)
[Out] -1/3/(a*d-b*c)*ln(b*x^3+a)+1/3/(a*d-b*c)*ln(d*x^3+c)
```

**maxima [A]** time = 0.48, size = 41, normalized size = 0.91

$$\frac{\log\left(bx^3+a\right)}{3\,(bc-ad)}-\frac{\log\left(dx^3+c\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] 1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)
```

**mupad [B]** time = 0.26, size = 602, normalized size = 13.38

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3\right)1i}{3\,a\,d-3\,b\,c}-\frac{\left(\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)-\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}-6\,b^3\,d^3\,x^3\right)1i}{3\,a\,d-3\,b\,c}}{\frac{\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3}{3\,a\,d-3\,b\,c}+\frac{\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)-\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}-6\,b^3\,d^3\,x^3}{3\,a\,d-3\,b\,c}}\right)2i}{3\,a\,d-3\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] -(atan(((((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b^
5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*
d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)*1i)/(3*a*d -
3*b*c) - (((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)*1i)/(3*a*d -
 3*b*c))/(((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)/(3*a*d - 3*b
*c) + ((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b^5*c
^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*d -
 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)/(3*a*d - 3*b*c))
)*2i)/(3*a*d - 3*b*c)
```

**sympy [B]** time = 2.47, size = 138, normalized size = 3.07

$$\frac{\log\left(x^3 + \frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3\,(ad-bc)} - \frac{\log\left(x^3 + \frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(3*(a*d - b*c))
```

# 3.114 $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=288

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{3\sqrt[3]{c}(bc-ad)}$$

**Rubi** [A] time = 0.14, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {482, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1
/3)*(b*c - a*d))) + (d^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*(b*c - a*d)) + (d^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c -
 a*d)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3
)*(b*c - a*d)) - (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\int \frac{x}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}(bc-ad)} + \frac{d^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3\sqrt[3]{c}(bc-ad)} - \frac{d^{2/3}\int \frac{\cdots}{c^{2/3}\cdots}}{3\sqrt[3]{\cdots}}$$

$$= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}(bc-ad)} + \frac{b^{2}\cdots}{\cdots}$$

$$= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^{\cdots}\right)}{6\sqrt[3]{a}(bc-ad)}$$

$$= -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\cdots}{3\sqrt[3]{\cdots}}$$

**Mathematica [A]** time = 0.14, size = 224, normalized size = 0.78

$$\frac{-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{\sqrt[3]{c}} - \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{c}} - \frac{2\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) -
(2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(1/3) + (
2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(1/3))
/(-6*b*c + 6*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[x/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 0.84, size = 201, normalized size = 0.70

$$\frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^{2}-cx\left(-\frac{d}{c}\right)^{\frac{2}{3}}-c\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)-2\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)-2\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3))
- 2*sqrt(3)*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3)) +
 (b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + (-d/c)^(1/3)*lo
g(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 2*(b/a)^(1/3)*log(b*x + a*(b
/a)^(2/3)) - 2*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)))/(b*c - a*d)
```

**giac [A]** time = 0.25, size = 290, normalized size = 1.01

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^{2}d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{2}-acd\right)}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,ab^{2}c-\sqrt{3}\,a^{2}bd}+\frac{\left(-cd^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^{2}d-\sqrt{3}\,acd^{2}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^{2}c-a^{2}bd\right)}-\frac{\left(-cd^{2}\right)^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{2}d-acd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(2/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c - sqrt(3)*
a^2*b*d) + (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^(2/3)*log(x^
2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2*d - a*c*d^2)
```

**maple [A]** time = 0.05, size = 222, normalized size = 0.77

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/(a*d-b*c)/(a/b)^(1/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d-b*c)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d-b*c)/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6
/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/(a*d-b*c)*3^(1
/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

**maxima [A]** time = 1.29, size = 265, normalized size = 0.92

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(bc-ad)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(bc-ad)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c - a*d
)*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(
1/3))/((b*c - a*d)*(c/d)^(1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) - 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)
^(2/3))/(b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3)) - 1/3*log(x + (a/b)^(1/3))/(b*c
*(a/b)^(1/3) - a*d*(a/b)^(1/3)) + 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(1/3)
 - a*d*(c/d)^(1/3))
```

**mupad** [**B**] time = 5.42, size = 982, normalized size = 3.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b*x + a^3*d^2*(b/(a*(a*d - b*c)^3))^(2/3) + a*b^2*c^2*(b/(a*(a*d - b*c)
^3))^(2/3) - 2*a^2*b*c*d*(b/(a*(a*d - b*c)^3))^(2/3))*(b/(27*a^4*d^3 - 27*a
*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3) + log(d*x + b^2*c^3*(-
d/(c*(a*d - b*c)^3))^(2/3) + a^2*c*d^2*(-d/(c*(a*d - b*c)^3))^(2/3) - 2*a*b
*c^2*d*(-d/(c*(a*d - b*c)^3))^(2/3))*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81*a^2
*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3) + (log(b^4*d^4*x - (b*(3^(1/2)*1i - 1)^
3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2
)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(216
*a*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^2*
b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 - (log(b^4*d^4*x + (b*(3^(1/2)*1i + 1
)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1
/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(2
16*a*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^
2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 + (log(b^4*d^4*x + (d*(3^(1/2)*1i -
 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^
(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/4))
/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81
*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2 - (log(b^4*d^4*x - (d*(3^(1/2)*1
i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*
(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/
4))/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 +
 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2
```

**sympy** [**A**] time = 14.51, size = 515, normalized size = 1.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**4*d**3 - 81*a**3*b*c*d**2 + 81*a**2*b**2*c**2*d - 27*a
*b**3*c**3) - b, Lambda(_t, _t*log(x + (243*_t**5*a**7*c*d**6 - 1458*_t**5*
a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860*_t**5*a**4*b**3*c*
*4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a**2*b**5*c**6*d + 24
3*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*b*c*d**3 + 18*_t**2
*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*b**4*c**4)/(a*b*d**
2 + b**2*c*d)))) + RootSum(_t**3*(27*a**3*c*d**3 - 81*a**2*b*c**2*d**2 + 81
*a*b**2*c**3*d - 27*b**3*c**4) + d, Lambda(_t, _t*log(x + (243*_t**5*a**7*c
*d**6 - 1458*_t**5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860
*_t**5*a**4*b**3*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a*
*2*b**5*c**6*d + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*
b*c*d**3 + 18*_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*
b**4*c**4)/(a*b*d**2 + b**2*c*d))))
```

## 3.115 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=288

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}\right.}{6c^{2/3}(bc-ad)}$$

**Rubi [A]** time = 0.14, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)}+\frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)} + \frac{b\int}{}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log}{3c^{2}}$$

**Mathematica [A]** time = 0.18, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} - \frac{2\sqrt{3}d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)), x]
```

**fricas** [A] time = 0.98, size = 254, normalized size = 0.88

$$-\frac{2\,\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+2\,\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+2\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)}{6\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

**giac** [A] time = 0.21, size = 278, normalized size = 0.97

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,abc-\sqrt{3}\,a^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^2-\sqrt{3}\,acd}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc-a^2d\right)}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2-acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

**maple** [A] time = 0.05, size = 222, normalized size = 0.77

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d-b*c)/(a/b)^(2/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(a*d-b*c)/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/
6/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/(a*d-b*c)/(c/
d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

**maxima** [A] time = 1.34, size = 293, normalized size = 1.02

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)
^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*
x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) -
a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2
/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

**mupad [B]**    time = 9.01, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

**sympy [A]**    time = 133.33, size = 447, normalized size = 1.55

$$\operatorname{RootSum}\left(t^3\left(27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3\right) + b^2, \left(t \mapsto t\log\left(x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162t^4a^5b^2c^4d^3 + 162t^4a^4b^3c^5d^2 - 243t^4a^3b^4c^6d + 81t^4a^2b^5c^7 - 3ta^4d^4 + 3ta^3bcd^3 + 3tab^3c^3d - 3tb^4c^4}{a^2bd^3 + b^3c^2d}\right)\right)\right) + \operatorname{RootSum}\left(t^3\left(27a^3c^2d^3 - 81a^2bc^3d^2 + 81ab^2c^4d - 27b^3c^5\right) - d^2, \left(t \mapsto t\log\left(x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162t^4a^5b^2c^4d^3 + 162t^4a^4b^3c^5d^2 - 243t^4a^3b^4c^6d + 81t^4a^2b^5c^7 - 3ta^4d^4 + 3ta^3bcd^3 + 3tab^3c^3d - 3tb^4c^4}{a^2bd^3 + b^3c^2d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*a
**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243
*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b*
*3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*
a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2
```

```
*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c*
*3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x + (8
1*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c
**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*
_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c*
*3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

### 3.116 $\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=62

$$-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}+\frac{\log(x)}{ac}$$

**Rubi [A]** time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^3])/(3*a*(b*c - a*d)) + (d*Log[c + d*x^3])/(3
*c*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^3\right)\\
&= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^3\right)+ad\log\left(c+dx^3\right)-3ad\log(x)+3bc\log(x)}{3abc^2-3a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3*
a*b*c^2 - 3*a^2*c*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 1.56, size = 54, normalized size = 0.87

$$-\frac{bc\log\left(bx^3+a\right)-ad\log\left(dx^3+c\right)-3\,(bc-ad)\log(x)}{3\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

**giac [A]** time = 0.18, size = 71, normalized size = 1.15

$$-\frac{b^2\log\left(\left|bx^3+a\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^3+c\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\log\left(|x|\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^2*log(abs(b*x^3 + a))/(a*b^2*c - a^2*b*d) + 1/3*d^2*log(abs(d*x^3 +
c))/(b*c^2*d - a*c*d^2) + log(abs(x))/(a*c)
```

**maple [A]** time = 0.05, size = 59, normalized size = 0.95

$$\frac{b\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,a}-\frac{d\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,c}+\frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*b/a/(a*d-b*c)*ln(b*x^3+a)-1/3*d/c/(a*d-b*c)*ln(d*x^3+c)+ln(x)/a/c
```

**maxima [A]** time = 0.54, size = 61, normalized size = 0.98

$$-\frac{b\log\left(bx^3+a\right)}{3\left(abc-a^2d\right)}+\frac{d\log\left(dx^3+c\right)}{3\left(bc^2-acd\right)}+\frac{\log\left(x^3\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*d
) + 1/3*log(x^3)/(a*c)
```

**mupad [B]** time = 2.84, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^3+a\right)}{3\,a^2\,d-3\,a\,b\,c}+\frac{d\,\ln\left(d\,x^3+c\right)}{3\,b\,c^2-3\,a\,c\,d}+\frac{\ln(x)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b*log(a + b*x^3))/(3*a^2*d - 3*a*b*c) + (d*log(c + d*x^3))/(3*b*c^2 - 3*a*
c*d) + log(x)/(a*c)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.117 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=299

$$-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}(bc-ad)}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}\right.}{6c^{4/3}(bc-ad)}$$

**Rubi [A]** time = 0.27, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}(bc-ad)}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{4/3}(bc-ad)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(4/3)*(b*c - a*d)) - (d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)) + (b^(4/3)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(4/3)*(b*c - a*d)) - (d^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^
(4/3)*(b*c - a*d)) - (b^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(6*a^(4/3)*(b*c - a*d)) + (d^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(6*c^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{acx}+\frac{\int \frac{x\left(-bc-ad-bdx^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{ac}$$

$$= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx}{(bc-ad)\left(a+bx^3\right)}-\frac{ad^2x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{ac}$$

$$= -\frac{1}{acx}-\frac{b^2\int \frac{x}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x}{c+dx^3}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{acx}+\frac{b^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}(bc-ad)}-\frac{b^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}(bc-ad)}-\frac{d^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{4/3}(bc-ad)}+\cdots$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x\cdots}}{6a^{4/3}(bc-ad)}\cdots$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots\right.}{6a^{4/3}(bc-a\cdots}$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}(bc-ad)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}\cdots$$

**Mathematica [A]**  time = 0.21, size = 244, normalized size = 0.82

$$\frac{\frac{b^{4/3}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}}-\frac{2b^{4/3}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}-\frac{2\sqrt{3}\,b^{4/3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{6b}{a}-\frac{d^{4/3}x\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{4/3}}+\frac{2d^{4/3}x\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{4/3}}+\frac{2\sqrt{3}\,d^{4/3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{4/3}}-\frac{6d}{c}}{6adx-6bcx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (2*Sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(4/3) + (2*Sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3
))/Sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (2*
d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 0.89, size = 238, normalized size = 0.80

$$-\frac{2\sqrt{3}\,bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\,adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)-adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^2-cx\left(\frac{d}{c}\right)^{\frac{2}{3}}+c\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)+2\,bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+2\,adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)+6\,bc-6\,ad}{6\left(abc^2-a^2cd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*
sqrt(3)) - 2*sqrt(3)*a*d*x*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(d/c)^(1/3) - 1
/3*sqrt(3)) - b*c*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1
/3)) - a*d*x*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 2*b
*c*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 2*a*d*x*(d/c)^(1/3)*log(d*x +
 c*(d/c)^(2/3)) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)
```

**giac [A]** time = 0.25, size = 305, normalized size = 1.02

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2bc-\sqrt{3}\,a^3d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc-a^3d\right)}+\frac{\left(-cd^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) + (-a*b^2)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c - s
qrt(3)*a^3*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(2/3)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(2/3)*log(x
^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/(a*c*x)
```

**maple [A]** time = 0.06, size = 257, normalized size = 0.86

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{b\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{b\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c}+\frac{d\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c}-\frac{d\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*b/a/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*b/a/(a*d-b*c)/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b/a/(a*d-b*c)*3^(1/2)/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d/c/(a*d-b*c)/(c/d)^(1/3)*ln(x
+(c/d)^(1/3))-1/6*d/c/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3
))-1/3*d/c/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*
x-1))-1/a/c/x
```

**maxima [A]** time = 1.20, size = 300, normalized size = 1.00

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc-a^{2}d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{2}-acd\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
- a^2*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c^2 - a*c*d)*(c/d)^(1/3)) - 1/6*b*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) + 1/6*d*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) + 1/3*
b*log(x + (a/b)^(1/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) - 1/3*d*log(
x + (c/d)^(1/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) - 1/(a*c*x)
```

**mupad [B]** time = 3.85, size = 716, normalized size = 2.39

$$\ln\left(b-a^{2}dx\left(-\frac{b^{4}}{a^{4}(ad-bc)^{3}}\right)^{1/3}+abcx\left(-\frac{b^{4}}{a^{4}(ad-bc)^{3}}\right)^{1/3}\right)\left(-\frac{b^{4}}{27a^{7}d^{3}-81a^{6}bcd^{2}+81a^{5}b^{2}c^{2}d-27a^{4}b^{3}c^{3}}\right)^{1/3}+\ln\left(d-bc^{2}x\left(\frac{d^{4}}{c^{4}(ad-bc)^{3}}\right)^{1/3}+acdx\left(\frac{d^{4}}{c^{4}(ad-bc)^{3}}\right)^{1/3}\right)\left(-\frac{d^{4}}{-27a^{3}c^{4}d^{3}+81a^{2}bc^{5}d^{2}-81ab^{2}c^{6}d+27b^{3}c^{7}}\right)^{1/3}-\frac{1}{acx}-\frac{\ln\left(b+2a^{2}dx\left(-\frac{b^{4}}{a^{4}(ad-bc)^{3}}\right)^{1/3}-2abcx\left(-\frac{b^{4}}{a^{4}(ad-bc)^{3}}\right)^{1/3}-\sqrt{3}\,b\,1\mathrm{i}\right)\left(-\frac{b^{4}}{27a^{7}d^{3}-81a^{6}bcd^{2}+81a^{5}b^{2}c^{2}d-27a^{4}b^{3}c^{3}}\right)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(b+2a^{2}dx\left(-\frac{b^{4}}{a^{4}(ad-bc)^{3}}\right)^{1/3}-2abcx\left(-\frac{b^{4}}{a^{4}(ad-bc)^{3}}\right)^{1/3}+\sqrt{3}\,b\,1\mathrm{i}\right)\left(-\frac{b^{4}}{27a^{7}d^{3}-81a^{6}bcd^{2}+81a^{5}b^{2}c^{2}d-27a^{4}b^{3}c^{3}}\right)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{\ln\left(d+2bc^{2}x\left(\frac{d^{4}}{c^{4}(ad-bc)^{3}}\right)^{1/3}-2acdx\left(\frac{d^{4}}{c^{4}(ad-bc)^{3}}\right)^{1/3}-\sqrt{3}\,d\,1\mathrm{i}\right)\left(-\frac{d^{4}}{-27a^{3}c^{4}d^{3}+81a^{2}bc^{5}d^{2}-81ab^{2}c^{6}d+27b^{3}c^{7}}\right)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(d+2bc^{2}x\left(\frac{d^{4}}{c^{4}(ad-bc)^{3}}\right)^{1/3}-2acdx\left(\frac{d^{4}}{c^{4}(ad-bc)^{3}}\right)^{1/3}+\sqrt{3}\,d\,1\mathrm{i}\right)\left(-\frac{d^{4}}{-27a^{3}c^{4}d^{3}+81a^{2}bc^{5}d^{2}-81ab^{2}c^{6}d+27b^{3}c^{7}}\right)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b - a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) + a*b*c*x*(-b^4/(a^4*(a*d
- b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 8
1*a^6*b*c*d^2))^(1/3) + log(d - b*c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3) + a
*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3
+ 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3) - 1/(a*c*x) - (log(b - 3^(1/2)*
b*1i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a
*d - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d
- 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(b + 3^(1/2)*b*1i + 2*a^
2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a*d - b*c)^3
))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 81*a^6*b*
c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(d - 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/
(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d
^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)
*(3^(1/2)*1i + 1))/2 + (log(d + 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/(c^4*(a*d - b
*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^
7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)*(3^(1/2)*1i
- 1))/2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.118 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=301

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x\right.}{6c^{5/3}(bc-ad)}$$

**Rubi [A]** time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 522, 200, 31, 634, 617, 204, 628}

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(2*a*c*x^2) + (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(5/3)*(b*c - a*d)) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(
Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)) - (b^(5/3)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(5/3)*(b*c - a*d)) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*
c^(5/3)*(b*c - a*d)) + (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(5/3)*(b*c - a*d)) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2])/(6*c^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{2acx^2}+\frac{\int \frac{-2(bc+ad)-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{2ac}$$

$$= -\frac{1}{2acx^2}-\frac{b^2\int \frac{1}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^3}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{2acx^2}-\frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}(bc-ad)}-\frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}(bc-ad)}+\frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{5/3}(bc-ad)}+\frac{d^2 \cdots}{\cdots}$$

$$= -\frac{1}{2acx^2}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)}+\frac{b^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}\cdots}}{6a^{5/3}(bc-ad)\cdots}$$

$$= -\frac{1}{2acx^2}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\cdots\right.}{6a^{5/3}(bc-\cdots}$$

$$= -\frac{1}{2acx^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}(bc-ad)}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{3a^{5/3}(bc-ad)}$$

**Mathematica [A]** time = 0.22, size = 259, normalized size = 0.86

$$\frac{\frac{2b^{5/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{b^{5/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}-\frac{2\sqrt{3}b^{5/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{3b}{a}-\frac{2d^{5/3}x^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}}+\frac{d^{5/3}x^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}}+\frac{2\sqrt{3}d^{5/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}}-\frac{3d}{c}}{6x^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (2*Sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(5/3) + (2*Sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/c^
```

```
(1/3))/Sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5/3)
 - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a+bx^3\right) \left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 2.62, size = 301, normalized size = 1.00

$$-\frac{2\sqrt{3}\,bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+2\sqrt{3}\,adx^2\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(-\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right)-bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-adx^2\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2+cdx\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(-\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+2\,bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+2\,adx^2\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx-c\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)+3\,bc-3\,ad}{6\left(abc^2-a^2cd\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)
^(2/3) - sqrt(3)*b)/b) + 2*sqrt(3)*a*d*x^2*(-d^2/c^2)^(1/3)*arctan(1/3*(2*s
qrt(3)*c*x*(-d^2/c^2)^(2/3) - sqrt(3)*d)/d) - b*c*x^2*(b^2/a^2)^(1/3)*log(b
^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - a*d*x^2*(-d^2/c^2)^
(1/3)*log(d^2*x^2 + c*d*x*(-d^2/c^2)^(1/3) + c^2*(-d^2/c^2)^(2/3)) + 2*b*c*
x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*a*d*x^2*(-d^2/c^2)^(1/
3)*log(d*x - c*(-d^2/c^2)^(1/3)) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*d)*x^2)
```

**giac [A]** time = 0.21, size = 309, normalized size = 1.03

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}-\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2bc-\sqrt{3}\,a^3d}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}-\frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc-a^3d\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)}-\frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) - (-a*b^2)^(1/3)
*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c -
 sqrt(3)*a^3*d) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/
(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(1/3)*b*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(1/3)
*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/2/(a*c*x^
2)
```

**maple [A]** time = 0.05, size = 257, normalized size = 0.85

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{b\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{b\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}c}-\frac{d\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}c}+\frac{d\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}c}-\frac{1}{2ac\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3/a*b/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/a*b/(a*d-b*c)/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/a*b/(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/2/a/c/x^2-1/3/c*d/(a*d-b*c)/(c/d)
^(2/3)*ln(x+(c/d)^(1/3))+1/6/c*d/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))-1/3/c*d/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
c/d)^(1/3)*x-1))
```

**maxima [A]** time = 1.14, size = 328, normalized size = 1.09

$$-\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^{2} d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}} - acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{b \log\left(x^{2} - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^{2} d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{d \log\left(x^{2} - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}} - acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{b \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}} - a^{2} d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{d \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}} - acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{1}{2\,acx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c*
(a/b)^(1/3) - a^2*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*c*(a/
b)^(2/3) - a^2*d*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/3*b*log(x + (a/b)^(1/3))/(a*b
*c*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) + 1/3*d*log(x + (c/d)^(1/3))/(b*c^2*(c/
d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/2/(a*c*x^2)
```

**mupad [B]** time = 11.83, size = 1829, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a
*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/9 + 9
*a^6*b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^
6*d^9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3 - 27*
a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3) + log(((-d^5/(c^5*(
a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^
5/(c^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2
+ b^2*c^2 + a*b*c*d))*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/9 + 9*a^6*b^9*c^11*
d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^6*d^9))/3 + 3*
a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c^5*d^3 +
81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(b^5/(a^
5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1
)*(a*d + b*c)*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(
a*d - b*c)^3))^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10
*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*
c^2))*(b^5/(27*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(b^5/(a^5*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^
2 + b^2*c^2 + a*b*c*d) + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)
*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8
- 9*a^11*b^4*c^6*d^9))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(2
7*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3)*(3^(
1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(
((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2
+ a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)
```

```
^4*(-d^5/(c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/3
6 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b
^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8
- 27*a^3*c^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i -
1))/2 - (log(((3^(1/2)*1i + 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*
1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d)
 + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-d^5/(
c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/36 - 9*a^6*
b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9
))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c
^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i + 1))/2 - 1/
(2*a*c*x^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.119 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)} dx$

**Optimal**. Leaf size=87

$$\frac{b^2 \log \left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2 \log \left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2 \log \left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2 \log \left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(3*a*c*x^3) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^3])/(3*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)} dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)\right.\\
&= -\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2 \log \left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{d^2 \log \left(c+dx^3\right)}{3c^2(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 88, normalized size = 1.01

$$-\frac{b^2 \log \left(a+bx^3\right)}{3a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2 \log \left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3])
/(3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 4.79, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^3\log\left(bx^3+a\right)-a^2d^2x^3\log\left(dx^3+c\right)-3\left(b^2c^2-a^2d^2\right)x^3\log(x)-abc^2+a^2cd}{3\left(a^2bc^3-a^3c^2d\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2 -
 a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)
```

**giac [A]** time = 0.19, size = 111, normalized size = 1.28

$$\frac{b^3\log\left(\left|bx^3+a\right|\right)}{3\left(a^2b^2c-a^3bd\right)}-\frac{d^3\log\left(\left|dx^3+c\right|\right)}{3\left(bc^3d-ac^2d^2\right)}-\frac{(bc+ad)\log\left(|x|\right)}{a^2c^2}+\frac{bcx^3+adx^3-ac}{3\,a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^3*log(abs(b*x^3 + a))/(a^2*b^2*c - a^3*b*d) - 1/3*d^3*log(abs(d*x^3 +
 c))/(b*c^3*d - a*c^2*d^2) - (b*c + a*d)*log(abs(x))/(a^2*c^2) + 1/3*(b*c*x
^3 + a*d*x^3 - a*c)/(a^2*c^2*x^3)
```

**maple [A]** time = 0.06, size = 87, normalized size = 1.00

$$-\frac{b^2\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,a^2}+\frac{d^2\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,c^2}-\frac{d\ln(x)}{a\,c^2}-\frac{b\ln(x)}{a^2c}-\frac{1}{3ac\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)+1/3*d^2/c^2/(a*d-b*c)*ln(d*x^3+c)-1/3/a/
c/x^3-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b
```

**maxima [A]** time = 0.49, size = 87, normalized size = 1.00

$$\frac{b^2\log\left(bx^3+a\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^3+c\right)}{3\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^3\right)}{3\,a^2c^2}-\frac{1}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 -
a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)
```

**mupad [B]** time = 3.22, size = 87, normalized size = 1.00

$$-\frac{b^2 \ln\left(b x^3 + a\right)}{3 \left(a^3 d - a^2 b c\right)} - \frac{d^2 \ln\left(d x^3 + c\right)}{3 \left(b c^3 - a c^2 d\right)} - \frac{1}{3 a c x^3} - \frac{\ln(x) \ (a d + b c)}{a^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] - (b^2*log(a + b*x^3))/(3*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^3))/(3*(b*c
^3 - a*c^2*d)) - 1/(3*a*c*x^3) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.120 $\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=318

$$\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\ldots\right)}{6c^{7/3}(bc-a\ldots}$$

**Rubi** [A] time = 0.38, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)}+\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(a^2*c^2*x) - (b^(7/3)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*(b*c - a*d)) + (d^(7/3)*ArcTan
[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d))
- (b^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*(b*c - a*d)) + (d^(7/3)*Log
[c^(1/3) + d^(1/3)*x])/(3*c^(7/3)*(b*c - a*d)) + (b^(7/3)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*(b*c - a*d)) - (d^(7/3)*Log[c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(7/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{4acx^4}+\frac{\int \frac{-4(bc+ad)-4bdx^3}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{4ac}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{x\left(-4\left(b^2c^2+abcd+a^2d^2\right)-4bd(bc+ad)x^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \left(-\frac{4b^3c^2x}{(bc-ad)\left(a+bx^3\right)}-\frac{4a^2d^3x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{x}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{x}{c+dx^3}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{8/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{7/3}(bc-ad)}+\frac{b^{8/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{7/3}(bc-ad)}+\frac{d^{8/3}\int \frac{1}{\sqrt[3]{c}+\cdots}}{3c^{7/3}(bc-\cdots}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\int \frac{-\sqrt[3]{a}\cdots}{a^{2/3}-\sqrt[3]{\cdots}}}{6a^{7/3}(\cdots}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\log\left(a^{2/\cdots}\right.}{6a\cdots}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{\cdots}\right.}{3a^{7/3}(bc\cdots}$$

**Mathematica** [A] time = 0.26, size = 282, normalized size = 0.89

$$\frac{\frac{4b^{7/3}x^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{7/3}}+\frac{4\sqrt{3}\,b^{7/3}x^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}-\frac{2b^{7/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{7/3}}-\frac{12b^2x^3}{a^2}+\frac{3b}{a}-\frac{4d^{7/3}x^4\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{7/3}}-\frac{4\sqrt{3}\,d^{7/3}x^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{7/3}}+\frac{2d^{7/3}x^4\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{7/3}}+\frac{12d^2x^3}{c^2}-\frac{3d}{c}}{12x^4(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*Sqrt[3]*b^(7/
3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (4*Sqrt[3]*d^
(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(7/3) + (4*b^(7/3)
*x^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) - (4*d^(7/3)*x^4*Log[c^(1/3) + d^(1/
3)*x])/c^(7/3) - (2*b^(7/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/a^(7/3) + (2*d^(7/3)*x^4*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
])/c^(7/3))/(12*(-(b*c) + a*d)*x^4)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas** [A] time = 3.41, size = 305, normalized size = 0.96

$$\frac{4\sqrt{3}\,b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}\,a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2\,b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)+2\,a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx^2-cx\left(-\frac{d}{c}\right)^{\frac{2}{3}}-c\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)-4\,b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)-4\,a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(dx+c\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)-3\,abc^2+3\,a^2cd+12\left(b^2c^2-a^2d^2\right)x^3}{12\left(a^2bc^3-a^3c^2d\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b^2*c^2*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) -
1/3*sqrt(3)) - 4*sqrt(3)*a^2*d^2*x^4*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/
c)^(1/3) + 1/3*sqrt(3)) + 2*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(
2/3) + a*(b/a)^(1/3)) + 2*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x^2 - c*x*(-d/c)^(
2/3) - c*(-d/c)^(1/3)) - 4*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3))
 - 4*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)) - 3*a*b*c^2 + 3*a^2
*c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

**giac** [A] time = 0.24, size = 328, normalized size = 1.03

$$-\frac{b^{3}\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^{3}bc-a^{4}d\right)}+\frac{d^{3}\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{4}-ac^{3}d\right)}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^{3}bc-\sqrt{3}a^{4}d}+\frac{\left(-cd^{2}\right)^{\frac{2}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^{4}-\sqrt{3}ac^{3}d}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}b\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^{3}bc-a^{4}d\right)}-\frac{\left(-cd^{2}\right)^{\frac{2}{3}}d\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{4}-ac^{3}d\right)}+\frac{4bcx^{3}+4adx^{3}-ac}{4a^{2}c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) - (-a*b^2)^(2/3
)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*c
- sqrt(3)*a^4*d) + (-c*d^2)^(2/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))
/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(2/3)*b*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2)^(2/3
)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/4*(4*b*c
*x^3 + 4*a*d*x^3 - a*c)/(a^2*c^2*x^4)
```

**maple** [A] time = 0.05, size = 291, normalized size = 0.92

$$-\frac{\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{b^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{b^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{\sqrt{3}\,d^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c^{2}}-\frac{d^{2}\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c^{2}}+\frac{d^{2}\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c^{2}}+\frac{d}{a\,c^{2}x}+\frac{b}{a^{2}cx}-\frac{1}{4ac\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*b^2/a^2/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*b^2/a^2/(a*d-b*c)/(
a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*b^2/a^2/(a*d-b*c)*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*d^2/c^2/(a*d-b*c)/(
c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6*d^2/c^2/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d)
^(1/3)*x+(c/d)^(2/3))+1/3*d^2/c^2/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)*arctan(1/3*
3^(1/2)*(2/(c/d)^(1/3)*x-1))-1/4/a/c/x^4+1/a/c^2/x*d+1/a^2/c/x*b
```

**maxima** [A] time = 1.13, size = 341, normalized size = 1.07

$$\frac{\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^{2}bc-a^{3}d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,d^{2}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{3}-ac^{2}d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{b^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d^{2}\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{b^{2}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d^{2}\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{4\left(bc+ad\right)x^{3}-ac}{4a^{2}c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c - a^3*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/((b*c^3 - a*c^2*d)*(c/d)^(1/3)) + 1/6*b^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3)) - 1/6*d
```

```
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d
)^(1/3)) - 1/3*b^2*log(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^
(1/3)) + 1/3*d^2*log(x + (c/d)^(1/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d)^(1
/3)) + 1/4*(4*(b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^4)
```

**mupad [B]** time = 11.37, size = 1734, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b
^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(b^7
/(a^7*(a*d - b*c)^3))^(2/3))*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^
11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13
*d^11))/9 + a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^
8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3) + log(((-d^7/(c^7*(a*d - b*c)^3))^(2/3
)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3
*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))*(-d^7
/(c^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^
5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/9 + a^13*b^9*c^13*d^9*x)*
(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1
/3) - (1/(4*a*c) - (x^3*(a*d + b*c))/(a^2*c^2))/x^4 + (log(((3^(1/2)*1i - 1
)^2*(b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*a^14*b^3*c^14*d^
3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i -
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a
^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 -
 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*b^9*c^13*d^9*x)*(b
^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3
)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(b^7/(a^7*(a*d - b*c)^3))^
(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d -
 b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^
4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/6 -
9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b^5*c^14*d^10 - 9*a^20
*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c
^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(-d^7/(c^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*
a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d
^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^
(2/3))/4)*(-d^7/(c^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^
14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*
b^9*c^13*d^9*x)*(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*
a*b^2*c^9*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(-d^7/(c
^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^
6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))/4)*(-d^7/(c^7*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b
^5*c^14*d^10 - 9*a^20*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(d^7/(27*b^
3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1/3)*(3^(1/2
)*1i + 1))/2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.121 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)} \, dx$

**Optimal**. Leaf size=321

$$-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{\phantom{c}}\right.}{6c^{8/3}(b}$$

**Rubi [A]** time = 0.46, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}+\frac{d^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{8/3}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(2*a^2*c^2*x^2) - (b^(8/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Ar
cTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*
d)) + (b^(8/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*(b*c - a*d)) - (d^(8/3)
*Log[c^(1/3) + d^(1/3)*x])/(3*c^(8/3)*(b*c - a*d)) - (b^(8/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Log[c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(8/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^3}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{5ac}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\int \frac{-10\left(b^2c^2+abcd+a^2d^2\right)-10bd(bc+ad)x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{10a^2c^2}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{1}{c+dx^3}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{8/3}(bc-ad)}+\frac{b^3\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{8/3}(bc-ad)}-\frac{d^3\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}}{3c^{8/3}(bc-ac}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{8/3}(bc-ad)}-\frac{b^{8/3}\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{}}}{6a^{8/3}(}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{8/3}(bc-ad)}-\frac{b^{8/3}\log\left(a^{2/}\right.}{6a}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}(bc-ad)}+\frac{d^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{}\right.}{3a^{8/3}(bc}$$

**Mathematica [A]** time = 0.22, size = 282, normalized size = 0.88

$$\frac{-\frac{10b^{8/3}x^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}+\frac{10\sqrt{3}b^{8/3}x^5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}}+\frac{5b^{8/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{8/3}}-\frac{15b^2x^3}{a^2}+\frac{6b}{a}+\frac{10d^{8/3}x^5\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{8/3}}-\frac{10\sqrt{3}d^{8/3}x^5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{8/3}}-\frac{5d^{8/3}x^5\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{8/3}}+\frac{15d^2x^3}{c^2}-\frac{6d}{c}}{30x^5(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*Sqrt[3]*b^(8
/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) - (10*Sqrt[3]*
d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (10*b^(8
/3)*x^5*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3) + d
^(1/3)*x])/c^(8/3) + (5*b^(8/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(8/3) - (5*d^(8/3)*x^5*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/c^(8/3))/(30*(-(b*c) + a*d)*x^5)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 1.04, size = 356, normalized size = 1.11

$$-\frac{10\sqrt{3}b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+10\sqrt{3}a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-5b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)-5a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+10b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+10a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)+6abc^2-6a^2cd-15\left(b^2c^2-a^2d^2\right)x^3}{30\left(a^2bc^3-a^3c^2d\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-
b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 10*sqrt(3)*a^2*d^2*x^5*(d^2/c^2)^(1/3)*arc
tan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - 5*b^2*c^2*x^5*(-b^
2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) -
 5*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d
^2/c^2)^(2/3)) + 10*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/
3)) + 10*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*a*b*c
^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^5)
```

**giac [A]** time = 0.21, size = 336, normalized size = 1.05

$$-\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}d^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^3bc-a^4d\right)}-\frac{\left(-cd^2\right)^{\frac{1}{3}}d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^4-ac^3d\right)}+\frac{5bcx^3+5adx^3-2ac}{10a^2c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) + (-a*b^2)^(1/3
)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*
c - sqrt(3)*a^4*d) - (-c*d^2)^(1/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(1/3)*b
^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2
```

```
)^(1/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/
10*(5*b*c*x^3 + 5*a*d*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

**maple [A]** time = 0.05, size = 293, normalized size = 0.91

$$-\frac{\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2}}-\frac{b^{2} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2}}+\frac{b^{2} \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} a^{2}}+\frac{\sqrt{3}\, d^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}} c^{2}}+\frac{d^{2} \ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}} c^{2}}-\frac{d^{2} \ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}} x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}} c^{2}}+\frac{d}{2a\,c^{2}x^{2}}+\frac{b}{2a^{2}c\,x^{2}}-\frac{1}{5ac\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/a^2*b^2/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/a^2*b^2/(a*d-b*c)/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/a^2*b^2/(a*d-b*c)/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/c^2*d^2/(a*d-b*c)/
(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/c^2*d^2/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d
)^(1/3)*x+(c/d)^(2/3))+1/3/c^2*d^2/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(c/d)^(1/3)*x-1))-1/5/a/c/x^5+1/2/a/c^2/x^2*d+1/2/a^2/c/x^2*b
```

**maxima [A]** time = 1.23, size = 369, normalized size = 1.15

$$\frac{\sqrt{3} b^{2} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3} d^{2} \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b^{2} \log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{d^{2} \log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{b^{2} \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^{2}bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{3}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{d^{2} \log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{5\left(bc+ad\right)x^{3}-2\,ac}{10\,a^{2}c^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^3*(c/d)^(1/3) - a*c^2*d*(
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*b^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) + 1/6*d^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/3*b^2*log(x +
(a/b)^(1/3))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) - 1/3*d^2*log(x + (c
/d)^(1/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/10*(5*(b*c + a*d)*
x^3 - 2*a*c)/(a^2*c^2*x^5)
```

**mupad [B]** time = 11.57, size = 1860, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^12*b^12*c
^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*c^16*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(-b^8/(a^8*(a*d - b*c)^3))^(1/3) + a^2*
d^2*x + b^2*c^2*x)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*c^12*d
^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*
c^2*d - 81*a^10*b*c*d^2))^(1/3) - (1/(5*a*c) - (x^3*(a*d + b*c))/(2*a^2*c^2
))/x^5 + log(((d^8/(c^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^1
2*b^12*c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*
c^16*d^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(d^8/(c^8*(a*d - b*c)^3))^(1/3)
 + a^2*d^2*x + b^2*c^2*x)*(d^8/(c^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*
c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^
2*b*c^9*d^2 - 81*a*b^2*c^10*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(-b^8/(a^8*(
a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*
c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^
```

```
3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(a^8*(a*d - b*c)^3))^(1/
3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13
*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b
^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81
*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/
2)*1i + 1)*(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b
^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)
 - (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(
a^8*(a*d - b*c)^3))^(1/3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12
*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c
^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^
3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i
 + 1))/2 + (log(((3^(1/2)*1i - 1)*(d^8/(c^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2
)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^
2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)
*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8/(c^8*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13
*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4
))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d^2 - 81*a*b^2*c^10*d
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(d^8/(c^8*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*
d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) - (81*a^19*b^3*c^19*d^3*(3^(1/2)
*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8
/(c^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d
^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*
x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d
^2 - 81*a*b^2*c^10*d))^(1/3)*(3^(1/2)*1i + 1))/2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

## 3.122 $\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=119

$$-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

**Rubi** [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(3*a^3*(b*c - a*d)) + (d^3*Log[
c + d*x^3])/(3*c^3*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^3}+\frac{-bc-ad}{a^2c^2x^2}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^4}{a^3(-bc+ad)(a+bx)}+\frac{}{c^3}\right.\right.\\ &= -\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log(x)}{a^3c^3}-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{d^3\log\left(c+\right.}{3c^3(bc-}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 119, normalized size = 1.00

$$\frac{b^3\log\left(a+bx^3\right)}{3a^3(ad-bc)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^3])/(3*a^3*(-(b*c) + a*d)) + (d^3
*Log[c + d*x^3])/(3*c^3*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(a + bx^3\right) \left(c + dx^3\right)}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 11.77, size = 127, normalized size = 1.07

$$-\frac{2\,b^3c^3x^6\log\left(bx^3+a\right)-2\,a^3d^3x^6\log\left(dx^3+c\right)-6\left(b^3c^3-a^3d^3\right)x^6\log(x)+a^2bc^3-a^3c^2d-2\left(ab^2c^3-a^3cd^2\right)x^3}{6\left(a^3bc^4-a^4c^3d\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3*x^6*log(b*x^3 + a) - 2*a^3*d^3*x^6*log(d*x^3 + c) - 6*(b^3*
c^3 - a^3*d^3)*x^6*log(x) + a^2*b*c^3 - a^3*c^2*d - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^6)
```

**giac [A]** time = 0.26, size = 165, normalized size = 1.39

$$-\frac{b^4\log\left(\left|bx^3+a\right|\right)}{3\left(a^3b^2c-a^4bd\right)}+\frac{d^4\log\left(\left|dx^3+c\right|\right)}{3\left(bc^4d-ac^3d^2\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(|x|\right)}{a^3c^3}-\frac{3\,b^2c^2x^6+3\,abcdx^6+3\,a^2d^2x^6-2\,abc^2x^3-2\,a^2cdx^3+a^2c^2}{6\,a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^4*log(abs(b*x^3 + a))/(a^3*b^2*c - a^4*b*d) + 1/3*d^4*log(abs(d*x^3
+ c))/(b*c^4*d - a*c^3*d^2) + (b^2*c^2 + a*b*c*d + a^2*d^2)*log(abs(x))/(a^
3*c^3) - 1/6*(3*b^2*c^2*x^6 + 3*a*b*c*d*x^6 + 3*a^2*d^2*x^6 - 2*a*b*c^2*x^3
 - 2*a^2*c*d*x^3 + a^2*c^2)/(a^3*c^3*x^6)
```

**maple [A]** time = 0.06, size = 124, normalized size = 1.04

$$\frac{b^3\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,a^3}-\frac{d^3\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,c^3}+\frac{d^2\ln(x)}{a\,c^3}+\frac{bd\ln(x)}{a^2c^2}+\frac{b^2\ln(x)}{a^3c}+\frac{d}{3a\,c^2x^3}+\frac{b}{3a^2c\,x^3}-\frac{1}{6ac\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*b^3/a^3/(a*d-b*c)*ln(b*x^3+a)-1/3*d^3/c^3/(a*d-b*c)*ln(d*x^3+c)-1/6/a/c
/x^6+1/3/a/c^2/x^3*d+1/3/a^2/c/x^3*b+1/a/c^3*ln(x)*d^2+1/a^2/c^2*ln(x)*b*d+
1/a^3/c*ln(x)*b^2
```

**maxima [A]** time = 0.52, size = 117, normalized size = 0.98

$$-\frac{b^3\log\left(bx^3+a\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\log\left(dx^3+c\right)}{3\left(bc^4-ac^3d\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^3\right)}{3\,a^3c^3}+\frac{2\,(bc+ad)x^3-ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(b*x^3 + a)/(a^3*b*c - a^4*d) + 1/3*d^3*log(d*x^3 + c)/(b*c^4 -
 a*c^3*d) + 1/3*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^3)/(a^3*c^3) + 1/6*(2*(
b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^6)
```

**mupad [B]** time = 3.21, size = 118, normalized size = 0.99

$$\frac{b^3\,\ln\left(b\,x^3+a\right)}{3\,a^4\,d-3\,a^3\,b\,c}-\frac{\frac{1}{6\,a\,c}-\frac{x^3\,\left(a\,d+b\,c\right)}{3\,a^2\,c^2}}{x^6}+\frac{d^3\,\ln\left(d\,x^3+c\right)}{3\,b\,c^4-3\,a\,c^3\,d}+\frac{\ln(x)\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b^3*log(a + b*x^3))/(3*a^4*d - 3*a^3*b*c) - (1/(6*a*c) - (x^3*(a*d + b*c))
/(3*a^2*c^2))/x^6 + (d^3*log(c + d*x^3))/(3*b*c^4 - 3*a*c^3*d) + (log(x)*(a
^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.123 $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$

**Optimal**. Leaf size=352

$$-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{a^2d^2+abcd+}{a^3c^3x}$$

**Rubi [A]** time = 0.50, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}+\frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)}-\frac{d^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)}-\frac{1}{7acx^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(7*a*c*x^7) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(a^3*c^3*x) + (b^(10/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(10/3)*(b*c - a*d)) - (d^(10/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)
/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(10/3)*(b*c - a*d)) + (b^(10/3)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(10/3)*(b*c - a*d)) - (d^(10/3)*Log[c^(1/3) + d^(1/3)*x
])/(3*c^(10/3)*(b*c - a*d)) - (b^(10/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(10/3)*(b*c - a*d)) + (d^(10/3)*Log[c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2])/(6*c^(10/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{7acx^7}+\frac{\int \frac{-7(bc+ad)-7bdx^3}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{7ac}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{\int \frac{-28\left(b^2c^2+abcd+a^2d^2\right)-28bd(bc+ad)x^3}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{28a^2c^2}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \frac{x\left(-28(bc+ad)\left(b^2c^2+a^2d^2\right)-28bd\left(b^2c^2+abcd\right.\right.}{\left(a+bx^3\right)\left(c+dx^3\right)}}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int\left(-\frac{28b^4c^3x}{(bc-ad)\left(a+bx^3\right)}-\frac{28a^3d^4x}{(-bc+ad)\left(c+dx^3\right)}\right)}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^4\int\frac{x}{a+bx^3}\,dx}{a^3(bc-ad)}+\frac{d^4\int\frac{x}{c+dx^3}\,dx}{c^3(bc-ad)}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{11/3}\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{10/3}(bc-ad)}-\frac{b^{11/3}\int\frac{\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}}{3a^{10/3}(b}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\right.}{3c^{10/3}(}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\right.}{3c^{10/3}(}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}(bc-ad)}-\frac{d^{10/3}\tan^{-}}{\sqrt{3}\,c^{10/}}$$

**Mathematica** [A] time = 0.24, size = 304, normalized size = 0.86

$$\frac{-\frac{28b^{10/3}x^7\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{10/3}}-\frac{28\sqrt{3}\,b^{10/3}x^7\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{10/3}}+\frac{14b^{10/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{10/3}}+\frac{84b^3x^6}{a^3}-\frac{21b^2x^3}{a^2}+\frac{12b}{a}+\frac{28d^{10/3}x^7\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{10/3}}+\frac{28\sqrt{3}\,d^{10/3}x^7\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{10/3}}-\frac{14d^{10/3}x^7\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{10/3}}-\frac{84d^3x^6}{c^3}+\frac{21d^2x^3}{c^2}-\frac{12d}{c}}{84x^7(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((12*b)/a - (12*d)/c - (21*b^2*x^3)/a^2 + (21*d^2*x^3)/c^2 + (84*b^3*x^6)/a
^3 - (84*d^3*x^6)/c^3 - (28*Sqrt[3]*b^(10/3)*x^7*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/a^(10/3) + (28*Sqrt[3]*d^(10/3)*x^7*ArcTan[(1 - (2*d^(1/
3)*x)/c^(1/3))/Sqrt[3]])/c^(10/3) - (28*b^(10/3)*x^7*Log[a^(1/3) + b^(1/3)*
x])/a^(10/3) + (28*d^(10/3)*x^7*Log[c^(1/3) + d^(1/3)*x])/c^(10/3) + (14*b^
(10/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(10/3) - (14*d
^(10/3)*x^7*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(10/3))/(84*(
-(b*c) + a*d)*x^7)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^8*(a + b*x^3)*(c + d*x^3)), x]
```

**fricas [A]** time = 1.20, size = 332, normalized size = 0.94

$$-\frac{28 \sqrt{3} b^3 c^3 x^7 \left(-\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3} \sqrt{3} x \left(-\frac{b}{a}\right)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) - 28 \sqrt{3} a^3 d^3 x^7 \left(\frac{d}{c}\right)^{\frac{1}{3}} \arctan\left(\frac{2}{3} \sqrt{3} x \left(\frac{d}{c}\right)^{\frac{1}{3}} - \frac{1}{3} \sqrt{3}\right) - 14 b^3 c^3 x^7 \left(-\frac{b}{a}\right)^{\frac{1}{3}} \log\left(b x^2 - a x \left(-\frac{b}{a}\right)^{\frac{2}{3}} - a \left(-\frac{b}{a}\right)^{\frac{1}{3}}\right) - 14 a^3 d^3 x^7 \left(\frac{d}{c}\right)^{\frac{1}{3}} \log\left(d x^2 - c x \left(\frac{d}{c}\right)^{\frac{2}{3}} + c \left(\frac{d}{c}\right)^{\frac{1}{3}}\right) + 28 b^3 c^3 x^7 \left(-\frac{b}{a}\right)^{\frac{1}{3}} \log\left(b x + a \left(-\frac{b}{a}\right)^{\frac{2}{3}}\right) + 28 a^3 d^3 x^7 \left(\frac{d}{c}\right)^{\frac{1}{3}} \log\left(d x + c \left(\frac{d}{c}\right)^{\frac{2}{3}}\right) + 84 \left(b^3 c^3 - a^3 d^3\right) x^6 + 12 a^2 b c^3 - 12 a^3 c^2 d - 21 \left(a b^2 c^3 - a^3 c d^2\right) x^3}{84 \left(a^3 b c^4 - a^4 c^3 d\right) x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^3*c^3*x^7*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3
) + 1/3*sqrt(3)) - 28*sqrt(3)*a^3*d^3*x^7*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*
(d/c)^(1/3) - 1/3*sqrt(3)) - 14*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x^2 - a*x*(-
b/a)^(2/3) - a*(-b/a)^(1/3)) - 14*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x^2 - c*x*(
d/c)^(2/3) + c*(d/c)^(1/3)) + 28*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 28*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x + c*(d/c)^(2/3)) + 84*(b^3*c^
3 - a^3*d^3)*x^6 + 12*a^2*b*c^3 - 12*a^3*c^2*d - 21*(a*b^2*c^3 - a^3*c*d^2)
*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^7)
```

**giac [A]** time = 0.22, size = 377, normalized size = 1.07

$$\frac{b^4 \left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 \left(a^4 b c - a^5 d\right)} - \frac{d^4 \left(-\frac{c}{d}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3 \left(b c^5 - a c^4 d\right)} + \frac{\left(-a b^2\right)^{\frac{2}{3}} b^2 \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3} a^4 b c - \sqrt{3} a^5 d} - \frac{\left(-c d^2\right)^{\frac{2}{3}} d^2 \arctan\left(\frac{\sqrt{3} \left(2 x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3} b c^5 - \sqrt{3} a c^4 d} - \frac{\left(-a b^2\right)^{\frac{2}{3}} b^2 \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(a^4 b c - a^5 d\right)} + \frac{\left(-c d^2\right)^{\frac{2}{3}} d^2 \log\left(x^2 + x \left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 \left(b c^5 - a c^4 d\right)} - \frac{28 b^2 c^2 x^6 + 28 a b c d x^6 + 28 a^2 d^2 x^6 - 7 a b c^2 x^3 - 7 a^2 c d x^3 + 4 a^2 c^2}{28 a^3 c^3 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^4*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*c - a^5*d) - 1/3*d^4
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^5 - a*c^4*d) + (-a*b^2)^(2/3)
*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^4*b*c
 - sqrt(3)*a^5*d) - (-c*d^2)^(2/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(sqrt(3)*b*c^5 - sqrt(3)*a*c^4*d) - 1/6*(-a*b^2)^(2/3)*b^
2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b*c - a^5*d) + 1/6*(-c*d^2)
^(2/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^5 - a*c^4*d) - 1/2
8*(28*b^2*c^2*x^6 + 28*a*b*c*d*x^6 + 28*a^2*d^2*x^6 - 7*a*b*c^2*x^3 - 7*a^2
*c*d*x^3 + 4*a^2*c^2)/(a^3*c^3*x^7)
```

**maple [A]** time = 0.05, size = 334, normalized size = 0.95

$$\frac{\sqrt{3}\, b^3 \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 (a d - b c) \left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} - \frac{b^3 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 (a d - b c) \left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} + \frac{b^3 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 (a d - b c) \left(\frac{a}{b}\right)^{\frac{1}{3}} a^3} - \frac{\sqrt{3}\, d^3 \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3 (a d - b c) \left(\frac{c}{d}\right)^{\frac{1}{3}} c^3} + \frac{d^3 \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 (a d - b c) \left(\frac{c}{d}\right)^{\frac{1}{3}} c^3} - \frac{d^3 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}} x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 (a d - b c) \left(\frac{c}{d}\right)^{\frac{1}{3}} c^3} - \frac{d^2}{a c^3 x} - \frac{b d}{a^2 c^2 x} - \frac{b^2}{a^3 c x} + \frac{d}{4 a c^2 x^4} + \frac{b}{4 a^2 c x^4} - \frac{1}{7 a c x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*b^3/a^3/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*b^3/a^3/(a*d-b*c)/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b^3/a^3/(a*d-b*c)*3^(1/2)
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d^3/c^3/(a*d-b*c)/
(c/d)^(1/3)*ln(x+(c/d)^(1/3))-1/6*d^3/c^3/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d
)^(1/3)*x+(c/d)^(2/3))-1/3*d^3/c^3/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)*arctan(1/3
*3^(1/2)*(2/(c/d)^(1/3)*x-1))-1/7/a/c/x^7+1/4/a/c^2/x^4*d+1/4/a^2/c/x^4*b-1
/a/c^3/x*d^2-1/a^2/c^2/x*b*d-1/a^3/c/x*b^2
```

**maxima [A]** time = 1.16, size = 376, normalized size = 1.07

$$-\frac{\sqrt{3} b^3 \arctan\left(\frac{\sqrt{3} \left(2 x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \left(a^3 b c - a^4 d\right) \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3} d^3 \arctan\left(\frac{\sqrt{3} \left(2 x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3 \left(b c^4 - a c^3 d\right) \left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b^3 \log\left(x^2 - x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(a^3 b c \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^4 d \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d^3 \log\left(x^2 - x \left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 \left(b c^4 \left(\frac{c}{d}\right)^{\frac{1}{3}} - a c^3 d \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{b^3 \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(a^3 b c \left(\frac{a}{b}\right)^{\frac{1}{3}} - a^4 d \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d^3 \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(b c^4 \left(\frac{c}{d}\right)^{\frac{1}{3}} - a c^3 d \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{28 \left(b^2 c^2 + a b c d + a^2 d^2\right) x^6 + 4 a^2 c^2 - 7 \left(a b c^2 + a^2 c d\right) x^3}{28 a^3 c^3 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^3*
b*c - a^4*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d^3*arctan(1/3*sqrt(3)*(2*x - (c/d)
^(1/3))/(c/d)^(1/3))/((b*c^4 - a*c^3*d)*(c/d)^(1/3)) - 1/6*b^3*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)^(1/3)) + 1/6*
d^3*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/
d)^(1/3)) + 1/3*b^3*log(x + (a/b)^(1/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)
^(1/3)) - 1/3*d^3*log(x + (c/d)^(1/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/d)^(
1/3)) - 1/28*(28*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^6 + 4*a^2*c^2 - 7*(a*b*c^2
 + a^2*c*d)*x^3)/(a^3*c^3*x^7)
```

**mupad [B]** time = 11.91, size = 1814, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8
+ b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(
-b^10/(a^10*(a*d - b*c)^3))^(2/3))*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/3 -
9*a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29
*b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3
 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3) + log(((d^
10/(c^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8
)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^1
0*(a*d - b*c)^3))^(2/3))*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/3 - 9*a^19*b^14
*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d
^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c
^10*d^3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3) - (1/(7*a*c) - (x^3*(
a*d + b*c))/(4*a^2*c^2) + (x^6*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^
7 - (log(((3^(1/2)*1i + 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^2
8*b^3*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a
*d - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14
*c^29*d^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d
^14))/36 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10
*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2
+ (log(((3^(1/2)*1i - 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*
b^3*c^28*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a*d
 - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c
^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^1
4))/36 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10*b
^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i +
1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3
*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b
*c)^3))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14*c^29*d
^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d^14))/3
6 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^
3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(
((3^(1/2)*1i - 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(2
7*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3*c^28
*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b*c)^3
))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c^29*d^4 +
9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^14))/36 - a
^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^3 + 8
1*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i - 1))/2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.124 $\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=148

$$\frac{a^5Ax^{m+1}}{m+1}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{b^4x^{m+16}(5aB+Ab)}{m+16}+\frac{5ab^3x^{m+}}{n}$$

**Rubi [A]** time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{a^5Ax^{m+1}}{m+1}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^4x^{m+16}(5aB+Ab)}{m+16}+\frac{b^5Bx^{m+19}}{m+19}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3*
b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/(1
0 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B)*x
^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5Ax^m+a^4(5Ab+aB)x^{3+m}+5a^3b(2Ab+aB)x^{6+m}+10a^2b^2(Ab+aB)x\right.\\ &= \frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+aB)x}{10+m}\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 137, normalized size = 0.93

$$x^{m+1}\left(\frac{a^5A}{m+1}+\frac{a^4x^3(aB+5Ab)}{m+4}+\frac{5a^3bx^6(aB+2Ab)}{m+7}+\frac{10a^2b^2x^9(aB+Ab)}{m+10}+\frac{b^4x^{15}(5aB+Ab)}{m+16}+\frac{5ab^3x^{12}(2aB+Ab)}{m+13}+\frac{b^5Bx^{18}}{m+19}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^5*A)/(1 + m) + (a^4*(5*A*b + a*B)*x^3)/(4 + m) + (5*a^3*b*(2*
A*b + a*B)*x^6)/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^9)/(10 + m) + (5*a*b^3*
(A*b + 2*a*B)*x^12)/(13 + m) + (b^4*(A*b + 5*a*B)*x^15)/(16 + m) + (b^5*B*x
^18)/(19 + m))
```

**IntegrateAlgebraic [F]** time = 0.35, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^3)^5*(A + B*x^3), x]
```

**fricas [B]** time = 0.91, size = 851, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5*
m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^19 + ((5*B*a*b^4 + A*b^5)*m^6 + 345800
*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 + A*b
^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2 + 1
12206*(5*B*a*b^4 + A*b^5)*m)*x^16 + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 170240
*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2*B*a^2
*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B*a^2*b^
3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^13 + 10*((B*a^3*b^2
+ A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3*b^2 + A*
a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^2 + A*a^2*
b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2 + A*a^2*b^
3)*m)*x^10 + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b + 316160*A*a^3
*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^3*b^2)*m^4 +
17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3*b^2)*m^2 + 24
3768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m^6 + 276640*B*a
^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B*a^5 + 5*A*a^4*b
)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*A*a^4*b)*m^2 + 39
6954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m^5 + 1905*A*a^5*m^
4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m + 1106560*A*a^5)*x)
*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 + 959070*m^2 + 18644
56*m + 1106560)
```

**giac [B]** time = 0.29, size = 1331, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^5*m^6*x^19*x^m + 51*B*b^5*m^5*x^19*x^m + 1005*B*b^5*m^4*x^19*x^m + 5*B
*a*b^4*m^6*x^16*x^m + A*b^5*m^6*x^16*x^m + 9605*B*b^5*m^3*x^19*x^m + 270*B*
a*b^4*m^5*x^16*x^m + 54*A*b^5*m^5*x^16*x^m + 45474*B*b^5*m^2*x^19*x^m + 555
0*B*a*b^4*m^4*x^16*x^m + 1110*A*b^5*m^4*x^16*x^m + 95064*B*b^5*m*x^19*x^m +
 10*B*a^2*b^3*m^6*x^13*x^m + 5*A*a*b^4*m^6*x^13*x^m + 54700*B*a*b^4*m^3*x^1
6*x^m + 10940*A*b^5*m^3*x^16*x^m + 58240*B*b^5*x^19*x^m + 570*B*a^2*b^3*m^5
*x^13*x^m + 285*A*a*b^4*m^5*x^13*x^m + 264645*B*a*b^4*m^2*x^16*x^m + 52929*
A*b^5*m^2*x^16*x^m + 12330*B*a^2*b^3*m^4*x^13*x^m + 6165*A*a*b^4*m^4*x^13*x
^m + 561030*B*a*b^4*m*x^16*x^m + 112206*A*b^5*m*x^16*x^m + 10*B*a^3*b^2*m^6
*x^10*x^m + 10*A*a^2*b^3*m^6*x^10*x^m + 126710*B*a^2*b^3*m^3*x^13*x^m + 633
55*A*a*b^4*m^3*x^13*x^m + 345800*B*a*b^4*x^16*x^m + 69160*A*b^5*x^16*x^m +
600*B*a^3*b^2*m^5*x^10*x^m + 600*A*a^2*b^3*m^5*x^10*x^m + 632460*B*a^2*b^3*
m^2*x^13*x^m + 316230*A*a*b^4*m^2*x^13*x^m + 13740*B*a^3*b^2*m^4*x^10*x^m +
 13740*A*a^2*b^3*m^4*x^10*x^m + 1368720*B*a^2*b^3*m*x^13*x^m + 684360*A*a*b
^4*m*x^13*x^m + 5*B*a^4*b*m^6*x^7*x^m + 10*A*a^3*b^2*m^6*x^7*x^m + 149600*B
*a^3*b^2*m^3*x^10*x^m + 149600*A*a^2*b^3*m^3*x^10*x^m + 851200*B*a^2*b^3*x^
13*x^m + 425600*A*a*b^4*x^13*x^m + 315*B*a^4*b*m^5*x^7*x^m + 630*A*a^3*b^2*
m^5*x^7*x^m + 783690*B*a^3*b^2*m^2*x^10*x^m + 783690*A*a^2*b^3*m^2*x^10*x^m
 + 7665*B*a^4*b*m^4*x^7*x^m + 15330*A*a^3*b^2*m^4*x^7*x^m + 1753800*B*a^3*b
^2*m*x^10*x^m + 1753800*A*a^2*b^3*m*x^10*x^m + B*a^5*m^6*x^4*x^m + 5*A*a^4*
b*m^6*x^4*x^m + 89845*B*a^4*b*m^3*x^7*x^m + 179690*A*a^3*b^2*m^3*x^7*x^m +
1106560*B*a^3*b^2*x^10*x^m + 1106560*A*a^2*b^3*x^10*x^m + 66*B*a^5*m^5*x^4*
x^m + 330*A*a^4*b*m^5*x^4*x^m + 510930*B*a^4*b*m^2*x^7*x^m + 1021860*A*a^3*
```

```
b^2*m^2*x^7*x^m + 1710*B*a^5*m^4*x^4*x^m + 8550*A*a^4*b*m^4*x^4*x^m + 12188
40*B*a^4*b*m*x^7*x^m + 2437680*A*a^3*b^2*m*x^7*x^m + A*a^5*m^6*x*x^m + 2186
0*B*a^5*m^3*x^4*x^m + 109300*A*a^4*b*m^3*x^4*x^m + 790400*B*a^4*b*x^7*x^m +
 1580800*A*a^3*b^2*x^7*x^m + 69*A*a^5*m^5*x*x^m + 140529*B*a^5*m^2*x^4*x^m
+ 702645*A*a^4*b*m^2*x^4*x^m + 1905*A*a^5*m^4*x*x^m + 396954*B*a^5*m*x^4*x^
m + 1984770*A*a^4*b*m*x^4*x^m + 26795*A*a^5*m^3*x*x^m + 276640*B*a^5*x^4*x^
m + 1383200*A*a^4*b*x^4*x^m + 201174*A*a^5*m^2*x*x^m + 757896*A*a^5*m*x*x^m
 + 1106560*A*a^5*x*x^m)/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 +
 959070*m^2 + 1864456*m + 1106560)
```

**maple** [**B**] time = 0.05, size = 1078, normalized size = 7.28

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] x^(m+1)*(B*b^5*m^6*x^18+51*B*b^5*m^5*x^18+1005*B*b^5*m^4*x^18+A*b^5*m^6*x^1
5+5*B*a*b^4*m^6*x^15+9605*B*b^5*m^3*x^18+54*A*b^5*m^5*x^15+270*B*a*b^4*m^5*
x^15+45474*B*b^5*m^2*x^18+1110*A*b^5*m^4*x^15+5550*B*a*b^4*m^4*x^15+95064*B
*b^5*m*x^18+5*A*a*b^4*m^6*x^12+10940*A*b^5*m^3*x^15+10*B*a^2*b^3*m^6*x^12+5
4700*B*a*b^4*m^3*x^15+58240*B*b^5*x^18+285*A*a*b^4*m^5*x^12+52929*A*b^5*m^2
*x^15+570*B*a^2*b^3*m^5*x^12+264645*B*a*b^4*m^2*x^15+6165*A*a*b^4*m^4*x^12+
112206*A*b^5*m*x^15+12330*B*a^2*b^3*m^4*x^12+561030*B*a*b^4*m*x^15+10*A*a^2
*b^3*m^6*x^9+63355*A*a*b^4*m^3*x^12+69160*A*b^5*x^15+10*B*a^3*b^2*m^6*x^9+1
26710*B*a^2*b^3*m^3*x^12+345800*B*a*b^4*x^15+600*A*a^2*b^3*m^5*x^9+316230*A
*a*b^4*m^2*x^12+600*B*a^3*b^2*m^5*x^9+632460*B*a^2*b^3*m^2*x^12+13740*A*a^2
*b^3*m^4*x^9+684360*A*a*b^4*m*x^12+13740*B*a^3*b^2*m^4*x^9+1368720*B*a^2*b^
3*m*x^12+10*A*a^3*b^2*m^6*x^6+149600*A*a^2*b^3*m^3*x^9+425600*A*a*b^4*x^12+
5*B*a^4*b*m^6*x^6+149600*B*a^3*b^2*m^3*x^9+851200*B*a^2*b^3*x^12+630*A*a^3*
b^2*m^5*x^6+783690*A*a^2*b^3*m^2*x^9+315*B*a^4*b*m^5*x^6+783690*B*a^3*b^2*m
^2*x^9+15330*A*a^3*b^2*m^4*x^6+1753800*A*a^2*b^3*m*x^9+7665*B*a^4*b*m^4*x^6
+1753800*B*a^3*b^2*m*x^9+5*A*a^4*b*m^6*x^3+179690*A*a^3*b^2*m^3*x^6+1106560
*A*a^2*b^3*x^9+B*a^5*m^6*x^3+89845*B*a^4*b*m^3*x^6+1106560*B*a^3*b^2*x^9+33
0*A*a^4*b*m^5*x^3+1021860*A*a^3*b^2*m^2*x^6+66*B*a^5*m^5*x^3+510930*B*a^4*b
*m^2*x^6+8550*A*a^4*b*m^4*x^3+2437680*A*a^3*b^2*m*x^6+1710*B*a^5*m^4*x^3+12
18840*B*a^4*b*m*x^6+A*a^5*m^6+109300*A*a^4*b*m^3*x^3+1580800*A*a^3*b^2*x^6+
21860*B*a^5*m^3*x^3+790400*B*a^4*b*x^6+69*A*a^5*m^5+702645*A*a^4*b*m^2*x^3+
140529*B*a^5*m^2*x^3+1905*A*a^5*m^4+1984770*A*a^4*b*m*x^3+396954*B*a^5*m*x^
3+26795*A*a^5*m^3+1383200*A*a^4*b*x^3+276640*B*a^5*x^3+201174*A*a^5*m^2+757
896*A*a^5*m+1106560*A*a^5)/(m+1)/(m+4)/(m+7)/(m+10)/(m+13)/(m+16)/(m+19)
```

**maxima** [**A**] time = 0.52, size = 205, normalized size = 1.39

$$\frac{Bb^5x^{m+19}}{m+19}+\frac{5\,Bab^4x^{m+16}}{m+16}+\frac{Ab^5x^{m+16}}{m+16}+\frac{10\,Ba^2b^3x^{m+13}}{m+13}+\frac{5\,Aab^4x^{m+13}}{m+13}+\frac{10\,Ba^3b^2x^{m+10}}{m+10}+\frac{10\,Aa^2b^3x^{m+10}}{m+10}+\frac{5\,Ba^4bx^{m+7}}{m+7}+\frac{10\,Aa^3b^2x^{m+7}}{m+7}+\frac{Ba^5x^{m+4}}{m+4}+\frac{5\,Aa^4bx^{m+4}}{m+4}+\frac{Aa^5x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b^5*x^(m + 19)/(m + 19) + 5*B*a*b^4*x^(m + 16)/(m + 16) + A*b^5*x^(m + 16
)/(m + 16) + 10*B*a^2*b^3*x^(m + 13)/(m + 13) + 5*A*a*b^4*x^(m + 13)/(m + 1
3) + 10*B*a^3*b^2*x^(m + 10)/(m + 10) + 10*A*a^2*b^3*x^(m + 10)/(m + 10) +
5*B*a^4*b*x^(m + 7)/(m + 7) + 10*A*a^3*b^2*x^(m + 7)/(m + 7) + B*a^5*x^(m +
 4)/(m + 4) + 5*A*a^4*b*x^(m + 4)/(m + 4) + A*a^5*x^(m + 1)/(m + 1)
```

**mupad** [**B**] time = 3.21, size = 559, normalized size = 3.78

$$\frac{B\,b^5\,x^m\,x^{19}\,\left(m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{a^4\,x^m\,x^4\,\left(5\,A\,b+B\,a\right)\,\left(m^6+66\,m^5+1710\,m^4+21860\,m^3+140529\,m^2+396954\,m+276640\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{b^4\,x^m\,x^{16}\,\left(A\,b+5\,B\,a\right)\,\left(m^6+54\,m^5+1110\,m^4+10940\,m^3+52929\,m^2+112206\,m+69160\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{A\,a^5\,x\,x^m\,\left(m^6+69\,m^5+1905\,m^4+26795\,m^3+201174\,m^2+757896\,m+1106560\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{10\,a^2\,b^2\,x^m\,x^{10}\,\left(A\,b+B\,a\right)\,\left(m^6+60\,m^5+1374\,m^4+14960\,m^3+78369\,m^2+175380\,m+110656\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{5\,a\,b^3\,x^m\,x^{13}\,\left(A\,b+2\,B\,a\right)\,\left(m^6+57\,m^5+1233\,m^4+12671\,m^3+63246\,m^2+136872\,m+85120\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}+\frac{5\,a^3\,b\,x^m\,x^7\,\left(2\,A\,b+B\,a\right)\,\left(m^6+63\,m^5+1533\,m^4+17969\,m^3+102186\,m^2+243768\,m+158080\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] (B*b^5*x^m*x^19*(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 +
 58240))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m
^6 + m^7 + 1106560) + (a^4*x^m*x^4*(5*A*b + B*a)*(396954*m + 140529*m^2 + 2
1860*m^3 + 1710*m^4 + 66*m^5 + m^6 + 276640))/(1864456*m + 959070*m^2 + 227
969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (b^4*x^m*x^16*(A
*b + 5*B*a)*(112206*m + 52929*m^2 + 10940*m^3 + 1110*m^4 + 54*m^5 + m^6 + 6
9160))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6
 + m^7 + 1106560) + (A*a^5*x*x^m*(757896*m + 201174*m^2 + 26795*m^3 + 1905*
m^4 + 69*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700
*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (10*a^2*b^2*x^m*x^10*(A*b + B*a
)*(175380*m + 78369*m^2 + 14960*m^3 + 1374*m^4 + 60*m^5 + m^6 + 110656))/(1
864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 +
1106560) + (5*a*b^3*x^m*x^13*(A*b + 2*B*a)*(136872*m + 63246*m^2 + 12671*m^
3 + 1233*m^4 + 57*m^5 + m^6 + 85120))/(1864456*m + 959070*m^2 + 227969*m^3
+ 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a^3*b*x^m*x^7*(2*A*b
+ B*a)*(243768*m + 102186*m^2 + 17969*m^3 + 1533*m^4 + 63*m^5 + m^6 + 15808
0))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 +
m^7 + 1106560)
```

**sympy [A]** time = 23.94, size = 5418, normalized size = 36.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**12
) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a**
5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**2*b
**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*log(x), Eq(m, -19)), (-A*a**5/(
15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2*b**3
/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*log(x) - B*a**5/(12*x**12) - 5*B*a
**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) + 5*B*a*b
**4*log(x) + B*b**5*x**3/3, Eq(m, -16)), (-A*a**5/(12*x**12) - 5*A*a**4*b/(
9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a*b**4*log
(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*B*a**3*b**
2/(3*x**3) + 10*B*a**2*b**3*log(x) + 5*B*a*b**4*x**3/3 + B*b**5*x**6/6, Eq(
m, -13)), (-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b**2/(3*x**3)
 + 10*A*a**2*b**3*log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 - B*a**5/(6*x*
*6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*log(x) + 10*B*a**2*b**3*x**3/3 +
 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, Eq(m, -10)), (-A*a**5/(6*x**6) - 5*A*a*
*4*b/(3*x**3) + 10*A*a**3*b**2*log(x) + 10*A*a**2*b**3*x**3/3 + 5*A*a*b**4*
x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*log(x) + 10*B*a**3*b*
*2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*b**5*x**12/12, Eq(
m, -7)), (-A*a**5/(3*x**3) + 5*A*a**4*b*log(x) + 10*A*a**3*b**2*x**3/3 + 5*
A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**9/9 + A*b**5*x**12/12 + B*a**5*log(x) +
5*B*a**4*b*x**3/3 + 5*B*a**3*b**2*x**6/3 + 10*B*a**2*b**3*x**9/9 + 5*B*a*b*
*4*x**12/12 + B*b**5*x**15/15, Eq(m, -4)), (A*a**5*log(x) + 5*A*a**4*b*x**3
/3 + 5*A*a**3*b**2*x**6/3 + 10*A*a**2*b**3*x**9/9 + 5*A*a*b**4*x**12/12 + A
*b**5*x**15/15 + B*a**5*x**3/3 + 5*B*a**4*b*x**6/6 + 10*B*a**3*b**2*x**9/9
+ 5*B*a**2*b**3*x**12/6 + B*a*b**4*x**15/3 + B*b**5*x**18/18, Eq(m, -1)), (
A*a**5*m**6*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 69*A*a**5*m**5*x*x**m/(m**7 + 70*m**6
 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560
) + 1905*A*a**5*m**4*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 26795*A*a**5*m**3*x*x**m/(m*
*7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456
*m + 1106560) + 201174*A*a**5*m**2*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 757896*A*a**5*
```

```
m*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 1106560*A*a**5*x*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*A*a*
*4*b*m**6*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3
+ 959070*m**2 + 1864456*m + 1106560) + 330*A*a**4*b*m**5*x**4*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 8550*A*a**4*b*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700
*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 109300*A*a**4*b*
m**3*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959
070*m**2 + 1864456*m + 1106560) + 702645*A*a**4*b*m**2*x**4*x**m/(m**7 + 70
*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 11
06560) + 1984770*A*a**4*b*m*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1383200*A*a**4*b*x
**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*A*a**3*b**2*m**6*x**7*x**m/(m**7 + 70*m**6 +
 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
+ 630*A*a**3*b**2*m**5*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 15330*A*a**3*b**2*m**4*
x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m
**2 + 1864456*m + 1106560) + 179690*A*a**3*b**2*m**3*x**7*x**m/(m**7 + 70*m
**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106
560) + 1021860*A*a**3*b**2*m**2*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 2437680*A*a**3
*b**2*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 1580800*A*a**3*b**2*x**7*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 10*A*a**2*b**3*m**6*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 600*A*a**2*b**
3*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 13740*A*a**2*b**3*m**4*x**10*x**m/(m**
7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*
m + 1106560) + 149600*A*a**2*b**3*m**3*x**10*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 783690
*A*a**2*b**3*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*A*a**2*b**3*m*x**10
*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2
+ 1864456*m + 1106560) + 1106560*A*a**2*b**3*x**10*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
5*A*a*b**4*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 22796
9*m**3 + 959070*m**2 + 1864456*m + 1106560) + 285*A*a*b**4*m**5*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864
456*m + 1106560) + 6165*A*a*b**4*m**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**
5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 63355*A
*a*b**4*m**3*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 316230*A*a*b**4*m**2*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864
456*m + 1106560) + 684360*A*a*b**4*m*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 425600*A
*a*b**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + A*b**5*m**6*x**16*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 54*A*b**5*m**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2
27969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1110*A*b**5*m**4*x**16*x*
*m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1
864456*m + 1106560) + 10940*A*b**5*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m
**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 52929
*A*b**5*m**2*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 112206*A*b**5*m*x**16*x**m/(m**7
 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
```

+ 1106560) + 69160*A*b**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + B*a**5*m**6*x**4*x
**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
1864456*m + 1106560) + 66*B*a**5*m**5*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5
+ 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1710*B*a
**5*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 21860*B*a**5*m**3*x**4*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 140529*B*a**5*m**2*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 396954*B*a**5*m*x
**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 276640*B*a**5*x**4*x**m/(m**7 + 70*m**6 + 1974*
m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*B*
a**4*b*m**6*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**
3 + 959070*m**2 + 1864456*m + 1106560) + 315*B*a**4*b*m**5*x**7*x**m/(m**7
+ 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
+ 1106560) + 7665*B*a**4*b*m**4*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 89845*B*a**4*b
*m**3*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95
9070*m**2 + 1864456*m + 1106560) + 510930*B*a**4*b*m**2*x**7*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 1218840*B*a**4*b*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 790400*B*a**4*b*x
**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*B*a**3*b**2*m**6*x**10*x**m/(m**7 + 70*m**6
+ 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
+ 600*B*a**3*b**2*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4
+ 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 13740*B*a**3*b**2*m**
4*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95907
0*m**2 + 1864456*m + 1106560) + 149600*B*a**3*b**2*m**3*x**10*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 783690*B*a**3*b**2*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 +
28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*B*
a**3*b**2*m*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m*
*3 + 959070*m**2 + 1864456*m + 1106560) + 1106560*B*a**3*b**2*x**10*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 10*B*a**2*b**3*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
+ 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 570*B*a*
*2*b**3*m**5*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 12330*B*a**2*b**3*m**4*x**13*x**
m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 18
64456*m + 1106560) + 126710*B*a**2*b**3*m**3*x**13*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
632460*B*a**2*b**3*m**2*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4
+ 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1368720*B*a**2*b**3*m
*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070
*m**2 + 1864456*m + 1106560) + 851200*B*a**2*b**3*x**13*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 5*B*a*b**4*m**6*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 270*B*a*b**4*m**5*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
1864456*m + 1106560) + 5550*B*a*b**4*m**4*x**16*x**m/(m**7 + 70*m**6 + 197
4*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 54
700*B*a*b**4*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 264645*B*a*b**4*m**2*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
1864456*m + 1106560) + 561030*B*a*b**4*m*x**16*x**m/(m**7 + 70*m**6 + 1974
*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 345
800*B*a*b**4*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m

```
**3 + 959070*m**2 + 1864456*m + 1106560) + B*b**5*m**6*x**19*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 51*B*b**5*m**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**
4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1005*B*b**5*m**4*x**
19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**
2 + 1864456*m + 1106560) + 9605*B*b**5*m**3*x**19*x**m/(m**7 + 70*m**6 + 19
74*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 4
5474*B*b**5*m**2*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 95064*B*b**5*m*x**19*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 58240*B*b**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 2870
0*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560), True))
```

## 3.125 $\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=71

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

**Rubi [A]** time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b + 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx &= \int \left(a^2Ax^m+a(2Ab+aB)x^{3+m}+b(Ab+2aB)x^{6+m}+b^2Bx^{9+m}\right) \, dx\\ &= \frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{4+m}}{4+m}+\frac{b(Ab+2aB)x^{7+m}}{7+m}+\frac{b^2Bx^{10+m}}{10+m}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 66, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2A}{m+1}+\frac{bx^6(2aB+Ab)}{m+7}+\frac{ax^3(aB+2Ab)}{m+4}+\frac{b^2Bx^9}{m+10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(4 + m) + (b*(A*b + 2*a*B)*x^6)/(7 + m) + (b^2*B*x^9)/(10 + m))
```

**IntegrateAlgebraic [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^3)^2*(A + B*x^3), x]
```

**fricas [B]** time = 0.89, size = 215, normalized size = 3.03

$$\frac{\left(\left(Bb^2m^3+12\,Bb^2m^2+39\,Bb^2m+28\,Bb^2\right)x^{10}+\left(\left(2\,Bab+Ab^2\right)m^3+80\,Bab+40\,Ab^2+15\left(2\,Bab+Ab^2\right)m^2+54\left(2\,Bab+Ab^2\right)m\right)x^7+\left(\left(Ba^2+2\,Aab\right)m^3+70\,Ba^2+140\,Aab+18\left(Ba^2+2\,Aab\right)m^2+87\left(Ba^2+2\,Aab\right)m\right)x^4+\left(Aa^2m^3+21\,Aa^2m^2+138\,Aa^2m+280\,Aa^2\right)x\right)x^m}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 12*B*b^2*m^2 + 39*B*b^2*m + 28*B*b^2)*x^10 + ((2*B*a*b + A*b^
2)*m^3 + 80*B*a*b + 40*A*b^2 + 15*(2*B*a*b + A*b^2)*m^2 + 54*(2*B*a*b + A*b
^2)*m)*x^7 + ((B*a^2 + 2*A*a*b)*m^3 + 70*B*a^2 + 140*A*a*b + 18*(B*a^2 + 2*
A*a*b)*m^2 + 87*(B*a^2 + 2*A*a*b)*m)*x^4 + (A*a^2*m^3 + 21*A*a^2*m^2 + 138*
A*a^2*m + 280*A*a^2)*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

**giac [B]** time = 0.20, size = 332, normalized size = 4.68

$$\frac{Bb^2m^3x^{10}x^m+12\,Bb^2m^2x^{10}x^m+39\,Bb^2mx^{10}x^m+2\,Babm^3x^7x^m+Ab^2m^3x^7x^m+28\,Bb^2x^{10}x^m+30\,Babm^2x^7x^m+15\,Ab^2m^2x^7x^m+108\,Babmx^7x^m+54\,Ab^2mx^7x^m+Ba^2m^3x^4x^m+2\,Aabm^3x^4x^m+80\,Babx^7x^m+40\,Ab^2x^7x^m+18\,Ba^2m^2x^4x^m+36\,Aabm^2x^4x^m+87\,Ba^2mx^4x^m+174\,Aabmx^4x^m+Aa^2m^3xx^m+70\,Ba^2x^4x^m+140\,Aabx^4x^m+21\,Aa^2m^2xx^m+138\,Aa^2mxx^m+280\,Aa^2xx^m}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^10*x^m + 12*B*b^2*m^2*x^10*x^m + 39*B*b^2*m*x^10*x^m + 2*B*a*b
*m^3*x^7*x^m + A*b^2*m^3*x^7*x^m + 28*B*b^2*x^10*x^m + 30*B*a*b*m^2*x^7*x^m
 + 15*A*b^2*m^2*x^7*x^m + 108*B*a*b*m*x^7*x^m + 54*A*b^2*m*x^7*x^m + B*a^2*
m^3*x^4*x^m + 2*A*a*b*m^3*x^4*x^m + 80*B*a*b*x^7*x^m + 40*A*b^2*x^7*x^m + 1
8*B*a^2*m^2*x^4*x^m + 36*A*a*b*m^2*x^4*x^m + 87*B*a^2*m*x^4*x^m + 174*A*a*b
*m*x^4*x^m + A*a^2*m^3*x*x^m + 70*B*a^2*x^4*x^m + 140*A*a*b*x^4*x^m + 21*A*
a^2*m^2*x*x^m + 138*A*a^2*m*x*x^m + 280*A*a^2*x*x^m)/(m^4 + 22*m^3 + 159*m^
2 + 418*m + 280)
```

**maple [B]** time = 0.05, size = 262, normalized size = 3.69

$$\frac{\left(B\,b^2m^3x^9+12B\,b^2m^2x^9+39B\,b^2m\,x^9+A\,b^2m^3x^6+2Bab\,m^3x^6+28b^2B\,x^9+15A\,b^2m^2x^6+30Bab\,m^2x^6+54A\,b^2m\,x^6+108Babm\,x^6+2Aab\,m^3x^3+40A\,b^2x^6+B\,a^2m^3x^3+80Bab\,x^6+36Aab\,m^2x^3+18B\,a^2m^2x^3+174Aabm\,x^3+87B\,a^2m\,x^3+A\,a^2m^3+140Aab\,x^3+70B\,a^2x^3+21A\,a^2m^2+138A\,a^2m+280a^2A\right)x^{m+1}}{(m+10)\,(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] x^(m+1)*(B*b^2*m^3*x^9+12*B*b^2*m^2*x^9+39*B*b^2*m*x^9+A*b^2*m^3*x^6+2*B*a*
b*m^3*x^6+28*B*b^2*x^9+15*A*b^2*m^2*x^6+30*B*a*b*m^2*x^6+54*A*b^2*m*x^6+108
*B*a*b*m*x^6+2*A*a*b*m^3*x^3+40*A*b^2*x^6+B*a^2*m^3*x^3+80*B*a*b*x^6+36*A*a
*b*m^2*x^3+18*B*a^2*m^2*x^3+174*A*a*b*m*x^3+87*B*a^2*m*x^3+A*a^2*m^3+140*A*
a*b*x^3+70*B*a^2*x^3+21*A*a^2*m^2+138*A*a^2*m+280*A*a^2)/(m+10)/(m+7)/(m+4)
/(m+1)
```

**maxima [A]** time = 0.49, size = 91, normalized size = 1.28

$$\frac{Bb^2x^{m+10}}{m+10}+\frac{2\,Babx^{m+7}}{m+7}+\frac{Ab^2x^{m+7}}{m+7}+\frac{Ba^2x^{m+4}}{m+4}+\frac{2\,Aabx^{m+4}}{m+4}+\frac{Aa^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b^2*x^(m + 10)/(m + 10) + 2*B*a*b*x^(m + 7)/(m + 7) + A*b^2*x^(m + 7)/(m
+ 7) + B*a^2*x^(m + 4)/(m + 4) + 2*A*a*b*x^(m + 4)/(m + 4) + A*a^2*x^(m + 1
)/(m + 1)
```

**mupad [B]** time = 2.72, size = 177, normalized size = 2.49

$$x^m\left(\frac{B\,b^2\,x^{10}\,\left(m^3+12\,m^2+39\,m+28\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{A\,a^2\,x\,\left(m^3+21\,m^2+138\,m+280\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{a\,x^4\,\left(2\,A\,b+B\,a\right)\,\left(m^3+18\,m^2+87\,m+70\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{b\,x^7\,\left(A\,b+2\,B\,a\right)\,\left(m^3+15\,m^2+54\,m+40\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^m*((B*b^2*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 + m^
4 + 280) + (A*a^2*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m^3
 + m^4 + 280) + (a*x^4*(2*A*b + B*a)*(87*m + 18*m^2 + m^3 + 70))/(418*m + 1
59*m^2 + 22*m^3 + m^4 + 280) + (b*x^7*(A*b + 2*B*a)*(54*m + 15*m^2 + m^3 +
40))/(418*m + 159*m^2 + 22*m^3 + m^4 + 280))
```

**sympy [A]** time = 6.31, size = 1057, normalized size = 14.89

$$\begin{cases} -\frac{Aa^2}{9x^9} - \frac{Aab}{3x^6} - \frac{Ab^2}{3x^3} - \frac{Ba^2}{6x^6} - \frac{2Bab}{3x^3} + Bb^2\log(x) & \text{for } m = -10 \\ -\frac{Aa^2}{6x^6} - \frac{2Aab}{3x^3} + Ab^2\log(x) - \frac{Ba^2}{3x^3} + 2Bab\log(x) + \frac{Bb^2x^3}{3} & \text{for } m = -7 \\ -\frac{Aa^2}{3x^3} + 2Aab\log(x) + \frac{Ab^2x^3}{3} + Ba^2\log(x) + \frac{2Babx^3}{3} + \frac{Bb^2x^6}{6} & \text{for } m = -4 \\ Aa^2\log(x) + \frac{2Aabx^3}{3} + \frac{Ab^2x^6}{6} + \frac{Ba^2x^3}{3} + \frac{Babx^6}{3} + \frac{Bb^2x^9}{9} & \text{for } m = -1 \\ \frac{Aa^2m^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{21Aa^2m^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{138Aa^2mxx^m}{m^4+22m^3+159m^2+418m+280} + \frac{280Aa^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{2Aabm^3x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{36Aabm^2x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{174Aabmx^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{140Aabx^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{Ab^2m^3x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{15Ab^2m^2x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{54Ab^2mx^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{40Ab^2x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{Ba^2m^3x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{18Ba^2m^2x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{87Ba^2mx^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{70Ba^2x^4x^m}{m^4+22m^3+159m^2+418m+280} + \frac{2Babm^3x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{30Babm^2x^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{108Babmx^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{80Babx^7x^m}{m^4+22m^3+159m^2+418m+280} + \frac{Bb^2m^3x^{10}x^m}{m^4+22m^3+159m^2+418m+280} + \frac{12Bb^2m^2x^{10}x^m}{m^4+22m^3+159m^2+418m+280} + \frac{39Bb^2mx^{10}x^m}{m^4+22m^3+159m^2+418m+280} + \frac{28Bb^2x^{10}x^m}{m^4+22m^3+159m^2+418m+280} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6*
x**6) - 2*B*a*b/(3*x**3) + B*b**2*log(x), Eq(m, -10)), (-A*a**2/(6*x**6) -
2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*b**
2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/3 +
B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*log(x)
+ 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b**2*x*
*9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m +
280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 13
8*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*A*a**2*x*
x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**4*x**m/(m*
*4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m/(m**4 + 22
*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 + 22*m**3 + 1
59*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 28
0) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 5
4*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 40*A*b**2*
x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a**2*m**3*x**4*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m**2*x**4*x**m/(m**4
 + 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*x**m/(m**4 + 22*m**
3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**
2 + 418*m + 280) + 2*B*a*b*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*
m + 280) + 30*B*a*b*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280
) + 108*B*a*b*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 80*B*
a*b*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*b**2*m**3*x**10
*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 12*B*b**2*m**2*x**10*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 39*B*b**2*m*x**10*x**m/(m**4 +
 22*m**3 + 159*m**2 + 418*m + 280) + 28*B*b**2*x**10*x**m/(m**4 + 22*m**3 +
 159*m**2 + 418*m + 280), True))
```

## 3.126 $\int x^m \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=45

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

**Rubi [A]** time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m))
/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int\left(aAx^m+(Ab+aB)x^{3+m}+bBx^{6+m}\right)\,dx\\ &= \frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 42, normalized size = 0.93

$$x^{m+1}\left(\frac{x^3(aB+Ab)}{m+4}+\frac{aA}{m+1}+\frac{bBx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^3)/(4 + m) + (b*B*x^6)/(7 + m))
```

**IntegrateAlgebraic [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^3)*(A + B*x^3), x]
```

**fricas [B]** time = 0.85, size = 92, normalized size = 2.04

$$\frac{\left(\left(Bbm^2+5\,Bbm+4\,Bb\right)x^7+\left((Ba+Ab)m^2+7\,Ba+7\,Ab+8\,(Ba+Ab)m\right)x^4+\left(Aam^2+11\,Aam+28\,Aa\right)x\right)x^m}{m^3+12\,m^2+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B*
a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39*m
 + 28)
```

**giac [B]** time = 0.17, size = 143, normalized size = 3.18

$$\frac{Bbm^2x^7x^m + 5\,Bbmx^7x^m + 4\,Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8\,Bamx^4x^m + 8\,Abmx^4x^m + 7\,Bax^4x^m + 7\,Abx^4x^m + Aam^2xx^m + 11\,Aamxx^m + 28\,Aaxx^m}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b*
m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x^4
*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39*m
+ 28)
```

**maple [B]** time = 0.04, size = 110, normalized size = 2.44

$$\frac{\left(Bb\,m^2x^6 + 5Bbm\,x^6 + 4Bb\,x^6 + Ab\,m^2x^3 + Ba\,m^2x^3 + 8Abm\,x^3 + 8Bam\,x^3 + 7Ab\,x^3 + 7Ba\,x^3 + Aa\,m^2 + 11Aam + 28Aa\right)x^{m+1}}{(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] x^(m+1)*(B*b*m^2*x^6+5*B*b*m*x^6+4*B*b*x^6+A*b*m^2*x^3+B*a*m^2*x^3+8*A*b*m*
x^3+8*B*a*m*x^3+7*A*b*x^3+7*B*a*x^3+A*a*m^2+11*A*a*m+28*A*a)/(m+7)/(m+4)/(m
+1)
```

**maxima [A]** time = 0.61, size = 53, normalized size = 1.18

$$\frac{Bbx^{m+7}}{m+7} + \frac{Bax^{m+4}}{m+4} + \frac{Abx^{m+4}}{m+4} + \frac{Aax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b*x^(m + 7)/(m + 7) + B*a*x^(m + 4)/(m + 4) + A*b*x^(m + 4)/(m + 4) + A*a
*x^(m + 1)/(m + 1)
```

**mupad [B]** time = 2.65, size = 95, normalized size = 2.11

$$x^m\,\left(\frac{x^4\,\left(A\,b+B\,a\right)\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}+\frac{B\,b\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28}+\frac{A\,a\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^m*((x^4*(A*b + B*a)*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (B*b*x^
7*(5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (A*a*x*(11*m + m^2 + 28))/(
39*m + 12*m^2 + m^3 + 28))
```

**sympy [A]** time = 2.89, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{6x^6} - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} + Bb\log(x) & \text{for } m = -7 \\ -\frac{Aa}{3x^3} + Ab\log(x) + Ba\log(x) + \frac{Bbx^3}{3} & \text{for } m = -4 \\ Aa\log(x) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} & \text{for } m = -1 \\ \frac{Aam^2xx^m}{m^3+12m^2+39m+28} + \frac{11Aamxx^m}{m^3+12m^2+39m+28} + \frac{28Aaxx^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^4x^m}{m^3+12m^2+39m+28} + \frac{7Abx^4x^m}{m^3+12m^2+39m+28} + \frac{Bam^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Bamx^4x^m}{m^3+12m^2+39m+28} + \frac{7Bax^4x^m}{m^3+12m^2+39m+28} + \frac{Bbm^2x^7x^m}{m^3+12m^2+39m+28} + \frac{5Bbmx^7x^m}{m^3+12m^2+39m+28} + \frac{4Bbx^7x^m}{m^3+12m^2+39m+28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m,
-7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)), (A
*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 28*A*a*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m/(m**
3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) +
 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(m**3 +
12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*B
*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m**3 + 12*m
**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*B*b*x
**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

## 3.127 $\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=39

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{7/2}+(Ab+aB)x^{13/2}+bBx^{19/2}\right)dx\\&=\frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{315}x^{9/2}\left(21x^3(aB+Ab)+35aA+15bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(9/2)*(35*a*A + 21*(A*b + a*B)*x^3 + 15*b*B*x^6))/315
```

**IntegrateAlgebraic [A]** time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{315}x^{9/2}\left(35aA+21aBx^3+21Abx^3+15bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(9/2)*(35*a*A + 21*A*b*x^3 + 21*a*B*x^3 + 15*b*B*x^6))/315
```

**fricas [A]** time = 0.88, size = 32, normalized size = 0.82

$$\frac{2}{315}\left(15\,Bbx^{10}+21\,(Ba+Ab)x^7+35\,Aax^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b*x^10 + 21*(B*a + A*b)*x^7 + 35*A*a*x^4)*sqrt(x)
```

**giac [A]** time = 0.17, size = 29, normalized size = 0.74

$$\frac{2}{21}\,Bbx^{\frac{21}{2}} + \frac{2}{15}\,Bax^{\frac{15}{2}} + \frac{2}{15}\,Abx^{\frac{15}{2}} + \frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b*x^(21/2) + 2/15*B*a*x^(15/2) + 2/15*A*b*x^(15/2) + 2/9*A*a*x^(9/2)
```

**maple [A]** time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(15Bb\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)x^{\frac{9}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/315*x^(9/2)*(15*B*b*x^6+21*A*b*x^3+21*B*a*x^3+35*A*a)
```

**maxima [A]** time = 0.45, size = 27, normalized size = 0.69

$$\frac{2}{21}\,Bbx^{\frac{21}{2}} + \frac{2}{15}\,(Ba + Ab)x^{\frac{15}{2}} + \frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b*x^(21/2) + 2/15*(B*a + A*b)*x^(15/2) + 2/9*A*a*x^(9/2)
```

**mupad [B]** time = 0.05, size = 31, normalized size = 0.79

$$\frac{2\,x^{9/2}\,\left(35\,A\,a+21\,A\,b\,x^3+21\,B\,a\,x^3+15\,B\,b\,x^6\right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(9/2)*(35*A*a + 21*A*b*x^3 + 21*B*a*x^3 + 15*B*b*x^6))/315
```

**sympy [A]** time = 20.86, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9} + \frac{2Abx^{\frac{15}{2}}}{15} + \frac{2Bax^{\frac{15}{2}}}{15} + \frac{2Bbx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(15/2)/15 + 2*B*a*x**(15/2)/15 + 2*B*b*x**(21/2
)/21
```

## 3.128 $\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=39

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{5/2}+(Ab+aB)x^{11/2}+bBx^{17/2}\right)\,dx\\&=\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{7/2}\left(133x^3(aB+Ab)+247aA+91bBx^6\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*x^(7/2)*(247*a*A + 133*(A*b + a*B)*x^3 + 91*b*B*x^6))/1729
```

**IntegrateAlgebraic [A]** time = 0.02, size = 41, normalized size = 1.05

$$\frac{2\left(247aAx^{7/2}+133aBx^{13/2}+133Abx^{13/2}+91bBx^{19/2}\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*(247*a*A*x^(7/2) + 133*A*b*x^(13/2) + 133*a*B*x^(13/2) + 91*b*B*x^(19/2)
))/1729
```

**fricas [A]** time = 1.32, size = 32, normalized size = 0.82

$$\frac{2}{1729}\left(91\,Bbx^9+133\,(Ba+Ab)x^6+247\,Aax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)
```

**giac [A]** time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{19}Bbx^{\frac{19}{2}} + \frac{2}{13}Bax^{\frac{13}{2}} + \frac{2}{13}Abx^{\frac{13}{2}} + \frac{2}{7}Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2)
```

**maple [A]** time = 0.04, size = 32, normalized size = 0.82

$$\frac{2\left(91Bb\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)x^{\frac{7}{2}}}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/1729*x^(7/2)*(91*B*b*x^6+133*A*b*x^3+133*B*a*x^3+247*A*a)
```

**maxima [A]** time = 0.47, size = 27, normalized size = 0.69

$$\frac{2}{19}Bbx^{\frac{19}{2}} + \frac{2}{13}(Ba + Ab)x^{\frac{13}{2}} + \frac{2}{7}Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/19*B*b*x^(19/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/7*A*a*x^(7/2)
```

**mupad [B]** time = 2.56, size = 31, normalized size = 0.79

$$\frac{2\,x^{7/2}\,\left(247\,A\,a+133\,A\,b\,x^3+133\,B\,a\,x^3+91\,B\,b\,x^6\right)}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(7/2)*(247*A*a + 133*A*b*x^3 + 133*B*a*x^3 + 91*B*b*x^6))/1729
```

**sympy [A]** time = 12.71, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7} + \frac{2Abx^{\frac{13}{2}}}{13} + \frac{2Bax^{\frac{13}{2}}}{13} + \frac{2Bbx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(19/2
)/19
```

## 3.129 $\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=39

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

**Rubi** [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{3/2}+(Ab+aB)x^{9/2}+bBx^{15/2}\right)\,dx\\&=\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{2}{935}x^{5/2}\left(85x^3(aB+Ab)+187aA+55bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(5/2)*(187*a*A + 85*(A*b + a*B)*x^3 + 55*b*B*x^6))/935
```

**IntegrateAlgebraic** [A] time = 0.02, size = 35, normalized size = 0.90

$$\frac{2}{935}x^{5/2}\left(187aA+85aBx^3+85Abx^3+55bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(5/2)*(187*a*A + 85*A*b*x^3 + 85*a*B*x^3 + 55*b*B*x^6))/935
```

**fricas** [A] time = 0.93, size = 32, normalized size = 0.82

$$\frac{2}{935}\left(55\,Bbx^8+85\,(Ba+Ab)x^5+187\,Aax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)
```

**giac** [A] time = 0.18, size = 29, normalized size = 0.74

$$\frac{2}{17}\,Bbx^{\frac{17}{2}} + \frac{2}{11}\,Bax^{\frac{11}{2}} + \frac{2}{11}\,Abx^{\frac{11}{2}} + \frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2)
```

**maple** [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(55Bb\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)x^{\frac{5}{2}}}{935}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/935*x^(5/2)*(55*B*b*x^6+85*A*b*x^3+85*B*a*x^3+187*A*a)
```

**maxima** [A] time = 0.45, size = 27, normalized size = 0.69

$$\frac{2}{17}\,Bbx^{\frac{17}{2}} + \frac{2}{11}\,(Ba + Ab)x^{\frac{11}{2}} + \frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/5*A*a*x^(5/2)
```

**mupad** [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{5/2}\,\left(187\,A\,a+85\,A\,b\,x^3+85\,B\,a\,x^3+55\,B\,b\,x^6\right)}{935}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(5/2)*(187*A*a + 85*A*b*x^3 + 85*B*a*x^3 + 55*B*b*x^6))/935
```

**sympy** [A] time = 6.74, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5} + \frac{2Abx^{\frac{11}{2}}}{11} + \frac{2Bax^{\frac{11}{2}}}{11} + \frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(17/2
)/17
```

## 3.130 $\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=39

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

**Rubi** [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aA\sqrt{x}+(Ab+aB)x^{7/2}+bBx^{13/2}\right)dx\\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{45}x^{3/2}\left(5x^3(aB+Ab)+15aA+3bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(3/2)*(15*a*A + 5*(A*b + a*B)*x^3 + 3*b*B*x^6))/45
```

**IntegrateAlgebraic** [A] time = 0.02, size = 41, normalized size = 1.05

$$\frac{2}{45}\left(15aAx^{3/2}+5aBx^{9/2}+5Abx^{9/2}+3bBx^{15/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*(15*a*A*x^(3/2) + 5*A*b*x^(9/2) + 5*a*B*x^(9/2) + 3*b*B*x^(15/2)))/45
```

**fricas** [A] time = 0.89, size = 30, normalized size = 0.77

$$\frac{2}{45}\left(3\,Bbx^7+5\,(Ba+Ab)x^4+15\,Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)
```

**giac [A]** time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{15}Bbx^{\frac{15}{2}} + \frac{2}{9}Bax^{\frac{9}{2}} + \frac{2}{9}Abx^{\frac{9}{2}} + \frac{2}{3}Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="giac")
[Out] 2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)
```

**maple [A]** time = 0.04, size = 32, normalized size = 0.82

$$\frac{2\left(3Bb\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)x^{\frac{3}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)*x^(1/2),x)
[Out] 2/45*x^(3/2)*(3*B*b*x^6+5*A*b*x^3+5*B*a*x^3+15*A*a)
```

**maxima [A]** time = 0.47, size = 27, normalized size = 0.69

$$\frac{2}{15}Bbx^{\frac{15}{2}} + \frac{2}{9}(Ba + Ab)x^{\frac{9}{2}} + \frac{2}{3}Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
[Out] 2/15*B*b*x^(15/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/3*A*a*x^(3/2)
```

**mupad [B]** time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{3/2}\,\left(15\,A\,a+5\,A\,b\,x^3+5\,B\,a\,x^3+3\,B\,b\,x^6\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3),x)
[Out] (2*x^(3/2)*(15*A*a + 5*A*b*x^3 + 5*B*a*x^3 + 3*B*b*x^6))/45
```

**sympy [A]** time = 3.37, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{3}{2}}}{3} + \frac{2Abx^{\frac{9}{2}}}{9} + \frac{2Bax^{\frac{9}{2}}}{9} + \frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)*x**(1/2),x)
[Out] 2*A*a*x**(3/2)/3 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(15/2)/15
```

## 3.131 $\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}} \, dx$

**Optimal**. Leaf size=37

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx &= \int \left(\frac{aA}{\sqrt{x}}+(Ab+aB)x^{5/2}+bBx^{11/2}\right)dx\\ &= 2aA\sqrt{x}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{13}bBx^{13/2}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 33, normalized size = 0.89

$$\frac{2}{91}\sqrt{x}\left(13x^3(aB+Ab)+91aA+7bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(91*a*A + 13*(A*b + a*B)*x^3 + 7*b*B*x^6))/91
```

**IntegrateAlgebraic [A]** time = 0.02, size = 41, normalized size = 1.11

$$\frac{2}{91}\left(91aA\sqrt{x}+13aBx^{7/2}+13Abx^{7/2}+7bBx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*(91*a*A*Sqrt[x] + 13*A*b*x^(7/2) + 13*a*B*x^(7/2) + 7*b*B*x^(13/2)))/91
```

**fricas [A]** time = 0.75, size = 29, normalized size = 0.78

$$\frac{2}{91}\left(7\,Bbx^6+13\,(Ba+Ab)x^3+91\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*sqrt(x)
```

**giac [A]** time = 0.15, size = 29, normalized size = 0.78

$$\frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,Bax^{\frac{7}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2*A*a*sqrt(x)
```

**maple [A]** time = 0.04, size = 32, normalized size = 0.86

$$\frac{2\left(7Bb\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa\right)\sqrt{x}}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/91*x^(1/2)*(7*B*b*x^6+13*A*b*x^3+13*B*a*x^3+91*A*a)
```

**maxima [A]** time = 0.50, size = 27, normalized size = 0.73

$$\frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,(Ba + Ab)x^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)
```

**mupad [B]** time = 2.59, size = 31, normalized size = 0.84

$$\frac{2\,\sqrt{x}\,\left(91\,A\,a+13\,A\,b\,x^3+13\,B\,a\,x^3+7\,B\,b\,x^6\right)}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(91*A*a + 13*A*b*x^3 + 13*B*a*x^3 + 7*B*b*x^6))/91
```

**sympy [A]** time = 2.30, size = 44, normalized size = 1.19

$$2Aa\sqrt{x} + \frac{2Abx^{\frac{7}{2}}}{7} + \frac{2Bax^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13
```

## 3.132 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{aA}{x^{3/2}}+(Ab+aB)x^{3/2}+bBx^{9/2}\right)dx\\ &= -\frac{2aA}{\sqrt{x}}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 35, normalized size = 0.95

$$\frac{2\left(-55aA+11aBx^3+11Abx^3+5bBx^6\right)}{55\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (2*(-55*a*A + 11*A*b*x^3 + 11*a*B*x^3 + 5*b*B*x^6))/(55*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.02, size = 35, normalized size = 0.95

$$\frac{2\left(-55aA+11aBx^3+11Abx^3+5bBx^6\right)}{55\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (2*(-55*a*A + 11*A*b*x^3 + 11*a*B*x^3 + 5*b*B*x^6))/(55*Sqrt[x])
```

**fricas** [A] time = 0.78, size = 29, normalized size = 0.78

$$\frac{2\left(5\,Bbx^{6}+11\,(Ba+Ab)x^{3}-55\,Aa\right)}{55\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
[Out] 2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/sqrt(x)
```

**giac** [A] time = 0.15, size = 29, normalized size = 0.78

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,Bax^{\frac{5}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="giac")
[Out] 2/11*B*b*x^(11/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) - 2*A*a/sqrt(x)
```

**maple** [A] time = 0.04, size = 32, normalized size = 0.86

$$-\frac{2\left(-5Bb\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa\right)}{55\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(3/2),x)
[Out] -2/55*(-5*B*b*x^6-11*A*b*x^3-11*B*a*x^3+55*A*a)/x^(1/2)
```

**maxima** [A] time = 0.46, size = 27, normalized size = 0.73

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
[Out] 2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)
```

**mupad** [B] time = 2.60, size = 31, normalized size = 0.84

$$\frac{22\,A\,b\,x^{3}-110\,A\,a+22\,B\,a\,x^{3}+10\,B\,b\,x^{6}}{55\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(3/2),x)
[Out] (22*A*b*x^3 - 110*A*a + 22*B*a*x^3 + 10*B*b*x^6)/(55*x^(1/2))
```

**sympy** [A] time = 2.77, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}}+\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Bax^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11
```

## 3.133 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx$

**Optimal**. Leaf size=39

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
[Out] (-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{aA}{x^{5/2}}+(Ab+aB)\sqrt{x}+bBx^{7/2}\right)dx\\ &= -\frac{2aA}{3x^{3/2}}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{9}bBx^{9/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 34, normalized size = 0.87

$$\frac{2\left(-3aA+3aBx^3+3Abx^3+bBx^6\right)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
[Out] (2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.02, size = 34, normalized size = 0.87

$$\frac{2\left(-3aA+3aBx^3+3Abx^3+bBx^6\right)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
[Out] (2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

**fricas [A]** time = 0.89, size = 28, normalized size = 0.72

$$\frac{2\left(Bbx^6+3\,(Ba+Ab)x^3-3\,Aa\right)}{9\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)
```

**giac [A]** time = 0.16, size = 29, normalized size = 0.74

$$\frac{2}{9}Bbx^{\frac{9}{2}}+\frac{2}{3}Bax^{\frac{3}{2}}+\frac{2}{3}Abx^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)
```

**maple [A]** time = 0.04, size = 32, normalized size = 0.82

$$-\frac{2\left(-Bb\,x^{6}-3Ab\,x^{3}-3Ba\,x^{3}+3Aa\right)}{9x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/9*(-B*b*x^6-3*A*b*x^3-3*B*a*x^3+3*A*a)/x^(3/2)
```

**maxima [A]** time = 0.48, size = 27, normalized size = 0.69

$$\frac{2}{9}Bbx^{\frac{9}{2}}+\frac{2}{3}(Ba+Ab)x^{\frac{3}{2}}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)
```

**mupad [B]** time = 0.04, size = 31, normalized size = 0.79

$$\frac{6\,A\,b\,x^3-6\,A\,a+6\,B\,a\,x^3+2\,B\,b\,x^6}{9\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(5/2),x)
```

```
[Out] (6*A*b*x^3 - 6*A*a + 6*B*a*x^3 + 2*B*b*x^6)/(9*x^(3/2))
```

**sympy [A]** time = 3.74, size = 46, normalized size = 1.18

$$-\frac{2Aa}{3x^{\frac{3}{2}}}+\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Bax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)/
9
```

## 3.134 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx$

**Optimal**. Leaf size=37

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

**Rubi [A]** time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{aA}{x^{7/2}}+\frac{Ab+aB}{\sqrt{x}}+bBx^{5/2}\right)dx\\ &= -\frac{2aA}{5x^{5/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{7}bBx^{7/2}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 36, normalized size = 0.97

$$\frac{2\left(5bx^3\left(7A+Bx^3\right)-7a\left(A-5Bx^3\right)\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (2*(-7*a*(A - 5*B*x^3) + 5*b*x^3*(7*A + B*x^3)))/(35*x^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.03, size = 35, normalized size = 0.95

$$\frac{2\left(-7aA+35aBx^3+35Abx^3+5bBx^6\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (2*(-7*a*A + 35*A*b*x^3 + 35*a*B*x^3 + 5*b*B*x^6))/(35*x^(5/2))
```

**fricas [A]** time = 0.75, size = 29, normalized size = 0.78

$$\frac{2\left(5\,Bbx^6+35\,(Ba+Ab)x^3-7\,Aa\right)}{35\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
[Out] 2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)
```

**giac [A]** time = 0.26, size = 29, normalized size = 0.78

$$\frac{2}{7}Bbx^{\frac{7}{2}} + 2Ba\sqrt{x} + 2Ab\sqrt{x} - \frac{2Aa}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="giac")
[Out] 2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)
```

**maple [A]** time = 0.04, size = 32, normalized size = 0.86

$$-\frac{2\left(-5Bbx^{6} - 35Abx^{3} - 35Bax^{3} + 7Aa\right)}{35x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(7/2),x)
[Out] -2/35*(-5*B*b*x^6-35*A*b*x^3-35*B*a*x^3+7*A*a)/x^(5/2)
```

**maxima [A]** time = 0.45, size = 27, normalized size = 0.73

$$\frac{2}{7}Bbx^{\frac{7}{2}} + 2(Ba + Ab)\sqrt{x} - \frac{2Aa}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
[Out] 2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)
```

**mupad [B]** time = 0.04, size = 30, normalized size = 0.81

$$\frac{2Abx^3 - \frac{2Aa}{5} + 2Bax^3 + \frac{2Bbx^6}{7}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(7/2),x)
[Out] (2*A*b*x^3 - (2*A*a)/5 + 2*B*a*x^3 + (2*B*b*x^6)/7)/x^(5/2)
```

**sympy [A]** time = 4.21, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)
[Out] -2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7
```

## 3.135 $\int x^{7/2} \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{21}bx^{21/2}(2aB+Ab)+\frac{2}{15}ax^{15/2}(aB+2Ab)+\frac{2}{27}b^2Bx^{27/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{21}bx^{21/2}(2aB+Ab)+\frac{2}{15}ax^{15/2}(aB+2Ab)+\frac{2}{27}b^2Bx^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)*
x^(21/2))/21 + (2*b^2*B*x^(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)dx&=\int\left(a^2Ax^{7/2}+a(2Ab+aB)x^{13/2}+b(Ab+2aB)x^{19/2}+b^2Bx^{25/2}\right)dx\\&=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{15}a(2Ab+aB)x^{15/2}+\frac{2}{21}b(Ab+2aB)x^{21/2}+\frac{2}{27}b^2Bx^{27/2}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 53, normalized size = 0.84

$$\frac{2}{945}x^{9/2}\left(105a^2A+45bx^6(2aB+Ab)+63ax^3(aB+2Ab)+35b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(105*a^2*A + 63*a*(2*A*b + a*B)*x^3 + 45*b*(A*b + 2*a*B)*x^6 + 3
5*b^2*B*x^9))/945
```

**IntegrateAlgebraic [A]** time = 0.03, size = 59, normalized size = 0.94

$$\frac{2}{945}x^{9/2}\left(105a^2A+63a^2Bx^3+126aAbx^3+90abBx^6+45Ab^2x^6+35b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(105*a^2*A + 126*a*A*b*x^3 + 63*a^2*B*x^3 + 45*A*b^2*x^6 + 90*a*
b*B*x^6 + 35*b^2*B*x^9))/945
```

**fricas** [A] time = 0.76, size = 56, normalized size = 0.89

$$\frac{2}{945}\left(35\,Bb^2x^{13}+45\left(2\,Bab+Ab^2\right)x^{10}+63\left(Ba^2+2\,Aab\right)x^7+105\,Aa^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^2*x^13 + 45*(2*B*a*b + A*b^2)*x^10 + 63*(B*a^2 + 2*A*a*b)*x^7
 + 105*A*a^2*x^4)*sqrt(x)
```

**giac** [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{2}{27}\,Bb^2x^{\frac{27}{2}}+\frac{4}{21}\,Babx^{\frac{21}{2}}+\frac{2}{21}\,Ab^2x^{\frac{21}{2}}+\frac{2}{15}\,Ba^2x^{\frac{15}{2}}+\frac{4}{15}\,Aabx^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 4/21*B*a*b*x^(21/2) + 2/21*A*b^2*x^(21/2) + 2/15*B*a^
2*x^(15/2) + 4/15*A*a*b*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

**maple** [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(35b^2B\,x^9+45A\,b^2x^6+90Bab\,x^6+126Aab\,x^3+63B\,a^2x^3+105a^2A\right)x^{\frac{9}{2}}}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/945*x^(9/2)*(35*B*b^2*x^9+45*A*b^2*x^6+90*B*a*b*x^6+126*A*a*b*x^3+63*B*a^
2*x^3+105*A*a^2)
```

**maxima** [A] time = 0.56, size = 51, normalized size = 0.81

$$\frac{2}{27}\,Bb^2x^{\frac{27}{2}}+\frac{2}{21}\left(2\,Bab+Ab^2\right)x^{\frac{21}{2}}+\frac{2}{15}\left(Ba^2+2\,Aab\right)x^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 2/21*(2*B*a*b + A*b^2)*x^(21/2) + 2/15*(B*a^2 + 2*A*a
*b)*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

**mupad** [B] time = 2.57, size = 51, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,a^2}{15}+\frac{4\,A\,b\,a}{15}\right)+x^{21/2}\left(\frac{2\,A\,b^2}{21}+\frac{4\,B\,a\,b}{21}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{27/2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(15/2)*((2*B*a^2)/15 + (4*A*a*b)/15) + x^(21/2)*((2*A*b^2)/21 + (4*B*a*b)
/21) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(27/2))/27
```

**sympy** [A] time = 47.81, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{21}{2}}}{21}+\frac{2Ba^2x^{\frac{15}{2}}}{15}+\frac{4Babx^{\frac{21}{2}}}{21}+\frac{2Bb^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(15/2)/15 + 2*A*b**2*x**(21/2)/21 + 2*B*a*
*2*x**(15/2)/15 + 4*B*a*b*x**(21/2)/21 + 2*B*b**2*x**(27/2)/27
```

## 3.136 $\int x^{5/2} \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2Ax^{5/2}+a(2Ab+aB)x^{11/2}+b(Ab+2aB)x^{17/2}+b^2Bx^{23/2}\right)\,dx\\ &= \frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{19}b(Ab+2aB)x^{19/2}+\frac{2}{25}b^2Bx^{25/2}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 63, normalized size = 1.00

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

**IntegrateAlgebraic [A]** time = 0.03, size = 69, normalized size = 1.10

$$\frac{2\left(6175a^2Ax^{7/2}+3325a^2Bx^{13/2}+6650aAbx^{13/2}+4550abBx^{19/2}+2275Ab^2x^{19/2}+1729b^2Bx^{25/2}\right)}{43225}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*(6175*a^2*A*x^(7/2) + 6650*a*A*b*x^(13/2) + 3325*a^2*B*x^(13/2) + 2275*A
*b^2*x^(19/2) + 4550*a*b*B*x^(19/2) + 1729*b^2*B*x^(25/2)))/43225
```

**fricas** [A] time = 0.76, size = 56, normalized size = 0.89

$$\frac{2}{43225}\left(1729\,Bb^2x^{12}+2275\left(2\,Bab+Ab^2\right)x^9+3325\left(Ba^2+2\,Aab\right)x^6+6175\,Aa^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^2*x^12 + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*a
*b)*x^6 + 6175*A*a^2*x^3)*sqrt(x)
```

**giac** [A] time = 0.18, size = 53, normalized size = 0.84

$$\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{4}{19}\,Babx^{\frac{19}{2}}+\frac{2}{19}\,Ab^2x^{\frac{19}{2}}+\frac{2}{13}\,Ba^2x^{\frac{13}{2}}+\frac{4}{13}\,Aabx^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 4/19*B*a*b*x^(19/2) + 2/19*A*b^2*x^(19/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

**maple** [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2\left(1729b^2B\,x^9+2275A\,b^2x^6+4550Bab\,x^6+6650Aab\,x^3+3325B\,a^2x^3+6175a^2A\right)x^{\frac{7}{2}}}{43225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/43225*x^(7/2)*(1729*B*b^2*x^9+2275*A*b^2*x^6+4550*B*a*b*x^6+6650*A*a*b*x^
3+3325*B*a^2*x^3+6175*A*a^2)
```

**maxima** [A] time = 0.54, size = 51, normalized size = 0.81

$$\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{2}{19}\left(2\,Bab+Ab^2\right)x^{\frac{19}{2}}+\frac{2}{13}\left(Ba^2+2\,Aab\right)x^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 2/19*(2*B*a*b + A*b^2)*x^(19/2) + 2/13*(B*a^2 + 2*A*a
*b)*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{19/2}\left(\frac{2\,A\,b^2}{19}+\frac{4\,B\,a\,b}{19}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(19/2)*((2*A*b^2)/19 + (4*B*a*b)
/19) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(25/2))/25
```

**sympy** [A] time = 29.39, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{19}{2}}}{19}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{19}{2}}}{19}+\frac{2Bb^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25
```

## 3.137 $\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{23}b^2Bx^{23/2}$$

**Rubi** [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{23}b^2Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2Ax^{3/2}+a(2Ab+aB)x^{9/2}+b(Ab+2aB)x^{15/2}+b^2Bx^{21/2}\right)dx\\ &= \frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{23}b^2Bx^{23/2}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(4301a^2A+1265bx^6(2aB+Ab)+1955ax^3(aB+2Ab)+935b^2Bx^9\right)}{21505}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(5/2)*(4301*a^2*A + 1955*a*(2*A*b + a*B)*x^3 + 1265*b*(A*b + 2*a*B)*x^
6 + 935*b^2*B*x^9))/21505
```

**IntegrateAlgebraic** [A] time = 0.03, size = 69, normalized size = 1.10

$$\frac{2\left(4301a^2Ax^{5/2}+1955a^2Bx^{11/2}+3910aAbx^{11/2}+2530abBx^{17/2}+1265Ab^2x^{17/2}+935b^2Bx^{23/2}\right)}{21505}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*(4301*a^2*A*x^(5/2) + 3910*a*A*b*x^(11/2) + 1955*a^2*B*x^(11/2) + 1265*A
*b^2*x^(17/2) + 2530*a*b*B*x^(17/2) + 935*b^2*B*x^(23/2)))/21505
```

**fricas** [A] time = 0.78, size = 56, normalized size = 0.89

$$\frac{2}{21505} \left(935 \, B b^{2} x^{11} + 1265 \, \left(2 \, B a b + A b^{2}\right) x^{8} + 1955 \, \left(B a^{2} + 2 \, A a b\right) x^{5} + 4301 \, A a^{2} x^{2}\right) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^2*x^11 + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a*
b)*x^5 + 4301*A*a^2*x^2)*sqrt(x)
```

**giac** [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{23} \, B b^{2} x^{\frac{23}{2}} + \frac{4}{17} \, B a b x^{\frac{17}{2}} + \frac{2}{17} \, A b^{2} x^{\frac{17}{2}} + \frac{2}{11} \, B a^{2} x^{\frac{11}{2}} + \frac{4}{11} \, A a b x^{\frac{11}{2}} + \frac{2}{5} \, A a^{2} x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

**maple** [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2 \left(935 b^{2} B \,x^{9}+1265 A \,b^{2} x^{6}+2530 B a b \,x^{6}+3910 A a b \,x^{3}+1955 B \,a^{2} x^{3}+4301 a^{2} A \right) x^{\frac{5}{2}}}{21505}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/21505*x^(5/2)*(935*B*b^2*x^9+1265*A*b^2*x^6+2530*B*a*b*x^6+3910*A*a*b*x^3
+1955*B*a^2*x^3+4301*A*a^2)
```

**maxima** [A] time = 0.50, size = 51, normalized size = 0.81

$$\frac{2}{23} \, B b^{2} x^{\frac{23}{2}} + \frac{2}{17} \, \left(2 \, B a b + A b^{2}\right) x^{\frac{17}{2}} + \frac{2}{11} \, \left(B a^{2} + 2 \, A a b\right) x^{\frac{11}{2}} + \frac{2}{5} \, A a^{2} x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/11*(B*a^2 + 2*A*a
*b)*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2}\,\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{17/2}\,\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b)
/17) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(23/2))/23
```

**sympy** [A] time = 21.96, size = 80, normalized size = 1.27

$$\frac{2 A a^{2} x^{\frac{5}{2}}}{5} + \frac{4 A a b x^{\frac{11}{2}}}{11} + \frac{2 A b^{2} x^{\frac{17}{2}}}{17} + \frac{2 B a^{2} x^{\frac{11}{2}}}{11} + \frac{4 B a b x^{\frac{17}{2}}}{17} + \frac{2 B b^{2} x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23
```

## 3.138 $\int \sqrt{x} \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2A\sqrt{x}+a(2Ab+aB)x^{7/2}+b(Ab+2aB)x^{13/2}+b^2Bx^{19/2}\right)dx\\ &= \frac{2}{3}a^2Ax^{3/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{21}b^2Bx^{21/2}\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 53, normalized size = 0.84

$$\frac{2}{315}x^{3/2}\left(105a^2A+21bx^6(2aB+Ab)+35ax^3(aB+2Ab)+15b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(105*a^2*A + 35*a*(2*A*b + a*B)*x^3 + 21*b*(A*b + 2*a*B)*x^6 + 1
5*b^2*B*x^9))/315
```

**IntegrateAlgebraic [A]** time = 0.04, size = 69, normalized size = 1.10

$$\frac{2}{315}\left(105a^2Ax^{3/2}+35a^2Bx^{9/2}+70aAbx^{9/2}+42abBx^{15/2}+21Ab^2x^{15/2}+15b^2Bx^{21/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*(105*a^2*A*x^(3/2) + 70*a*A*b*x^(9/2) + 35*a^2*B*x^(9/2) + 21*A*b^2*x^(1
5/2) + 42*a*b*B*x^(15/2) + 15*b^2*B*x^(21/2)))/315
```

**fricas** [A] time = 0.63, size = 54, normalized size = 0.86

$$\frac{2}{315}\left(15\,Bb^2x^{10} + 21\left(2\,Bab + Ab^2\right)x^7 + 35\left(Ba^2 + 2\,Aab\right)x^4 + 105\,Aa^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^2*x^10 + 21*(2*B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2*A*a*b)*x^4
+ 105*A*a^2*x)*sqrt(x)
```

**giac** [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}} + \frac{4}{15}\,Babx^{\frac{15}{2}} + \frac{2}{15}\,Ab^2x^{\frac{15}{2}} + \frac{2}{9}\,Ba^2x^{\frac{9}{2}} + \frac{4}{9}\,Aabx^{\frac{9}{2}} + \frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

**maple** [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2\left(15b^2B\,x^9 + 21A\,b^2x^6 + 42Bab\,x^6 + 70Aab\,x^3 + 35B\,a^2x^3 + 105a^2A\right)x^{\frac{3}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/315*x^(3/2)*(15*B*b^2*x^9+21*A*b^2*x^6+42*B*a*b*x^6+70*A*a*b*x^3+35*B*a^2
*x^3+105*A*a^2)
```

**maxima** [A] time = 0.54, size = 51, normalized size = 0.81

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}} + \frac{2}{15}\left(2\,Bab + Ab^2\right)x^{\frac{15}{2}} + \frac{2}{9}\left(Ba^2 + 2\,Aab\right)x^{\frac{9}{2}} + \frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/9*(B*a^2 + 2*A*a*
b)*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

**mupad** [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^2}{9} + \frac{4\,A\,b\,a}{9}\right) + x^{15/2}\left(\frac{2\,A\,b^2}{15} + \frac{4\,B\,a\,b}{15}\right) + \frac{2\,A\,a^2\,x^{3/2}}{3} + \frac{2\,B\,b^2\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)/15
) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(21/2))/21
```

**sympy** [A] time = 5.31, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{3}{2}}}{3} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21
```

## 3.139 $\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=61

$$2a^2A\sqrt{x}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2A\sqrt{x}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13/
2))/13 + (2*b^2*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2A}{\sqrt{x}}+a(2Ab+aB)x^{5/2}+b(Ab+2aB)x^{11/2}+b^2Bx^{17/2}\right)dx\\ &= 2a^2A\sqrt{x}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{19}b^2Bx^{19/2}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 53, normalized size = 0.87

$$\frac{2\sqrt{x}\left(1729a^2A+133bx^6(2aB+Ab)+247ax^3(aB+2Ab)+91b^2Bx^9\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(1729*a^2*A + 247*a*(2*A*b + a*B)*x^3 + 133*b*(A*b + 2*a*B)*x^6
+ 91*b^2*B*x^9))/1729
```

**IntegrateAlgebraic [A]** time = 0.03, size = 69, normalized size = 1.13

$$\frac{2\left(1729a^2A\sqrt{x}+247a^2Bx^{7/2}+494aAbx^{7/2}+266abBx^{13/2}+133Ab^2x^{13/2}+91b^2Bx^{19/2}\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*(1729*a^2*A*Sqrt[x] + 494*a*A*b*x^(7/2) + 247*a^2*B*x^(7/2) + 133*A*b^2*
x^(13/2) + 266*a*b*B*x^(13/2) + 91*b^2*B*x^(19/2)))/1729
```

**fricas [A]** time = 0.82, size = 53, normalized size = 0.87

$$\frac{2}{1729}\left(91\,Bb^{2}x^{9}+133\left(2\,Bab+Ab^{2}\right)x^{6}+247\left(Ba^{2}+2\,Aab\right)x^{3}+1729\,Aa^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x^
3 + 1729*A*a^2)*sqrt(x)
```

**giac [A]** time = 0.23, size = 53, normalized size = 0.87

$$\frac{2}{19}\,Bb^{2}x^{\frac{19}{2}}+\frac{4}{13}\,Babx^{\frac{13}{2}}+\frac{2}{13}\,Ab^{2}x^{\frac{13}{2}}+\frac{2}{7}\,Ba^{2}x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+2\,Aa^{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)
```

**maple [A]** time = 0.05, size = 56, normalized size = 0.92

$$\frac{2\left(91b^{2}B\,x^{9}+133A\,b^{2}x^{6}+266Bab\,x^{6}+494Aab\,x^{3}+247B\,a^{2}x^{3}+1729a^{2}A\right)\sqrt{x}}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/1729*x^(1/2)*(91*B*b^2*x^9+133*A*b^2*x^6+266*B*a*b*x^6+494*A*a*b*x^3+247*
B*a^2*x^3+1729*A*a^2)
```

**maxima [A]** time = 0.51, size = 51, normalized size = 0.84

$$\frac{2}{19}\,Bb^{2}x^{\frac{19}{2}}+\frac{2}{13}\left(2\,Bab+Ab^{2}\right)x^{\frac{13}{2}}+\frac{2}{7}\left(Ba^{2}+2\,Aab\right)x^{\frac{7}{2}}+2\,Aa^{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/7*(B*a^2 + 2*A*a*
b)*x^(7/2) + 2*A*a^2*sqrt(x)
```

**mupad [B]** time = 0.05, size = 51, normalized size = 0.84

$$x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right)+x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right)+2\,A\,a^2\,\sqrt{x}+\frac{2\,B\,b^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/13
) + 2*A*a^2*x^(1/2) + (2*B*b^2*x^(19/2))/19
```

**sympy** [A] time = 9.22, size = 78, normalized size = 1.28

$$2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{7}{2}}}{7} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{7}{2}}}{7} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x*
*(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19
```

## 3.140 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx$

**Optimal**. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(
11/2))/11 + (2*b^2*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{a^2A}{x^{3/2}}+a(2Ab+aB)x^{3/2}+b(Ab+2aB)x^{9/2}+b^2Bx^{15/2}\right)dx\\ &= -\frac{2a^2A}{\sqrt{x}}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{17}b^2Bx^{17/2}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 60, normalized size = 0.98

$$\frac{-374a^2(5A-Bx^3)+68abx^3(11A+5Bx^3)+10b^2x^6(17A+11Bx^3)}{935\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-374*a^2*(5*A - B*x^3) + 68*a*b*x^3*(11*A + 5*B*x^3) + 10*b^2*x^6*(17*A +
11*B*x^3))/(935*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-935a^2A+187a^2Bx^3+374aAbx^3+170abBx^6+85Ab^2x^6+55b^2Bx^9\right)}{935\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (2*(-935*a^2*A + 374*a*A*b*x^3 + 187*a^2*B*x^3 + 85*A*b^2*x^6 + 170*a*b*B*x
^6 + 55*b^2*B*x^9))/(935*Sqrt[x])
```

**fricas [A]** time = 0.71, size = 53, normalized size = 0.87

$$\frac{2\left(55\,Bb^2x^9+85\left(2\,Bab+Ab^2\right)x^6+187\left(Ba^2+2\,Aab\right)x^3-935\,Aa^2\right)}{935\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3
- 935*A*a^2)/sqrt(x)
```

**giac [A]** time = 0.15, size = 53, normalized size = 0.87

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{5}\,Ba^2x^{\frac{5}{2}}+\frac{4}{5}\,Aabx^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^2
*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)
```

**maple [A]** time = 0.05, size = 56, normalized size = 0.92

$$-\frac{2\left(-55b^2B\,x^9-85A\,b^2x^6-170Bab\,x^6-374Aab\,x^3-187B\,a^2x^3+935a^2A\right)}{935\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x)
```

```
[Out] -2/935*(-55*B*b^2*x^9-85*A*b^2*x^6-170*B*a*b*x^6-374*A*a*b*x^3-187*B*a^2*x^
3+935*A*a^2)/x^(1/2)
```

**maxima [A]** time = 0.54, size = 51, normalized size = 0.84

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{2}{11}\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{5}\left(Ba^2+2\,Aab\right)x^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/5*(B*a^2 + 2*A*a*
b)*x^(5/2) - 2*A*a^2/sqrt(x)
```

**mupad [B]** time = 0.05, size = 51, normalized size = 0.84

$$x^{5/2}\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right)-\frac{2\,A\,a^2}{\sqrt{x}}+\frac{2\,B\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/11
) - (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(17/2))/17
```

**sympy** [**A**] time = 7.80, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Ba^2x^{\frac{5}{2}}}{5}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*x
**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17
```

## 3.141 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx$

**Optimal**. Leaf size=63

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

**Rubi [A]** time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)
*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{a^2A}{x^{5/2}}+a(2Ab+aB)\sqrt{x}+b(Ab+2aB)x^{7/2}+b^2Bx^{13/2}\right)dx\\ &= -\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{15}b^2Bx^{15/2}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 57, normalized size = 0.90

$$\frac{-30a^2\left(A-Bx^3\right)+20abx^3\left(3A+Bx^3\right)+2b^2x^6\left(5A+3Bx^3\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-30*a^2*(A - B*x^3) + 20*a*b*x^3*(3*A + B*x^3) + 2*b^2*x^6*(5*A + 3*B*x^3)
)/(45*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 59, normalized size = 0.94

$$\frac{2\left(-15a^2A+15a^2Bx^3+30aAbx^3+10abBx^6+5Ab^2x^6+3b^2Bx^9\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-15*a^2*A + 30*a*A*b*x^3 + 15*a^2*B*x^3 + 5*A*b^2*x^6 + 10*a*b*B*x^6 +
3*b^2*B*x^9))/(45*x^(3/2))
```

**fricas [A]** time = 0.77, size = 53, normalized size = 0.84

$$\frac{2\left(3\,Bb^{2}x^{9}+5\left(2\,Bab+Ab^{2}\right)x^{6}+15\left(Ba^{2}+2\,Aab\right)x^{3}-15\,Aa^{2}\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 15
*A*a^2)/x^(3/2)
```

**giac [A]** time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{15}\,Bb^{2}x^{\frac{15}{2}}+\frac{4}{9}\,Babx^{\frac{9}{2}}+\frac{2}{9}\,Ab^{2}x^{\frac{9}{2}}+\frac{2}{3}\,Ba^{2}x^{\frac{3}{2}}+\frac{4}{3}\,Aabx^{\frac{3}{2}}-\frac{2\,Aa^{2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

**maple [A]** time = 0.05, size = 56, normalized size = 0.89

$$-\frac{2\left(-3b^{2}B\,x^{9}-5A\,b^{2}x^{6}-10Bab\,x^{6}-30Aab\,x^{3}-15B\,a^{2}x^{3}+15a^{2}A\right)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/45*(-3*B*b^2*x^9-5*A*b^2*x^6-10*B*a*b*x^6-30*A*a*b*x^3-15*B*a^2*x^3+15*A
*a^2)/x^(3/2)
```

**maxima [A]** time = 0.51, size = 51, normalized size = 0.81

$$\frac{2}{15}\,Bb^{2}x^{\frac{15}{2}}+\frac{2}{9}\left(2\,Bab+Ab^{2}\right)x^{\frac{9}{2}}+\frac{2}{3}\left(Ba^{2}+2\,Aab\right)x^{\frac{3}{2}}-\frac{2\,Aa^{2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b)
*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

**mupad [B]** time = 0.05, size = 51, normalized size = 0.81

$$x^{3/2}\left(\frac{2\,B\,a^{2}}{3}+\frac{4\,A\,b\,a}{3}\right)+x^{9/2}\left(\frac{2\,A\,b^{2}}{9}+\frac{4\,B\,a\,b}{9}\right)-\frac{2\,A\,a^{2}}{3\,x^{3/2}}+\frac{2\,B\,b^{2}\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9) -
 (2*A*a^2)/(3*x^(3/2)) + (2*B*b^2*x^(15/2))/15
```

**sympy [A]** time = 11.74, size = 80, normalized size = 1.27

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a**
2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15
```

## 3.142 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx$

**Optimal**. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

**Rubi** [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{a^2A}{x^{7/2}}+\frac{a(2Ab+aB)}{\sqrt{x}}+b(Ab+2aB)x^{5/2}+b^2Bx^{11/2}\right)dx\\ &= -\frac{2a^2A}{5x^{5/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{13}b^2Bx^{13/2}\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 57, normalized size = 0.93

$$\frac{2\left(-91a^2\left(A-5Bx^3\right)+130abx^3\left(7A+Bx^3\right)+5b^2x^6\left(13A+7Bx^3\right)\right)}{455x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (2*(-91*a^2*(A - 5*B*x^3) + 130*a*b*x^3*(7*A + B*x^3) + 5*b^2*x^6*(13*A + 7
*B*x^3)))/(455*x^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-91a^2A+455a^2Bx^3+910aAbx^3+130abBx^6+65Ab^2x^6+35b^2Bx^9\right)}{455x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (2*(-91*a^2*A + 910*a*A*b*x^3 + 455*a^2*B*x^3 + 65*A*b^2*x^6 + 130*a*b*B*x^
6 + 35*b^2*B*x^9))/(455*x^(5/2))
```

**fricas [A]** time = 0.88, size = 53, normalized size = 0.87

$$\frac{2\left(35\,Bb^2x^9+65\left(2\,Bab+Ab^2\right)x^6+455\left(Ba^2+2\,Aab\right)x^3-91\,Aa^2\right)}{455\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3
- 91*A*a^2)/x^(5/2)
```

**giac [A]** time = 0.15, size = 53, normalized size = 0.87

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{7}\,Babx^{\frac{7}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}+2\,Ba^2\sqrt{x}+4\,Aab\sqrt{x}-\frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt(
x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)
```

**maple [A]** time = 0.05, size = 56, normalized size = 0.92

$$-\frac{2\left(-35b^2B\,x^9-65A\,b^2x^6-130Bab\,x^6-910Aab\,x^3-455B\,a^2x^3+91a^2A\right)}{455x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x)
```

```
[Out] -2/455*(-35*B*b^2*x^9-65*A*b^2*x^6-130*B*a*b*x^6-910*A*a*b*x^3-455*B*a^2*x^
3+91*A*a^2)/x^(5/2)
```

**maxima [A]** time = 0.46, size = 51, normalized size = 0.84

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{2}{7}\left(2\,Bab+Ab^2\right)x^{\frac{7}{2}}+2\left(Ba^2+2\,Aab\right)\sqrt{x}-\frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2*(B*a^2 + 2*A*a*b)*s
qrt(x) - 2/5*A*a^2/x^(5/2)
```

**mupad [B]** time = 0.05, size = 51, normalized size = 0.84

$$\sqrt{x}\,\left(2\,B\,a^2+4\,A\,b\,a\right)+x^{7/2}\,\left(\frac{2\,A\,b^2}{7}+\frac{4\,B\,a\,b}{7}\right)-\frac{2\,A\,a^2}{5\,x^{5/2}}+\frac{2\,B\,b^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) - (2*A*a^
2)/(5*x^(5/2)) + (2*B*b^2*x^(13/2))/13
```

<u>**sympy**</u> **[A]** time = 18.41, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{5x^{\frac{5}{2}}}+4Aab\sqrt{x}+\frac{2Ab^2x^{\frac{7}{2}}}{7}+2Ba^2\sqrt{x}+\frac{4Babx^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13
```

## 3.143 $\int x^{7/2} \left(a+bx^3\right)^3 \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=85

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*B
)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx=\int\left(a^3Ax^{7/2}+a^2(3Ab+aB)x^{13/2}+3ab(Ab+aB)x^{19/2}+b^2(Ab+3aB)x^{25/2}+b\right.$$
$$=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2(3Ab+aB)x^{15/2}+\frac{2}{7}ab(Ab+aB)x^{21/2}+\frac{2}{27}b^2(Ab+3aB)x^{27/}$$

**Mathematica [A]** time = 0.06, size = 71, normalized size = 0.84

$$\frac{2x^{9/2}\left(1155a^3A+693a^2x^3(aB+3Ab)+385b^2x^9(3aB+Ab)+1485abx^6(aB+Ab)+315b^3Bx^{12}\right)}{10395}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(1155*a^3*A + 693*a^2*(3*A*b + a*B)*x^3 + 1485*a*b*(A*b + a*B)*x
^6 + 385*b^2*(A*b + 3*a*B)*x^9 + 315*b^3*B*x^12))/10395
```

**IntegrateAlgebraic [A]** time = 0.04, size = 97, normalized size = 1.14

$$\frac{2\left(1155a^3Ax^{9/2}+693a^3Bx^{15/2}+2079a^2Abx^{15/2}+1485a^2bBx^{21/2}+1485aAb^2x^{21/2}+1155ab^2Bx^{27/2}+385Ab^3x^{27/2}+315b^3Bx^{33/2}\right)}{10395}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*(1155*a^3*A*x^(9/2) + 2079*a^2*A*b*x^(15/2) + 693*a^3*B*x^(15/2) + 1485*
a*A*b^2*x^(21/2) + 1485*a^2*b*B*x^(21/2) + 385*A*b^3*x^(27/2) + 1155*a*b^2*
B*x^(27/2) + 315*b^3*B*x^(33/2)))/10395
```

**fricas** [A] time = 0.72, size = 78, normalized size = 0.92

$$\frac{2}{10395}\left(315\,Bb^3x^{16}+385\left(3\,Bab^2+Ab^3\right)x^{13}+1485\left(Ba^2b+Aab^2\right)x^{10}+1155\,Aa^3x^4+693\left(Ba^3+3\,Aa^2b\right)x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/10395*(315*B*b^3*x^16 + 385*(3*B*a*b^2 + A*b^3)*x^13 + 1485*(B*a^2*b + A*
a*b^2)*x^10 + 1155*A*a^3*x^4 + 693*(B*a^3 + 3*A*a^2*b)*x^7)*sqrt(x)
```

**giac** [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{33}\,Bb^3x^{\frac{33}{2}}+\frac{2}{9}\,Bab^2x^{\frac{27}{2}}+\frac{2}{27}\,Ab^3x^{\frac{27}{2}}+\frac{2}{7}\,Ba^2bx^{\frac{21}{2}}+\frac{2}{7}\,Aab^2x^{\frac{21}{2}}+\frac{2}{15}\,Ba^3x^{\frac{15}{2}}+\frac{2}{5}\,Aa^2bx^{\frac{15}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/9*B*a*b^2*x^(27/2) + 2/27*A*b^3*x^(27/2) + 2/7*B*a^
2*b*x^(21/2) + 2/7*A*a*b^2*x^(21/2) + 2/15*B*a^3*x^(15/2) + 2/5*A*a^2*b*x^(
15/2) + 2/9*A*a^3*x^(9/2)
```

**maple** [A] time = 0.04, size = 80, normalized size = 0.94

$$\frac{2\left(315B\,b^3x^{12}+385x^9A\,b^3+1155x^9Ba\,b^2+1485x^6Aa\,b^2+1485x^6B\,a^2b+2079x^3A\,a^2b+693B\,a^3x^3+1155A\,a^3\right)x^{\frac{9}{2}}}{10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/10395*x^(9/2)*(315*B*b^3*x^12+385*A*b^3*x^9+1155*B*a*b^2*x^9+1485*A*a*b^2
*x^6+1485*B*a^2*b*x^6+2079*A*a^2*b*x^3+693*B*a^3*x^3+1155*A*a^3)
```

**maxima** [A] time = 0.57, size = 73, normalized size = 0.86

$$\frac{2}{33}\,Bb^3x^{\frac{33}{2}}+\frac{2}{27}\left(3\,Bab^2+Ab^3\right)x^{\frac{27}{2}}+\frac{2}{7}\left(Ba^2b+Aab^2\right)x^{\frac{21}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2}{15}\left(Ba^3+3\,Aa^2b\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/27*(3*B*a*b^2 + A*b^3)*x^(27/2) + 2/7*(B*a^2*b + A*
a*b^2)*x^(21/2) + 2/9*A*a^3*x^(9/2) + 2/15*(B*a^3 + 3*A*a^2*b)*x^(15/2)
```

**mupad** [B] time = 2.52, size = 69, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,a^3}{15}+\frac{2\,A\,b\,a^2}{5}\right)+x^{27/2}\left(\frac{2\,A\,b^3}{27}+\frac{2\,B\,a\,b^2}{9}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{33/2}}{33}+\frac{2\,a\,b\,x^{21/2}\,\left(A\,b+B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(15/2)*((2*B*a^3)/15 + (2*A*a^2*b)/5) + x^(27/2)*((2*A*b^3)/27 + (2*B*a*b
^2)/9) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(33/2))/33 + (2*a*b*x^(21/2)*(A*b
 + B*a))/7
```

**sympy** [A] time = 92.99, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{2Aa^2bx^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{21}{2}}}{7}+\frac{2Ab^3x^{\frac{27}{2}}}{27}+\frac{2Ba^3x^{\frac{15}{2}}}{15}+\frac{2Ba^2bx^{\frac{21}{2}}}{7}+\frac{2Bab^2x^{\frac{27}{2}}}{9}+\frac{2Bb^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 2*A*a**2*b*x**(15/2)/5 + 2*A*a*b**2*x**(21/2)/7 + 2*A
*b**3*x**(27/2)/27 + 2*B*a**3*x**(15/2)/15 + 2*B*a**2*b*x**(21/2)/7 + 2*B*a
*b**2*x**(27/2)/9 + 2*B*b**3*x**(33/2)/33
```

## 3.144 $\int x^{5/2} \left(a + bx^3\right)^3 \left(A + Bx^3\right) dx$

**Optimal**. Leaf size=85

$$\frac{2}{7}a^3Ax^{7/2} + \frac{2}{13}a^2x^{13/2}(aB + 3Ab) + \frac{2}{25}b^2x^{25/2}(3aB + Ab) + \frac{6}{19}abx^{19/2}(aB + Ab) + \frac{2}{31}b^3Bx^{31/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{13}a^2x^{13/2}(aB + 3Ab) + \frac{2}{7}a^3Ax^{7/2} + \frac{2}{25}b^2x^{25/2}(3aB + Ab) + \frac{6}{19}abx^{19/2}(aB + Ab) + \frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2} \left(a + bx^3\right)^3 \left(A + Bx^3\right) dx &= \int \left(a^3Ax^{5/2} + a^2(3Ab + aB)x^{11/2} + 3ab(Ab + aB)x^{17/2} + b^2(Ab + 3aB)x^{23/2}\right.\\ &= \frac{2}{7}a^3Ax^{7/2} + \frac{2}{13}a^2(3Ab + aB)x^{13/2} + \frac{6}{19}ab(Ab + aB)x^{19/2} + \frac{2}{25}b^2(Ab + 3aB)\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 85, normalized size = 1.00

$$\frac{2}{7}a^3Ax^{7/2} + \frac{2}{13}a^2x^{13/2}(aB + 3Ab) + \frac{2}{25}b^2x^{25/2}(3aB + Ab) + \frac{6}{19}abx^{19/2}(aB + Ab) + \frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

**IntegrateAlgebraic [A]** time = 0.04, size = 97, normalized size = 1.14

$$\frac{2\left(191425a^3Ax^{7/2} + 103075a^3Bx^{13/2} + 309225a^2Abx^{13/2} + 211575a^2bBx^{19/2} + 211575aAb^2x^{19/2} + 160797ab^2Bx^{25/2} + 53599Ab^3x^{25/2} + 43225b^3Bx^{31/2}\right)}{1339975}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*(191425*a^3*A*x^(7/2) + 309225*a^2*A*b*x^(13/2) + 103075*a^3*B*x^(13/2)
+ 211575*a*A*b^2*x^(19/2) + 211575*a^2*b*B*x^(19/2) + 53599*A*b^3*x^(25/2)
+ 160797*a*b^2*B*x^(25/2) + 43225*b^3*B*x^(31/2)))/1339975
```

**fricas [A]** time = 0.82, size = 78, normalized size = 0.92

$$\frac{2}{1339975}\left(43225\,Bb^{3}x^{15}+53599\left(3\,Bab^{2}+Ab^{3}\right)x^{12}+211575\left(Ba^{2}b+Aab^{2}\right)x^{9}+191425\,Aa^{3}x^{3}+103075\left(Ba^{3}+3\,Aa^{2}b\right)x^{6}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1339975*(43225*B*b^3*x^15 + 53599*(3*B*a*b^2 + A*b^3)*x^12 + 211575*(B*a^
2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*sqr
t(x)
```

**giac [A]** time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{31}\,Bb^{3}x^{\frac{31}{2}}+\frac{6}{25}\,Bab^{2}x^{\frac{25}{2}}+\frac{2}{25}\,Ab^{3}x^{\frac{25}{2}}+\frac{6}{19}\,Ba^{2}bx^{\frac{19}{2}}+\frac{6}{19}\,Aab^{2}x^{\frac{19}{2}}+\frac{2}{13}\,Ba^{3}x^{\frac{13}{2}}+\frac{6}{13}\,Aa^{2}bx^{\frac{13}{2}}+\frac{2}{7}\,Aa^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 6/25*B*a*b^2*x^(25/2) + 2/25*A*b^3*x^(25/2) + 6/19*B*
a^2*b*x^(19/2) + 6/19*A*a*b^2*x^(19/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b
*x^(13/2) + 2/7*A*a^3*x^(7/2)
```

**maple [A]** time = 0.04, size = 80, normalized size = 0.94

$$\frac{2\left(43225B\,b^{3}x^{12}+53599x^{9}A\,b^{3}+160797x^{9}Ba\,b^{2}+211575x^{6}Aa\,b^{2}+211575x^{6}B\,a^{2}b+309225x^{3}A\,a^{2}b+103075B\,a^{3}x^{3}+191425A\,a^{3}\right)x^{\frac{7}{2}}}{1339975}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/1339975*x^(7/2)*(43225*B*b^3*x^12+53599*A*b^3*x^9+160797*B*a*b^2*x^9+2115
75*A*a*b^2*x^6+211575*B*a^2*b*x^6+309225*A*a^2*b*x^3+103075*B*a^3*x^3+19142
5*A*a^3)
```

**maxima [A]** time = 0.50, size = 73, normalized size = 0.86

$$\frac{2}{31}\,Bb^{3}x^{\frac{31}{2}}+\frac{2}{25}\left(3\,Bab^{2}+Ab^{3}\right)x^{\frac{25}{2}}+\frac{6}{19}\left(Ba^{2}b+Aab^{2}\right)x^{\frac{19}{2}}+\frac{2}{7}\,Aa^{3}x^{\frac{7}{2}}+\frac{2}{13}\left(Ba^{3}+3\,Aa^{2}b\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 2/25*(3*B*a*b^2 + A*b^3)*x^(25/2) + 6/19*(B*a^2*b + A
*a*b^2)*x^(19/2) + 2/7*A*a^3*x^(7/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

**mupad [B]** time = 0.03, size = 69, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{25/2}\left(\frac{2\,A\,b^3}{25}+\frac{6\,B\,a\,b^2}{25}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{31/2}}{31}+\frac{6\,a\,b\,x^{19/2}\,\left(A\,b+B\,a\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(25/2)*((2*A*b^3)/25 + (6*B*a*
b^2)/25) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(31/2))/31 + (6*a*b*x^(19/2)*(A
*b + B*a))/19
```

**sympy [A]** time = 58.70, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{25}{2}}}{25}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{19}{2}}}{19}+\frac{6Bab^2x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 + 2
*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 + 6*
B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31
```

## 3.145 $\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx$

**Optimal**. Leaf size=85

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {448}

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx=\int\left(a^3Ax^{3/2}+a^2(3Ab+aB)x^{9/2}+3ab(Ab+aB)x^{15/2}+b^2(Ab+3aB)x^{21/2}+b^3\right.$$
$$=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{23}b^2(Ab+3aB)x^{23}$$

**Mathematica [A]** time = 0.08, size = 85, normalized size = 1.00

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{23}b^2x^{23/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{29}b^3Bx^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

**IntegrateAlgebraic [A]** time = 0.04, size = 97, normalized size = 1.14

$$\frac{2\left(124729a^3Ax^{5/2}+56695a^3Bx^{11/2}+170085a^2Abx^{11/2}+110055a^2bBx^{17/2}+110055aAb^2x^{17/2}+81345ab^2Bx^{23/2}+27115Ab^3x^{23/2}+21505b^3Bx^{29/2}\right)}{623645}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*(124729*a^3*A*x^(5/2) + 170085*a^2*A*b*x^(11/2) + 56695*a^3*B*x^(11/2) +
 110055*a*A*b^2*x^(17/2) + 110055*a^2*b*B*x^(17/2) + 27115*A*b^3*x^(23/2) +
 81345*a*b^2*B*x^(23/2) + 21505*b^3*B*x^(29/2)))/623645
```

**fricas** [A] time = 0.83, size = 78, normalized size = 0.92

$$\frac{2}{623645}\left(21505\,Bb^3x^{14} + 27115\left(3\,Bab^2 + Ab^3\right)x^{11} + 110055\left(Ba^2b + Aab^2\right)x^8 + 124729\,Aa^3x^2 + 56695\left(Ba^3 + 3\,Aa^2b\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/623645*(21505*B*b^3*x^14 + 27115*(3*B*a*b^2 + A*b^3)*x^11 + 110055*(B*a^2
*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(
x)
```

**giac** [A] time = 0.17, size = 77, normalized size = 0.91

$$\frac{2}{29}\,Bb^3x^{\frac{29}{2}} + \frac{6}{23}\,Bab^2x^{\frac{23}{2}} + \frac{2}{23}\,Ab^3x^{\frac{23}{2}} + \frac{6}{17}\,Ba^2bx^{\frac{17}{2}} + \frac{6}{17}\,Aab^2x^{\frac{17}{2}} + \frac{2}{11}\,Ba^3x^{\frac{11}{2}} + \frac{6}{11}\,Aa^2bx^{\frac{11}{2}} + \frac{2}{5}\,Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 6/23*B*a*b^2*x^(23/2) + 2/23*A*b^3*x^(23/2) + 6/17*B*
a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b
*x^(11/2) + 2/5*A*a^3*x^(5/2)
```

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(21505B\,b^3x^{12} + 27115x^9A\,b^3 + 81345x^9Ba\,b^2 + 110055x^6Aa\,b^2 + 110055x^6B\,a^2b + 170085x^3A\,a^2b + 56695B\,a^3x^3 + 124729A\,a^3\right)x^{\frac{5}{2}}}{623645}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/623645*x^(5/2)*(21505*B*b^3*x^12+27115*A*b^3*x^9+81345*B*a*b^2*x^9+110055
*A*a*b^2*x^6+110055*B*a^2*b*x^6+170085*A*a^2*b*x^3+56695*B*a^3*x^3+124729*A
*a^3)
```

**maxima** [A] time = 0.48, size = 73, normalized size = 0.86

$$\frac{2}{29}\,Bb^3x^{\frac{29}{2}} + \frac{2}{23}\left(3\,Bab^2 + Ab^3\right)x^{\frac{23}{2}} + \frac{6}{17}\left(Ba^2b + Aab^2\right)x^{\frac{17}{2}} + \frac{2}{5}\,Aa^3x^{\frac{5}{2}} + \frac{2}{11}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 2/23*(3*B*a*b^2 + A*b^3)*x^(23/2) + 6/17*(B*a^2*b + A
*a*b^2)*x^(17/2) + 2/5*A*a^3*x^(5/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

**mupad** [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,a^3}{11} + \frac{6\,A\,b\,a^2}{11}\right) + x^{23/2}\left(\frac{2\,A\,b^3}{23} + \frac{6\,B\,a\,b^2}{23}\right) + \frac{2\,A\,a^3\,x^{5/2}}{5} + \frac{2\,B\,b^3\,x^{29/2}}{29} + \frac{6\,a\,b\,x^{17/2}\,\left(A\,b + B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(23/2)*((2*A*b^3)/23 + (6*B*a*
b^2)/23) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(29/2))/29 + (6*a*b*x^(17/2)*(A
*b + B*a))/17
```

**sympy [A]** time = 39.60, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{23}{2}}}{23}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 + 6*
B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29
```

## 3.146 $\int \sqrt{x} \left(a+bx^3\right)^3 \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=85

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

**Rubi** [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{3}a^3Ax^{3/2}+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)*
x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx &= \int \left(a^3A\sqrt{x}+a^2(3Ab+aB)x^{7/2}+3ab(Ab+aB)x^{13/2}+b^2(Ab+3aB)x^{19/2}+\right.\\ &= \frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{21}b^2(Ab+3aB)x^{21}\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 71, normalized size = 0.84

$$\frac{2}{945}x^{3/2}\left(315a^3A+105a^2x^3(aB+3Ab)+45b^2x^9(3aB+Ab)+189abx^6(aB+Ab)+35b^3Bx^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(315*a^3*A + 105*a^2*(3*A*b + a*B)*x^3 + 189*a*b*(A*b + a*B)*x^6
 + 45*b^2*(A*b + 3*a*B)*x^9 + 35*b^3*B*x^12))/945
```

**IntegrateAlgebraic** [A] time = 0.05, size = 97, normalized size = 1.14

$$\frac{2}{945}\left(315a^3Ax^{3/2}+105a^3Bx^{9/2}+315a^2Abx^{9/2}+189a^2bBx^{15/2}+189aAb^2x^{15/2}+135ab^2Bx^{21/2}+45Ab^3x^{21/2}+35b^3Bx^{27/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*(315*a^3*A*x^(3/2) + 315*a^2*A*b*x^(9/2) + 105*a^3*B*x^(9/2) + 189*a*A*b
^2*x^(15/2) + 189*a^2*b*B*x^(15/2) + 45*A*b^3*x^(21/2) + 135*a*b^2*B*x^(21/
2) + 35*b^3*B*x^(27/2)))/945
```

**fricas** [A] time = 0.80, size = 76, normalized size = 0.89

$$\frac{2}{945}\left(35\,Bb^3x^{13}+45\left(3\,Bab^2+Ab^3\right)x^{10}+189\left(Ba^2b+Aab^2\right)x^7+315\,Aa^3x+105\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^3*x^13 + 45*(3*B*a*b^2 + A*b^3)*x^10 + 189*(B*a^2*b + A*a*b^2
)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

**giac** [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{27}\,Bb^3x^{\frac{27}{2}}+\frac{2}{7}\,Bab^2x^{\frac{21}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{2}{5}\,Ba^2bx^{\frac{15}{2}}+\frac{2}{5}\,Aab^2x^{\frac{15}{2}}+\frac{2}{9}\,Ba^3x^{\frac{9}{2}}+\frac{2}{3}\,Aa^2bx^{\frac{9}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 2/5*B*a^
2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(9/
2) + 2/3*A*a^3*x^(3/2)
```

**maple** [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(35B\,b^3x^{12}+45x^9A\,b^3+135x^9Ba\,b^2+189x^6Aa\,b^2+189x^6B\,a^2b+315x^3A\,a^2b+105B\,a^3x^3+315A\,a^3\right)x^{\frac{3}{2}}}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/945*x^(3/2)*(35*B*b^3*x^12+45*A*b^3*x^9+135*B*a*b^2*x^9+189*A*a*b^2*x^6+1
89*B*a^2*b*x^6+315*A*a^2*b*x^3+105*B*a^3*x^3+315*A*a^3)
```

**maxima** [A] time = 0.67, size = 73, normalized size = 0.86

$$\frac{2}{27}\,Bb^3x^{\frac{27}{2}}+\frac{2}{21}\left(3\,Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{2}{5}\left(Ba^2b+Aab^2\right)x^{\frac{15}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}+\frac{2}{9}\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/3*A*a^3*x^(3/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

**mupad** [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right)+\frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{27/2}}{27}+\frac{2\,a\,b\,x^{15/2}\,\left(A\,b+B\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*b^2
)/7) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(27/2))/27 + (2*a*b*x^(15/2)*(A*b +
 B*a))/5
```

**sympy** [A] time = 6.34, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A*
b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*b*
*2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27
```

## 3.147 $\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=83

$$2a^3A\sqrt{x}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

**Rubi** [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{\sqrt{x}}\,dx &= \int \left(\frac{a^3A}{\sqrt{x}}+a^2(3Ab+aB)x^{5/2}+3ab(Ab+aB)x^{11/2}+b^2(Ab+3aB)x^{17/2}+b^3Bx^{23/2}\right)dx\\ &= 2a^3A\sqrt{x}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}+\frac{2}{25}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 83, normalized size = 1.00

$$2a^3A\sqrt{x}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

**IntegrateAlgebraic** [A] time = 0.04, size = 97, normalized size = 1.17

$$\frac{2\left(43225a^3A\sqrt{x}+6175a^3Bx^{7/2}+18525a^2Abx^{7/2}+9975a^2bBx^{13/2}+9975aAb^2x^{13/2}+6825ab^2Bx^{19/2}+2275Ab^3x^{19/2}+1729b^3Bx^{25/2}\right)}{43225}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*(43225*a^3*A*Sqrt[x] + 18525*a^2*A*b*x^(7/2) + 6175*a^3*B*x^(7/2) + 9975
*a*A*b^2*x^(13/2) + 9975*a^2*b*B*x^(13/2) + 2275*A*b^3*x^(19/2) + 6825*a*b^
2*B*x^(19/2) + 1729*b^3*B*x^(25/2)))/43225
```

**fricas** [A] time = 0.80, size = 75, normalized size = 0.90

$$\frac{2}{43225} \left(1729\, Bb^3x^{12} + 2275\left(3\, Bab^2 + Ab^3\right)x^9 + 9975\left(Ba^2b + Aab^2\right)x^6 + 43225\, Aa^3 + 6175\left(Ba^3 + 3\, Aa^2b\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^3*x^12 + 2275*(3*B*a*b^2 + A*b^3)*x^9 + 9975*(B*a^2*b + A
*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

**giac** [A] time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{25}\, Bb^3x^{\frac{25}{2}} + \frac{6}{19}\, Bab^2x^{\frac{19}{2}} + \frac{2}{19}\, Ab^3x^{\frac{19}{2}} + \frac{6}{13}\, Ba^2bx^{\frac{13}{2}} + \frac{6}{13}\, Aab^2x^{\frac{13}{2}} + \frac{2}{7}\, Ba^3x^{\frac{7}{2}} + \frac{6}{7}\, Aa^2bx^{\frac{7}{2}} + 2\, Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^
(7/2) + 2*A*a^3*sqrt(x)
```

**maple** [A] time = 0.04, size = 80, normalized size = 0.96

$$\frac{2\left(1729 B\, b^3x^{12} + 2275x^9 A\, b^3 + 6825x^9 Ba\, b^2 + 9975x^6 Aa\, b^2 + 9975x^6 B\, a^2b + 18525x^3 A\, a^2b + 6175 B\, a^3x^3 + 43225 A\, a^3\right)\sqrt{x}}{43225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/43225*x^(1/2)*(1729*B*b^3*x^12+2275*A*b^3*x^9+6825*B*a*b^2*x^9+9975*A*a*b
^2*x^6+9975*B*a^2*b*x^6+18525*A*a^2*b*x^3+6175*B*a^3*x^3+43225*A*a^3)
```

**maxima** [A] time = 0.56, size = 73, normalized size = 0.88

$$\frac{2}{25}\, Bb^3x^{\frac{25}{2}} + \frac{2}{19}\left(3\, Bab^2 + Ab^3\right)x^{\frac{19}{2}} + \frac{6}{13}\left(Ba^2b + Aab^2\right)x^{\frac{13}{2}} + 2\, Aa^3\sqrt{x} + \frac{2}{7}\left(Ba^3 + 3\, Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2*A*a^3*sqrt(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

**mupad** [B] time = 0.03, size = 69, normalized size = 0.83

$$x^{7/2}\left(\frac{2\, B\, a^3}{7} + \frac{6\, A\, b\, a^2}{7}\right) + x^{19/2}\left(\frac{2\, A\, b^3}{19} + \frac{6\, B\, a\, b^2}{19}\right) + 2\, A\, a^3\,\sqrt{x} + \frac{2\, B\, b^3\, x^{25/2}}{25} + \frac{6\, a\, b\, x^{13/2}\,\left(A\, b + B\, a\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*b^2
)/19) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(13/2)*(A*b + B*
a))/13
```

**sympy** [A] time = 23.64, size = 112, normalized size = 1.35

$$2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{7}{2}}}{7} + \frac{6Aab^2x^{\frac{13}{2}}}{13} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{7}{2}}}{7} + \frac{6Ba^2bx^{\frac{13}{2}}}{13} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b*
*3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b**
2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25
```

## 3.148 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx$

**Optimal**. Leaf size=83

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

**Rubi** [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2x^{5/2}(aB+3Ab)-\frac{2a^3A}{\sqrt{x}}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x
^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = \int \left(\frac{a^3A}{x^{3/2}}+a^2(3Ab+aB)x^{3/2}+3ab(Ab+aB)x^{9/2}+b^2(Ab+3aB)x^{15/2}+b^3Bx^{21}\right.$$

$$= -\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/2}+\frac{2}{}$$

**Mathematica** [A] time = 0.04, size = 81, normalized size = 0.98

$$\frac{-8602a^3\left(5A-Bx^3\right)+2346a^2bx^3\left(11A+5Bx^3\right)+690ab^2x^6\left(17A+11Bx^3\right)+110b^3x^9\left(23A+17Bx^3\right)}{21505\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-8602*a^3*(5*A - B*x^3) + 2346*a^2*b*x^3*(11*A + 5*B*x^3) + 690*a*b^2*x^6*
(17*A + 11*B*x^3) + 110*b^3*x^9*(23*A + 17*B*x^3))/(21505*Sqrt[x])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 83, normalized size = 1.00

$$\frac{2\left(-21505a^3A+4301a^3Bx^3+12903a^2Abx^3+5865a^2bBx^6+5865aAb^2x^6+3795ab^2Bx^9+1265Ab^3x^9+935b^3Bx^{12}\right)}{21505\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (2*(-21505*a^3*A + 12903*a^2*A*b*x^3 + 4301*a^3*B*x^3 + 5865*a*A*b^2*x^6 +
5865*a^2*b*B*x^6 + 1265*A*b^3*x^9 + 3795*a*b^2*B*x^9 + 935*b^3*B*x^12))/(21
505*Sqrt[x])
```

**fricas [A]** time = 1.00, size = 75, normalized size = 0.90

$$\frac{2\left(935\,Bb^3x^{12}+1265\left(3\,Bab^2+Ab^3\right)x^9+5865\left(Ba^2b+Aab^2\right)x^6-21505\,Aa^3+4301\left(Ba^3+3\,Aa^2b\right)x^3\right)}{21505\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A*
a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)
```

**giac [A]** time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{6}{17}\,Bab^2x^{\frac{17}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{6}{11}\,Ba^2bx^{\frac{11}{2}}+\frac{6}{11}\,Aab^2x^{\frac{11}{2}}+\frac{2}{5}\,Ba^3x^{\frac{5}{2}}+\frac{6}{5}\,Aa^2bx^{\frac{5}{2}}-\frac{2\,Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B*
a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^
(5/2) - 2*A*a^3/sqrt(x)
```

**maple [A]** time = 0.04, size = 80, normalized size = 0.96

$$-\frac{2\left(-935B\,b^3x^{12}-1265x^9A\,b^3-3795x^9Ba\,b^2-5865x^6Aa\,b^2-5865x^6B\,a^2b-12903x^3A\,a^2b-4301B\,a^3x^3+21505A\,a^3\right)}{21505\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x)
```

```
[Out] -2/21505*(-935*B*b^3*x^12-1265*A*b^3*x^9-3795*B*a*b^2*x^9-5865*A*a*b^2*x^6-
5865*B*a^2*b*x^6-12903*A*a^2*b*x^3-4301*B*a^3*x^3+21505*A*a^3)/x^(1/2)
```

**maxima [A]** time = 0.56, size = 73, normalized size = 0.88

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{2}{17}\left(3\,Bab^2+Ab^3\right)x^{\frac{17}{2}}+\frac{6}{11}\left(Ba^2b+Aab^2\right)x^{\frac{11}{2}}-\frac{2\,Aa^3}{\sqrt{x}}+\frac{2}{5}\left(Ba^3+3\,Aa^2b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/11*(B*a^2*b + A
*a*b^2)*x^(11/2) - 2*A*a^3/sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

**mupad [B]** time = 0.03, size = 69, normalized size = 0.83

$$x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right)+x^{17/2}\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right)-\frac{2\,A\,a^3}{\sqrt{x}}+\frac{2\,B\,b^3\,x^{23/2}}{23}+\frac{6\,a\,b\,x^{11/2}\,\left(A\,b+B\,a\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^2
)/17) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(23/2))/23 + (6*a*b*x^(11/2)*(A*b +
B*a))/11
```

**sympy** [A] time = 19.49, size = 112, normalized size = 1.35

$$-\frac{2Aa^3}{\sqrt{x}}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{6Aab^2x^{\frac{11}{2}}}{11}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{5}{2}}}{5}+\frac{6Ba^2bx^{\frac{11}{2}}}{11}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*b
**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*b*
*2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23
```

## 3.149 $\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} \, dx$

**Optimal**. Leaf size=85

$$-\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2x^{3/2}(aB+3Ab)-\frac{2a^3A}{3x^{3/2}}+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a*
B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} \, dx &= \int \left(\frac{a^3A}{x^{5/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{13/2}+b^3Bx^{19/2}\right) \\ &= -\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{21}b^3B\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 77, normalized size = 0.91

$$\frac{2\left(-35a^3\left(A-Bx^3\right)+35a^2bx^3\left(3A+Bx^3\right)+7ab^2x^6\left(5A+3Bx^3\right)+b^3x^9\left(7A+5Bx^3\right)\right)}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A + 3
*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.04, size = 83, normalized size = 0.98

$$\frac{2\left(-35a^3A+35a^3Bx^3+105a^2Abx^3+35a^2bBx^6+35aAb^2x^6+21ab^2Bx^9+7Ab^3x^9+5b^3Bx^{12}\right)}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-35*a^3*A + 105*a^2*A*b*x^3 + 35*a^3*B*x^3 + 35*a*A*b^2*x^6 + 35*a^2*b*
B*x^6 + 7*A*b^3*x^9 + 21*a*b^2*B*x^9 + 5*b^3*B*x^12))/(105*x^(3/2))
```

**fricas [A]** time = 0.79, size = 75, normalized size = 0.88

$$\frac{2\left(5\,Bb^3x^{12}+7\left(3\,Bab^2+Ab^3\right)x^9+35\left(Ba^2b+Aab^2\right)x^6-35\,Aa^3+35\left(Ba^3+3\,Aa^2b\right)x^3\right)}{105\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x^
6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)
```

**giac [A]** time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{5}\,Bab^2x^{\frac{15}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{3}\,Ba^2bx^{\frac{9}{2}}+\frac{2}{3}\,Aab^2x^{\frac{9}{2}}+\frac{2}{3}\,Ba^3x^{\frac{3}{2}}+2\,Aa^2bx^{\frac{3}{2}}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a^
2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) -
 2/3*A*a^3/x^(3/2)
```

**maple [A]** time = 0.04, size = 80, normalized size = 0.94

$$-\frac{2\left(-5B\,b^3x^{12}-7x^9A\,b^3-21x^9Ba\,b^2-35x^6Aa\,b^2-35x^6B\,a^2b-105x^3A\,a^2b-35B\,a^3x^3+35A\,a^3\right)}{105x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/105*(-5*B*b^3*x^12-7*A*b^3*x^9-21*B*a*b^2*x^9-35*A*a*b^2*x^6-35*B*a^2*b*
x^6-105*A*a^2*b*x^3-35*B*a^3*x^3+35*A*a^3)/x^(3/2)
```

**maxima [A]** time = 0.64, size = 73, normalized size = 0.86

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{15}\left(3\,Bab^2+Ab^3\right)x^{\frac{15}{2}}+\frac{2}{3}\left(Ba^2b+Aab^2\right)x^{\frac{9}{2}}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}+\frac{2}{3}\left(Ba^3+3\,Aa^2b\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) - 2/3*A*a^3/x^(3/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

**mupad [B]** time = 0.03, size = 69, normalized size = 0.81

$$x^{3/2}\left(\frac{2\,B\,a^3}{3}+2\,A\,b\,a^2\right)+x^{15/2}\left(\frac{2\,A\,b^3}{15}+\frac{2\,B\,a\,b^2}{5}\right)-\frac{2\,A\,a^3}{3\,x^{3/2}}+\frac{2\,B\,b^3\,x^{21/2}}{21}+\frac{2\,a\,b\,x^{9/2}\,\left(A\,b+B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2)/5)
 - (2*A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(21/2))/21 + (2*a*b*x^(9/2)*(A*b + B*
a))/3
```

**sympy** [A] time = 28.37, size = 112, normalized size = 1.32

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}} + 2Aa^2bx^{\frac{3}{2}} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{15}{2}}}{15} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{2Bab^2x^{\frac{15}{2}}}{5} + \frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A*
b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b**
2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21
```

## 3.150 $\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} dx$

**Optimal**. Leaf size=83

$$-\frac{2a^3A}{5x^{5/2}} + 2a^2\sqrt{x}(aB+3Ab) + \frac{2}{13}b^2x^{13/2}(3aB+Ab) + \frac{6}{7}abx^{7/2}(aB+Ab) + \frac{2}{19}b^3Bx^{19/2}$$

**Rubi [A]** time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$2a^2\sqrt{x}(aB+3Ab) - \frac{2a^3A}{5x^{5/2}} + \frac{2}{13}b^2x^{13/2}(3aB+Ab) + \frac{6}{7}abx^{7/2}(aB+Ab) + \frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} dx &= \int \left(\frac{a^3A}{x^{7/2}} + \frac{a^2(3Ab+aB)}{\sqrt{x}} + 3ab(Ab+aB)x^{5/2} + b^2(Ab+3aB)x^{11/2} + b^3Bx^{17/2}\right) \\ &= -\frac{2a^3A}{5x^{5/2}} + 2a^2(3Ab+aB)\sqrt{x} + \frac{6}{7}ab(Ab+aB)x^{7/2} + \frac{2}{13}b^2(Ab+3aB)x^{13/2} + \frac{2}{19}b \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 78, normalized size = 0.94

$$\frac{-3458a^3(A-5Bx^3) + 7410a^2bx^3(7A+Bx^3) + 570ab^2x^6(13A+7Bx^3) + 70b^3x^9(19A+13Bx^3)}{8645x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(13
*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 83, normalized size = 1.00

$$\frac{2(-1729a^3A + 8645a^3Bx^3 + 25935a^2Abx^3 + 3705a^2bBx^6 + 3705aAb^2x^6 + 1995ab^2Bx^9 + 665Ab^3x^9 + 455b^3Bx^{12})}{8645x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (2*(-1729*a^3*A + 25935*a^2*A*b*x^3 + 8645*a^3*B*x^3 + 3705*a*A*b^2*x^6 + 3
705*a^2*b*B*x^6 + 665*A*b^3*x^9 + 1995*a*b^2*B*x^9 + 455*b^3*B*x^12))/(8645
*x^(5/2))
```

**fricas [A]** time = 0.68, size = 75, normalized size = 0.90

$$\frac{2\left(455\,Bb^{3}x^{12}+665\left(3\,Bab^{2}+Ab^{3}\right)x^{9}+3705\left(Ba^{2}b+Aab^{2}\right)x^{6}-1729\,Aa^{3}+8645\left(Ba^{3}+3\,Aa^{2}b\right)x^{3}\right)}{8645\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a*
b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)
```

**giac [A]** time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{19}\,Bb^{3}x^{\frac{19}{2}}+\frac{6}{13}\,Bab^{2}x^{\frac{13}{2}}+\frac{2}{13}\,Ab^{3}x^{\frac{13}{2}}+\frac{6}{7}\,Ba^{2}bx^{\frac{7}{2}}+\frac{6}{7}\,Aab^{2}x^{\frac{7}{2}}+2\,Ba^{3}\sqrt{x}+6\,Aa^{2}b\sqrt{x}-\frac{2\,Aa^{3}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) -
2/5*A*a^3/x^(5/2)
```

**maple [A]** time = 0.04, size = 80, normalized size = 0.96

$$-\frac{2\left(-455B\,b^{3}x^{12}-665x^{9}A\,b^{3}-1995x^{9}Ba\,b^{2}-3705x^{6}Aa\,b^{2}-3705x^{6}B\,a^{2}b-25935x^{3}A\,a^{2}b-8645B\,a^{3}x^{3}+1729A\,a^{3}\right)}{8645x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x)
```

```
[Out] -2/8645*(-455*B*b^3*x^12-665*A*b^3*x^9-1995*B*a*b^2*x^9-3705*A*a*b^2*x^6-37
05*B*a^2*b*x^6-25935*A*a^2*b*x^3-8645*B*a^3*x^3+1729*A*a^3)/x^(5/2)
```

**maxima [A]** time = 0.55, size = 73, normalized size = 0.88

$$\frac{2}{19}\,Bb^{3}x^{\frac{19}{2}}+\frac{2}{13}\left(3\,Bab^{2}+Ab^{3}\right)x^{\frac{13}{2}}+\frac{6}{7}\left(Ba^{2}b+Aab^{2}\right)x^{\frac{7}{2}}-\frac{2\,Aa^{3}}{5\,x^{\frac{5}{2}}}+2\left(Ba^{3}+3\,Aa^{2}b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2/5*A*a^3/x^(5/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

**mupad [B]** time = 0.03, size = 69, normalized size = 0.83

$$\sqrt{x}\,\left(2\,B\,a^{3}+6\,A\,b\,a^{2}\right)+x^{13/2}\,\left(\frac{2\,A\,b^{3}}{13}+\frac{6\,B\,a\,b^{2}}{13}\right)-\frac{2\,A\,a^{3}}{5\,x^{5/2}}+\frac{2\,B\,b^{3}\,x^{19/2}}{19}+\frac{6\,a\,b\,x^{7/2}\,\left(A\,b+B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2)/13) -
(2*A*a^3)/(5*x^(5/2)) + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(7/2)*(A*b + B*a))
/7
```

**sympy [A]** time = 29.28, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{5x^{\frac{5}{2}}} + 6Aa^2b\sqrt{x} + \frac{6Aab^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{13}{2}}}{13} + 2Ba^3\sqrt{x} + \frac{6Ba^2bx^{\frac{7}{2}}}{7} + \frac{6Bab^2x^{\frac{13}{2}}}{13} + \frac{2Bb^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b
**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(19/2)/19
```

## 3.151 $\int \frac{x^{7/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

**Optimal**. Leaf size=73

$$-\frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{9/2}}{9b}$$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 321, 329, 275, 205}

$$\frac{2x^{3/2}(Ab-aB)}{3b^2}-\frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(9/2))/(9*b) - (2*Sqrt[a]*(A*b - a
*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{9/2}}{9b} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{9b}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{b^2}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3b^2}$$
$$= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

**Mathematica [A]** time = 0.12, size = 67, normalized size = 0.92

$$\frac{2\sqrt{a}\,(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}} + \frac{2x^{3/2}\left(-3aB+3Ab+bBx^3\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 71, normalized size = 0.97

$$\frac{2\left(a^{3/2}B-\sqrt{a}\,Ab\right)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}} + \frac{2x^{3/2}\left(-3aB+3Ab+bBx^3\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*(-(Sqrt[a]*A*b) + a^(3/2
)*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

**fricas [A]** time = 0.68, size = 143, normalized size = 1.96

$$\left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^3-2\,bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^3+a}\right)-2\left(Bbx^4-3\,(Ba-Ab)x\right)\sqrt{x}}{9\,b^2},\ \frac{2\left(3\,(Ba-Ab)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{a}\right)+\left(Bbx^4-3\,(Ba-Ab)x\right)\sqrt{x}\right)}{9\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(b
*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*b)
*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*sqrt
(x))/b^2]
```

**giac [A]** time = 0.17, size = 64, normalized size = 0.88

$$\frac{2\left(Ba^2 - Aab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,b^2} + \frac{2\left(Bb^2x^{\frac{9}{2}} - 3\,Babx^{\frac{3}{2}} + 3\,Ab^2x^{\frac{3}{2}}\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b^
2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3
```

**maple [A]** time = 0.05, size = 78, normalized size = 1.07

$$\frac{2B\,x^{\frac{9}{2}}}{9b} - \frac{2Aa\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b} + \frac{2B\,a^2\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b^2} + \frac{2A\,x^{\frac{3}{2}}}{3b} - \frac{2Ba\,x^{\frac{3}{2}}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/9*B*x^(9/2)/b+2/3/b*A*x^(3/2)-2/3/b^2*B*a*x^(3/2)-2/3*a/b/(a*b)^(1/2)*arc
tan(x^(3/2)*b/(a*b)^(1/2))*A+2/3*a^2/b^2/(a*b)^(1/2)*arctan(x^(3/2)*b/(a*b)
^(1/2))*B
```

**maxima [A]** time = 1.32, size = 58, normalized size = 0.79

$$\frac{2\left(Ba^2 - Aab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,b^2} + \frac{2\left(Bbx^{\frac{9}{2}} - 3\,(Ba - Ab)x^{\frac{3}{2}}\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b*
x^(9/2) - 3*(B*a - A*b)*x^(3/2))/b^2
```

**mupad [B]** time = 2.61, size = 111, normalized size = 1.52

$$x^{3/2}\left(\frac{2\,A}{3\,b} - \frac{2\,B\,a}{3\,b^2}\right) + \frac{2\,B\,x^{9/2}}{9\,b} - \frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{72\,b^{3/2}\,x^{3/2}\left(A^2\,a^2\,b^2 - 2\,A\,B\,a^3\,b + B^2\,a^4\right)}{\sqrt{a}\left(72\,A\,a^2\,b^2 - 72\,B\,a^3\,b\right)\left(A\,b - B\,a\right)}\right)\left(A\,b - B\,a\right)}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(9/2))/(9*b) - (2*a^(1/2)*
atan((72*b^(3/2)*x^(3/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/(a^(1/2)*(7
2*A*a^2*b^2 - 72*B*a^3*b)*(A*b - B*a)))*(A*b - B*a))/(3*b^(5/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Timed out
```

## 3.152 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

**Optimal**. Leaf size=288

$$\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}+\frac{\sqrt[6]{a}\,(Ab$$

**Rubi [A]** time = 0.52, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 209, 634, 618, 204, 628, 205}

$$\frac{2\sqrt{x}\,(Ab-aB)}{b^2}+\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}+\frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3b^{13/6}}-\frac{2\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}+\frac{2Bx^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(7/2))/(7*b) + (a^(1/6)*(A*b - a*B)*Ar
cTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (a^(1/6)*(A*b -
 a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (2*a^(1
/6)*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + (a^(1/6)*
(A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*
Sqrt[3]*b^(13/6)) - (a^(1/6)*(A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{7b}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{\left(2\sqrt[6]{a}\,(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3b^2} - \cdots$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\left(\sqrt[6]{a}\,(Ab-aB)\right)\text{Subst}\left(\int \cdots\right)}{2\sqrt{3}\,\cdots}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\cdots\right)}{2\sqrt{3}\,b^{13/6}\cdots}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\cdots\right)}{3b^{13/6}}$$

**Mathematica [C]** time = 0.09, size = 54, normalized size = 0.19

$$\frac{2\sqrt{x}\left((7aB-7Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)-7aB+7Ab+bBx^3\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3 + (-7*A*b + 7*a*B)*Hypergeometric2F1[1/
6, 1, 7/6, -((b*x^3)/a)]))/(7*b^2)
```

**IntegrateAlgebraic [A]** time = 0.21, size = 193, normalized size = 0.67

$$\frac{2\left(a^{7/6}B-\sqrt[6]{a}\,Ab\right)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}}+\frac{\left(\sqrt[6]{a}\,Ab-a^{7/6}B\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{3b^{13/6}}+\frac{\left(a^{7/6}B-\sqrt[6]{a}\,Ab\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,b^{13/6}}+\frac{2\sqrt{x}\left(-7aB+7Ab+bBx^3\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3))/(7*b^2) + (2*(-(a^(1/6)*A*b) + a^(7/6
)*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + ((a^(1/6)*A*b - a^(7
/6)*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(3*b^(13/6)
) + ((-(a^(1/6)*A*b) + a^(7/6)*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])
/(a^(1/3) + b^(1/3)*x)])/(Sqrt[3]*b^(13/6))
```

**fricas [B]** time = 1.07, size = 2433, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/42*(28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*
A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4
*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*
a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a*b^2 - A*b^3)*
sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^
3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6))*b^11*(-(
B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*
B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/6) + 2*sqrt(3)*(B*a*b^1
1 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) - sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4
*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6))/(B^6*a^7 - 6*A*B^
5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*
A^5*B*a^2*b^5 + A^6*a*b^6)) + 28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 1
5*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b
^5 + A^6*a*b^6)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*
b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6
)/b^13)^(1/6))*b^11*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) + 2*sqrt(3)*(B*a*b^11 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*
A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5
 + A^6*a*b^6)/b^13)^(5/6) + sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a
^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*
```

```
b^6))/(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 +
15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)) - 7*b^2*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*
B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x +
4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2
 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b
^13)^(1/6)) + 7*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/
6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a
^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7
- 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*
b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) + 14*b^2*(-(B^6*a^7 - 6*A*B
^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6
*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b
+ 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^
2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 14*b^2*(-(B^6*a^7 -
 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b
^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-b^2*(-(B^6*a^7 - 6*A*B^5
*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A
^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 12*(B*b*x^3
- 7*B*a + 7*A*b)*sqrt(x))/b^2
```

**giac [A]** time = 0.21, size = 289, normalized size = 1.00

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}} Ba-\left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^3}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}} Ba-\left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^3}+\frac{\left(\left(ab^5\right)^{\frac{1}{6}} Ba-\left(ab^5\right)^{\frac{1}{6}} Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b^3}+\frac{\left(\left(ab^5\right)^{\frac{1}{6}} Ba-\left(ab^5\right)^{\frac{1}{6}} Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b^3}+\frac{2\left(\left(ab^5\right)^{\frac{1}{6}} Ba-\left(ab^5\right)^{\frac{1}{6}} Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b^3}+\frac{2\left(Bb^6x^{\frac{7}{2}}-7\,Bab^5\sqrt{x}+7\,Ab^6\sqrt{x}\right)}{7b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(
x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-
(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/6)*B*a
 - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x^(7/2)
- 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7
```

**maple [A]** time = 0.18, size = 377, normalized size = 1.31

$$\frac{2Bx^{\frac{7}{2}}}{7b}-\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3b}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}Ba\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}Ba\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}Ba\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3b^2}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}Ba\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^2}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}Ba\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b^2}+\frac{2A\sqrt{x}}{b}-\frac{2Ba\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/7*B*x^(7/2)/b+2/b*A*x^(1/2)-2/b^2*B*a*x^(1/2)-2/3/b*(a/b)^(1/6)*arctan(x^
(1/2)/(a/b)^(1/6))*A+2/3*a/b^2*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B+1/
6/b*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))*A-1/6
*a/b^2*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))*B-
1/3/b*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))*A+1/3*a/b^2*(a/b)^
(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))*B-1/6/b*3^(1/2)*(a/b)^(1/6)*ln
(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+1/6*a/b^2*3^(1/2)*(a/b)^(1/6)
*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-1/3/b*(a/b)^(1/6)*arctan(2
*x^(1/2)/(a/b)^(1/6)+3^(1/2))*A+1/3*a/b^2*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b
)^(1/6)+3^(1/2))*B
```

**maxima** [A] time = 1.17, size = 295, normalized size = 1.02

$$\frac{\left(\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)a}{6b^2}+\frac{2\left(Bbx^{\frac{7}{2}}-7(Ba-Ab)\sqrt{x}\right)}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))*a/b^2 + 2/7*(B*b*x^(7/2) - 7*(B*a - A*b)*sqrt(x
))/b^2
```

**mupad** [B] time = 2.89, size = 1933, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(7/2))/(7*b) + ((-a)^(1/6)*atan(((
(-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*
b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*(A*b - B*a)*(B
^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b
^(13/6)) + ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*
A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*(A
*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/
6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*
a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(
-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*
b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a
^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^
3 + (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A
^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-
a)^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*
(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^
3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*
a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((
-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^
4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a
)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2
*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/
2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2
*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)
*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*
a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a
^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b -
 B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))
/(3*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-a)
^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B
^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3)
)/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^
```

```
4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((-a
)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*
b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^
(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a
^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/2)
*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a
^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1
i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^
5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b -
 B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7
*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B
*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(
3*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*b^(13/6))
```

**sympy [A]** time = 177.80, size = 881, normalized size = 3.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2*A*
sqrt(x) + 2*B*x**(7/2)/7)/b, Eq(a, 0)), ((2*A*x**(7/2)/7 + 2*B*x**(13/2)/13
)/a, Eq(b, 0)), ((-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log(-(-1)**(1/6)*a**(1
/6)*(1/b)**(1/6) + sqrt(x))/(3*b) - (-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log
((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b) + (-1)**(1/6)*A*a**(1/6
)*(1/b)**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(
1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*A*a**(1/6)*(1/b)**(1/
6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)
*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*sqrt(3)*A*a**(1/6)*(1/b)**(1/6)*at
an(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*
b) + (-1)**(1/6)*sqrt(3)*A*a**(1/6)*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(
5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b) + 2*A*sqrt(x)/b - (-1
)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + s
qrt(x))/(3*b**2) + (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(
1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2) - (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)
*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*
(1/b)**(1/3) + 4*x)/(6*b**2) + (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log(4*(-
1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/
3) + 4*x)/(6*b**2) + (-1)**(1/6)*sqrt(3)*B*a**(7/6)*(1/b)**(1/6)*atan(sqrt(
3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2) -
(-1)**(1/6)*sqrt(3)*B*a**(7/6)*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*
sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2) - 2*B*a*sqrt(x)/b**2 +
2*B*x**(7/2)/(7*b), True))
```

## 3.153 $\int \frac{x^{3/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

**Optimal**. Leaf size=270

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}-\frac{(Ab-aB)\tan^{-1}}{3\sqrt[6]{a}}$$

**Rubi [A]** time = 0.55, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {459, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}\,b^{11/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(5/2))/(5*b) - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt
[x])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6)) - ((A*b - a*
B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a
^(1/6)*b^(11/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{\left(4\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3b^{5/3}} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}}\right.}{3\sqrt[6]{a}\,b^{5/3}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right.}{2\sqrt{3}\,\sqrt[6]{a}\,b^{11/6}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,\sqrt[6]{a}\,b^{11/6}} - \frac{(A}{}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{2(Ab-a}{3}$$

**Mathematica [C]** time = 0.07, size = 45, normalized size = 0.17

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\right)}{5ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(5/2)*(a*B + (A*b - a*B)*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)]
))/(5*a*b)
```

**IntegrateAlgebraic [A]** time = 0.23, size = 167, normalized size = 0.62

$$-\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{3\sqrt[6]{a}\,b^{11/6}}+\frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\sqrt[6]{a}\,b^{11/6}}+\frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(5/2))/(5*b) - (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/
(3*a^(1/6)*b^(11/6)) + ((-(A*b) + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6
)*b^(1/6)*Sqrt[x])])/(3*a^(1/6)*b^(11/6)) + ((-(A*b) + a*B)*ArcTanh[(Sqrt[3
]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(Sqrt[3]*a^(1/6)*b^(11/6
))
```

**fricas [B]** time = 1.08, size = 3635, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(12*B*x^(5/2) - 20*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^6*b^9 - 5*A*B^4*a^5*b^10
 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^1
4)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + (B^1
0*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^
4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3
*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^7*b^7 -
6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3
*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*
B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6
*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^1
1))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 -
 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5
*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A
^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b +
15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^
5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 20*sqrt(3)*b*(-(B^
6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^
2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*arctan(1/3*(sqrt(3)*sq
rt(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^
3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b
+ 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*
b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8
*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5
+ 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B
*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b
^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b
^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2
```

```
 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 - 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^
6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1
/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a
^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5
*B*a*b^5 + A^6*b^6)) + 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^
11))^(1/6)*log(a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5
/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 +
5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A
^6*b^6)/(a*b^11))^(1/6)*log(-a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*
a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6
)/(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*
B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*b*(-(B^6*a^6 - 6*A*B^5*
a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^
5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10
+ 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^
10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A
^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^
3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7
 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*
a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a*b^11))^(2/3)) + 5*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a*b^11))^(1/6)*log(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11
 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^
9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*
A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^
2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 +
15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^
2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2
/3)))/b
```

**giac [A]** time = 1.35, size = 280, normalized size = 1.04

$$\frac{2\,Bx^{\frac{5}{2}}}{5\,b}-\frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab}+\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^{6}}-\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^{6}}-\frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^{6}}-\frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/5*B*x^(5/2)/b - 2/3*(B*a*(a/b)^(5/6) - A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a
/b)^(1/6))/(a*b) + 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^6) - 1/6*sqrt(3)*((a*b^
5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a
/b)^(1/3))/(a*b^6) - 1/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^6) - 1/3*((a*b^5)^(5/6)*B*
a - (a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6
))/(a*b^6)
```

**maple [A]** time = 0.16, size = 356, normalized size = 1.32

$$\frac{2Bx^{\frac{5}{2}}}{5b}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}A\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}A\ln\left(-x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{2A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b}+\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b}+\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b}-\frac{2Ba\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b^{2}}-\frac{Ba\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b^{2}}-\frac{Ba\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b^{2}}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(-x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/5*B*x^(5/2)/b+2/3/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A-2/3/b^2/(
a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B*a+1/6/a*3^(1/2)*(a/b)^(5/6)*ln(-
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A-1/6/b*3^(1/2)*(a/b)^(5/6)*ln(-
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+1/3/b/(a/b)^(1/6)*arctan(2/(a/
b)^(1/6)*x^(1/2)-3^(1/2))*A-1/3/b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2
)-3^(1/2))*B*a-1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(
a/b)^(1/3))*A+1/6/b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a
/b)^(1/3))*B+1/3/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A-1/3/
b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B*a
```

**maxima [A]** time = 1.14, size = 212, normalized size = 0.79

$$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/5*B*x^(5/2)/b + 1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt
(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)
*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqr
t(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x
))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3
)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

**mupad [B]** time = 2.85, size = 1640, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (2*B*x^(5/2))/(5*b) + (atan((((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 +
96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 +
 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(
1/3)*b^(11/3)) + ((A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5
*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5
*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3
)))/(((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*
B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*
A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - ((A*b - B*
a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^
(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(
27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*(A*b - B*a)*2i)/(3*(-a)^(
1/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3
*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*
b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/
2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96
```

```
*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b
^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-
a)^(1/3)*b^(11/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^
3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)
/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3
))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/
2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B
*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 8
64*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*
b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
+ (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a
^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A
*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^
(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b
 - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2
+ (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^
5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/
3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6
+ 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
- B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/
6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a
)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
- 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
```

**sympy [A]** time = 64.30, size = 857, normalized size = 3.17

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} & \text{for}\: a = 0 \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} & \text{for}\: b = 0 \\ -\frac{(-1)^{\frac{5}{6}}A\log\left(-\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}A\log\left(\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}A\log\left(-4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}A\log\left(4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}\sqrt{3}A\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}\sqrt{3}A\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}Ba^{\frac{5}{6}}\log\left(-\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3b^{2}\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}Ba^{\frac{5}{6}}\log\left(\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3b^{2}\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}Ba^{\frac{5}{6}}\log\left(-4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6b^{2}\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}Ba^{\frac{5}{6}}\log\left(4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6b^{2}\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}\sqrt{3}Ba^{\frac{5}{6}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3b^{2}\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}\sqrt{3}Ba^{\frac{5}{6}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3b^{2}\sqrt[6]{\frac{1}{b}}}+\frac{2Bx^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*
A/sqrt(x) + 2*B*x**(5/2)/5)/b, Eq(a, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(11/2)/
11)/a, Eq(b, 0)), (-(-1)**(5/6)*A*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) +
sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*A*log((-1)**(1/6)*a**(1/
6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*A*log(
-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)
**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*A*log(4*(-1)**(1/6
)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x
)/(6*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 - 2*(-
1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(
1/6)) + (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x
)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*B*a*
*(5/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*(1/b)**(1/
6)) - (-1)**(5/6)*B*a**(5/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x
))/(3*b**2*(1/b)**(1/6)) + (-1)**(5/6)*B*a**(5/6)*log(-4*(-1)**(1/6)*a**(1/
6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**
2*(1/b)**(1/6)) - (-1)**(5/6)*B*a**(5/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)
*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**2*(1/b)**(
1/6)) + (-1)**(5/6)*sqrt(3)*B*a**(5/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(
3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2*(1/b)**(1/6)) - (-1)**(5/6)*s
qrt(3)*B*a**(5/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6
)*(1/b)**(1/6)))/(3*b**2*(1/b)**(1/6)) + 2*B*x**(5/2)/(5*b), True))
```

## 3.154 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{a+bx^3}\,dx$

**Optimal**. Leaf size=53

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

**Rubi [A]** time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 329, 275, 205}

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*
Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{2Bx^{3/2}}{3b}-\frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3b}\\ &= \frac{2Bx^{3/2}}{3b}-\frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3b}\\ &= \frac{2Bx^{3/2}}{3b}+\frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b}\\ &= \frac{2Bx^{3/2}}{3b}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 52, normalized size = 0.98

$$\frac{2}{3}\left(\frac{Bx^{3/2}}{b}-\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*((B*x^(3/2))/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(Sqr
t[a]*b^(3/2))))/3
```

**IntegrateAlgebraic [A]** time = 0.05, size = 53, normalized size = 1.00

$$\frac{2Bx^{3/2}}{3b}-\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) - (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/
(3*Sqrt[a]*b^(3/2))
```

**fricas [A]** time = 0.81, size = 108, normalized size = 2.04

$$\left[\frac{2\,Babx^{\frac{3}{2}}+(Ba-Ab)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)}{3\,ab^2},\ \frac{2\left(Babx^{\frac{3}{2}}-(Ba-Ab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)\right)}{3\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^
(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a*b
)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]
```

**giac [A]** time = 0.17, size = 39, normalized size = 0.74

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

**maple** [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}-\frac{2Ba\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b}+\frac{2B\,x^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/3*B*x^(3/2)/b+2/3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A-2/3/b/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*B*a
```

**maxima** [A] time = 1.21, size = 39, normalized size = 0.74

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

**mupad** [B] time = 2.60, size = 93, normalized size = 1.75

$$\frac{2\,B\,x^{3/2}}{3\,b}-\frac{2\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(24\,A^2\,b^3-48\,A\,B\,a\,b^2+24\,B^2\,a^2\,b\right)}{\left(72\,B\,a^2\,b^2-72\,A\,a\,b^3\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (2*B*x^(3/2))/(3*b) - (2*atan((3*a^(1/2)*b^(3/2)*x^(3/2)*(24*A^2*b^3 + 24*B
^2*a^2*b - 48*A*B*a*b^2))/((72*B*a^2*b^2 - 72*A*a*b^3)*(A*b - B*a)))*(A*b -
 B*a))/(3*a^(1/2)*b^(3/2))
```

**sympy** [A] time = 22.39, size = 537, normalized size = 10.13

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) & \text{for}\ a=0 \wedge b=0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} & \text{for}\ a=0 \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} & \text{for}\ b=0 \\ -\frac{iA\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}+\frac{iA\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}+\frac{iA\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}-\frac{iA\log\left(4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}+\frac{iB\sqrt{a}\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3b^2\sqrt{\frac{1}{b}}}-\frac{iB\sqrt{a}\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3b^2\sqrt{\frac{1}{b}}}-\frac{iB\sqrt{a}\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3b^2\sqrt{\frac{1}{b}}}+\frac{iB\sqrt{a}\log\left(4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3b^2\sqrt{\frac{1}{b}}}+\frac{2Bx^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)),
((-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3)/b, Eq(a, 0)), ((2*A*x**(3/2)/3 + 2*B*
x**(9/2)/9)/a, Eq(b, 0)), (-I*A*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sq
rt(x))/(3*sqrt(a)*b*sqrt(1/b)) + I*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6)
+ sqrt(x))/(3*sqrt(a)*b*sqrt(1/b)) + I*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x
```

```
)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sq
rt(1/b)) - I*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1
/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)) + I*B*sqrt(a)*log(
-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*sqrt(1/b)) - I*B*sqrt
(a)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*sqrt(1/b)) - I
*B*sqrt(a)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)
*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*b**2*sqrt(1/b)) + I*B*sqrt(a)*log(4*(-1)**
(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) +
 4*x)/(3*b**2*sqrt(1/b)) + 2*B*x**(3/2)/(3*b), True))
```

## 3.155 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=268

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}}-\frac{(Ab-aB)\tan}{3a}$$

**Rubi** [A] time = 0.47, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{5/6}b^{7/6}}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)
])/(3*a^(5/6)*b^(7/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(5/6)*b^(7/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(5/6)*b^(7/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(
1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6)) + ((A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b
^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)}\,dx &= \frac{2B\sqrt{x}}{b}-\frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{b}\\
&= \frac{2B\sqrt{x}}{b}-\frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{2B\sqrt{x}}{b}+\frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx,x,\sqrt{x}\right)}{3a^{5/6}b}+\frac{(2(Ab-aB))\,\text{Subst}\left(\int \cdots\right.}{}\\
&= \frac{2B\sqrt{x}}{b}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}-\frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}}\\
&= \frac{2B\sqrt{x}}{b}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}}+\frac{(Ab\cdots}{}\\
&= \frac{2B\sqrt{x}}{b}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{2(Ab-aB\cdots}{3\cdots}
\end{aligned}
$$

**Mathematica** [C] time = 0.03, size = 43, normalized size = 0.16

$$
\frac{2\sqrt{x}\left((Ab-aB)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+aB\right)}{ab}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[x]*(a*B + (A*b - a*B)*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)])
)/(a*b)
```

**IntegrateAlgebraic [A]** time = 0.25, size = 166, normalized size = 0.62

$$-\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{3a^{5/6}b^{7/6}}-\frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^{5/6}b^{7/6}}+\frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*B*Sqrt[x])/b - (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*a
^(5/6)*b^(7/6)) + ((-(A*b) + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(
1/6)*Sqrt[x])])/(3*a^(5/6)*b^(7/6)) - ((-(A*b) + a*B)*ArcTanh[(Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(Sqrt[3]*a^(5/6)*b^(7/6))
```

**fricas [B]** time = 1.00, size = 2424, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^
6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^2*b - A*a*b^
2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6))*a^4*
b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + 2*sqrt(3)*
(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a^5*b^7))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6
 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b +
 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^
6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^
2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b +
A^2*b^2)*x - (B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A
^6*b^6)/(a^5*b^7))^(1/6))*a^4*b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^7))^(5/6) + 2*sqrt(3)*(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*
a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^
5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - b*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6
 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b +
 A^2*b^2)*x + 4*(B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15
*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
+ A^6*b^6)/(a^5*b^7))^(1/6)) + b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^
4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/
(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
```

```
^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^2*b - A*a*b^2
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) + 2*b
*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*
A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(a*b*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqrt(x))
- 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3
+ 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-a*b*(
-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^
4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqr
t(x)) + 12*B*sqrt(x))/b
```

**giac** [A]  time = 0.21, size = 280, normalized size = 1.04

$$\frac{2\,B\sqrt{x}}{b} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^2} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^2} - \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{2\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqr
t(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5)^
(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(
3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a -
 (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/
(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^
(1/6))/(a*b^2)
```

**maple** [A]  time = 0.15, size = 353, normalized size = 1.32

$$\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{3a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{3a} + \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a} - \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(-x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a} - \frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{3b} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{3b} - \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b} + \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(-x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6b} + \frac{2B\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)/x^(1/2),x)
```

```
[Out] 2*B*x^(1/2)/b+2/3/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A-2/3/b*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A+1/6/b*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/
6)*x^(1/2)-3^(1/2))*A-1/3/b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2
))*B+1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3)
)*A-1/6/b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))
*B+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A-1/3/b*(a/b)^(1
/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B
```

**maxima** [A]  time = 1.42, size = 278, normalized size = 1.04

$$\frac{2\,B\sqrt{x}}{b} - \frac{\frac{\sqrt{3}\,(Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}\,(Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x} + b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}} - Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}} - Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}} + 2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}} - Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}} - 2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x
) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(
3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*
a*b^(1/3) - A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/
```

```
3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3
))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3
)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^
(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

**mupad [B]** time = 2.88, size = 1915, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)),x)
```

```
[Out] (2*B*x^(1/2))/b + (atan((((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2
*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b
^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6
)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/6)) + ((x^(1/2)*(96*A^4*b^5
+ 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2)
 + ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*
A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/
6)))/(((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*
B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b -
B*a))/(3*(-a)^(5/6)*b^(7/6)) - ((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A
^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((A*b - B*a)*(288*A
^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a
)^(5/6)*b^(7/6)))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6))))*(A*b - B*a)*2i)/(3*
(-a)^(5/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(
96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B
^3*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*
a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1
i)/(3*(-a)^(5/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(9
6*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^
3*a^3*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a
^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i
)/(3*(-a)^(5/6)*b^(7/6)))/((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96
*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3
*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^
4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3
*(-a)^(5/6)*b^(7/6)) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4
*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3
*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a
)^(5/6)*b^(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(
7/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 9
6*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) -
(((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*
A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/
6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96
*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (
((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A
*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/6
)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*
B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((
(3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*
B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^
(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*
a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (((3^(
1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*
a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^(7/6
```

```
))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(7/6))
```

<u>**sympy**</u> **[A]** time = 23.16, size = 833, normalized size = 3.11

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}}{b} & \text{for } a = 0 \\ \frac{2A\sqrt{x} + \frac{2Bx^{\frac{7}{2}}}{7}}{a} & \text{for } b = 0 \\ -\frac{\sqrt[6]{-1}A\sqrt[6]{\frac{1}{b}}\log\left(-\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}} + \sqrt{x}\right)}{3a^{\frac{5}{6}}} + \frac{\sqrt[6]{-1}A\sqrt[6]{\frac{1}{b}}\log\left(\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}} + \sqrt{x}\right)}{3a^{\frac{5}{6}}} - \frac{\sqrt[6]{-1}A\sqrt[6]{\frac{1}{b}}\log\left(-4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + 4x\right)}{6a^{\frac{5}{6}}} + \frac{\sqrt[6]{-1}A\sqrt[6]{\frac{1}{b}}\log\left(4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + 4x\right)}{6a^{\frac{5}{6}}} + \frac{\sqrt[6]{-1}\sqrt{3}A\sqrt[6]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3a^{\frac{5}{6}}} - \frac{\sqrt[6]{-1}\sqrt{3}A\sqrt[6]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3} + \frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3a^{\frac{5}{6}}} + \frac{\sqrt[6]{-1}B\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}\log\left(-\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}} + \sqrt{x}\right)}{3b} - \frac{\sqrt[6]{-1}B\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}\log\left(\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}} + \sqrt{x}\right)}{3b} + \frac{\sqrt[6]{-1}B\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}\log\left(-4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + 4x\right)}{6b} - \frac{\sqrt[6]{-1}B\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}\log\left(4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + 4x\right)}{6b} - \frac{\sqrt[6]{-1}\sqrt{3}B\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3b} + \frac{\sqrt[6]{-1}\sqrt{3}B\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3} + \frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3b} + \frac{2B\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/
7)/a, Eq(b, 0)), (-(-1)**(1/6)*A*(1/b)**(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/
b)**(1/6) + sqrt(x))/(3*a**(5/6)) + (-1)**(1/6)*A*(1/b)**(1/6)*log((-1)**(1
/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(5/6)) - (-1)**(1/6)*A*(1/b)**(1
/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/
3)*(1/b)**(1/3) + 4*x)/(6*a**(5/6)) + (-1)**(1/6)*A*(1/b)**(1/6)*log(4*(-1)
**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3)
 + 4*x)/(6*a**(5/6)) + (-1)**(1/6)*sqrt(3)*A*(1/b)**(1/6)*atan(sqrt(3)/3 -
2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(5/6)) - (-1
)**(1/6)*sqrt(3)*A*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt
(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(5/6)) + (-1)**(1/6)*B*a**(1/6)*(1/b)*
*(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b) - (-1)**(1/6
)*B*a**(1/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/
(3*b) + (-1)**(1/6)*B*a**(1/6)*(1/b)**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqr
t(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)
**(1/6)*B*a**(1/6)*(1/b)**(1/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(
1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*sqrt(
3)*B*a**(1/6)*(1/b)**(1/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(
3*a**(1/6)*(1/b)**(1/6)))/(3*b) + (-1)**(1/6)*sqrt(3)*B*a**(1/6)*(1/b)**(1/
6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6))
)/(3*b) + 2*B*sqrt(x)/b, True))
```

## 3.156 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

**Optimal**. Leaf size=268

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\tan}{3a}$$

**Rubi** [A] time = 0.55, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{7/6}b^{5/6}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(7/6)*b^(5/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6)*b^(5/6)) + ((A*b - a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6
)*b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{7/6}b^{2/3}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \cdots\right.}{3\cdots}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}\,a^{7/6}b^{5/6}} \cdots$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\,a^{7/6}b^{5/6}} + \frac{(Ab\cdots}{\cdots}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{2(Ab-aB\cdots}{3\cdots}$$

**Mathematica [C]** time = 0.01, size = 46, normalized size = 0.17

$$\frac{2\left(x^3(aB-Ab)\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)-5aA\right)}{5a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (2*(-5*a*A + (-(A*b) + a*B)*x^3*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a
)]))/(5*a^2*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.24, size = 166, normalized size = 0.62

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{3a^{7/6}b^{5/6}}-\frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}\,a^{7/6}b^{5/6}}-\frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(
3*a^(7/6)*b^(5/6)) + ((A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(
1/6)*Sqrt[x])])/(3*a^(7/6)*b^(5/6)) - ((-(A*b) + a*B)*ArcTanh[(Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(Sqrt[3]*a^(7/6)*b^(5/6))
```

**fricas [B]** time = 1.15, size = 3663, normalized size = 13.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3
*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/
6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3
*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B
^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B
^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) + (B^10*a^10 - 10*A*
B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4
- 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*
B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^11*b^3 - 6*A*B^5*a^10*
b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*
B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5
))^(2/3))*a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) +
2*sqrt(3)*(B^5*a^6*b - 5*A*B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^
3*b^4 + 5*A^4*B*a^2*b^5 - A^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^
4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 1
5*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*arctan(1/3*(sqrt(3)*sqrt(-4*(
B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 +
 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2
*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^
6*b^6)/(a^7*b^5))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^
2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A
^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9
+ A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 -
20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-
(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4
*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3))*a*b*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^6*b - 5*A*
```

```
B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^3*b^4 + 5*A^4*B*a^2*b^5 - A
^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6
) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b
^5))^(1/6)*log(a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20
*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))
^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3
 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 -
 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(4*(B^5*a^11*b^4 - 5*A*B^4
*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5
*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6
) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^
3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A
^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^
6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 1
5*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3)) - a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(1/6)*log(-4*(B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*
B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(
-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^
4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) + 4*(B^10*a^10 -
10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6
*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45
*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^
5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 -
 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a^7*b^5))^(2/3)) - 12*A*sqrt(x))/(a*x)
```

**giac [A]** time = 0.41, size = 280, normalized size = 1.04

$$-\frac{2\,A}{a\sqrt{x}} - \frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}} Ba - \left(ab^{5}\right)^{\frac{5}{6}} Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b^{5}} + \frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}} Ba - \left(ab^{5}\right)^{\frac{5}{6}} Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b^{5}} + \frac{\left(\left(ab^{5}\right)^{\frac{5}{6}} Ba - \left(ab^{5}\right)^{\frac{5}{6}} Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b^{5}} + \frac{\left(\left(ab^{5}\right)^{\frac{5}{6}} Ba - \left(ab^{5}\right)^{\frac{5}{6}} Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b^{5}} + \frac{2\left(\left(ab^{5}\right)^{\frac{5}{6}} Ba - \left(ab^{5}\right)^{\frac{5}{6}} Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^5) + 1/6*sqrt(3)*((a*
b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x +
(a/b)^(1/3))/(a^2*b^5) + 1/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan
((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^5) + 1/3*((a*b^5)^(5
/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b
)^(1/6))/(a^2*b^5) + 2/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(sqr
t(x)/(a/b)^(1/6))/(a^2*b^5)
```

**maple [A]** time = 0.16, size = 355, normalized size = 1.32

$$-\frac{2A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}a}-\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}a}-\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}Ab\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}Ab\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{2B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b}+\frac{B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b}+\frac{B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}b}-\frac{2A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a),x)
```

```
[Out] -2/3/a/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+2/3/b/(a/b)^(1/6)*arctan
(1/(a/b)^(1/6)*x^(1/2))*B-1/6/a^2*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(
1/6)*x^(1/2)-(a/b)^(1/3))*A*b+1/6/a*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)-(a/b)^(1/3))*B-1/3/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2
)-3^(1/2))*A+1/3/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B+1/6/
a^2*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A*b-1
/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-1/
3/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+1/3/b/(a/b)^(1/6)*a
rctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B-2*A/a/x^(1/2)
```

**maxima [A]** time = 1.34, size = 212, normalized size = 0.79

$$-\frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6a}-\frac{2A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x)
+ b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6
) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)
)) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^
(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2*A/(a*sqrt(x))
```

**mupad [B]** time = 2.86, size = 1700, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)),x)
```

```
[Out] (atan((((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4
 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^1
2*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3
)) + ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 -
 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*
b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3))
)/(((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 9
6*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^
4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - ((
A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2
*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1
728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*(A*b -
B*a)*2i)/(3*(-a)^(7/6)*b^(5/6)) - (2*A)/(a*x^(1/2)) + (atan(((((3^(1/2)*1i)
/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11
*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864
*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/
6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32
```

```
*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x
^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*
b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3))
)/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^
6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)
*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27
*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*
b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B
*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 +
 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/
3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6))
 + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3
*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2
 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^
5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 +
1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4
- 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*
a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*
1i)/((-a)^(7/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*
a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2
)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 -
 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^
(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A
*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7
/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i
)/(3*(-a)^(7/6)*b^(5/6))
```

**sympy [A]** time = 59.23, size = 836, normalized size = 3.12

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{\sqrt{x}}\right) & \text{for } a=0\wedge b=0 \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{\sqrt{x}}}{b} & \text{for } a=0 \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} & \text{for } b=0 \\ -\frac{2A}{a\sqrt{x}}+\frac{(-1)^{\frac{5}{6}}A\log\left(-\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}A\log\left(\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}A\log\left(-4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}A\log\left(4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}\sqrt{3}A\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}\sqrt{3}A\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}B\log\left(-\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}B\log\left(\sqrt[6]{-1}\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}B\log\left(-4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}B\log\left(4\sqrt[6]{-1}\sqrt[6]{a}\sqrt{x}\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+4x\right)}{6\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}\sqrt{3}B\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}\sqrt{3}B\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2(-1)^{\frac{5}{6}}\sqrt{3}\sqrt{x}}{3\sqrt[6]{a}\sqrt[6]{\frac{1}{b}}}\right)}{3\sqrt[6]{a}b\sqrt[6]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)
/5)/a, Eq(b, 0)), (-2*A/(a*sqrt(x)) + (-1)**(5/6)*A*log(-(-1)**(1/6)*a**(1/
6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*A*log((-
1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)*(1/b)**(1/6)) + (-1)
**(5/6)*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*
a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*A*log(
4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + 4*x)/(6*a**(7/6)*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3
)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(7/6)*
(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3
)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6
)*B*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**
(1/6)) + (-1)**(5/6)*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*
a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*B*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x
)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(
1/b)**(1/6)) + (-1)**(5/6)*B*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6
) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6))
- (-1)**(5/6)*sqrt(3)*B*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a
**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*B*
atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(
3*a**(1/6)*b*(1/b)**(1/6)), True))
```

## 3.157 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=53

$$-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 329, 275, 205}

$$-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{3ax^{3/2}} - \frac{\left(2\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3a}$$
$$= -\frac{2A}{3ax^{3/2}} - \frac{\left(4\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a}$$
$$= -\frac{2A}{3ax^{3/2}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3a}$$
$$= -\frac{2A}{3ax^{3/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

**Mathematica [A]** time = 0.05, size = 53, normalized size = 1.00

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*a^(3/2)*Sqrt[b])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 53, normalized size = 1.00

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*a^(3/2)*Sqrt[b])
```

**fricas [A]** time = 0.78, size = 120, normalized size = 2.26

$$\left[\frac{(Ba-Ab)\sqrt{-ab}\,x^2\log\left(\frac{bx^3+2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,Aab\sqrt{x}}{3\,a^2bx^2}, \frac{2\left((Ba-Ab)\sqrt{ab}\,x^2\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-Aab\sqrt{x}\right)}{3\,a^2bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*((B*a - A*b)*sqrt(-a*b)*x^2*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*
x^3 + a)) - 2*A*a*b*sqrt(x))/(a^2*b*x^2), 2/3*((B*a - A*b)*sqrt(a*b)*x^2*ar
ctan(sqrt(a*b)*x^(3/2)/a) - A*a*b*sqrt(x))/(a^2*b*x^2)]
```

**giac [A]** time = 0.17, size = 39, normalized size = 0.74

$$\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a} - \frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

**maple** [A] time = 0.06, size = 53, normalized size = 1.00

$$-\frac{2Ab\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;a}+\frac{2B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}-\frac{2A}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a),x)
```

```
[Out] -2/3*A/a/x^(3/2)-2/3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A*b+2/3/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

**maxima** [A] time = 1.40, size = 39, normalized size = 0.74

$$\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}-\frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

**mupad** [B] time = 0.10, size = 102, normalized size = 1.92

$$-\frac{2\,A}{3\,a\,x^{3/2}}-\frac{2\,\mathrm{atan}\left(\frac{3\,a^{3/2}\,\sqrt{b}\,x^{3/2}\,\left(24\,A^2\,a^3\,b^5-48\,A\,B\,a^4\,b^4+24\,B^2\,a^5\,b^3\right)}{\left(A\,b-B\,a\right)\,\left(72\,A\,a^5\,b^4-72\,B\,a^6\,b^3\right)}\right)\,\left(A\,b-B\,a\right)}{3\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(3*a*x^(3/2)) - (2*atan((3*a^(3/2)*b^(1/2)*x^(3/2)*(24*A^2*a^3*b^5
+ 24*B^2*a^5*b^3 - 48*A*B*a^4*b^4))/((A*b - B*a)*(72*A*a^5*b^4 - 72*B*a^6*b
^3)))*(A*b - B*a))/(3*a^(3/2)*b^(1/2))
```

**sympy** [A] time = 156.58, size = 527, normalized size = 9.94

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) & \text{for}\: a=0\wedge b=0 \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} & \text{for}\: a=0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} & \text{for}\: b=0 \\ -\frac{2A}{3ax^{\frac{3}{2}}}+\frac{iA\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{iA\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{iA\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{iA\log\left(4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{iB\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}+\frac{iB\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}+\frac{iB\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}-\frac{iB\log\left(4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-2*A/(3*a*x**(3/2)) + I*A*log(-(-1)**(1/6
```

```
)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(3/2)*sqrt(1/b)) - I*A*log((-1)**(
1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(3/2)*sqrt(1/b)) - I*A*log(-4*(
-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1
/3) + 4*x)/(3*a**(3/2)*sqrt(1/b)) + I*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*
(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*a**(3/2)*sqrt(
1/b)) - I*B*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a)*b*
sqrt(1/b)) + I*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a
)*b*sqrt(1/b)) + I*B*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(
-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)) - I*B*log(4
*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**
(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)), True))
```

## 3.158 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

**Optimal**. Leaf size=270

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\tan^{-}}{3a^{11}}$$

**Rubi** [A] time = 0.48, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^
(1/6)])/(3*a^(11/6)*b^(1/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqr
t[x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(11/6)*b^(1/6)) - ((A*b - a
*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*
a^(11/6)*b^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{5ax^{5/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{\left(4\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{5a}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{3a^{11/6}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{6a^{5/3}}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\,a^{11/6}\sqrt[6]{b}} - \frac{(A}{}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{2(Ab-a}{3}$$

**Mathematica [C]** time = 0.02, size = 47, normalized size = 0.17

$$\frac{10x^3(aB-Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)-2aA}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*a*A + 10*(-(A*b) + a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)
])/(5*a^2*x^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.19, size = 167, normalized size = 0.62

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{3a^{11/6}\sqrt[6]{b}}+\frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])
/(3*a^(11/6)*b^(1/6)) + ((A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*
b^(1/6)*Sqrt[x])])/(3*a^(11/6)*b^(1/6)) + ((-(A*b) + a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(Sqrt[3]*a^(11/6)*b^(1/6))
```

**fricas [B]** time = 0.95, size = 2424, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))
^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^3 - A*a^2*b
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6))*a^9*b*
(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A
^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5/6) + 2*sqrt(3)*(B*a^
10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*
B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
6*A^5*B*a*b^5 + A^6*b^6)) + 20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^11*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
^11*b))^(1/6))*a^9*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5
/6) + 2*sqrt(3)*(B*a^10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b +
 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^11*b))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 5*a*x^3*(-(B^6*a^6 - 6*A*B
^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6
*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + 4*
(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(1/6)) + 5*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
```

```
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/
6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a
^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 10*a*x^3*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(a^2*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) - 10*a*x^3*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-a^2*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) + 12*A*sqr
t(x))/(a*x^3)
```

**giac** [A]    time = 0.21, size = 280, normalized size = 1.04

$$\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{1}{6}} Ba - \left(ab^{5}\right)^{\frac{1}{6}} Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b} - \frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{1}{6}} Ba - \left(ab^{5}\right)^{\frac{1}{6}} Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^{2}b} + \frac{\left(\left(ab^{5}\right)^{\frac{1}{6}} Ba - \left(ab^{5}\right)^{\frac{1}{6}} Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b} + \frac{\left(\left(ab^{5}\right)^{\frac{1}{6}} Ba - \left(ab^{5}\right)^{\frac{1}{6}} Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b} + \frac{2\left(\left(ab^{5}\right)^{\frac{1}{6}} Ba - \left(ab^{5}\right)^{\frac{1}{6}} Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^{2}b} - \frac{2\,A}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b
^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b)
+ 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) +
 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 2/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) -
 2/5*A/(a*x^(5/2))
```

**maple** [A]    time = 0.18, size = 358, normalized size = 1.33

$$-\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a^{2}} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{3a^{2}} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{3a^{2}} - \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\ln\left(x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^{2}} + \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\ln\left(-x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^{2}} + \frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}} B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}} B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{3a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}} B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{3a} + \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} B\ln\left(x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a} - \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} B\ln\left(-x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a} - \frac{2A}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a),x)
```

```
[Out] -2/5*A/a/x^(5/2)-2/3/a^2*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A*b+2/3/
a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B+1/6/a^2*3^(1/2)*(a/b)^(1/6)*l
n(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A*b-1/6/a*3^(1/2)*(a/b)^(1/6)
*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B-1/3/a^2*(a/b)^(1/6)*arcta
n(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A*b+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)
*x^(1/2)-3^(1/2))*B-1/6/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^
(1/2)+(a/b)^(1/3))*A*b+1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x
^(1/2)+(a/b)^(1/3))*B-1/3/a^2*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1
/2))*A*b+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B
```

**maxima** [A]    time = 1.42, size = 278, normalized size = 1.03

$$\frac{\frac{\sqrt{3}\,(Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}\,(Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{6\,a} - \frac{2\,A}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2/5*A/(a*x^(5/2))
```

**mupad [B]** time = 2.91, size = 2023, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(5*a*x^(5/2)) - (atan((((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 +
 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B*a
)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b
^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + ((x
^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a
^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*
b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
 - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/(((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^
9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*
b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*
B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) -
 ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B
^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a
^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*
(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/
6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*
b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(
1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B
^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(
3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 +
 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b
^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b
^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
- B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A
^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384
*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 28
8*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(
1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1
/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*
b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7
*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(
11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)
*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^
3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A
^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*
(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*
1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^
7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(A*b -
 B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a
^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/
((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^
2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 +
```

```
 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 -
 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b
^(1/6)) - (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5
 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2
)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*
a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)
^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^
(1/6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

## 3.159 $\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=95

$$\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}-\frac{x^{3/2}(Ab-3aB)}{3ab^2}+\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

**Rubi** [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 321, 329, 275, 205}

$$-\frac{x^{3/2}(Ab-3aB)}{3ab^2}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}+\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 3*a*B)*x^(3/2))/(3*a*b^2) + ((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x^
3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{\left(-\frac{3Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{3ab}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.18, size = 77, normalized size = 0.81

$$
\frac{\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{b}\,x^{3/2}\left(3aB-Ab+2bBx^3\right)}{a+bx^3}}{3b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((Sqrt[b]*x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(a + b*x^3) + ((A*b - 3*a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[a])/(3*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 77, normalized size = 0.81

$$
\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}+\frac{x^{3/2}\left(3aB-Ab+2bBx^3\right)}{3b^2\left(a+bx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(3*b^2*(a + b*x^3)) + ((A*b - 3*a*B)
*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

**fricas [A]** time = 0.72, size = 222, normalized size = 2.34

$$
\left[\frac{\left(\left(3\,Bab-Ab^2\right)x^3+3\,Ba^2-Aab\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\left(2\,Bab^2x^4+\left(3\,Ba^2b-Aab^2\right)x\right)\sqrt{x}}{6\left(ab^4x^3+a^2b^3\right)},-\frac{\left(\left(3\,Bab-Ab^2\right)x^3+3\,Ba^2-Aab\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-\left(2\,Bab^2x^4+\left(3\,Ba^2b-Aab^2\right)x\right)\sqrt{x}}{3\left(ab^4x^3+a^2b^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*s
qrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a*b
^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*B*a
```

```
^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3*B*a
^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]
```

**giac [A]** time = 0.18, size = 68, normalized size = 0.72

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba - Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^2} + \frac{Bax^{\frac{3}{2}} - Abx^{\frac{3}{2}}}{3\left(bx^3 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b
)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)
```

**maple [A]** time = 0.06, size = 93, normalized size = 0.98

$$-\frac{A\,x^{\frac{3}{2}}}{3\left(b\,x^3 + a\right)b} + \frac{Ba\,x^{\frac{3}{2}}}{3\left(b\,x^3 + a\right)b^2} + \frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b} - \frac{Ba\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} + \frac{2B\,x^{\frac{3}{2}}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 2/3*B*x^(3/2)/b^2-1/3/b*x^(3/2)/(b*x^3+a)*A+1/3/b^2*x^(3/2)/(b*x^3+a)*B*a+1
/3/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A-1/b^2/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x^(3/2))*B*a
```

**maxima [A]** time = 1.24, size = 68, normalized size = 0.72

$$\frac{(Ba - Ab)x^{\frac{3}{2}}}{3\left(b^3x^3 + ab^2\right)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba - Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^(3/2)/(b^3*x^3 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a
- A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

**mupad [B]** time = 2.65, size = 116, normalized size = 1.22

$$\frac{2\,B\,x^{3/2}}{3\,b^2} - \frac{x^{3/2}\,\left(\frac{A\,b}{3} - \frac{B\,a}{3}\right)}{b^3\,x^3 + a\,b^2} + \frac{\mathrm{atan}\left(\frac{36\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2 - 6\,A\,B\,a\,b + 9\,B^2\,a^2\right)}{(A\,b - 3\,B\,a)\,\left(36\,A\,a\,b^2 - 108\,B\,a^2\,b\right)}\right)\,(A\,b - 3\,B\,a)}{3\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (
atan((36*a^(1/2)*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 - 6*A*B*a*b))/((A*b -
 3*B*a)*(36*A*a*b^2 - 108*B*a^2*b)))*(A*b - 3*B*a))/(3*a^(1/2)*b^(5/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.160 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=312

$$-\frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}-\frac{(Ab-7aB)}{}$$

**Rubi [A]** time = 0.50, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}-\frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}}-\frac{\sqrt{x}(Ab-7aB)}{3ab^2}+\frac{x^{7/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 7*a*B)*Sqrt[x])/(3*a*b^2) + ((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x^
3)) - ((A*b - 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(
5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6
)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)
])/(9*a^(5/6)*b^(13/6)) - ((A*b - 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/
6)*b^(13/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{\left(-\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{6b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{\right.}{9a^{5/6}b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\,\text{Subst}\left(\right.}{1}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\log\left(\sqrt[3]{a}\right.}{12\sqrt{}}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} - \frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\,\text{ta}}{18}$$

**Mathematica [C]** time = 0.09, size = 76, normalized size = 0.24

$$\frac{\sqrt{x}\left(\left(a+bx^3\right)(Ab-7aB)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+a\left(7aB-Ab+6bBx^3\right)\right)}{3ab^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[x]*(a*(-(A*b) + 7*a*B + 6*b*B*x^3) + (A*b - 7*a*B)*(a + b*x^3)*Hyperg
eometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(3*a*b^2*(a + b*x^3))
```

**IntegrateAlgebraic [A]** time = 0.65, size = 196, normalized size = 0.63

$$\frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} + \frac{(7aB-Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{18a^{5/6}b^{13/6}} - \frac{(7aB-Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{5/6}b^{13/6}} + \frac{\sqrt{x}\left(7aB-Ab+6bBx^3\right)}{3b^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[x]*(-(A*b) + 7*a*B + 6*b*B*x^3))/(3*b^2*(a + b*x^3)) + ((A*b - 7*a*B)
*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(5/6)*b^(13/6)) + ((-(A*b) + 7*a*B
)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(18*a^(5/6)*b^(1
3/6)) - ((-(A*b) + 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3
) + b^(1/3)*x)])/(6*Sqrt[3]*a^(5/6)*b^(13/6))
```

**fricas [B]** time = 0.84, size = 2566, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b +
36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5
*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-
(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3
*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3
) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)
*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*
B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(
1/6))*a^4*b^11*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b
^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^13))^(5/6) + 2*sqrt(3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(11764
9*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b
^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) - sq
rt(3)*(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A
^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B
^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3
 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*(b^3*x^3 + a
*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860
*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^1
3))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B
^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b
^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b
 + A^2*b^2)*x - (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6))*a^4*b^11*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + 2*sqrt(
3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5
*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 4
2*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 - 1008
42*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2
*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B^6*a^6 - 100842*A*B^5*a^5*b
 + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A
^5*B*a*b^5 + A^6*b^6)) - (b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5
*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4
 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^
6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735
*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^
2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + (b^3*x
^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2
 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a
^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*
A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x - (
7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 360
15*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*
a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6
 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*
A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a*b^2*(-(
117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*
a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)
 - (7*B*a - A*b)*sqrt(x)) - 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-a*b^2*(-(117649*B^
```

```
6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a
 - A*b)*sqrt(x)) + 12*(6*B*b*x^3 + 7*B*a - A*b)*sqrt(x))/(b^3*x^3 + a*b^2)
```

**giac [A]** time = 0.33, size = 313, normalized size = 1.00

$$\frac{2\,B\sqrt{x}}{b^2}-\frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}Ba-(ab^5)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3}+\frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}Ba-(ab^5)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\left(bx^3+a\right)b^2}-\frac{\left(7\,(ab^5)^{\frac{1}{6}}Ba-(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3}-\frac{\left(7\,(ab^5)^{\frac{1}{6}}Ba-(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3}-\frac{\left(7\,(ab^5)^{\frac{1}{6}}Ba-(ab^5)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/36*sqrt(3)*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*lo
g(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^3) + 1/36*sqrt(3)*(7*
(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x
 + (a/b)^(1/3))/(a*b^3) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*b^2)
 - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) -
 1/9*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/
(a*b^3)
```

**maple [A]** time = 0.17, size = 405, normalized size = 1.30

$$-\frac{A\sqrt{x}}{3\left(bx^3+a\right)b}+\frac{Ba\sqrt{x}}{3\left(bx^3+a\right)b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18ab}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36ab}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36ab}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9b^2}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18b^2}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18b^2}-\frac{7\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36b^2}+\frac{7\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36b^2}+\frac{2B\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 2*B/b^2*x^(1/2)-1/3/b*x^(1/2)/(b*x^3+a)*A+1/3/b^2*x^(1/2)/(b*x^3+a)*B*a-7/9
/b^2*B*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+7/36/b^2*B*3^(1/2)*(a/b)^(
1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-7/18/b^2*B*(a/b)^(1/6)*
arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-7/36/b^2*B*3^(1/2)*(a/b)^(1/6)*ln(x+3
^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-7/18/b^2*B*(a/b)^(1/6)*arctan(2/(a/
b)^(1/6)*x^(1/2)+3^(1/2))+1/9/b*A/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2
))-1/36/b*A/a*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(
1/3))+1/18/b*A/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+1/36/b*A
/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/18/b
*A/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))
```

**maxima [A]** time = 1.35, size = 311, normalized size = 1.00

$$\frac{(Ba-Ab)\sqrt{x}}{3\left(b^3x^3+ab^2\right)}+\frac{2\,B\sqrt{x}}{b^2}-\frac{\frac{\sqrt{3}\,(7\,Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(7\,Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(7\,Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*sqrt(x)/(b^3*x^3 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/36*(sqrt(3)
*(7*B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) - sqrt(3)*(7*B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(
x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) - A*b^(4/3))
*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3
)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a
^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(sq
rt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3
)*sqrt(a^(1/3)*b^(1/3))))/b^2
```

**mupad [B]** time = 2.89, size = 1884, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (atan
(((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^
3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7
*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 +
 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 1
47*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)
)/(18*(-a)^(5/6)*b^(13/6)) - (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2
*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A*b - 7*B
*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a
)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*(A*b - 7*B*a)*
1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*((2*x^(1/2)*(A
^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a
*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^4 - A^
3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A
*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((2*x^(1
/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A
^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^
4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6
)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2)*1i)/2 - 1/2)*((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*
B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b
^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)
*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3
*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(3
43*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6
)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 + 1/
2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*
(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5
/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/
2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A
*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7
*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(
-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2
)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 -
1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(
1/2)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2
 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)
*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^
2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.161 $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=289

$$\frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\tan}{18a^{7}}$$

**Rubi [A]** time = 0.55, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}}+\frac{(5aB+Ab)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{7/6}b^{11/6}}+\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}}+\frac{x^{5/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(7/6)*b^(11/6)) + ((A*b + 5*a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(7/6)*b^(11/6)) - ((A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{\left(2\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{9a^{7/6}b^{5/3}} + \frac{(Ab+5aB)\,\text{Su}}{}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{b}\,}\right.}{12\sqrt{3}\,a^{7/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\right.}{12\sqrt{3}\,a^{7/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} - \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right.}{18a^{7/6}b^{11/6}}$$

**Mathematica [C]** time = 0.10, size = 62, normalized size = 0.21

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (A*b - a*B)
*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)]))/(5*a^2*b)
```

**IntegrateAlgebraic [A]** time = 0.57, size = 190, normalized size = 0.66

$$\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}}+\frac{(-5aB-Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{18a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{7/6}b^{11/6}}-\frac{x^{5/2}(aB-Ab)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((-(A*b) + a*B)*x^(5/2))/(a*b*(a + b*x^3)) + ((A*b + 5*a*B)*ArcTan[(b^
(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(7/6)*b^(11/6)) + ((-(A*b) - 5*a*B)*ArcTan[(a
^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(18*a^(7/6)*b^(11/6)) - ((A
*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)
])/(6*Sqrt[3]*a^(7/6)*b^(11/6))
```

**fricas [B]** time = 1.13, size = 3787, normalized size = 13.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(B*a - A*b)*x^(5/2) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*
a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375
*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9
*b^11 + 250*A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) +
 (9765625*B^10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 937
5000*A^3*B^7*a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 1
31250*A^6*B^4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A
^9*B*a*b^9 + A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 93
75*A^2*B^4*a^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*
B*a^6*b^12 + A^6*a^5*b^13)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*
B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 +
 A^6*b^6)/(a^7*b^11))^(2/3))*a*b^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9
375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*
B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6
 + A^5*a*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a
^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*
b^6)/(a^7*b^11))^(1/6) + sqrt(3)*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*
A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b
^5 + A^6*b^6))/(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) + 4
*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^
2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^11*b^
9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*A^3*B^2*a^8*b^12 + 2
5*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5
*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30
*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^10*a^10 + 19531250*A
*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A
```

```
^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^
7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (156
25*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^
3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(-(15
625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b
^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(2/3))*a*b
^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B
^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1
/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b
^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6 + A^5*a*b^7)*sqrt(x)*(-(15625*B
^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 +
375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - sqrt(3)
*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a
^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6))/(15625*B^6*a^6 +
18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B
^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) - 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^
6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 3
75*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(a^6*b^
9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^
3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/
6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*
a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(-(156
25*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-
a^6*b^9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*
A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11
))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^
3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*l
og((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*
A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a
^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*
A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^
10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7
*a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^
4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 +
 A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a
^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 +
 A^6*a^5*b^13)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2
+ 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a
^7*b^11))^(2/3)) + (a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b
 + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A
^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-(3125*B^5*a^11*b^9 + 3125*A*B^
4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b
^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2
*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5
+ A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^10*a^10 + 19531250*A*B^9*a^9*b +
17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A^4*B^6*a^6*b^
4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7
 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (15625*B^6*a^11*b
^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^3*a^8*b^10 +
375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(-(15625*B^6*a^6 +
 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*
B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(2/3)))/(a*b^2*x^3 + a^
2*b)
```

**giac** [A] time = 1.38, size = 302, normalized size = 1.04

$$\frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}} + Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b} - \frac{Bax^{\frac{5}{2}} - Abx^{\frac{5}{2}}}{3\left(bx^3+a\right)ab} - \frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{5}{6}}Ba + \left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6} + \frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{5}{6}}Ba + \left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6} + \frac{\left(5\left(ab^5\right)^{\frac{5}{6}}Ba + \left(ab^5\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^6} + \frac{\left(5\left(ab^5\right)^{\frac{5}{6}}Ba + \left(ab^5\right)^{\frac{5}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*B*a*(a/b)^(5/6) + A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*
b) - 1/3*(B*a*x^(5/2) - A*b*x^(5/2))/((b*x^3 + a)*a*b) - 1/36*sqrt(3)*(5*(a
*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x +
(a/b)^(1/3))/(a^2*b^6) + 1/36*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*
A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6) + 1/18*(
5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^2*b^6) + 1/18*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^6)
```

**maple [A]** time = 0.16, size = 387, normalized size = 1.34

$$\frac{(Ab-Ba)\,x^{\frac{5}{2}}}{3\left(b x^3+a\right)ab}+\frac{A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{6}}ab}+\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}ab}+\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}ab}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}A\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^2}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}A\ln\left(-x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^2}-\frac{5\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36ab}+\frac{5\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(-x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36ab}+\frac{5B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{6}}b^2}+\frac{5B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}b^2}+\frac{5B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/9/b/a/(a/b)^(1/6)*arctan(1/(a/b)^(1/6
)*x^(1/2))*A+5/9/b^2/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B+1/36/a^2*3
^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A+5/36/b/
a*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+1/18
/b/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+5/18/b^2/(a/b)^(1/
6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B-1/36/a^2*3^(1/2)*(a/b)^(5/6)*ln(
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-5/36/b/a*3^(1/2)*(a/b)^(5/6)*l
n(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/18/b/a/(a/b)^(1/6)*arctan(
2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+5/18/b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*
x^(1/2)+3^(1/2))*B
```

**maxima [A]** time = 1.47, size = 235, normalized size = 0.81

$$-\frac{(Ba-Ab)x^{\frac{5}{2}}}{3\left(ab^2x^3+a^2b\right)}-\frac{(5\,Ba+Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{36\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(5/2)/(a*b^2*x^3 + a^2*b) - 1/36*(5*B*a + A*b)*(sqrt(3)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)
) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqr
t(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^
(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(
1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*s
qrt(a^(1/3)*b^(1/3))))/(a*b)
```

**mupad [B]** time = 2.87, size = 1578, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (x^(5/2)*(A*b - B*a))/(3*a*b*(a + b*x^3)) - (atan(((((3^(1/2)*1i)/2 - 1/2)^
2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 +
```

```
600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)
^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3
 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^
3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/((((3^(1/2)*1i
)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a
^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A
^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(
324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^
3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*
B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*((3^(1/2
)*1i)/2 - 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (atan(((((3^(1/2
)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B
^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(
24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))
)*1i)/(324*(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2
*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))
/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/
3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
 + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6
)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b +
5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^
2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a
^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11
/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (
atan((((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b +
 20*A^2*B*a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)) -
 ((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 + (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A
*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/(((A*
b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*
a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^
2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + ((A*b + 5*B*
a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 +
(x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/
(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*(A*b + 5*B*a)*1i)/(9
*(-a)^(7/6)*b^(11/6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)
[Out] Timed out
```

## 3.162 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=71

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

**Rubi [A]** time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 329, 275, 205}

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

**Mathematica** [A] time = 0.06, size = 71, normalized size = 1.00

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} + \frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.10, size = 71, normalized size = 1.00

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} - \frac{x^{3/2}(aB-Ab)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((-(A*b) + a*B)*x^(3/2))/(a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt
[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

**fricas** [A] time = 0.80, size = 190, normalized size = 2.68

$$\left[-\frac{2\left(Ba^2b-Aab^2\right)x^{\frac{3}{2}} + \left(\left(Bab+Ab^2\right)x^3 + Ba^2 + Aab\right)\sqrt{-ab}\log\left(\frac{bx^3-2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)}{6\left(a^2b^3x^3+a^3b^2\right)}, -\frac{\left(Ba^2b-Aab^2\right)x^{\frac{3}{2}} - \left(\left(Bab+Ab^2\right)x^3 + Ba^2 + Aab\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)}{3\left(a^2b^3x^3+a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(B*a^2*b - A*a*b^2)*x^(3/2) + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*b
)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a^2*b^3*
x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^(3/2) - ((B*a*b + A*b^2)*x^3 +
B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a^2*b^3*x^3 + a^3*b^
2)]
```

**giac [A]** time = 0.28, size = 63, normalized size = 0.89

$$\frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,ab}-\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\left(bx^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^(3
/2) - A*b*x^(3/2))/((b*x^3 + a)*a*b)
```

**maple [A]** time = 0.06, size = 74, normalized size = 1.04

$$\frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;a}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b}+\frac{(Ab-Ba)\,x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x^(3/2))*A+1/3/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

**maxima [A]** time = 1.40, size = 61, normalized size = 0.86

$$-\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(3/2)/(a*b^2*x^3 + a^2*b) + 1/3*(B*a + A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

**mupad [B]** time = 0.14, size = 115, normalized size = 1.62

$$\frac{B\,a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,b^2\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,a\,b\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,\sqrt{a}\,b^{3/2}\,x^{3/2}-B\,a^{3/2}\,\sqrt{b}\,x^{3/2}+B\,a\,b\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\,a^{5/2}\,b^{3/2}+3\,a^{3/2}\,b^{5/2}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*a^2*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*b^2*x^3*atan((b^(1/2)*x^(3/2))/a
^(1/2)) + A*a*b*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*a^(1/2)*b^(3/2)*x^(3/2)
 - B*a^(3/2)*b^(1/2)*x^(3/2) + B*a*b*x^3*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(
3*a^(5/2)*b^(3/2) + 3*a^(3/2)*b^(5/2)*x^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.163 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=289

$$-\frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}-\frac{(aB+5Ab}{}$$

**Rubi [A]** time = 0.47, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}-\frac{(aB+5Ab)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}}+\frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b +
 a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(11/6)*b^(7/6)) - ((5*A*b + a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
```

```
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{\left(2\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{(5Ab+aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{9a^{11/6}b} + \frac{(5Ab+aB)\,\text{Subs}}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,d\right.}{12\sqrt{3}\,a^{11/6}b^{7/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}\right.}{12\sqrt{3}\,a^{11/6}b^{7/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} - \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} +$$

**Mathematica [C]** time = 0.03, size = 68, normalized size = 0.24

$$\frac{\sqrt{x}\left(\left(a+bx^3\right)(aB+5Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+a(Ab-aB)\right)}{3a^2b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] (Sqrt[x]*(a*(A*b - a*B) + (5*A*b + a*B)*(a + b*x^3)*Hypergeometric2F1[1/6,
1, 7/6, -((b*x^3)/a)]))/(3*a^2*b*(a + b*x^3))
```

**IntegrateAlgebraic [A]** time = 0.57, size = 190, normalized size = 0.66

$$\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}}+\frac{(-aB-5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{18a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{11/6}b^{7/6}}-\frac{\sqrt{x}(aB-Ab)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] -1/3*((-(A*b) + a*B)*Sqrt[x])/(a*b*(a + b*x^3)) + ((5*A*b + a*B)*ArcTan[(b^
(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(11/6)*b^(7/6)) + ((-5*A*b - a*B)*ArcTan[(a^(
1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(18*a^(11/6)*b^(7/6)) + ((5*A
*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])
/(6*Sqrt[3]*a^(11/6)*b^(7/6))
```

**fricas [B]** time = 1.07, size = 2555, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B
^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^
5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^2*(-(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) +
(B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6))*a
^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3
*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)
)^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sqrt(x)*(-(B^6*a^6 + 30*A*B^
5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4
 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(5/6) + sqrt(3)*(B^6*a^6
+ 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^
2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(B^6*a^6 + 30*A*B^5*a^5*b +
 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*
A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 +
30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*
a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x - (B*a^3*b
+ 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^7))^(1/6))*a^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*
a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 +
 15625*A^6*b^6)/(a^11*b^7))^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sq
rt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*
b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))
^(5/6) - sqrt(3)*(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3
*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
```

```
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)) + (a*b^2*x^3 + a^2*
b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1
/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^
3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(
a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a
^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^
3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(
a^11*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A
^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*
a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a
*b + 25*A^2*b^2)*x - (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)) + 2*(a*b^2*x^3 + a^2
*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(
1/6)*log(a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3
*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a
^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) - 2*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6
 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B
^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(-a^2*
b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
+ 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/
6) + (B*a + 5*A*b)*sqrt(x)) - 12*(B*a - A*b)*sqrt(x))/(a*b^2*x^3 + a^2*b)
```

**giac** [A] time = 0.31, size = 302, normalized size = 1.04

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2}-\frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\left(bx^3+a\right)ab}+\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2}+\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2}+\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*
(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a
+ 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/
(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((a*b^
5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x)
)/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*a
rctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/9*((a*b
^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^2)
```

**maple** [A] time = 0.16, size = 387, normalized size = 1.34

$$\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9a^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18a^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18a^2}+\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^2}-\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18ab}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36ab}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36ab}+\frac{(Ab-Ba)\sqrt{x}}{3\left(b\,x^3+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x)
```

```
[Out] 1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+5/9/a^2*(a/b)^(1/6)*arctan(1/(a/b)^(1/6
)*x^(1/2))*A+1/9/b/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-5/36/a^2*3
^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A-1/36/b/
a*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+5/18
/a^2*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+1/18/b/a*(a/b)^(1/
6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B+5/36/a^2*3^(1/2)*(a/b)^(1/6)*ln(
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+1/36/b/a*3^(1/2)*(a/b)^(1/6)*l
n(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/18/a^2*(a/b)^(1/6)*arctan(
```

```
2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+1/18/b/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*
x^(1/2)+3^(1/2))*B
```

**maxima [A]** time = 1.20, size = 301, normalized size = 1.04

$$-\frac{(Ba-Ab)\sqrt{x}}{3\left(ab^2x^3+a^2b\right)}+\frac{\frac{\sqrt{3}(Ba+5\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(Ba+5\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*sqrt(x)/(a*b^2*x^3 + a^2*b) + 1/36*(sqrt(3)*(B*a + 5*A*b)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)
) - sqrt(3)*(B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)
*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2
*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6)
+ 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)
)) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b
^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*
b^(1/3))))/(a*b)
```

**mupad [B]** time = 2.92, size = 1922, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan(((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^
3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 + B
^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*
(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^
2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(
7/6)))/((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A
^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 +
B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))
*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6)) - (((2*x^(1/2)*(625*A^4*b^5 + B^4*
a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4)
 + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*
a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6))
))*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/
2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3
 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 - 1/2
)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*
b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((3^(1/2)*1
i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^
2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i
)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A
*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)))/((((
3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 1
50*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3
^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b
^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))
 - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4
*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) +
(2*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2
*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^
```

```
(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) +
 (atan(((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 7
5*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(
11/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4
*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2
))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^
3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(
18*(-a)^(11/6)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)
*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^
3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5
 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6
))))/(18*(-a)^(11/6)*b^(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x
^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 2
0*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A
^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*
b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*
1i)/(9*(-a)^(11/6)*b^(7/6)) + (x^(1/2)*(A*b - B*a))/(3*a*b*(a + b*x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

## 3.164 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=318

$$-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB}{}$$

**Rubi [A]** time = 0.69, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}}-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] -(7*A*b - a*B)/(3*a^2*b*Sqrt[x]) + (A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3))
+ ((7*A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(13/6
)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(18*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(
9*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*
Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*
b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} + \frac{\left(\frac{7Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{6a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\,\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right.}{9a^{13/6}b^{2/3}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\,\text{Subst}\left(\int \right.}{12\sqrt{}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\log\left(\sqrt[3]{a}-\right.}{12\sqrt{3}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} + \frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\tan}{18a}$$

**Mathematica** [C] time = 0.14, size = 70, normalized size = 0.22

$$\frac{2\left(x^3(aB-Ab)\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)-Abx^3\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)-5aA\right)}{5a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] (2*(-5*a*A - A*b*x^3*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (-(A*b
) + a*B)*x^3*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)]))/(5*a^3*Sqrt[x]
)
```

**IntegrateAlgebraic** [A] time = 0.61, size = 197, normalized size = 0.62

$$\frac{(aB-7Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} + \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{18a^{13/6}b^{5/6}} - \frac{(aB-7Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{13/6}b^{5/6}} + \frac{-6aA+aBx^3-7Abx^3}{3a^2\sqrt{x}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] (-6*a*A - 7*A*b*x^3 + a*B*x^3)/(3*a^2*Sqrt[x]*(a + b*x^3)) + ((-7*A*b + a*B
)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*
ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(18*a^(13/6)*b^(5/
6)) - ((-7*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) +
b^(1/3)*x)])/(6*Sqrt[3]*a^(13/6)*b^(5/6))
```

**fricas** [B] time = 0.97, size = 3798, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B
^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*
b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^1
6*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 +
12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^
5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 -
100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*
B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a
^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*
B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249
*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 -
6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 11
7649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*
b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 1
17649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^2
+ 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 16807*
A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 68
60*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^
6*b^6)/(a^13*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*
a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5
 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)) + 4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*
B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(B^5*a
^16*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7
+ 12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*
a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4
- 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*
A*B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6
*a^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^
7*B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 2824752
49*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5
- 6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 +
117649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^
2 + 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 1680
7*A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 -
6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*
A^6*b^6)/(a^13*b^5))^(1/6) + sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^
4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b
^5 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)) - 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(a^11*b^4*(-(B^6*a^6 - 42*A*B^5*a^5*b
 + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100
842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4*a
^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 168
```

07*A^5*b^5)*sqrt(x)) + 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b +
735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-a^11*b^4*(-(B^6*a^6 -
42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^
2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a
^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^
4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A
*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2
*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log((B^5*a^16
*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 1
2005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5
*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 1
00842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*B
^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^
6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B
^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*
A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 - 6
860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 117
649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A
^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^
6)/(a^13*b^5))^(2/3)) - (a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 7
35*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*
A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-(B^5*a^16*b^4 - 35*A*B
^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 12005*A^4*B*a^
12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*
B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*B^9*a^9*b + 22
05*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^6*b^4 - 42353
64*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B^3*a^3*b^7 +
259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*A^10*b^10)*x
- (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 - 6860*A^3*B^3*a
^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 117649*A^6*a^9*b
^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^
3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5)
)^(2/3)) + 12*((B*a - 7*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x)

**giac [A]** time = 0.47, size = 307, normalized size = 0.97

$$\frac{Bax^{3} - 7\,Abx^{3} - 6\,Aa}{3\,\left(bx^{\frac{7}{2}} + a\sqrt{x}\right)a^{2}} - \frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba - 7\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{3}b^{5}} + \frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba - 7\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^{3}b^{5}} + \frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba - 7\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{3}b^{5}} + \frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba - 7\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^{3}b^{5}} + \frac{\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba - 7\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^{3}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")

[Out] 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*A*a)/((b*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt
(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/
6) + x + (a/b)^(1/3))/(a^3*b^5) + 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^
5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5)
+ 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^5) + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5
)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^
5) + 1/9*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/
6))/(a^3*b^5)

**maple [A]** time = 0.16, size = 401, normalized size = 1.26

$$-\frac{Ab\,x^{\frac{5}{2}}}{3\left(b\,x^{3}+a\right)a^{2}} + \frac{B\,x^{\frac{5}{2}}}{3\left(b\,x^{3}+a\right)a} - \frac{7A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{6}}a^{2}} - \frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}a^{2}} - \frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}a^{2}} + \frac{7\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}Ab\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^{3}} - \frac{7\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}Ab\ln\left(-x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^{3}} + \frac{B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{6}}ab} + \frac{B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}ab} + \frac{B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}ab} - \frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^{2}} + \frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(-x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^{2}} - \frac{2A}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(5/2)/(b*x^3+a)*A*b+1/3/a*x^(5/2)/(b*x^3+a)*B-7/9/a^2*A/(a/b)^(1
/6)*arctan(1/(a/b)^(1/6)*x^(1/2))-7/36/a^3*A*b*3^(1/2)*(a/b)^(5/6)*ln(-x+3^
(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-7/18/a^2*A/(a/b)^(1/6)*arctan(2/(a/b
)^(1/6)*x^(1/2)-3^(1/2))+7/36/a^3*A*b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b
)^(1/6)*x^(1/2)+(a/b)^(1/3))-7/18/a^2*A/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^
(1/2)+3^(1/2))+1/9/a*B/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+1/36/a^2
*B*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))+1/18/
a*B/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-1/36/a^2*B*3^(1/2)*
(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/18/a*B/b/(a/b)^
(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2*A/a^2/x^(1/2)
```

**maxima** [A] time = 1.16, size = 240, normalized size = 0.75

$$\frac{(Ba-7\,Ab)x^3-6\,Aa}{3\left(a^2bx^{\frac{7}{2}}+a^3\sqrt{x}\right)}-\frac{(Ba-7\,Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\,\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\,\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\,\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{36\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 7*A*b)*x^3 - 6*A*a)/(a^2*b*x^(7/2) + a^3*sqrt(x)) - 1/36*(B*a -
 7*A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)
*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(
1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(
1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

**mupad** [B] time = 2.91, size = 1757, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan((((7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*
a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17
*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/
6)))*1i)/((-a)^(13/3)*b^(5/3)) + ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*
B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b
 - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5
))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/(((7*A*b - B*a)^2
*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*
a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*
b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5
/3)) - ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*
a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17
*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/
6))))/((-a)^(13/3)*b^(5/3))))*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) - (
(2*A)/a + (x^3*(7*A*b - B*a))/(3*a^2))/(a*x^(1/2) + b*x^(7/2)) + (atan(((((
3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^
6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 -
 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*
A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3
^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3
 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 -
1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A
```

```
*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^
(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6
- 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1
/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*
B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2
)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 17
01*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*
(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^
18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)
/2 - 1/2)*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)
/2 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*
B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*
b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^
5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3^(1/2)*1i)/
2 + 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B
^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b
 - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5
))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^(1/2)*1i)/2
 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^
2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b
- B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5)
)/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1
/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^
17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*
a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5
832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(
7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)
[Out] Timed out
```

## 3.165 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=96

$$-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}+\frac{aB-3Ab}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

**Rubi [A]** time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 325, 329, 275, 205}

$$-\frac{3Ab-aB}{3a^2bx^{3/2}}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] -(3*A*b - a*B)/(3*a^2*b*x^(3/2)) + (A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3))
- ((3*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)} + \frac{\left(\frac{9Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{3Ab-aB}{3a^2bx^{3/2}} + \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)} - \frac{(3Ab-aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2a^2}$$

$$= -\frac{3Ab-aB}{3a^2bx^{3/2}} + \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)} - \frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{a^2}$$

$$= -\frac{3Ab-aB}{3a^2bx^{3/2}} + \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)} - \frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3a^2}$$

$$= -\frac{3Ab-aB}{3a^2bx^{3/2}} + \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)} - \frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

**Mathematica [A]** time = 0.13, size = 79, normalized size = 0.82

$$\frac{\frac{(aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}\left(-2aA+aBx^3-3Abx^3\right)}{x^{3/2}\left(a+bx^3\right)}}{3a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] ((Sqrt[a]*(-2*a*A - 3*A*b*x^3 + a*B*x^3))/(x^(3/2)*(a + b*x^3)) + ((-3*A*b
+ a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[b])/(3*a^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 79, normalized size = 0.82

$$\frac{(aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}} + \frac{-2aA+aBx^3-3Abx^3}{3a^2x^{3/2}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] (-2*a*A - 3*A*b*x^3 + a*B*x^3)/(3*a^2*x^(3/2)*(a + b*x^3)) + ((-3*A*b + a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

**fricas [A]** time = 0.82, size = 232, normalized size = 2.42

$$\left[\frac{\left(\left(Bab-3Ab^2\right)x^5+\left(Ba^2-3Aab\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^3+2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\left(2Aa^2b-\left(Ba^2b-3Aab^2\right)x^3\right)\sqrt{x}}{6\left(a^3b^2x^5+a^4bx^2\right)}, \frac{\left(\left(Bab-3Ab^2\right)x^5+\left(Ba^2-3Aab\right)x^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-\left(2Aa^2b-\left(Ba^2b-3Aab^2\right)x^3\right)\sqrt{x}}{3\left(a^3b^2x^5+a^4bx^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^3
 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a
*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x^5
```

```
+ (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a^2*b
 - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]
```

**giac** [**A**] time = 0.16, size = 66, normalized size = 0.69

$$\frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a^2}+\frac{Bax^3-3\,Abx^3-2\,Aa}{3\left(bx^{\frac{9}{2}}+ax^{\frac{3}{2}}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x^
3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)
```

**maple** [**A**] time = 0.07, size = 93, normalized size = 0.97

$$-\frac{Ab\,x^{\frac{3}{2}}}{3\left(b\,x^3+a\right)a^2}+\frac{B\,x^{\frac{3}{2}}}{3\left(b\,x^3+a\right)a}-\frac{Ab\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;a}-\frac{2A}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(3/2)/(b*x^3+a)*A*b+1/3/a*x^(3/2)/(b*x^3+a)*B-1/a^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x^(3/2))*A*b+1/3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x^(3/2))*B-2/3*A/a^2/x^(3/2)
```

**maxima** [**A**] time = 1.19, size = 67, normalized size = 0.70

$$\frac{(Ba-3\,Ab)x^3-2\,Aa}{3\left(a^2bx^{\frac{9}{2}}+a^3x^{\frac{3}{2}}\right)}+\frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 3*A*b)*x^3 - 2*A*a)/(a^2*b*x^(9/2) + a^3*x^(3/2)) + 1/3*(B*a -
3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

**mupad** [**B**] time = 0.15, size = 139, normalized size = 1.45

$$-\frac{2\,A\,a^{3/2}\,\sqrt{b}-B\,a^2\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,b^2\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,\sqrt{a}\,b^{3/2}\,x^3-B\,a^{3/2}\,\sqrt{b}\,x^3+3\,A\,a\,b\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)-B\,a\,b\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\,a^{7/2}\,\sqrt{b}\,x^{3/2}+3\,a^{5/2}\,b^{3/2}\,x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x)
```

```
[Out] -(2*A*a^(3/2)*b^(1/2) - B*a^2*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A
*b^2*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A*a^(1/2)*b^(3/2)*x^3 - B*
a^(3/2)*b^(1/2)*x^3 + 3*A*a*b*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) - B*a
*b*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(3*a^(7/2)*b^(1/2)*x^(3/2) + 3*
a^(5/2)*b^(3/2)*x^(9/2))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
[Out] Timed out
```

## 3.166 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=318

$$\frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}+\frac{(11Ab-5a}{}$$

**Rubi [A]** time = 0.50, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}+\frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}}+\frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] -(11*A*b - 5*a*B)/(15*a^2*b*x^(5/2)) + (A*b - a*B)/(3*a*b*x^(5/2)*(a + b*x^
3)) + ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*
a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(18*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x]
)/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(17/6)*b^(1/6)) - ((11*
A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(1
2*Sqrt[3]*a^(17/6)*b^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{\left(\frac{11Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{6a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right.}{9a^{17/6}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\,\text{Subst}\,(}{}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\log\left(\sqrt[3]{\right.}{12\sqrt{}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)}{1}$$

**Mathematica [C]** time = 0.11, size = 74, normalized size = 0.23

$$\frac{5x^3(5aB-11Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+\frac{a\left(-6aA+5aBx^3-11Abx^3\right)}{a+bx^3}}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] ((a*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(a + b*x^3) + 5*(-11*A*b + 5*a*B)*x^
3*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)])/(15*a^3*x^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.60, size = 200, normalized size = 0.63

$$\frac{(5aB-11Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}}+\frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{18a^{17/6}\sqrt[6]{b}}+\frac{(5aB-11Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{6\sqrt{3}a^{17/6}\sqrt[6]{b}}+\frac{-6aA+5aBx^3-11Abx^3}{15a^2x^{5/2}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] (-6*a*A - 11*A*b*x^3 + 5*a*B*x^3)/(15*a^2*x^(5/2)*(a + b*x^3)) + ((-11*A*b
+ 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b
 - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(18*a^(1
7/6)*b^(1/6)) + ((-11*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x]
)/(a^(1/3) + b^(1/3)*x)])/(6*Sqrt[3]*a^(17/6)*b^(1/6))
```

**fricas [B]** time = 1.09, size = 2584, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/180*(20*sqrt(3)*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^
5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a
^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4
*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*
B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 12
1*A^2*b^2)*x + (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B
^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*
B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*
b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500
*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(
-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6
*b^6)/(a^17*b))^(5/6) - sqrt(3)*(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11343
75*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 48
31530*A^5*B*a*b^5 + 1771561*A^6*b^6))/(15625*B^6*a^6 - 206250*A*B^5*a^5*b +
 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^
4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b*x^6 + a^3*x
^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 33275
00*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 177156
1*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6
- 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 +
5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^
(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)
*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 -
3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1
771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5
*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^
2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*
(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b
+ 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b
^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + sqrt(3)*(1562
5*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*
a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6))
/(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6
*b^6)) - 5*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11
34375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 -
 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^6*(-(15625*B^
6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*
b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^1
7*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x + (5*B*a^4 - 11*A*
a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*
b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b
^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 5*(a^2*b*x^6 + a^3*x^3)*(-(15625*B
^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3
*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^
17*b))^(1/6)*log(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^
4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5
*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 1
21*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*
B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4
*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 10
*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*
B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A
```

```
^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^3*(-(15625*B^6*a^6 - 20
6250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490
375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6
) - (5*B*a - 11*A*b)*sqrt(x)) - 10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 -
 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5
490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(
1/6)*log(-a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b
^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^
5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 12*((5*B
*a - 11*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^6 + a^3*x^3)
```

**giac** [A] time = 0.22, size = 313, normalized size = 0.98

$$\frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^3b}-\frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^3b}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\left(bx^3+a\right)a^2}+\frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18a^3b}+\frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18a^3b}+\frac{\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9a^3b}-\frac{2A}{5a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B
*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*(5*(
a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/9*(
5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^
3*b) - 2/5*A/(a^2*x^(5/2))
```

**maple** [A] time = 0.16, size = 395, normalized size = 1.24

$$-\frac{Ab\sqrt{x}}{3\left(bx^3+a\right)a^2}+\frac{B\sqrt{x}}{3\left(bx^3+a\right)a}-\frac{11\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9a^3}-\frac{11\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18a^3}-\frac{11\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18a^3}-\frac{11\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^3}+\frac{11\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^3}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9a^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18a^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18a^2}+\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^2}-\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36a^2}-\frac{2A}{5a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(1/2)/(b*x^3+a)*A*b+1/3/a*x^(1/2)/(b*x^3+a)*B-11/9/a^3*A*b*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(-
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan
(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan(2/(a/
b)^(1/6)*x^(1/2)+3^(1/2))+5/9/a^2*B*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2
))-5/36/a^2*B*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(
1/3))+5/18/a^2*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+5/36/a^2
*B*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+5/18/a
^2*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2/5/a^2*A/x^(5/2)
```

**maxima** [A] time = 1.19, size = 312, normalized size = 0.98

$$\frac{(5Ba-11Ab)x^3-6Aa}{15\left(a^2bx^{\frac{11}{2}}+a^3x^{\frac{5}{2}}\right)}+\frac{\frac{\sqrt{3}(5Ba-11Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(5Ba-11Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(5Bab^{\frac{1}{3}}-11Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(5Ba^{\frac{4}{3}}b^{\frac{1}{3}}-11Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(5Ba^{\frac{4}{3}}b^{\frac{1}{3}}-11Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*((5*B*a - 11*A*b)*x^3 - 6*A*a)/(a^2*b*x^(11/2) + a^3*x^(5/2)) + 1/36*(
sqrt(3)*(5*B*a - 11*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 11*A*b)*log(-sqrt(3)*a^(1/6)*
b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3)
```

```
 - 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3)*b^(4/3)
)*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)
))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3
)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/
3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

**mupad [B]** time = 2.96, size = 2080, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x)
```

```
[Out] - ((2*A)/(5*a) + (x^3*(11*A*b - 5*B*a))/(15*a^2))/(a*x^(5/2) + b*x^(11/2))
- (atan((((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*
A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11
*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^
2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b -
5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + ((x^(1/2)*(21346578*A^4*a^10*b^9 + 91
1250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38
811960*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500
*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a
)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)))/(((x^(1/2
)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7
 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11*A*b - 5*B*a)*(34
930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 4763
2860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^
(17/6)*b^(1/6)) - ((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 +
26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^
8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 2165
1300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(
11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6))))*(11*A*b - 5*B*a)*1i)/(9*(-a)^(1
7/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*
(-a)^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*
(-a)^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a
)^(17/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(349
30764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632
860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(
17/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)
*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)
^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
```

```
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)
^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(1
7/6)*b^(1/6)) - (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 9
11250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 3
8811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493076
4*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*
A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6
)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)*b^(
1/6))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.167 $\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=104

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}(3aB+Ab)}{12ab^2\left(a+bx^3\right)}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 288, 329, 275, 205}

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}(3aB+Ab)}{12ab^2\left(a+bx^3\right)}+\frac{x^{9/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) - ((A*b + 3*a*B)*x^(3/2))/(12*a
*b^2*(a + b*x^3)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{3Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8ab^2}\\
&= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4ab^2}\\
&= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12ab^2}\\
&= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}
\end{aligned}
$$

**Mathematica** [A] time = 0.19, size = 93, normalized size = 0.89

$$
\frac{\frac{\sqrt{a}\sqrt{b}\,x^{3/2}\left(-3a^2B-ab\left(A+5Bx^3\right)+Ab^2x^3\right)}{\left(a+bx^3\right)^2}+(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - a*b*(A + 5*B*x^3)))/(a +
b*x^3)^2 + (A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(
5/2))
```

**IntegrateAlgebraic** [A] time = 0.17, size = 92, normalized size = 0.88

$$
\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}\left(3a^2B+aAb+5abBx^3-Ab^2x^3\right)}{12ab^2\left(a+bx^3\right)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/12*(x^(3/2)*(a*A*b + 3*a^2*B - A*b^2*x^3 + 5*a*b*B*x^3))/(a*b^2*(a + b*x
^3)^2) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(5
/2))
```

**fricas** [A] time = 0.77, size = 314, normalized size = 3.02

$$
\left[-\frac{\left(\left(3Bab^2+Ab^3\right)x^6+3Ba^3+Aa^2b+2\left(3Ba^2b+Aab^2\right)x^3\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\left(\left(5Ba^2b^2-Aab^3\right)x^4+\left(3Ba^3b+Aa^2b^2\right)x\right)\sqrt{x}}{24\left(a^2b^5x^6+2a^3b^4x^3+a^4b^3\right)},\ \frac{\left(\left(3Bab^2+Ab^3\right)x^6+3Ba^3+Aa^2b+2\left(3Ba^2b+Aab^2\right)x^3\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-\left(\left(5Ba^2b^2-Aab^3\right)x^4+\left(3Ba^3b+Aa^2b^2\right)x\right)\sqrt{x}}{12\left(a^2b^5x^6+2a^3b^4x^3+a^4b^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2
*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2*b^
5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a
^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

**giac** [A] time = 0.21, size = 84, normalized size = 0.81

$$\frac{(3\,Ba + Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,ab^2} - \frac{5\,Babx^{\frac{9}{2}} - Ab^2x^{\frac{9}{2}} + 3\,Ba^2x^{\frac{3}{2}} + Aabx^{\frac{3}{2}}}{12\,\left(bx^3 + a\right)^2 ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/12*(5*
B*a*b*x^(9/2) - A*b^2*x^(9/2) + 3*B*a^2*x^(3/2) + A*a*b*x^(3/2))/((b*x^3 +
a)^2*a*b^2)
```

**maple** [A] time = 0.07, size = 96, normalized size = 0.92

$$\frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\sqrt{ab}\;ab} + \frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^2} + \frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab} - \frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{\left(b\,x^3 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2/3*(1/8*(A*b-5*B*a)/a/b*x^(9/2)-1/8*(A*b+3*B*a)/b^2*x^(3/2))/(b*x^3+a)^2+1
/12/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A+1/4/b^2/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

**maxima** [A] time = 1.43, size = 96, normalized size = 0.92

$$-\frac{\left(5\,Bab - Ab^2\right)x^{\frac{9}{2}} + \left(3\,Ba^2 + Aab\right)x^{\frac{3}{2}}}{12\,\left(ab^4x^6 + 2\,a^2b^3x^3 + a^3b^2\right)} + \frac{(3\,Ba + Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*((5*B*a*b - A*b^2)*x^(9/2) + (3*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^6 +
2*a^2*b^3*x^3 + a^3*b^2) + 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(
sqrt(a*b)*a*b^2)
```

**mupad** [B] time = 2.76, size = 133, normalized size = 1.28

$$\frac{\mathrm{atan}\left(\frac{9\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2+6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{\sqrt{a}\,\left(9\,A\,b^2+27\,B\,a\,b\right)\,\left(A\,b+3\,B\,a\right)}\right)\,\left(A\,b+3\,B\,a\right)}{12\,a^{3/2}\,b^{5/2}} - \frac{\frac{x^{3/2}\,\left(A\,b+3\,B\,a\right)}{12\,b^2} - \frac{x^{9/2}\,\left(A\,b-5\,B\,a\right)}{12\,a\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((9*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 + 6*A*B*a*b))/(a^(1/2)*(9*A*b
^2 + 27*B*a*b)*(A*b + 3*B*a)))*(A*b + 3*B*a))/(12*a^(3/2)*b^(5/2)) - ((x^(3
/2)*(A*b + 3*B*a))/(12*b^2) - (x^(9/2)*(A*b - 5*B*a))/(12*a*b))/(a^2 + b^2*
x^6 + 2*a*b*x^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

## 3.168 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=327

$$-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}-\frac{(7aB+5\ldots}{}$$

**Rubi [A]** time = 0.49, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 288, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}-\frac{(7aB+5Ab)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}}-\frac{\sqrt{x}(7aB+5Ab)}{36ab^2\left(a+bx^3\right)}+\frac{x^{7/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) - ((5*A*b + 7*a*B)*Sqrt[x])/(36
*a*b^2*(a + b*x^3)) - ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) - ((5*A*b + 7*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11
/6)*b^(13/6)) + ((5*A*b + 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} + \frac{\left(\frac{5Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \right.}{108a^{11/6}b^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\,\text{Subs}}{}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\log\left(\right.}{144}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} - \frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB}{}$$

**Mathematica [C]** time = 0.11, size = 92, normalized size = 0.28

$$\frac{\sqrt{x}\left(a\left(-7a^2B-ab\left(5A+13Bx^3\right)+Ab^2x^3\right)+\left(a+bx^3\right)^2(7aB+5Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{36a^2b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (Sqrt[x]*(a*(-7*a^2*B + A*b^2*x^3 - a*b*(5*A + 13*B*x^3)) + (5*A*b + 7*a*B)
*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^2*b^2*(
a + b*x^3)^2)
```

**IntegrateAlgebraic [A]** time = 0.78, size = 213, normalized size = 0.65

$$\frac{(7aB+5Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} + \frac{(-7aB-5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{216a^{11/6}b^{13/6}} + \frac{(7aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{11/6}b^{13/6}} - \frac{\sqrt{x}\left(7a^2B+5aAb+13abBx^3-Ab^2x^3\right)}{36ab^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/36*(Sqrt[x]*(5*a*A*b + 7*a^2*B - A*b^2*x^3 + 13*a*b*B*x^3))/(a*b^2*(a +
b*x^3)^2) + ((5*A*b + 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/
6)*b^(13/6)) + ((-5*A*b - 7*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1
/6)*Sqrt[x])])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(72*Sqrt[3]*a^(11/6)*b^(13
/6))
```

**fricas [B]** time = 0.79, size = 2714, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(4*sqrt(3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 +
504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4593
75*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
+ 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
+ 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70
*A*B*a*b + 25*A^2*b^2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/6))*a^9*b^11*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*
a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*
b^5 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^
9*b^12)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 + 504210*A*B
^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2
*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*
B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^
2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^4*x^6 + 2
*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2
*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5
*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4
*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^
6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B
*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 90
0375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 13
1250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6))*a^9*b^11*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^9*b^12)*sqrt(x)*(-(117649*B^6*a
^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 +
 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^
(5/6) - sqrt(3)*(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b
^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 +
 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6
 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4
59375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1
/6)*log(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^
2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5
*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a
^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)) - (a*b^4*x
^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 9003
75*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 1312
50*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(a^4*b^4*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B*a^3*b^2 + 5*A*a^
2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
```

```
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)/(a^11*b^13))^(1/6)) + 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b
^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^13))^(1/6)*log(a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
 + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)
*sqrt(x)) - 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504
210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*
A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*lo
g(-a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2
+ 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 12*((13*B*a*b
- A*b^2)*x^3 + 7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3
*b^2)
```

**giac** [A] time = 0.23, size = 328, normalized size = 1.00

$$\frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}-\frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}+\frac{\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3}+\frac{\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3}+\frac{\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^2b^3}-\frac{13\,Babx^{\frac{7}{2}}-Ab^2x^{\frac{7}{2}}+7\,Ba^2\sqrt{x}+5\,Aab\sqrt{x}}{36\left(bx^3+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1/6
)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(
1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(
(sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a*b^5)
^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))
/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)
*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A*b^2*x^(
7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)
```

**maple** [A] time = 0.16, size = 416, normalized size = 1.27

$$\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108a^2b}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216a^2b}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216a^2b}-\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^2b}+\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^2b}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108a\,b^2}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216a\,b^2}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216a\,b^2}-\frac{7\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a\,b^2}+\frac{7\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a\,b^2}+\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^2}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)^
2+5/108/b/a^2*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+7/108/b^2/a*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x
-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-7/432/b^2/a*3^(1/2)*(a/b)^(1/6)
*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*ar
ctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2/(a/b
)^(1/6)*x^(1/2)-3^(1/2))*B+5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/
b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+7/432/b^2/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2
)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*arctan(2/(a/b)
^(1/6)*x^(1/2)+3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1
/2)+3^(1/2))*B
```

**maxima** [A] time = 1.40, size = 341, normalized size = 1.04

$$-\frac{\left(13\,Bab-Ab^2\right)x^{\frac{7}{2}}+\left(7\,Ba^2+5\,Aab\right)\sqrt{x}}{36\left(ab^4x^6+2\,a^2b^3x^3+a^3b^2\right)}+\frac{\frac{\sqrt{3}\left(7\,Ba+5\,Ab\right)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\left(7\,Ba+5\,Ab\right)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(7\,Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(7\,Ba^{\frac{4}{3}}b^{\frac{1}{3}}+5\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{432\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*((13*B*a*b - A*b^2)*x^(7/2) + (7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6
 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/432*(sqrt(3)*(7*B*a + 5*A*b)*log(sqrt(3)*a^
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(7
*B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/s
qrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*B*a^(4
/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1
/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(
7*B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
 - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3
))))/(a*b^2)
```

**mupad [B]** time = 2.98, size = 1944, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 52
5*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 240
1*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(2
79936*a^4*b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) - ((((5*A*b
+ 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(
279936*(-a)^(23/6)*b^(19/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*
A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(
5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(((((5*A*b + 7*B*a)*(125*A^3
*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6
)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 +
 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(
216*(-a)^(11/6)*b^(13/6)) + ((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 +
 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) + (x^(1/
2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b +
3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(1
3/6))))*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) - ((x^(1/2)*(5*A*b +
 7*B*a))/(36*b^2) - (x^(7/2)*(A*b - 13*B*a))/(36*a*b))/(a^2 + b^2*x^6 + 2*a
*b*x^3) + (atan(((((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*
a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*
a^4*b^3) - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a
^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*
A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((x^(
1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b
+ 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*
B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(2799
36*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(
(((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^
2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(
1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^
2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(21
6*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 +
2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))
/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 +
343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19
/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*
(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) + (atan(((((3^(1/2)*1i)/2 +
1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*
B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(
5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b
^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^
```

```
(13/6)) + (((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7
350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3
) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 73
5*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7
*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/((((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(6
25*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*
A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(1
25*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)
^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)
*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2
+ 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2
 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*
A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11
/6)*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)
*b^(13/6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)
[Out] Timed out
```

## 3.169 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=327

$$\frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)}{21}$$

**Rubi [A]** time = 0.60, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}+\frac{(5aB+7Ab)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{13/6}b^{11/6}}+\frac{(5aB+7Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}+\frac{x^{5/2}(5aB+7Ab)}{36a^2b\left(a+bx^3\right)}+\frac{x^{5/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*x^(5/2))/(36
*a^2*b*(a + b*x^3)) - ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13
/6)*b^(11/6)) - ((7*A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13/6)*b^(11/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{7Ab}{2}+\frac{5aB}{2}\right)\int\frac{x^{3/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\int\frac{x^{3/2}}{a+bx^3}\,dx}{72a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\,\text{Subst}\left(\int\frac{x^4}{a+bx^6}\,dx,x,\sqrt{x}\right)}{36a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\,\text{Subst}\left(\int\frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right.}{108a^{13/6}b^{5/3}}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\,\text{Subst}\left(\right.}{14}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\log\left(\sqrt[3]{a}\right.}{144\sqrt{}}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}-\frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\,\text{t}}{216}$$

**Mathematica [C]** time = 0.06, size = 62, normalized size = 0.19

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},3;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)] + (A*b - a*B)
*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3)/a)]))/(5*a^3*b)
```

**IntegrateAlgebraic [A]** time = 0.74, size = 212, normalized size = 0.65

$$\frac{(5aB+7Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}+\frac{(-5aB-7Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{216a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{13/6}b^{11/6}}-\frac{x^{5/2}\left(a^2B-13aAb-5abBx^3-7Ab^2x^3\right)}{36a^2b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/36*(x^(5/2)*(-13*a*A*b + a^2*B - 7*A*b^2*x^3 - 5*a*b*B*x^3))/(a^2*b*(a +
 b*x^3)^2) + ((7*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13
/6)*b^(11/6)) + ((-7*A*b - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(
1/6)*Sqrt[x])])/(216*a^(13/6)*b^(11/6)) - ((7*A*b + 5*a*B)*ArcTanh[(Sqrt[3]
*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(72*Sqrt[3]*a^(13/6)*b^(1
1/6))
```

**fricas [B]** time = 0.81, size = 3951, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/432*(4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9003
75*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*sqrt((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 6
1250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 +
16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 13
6718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^
3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*
A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8
+ 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 1
31250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 +
 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(
-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*
B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)
/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^7
*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*b^5
+ 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 13125
0*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^
4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) + s
qrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^
6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A
^6*b^6)) + 4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6
 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9
00375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10
 + 61250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^1
3 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10
+ 136718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^
7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431
250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*
b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7
 + 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^
10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^1
3)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*
A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*
b^6)/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5
*a^7*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*
b^5 + 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
 - sqrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8
57500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 11764
9*A^6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
```

```
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 1176
49*A^6*b^6)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
*log(a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^
2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 +
117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61
250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5
*b^5)*sqrt(x)) + 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900
375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/
6)*log(-a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5
 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b +
 61250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*
A^5*b^5)*sqrt(x)) - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6
+ 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90
0375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1
/6)*log((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 61250*A^2*B^3*a^14*b^1
1 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 + 16807*A^5*a^11*b^14)*
sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85
7500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649
*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 136718750*A*B^9*a^9*b +
 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^3 + 7878281250*A^4*B^
6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*A^6*B^4*a^4*b^6 + 123
53145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8 + 2017680350*A^9*B*a*
b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 131250*A*B^5*a^14*b^8
+ 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 + 900375*A^4*B^2*a^11*
b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(-(15625*B^6*a^6 + 131
250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*
A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(2/3))
+ (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5
*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b
^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(-(3125*B^5
*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 61250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2
*a^13*b^12 + 60025*A^4*B*a^12*b^13 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*
B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*
b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b
^11))^(5/6) + (9765625*B^10*a^10 + 136718750*A*B^9*a^9*b + 861328125*A^2*B^
8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235
512500*A^5*B^5*a^5*b^5 + 15441431250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*
a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A
^10*b^10)*x - (15625*B^6*a^15*b^7 + 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*
a^13*b^9 + 857500*A^3*B^3*a^12*b^10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5
*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b +
 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 +
 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(2/3)) - 12*((5*B*a*b +
7*A*b^2)*x^5 - (B*a^2 - 13*A*a*b)*x^2)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^
3 + a^4*b)
```

**giac** [**A**] time = 0.37, size = 328, normalized size = 1.00

$$\frac{5 Babx^{\frac{11}{2}} + 7 Ab^2x^{\frac{11}{2}} - Ba^2x^{\frac{5}{2}} + 13 Aabx^{\frac{5}{2}}}{36 \left(bx^3 + a\right)^2 a^2 b} - \frac{\sqrt{3}\left(5 \left(ab^5\right)^{\frac{5}{6}} Ba + 7 \left(ab^5\right)^{\frac{5}{6}} Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^3 b^6} + \frac{\sqrt{3}\left(5 \left(ab^5\right)^{\frac{5}{6}} Ba + 7 \left(ab^5\right)^{\frac{5}{6}} Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^3 b^6} + \frac{\left(5 \left(ab^5\right)^{\frac{5}{6}} Ba + 7 \left(ab^5\right)^{\frac{5}{6}} Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216 a^3 b^6} + \frac{\left(5 \left(ab^5\right)^{\frac{5}{6}} Ba + 7 \left(ab^5\right)^{\frac{5}{6}} Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216 a^3 b^6} + \frac{\left(5 \left(ab^5\right)^{\frac{5}{6}} Ba + 7 \left(ab^5\right)^{\frac{5}{6}} Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108 a^3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/36*(5*B*a*b*x^(11/2) + 7*A*b^2*x^(11/2) - B*a^2*x^(5/2) + 13*A*a*b*x^(5/2
))/((b*x^3 + a)^2*a^2*b) - 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(
```

```
5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/
432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/216*(5*(a*b^5)^(5/6)*B*a + 7
*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(
a^3*b^6) + 1/216*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*arctan(-(sqrt(
3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^6) + 1/108*(5*(a*b^5)^(5/6)
*B*a + 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b^6)
```

**maple** [A] time = 0.16, size = 411, normalized size = 1.26

$$\frac{7A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\left(\frac{a}{b}\right)^{\frac{1}{6}}a^{2}b}+\frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216\left(\frac{a}{b}\right)^{\frac{1}{6}}a^{2}b}+\frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216\left(\frac{a}{b}\right)^{\frac{1}{6}}a^{2}b}+\frac{7\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,A\ln\left(x-\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^{3}}-\frac{7\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}A\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^{3}}+\frac{5B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\left(\frac{a}{b}\right)^{\frac{1}{6}}a\,b^{2}}+\frac{5B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216\left(\frac{a}{b}\right)^{\frac{1}{6}}a\,b^{2}}+\frac{5B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216\left(\frac{a}{b}\right)^{\frac{1}{6}}a\,b^{2}}+\frac{5\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,B\ln\left(x-\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^{2}b}-\frac{5\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}B\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^{2}b}+\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^{2}}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{\left(b\,x^{3}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2*(1/72*(7*A*b+5*B*a)/a^2*x^(11/2)+1/72*(13*A*b-B*a)/a/b*x^(5/2))/(b*x^3+a)
^2+7/108/a^2/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+5/108/a/b^2/(a/b
)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B+7/432/a^3*(a/b)^(5/6)*3^(1/2)*ln(x-
3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+5/432/a^2/b*(a/b)^(5/6)*3^(1/2)*
ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+7/216/a^2/b/(a/b)^(1/6)*arc
tan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+5/216/a/b^2/(a/b)^(1/6)*arctan(2/(a/b)
^(1/6)*x^(1/2)-3^(1/2))*B-7/432/a^3*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^
(1/6)*x^(1/2)+(a/b)^(1/3))*A-5/432/a^2/b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(
a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+7/216/a^2/b/(a/b)^(1/6)*arctan(2/(a/b)^(1
/6)*x^(1/2)+3^(1/2))*A+5/216/a/b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)
+3^(1/2))*B
```

**maxima** [A] time = 1.23, size = 271, normalized size = 0.83

$$\frac{\left(5\,Bab+7\,Ab^{2}\right)x^{\frac{11}{2}}-\left(Ba^{2}-13\,Aab\right)x^{\frac{5}{2}}}{36\left(a^{2}b^{3}x^{6}+2\,a^{3}b^{2}x^{3}+a^{4}b\right)}-\frac{(5\,Ba+7\,Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\,\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\,\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\,\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*((5*B*a*b + 7*A*b^2)*x^(11/2) - (B*a^2 - 13*A*a*b)*x^(5/2))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) - 1/432*(5*B*a + 7*A*b)*(sqrt(3)*log(sqrt(3)*a^
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*lo
g(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6))
 - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1
/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
- 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)))
 - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^
(1/3))))/(a^2*b)
```

**mupad** [B] time = 2.89, size = 1672, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(11/2)*(7*A*b + 5*B*a))/(36*a^2) + (x^(5/2)*(13*A*b - B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) + (atan(((((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^
2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B*a)*(49*A^2*b^
4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*A*b + 5
*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)) - (((343*A^3*b^3 + 125*B^3*a^3 + 5
25*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b + 5*B*a)*(49
```

```
*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*
A*b + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((343*A^3*b^3 + 125*B^3*
a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B
*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6
)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3)) + (((343*A^3*b^3 + 125*
B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b +
 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(
11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3))))*(7*A*b + 5*B*a)*1
i)/(108*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5
*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1
296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25
*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^
(13/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^
3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*
((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*
B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((
((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 52
5*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/
2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)
^(19/6)*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^
2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2
*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*
A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46
656*(-a)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*1i)/(108
*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2
*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3
) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^
2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*
b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125
*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/
2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3
))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((3^(1/
2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2
*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A
*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)
*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*
b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^
2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4
 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46656*(-a
)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*1i)/(108*(-a)^(
13/6)*b^(11/6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

## 3.170 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=104

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}+\frac{x^{3/2}(aB+3Ab)}{12a^2b\left(a+bx^3\right)}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

**Rubi** [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 290, 329, 275, 205}

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}+\frac{x^{3/2}(aB+3Ab)}{12a^2b\left(a+bx^3\right)}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*x^(3/2))/(12*a
^2*b*(a + b*x^3)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
```

```
, -(n*(p + 1))]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{9Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.11, size = 94, normalized size = 0.90

$$
\frac{\frac{\sqrt{a}\sqrt{b}\,x^{3/2}\left(-a^2B+ab\left(5A+Bx^3\right)+3Ab^2x^3\right)}{\left(a+bx^3\right)^2}+(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-(a^2*B) + 3*A*b^2*x^3 + a*b*(5*A + B*x^3)))/(a
+ b*x^3)^2 + (3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b
^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 92, normalized size = 0.88

$$
\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}-\frac{x^{3/2}\left(a^2B-5aAb-abBx^3-3Ab^2x^3\right)}{12a^2b\left(a+bx^3\right)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/12*(x^(3/2)*(-5*a*A*b + a^2*B - 3*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a + b*
x^3)^2) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b^(
3/2))
```

**fricas [A]** time = 0.57, size = 313, normalized size = 3.01

$$
\left[-\frac{\left(\left(Bab^2+3\,Ab^3\right)x^6+Ba^3+3\,Aa^2b+2\left(Ba^2b+3\,Aab^2\right)x^3\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\left(\left(Ba^2b^2+3\,Aab^3\right)x^4-\left(Ba^3b-5\,Aa^2b^2\right)x\right)\sqrt{x}}{24\left(a^3b^4x^6+2\,a^4b^3x^3+a^5b^2\right)},\ \frac{\left(\left(Bab^2+3\,Ab^3\right)x^6+Ba^3+3\,Aa^2b+2\left(Ba^2b+3\,Aab^2\right)x^3\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)+\left(\left(Ba^2b^2+3\,Aab^3\right)x^4-\left(Ba^3b-5\,Aa^2b^2\right)x\right)\sqrt{x}}{12\left(a^3b^4x^6+2\,a^4b^3x^3+a^5b^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2
*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 +
3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a
^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

**giac** [A] time = 0.18, size = 84, normalized size = 0.81

$$\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,a^2b}+\frac{Babx^{\frac{9}{2}}+3\,Ab^2x^{\frac{9}{2}}-Ba^2x^{\frac{3}{2}}+5\,Aabx^{\frac{3}{2}}}{12\left(bx^3+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/12*(B*
a*b*x^(9/2) + 3*A*b^2*x^(9/2) - B*a^2*x^(3/2) + 5*A*a*b*x^(3/2))/((b*x^3 +
a)^2*a^2*b)
```

**maple** [A] time = 0.07, size = 97, normalized size = 0.93

$$\frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^2}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\sqrt{ab}\;ab}+\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x)
```

```
[Out] 2/3*(1/8*(3*A*b+B*a)/a^2*x^(9/2)+1/8*(5*A*b-B*a)/a/b*x^(3/2))/(b*x^3+a)^2+1
/4/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A+1/12/a/b/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

**maxima** [A] time = 1.27, size = 96, normalized size = 0.92

$$\frac{\left(Bab+3\,Ab^2\right)x^{\frac{9}{2}}-\left(Ba^2-5\,Aab\right)x^{\frac{3}{2}}}{12\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*((B*a*b + 3*A*b^2)*x^(9/2) - (B*a^2 - 5*A*a*b)*x^(3/2))/(a^2*b^3*x^6 +
 2*a^3*b^2*x^3 + a^4*b) + 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(s
qrt(a*b)*a^2*b)
```

**mupad** [B] time = 2.71, size = 136, normalized size = 1.31

$$\frac{\frac{x^{9/2}\,(3\,A\,b+B\,a)}{12\,a^2}+\frac{x^{3/2}\,(5\,A\,b-B\,a)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\mathrm{atan}\left(\frac{b^{3/2}\,x^{3/2}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{\sqrt{a}\,(3\,A\,b+B\,a)\,\left(3\,A\,b^3+B\,a\,b^2\right)}\right)(3\,A\,b+B\,a)}{12\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(9/2)*(3*A*b + B*a))/(12*a^2) + (x^(3/2)*(5*A*b - B*a))/(12*a*b))/(a^2
+ b^2*x^6 + 2*a*b*x^3) + (atan((b^(3/2)*x^(3/2)*(9*A^2*b^3 + B^2*a^2*b + 6*
A*B*a*b^2))/(a^(1/2)*(3*A*b + B*a)*(3*A*b^3 + B*a*b^2)))*(3*A*b + B*a))/(12
*a^(5/2)*b^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

## 3.171 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=321

$$-\frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}-\frac{5(aB+}{}$$

**Rubi** [A] time = 0.52, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}-\frac{5(aB+11Ab)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}}+\frac{\sqrt{x}(aB+11Ab)}{36a^2b\left(a+bx^3\right)}+\frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*Sqrt[x])/(36*
a^2*b*(a + b*x^3)) - (5*(11*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17
/6)*b^(7/6)) + (5*(11*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)^2}\,dx}{12ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{(5(11Ab+aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{(5(11Ab+aB))\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{36a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{(5(11Ab+aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,\ldots}{108a^{17/6}b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}}-\frac{(5(11Ab+aB))\,\text{Su}\ldots}{\ldots}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}}-\frac{5(11Ab+aB)\log\ldots}{1\ldots}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}-\frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}}+\frac{5(11Ab+\ldots}{\ldots}$$

**Mathematica [C]** time = 0.05, size = 91, normalized size = 0.28

$$\frac{\sqrt{x}\left(a\left(-5a^2B+ab\left(17A+Bx^3\right)+11Ab^2x^3\right)+5\left(a+bx^3\right)^2(aB+11Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{36a^3b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] (Sqrt[x]*(a*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3)) + 5*(11*A*b + a*
B)*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^3*b*(
a + b*x^3)^2)
```

**IntegrateAlgebraic [A]** time = 0.79, size = 210, normalized size = 0.65

$$\frac{5(aB+11Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}}-\frac{5(aB+11Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{216a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{17/6}b^{7/6}}-\frac{\sqrt{x}\left(5a^2B-17aAb-abBx^3-11Ab^2x^3\right)}{36a^2b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] -1/36*(Sqrt[x]*(-17*a*A*b + 5*a^2*B - 11*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a
+ b*x^3)^2) + (5*(11*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(
17/6)*b^(7/6)) - (5*(11*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^
(1/6)*Sqrt[x])])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTanh[(Sqrt[3
]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(72*Sqrt[3]*a^(17/6)*b^(
7/6))
```

**fricas [B]** time = 1.00, size = 2674, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*(20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B
^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^
2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2
+ 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 177
1561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B*a*b + 121*A^2*b^2)*x +
(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*(-(B^6*a^6 + 66*A*B^5*a^
5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4
 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - 2*sqrt(3)*(B*a
^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) + sqrt(3)*(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^
2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5
*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3
*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b
 + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B
*a*b + 121*A^2*b^2)*x - (B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*
(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3
+ 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7)
)^(5/6) - 2*sqrt(3)*(B*a^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - sqrt(3)*
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 +
66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*
B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 5*(a^2*b^3*x^6 + 2*a
^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26
620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561
*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 18
15*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 96630
6*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*a^2 + 22*A*B*a
*b + 121*A^2*b^2)*x + 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A
*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*
a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)) - 5*(a^2
*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^
4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a
*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*
a^2 + 22*A*B*a*b + 121*A^2*b^2)*x - 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(
B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2
19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(
1/6)) + 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
```

```
966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*a^3*b*(-(B^6
*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2196
15*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6
) + 5*(B*a + 11*A*b)*sqrt(x)) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^
(1/6)*log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 266
20*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b^7))^(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) + 12*((B*a*b + 11*A*
b^2)*x^3 - 5*B*a^2 + 17*A*a*b)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*
b)
```

**giac** [**A**] time = 0.24, size = 322, normalized size = 1.00

$$\frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2}-\frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2}+\frac{5\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2}+\frac{5\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2}+\frac{5\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b^2}+\frac{Babx^{\frac{7}{2}}+11\,Ab^2x^{\frac{7}{2}}-5\,Ba^2\sqrt{x}+17\,Aab\sqrt{x}}{36\left(bx^3+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 5/432*sqrt(3)*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^2) - 5/432*sqrt(3)*((a*b^5)^(1/6)*B
*a + 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b^2) + 5/216*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^2) + 5/216*((a*b^5)^(1/6
)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/
b)^(1/6))/(a^3*b^2) + 5/108*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arct
an(sqrt(x)/(a/b)^(1/6))/(a^3*b^2) + 1/36*(B*a*b*x^(7/2) + 11*A*b^2*x^(7/2)
- 5*B*a^2*sqrt(x) + 17*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^2*b)
```

**maple** [**A**] time = 0.17, size = 407, normalized size = 1.27

$$\frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108a^3}+\frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216a^3}+\frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216a^3}+\frac{55\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^3}-\frac{55\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^3}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108a^2b}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216a^2b}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}B\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216a^2b}+\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^2b}-\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}B\ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^2b}+\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^2}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{\left(bx^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x)
```

```
[Out] 2*(1/72*(11*A*b+B*a)/a^2*x^(7/2)+1/72*(17*A*b-5*B*a)/a/b*x^(1/2))/(b*x^3+a)
^2+55/108/a^3*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+5/108/a^2/b*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-55/432/a^3*3^(1/2)*(a/b)^(1/6)*ln(-x
+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A-5/432/a^2/b*3^(1/2)*(a/b)^(1/6)
*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+55/216/a^3*(a/b)^(1/6)*ar
ctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+5/216/a^2/b*(a/b)^(1/6)*arctan(2/(a/b
)^(1/6)*x^(1/2)-3^(1/2))*B+55/432/a^3*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b
)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+5/432/a^2/b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)
*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+55/216/a^3*(a/b)^(1/6)*arctan(2/(a/b)^(
1/6)*x^(1/2)+3^(1/2))*A+5/216/a^2/b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2
)+3^(1/2))*B
```

**maxima** [**A**] time = 1.21, size = 336, normalized size = 1.05

$$\frac{\left(Bab+11\,Ab^2\right)x^{\frac{7}{2}}-\left(5\,Ba^2-17\,Aab\right)\sqrt{x}}{36\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}+\frac{5\left(\frac{\sqrt{3}\,(Ba+11\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(Ba+11\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}+11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+11\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}+11\,Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/36*((B*a*b + 11*A*b^2)*x^(7/2) - (5*B*a^2 - 17*A*a*b)*sqrt(x))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) + 5/432*(sqrt(3)*(B*a + 11*A*b)*log(sqrt(3)*a^(
```

```
1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*
a + 11*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt
(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b
^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*
sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^
(4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*
b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/
(a^2*b)
```

**mupad [B]** time = 2.95, size = 1952, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^3),x)

[Out] ((x^(7/2)*(11*A*b + B*a))/(36*a^2) + (x^(1/2)*(17*A*b - 5*B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) - (atan(((((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4
*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^
8) - (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33
*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)
^(17/6)*b^(7/6)) + (((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*
a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b
 + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)^(17/6)*b^(7/6)))
/((5*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*
A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*(11*A*b + B*a)*(1331*A
^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(4
7/6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)) - (5*((625*x^(1/2)
*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A
*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b
^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11
*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6))))*(11*A*b + B*a)*5i)/(108*(-a)^(17/6
)*b^(7/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(1
4641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^
3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331
*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^
(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(1
1*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3
 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/
2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*
A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6))
)/((5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 +
B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(27
9936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*
a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))
))/(216*(-a)^(17/6)*b^(7/6)) - (5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((6
25*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*
b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b
 + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2
- 1/2)*(11*A*b + B*a)*5i)/(108*(-a)^(17/6)*b^(7/6)) - (atan(((((3^(1/2)*1i)
/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2
*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((
3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B
*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17
/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*
A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3
*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*
```

```
b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6
)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)))/((5*((3^(1/2)*1i)/2 + 1/2)*(11*A
*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 +
5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 +
 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B
^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6)) - (5*
((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^
4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a
^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^
2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(21
6*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*5i)/(108*(-a
)^(17/6)*b^(7/6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

## 3.172 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

**Optimal**. Leaf size=351

$$-\frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{19/6}b^{5/6}}+\frac{7(13Ab-}{}$$

**Rubi [A]** time = 0.62, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] (-7*(13*A*b - a*B))/(36*a^3*b*Sqrt[x]) + (A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*
x^3)^2) + (13*A*b - a*B)/(36*a^2*b*Sqrt[x]*(a + b*x^3)) + (7*(13*A*b - a*B)
*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7
*(13*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6
)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(1
9/6)*b^(5/6)) - (7*(13*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqr
t[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6)) + (7*(13*A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6
)*b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{\left(\frac{13Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} + \frac{(7(13Ab-aB))\int \frac{1}{x^{3/2}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{(7(13Ab-aB))\int \frac{x^{3/2}}{a+bx^3}\,dx}{72a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{(7(13Ab-aB))\,\text{Subst}\left(\int \frac{}{a\cdot}\right.}{36a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{(7(13Ab-aB))\,\text{Subst}\left(\int \frac{}{\sqrt[3]{}}\right.}{108a}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right.}{108a^{19/6}b^{5/6}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right.}{108a^{19/6}b^{5/6}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} + \frac{7(13Ab-aB)\tan^{-1}\left(\sqrt{3}-\right.}{216a^{19/6}b^{5/6}}$$

**Mathematica [C]** time = 0.13, size = 113, normalized size = 0.32

$$2\left(-\frac{x^{5/2}(Ab-aB)\,{}_2F_1\left(\frac{5}{6},3;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4} - \frac{Abx^{5/2}\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4} - \frac{Abx^{5/2}\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4} - \frac{A}{a^3\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] 2*(-(A/(a^3*Sqrt[x])) - (A*b*x^(5/2)*Hypergeometric2F1[5/6, 1, 11/6, -((b*x
^3)/a)])/(5*a^4) - (A*b*x^(5/2)*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a
)])/(5*a^4) - ((A*b - a*B)*x^(5/2)*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3
)/a)])/(5*a^4))
```

**IntegrateAlgebraic [A]** time = 0.63, size = 219, normalized size = 0.62

$$\frac{7(aB-13Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} - \frac{7(aB-13Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{216a^{19/6}b^{5/6}} - \frac{7(aB-13Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}a^{19/6}b^{5/6}} + \frac{-72a^2A+13a^2Bx^3-169aAbx^3+7abBx^6-91Ab^2x^6}{36a^3\sqrt{x}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] (-72*a^2*A - 169*a*A*b*x^3 + 13*a^2*B*x^3 - 91*A*b^2*x^6 + 7*a*b*B*x^6)/(36
*a^3*Sqrt[x]*(a + b*x^3)^2) + (7*(-13*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a
```

```
^(1/6)])/(108*a^(19/6)*b^(5/6)) - (7*(-13*A*b + a*B)*ArcTan[(a^(1/3) - b^(1
/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(216*a^(19/6)*b^(5/6)) - (7*(-13*A*b + a
*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(72*S
qrt[3]*a^(19/6)*b^(5/6))
```

**fricas [B]** time = 0.97, size = 3904, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5
*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*
b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^6 -
21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*sqrt(
x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b
^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*
b^5))^(5/6) + (B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8*b^2 - 263640*
A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*a^5*b^5 + 1013
629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 36707882445*A^8*B^2*a
^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*x - (B^6*a^19*b
^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 4
28415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(
B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 4
28415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^
(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3
*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b
^6)/(a^19*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*
B^3*a^6*b^3 - 21970*A^3*B^2*a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3
*b^6)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a
^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b
^5 + 4826809*A^6*b^6))/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 4
3940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48268
09*A^6*b^6)) + 28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 -
78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*
B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arct
an(1/50421*(2*sqrt(3)*sqrt(-282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1
690*A^2*B^3*a^19*b^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371
293*A^5*a^16*b^9)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^
2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 +
4826809*A^6*b^6)/(a^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b
 + 7605*A^2*B^8*a^8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4
- 93565836*A^5*B^5*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^
3*a^3*b^7 + 36707882445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858
491849*A^10*b^10)*x - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^
2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758
*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*
A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*
A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B
^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^
2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 33614*sq
rt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*B^3*a^6*b^3 - 21970*A^3*B^2*
a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3*b^6)*sqrt(x)*(-(B^6*a^6 - 7
8*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B
^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 168
07*sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3
```

```
*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6))
/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 +
 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)) - 14*(a^3
*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*
a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*
b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807*a^16*b^4*(-(B^6*a^6 - 78
*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^
2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 1680
7*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3
+ 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 14*(a^3*b^2*x^7 + 2*a^4*b
*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6)*log(-16807*a^16*b^4*(-(B^6*a^6 - 78*A*B^5*a^5*b + 25
35*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 22277
58*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A
*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*
b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B
^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 42
8415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(
1/6)*log(282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^
6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*s
qrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a
^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a
^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8
*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*
a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 3670788
2445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*x
 - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 43
940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 + 4
826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 4
3940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48268
09*A^6*b^6)/(a^19*b^5))^(2/3)) - 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B
^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 42
8415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(
1/6)*log(-282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b
^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*
sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*
a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(
a^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^
8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5
*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 367078
82445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*
x - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 4
3940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 +
4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 -
43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826
809*A^6*b^6)/(a^19*b^5))^(2/3)) + 12*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2
- 13*A*a*b)*x^3 - 72*A*a^2)*sqrt(x))/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

**giac** [A] time = 0.48, size = 329, normalized size = 0.94

$$-\frac{2\,A}{a^{3}\sqrt{x}}+\frac{7\,Babx^{\frac{11}{2}}-19\,Ab^{2}x^{\frac{11}{2}}+13\,Ba^{2}x^{\frac{5}{2}}-25\,Aabx^{\frac{5}{2}}}{36\,{\left(bx^{3}+a\right)}^{2}a^{3}}-\frac{7\,\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-13\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^{4}b^{5}}+\frac{7\,\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-13\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^{4}b^{5}}+\frac{7\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-13\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^{4}b^{5}}+\frac{7\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-13\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^{4}b^{5}}+\frac{7\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-13\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^{4}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -2*A/(a^3*sqrt(x)) + 1/36*(7*B*a*b*x^(11/2) - 19*A*b^2*x^(11/2) + 13*B*a^2*
x^(5/2) - 25*A*a*b*x^(5/2))/((b*x^3 + a)^2*a^3) - 7/432*sqrt(3)*((a*b^5)^(5
/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
```

```
^(1/3))/(a^4*b^5) + 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b
)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) + 7/216*((a
*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqr
t(x))/(a/b)^(1/6))/(a^4*b^5) + 7/216*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^4*b^5) + 7/1
08*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(
a^4*b^5)
```

**maple** [A] time = 0.17, size = 441, normalized size = 1.26

$$-\frac{19 A b^2 x^{\frac{11}{2}}}{36 \left(b x^3+a\right)^2 a^3}+\frac{7 B b x^{\frac{11}{2}}}{36 \left(b x^3+a\right)^2 a^2}-\frac{25 A b x^{\frac{5}{2}}}{36 \left(b x^3+a\right)^2 a^2}+\frac{13 B x^{\frac{5}{2}}}{36 \left(b x^3+a\right)^2 a}-\frac{91 A \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^3}-\frac{91 A \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^3}-\frac{91 A \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^3}+\frac{91 \sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}} A b \ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^4}-\frac{91 \sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}} A b \ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^4}+\frac{7 B \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^2 b}+\frac{7 B \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^2 b}+\frac{7 B \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^2 b}-\frac{7 \sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}} B \ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^3}+\frac{7 \sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}} B \ln\left(-x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^3}-\frac{2 A}{a^3 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x)
```

```
[Out] -19/36/a^3/(b*x^3+a)^2*x^(11/2)*b^2*A+7/36/a^2/(b*x^3+a)^2*x^(11/2)*B*b-25/
36/a^2/(b*x^3+a)^2*A*x^(5/2)*b+13/36/a/(b*x^3+a)^2*B*x^(5/2)-91/108/a^3*A/(
a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))-91/432/a^4*A*b*3^(1/2)*(a/b)^(5/6)
*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-91/216/a^3*A/(a/b)^(1/6)*ar
ctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+91/432/a^4*A*b*3^(1/2)*(a/b)^(5/6)*ln(x
+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-91/216/a^3*A/(a/b)^(1/6)*arctan(2
/(a/b)^(1/6)*x^(1/2)+3^(1/2))+7/108/a^2*B/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6
)*x^(1/2))+7/432/a^3*B*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2
)-(a/b)^(1/3))+7/216/a^2*B/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/
2))-7/432/a^3*B*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^
(1/3))+7/216/a^2*B/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2*A/
a^3/x^(1/2)
```

**maxima** [A] time = 1.29, size = 273, normalized size = 0.78

$$\frac{7\left(Bab-13\,Ab^2\right)x^6+13\left(Ba^2-13\,Aab\right)x^3-72\,Aa^2}{36\left(a^3b^2x^{\frac{13}{2}}+2\,a^4bx^{\frac{7}{2}}+a^5\sqrt{x}\right)}-\frac{7\left(Ba-13\,Ab\right)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{4\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)/(a^3
*b^2*x^(13/2) + 2*a^4*b*x^(7/2) + a^5*sqrt(x)) - 7/432*(B*a - 13*A*b)*(sqrt
(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(
5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))
/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3
)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt
(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/
3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

**mupad** [B] time = 2.91, size = 1786, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x)
```

```
[Out] (atan((((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^2
1*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343
*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*
B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/
6))*1i)/((-a)^(19/3)*b^(5/3)) + ((13*A*b - B*a)^2*(62019785528064*A^3*a^21
*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 1431225819
```

```
8784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a
^24*b^6 + 829406312792064*B^2*a^26*b^4 - 2156456413259366 4*A*B*a^25*b^5))/(
10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/(((13*A*b - B*a)^
2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A
*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)
*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564
132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5
/3)) - ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^2
4*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343
*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*
B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/
6))))/((-a)^(19/3)*b^(5/3))))*(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6))
 - ((2*A)/a + (13*x^3*(13*A*b - B*a))/(36*a^2) + (7*b*x^6*(13*A*b - B*a))/(3
6*a^3))/(a^2*x^(1/2) + b^2*x^(13/2) + 2*a*b*x^(7/2)) + (atan(((((3^(1/2)*1i
)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^
3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 +
 (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2
*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))
/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)
/2 - 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3
*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 +
(343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*
a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/
(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/
2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*
a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (
343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a
^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(
10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 -
1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24
*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*
x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*
b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(1007
7696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*
(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)/2 + 1/
2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b
^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^
(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^
6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(100776
96*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)/2 + 1/2
)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^
3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6
 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(1007769
6*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)
^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6
 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1
/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6
+ 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696
*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(
13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1
100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*
((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 82
9406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a
)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(13*A*b
- B*a)*7i)/(108*(-a)^(19/6)*b^(5/6))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

## 3.173 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=129

$$-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}+\frac{aB-5Ab}{4a^3bx^{3/2}}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

**Rubi** [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 290, 325, 329, 275, 205}

$$\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{5Ab-aB}{4a^3bx^{3/2}}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] -(5*A*b - a*B)/(4*a^3*b*x^(3/2)) + (A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)^2
) + (5*A*b - a*B)/(12*a^2*b*x^(3/2)*(a + b*x^3)) - ((5*A*b - a*B)*ArcTan[(S
qrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{\left(\frac{15Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} + \frac{(3(5Ab-aB))\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{8a^2b}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(3(5Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(3(5Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{a+b\ldots}\right.}{4a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(5Ab-aB)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,\ldots\right.}{4a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

**Mathematica [A]** time = 0.19, size = 102, normalized size = 0.79

$$\frac{(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}} + \frac{a^2\left(5Bx^3-8A\right)+a\left(3bBx^6-25Abx^3\right)-15Ab^2x^6}{12a^3x^{3/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] (-15*A*b^2*x^6 + a^2*(-8*A + 5*B*x^3) + a*(-25*A*b*x^3 + 3*b*B*x^6))/(12*a^
3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]
])/(4*a^(7/2)*Sqrt[b])
```

**IntegrateAlgebraic [A]** time = 0.17, size = 102, normalized size = 0.79

$$\frac{(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}} + \frac{-8a^2A+5a^2Bx^3-25aAbx^3+3abBx^6-15Ab^2x^6}{12a^3x^{3/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] (-8*a^2*A - 25*a*A*b*x^3 + 5*a^2*B*x^3 - 15*A*b^2*x^6 + 3*a*b*B*x^6)/(12*a^
3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]
])/(4*a^(7/2)*Sqrt[b])
```

**fricas** [A] time = 0.83, size = 347, normalized size = 2.69

$$\left[\frac{3\left(\left(Bab^{2}-5\,Ab^{3}\right)x^{8}+2\left(Ba^{2}b-5\,Aab^{2}\right)x^{5}+\left(Ba^{3}-5\,Aa^{2}b\right)x^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx^{3}+2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^{3}+a}\right)+2\left(3\left(Ba^{2}b^{2}-5\,Aab^{3}\right)x^{6}-8\,Aa^{3}b+5\left(Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{3}\right)\sqrt{x}}{24\left(a^{4}b^{3}x^{8}+2\,a^{5}b^{2}x^{5}+a^{6}bx^{2}\right)},\ \frac{3\left(\left(Bab^{2}-5\,Ab^{3}\right)x^{8}+2\left(Ba^{2}b-5\,Aab^{2}\right)x^{5}+\left(Ba^{3}-5\,Aa^{2}b\right)x^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)+\left(3\left(Ba^{2}b^{2}-5\,Aab^{3}\right)x^{6}-8\,Aa^{3}b+5\left(Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{3}\right)\sqrt{x}}{12\left(a^{4}b^{3}x^{8}+2\,a^{5}b^{2}x^{5}+a^{6}bx^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 -
5*A*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 +
a)) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b
^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((B*a*
b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8
*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2
*x^5 + a^6*b*x^2)]
```

**giac** [A] time = 0.18, size = 88, normalized size = 0.68

$$\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}-\frac{2\,A}{3\,a^{3}x^{\frac{3}{2}}}+\frac{3\,Babx^{\frac{9}{2}}-7\,Ab^{2}x^{\frac{9}{2}}+5\,Ba^{2}x^{\frac{3}{2}}-9\,Aabx^{\frac{3}{2}}}{12\left(bx^{3}+a\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(B*a - 5*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/3*A/(a^3*
x^(3/2)) + 1/12*(3*B*a*b*x^(9/2) - 7*A*b^2*x^(9/2) + 5*B*a^2*x^(3/2) - 9*A*
a*b*x^(3/2))/((b*x^3 + a)^2*a^3)
```

**maple** [A] time = 0.07, size = 133, normalized size = 1.03

$$-\frac{7A\,b^{2}x^{\frac{9}{2}}}{12\left(b\,x^{3}+a\right)^{2}a^{3}}+\frac{Bb\,x^{\frac{9}{2}}}{4\left(b\,x^{3}+a\right)^{2}a^{2}}-\frac{3Ab\,x^{\frac{3}{2}}}{4\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{5B\,x^{\frac{3}{2}}}{12\left(b\,x^{3}+a\right)^{2}a}-\frac{5Ab\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^{3}}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^{2}}-\frac{2A}{3a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x)
```

```
[Out] -7/12/a^3/(b*x^3+a)^2*x^(9/2)*b^2*A+1/4/a^2/(b*x^3+a)^2*x^(9/2)*B*b-3/4/a^2
/(b*x^3+a)^2*A*x^(3/2)*b+5/12/a/(b*x^3+a)^2*B*x^(3/2)-5/4/a^3/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x^(3/2))*A*b+1/4/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x^(3/2))*B-2/3/a^3*A/x^(3/2)
```

**maxima** [A] time = 1.35, size = 100, normalized size = 0.78

$$\frac{3\left(Bab-5\,Ab^{2}\right)x^{6}+5\left(Ba^{2}-5\,Aab\right)x^{3}-8\,Aa^{2}}{12\left(a^{3}b^{2}x^{\frac{15}{2}}+2\,a^{4}bx^{\frac{9}{2}}+a^{5}x^{\frac{3}{2}}\right)}+\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(B*a*b - 5*A*b^2)*x^6 + 5*(B*a^2 - 5*A*a*b)*x^3 - 8*A*a^2)/(a^3*b^2
*x^(15/2) + 2*a^4*b*x^(9/2) + a^5*x^(3/2)) + 1/4*(B*a - 5*A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

**mupad [B]** time = 2.73, size = 163, normalized size = 1.26

$$-\frac{\frac{2A}{3a}+\frac{5x^3(5Ab-Ba)}{12a^2}+\frac{bx^6(5Ab-Ba)}{4a^3}}{a^2x^{3/2}+b^2x^{15/2}+2abx^{9/2}}-\frac{\mathrm{atan}\left(\frac{8a^{7/2}\sqrt{b}\,x^{3/2}\left(86400A^2a^9b^5-34560ABa^{10}b^4+3456B^2a^{11}b^3\right)}{(5Ab-Ba)\left(138240Aa^{13}b^4-27648Ba^{14}b^3\right)}\right)(5Ab-Ba)}{4a^{7/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(3*a) + (5*x^3*(5*A*b - B*a))/(12*a^2) + (b*x^6*(5*A*b - B*a))/(4*
a^3))/(a^2*x^(3/2) + b^2*x^(15/2) + 2*a*b*x^(9/2)) - (atan((8*a^(7/2)*b^(1/
2)*x^(3/2)*(86400*A^2*a^9*b^5 + 3456*B^2*a^11*b^3 - 34560*A*B*a^10*b^4))/((
5*A*b - B*a)*(138240*A*a^13*b^4 - 27648*B*a^14*b^3)))*(5*A*b - B*a))/(4*a^(
7/2)*b^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

## 3.174 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

**Optimal**. Leaf size=351

$$\frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{23/6}\sqrt[6]{b}}+\frac{11(17A}{}$$

**Rubi** [A] time = 0.54, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 209, 634, 618, 204, 628, 205}

$$\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{23/6}\sqrt[6]{b}}+\frac{11(17Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] (-11*(17*A*b - 5*a*B))/(180*a^3*b*x^(5/2)) + (A*b - a*B)/(6*a*b*x^(5/2)*(a
+ b*x^3)^2) + (17*A*b - 5*a*B)/(36*a^2*b*x^(5/2)*(a + b*x^3)) + (11*(17*A*b
 - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1
/6)) - (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/(108*a^(23/6)*b^(1/6)) + (11*(17*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6)) - (11*(
17*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])
/(144*Sqrt[3]*a^(23/6)*b^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{\left(\frac{17Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{(11(17Ab-5aB))\int \frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))\int \frac{}{\sqrt{x}\,(a}}{72a^3}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))\,\text{Subst}}{36a^3}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))\,\text{Subst}}{}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\tan^{-1}\left(\right.}{108a^{23/6}\sqrt[6]{b}}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\tan^{-1}\left(\right.}{108a^{23/6}\sqrt[6]{b}}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{11(17Ab-5aB)\tan^{-1}\left(\right.}{216a^{23/6}\sqrt[6]{b}}$$

**Mathematica [C]** time = 0.12, size = 96, normalized size = 0.27

$$\frac{\frac{a\left(a^2\left(85Bx^3-72A\right)+a\left(55bBx^6-289Abx^3\right)-187Ab^2x^6\right)}{\left(a+bx^3\right)^2}+55x^3(5aB-17Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)}{180a^4x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] ((a*(-187*A*b^2*x^6 + a^2*(-72*A + 85*B*x^3) + a*(-289*A*b*x^3 + 55*b*B*x^6
)))/(a + b*x^3)^2 + 55*(-17*A*b + 5*a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6,
 -((b*x^3)/a)])/(180*a^4*x^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.64, size = 222, normalized size = 0.63

$$\frac{11(5aB-17Ab)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}}-\frac{11(5aB-17Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{216a^{23/6}\sqrt[6]{b}}+\frac{11(5aB-17Ab)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}{\sqrt[3]{a}+\sqrt[3]{b}x}\right)}{72\sqrt{3}\,a^{23/6}\sqrt[6]{b}}+\frac{-72a^2A+85a^2Bx^3-289aAbx^3+55abBx^6-187Ab^2x^6}{180a^3x^{5/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] (-72*a^2*A - 289*a*A*b*x^3 + 85*a^2*B*x^3 - 187*A*b^2*x^6 + 55*a*b*B*x^6)/(
180*a^3*x^(5/2)*(a + b*x^3)^2) + (11*(-17*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt
```

```
[x])/a^(1/6)])/(108*a^(23/6)*b^(1/6)) - (11*(-17*A*b + 5*a*B)*ArcTan[(a^(1/
3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/(216*a^(23/6)*b^(1/6)) + (11*(-
17*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3
)*x)])/(72*Sqrt[3]*a^(23/6)*b^(1/6))
```

**fricas [B]** time = 1.33, size = 2690, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2160*(220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6
 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3
+ 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23
*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a
^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^
2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + (25*
B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x + (5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(
15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3
*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A
^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 270
9375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
- 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt(3)*(5*B
*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27
09375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) - sqrt(3)*(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 122825
00*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 2413
7569*A^6*b^6)) + 220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625
*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*
a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^
6)/(a^23*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750
*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132037
5*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3
) + (25*B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x - (5*B*a^5 - 17*A*a^4*b)*sqr
t(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 1228
2500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24
137569*A^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5
*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*
a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt
(3)*(5*B*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^
5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2
*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + sqrt(
3)*(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500
*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 241375
69*A^6*b^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2
- 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^
5 + 24137569*A^6*b^6)) - 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27093
75*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 -
42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2
- 170*A*B*a*b + 289*A^2*b^2)*x + 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6)/(a^23*b))^(1/6)) + 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^
```

```
6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3
*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/
(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375
*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42
595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2 -
170*A*B*a*b + 289*A^2*b^2)*x - 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)) + 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6
*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*
b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(
a^23*b))^(1/6)*log(11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A
^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 4259
5710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*
sqrt(x)) - 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318
750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132
0375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(
1/6)*log(-11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^
4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*
B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x))
- 12*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2)
*sqrt(x))/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)
```

**giac** [A] time = 0.24, size = 334, normalized size = 0.95

$$\frac{11 \sqrt{3}\left(5 \left(ab^5\right)^{\frac{1}{6}} Ba - 17 \left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\, a^4 b} - \frac{11 \sqrt{3}\left(5 \left(ab^5\right)^{\frac{1}{6}} Ba - 17 \left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\, a^4 b} + \frac{11\left(5 \left(ab^5\right)^{\frac{1}{6}} Ba - 17 \left(ab^5\right)^{\frac{1}{6}} Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} + 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\, a^4 b} + \frac{11\left(5 \left(ab^5\right)^{\frac{1}{6}} Ba - 17 \left(ab^5\right)^{\frac{1}{6}} Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} - 2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\, a^4 b} + \frac{11\left(5 \left(ab^5\right)^{\frac{1}{6}} Ba - 17 \left(ab^5\right)^{\frac{1}{6}} Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\, a^4 b} + \frac{11\, Babx^{\frac{7}{2}} - 23\, Ab^2 x^{\frac{7}{2}} + 17\, Ba^2\sqrt{x} - 29\, Aab\sqrt{x}}{36\left(bx^3 + a\right)^2 a^3} - \frac{2\, A}{5\, a^3 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqr
t(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(1/
6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arcta
n((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(a*b^5
)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x
))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*
b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 23*A*b^2*
x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3) - 2/5*A/
(a^3*x^(5/2))
```

**maple** [A] time = 0.17, size = 435, normalized size = 1.24

$$-\frac{23 A\, b^2 x^{\frac{7}{2}}}{36\left(b\, x^3 + a\right)^2 a^3} + \frac{11 B b\, x^{\frac{7}{2}}}{36\left(b\, x^3 + a\right)^2 a^2} - \frac{29 A b\sqrt{x}}{36\left(b\, x^3 + a\right)^2 a^2} + \frac{17 B\sqrt{x}}{36\left(b\, x^3 + a\right)^2 a} - \frac{187\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108 a^4} - \frac{187\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{216 a^4} - \frac{187\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{216 a^4} - \frac{187\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab \ln\left(x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^4} + \frac{187\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab \ln\left(-x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^4} + \frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}} B \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108 a^3} + \frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}} B \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{216 a^3} + \frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}} B \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{216 a^3} + \frac{55\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} B \ln\left(x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^3} - \frac{55\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} B \ln\left(-x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432 a^3} - \frac{2 A}{5 a^3 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x)
```

```
[Out] -23/36/a^3/(b*x^3+a)^2*x^(7/2)*b^2*A+11/36/a^2/(b*x^3+a)^2*x^(7/2)*B*b-29/3
6/a^2/(b*x^3+a)^2*A*x^(1/2)*b+17/36/a/(b*x^3+a)^2*B*x^(1/2)-187/108/a^4*A*b
*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+187/432/a^4*A*b*3^(1/2)*(a/b)^(1
/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1
/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-187/432/a^4*A*b*3^(1/2)*(a/b)^(1/
6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1/6
)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))+55/108/a^3*B*(a/b)^(1/6)*arctan(1/(
a/b)^(1/6)*x^(1/2))-55/432/a^3*B*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1
/6)*x^(1/2)-(a/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1
/2)-3^(1/2))+55/432/a^3*B*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1
/2)+(a/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1
/2))-2/5*A/a^3/x^(5/2)
```

**maxima** [A] time = 1.34, size = 346, normalized size = 0.99

$$\frac{11\left(5 Bab - 17 Ab^2\right)x^6 + 17\left(5 Ba^2 - 17 Aab\right)x^3 - 72 Aa^2}{180\left(a^3b^2x^{\frac{17}{2}} + 2 a^4bx^{\frac{11}{2}} + a^5x^{\frac{5}{2}}\right)} + \frac{11\left(\frac{\sqrt{3}(5Ba-17Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(5Ba-17Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(5Bab^{\frac{1}{3}}-17Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(5Ba^{\frac{4}{3}}b^{\frac{1}{3}}-17Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(5Ba^{\frac{4}{3}}b^{\frac{1}{3}}-17Aa^{\frac{1}{3}}b^{\frac{4}{3}}\right)\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}-2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{ab^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{432a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/180*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2
)/(a^3*b^2*x^(17/2) + 2*a^4*b*x^(11/2) + a^5*x^(5/2)) + 11/432*(sqrt(3)*(5*
B*a - 17*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a
^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 17*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqr
t(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3) - 17*A*b^(
4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a
^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*arctan((s
qrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/
3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*
arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3))
)/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

**mupad** [B] time = 2.96, size = 2109, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(5*a) + (17*x^3*(17*A*b - 5*B*a))/(180*a^2) + (11*b*x^6*(17*A*b -
5*B*a))/(180*a^3))/(a^2*x^(5/2) + b^2*x^(17/2) + 2*a*b*x^(11/2)) - (atan(((
(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 +
 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5
21928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(51243917694905548
8*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*
a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(
17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)) + ((x^(1/2)*(443639472636450
816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B
^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a
^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837
801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 45215221495504
8960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a)*11i)/(216
*(-a)^(23/6)*b^(1/6)))/((11*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b
 - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5
 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/
(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6)) - (1
1*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5
 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 -
 521928791337000960*A^3*B*a^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055
488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^
2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))
*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6))))*(17*A*b - 5*B*a)*11i)/(108*(
-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1
/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 2302
62702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928
791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*
(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 1329859
45575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^
(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(
```

```
17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331981981056000
0*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*
B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1
/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B
^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*
a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)))/((11*
((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^
15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^
7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) -
 (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b
^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4
52152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6))))/(216*(-a)^(2
3/6)*b^(1/6)) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(44363
9472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 23026270206044
1600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219287913370009
60*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176
949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 13298594557501440
0*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(
1/6))))/(216*(-a)^(23/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)
*11i)/(108*(-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(17*A*b -
5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^1
9*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*
b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A
*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b
^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7)
)/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i
)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/
2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 1303783
7801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4521522149550
48960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(
1/6)))/((11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(4436394726364
50816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2
*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B
*a^16*b^8) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(51243917694905548
8*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*
a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6))))/
(216*(-a)^(23/6)*b^(1/6)) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^
(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 23
0262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219
28791337000960*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a
)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 13298
5945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a
)^(23/6)*b^(1/6))))/(216*(-a)^(23/6)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(17*
A*b - 5*B*a)*11i)/(108*(-a)^(23/6)*b^(1/6))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

## 3.175 $\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

**Optimal**. Leaf size=103

$$\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

**Rubi** [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b
*x^3)^(5/2))/(15*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*B
*(a + b*x^3)^(9/2))/(27*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)dx&=\frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx}\,(A+Bx)\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}+\frac{(Ab-3\right.\right.\\&=\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(a+\right.}{21b^4}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 75, normalized size = 0.73

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-16a^3B+24a^2b\left(A+Bx^3\right)-6ab^2x^3\left(6A+5Bx^3\right)+5b^3x^6\left(9A+7Bx^3\right)\right)}{945b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A + 
 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 80, normalized size = 0.78

$$-\frac{2\left(a+bx^3\right)^{3/2}\left(16a^3B-24a^2Ab-24a^2bBx^3+36aAb^2x^3+30ab^2Bx^6-45Ab^3x^6-35b^3Bx^9\right)}{945b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (-2*(a + b*x^3)^(3/2)*(-24*a^2*A*b + 16*a^3*B + 36*a*A*b^2*x^3 - 24*a^2*b*B
*x^3 - 45*A*b^3*x^6 + 30*a*b^2*B*x^6 - 35*b^3*B*x^9))/(945*b^4)
```

**fricas [A]** time = 0.79, size = 99, normalized size = 0.96

$$\frac{2\left(35\,Bb^4x^{12}+5\left(Bab^3+9\,Ab^4\right)x^9-3\left(2\,Ba^2b^2-3\,Aab^3\right)x^6-16\,Ba^4+24\,Aa^3b+4\left(2\,Ba^3b-3\,Aa^2b^2\right)x^3\right)\sqrt{bx^3+a}}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*b
^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b*x
^3 + a)/b^4
```

**giac [A]** time = 0.16, size = 104, normalized size = 1.01

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}B-135\left(bx^3+a\right)^{\frac{7}{2}}Ba+189\left(bx^3+a\right)^{\frac{5}{2}}Ba^2-105\left(bx^3+a\right)^{\frac{3}{2}}Ba^3+45\left(bx^3+a\right)^{\frac{7}{2}}Ab-126\left(bx^3+a\right)^{\frac{5}{2}}Aab+105\left(bx^3+a\right)^{\frac{3}{2}}Aa^2b\right)}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 135*(b*x^3 + a)^(7/2)*B*a + 189*(b*x^3 + a)
^(5/2)*B*a^2 - 105*(b*x^3 + a)^(3/2)*B*a^3 + 45*(b*x^3 + a)^(7/2)*A*b - 126
*(b*x^3 + a)^(5/2)*A*a*b + 105*(b*x^3 + a)^(3/2)*A*a^2*b)/b^4
```

**maple [A]** time = 0.05, size = 77, normalized size = 0.75

$$\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(35B\,x^9b^3+45A\,b^3x^6-30Ba\,b^2x^6-36Aa\,b^2x^3+24B\,a^2b\,x^3+24A\,a^2b-16B\,a^3\right)}{945b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/945*(b*x^3+a)^(3/2)*(35*B*b^3*x^9+45*A*b^3*x^6-30*B*a*b^2*x^6-36*A*a*b^2*
x^3+24*B*a^2*b*x^3+24*A*a^2*b-16*B*a^3)/b^4
```

**maxima [A]** time = 0.62, size = 118, normalized size = 1.15

$$\frac{2}{945}\,B\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(bx^3+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(bx^3+a\right)^{\frac{3}{2}}a^3}{b^4}\right)+\frac{2}{315}\,A\left(\frac{15\left(bx^3+a\right)^{\frac{7}{2}}}{b^3}-\frac{42\left(bx^3+a\right)^{\frac{5}{2}}a}{b^3}+\frac{35\left(bx^3+a\right)^{\frac{3}{2}}a^2}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x^
3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^3
+ a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b^3)
```

**mupad [B]** time = 2.72, size = 154, normalized size = 1.50

$$\frac{2\,B\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^9\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{21\,b}+\frac{8\,a^2\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^3}+\frac{x^6\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{15\,b}-\frac{4\,a\,x^3\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^12*(a + b*x^3)^(1/2))/27 + (x^9*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/9
))/(21*b) + (8*a^2*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^(1
/2))/(45*b^3) + (x^6*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^
(1/2))/(15*b) - (4*a*x^3*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x
^3)^(1/2))/(45*b^2)
```

**sympy [A]** time = 4.17, size = 219, normalized size = 2.13

$$\begin{cases} \frac{16Aa^3\sqrt{a+bx^3}}{315b^3}-\frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Aax^6\sqrt{a+bx^3}}{105b}+\frac{2Ax^9\sqrt{a+bx^3}}{21}-\frac{32Ba^4\sqrt{a+bx^3}}{945b^4}+\frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3}-\frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^9\sqrt{a+bx^3}}{189b}+\frac{2Bx^{12}\sqrt{a+bx^3}}{27} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a + b
*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a +
 b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqrt(a
 + b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x
**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (s
qrt(a)*(A*x**9/9 + B*x**12/12), True))
```

# 3.176 $\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

**Optimal**. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)
^(5/2))/(15*b^3) + (2*B*(a + b*x^3)^(7/2))/(21*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)dx&=\frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx}\,(A+Bx)\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2}+\frac{(Ab-2aB)(a+bx)^{3/2}}{b^2}+\frac{B(a+bx)^{5/2}}{b^2}\right)dx,\right.\\&=-\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 57, normalized size = 0.78

$$\frac{2\left(a+bx^3\right)^{3/2}\left(8a^2B-2ab\left(7A+6Bx^3\right)+3b^2x^3\left(7A+5Bx^3\right)\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(8*a^2*B + 3*b^2*x^3*(7*A + 5*B*x^3) - 2*a*b*(7*A + 6*
B*x^3)))/(315*b^3)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 56, normalized size = 0.77

$$\frac{2\left(a+bx^3\right)^{3/2}\left(8a^2B-14aAb-12abBx^3+21Ab^2x^3+15b^2Bx^6\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^3 - 12*a*b*B*x^3 + 1
5*b^2*B*x^6))/(315*b^3)
```

**fricas** [A] time = 0.83, size = 75, normalized size = 1.03

$$\frac{2\left(15\,Bb^3x^9+3\left(Bab^2+7\,Ab^3\right)x^6+8\,Ba^3-14\,Aa^2b-\left(4\,Ba^2b-7\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3
```

**giac** [A] time = 0.18, size = 73, normalized size = 1.00

$$\frac{2\left(15\left(bx^3+a\right)^{\frac{7}{2}}B-42\left(bx^3+a\right)^{\frac{5}{2}}Ba+35\left(bx^3+a\right)^{\frac{3}{2}}Ba^2+21\left(bx^3+a\right)^{\frac{5}{2}}Ab-35\left(bx^3+a\right)^{\frac{3}{2}}Aab\right)}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(15*(b*x^3 + a)^(7/2)*B - 42*(b*x^3 + a)^(5/2)*B*a + 35*(b*x^3 + a)^(
3/2)*B*a^2 + 21*(b*x^3 + a)^(5/2)*A*b - 35*(b*x^3 + a)^(3/2)*A*a*b)/b^3
```

**maple** [A] time = 0.04, size = 53, normalized size = 0.73

$$-\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(-15B\,b^2x^6-21A\,b^2x^3+12Bab\,x^3+14Aab-8B\,a^2\right)}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] -2/315*(b*x^3+a)^(3/2)*(-15*B*b^2*x^6-21*A*b^2*x^3+12*B*a*b*x^3+14*A*a*b-8*
B*a^2)/b^3
```

**maxima** [A] time = 0.51, size = 84, normalized size = 1.15

$$\frac{2}{315}\,B\left(\frac{15\left(bx^3+a\right)^{\frac{7}{2}}}{b^3}-\frac{42\left(bx^3+a\right)^{\frac{5}{2}}a}{b^3}+\frac{35\left(bx^3+a\right)^{\frac{3}{2}}a^2}{b^3}\right)+\frac{2}{45}\,A\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3
+ a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)
*a/b^2)
```

**mupad [B]** time = 2.66, size = 114, normalized size = 1.56

$$\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2}+\frac{x^3\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^9*(a + b*x^3)^(1/2))/21 + (x^6*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/7)
)/(15*b) - (2*a*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2)
)/(9*b^2) + (x^3*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2
))/(9*b)
```

**sympy [A]** time = 1.82, size = 168, normalized size = 2.30

$$\begin{cases} -\frac{4Aa^2\sqrt{a+bx^3}}{45b^2}+\frac{2Aax^3\sqrt{a+bx^3}}{45b}+\frac{2Ax^6\sqrt{a+bx^3}}{15}+\frac{16Ba^3\sqrt{a+bx^3}}{315b^3}-\frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^6\sqrt{a+bx^3}}{105b}+\frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x**
3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(315*
b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a + b*x
**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**6/6
+ B*x**9/9), True))
```

## 3.177 $\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

**Optimal**. Leaf size=46

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

**Rubi** [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^2) + (2*B*(a + b*x^3)^(5/2))/(15*b^2
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int\sqrt{a+bx}\,(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)dx,x,x^3\right)\\ &= \frac{2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-2aB+5Ab+3bBx^3\right)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-2aB+5Ab+3bBx^3\right)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

**fricas [A]** time = 0.71, size = 50, normalized size = 1.09

$$\frac{2\left(3\,Bb^2x^6+\left(Bab+5\,Ab^2\right)x^3-2\,Ba^2+5\,Aab\right)\sqrt{bx^3+a}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3 +
 a)/b^2
```

**giac [A]** time = 0.15, size = 44, normalized size = 0.96

$$\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}B-5\left(bx^3+a\right)^{\frac{3}{2}}Ba+5\left(bx^3+a\right)^{\frac{3}{2}}Ab\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 2/45*(3*(b*x^3 + a)^(5/2)*B - 5*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)
*A*b)/b^2
```

**maple [A]** time = 0.04, size = 31, normalized size = 0.67

$$\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(3Bb\,x^3+5Ab-2Ba\right)}{45b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x)
[Out] 2/45*(b*x^3+a)^(3/2)*(3*B*b*x^3+5*A*b-2*B*a)/b^2
```

**maxima [A]** time = 0.59, size = 49, normalized size = 1.07

$$\frac{2}{45}\,B\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}a}{b^2}\right)+\frac{2\left(bx^3+a\right)^{\frac{3}{2}}A}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3 +
 a)^(3/2)*A/b
```

**mupad [B]** time = 2.60, size = 44, normalized size = 0.96

$$\frac{6\,B\,\left(b\,x^3+a\right)^{5/2}+10\,A\,b\,\left(b\,x^3+a\right)^{3/2}-10\,B\,a\,\left(b\,x^3+a\right)^{3/2}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (6*B*(a + b*x^3)^(5/2) + 10*A*b*(a + b*x^3)^(3/2) - 10*B*a*(a + b*x^3)^(3/2
))/(45*b^2)
```

**sympy [A]** time = 0.74, size = 117, normalized size = 2.54

$$\begin{cases} \frac{2Aa\sqrt{a+bx^3}}{9b} + \frac{2Ax^3\sqrt{a+bx^3}}{9} - \frac{4Ba^2\sqrt{a+bx^3}}{45b^2} + \frac{2Bax^3\sqrt{a+bx^3}}{45b} + \frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*B
*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*B
*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), True)
)
```

## 3.178 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x}\,dx$

**Optimal**. Leaf size=64

$$\frac{2}{3}A\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}$$

**Rubi [A]** time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$\frac{2}{3}A\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*A*Sqrt[a + b*x^3])/3 + (2*B*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*A*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x}\,dx, x, x^3\right)\\
&= \frac{2B\left(a+bx^3\right)^{3/2}}{9b} + \frac{1}{3}A\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B\left(a+bx^3\right)^{3/2}}{9b} + \frac{1}{3}(aA)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B\left(a+bx^3\right)^{3/2}}{9b} + \frac{(2aA)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B\left(a+bx^3\right)^{3/2}}{9b} - \frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.05, size = 60, normalized size = 0.94

$$
\frac{2}{9}\left(\frac{\sqrt{a+bx^3}\left(B\left(a+bx^3\right)+3Ab\right)}{b} - 3\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*((Sqrt[a + b*x^3]*(3*A*b + B*(a + b*x^3)))/b - 3*Sqrt[a]*A*ArcTanh[Sqrt[
a + b*x^3]/Sqrt[a]]))/9
```

**IntegrateAlgebraic [A]** time = 0.05, size = 61, normalized size = 0.95

$$
\frac{2\sqrt{a+bx^3}\left(aB+3Ab+bBx^3\right)}{9b} - \frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*A*b + a*B + b*B*x^3))/(9*b) - (2*Sqrt[a]*A*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/3
```

**fricas [A]** time = 0.77, size = 125, normalized size = 1.95

$$
\left[\frac{3\,A\sqrt{a}\,b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(Bbx^3+Ba+3\,Ab\right)\sqrt{bx^3+a}}{9\,b}, \frac{2\left(3\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(Bbx^3+Ba+3\,Ab\right)\sqrt{bx^3+a}\right)}{9\,b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/9*(3*A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*
(B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b, 2/9*(3*A*sqrt(-a)*b*arctan(sqrt
(b*x^3 + a)*sqrt(-a)/a) + (B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b]
```

**giac [A]** time = 0.15, size = 61, normalized size = 0.95

$$\frac{2\,Aa\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^2+3\,\sqrt{bx^3+a}\,Ab^3\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*((b*x^3 + a)^(3/2)*
B*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3
```

**maple [A]** time = 0.05, size = 50, normalized size = 0.78

$$\left(-\frac{2\sqrt{a}\;\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}+\frac{2\sqrt{b\,x^3+a}}{3}\right)A+\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}B}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x,x)
```

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b+A*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/
3*(b*x^3+a)^(1/2))
```

**maxima [A]** time = 1.20, size = 67, normalized size = 1.05

$$\frac{1}{3}\left(\sqrt{a}\,\log\left(\frac{\sqrt{bx^3+a}\,-\sqrt{a}}{\sqrt{bx^3+a}\,+\sqrt{a}}\right)+2\,\sqrt{bx^3+a}\,\right)A+\frac{2\left(bx^3+a\right)^{\frac{3}{2}}B}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
 2*sqrt(b*x^3 + a))*A + 2/9*(b*x^3 + a)^(3/2)*B/b
```

**mupad [B]** time = 2.71, size = 80, normalized size = 1.25

$$\frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9}+\frac{\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{3}\right)}{3\,b}+\frac{A\,\sqrt{a}\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x,x)
```

```
[Out] (2*B*x^3*(a + b*x^3)^(1/2))/9 + ((a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/3))/(3*
b) + (A*a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a
^(1/2)))/x^6))/3
```

**sympy [A]** time = 25.83, size = 76, normalized size = 1.19

$$-\frac{A\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+bx^3}\right)}{3}-\frac{B\left(\begin{cases}-\sqrt{a}\,x^3 & \text{for}\ b=0\\-\frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -A*(-2*a*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*x**3))/3 -
 B*Piecewise((-sqrt(a)*x**3, Eq(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), True
))/3
```

## 3.179 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx$

**Optimal**. Leaf size=84

$$\frac{\sqrt{a+bx^3}\,(2aB+Ab)}{3a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{a+bx^3}\,(2aB+Ab)}{3a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(3*a) - (A*(a + b*x^3)^(3/2))/(3*a*x^3) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^2}\,dx,x,x^3\right)\\ &= -\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{(Ab+2aB)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^3\right)}{6a}\\ &= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{1}{6}(Ab+2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.\\ &= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{(Ab+2aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a\ }\right.}{3b}\\ &= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}-\frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 63, normalized size = 0.75

$$\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\left(2Bx^3-A\right)}{x^3}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
[Out] ((Sqrt[a + b*x^3]*(-A + 2*B*x^3))/x^3 - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x
^3]/Sqrt[a]])/Sqrt[a])/3
```

**IntegrateAlgebraic [A]** time = 0.11, size = 65, normalized size = 0.77

$$\frac{\sqrt{a+bx^3}\left(2Bx^3-A\right)}{3x^3}+\frac{(-2aB-Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
[Out] (Sqrt[a + b*x^3]*(-A + 2*B*x^3))/(3*x^3) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

**fricas [A]** time = 0.90, size = 143, normalized size = 1.70

$$\left[\frac{(2\,Ba+Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(2\,Bax^3-Aa\right)\sqrt{bx^3+a}}{6\,ax^3},\ \frac{(2\,Ba+Ab)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(2\,Bax^3-Aa\right)\sqrt{bx^3+a}}{3\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*
a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b)*
sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*a*x^3 - A*a)*sqrt(b*
x^3 + a))/(a*x^3)]
```

**giac** [A] time = 0.17, size = 68, normalized size = 0.81

$$\frac{2\sqrt{bx^3+a}\,Bb + \frac{\left(2\,Bab+Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^3+a}\,Ab}{x^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(b*x^3 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^3 + a)*A*b/x^3)/b
```

**maple** [A] time = 0.05, size = 72, normalized size = 0.86

$$\left(-\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}} - \frac{\sqrt{b\,x^3+a}}{3x^3}\right)A + \left(-\frac{2\sqrt{a}\ \mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3} + \frac{2\sqrt{b\,x^3+a}}{3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] A*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
+B*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))
```

**maxima** [A] time = 1.26, size = 107, normalized size = 1.27

$$\frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{bx^3+a}}{x^3}\right)A + \frac{1}{3}\left(\sqrt{a}\ \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\sqrt{bx^3+a}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
 - 2*sqrt(b*x^3 + a)/x^3)*A + 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a))) + 2*sqrt(b*x^3 + a))*B
```

**mupad** [B] time = 2.93, size = 76, normalized size = 0.90

$$\frac{2\,B\,\sqrt{b\,x^3+a}}{3} - \frac{A\,\sqrt{b\,x^3+a}}{3\,x^3} + \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(\frac{A\,b}{2}+B\,a\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^4,x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/3 - (A*(a + b*x^3)^(1/2))/(3*x^3) + (log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*((A*b)/2 + B*a))
/(3*a^(1/2))
```

**sympy [A]** time = 43.66, size = 134, normalized size = 1.60

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}-\frac{2B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2B\sqrt{b}\,x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*x
**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2
*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3*sq
rt(a/(b*x**3) + 1))
```

## 3.180 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^7}\,dx$

**Optimal**. Leaf size=88

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{\sqrt{a+bx^3}\,(Ab-4aB)}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}$$

**Rubi [A]** time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{\sqrt{a+bx^3}\,(Ab-4aB)}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a*x^3) - (A*(a + b*x^3)^(3/2))/(6*a*x^6
) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^7}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2}+2aB\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^3\right)}{6a}$$
$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} - \frac{(b(Ab-4aB))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x\right.}{24a}$$
$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} - \frac{(Ab-4aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{12a}$$
$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} + \frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

**Mathematica [A]** time = 0.08, size = 93, normalized size = 1.06

$$\frac{-\left(a+bx^3\right)\left(2a\left(A+2Bx^3\right)+Abx^3\right)-bx^6\sqrt{\frac{bx^3}{a}+1}\,(4aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{12ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] (-((a + b*x^3)*(A*b*x^3 + 2*a*(A + 2*B*x^3))) - b*(-(A*b) + 4*a*B)*x^6*Sqrt
[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]])/(12*a*x^6*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 0.12, size = 79, normalized size = 0.90

$$\frac{\left(Ab^2-4abB\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{\sqrt{a+bx^3}\left(-2aA-4aBx^3-Abx^3\right)}{12ax^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*a*A - A*b*x^3 - 4*a*B*x^3))/(12*a*x^6) + ((A*b^2 - 4*a
*b*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

**fricas [A]** time = 0.76, size = 172, normalized size = 1.95

$$\left[-\frac{\left(4\,Bab-Ab^2\right)\sqrt{a}\,x^6\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(\left(4\,Ba^2+Aab\right)x^3+2\,Aa^2\right)\sqrt{bx^3+a}}{24\,a^2x^6}, \frac{\left(4\,Bab-Ab^2\right)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)-\left(\left(4\,Ba^2+Aab\right)x^3+2\,Aa^2\right)\sqrt{bx^3+a}}{12\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - A*b^2)*sqrt(a)*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a
) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x
```

```
^6), 1/12*((4*B*a*b - A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a
) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

**giac [A]** time = 0.20, size = 120, normalized size = 1.36

$$\frac{\frac{\left(4 Bab^2-Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}\,Ba^2b^2+\left(bx^3+a\right)^{\frac{3}{2}}Ab^3+\sqrt{bx^3+a}\,Aab^3}{ab^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - (
4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^(3/
2)*A*b^3 + sqrt(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b
```

**maple [A]** time = 0.05, size = 96, normalized size = 1.09

$$\left(\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^3+a}\;b}{12a\,x^3}-\frac{\sqrt{b\,x^3+a}}{6x^6}\right)A+\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b\,x^3+a}}{3x^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] B*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
+A*(1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^3+a)^(1/2)/x
^6-1/12*(b*x^3+a)^(1/2)/a*b/x^3)
```

**maxima [B]** time = 1.32, size = 158, normalized size = 1.80

$$-\frac{1}{24}\left(\frac{b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}b^2+\sqrt{bx^3+a}\,ab^2\right)}{\left(bx^3+a\right)^2a-2\left(bx^3+a\right)a^2+a^3}\right)A+\frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{bx^3+a}}{x^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a -
 2*(b*x^3 + a)*a^2 + a^3))*A + 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt
(b*x^3 + a) + sqrt(a)))/sqrt(a) - 2*sqrt(b*x^3 + a)/x^3)*B
```

**mupad [B]** time = 3.12, size = 93, normalized size = 1.06

$$\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\left(A\,b-4\,B\,a\right)}{24\,a^{3/2}}-\frac{\left(4\,B\,a^2+A\,b\,a\right)\sqrt{b\,x^3+a}}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^7,x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)
*(A*b - 4*B*a))/(24*a^(3/2)) - ((4*B*a^2 + A*a*b)*(a + b*x^3)^(1/2))/(12*a^
2*x^3) - (A*(a + b*x^3)^(1/2))/(6*x^6)
```

**sympy [B]** time = 129.14, size = 160, normalized size = 1.82

$$-\frac{Aa}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*sqrt(b)/(4*x**(9/2)*sqr
t(a/(b*x**3) + 1)) - A*b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + A*b*
*2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x*
*3) + 1)/(3*x**(3/2)) - B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

## 3.181 $\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=103

$$\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

**Rubi [A]** time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
b*x^3)^(7/2))/(21*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(27*b^4) + (2*
B*(a + b*x^3)^(11/2))/(33*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3}+\frac{(Ab-\right.\right.\\ &= \frac{2a^2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2(Ab-3aB)\left(a+b\right.}{27b^4}\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 78, normalized size = 0.76

$$\frac{2\left(a+bx^3\right)^{5/2}\left(-48a^3B+8a^2b\left(11A+15Bx^3\right)-10ab^2x^3\left(22A+21Bx^3\right)+35b^3x^6\left(11A+9Bx^3\right)\right)}{10395b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

[Out] (2*(a + b*x^3)^(5/2)*(-48*a^3*B + 35*b^3*x^6*(11*A + 9*B*x^3) + 8*a^2*b*(11*A + 15*B*x^3) - 10*a*b^2*x^3*(22*A + 21*B*x^3)))/(10395*b^4)

**IntegrateAlgebraic [A]** time = 0.06, size = 80, normalized size = 0.78

$$-\frac{2\left(a+bx^3\right)^{5/2}\left(48a^3B-88a^2Ab-120a^2bBx^3+220aAb^2x^3+210ab^2Bx^6-385Ab^3x^6-315b^3Bx^9\right)}{10395b^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]

[Out] (-2*(a + b*x^3)^(5/2)*(-88*a^2*A*b + 48*a^3*B + 220*a*A*b^2*x^3 - 120*a^2*b*B*x^3 - 385*A*b^3*x^6 + 210*a*b^2*B*x^6 - 315*b^3*B*x^9))/(10395*b^4)

**fricas [A]** time = 1.11, size = 124, normalized size = 1.20

$$\frac{2\left(315\,Bb^5x^{15}+35\left(12\,Bab^4+11\,Ab^5\right)x^{12}+5\left(3\,Ba^2b^3+110\,Aab^4\right)x^9-3\left(6\,Ba^3b^2-11\,Aa^2b^3\right)x^6-48\,Ba^5+88\,Aa^4b+4\left(6\,Ba^4b-11\,Aa^3b^2\right)x^3\right)\sqrt{bx^3+a}}{10395\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")

[Out] 2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3 + 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A*a^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4

**giac [A]** time = 0.16, size = 104, normalized size = 1.01

$$\frac{2\left(315\left(bx^3+a\right)^{\frac{11}{2}}B-1155\left(bx^3+a\right)^{\frac{9}{2}}Ba+1485\left(bx^3+a\right)^{\frac{7}{2}}Ba^2-693\left(bx^3+a\right)^{\frac{5}{2}}Ba^3+385\left(bx^3+a\right)^{\frac{9}{2}}Ab-990\left(bx^3+a\right)^{\frac{7}{2}}Aab+693\left(bx^3+a\right)^{\frac{5}{2}}Aa^2b\right)}{10395\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")

[Out] 2/10395*(315*(b*x^3 + a)^(11/2)*B - 1155*(b*x^3 + a)^(9/2)*B*a + 1485*(b*x^3 + a)^(7/2)*B*a^2 - 693*(b*x^3 + a)^(5/2)*B*a^3 + 385*(b*x^3 + a)^(9/2)*A*b - 990*(b*x^3 + a)^(7/2)*A*a*b + 693*(b*x^3 + a)^(5/2)*A*a^2*b)/b^4

**maple [A]** time = 0.05, size = 77, normalized size = 0.75

$$\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(315B\,x^9b^3+385A\,b^3x^6-210Ba\,b^2x^6-220Aa\,b^2x^3+120B\,a^2b\,x^3+88A\,a^2b-48B\,a^3\right)}{10395b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x)

[Out] 2/10395*(b*x^3+a)^(5/2)*(315*B*b^3*x^9+385*A*b^3*x^6-210*B*a*b^2*x^6-220*A*a*b^2*x^3+120*B*a^2*b*x^3+88*A*a^2*b-48*B*a^3)/b^4

**maxima [A]** time = 0.61, size = 118, normalized size = 1.15

$$\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)A+\frac{2}{3465}\left(\frac{105\left(bx^3+a\right)^{\frac{11}{2}}}{b^4}-\frac{385\left(bx^3+a\right)^{\frac{9}{2}}a}{b^4}+\frac{495\left(bx^3+a\right)^{\frac{7}{2}}a^2}{b^4}-\frac{231\left(bx^3+a\right)^{\frac{5}{2}}a^3}{b^4}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")

[Out] 2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a)^

```
(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/b^4
)*B
```

**mupad [B]** time = 2.65, size = 206, normalized size = 2.00

$$\frac{20\,A\,a\,x^9\,\sqrt{b\,x^3+a}}{189}+\frac{2\,A\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{8\,B\,a\,x^{12}\,\sqrt{b\,x^3+a}}{99}+\frac{2\,B\,b\,x^{15}\,\sqrt{b\,x^3+a}}{33}+\frac{16\,A\,a^4\,\sqrt{b\,x^3+a}}{945\,b^3}-\frac{32\,B\,a^5\,\sqrt{b\,x^3+a}}{3465\,b^4}-\frac{8\,A\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^2}+\frac{2\,A\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b}+\frac{16\,B\,a^4\,x^3\,\sqrt{b\,x^3+a}}{3465\,b^3}-\frac{4\,B\,a^3\,x^6\,\sqrt{b\,x^3+a}}{1155\,b^2}+\frac{2\,B\,a^2\,x^9\,\sqrt{b\,x^3+a}}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (20*A*a*x^9*(a + b*x^3)^(1/2))/189 + (2*A*b*x^12*(a + b*x^3)^(1/2))/27 + (8
*B*a*x^12*(a + b*x^3)^(1/2))/99 + (2*B*b*x^15*(a + b*x^3)^(1/2))/33 + (16*A
*a^4*(a + b*x^3)^(1/2))/(945*b^3) - (32*B*a^5*(a + b*x^3)^(1/2))/(3465*b^4)
 - (8*A*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2) + (2*A*a^2*x^6*(a + b*x^3)^(1/
2))/(315*b) + (16*B*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3) - (4*B*a^3*x^6*(a
 + b*x^3)^(1/2))/(1155*b^2) + (2*B*a^2*x^9*(a + b*x^3)^(1/2))/(693*b)
```

**sympy [A]** time = 8.44, size = 267, normalized size = 2.59

$$\begin{cases} \frac{16Aa^4\sqrt{a+bx^3}}{945b^3}-\frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Aa^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Aax^9\sqrt{a+bx^3}}{189}+\frac{2Abx^{12}\sqrt{a+bx^3}}{27}-\frac{32Ba^5\sqrt{a+bx^3}}{3465b^4}+\frac{16Ba^4x^3\sqrt{a+bx^3}}{3465b^3}-\frac{4Ba^3x^6\sqrt{a+bx^3}}{1155b^2}+\frac{2Ba^2x^9\sqrt{a+bx^3}}{693b}+\frac{8Bax^{12}\sqrt{a+bx^3}}{99}+\frac{2Bbx^{15}\sqrt{a+bx^3}}{33} & \text{for } b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((16*A*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*A*a**3*x**3*sqrt(a + b
*x**3)/(945*b**2) + 2*A*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*A*a*x**9*sq
rt(a + b*x**3)/189 + 2*A*b*x**12*sqrt(a + b*x**3)/27 - 32*B*a**5*sqrt(a + b
*x**3)/(3465*b**4) + 16*B*a**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4*B*a**3
*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*B*a**2*x**9*sqrt(a + b*x**3)/(693*b)
 + 8*B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33, Ne(b,
 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))
```

## 3.182 $\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{7/2}(Ab-2aB)}{21b^3}-\frac{2a\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{7/2}(Ab-2aB)}{21b^3}-\frac{2a\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3
)^(7/2))/(21*b^3) + (2*B*(a + b*x^3)^(9/2))/(27*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx &= \frac{1}{3}\text{Subst}\left(\int x(a+bx)^{3/2}(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2}+\frac{(Ab-2aB)(a+bx)^{5/2}}{b^2}+\frac{B(a+bx)}{b^2}\right.\right.\\ &= -\frac{2a(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{7/2}}{21b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 57, normalized size = 0.78

$$\frac{2\left(a+bx^3\right)^{5/2}\left(8a^2B-2ab\left(9A+10Bx^3\right)+5b^2x^3\left(9A+7Bx^3\right)\right)}{945b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

[Out] (2*(a + b*x^3)^(5/2)*(8*a^2*B + 5*b^2*x^3*(9*A + 7*B*x^3) - 2*a*b*(9*A + 10*B*x^3)))/(945*b^3)

**IntegrateAlgebraic [A]** time = 0.04, size = 56, normalized size = 0.77

$$\frac{2\left(a+bx^3\right)^{5/2}\left(8a^2B-18aAb-20abBx^3+45Ab^2x^3+35b^2Bx^6\right)}{945b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]

[Out] (2*(a + b*x^3)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^3 - 20*a*b*B*x^3 + 35*b^2*B*x^6))/(945*b^3)

**fricas [A]** time = 0.82, size = 99, normalized size = 1.36

$$\frac{2\left(35\,Bb^4x^{12}+5\left(10\,Bab^3+9\,Ab^4\right)x^9+3\left(Ba^2b^2+24\,Aab^3\right)x^6+8\,Ba^4-18\,Aa^3b-\left(4\,Ba^3b-9\,Aa^2b^2\right)x^3\right)\sqrt{bx^3+a}}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")

[Out] 2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*a*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*x^3 + a)/b^3

**giac [A]** time = 0.18, size = 73, normalized size = 1.00

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}B-90\left(bx^3+a\right)^{\frac{7}{2}}Ba+63\left(bx^3+a\right)^{\frac{5}{2}}Ba^2+45\left(bx^3+a\right)^{\frac{7}{2}}Ab-63\left(bx^3+a\right)^{\frac{5}{2}}Aab\right)}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")

[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 90*(b*x^3 + a)^(7/2)*B*a + 63*(b*x^3 + a)^(5/2)*B*a^2 + 45*(b*x^3 + a)^(7/2)*A*b - 63*(b*x^3 + a)^(5/2)*A*a*b)/b^3

**maple [A]** time = 0.05, size = 53, normalized size = 0.73

$$-\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(-35B\,b^2x^6-45A\,b^2x^3+20Bab\,x^3+18Aab-8B\,a^2\right)}{945b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x)

[Out] -2/945*(b*x^3+a)^(5/2)*(-35*B*b^2*x^6-45*A*b^2*x^3+20*B*a*b*x^3+18*A*a*b-8*B*a^2)/b^3

**maxima [A]** time = 0.61, size = 84, normalized size = 1.15

$$\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)A+\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*(
b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^
2/b^3)*B
```

**mupad [B]** time = 2.72, size = 211, normalized size = 2.89

$$\frac{x^6\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{15\,b}-\frac{2\,a\left(2\,A\,a^2-\frac{4\,a\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\sqrt{b\,x^3+a}}{9\,b^2}+\frac{2\,B\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^3\left(2\,A\,a^2-\frac{4\,a\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\sqrt{b\,x^3+a}}{9\,b}+\frac{x^9\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)\sqrt{b\,x^3+a}}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (x^6*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9))/
(7*b)))/(15*b) - (2*a*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 +
(20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b^2) + (2*B*b*x^12*(a +
 b*x^3)^(1/2))/27 + (x^3*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2
 + (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b) + (x^9*(2*A*b^2 +
 (20*B*a*b)/9)*(a + b*x^3)^(1/2))/(21*b)
```

**sympy [A]** time = 4.75, size = 216, normalized size = 2.96

$$\begin{cases} -\frac{4Aa^3\sqrt{a+bx^3}}{105b^2}+\frac{2Aa^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Aax^6\sqrt{a+bx^3}}{105}+\frac{2Abx^9\sqrt{a+bx^3}}{21}+\frac{16Ba^4\sqrt{a+bx^3}}{945b^3}-\frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Ba^2x^6\sqrt{a+bx^3}}{315b}+\frac{20Bax^9\sqrt{a+bx^3}}{189}+\frac{2Bbx^{12}\sqrt{a+bx^3}}{27} & \text{for } b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a + b
*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b*x
**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a + b*
x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9*sqr
t(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*(
A*x**6/6 + B*x**9/9), True))
```

## 3.183 $\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=46

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^2) + (2*B*(a + b*x^3)^(7/2))/(21*b^
2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx &= \frac{1}{3}\operatorname{Subst}\left(\int (a+bx)^{3/2}(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)dx,x,x^3\right)\\ &= \frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{5/2}\left(-2aB+7Ab+5bBx^3\right)}{105b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{5/2}\left(-2aB+7Ab+5bBx^3\right)}{105b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

**fricas** [A] time = 1.02, size = 73, normalized size = 1.59

$$\frac{2\left(5\,Bb^3x^9+\left(8\,Bab^2+7\,Ab^3\right)x^6-2\,Ba^3+7\,Aa^2b+\left(Ba^2b+14\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*a
^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2
```

**giac** [A] time = 0.17, size = 44, normalized size = 0.96

$$\frac{2\left(5\left(bx^3+a\right)^{\frac{7}{2}}B-7\left(bx^3+a\right)^{\frac{5}{2}}Ba+7\left(bx^3+a\right)^{\frac{5}{2}}Ab\right)}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)*B - 7*(b*x^3 + a)^(5/2)*B*a + 7*(b*x^3 + a)^(5/2
)*A*b)/b^2
```

**maple** [A] time = 0.05, size = 31, normalized size = 0.67

$$\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(5Bb\,x^3+7Ab-2Ba\right)}{105b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] 2/105*(b*x^3+a)^(5/2)*(5*B*b*x^3+7*A*b-2*B*a)/b^2
```

**maxima** [A] time = 0.49, size = 49, normalized size = 1.07

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}A}{15\,b}+\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)
^(5/2)*a/b^2)*B
```

**mupad [B]** time = 3.35, size = 150, normalized size = 3.26

$$\frac{\left(2\,A\,a^2-\frac{2\,a\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{3\,b}\right)\,\sqrt{b\,x^3+a}}{3\,b}+\frac{x^3\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{9\,b}+\frac{2\,B\,b\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)\,\sqrt{b\,x^3+a}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] ((2*A*a^2 - (2*a*(2*B*a^2 + 4*A*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b))
)/(3*b))*(a + b*x^3)^(1/2))/(3*b) + (x^3*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a
*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)))/(9*b) + (2*B*b*x^9*(a + b*x^3)^
(1/2))/21 + (x^6*(2*A*b^2 + (16*B*a*b)/7)*(a + b*x^3)^(1/2))/(15*b)
```

**sympy [A]** time = 2.52, size = 165, normalized size = 3.59

$$\begin{cases} \frac{2Aa^2\sqrt{a+bx^3}}{15b}+\frac{4Aax^3\sqrt{a+bx^3}}{15}+\frac{2Abx^6\sqrt{a+bx^3}}{15}-\frac{4Ba^3\sqrt{a+bx^3}}{105b^2}+\frac{2Ba^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Bax^6\sqrt{a+bx^3}}{105}+\frac{2Bbx^9\sqrt{a+bx^3}}{21} & \text{for } b\neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/1
5 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2) +
 2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/105
+ 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**6/6
), True))
```

## 3.184 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} dx$

**Optimal**. Leaf size=81

$$-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}$$

**Rubi [A]** time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*Sqrt[a + b*x^3])/3 + (2*A*(a + b*x^3)^(3/2))/9 + (2*B*(a + b*x^3)^(5
/2))/(15*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx,x,x^3\right)\\
&= \frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{1}{3}A\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^3\right)\\
&= \frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{1}{3}(aA)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^3\right)\\
&= \frac{2}{3}aA\sqrt{a+bx^3}+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{1}{3}\left(a^2A\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+
\right.\right.\\
&= \frac{2}{3}aA\sqrt{a+bx^3}+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{\left(2a^2A\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.\right.}{3b}\\
&= \frac{2}{3}aA\sqrt{a+bx^3}+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.08, size = 80, normalized size = 0.99

$$
\frac{2\left(-15a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+5Ab\left(a+bx^3\right)^{3/2}+15aAb\sqrt{a+bx^3}+3B\left(a+bx^3\right)^{5/2}\right)}{45b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*(15*a*A*b*Sqrt[a + b*x^3] + 5*A*b*(a + b*x^3)^(3/2) + 3*B*(a + b*x^3)^(5
/2) - 15*a^(3/2)*A*b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/(45*b)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 85, normalized size = 1.05

$$
\frac{2\sqrt{a+bx^3}\left(3a^2B+20aAb+6abBx^3+5Ab^2x^3+3b^2Bx^6\right)}{45b}-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^3 + 6*a*b*B*x^3 + 3*b^2*
B*x^6))/(45*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

**fricas [A]** time = 0.82, size = 172, normalized size = 2.12

$$
\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Bb^2x^6+\left(6\,Bab+5\,Ab^2\right)x^3+3\,Ba^2+20\,Aab\right)\sqrt{bx^3+a}}{45\,b},\ \frac{2\left(15\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(3\,Bb^2x^6+\left(6\,Bab+5\,Ab^2\right)x^3+3\,Ba^2+20\,Aab\right)\sqrt{bx^3+a}\right)}{45\,b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] [1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 +
```

```
a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*B*b
^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b]
```

**giac [A]** time = 0.16, size = 80, normalized size = 0.99

$$\frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}Bb^4+5\left(bx^3+a\right)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx^3+a}\,Aab^5\right)}{45\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^(
5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5
```

**maple [A]** time = 0.05, size = 66, normalized size = 0.81

$$\left(\frac{2\sqrt{b\,x^3+a}\;b\,x^3}{9}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}+\frac{8\sqrt{b\,x^3+a}\;a}{9}\right)A+\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}B}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x)
```

```
[Out] 2/15*B*(b*x^3+a)^(5/2)/b+A*(2/9*(b*x^3+a)^(1/2)*b*x^3+8/9*(b*x^3+a)^(1/2)*a
-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

**maxima [A]** time = 1.26, size = 80, normalized size = 0.99

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}B}{15\,b}+\frac{1}{9}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\left(bx^3+a\right)^{\frac{3}{2}}+6\,\sqrt{bx^3+a}\,a\right)A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*B/b + 1/9*(3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a))
/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3/2) + 6*sqrt(b*x^3 + a)*a)*
A
```

**mupad [B]** time = 2.79, size = 131, normalized size = 1.62

$$\frac{A\,a^{3/2}\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}+\frac{\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{2\,a\,\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)}{3\,b}\right)}{3\,b}+\frac{2\,B\,b\,x^6\,\sqrt{b\,x^3+a}}{15}+\frac{x^3\,\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x,x)
```

```
[Out] (A*a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2
)))/x^6))/3 + ((a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (2*a*(2*A*b^2 + (12*B
*a*b)/5))/(3*b)))/(3*b) + (2*B*b*x^6*(a + b*x^3)^(1/2))/15 + (x^3*(2*A*b^2
+ (12*B*a*b)/5)*(a + b*x^3)^(1/2))/(9*b)
```

**sympy [A]** time = 66.50, size = 82, normalized size = 1.01

$$\frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2Aa\sqrt{a+bx^3}}{3}+\frac{2A\left(a+bx^3\right)^{\frac{3}{2}}}{9}+\frac{2B\left(a+bx^3\right)^{\frac{5}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

```
[Out] 2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*sqrt(-a)) + 2*A*a*sqrt(a + b*x*
*3)/3 + 2*A*(a + b*x**3)**(3/2)/9 + 2*B*(a + b*x**3)**(5/2)/(15*b)
```

## 3.185 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx$

**Optimal**. Leaf size=110

$$\frac{(a+bx^3)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}\,(2aB+3Ab)-\frac{1}{3}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{A(a+bx^3)^{5/2}}{3ax^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{(a+bx^3)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}\,(2aB+3Ab)-\frac{1}{3}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{A(a+bx^3)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^3])/3 + ((3*A*b + 2*a*B)*(a + b*x^3)^(3/2))/(
9*a) - (A*(a + b*x^3)^(5/2))/(3*a*x^3) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^4}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^3\right)}{3a}$$

$$= \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{1}{6}(3Ab+2aB)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\, \ldots\right.$$

$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{1}{6}(a(3A\ldots$$

$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{(a(3Ab\ldots}{\ldots}$$

$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} - \frac{1}{3}\sqrt{a}\,(3A\ldots$$

**Mathematica [A]** time = 0.09, size = 80, normalized size = 0.73

$$\frac{1}{9}\left(\frac{\sqrt{a+bx^3}\left(-3aA+8aBx^3+6Abx^3+2bBx^6\right)}{x^3} - 3\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/x^3 - 3*Sqr
t[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/9
```

**IntegrateAlgebraic [A]** time = 0.11, size = 85, normalized size = 0.77

$$\frac{1}{3}\left(-2a^{3/2}B-3\sqrt{a}\,Ab\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{\sqrt{a+bx^3}\left(-3aA+8aBx^3+6Abx^3+2bBx^6\right)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/(9*x^3) + ((
-3*Sqrt[a]*A*b - 2*a^(3/2)*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

**fricas [A]** time = 0.57, size = 169, normalized size = 1.54

$$\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(2\,Bbx^6+2\,(4\,Ba+3\,Ab)x^3-3\,Aa\right)\sqrt{bx^3+a}}{18\,x^3}, \frac{3\,(2\,Ba+3\,Ab)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(2\,Bbx^6+2\,(4\,Ba+3\,Ab)x^3-3\,Aa\right)\sqrt{bx^3+a}}{9\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a)
 + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a
))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]
```

**giac [A]** time = 0.20, size = 103, normalized size = 0.94

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}Bb+6\sqrt{bx^3+a}\,Bab+6\sqrt{bx^3+a}\,Ab^2+\frac{3\left(2\,Ba^2b+3\,Aab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\sqrt{bx^3+a}\,Aab}{x^3}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(2*(b*x^3 + a)^(3/2)*B*b + 6*sqrt(b*x^3 + a)*B*a*b + 6*sqrt(b*x^3 + a)*
A*b^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a)
 - 3*sqrt(b*x^3 + a)*A*a*b/x^3)/b
```

**maple [A]** time = 0.05, size = 101, normalized size = 0.92

$$\left(-\sqrt{a}\,b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\frac{2\sqrt{b\,x^3+a}\,b}{3}-\frac{\sqrt{b\,x^3+a}\,a}{3x^3}\right)A+\left(\frac{2\sqrt{b\,x^3+a}\,b\,x^3}{9}-\frac{2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}+\frac{8\sqrt{b\,x^3+a}\,a}{9}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x)
```

```
[Out] A*(-1/3*(b*x^3+a)^(1/2)*a/x^3+2/3*(b*x^3+a)^(1/2)*b-b*arctanh((b*x^3+a)^(1/
2)/a^(1/2))*a^(1/2))+B*(2/9*(b*x^3+a)^(1/2)*b*x^3+8/9*(b*x^3+a)^(1/2)*a-2/3
*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

**maxima [A]** time = 1.44, size = 134, normalized size = 1.22

$$\frac{1}{6}\left(3\sqrt{a}\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+4\sqrt{bx^3+a}\,b-\frac{2\sqrt{bx^3+a}\,a}{x^3}\right)A+\frac{1}{9}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\left(bx^3+a\right)^{\frac{3}{2}}+6\sqrt{bx^3+a}\,a\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
)) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a)*a/x^3)*A + 1/9*(3*a^(3/2)*log(
(sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3
/2) + 6*sqrt(b*x^3 + a)*a)*B
```

**mupad [B]** time = 3.38, size = 111, normalized size = 1.01

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(3\,A\,b+2\,B\,a\right)\sqrt{\frac{a}{4}}}{3}+\frac{\left(2\,A\,b^2+\frac{8\,B\,a\,b}{3}\right)\sqrt{b\,x^3+a}}{3\,b}-\frac{A\,a\sqrt{b\,x^3+a}}{3\,x^3}+\frac{2\,B\,b\,x^3\sqrt{b\,x^3+a}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^4,x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(
3*A*b + 2*B*a)*(a/4)^(1/2))/3 + ((2*A*b^2 + (8*B*a*b)/3)*(a + b*x^3)^(1/2))
/(3*b) - (A*a*(a + b*x^3)^(1/2))/(3*x^3) + (2*B*b*x^3*(a + b*x^3)^(1/2))/9
```

**sympy [A]** time = 58.40, size = 223, normalized size = 2.03

$$-A\sqrt{a}\,b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)-\frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}-\frac{2Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba^2}{3\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ba\sqrt{b}\,x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}+Bb\left(\begin{cases}\frac{\sqrt{a}\,x^3}{3} & \text{for } b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - A*a*sqrt(b)*sqrt(a/(b*x**3
) + 1)/(3*x**(3/2)) + 2*A*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*A
*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - 2*B*a**(3/2)*asinh(sqrt(a)/(s
qrt(b)*x**(3/2)))/3 + 2*B*a**2/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) +
2*B*a*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + B*b*Piecewise((sqrt(a)*x*
*3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

## 3.186 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^7}\,dx$

**Optimal**. Leaf size=115

$$-\frac{\left(a+bx^3\right)^{3/2}(4aB+Ab)}{12ax^3}+\frac{b\sqrt{a+bx^3}\,(4aB+Ab)}{4a}-\frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6}$$

**Rubi [A]** time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{\left(a+bx^3\right)^{3/2}(4aB+Ab)}{12ax^3}+\frac{b\sqrt{a+bx^3}\,(4aB+Ab)}{4a}-\frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] (b*(A*b + 4*a*B)*Sqrt[a + b*x^3])/(4*a) - ((A*b + 4*a*B)*(a + b*x^3)^(3/2))
/(12*a*x^3) - (A*(a + b*x^3)^(5/2))/(6*a*x^6) - (b*(A*b + 4*a*B)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^7}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(Ab+4aB)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^3\right)}{12a}$$
$$= -\frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(b(Ab+4aB))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, \right.}{8a}$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{8}(b(Ab+4$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{4}(Ab+4a$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} - \frac{b(Ab+4aB}{}$$

**Mathematica [C]** time = 0.04, size = 59, normalized size = 0.51

$$\frac{\left(a+bx^3\right)^{5/2}\left(bx^6(4aB+Ab)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\right)-5a^2A\right)}{30a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] ((a + b*x^3)^(5/2)*(-5*a^2*A + b*(A*b + 4*a*B)*x^6*Hypergeometric2F1[2, 5/2
, 7/2, 1 + (b*x^3)/a]))/(30*a^3*x^6)
```

**IntegrateAlgebraic [A]** time = 0.15, size = 84, normalized size = 0.73

$$\frac{\left(-4abB-Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} + \frac{\sqrt{a+bx^3}\left(-2aA-4aBx^3-5Abx^3+8bBx^6\right)}{12x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*a*A - 5*A*b*x^3 - 4*a*B*x^3 + 8*b*B*x^6))/(12*x^6) + (
(-(A*b^2) - 4*a*b*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

**fricas** [A] time = 0.89, size = 191, normalized size = 1.66

$$\left[\frac{3\left(4\,Bab+Ab^{2}\right)\sqrt{a}\,x^{6}\log\left(\frac{bx^{3}-2\,\sqrt{bx^{3}+a}\,\sqrt{a}+2\,a}{x^{3}}\right)+2\left(8\,Babx^{6}-\left(4\,Ba^{2}+5\,Aab\right)x^{3}-2\,Aa^{2}\right)\sqrt{bx^{3}+a}}{24\,ax^{6}},\ \frac{3\left(4\,Bab+Ab^{2}\right)\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{bx^{3}+a}\,\sqrt{-a}}{a}\right)+\left(8\,Babx^{6}-\left(4\,Ba^{2}+5\,Aab\right)x^{3}-2\,Aa^{2}\right)\sqrt{bx^{3}+a}}{12\,ax^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(
a) + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt(b
*x^3 + a))/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^
3 + a)*sqrt(-a)/a) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt
(b*x^3 + a))/(a*x^6)]
```

**giac** [A] time = 0.18, size = 131, normalized size = 1.14

$$\frac{8\,\sqrt{bx^{3}+a}\,Bb^{2}+\frac{3\left(4\,Bab^{2}+Ab^{3}\right)\arctan\left(\frac{\sqrt{bx^{3}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4\left(bx^{3}+a\right)^{\frac{3}{2}}Bab^{2}-4\,\sqrt{bx^{3}+a}\,Ba^{2}b^{2}+5\left(bx^{3}+a\right)^{\frac{3}{2}}Ab^{3}-3\,\sqrt{bx^{3}+a}\,Aab^{3}}{b^{2}x^{6}}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(8*sqrt(b*x^3 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^3 + a
)/sqrt(-a))/sqrt(-a) - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a
^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^3 + a)*A*a*b^3)/(b^2*x^6))/
b
```

**maple** [A] time = 0.05, size = 107, normalized size = 0.93

$$\left(-\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{5\sqrt{b\,x^{3}+a}\;b}{12x^{3}}-\frac{\sqrt{b\,x^{3}+a}\;a}{6x^{6}}\right)A+\left(-\sqrt{a}\;b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\frac{2\sqrt{b\,x^{3}+a}\;b}{3}-\frac{\sqrt{b\,x^{3}+a}\;a}{3x^{3}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x)
```

```
[Out] B*(-1/3*(b*x^3+a)^(1/2)*a/x^3+2/3*(b*x^3+a)^(1/2)*b-b*arctanh((b*x^3+a)^(1/
2)/a^(1/2))*a^(1/2))+A*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*(b*x^3+a)^(1/2)*b/x
^3-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
```

**maxima** [A] time = 1.31, size = 171, normalized size = 1.49

$$\frac{1}{24}\left(\frac{3\,b^{2}\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(5\left(bx^{3}+a\right)^{\frac{3}{2}}b^{2}-3\,\sqrt{bx^{3}+a}\,ab^{2}\right)}{\left(bx^{3}+a\right)^{2}-2\left(bx^{3}+a\right)a+a^{2}}\right)A+\frac{1}{6}\left(3\,\sqrt{a}\,b\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)+4\,\sqrt{bx^{3}+a}\,b-\frac{2\,\sqrt{bx^{3}+a}\,a}{x^{3}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^
2 - 2*(b*x^3 + a)*a + a^2))*A + 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a
)*a/x^3)*B
```

**mupad [B]** time = 3.47, size = 110, normalized size = 0.96

$$\frac{2\,B\,b\,\sqrt{b\,x^3+a}}{3}-\frac{\sqrt{b\,x^3+a}\,\left(4\,B\,a^3+5\,A\,b\,a^2\right)}{12\,a^2\,x^3}-\frac{A\,a\,\sqrt{b\,x^3+a}}{6\,x^6}+\frac{b\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(A\,b+4\,B\,a\right)}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^7,x)
```

```
[Out] (2*B*b*(a + b*x^3)^(1/2))/3 - ((a + b*x^3)^(1/2)*(4*B*a^3 + 5*A*a^2*b))/(12
*a^2*x^3) - (A*a*(a + b*x^3)^(1/2))/(6*x^6) + (b*log((((a + b*x^3)^(1/2) -
a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(A*b + 4*B*a))/(8*a^(1/2))
```

**sympy [B]** time = 153.11, size = 243, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Aa\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}-B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Ba\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Bb^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/2
)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*
b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - B
*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

## 3.187 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=103

$$\frac{2a^2\sqrt{a+bx^3}\,(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4}-\frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

**Rubi** [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\sqrt{a+bx^3}\,(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4}-\frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x
^3)^(3/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*B*(a
 + b*x^3)^(7/2))/(21*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}}+\frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3}+\frac{(Ab-3aB)(a+bx)^{3/2}}{b^3}+\frac{B(\right.\right.\\ &= \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2B}{}\end{aligned}$$

**Mathematica** [A] time = 0.08, size = 78, normalized size = 0.76

$$\frac{2\sqrt{a+bx^3}\left(-48a^3B+8a^2b\left(7A+3Bx^3\right)-2ab^2x^3\left(14A+9Bx^3\right)+3b^3x^6\left(7A+5Bx^3\right)\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x^3) + 3*b^3*x^6*(7*A +
5*B*x^3) - 2*a*b^2*x^3*(14*A + 9*B*x^3)))/(315*b^4)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 80, normalized size = 0.78

$$-\frac{2\sqrt{a+bx^3}\left(48a^3B-56a^2Ab-24a^2bBx^3+28aAb^2x^3+18ab^2Bx^6-21Ab^3x^6-15b^3Bx^9\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*Sqrt[a + b*x^3]*(-56*a^2*A*b + 48*a^3*B + 28*a*A*b^2*x^3 - 24*a^2*b*B*x
^3 - 21*A*b^3*x^6 + 18*a*b^2*B*x^6 - 15*b^3*B*x^9))/(315*b^4)
```

**fricas [A]** time = 0.92, size = 76, normalized size = 0.74

$$\frac{2\left(15\,Bb^3x^9-3\left(6\,Bab^2-7\,Ab^3\right)x^6-48\,Ba^3+56\,Aa^2b+4\left(6\,Ba^2b-7\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

**giac [A]** time = 0.16, size = 101, normalized size = 0.98

$$-\frac{2\left(Ba^3-Aa^2b\right)\sqrt{bx^3+a}}{3\,b^4}+\frac{2\left(15\left(bx^3+a\right)^{\frac{7}{2}}B-63\left(bx^3+a\right)^{\frac{5}{2}}Ba+105\left(bx^3+a\right)^{\frac{3}{2}}Ba^2+21\left(bx^3+a\right)^{\frac{5}{2}}Ab-70\left(bx^3+a\right)^{\frac{3}{2}}Aab\right)}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^3 - A*a^2*b)*sqrt(b*x^3 + a)/b^4 + 2/315*(15*(b*x^3 + a)^(7/2)*B
- 63*(b*x^3 + a)^(5/2)*B*a + 105*(b*x^3 + a)^(3/2)*B*a^2 + 21*(b*x^3 + a)^(
5/2)*A*b - 70*(b*x^3 + a)^(3/2)*A*a*b)/b^4
```

**maple [A]** time = 0.04, size = 77, normalized size = 0.75

$$\frac{2\sqrt{b\,x^3+a}\,\left(15B\,x^9b^3+21A\,b^3x^6-18Ba\,b^2x^6-28Aa\,b^2x^3+24B\,a^2b\,x^3+56A\,a^2b-48B\,a^3\right)}{315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/315*(b*x^3+a)^(1/2)*(15*B*b^3*x^9+21*A*b^3*x^6-18*B*a*b^2*x^6-28*A*a*b^2*
x^3+24*B*a^2*b*x^3+56*A*a^2*b-48*B*a^3)/b^4
```

**maxima [A]** time = 0.59, size = 118, normalized size = 1.15

$$\frac{2}{105}\,B\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^4}-\frac{21\left(bx^3+a\right)^{\frac{5}{2}}a}{b^4}+\frac{35\left(bx^3+a\right)^{\frac{3}{2}}a^2}{b^4}-\frac{35\sqrt{bx^3+a}\,a^3}{b^4}\right)+\frac{2}{45}\,A\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^3}-\frac{10\left(bx^3+a\right)^{\frac{3}{2}}a}{b^3}+\frac{15\sqrt{bx^3+a}\,a^2}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*B*(5*(b*x^3 + a)^(7/2)/b^4 - 21*(b*x^3 + a)^(5/2)*a/b^4 + 35*(b*x^3 +
 a)^(3/2)*a^2/b^4 - 35*sqrt(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^(5/
2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3)
```

**mupad [B]** time = 2.68, size = 104, normalized size = 1.01

$$\frac{8\,a^2\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^3}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{15\,b}+\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21\,b}-\frac{4\,a\,x^3\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (8*a^2*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^3) + (x^6*(a + b*x^3
)^(1/2)*(2*A - (12*B*a)/(7*b)))/(15*b) + (2*B*x^9*(a + b*x^3)^(1/2))/(21*b)
 - (4*a*x^3*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^2)
```

**sympy [A]** time = 3.44, size = 175, normalized size = 1.70

$$\begin{cases} \frac{16Aa^2\sqrt{a+bx^3}}{45b^3} - \frac{8Aax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Ax^6\sqrt{a+bx^3}}{15b} - \frac{32Ba^3\sqrt{a+bx^3}}{105b^4} + \frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4Bax^6\sqrt{a+bx^3}}{35b^2} + \frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x**
3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x**3
)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*sqrt
(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), ((A*x
**9/9 + B*x**12/12)/sqrt(a), True))
```

## 3.188 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-2aB)}{9b^3}-\frac{2a\sqrt{a+bx^3}\,(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-2aB)}{9b^3}-\frac{2a\sqrt{a+bx^3}\,(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(
3/2))/(9*b^3) + (2*B*(a + b*x^3)^(5/2))/(15*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}}+\frac{(Ab-2aB)\sqrt{a+bx}}{b^2}+\frac{B(a+bx)^{3/2}}{b^2}\right)dx, x, x^3\right)\\
&= -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 56, normalized size = 0.77

$$\frac{2\sqrt{a+bx^3}\left(8a^2B-2ab\left(5A+2Bx^3\right)+b^2x^3\left(5A+3Bx^3\right)\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(8*a^2*B - 2*a*b*(5*A + 2*B*x^3) + b^2*x^3*(5*A + 3*B*x^
3)))/(45*b^3)
```

**IntegrateAlgebraic [A]** time = 0.04, size = 56, normalized size = 0.77

$$\frac{2\sqrt{a+bx^3}\left(8a^2B-10aAb-4abBx^3+5Ab^2x^3+3b^2Bx^6\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^3 - 4*a*b*B*x^3 + 3*b^2
*B*x^6))/(45*b^3)
```

**fricas [A]** time = 0.81, size = 52, normalized size = 0.71

$$\frac{2\left(3\,Bb^2x^6-\left(4\,Bab-5\,Ab^2\right)x^3+8\,Ba^2-10\,Aab\right)\sqrt{bx^3+a}}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x^
3 + a)/b^3
```

**giac [A]** time = 0.16, size = 70, normalized size = 0.96

$$\frac{2\,\sqrt{bx^3+a}\left(Ba^2-Aab\right)}{3\,b^3}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}B-10\left(bx^3+a\right)^{\frac{3}{2}}Ba+5\left(bx^3+a\right)^{\frac{3}{2}}Ab\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*(B*a^2 - A*a*b)/b^3 + 2/45*(3*(b*x^3 + a)^(5/2)*B - 10*
(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^3
```

**maple [A]** time = 0.05, size = 53, normalized size = 0.73

$$-\frac{2\sqrt{b\,x^3+a}\,\left(-3B\,b^2x^6-5A\,b^2x^3+4Bab\,x^3+10Aab-8B\,a^2\right)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/45*(b*x^3+a)^(1/2)*(-3*B*b^2*x^6-5*A*b^2*x^3+4*B*a*b*x^3+10*A*a*b-8*B*a^
2)/b^3
```

**maxima [A]** time = 0.45, size = 83, normalized size = 1.14

$$\frac{2}{45}\,B\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^3}-\frac{10\left(bx^3+a\right)^{\frac{3}{2}}a}{b^3}+\frac{15\,\sqrt{bx^3+a}\,a^2}{b^3}\right)+\frac{2}{9}\,A\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{bx^3+a}\,a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^
3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)
```

**mupad [B]** time = 2.65, size = 52, normalized size = 0.71

$$\frac{2\sqrt{b\,x^3+a}\,\left(8\,B\,a^2-4\,B\,a\,b\,x^3-10\,A\,a\,b+3\,B\,b^2\,x^6+5\,A\,b^2\,x^3\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(1/2)*(8*B*a^2 + 5*A*b^2*x^3 + 3*B*b^2*x^6 - 10*A*a*b - 4*B*
a*b*x^3))/(45*b^3)
```

**sympy [A]** time = 1.82, size = 124, normalized size = 1.70

$$\begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2} + \frac{2Ax^3\sqrt{a+bx^3}}{9b} + \frac{16Ba^2\sqrt{a+bx^3}}{45b^3} - \frac{8Bax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9*
b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(45
*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**9/9
)/sqrt(a), True))
```

## 3.189 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=46

$$\frac{2\sqrt{a+bx^3}\,(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\sqrt{a+bx^3}\,(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^2) + (2*B*(a + b*x^3)^(3/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)dx,x,x^3\right)\\ &= \frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 33, normalized size = 0.72

$$\frac{2\sqrt{a+bx^3}\left(-2aB+3Ab+bBx^3\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)
```

**IntegrateAlgebraic** [A] time = 0.03, size = 33, normalized size = 0.72

$$\frac{2\sqrt{a+bx^3}\left(-2aB+3Ab+bBx^3\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)
```

**fricas** [A] time = 1.11, size = 29, normalized size = 0.63

$$\frac{2\left(Bbx^3-2\,Ba+3\,Ab\right)\sqrt{bx^3+a}}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2
```

**giac** [A] time = 0.16, size = 38, normalized size = 0.83

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}B}{9\,b^2}-\frac{2\,\sqrt{bx^3+a}\,(Ba-Ab)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*B/b^2 - 2/3*sqrt(b*x^3 + a)*(B*a - A*b)/b^2
```

**maple** [A] time = 0.04, size = 30, normalized size = 0.65

$$\frac{2\sqrt{b\,x^3+a}\,\left(Bb\,x^3+3Ab-2Ba\right)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/9*(b*x^3+a)^(1/2)*(B*b*x^3+3*A*b-2*B*a)/b^2
```

**maxima** [A] time = 0.46, size = 48, normalized size = 1.04

$$\frac{2}{9}\,B\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{bx^3+a}\,a}{b^2}\right)+\frac{2\,\sqrt{bx^3+a}\,A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 +
a)*A/b
```

**mupad** [B] time = 2.60, size = 29, normalized size = 0.63

$$\frac{2\,\sqrt{b\,x^3+a}\,\left(B\,b\,x^3+3\,A\,b-2\,B\,a\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(1/2)*(3*A*b - 2*B*a + B*b*x^3))/(9*b^2)
```

**sympy** [A] time = 0.98, size = 75, normalized size = 1.63

$$\begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) + 2
*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a),
True))
```

## 3.190 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=48

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

**Rubi [A]** time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx,x,x^3\right)$$
$$= \frac{2B\sqrt{a+bx^3}}{3b} + \frac{1}{3}A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)$$
$$= \frac{2B\sqrt{a+bx^3}}{3b} + \frac{(2A)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}$$
$$= \frac{2B\sqrt{a+bx^3}}{3b} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

**Mathematica [A]** time = 0.05, size = 48, normalized size = 1.00

$$\frac{2B\sqrt{a+bx^3}}{3b} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

**IntegrateAlgebraic [A]** time = 0.04, size = 48, normalized size = 1.00

$$\frac{2B\sqrt{a+bx^3}}{3b} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

**fricas [A]** time = 1.01, size = 105, normalized size = 2.19

$$\left[\frac{A\sqrt{a}\,b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,\sqrt{bx^3+a}\,Ba}{3\,ab}, \frac{2\left(A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx^3+a}\,Ba\right)}{3\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] [1/3*(A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*sq
rt(b*x^3 + a)*B*a)/(a*b), 2/3*(A*sqrt(-a)*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + sqrt(b*x^3 + a)*B*a)/(a*b)]
```

**giac [A]** time = 0.19, size = 40, normalized size = 0.83

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} + \frac{2\,\sqrt{bx^3+a}\,B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)*B/b
```

**maple** [A] time = 0.04, size = 37, normalized size = 0.77

$$-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2\sqrt{b\,x^{3}+a}\;B}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

**maxima** [A] time = 1.08, size = 54, normalized size = 1.12

$$\frac{A\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2\sqrt{bx^{3}+a}\,B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
+ 2/3*sqrt(b*x^3 + a)*B/b
```

**mupad** [B] time = 2.72, size = 57, normalized size = 1.19

$$\frac{2\,B\,\sqrt{b\,x^{3}+a}}{3\,b}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^{3}+a}-\sqrt{a}\right)^{3}\left(\sqrt{b\,x^{3}+a}+\sqrt{a}\right)}{x^{6}}\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(1/2)),x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/(3*b) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a
 + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(1/2))
```

**sympy** [A] time = 11.27, size = 65, normalized size = 1.35

$$\frac{2A\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+bx^{3}}}\right)}{3a\sqrt{-\frac{1}{a}}}-\frac{B\left(\begin{cases}-\frac{x^{3}}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^{3}}}{b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**3)))/(3*a*sqrt(-1/a)) - B*Piecewise((-
x**3/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**3)/b, True))/3
```

## 3.191 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=58

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(3*a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)$$
$$= -\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a}$$
$$= -\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{\left(2\left(-\frac{Ab}{2}+aB\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab}$$
$$= -\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

**Mathematica** [A] time = 0.07, size = 57, normalized size = 0.98

$$\frac{1}{3}\left(\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{A\sqrt{a+bx^3}}{ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
[Out] (-((A*Sqrt[a + b*x^3])/(a*x^3)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/a^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.07, size = 58, normalized size = 1.00

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{A\sqrt{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
[Out] -1/3*(A*Sqrt[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(3*a^(3/2))
```

**fricas** [A] time = 1.51, size = 126, normalized size = 2.17

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,\sqrt{bx^3+a}\,Aa}{6\,a^2x^3}, \frac{(2\,Ba-Ab)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)-\sqrt{bx^3+a}\,Aa}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] [-1/6*((2*B*a - A*b)*sqrt(a)*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) + 2*sqrt(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*sqrt(-a)*x^
3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^3 + a)*A*a)/(a^2*x^3)]
```

**giac** [A] time = 0.16, size = 62, normalized size = 1.07

$$\frac{\frac{\left(2\,Bab-Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} - \frac{\sqrt{bx^3+a}\,Ab}{ax^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^3 + a)*A*b/(a*x^3))/b
```

**maple** [A] time = 0.05, size = 62, normalized size = 1.07

$$-\frac{2B\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{3a\,x^{3}}\right)A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x)
```

```
[Out] A*(1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3
)-2/3*B*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

**maxima** [B] time = 1.17, size = 109, normalized size = 1.88

$$-\frac{1}{6}A\left(\frac{2\sqrt{bx^{3}+a}\,b}{\left(bx^{3}+a\right)a-a^{2}}+\frac{b\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)+\frac{B\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/3*B*log((sqrt(b*x^3 +
a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

**mupad** [B] time = 2.89, size = 67, normalized size = 1.16

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(A\,b-2\,B\,a)}{6\,a^{3/2}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(1/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
A*b - 2*B*a))/(6*a^(3/2)) - (A*(a + b*x^3)^(1/2))/(3*a*x^3)
```

**sympy** [A] time = 31.83, size = 80, normalized size = 1.38

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^{3}}+1}}{3ax^{\frac{3}{2}}}+\frac{Ab\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}-\frac{2B\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)
)
```

## 3.192 $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=90

$$-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}+\frac{\sqrt{a+bx^3}\,(3Ab-4aB)}{12a^2x^3}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

**Rubi [A]** time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}\,(3Ab-4aB)}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(6*a*x^6) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a^2*
x^3) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{\left(-\frac{3Ab}{2}+2aB\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^3\right)}{6a}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(b(3Ab-4aB))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{24a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(3Ab-4aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{12a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.29, size = 81, normalized size = 0.90

$$
\frac{\sqrt{a+bx^3}\left(b\left(2aB-\frac{3Ab}{2}\right)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\frac{a}{bx^3}\right)-\frac{a^2A}{x^6}\right)}{6a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-((a^2*A)/x^6) + b*((-3*A*b)/2 + 2*a*B)*(-(a/(b*x^3)) + A
rcTanh[Sqrt[1 + (b*x^3)/a]]/Sqrt[1 + (b*x^3)/a])))/(6*a^3)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 80, normalized size = 0.89

$$
\frac{\left(4abB-3Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}+\frac{\sqrt{a+bx^3}\left(-2aA-4aBx^3+3Abx^3\right)}{12a^2x^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*a*A + 3*A*b*x^3 - 4*a*B*x^3))/(12*a^2*x^6) + ((-3*A*b^
2 + 4*a*b*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

**fricas [A]** time = 1.08, size = 173, normalized size = 1.92

$$
\left[-\frac{\left(4\,Bab-3\,Ab^2\right)\sqrt{a}\,x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,\left(\left(4\,Ba^2-3\,Aab\right)x^3+2\,Aa^2\right)\sqrt{bx^3+a}}{24\,a^3x^6}, -\frac{\left(4\,Bab-3\,Ab^2\right)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(\left(4\,Ba^2-3\,Aab\right)x^3+2\,Aa^2\right)\sqrt{bx^3+a}}{12\,a^3x^6}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a
```

```
^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqr
t(-a)/a) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6)]
```

**giac [A]** time = 0.19, size = 121, normalized size = 1.34

$$-\frac{\frac{\left(4\,Bab^2-3\,Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}\,Ba^2b^2-3\left(bx^3+a\right)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx^3+a}\,Aab^3}{a^2b^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3 +
 a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b
```

**maple [A]** time = 0.08, size = 102, normalized size = 1.13

$$\left(-\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}+\frac{\sqrt{b\,x^3+a}\;b}{4a^2x^3}-\frac{\sqrt{b\,x^3+a}}{6a\,x^6}\right)A+\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^3+a}}{3a\,x^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3
)+A*(-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*(b*x^3+a)^(1/2)/
a/x^6+1/4*(b*x^3+a)^(1/2)/a^2*b/x^3)
```

**maxima [B]** time = 1.20, size = 178, normalized size = 1.98

$$-\frac{1}{6}B\left(\frac{2\sqrt{bx^3+a}\,b}{\left(bx^3+a\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)+\frac{1}{24}A\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\left(bx^3+a\right)^{\frac{3}{2}}b^2-5\sqrt{bx^3+a}\,ab^2\right)}{\left(bx^3+a\right)^2a^2-2\left(bx^3+a\right)a^3+a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*B*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/24*A*(3*b^2*log((sqrt(
b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*(b*x^3 +
a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(b*x^3 + a)*
a^3 + a^4))
```

**mupad [B]** time = 2.99, size = 95, normalized size = 1.06

$$\frac{\sqrt{b\,x^3+a}\,\left(3\,A\,b-4\,B\,a\right)}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,A\,b-4\,B\,a\right)}{24\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(1/2)),x)
```

```
[Out] ((a + b*x^3)^(1/2)*(3*A*b - 4*B*a))/(12*a^2*x^3) - (A*(a + b*x^3)^(1/2))/(6
*a*x^6) + (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1
/2)))/x^6)*(3*A*b - 4*B*a))/(24*a^(5/2))
```

**sympy [B]** time = 71.18, size = 163, normalized size = 1.81

$$-\frac{A}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{Ab^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + A*sqrt(b)/(12*a*x**(9/2)*sq
rt(a/(b*x**3) + 1)) + A*b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A
*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*
x**3) + 1)/(3*a*x**(3/2)) + B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a**(3/
2))
```

## 3.193 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\left(a+bx^3\right)^{3/2}(Ab-3aB)}{9b^4}-\frac{2a\sqrt{a+bx^3}\,(2Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

**Rubi [A]** time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\left(a+bx^3\right)^{3/2}(Ab-3aB)}{9b^4}-\frac{2a\sqrt{a+bx^3}\,(2Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(3*b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*B*(a +
 b*x^3)^(5/2))/(15*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}}+\frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}}+\frac{(Ab-3aB)\sqrt{a+bx}}{b^3}+\frac{B(a+bx)^{3/2}}{b^3}\right)dx\right.\\
&= -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}-\frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 77, normalized size = 0.75

$$\frac{2\left(48a^3B-8a^2b\left(5A-3Bx^3\right)-2ab^2x^3\left(10A+3Bx^3\right)+b^3x^6\left(5A+3Bx^3\right)\right)}{45b^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2*
x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 80, normalized size = 0.78

$$\frac{2\left(48a^3B-40a^2Ab+24a^2bBx^3-20aAb^2x^3-6ab^2Bx^6+5Ab^3x^6+3b^3Bx^9\right)}{45b^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-40*a^2*A*b + 48*a^3*B - 20*a*A*b^2*x^3 + 24*a^2*b*B*x^3 + 5*A*b^3*x^6
- 6*a*b^2*B*x^6 + 3*b^3*B*x^9))/(45*b^4*Sqrt[a + b*x^3])
```

**fricas [A]** time = 1.04, size = 88, normalized size = 0.85

$$\frac{2\left(3\,Bb^3x^9-\left(6\,Bab^2-5\,Ab^3\right)x^6+48\,Ba^3-40\,Aa^2b+4\left(6\,Ba^2b-5\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{45\left(b^5x^3+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^3*x^9 - (6*B*a*b^2 - 5*A*b^3)*x^6 + 48*B*a^3 - 40*A*a^2*b + 4*(
6*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^5*x^3 + a*b^4)
```

**giac [A]** time = 0.17, size = 114, normalized size = 1.11

$$\frac{2\left(Ba^3-Aa^2b\right)}{3\sqrt{bx^3+a}\,b^4}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}Bb^{16}-15\left(bx^3+a\right)^{\frac{3}{2}}Bab^{16}+45\sqrt{bx^3+a}\,Ba^2b^{16}+5\left(bx^3+a\right)^{\frac{3}{2}}Ab^{17}-30\sqrt{bx^3+a}\,Aab^{17}\right)}{45\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^3 - A*a^2*b)/(sqrt(b*x^3 + a)*b^4) + 2/45*(3*(b*x^3 + a)^(5/2)*B*b
^16 - 15*(b*x^3 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^3 + a)*B*a^2*b^16 + 5*(b*
x^3 + a)^(3/2)*A*b^17 - 30*sqrt(b*x^3 + a)*A*a*b^17)/b^20
```

**maple [A]** time = 0.04, size = 77, normalized size = 0.75

$$-\frac{2\left(-3B\,x^9b^3-5A\,b^3x^6+6Ba\,b^2x^6+20Aa\,b^2x^3-24B\,a^2b\,x^3+40A\,a^2b-48B\,a^3\right)}{45\sqrt{b\,x^3+a}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/45/(b*x^3+a)^(1/2)*(-3*B*b^3*x^9-5*A*b^3*x^6+6*B*a*b^2*x^6+20*A*a*b^2*x^
3-24*B*a^2*b*x^3+40*A*a^2*b-48*B*a^3)/b^4
```

**maxima [A]** time = 0.59, size = 116, normalized size = 1.13

$$\frac{2}{15}B\left(\frac{\left(bx^3+a\right)^{\frac{5}{2}}}{b^4}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}a}{b^4}+\frac{15\sqrt{bx^3+a}\,a^2}{b^4}+\frac{5\,a^3}{\sqrt{bx^3+a}\,b^4}\right)+\frac{2}{9}A\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^3}-\frac{6\sqrt{bx^3+a}\,a}{b^3}-\frac{3\,a^2}{\sqrt{bx^3+a}\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*((b*x^3 + a)^(5/2)/b^4 - 5*(b*x^3 + a)^(3/2)*a/b^4 + 15*sqrt(b*x^3 +
 a)*a^2/b^4 + 5*a^3/(sqrt(b*x^3 + a)*b^4)) + 2/9*A*((b*x^3 + a)^(3/2)/b^3 -
 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3 + a)*b^3))
```

**mupad [B]** time = 2.77, size = 152, normalized size = 1.48

$$\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(B\,a^2-A\,a\,b\right)}{b^3}-\frac{2\,a\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{3\,b}\right)}{3\,b}+\frac{x^3\,\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{9\,b}-\frac{a^2\,\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)}{b^2\,\sqrt{b\,x^3+a}}+\frac{2\,B\,x^6\,\sqrt{b\,x^3+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(B*a^2 - A*a*b))/b^3 - (2*a*((2*(A*b^2 - B*a*b))/b^3
 - (8*B*a)/(5*b^2)))/(3*b)))/(3*b) + (x^3*(a + b*x^3)^(1/2)*((2*(A*b^2 - B*
a*b))/b^3 - (8*B*a)/(5*b^2)))/(9*b) - (a^2*((2*A)/(3*b) - (2*B*a)/(3*b^2)))
/(b^2*(a + b*x^3)^(1/2)) + (2*B*x^6*(a + b*x^3)^(1/2))/(15*b^2)
```

**sympy [A]** time = 3.90, size = 175, normalized size = 1.70

$$\begin{cases} -\frac{16Aa^2}{9b^3\sqrt{a+bx^3}}-\frac{8Aax^3}{9b^2\sqrt{a+bx^3}}+\frac{2Ax^6}{9b\sqrt{a+bx^3}}+\frac{32Ba^3}{15b^4\sqrt{a+bx^3}}+\frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}}-\frac{4Bax^6}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^9}{15b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(a
 + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a +
 b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b**2
*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/
9 + B*x**12/12)/a**(3/2), True))
```

## 3.194 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=73

$$\frac{2\sqrt{a+bx^3}\,(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\sqrt{a+bx^3}\,(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/(3*b^3) + (2*B*(a + b*x^3)^(3/2))/(9*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}}+\frac{Ab-2aB}{b^2\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^2}\right)dx,x,x^3\right)\\ &= \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 55, normalized size = 0.75

$$\frac{2\left(-8a^2B+a\left(6Ab-4bBx^3\right)+b^2x^3\left(3A+Bx^3\right)\right)}{9b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-8*a^2*B + b^2*x^3*(3*A + B*x^3) + a*(6*A*b - 4*b*B*x^3)))/(9*b^3*Sqrt[
a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 56, normalized size = 0.77

$$-\frac{2\left(8a^2B-6aAb+4abBx^3-3Ab^2x^3-b^2Bx^6\right)}{9b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(-6*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 4*a*b*B*x^3 - b^2*B*x^6))/(9*b^3*Sq
rt[a + b*x^3])
```

**fricas [A]** time = 0.98, size = 63, normalized size = 0.86

$$\frac{2\left(Bb^2x^6-\left(4\,Bab-3\,Ab^2\right)x^3-8\,Ba^2+6\,Aab\right)\sqrt{bx^3+a}}{9\left(b^4x^3+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^2*x^6 - (4*B*a*b - 3*A*b^2)*x^3 - 8*B*a^2 + 6*A*a*b)*sqrt(b*x^3 +
a)/(b^4*x^3 + a*b^3)
```

**giac [A]** time = 0.18, size = 77, normalized size = 1.05

$$-\frac{2\left(Ba^2-Aab\right)}{3\,\sqrt{bx^3+a}\,b^3}+\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^6-6\,\sqrt{bx^3+a}\,Bab^6+3\,\sqrt{bx^3+a}\,Ab^7\right)}{9\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^2 - A*a*b)/(sqrt(b*x^3 + a)*b^3) + 2/9*((b*x^3 + a)^(3/2)*B*b^6 -
 6*sqrt(b*x^3 + a)*B*a*b^6 + 3*sqrt(b*x^3 + a)*A*b^7)/b^9
```

**maple [A]** time = 0.05, size = 52, normalized size = 0.71

$$\frac{\frac{2}{9}B\,b^2x^6+\frac{2}{3}A\,b^2x^3-\frac{8}{9}Bab\,x^3+\frac{4}{3}Aab-\frac{16}{9}B\,a^2}{\sqrt{b\,x^3+a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/9/(b*x^3+a)^(1/2)*(B*b^2*x^6+3*A*b^2*x^3-4*B*a*b*x^3+6*A*a*b-8*B*a^2)/b^3
```

**maxima [A]** time = 0.65, size = 81, normalized size = 1.11

$$\frac{2}{9}\,B\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^3}-\frac{6\,\sqrt{bx^3+a}\,a}{b^3}-\frac{3\,a^2}{\sqrt{bx^3+a}\,b^3}\right)+\frac{2}{3}\,A\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^3 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3
+ a)*b^3)) + 2/3*A*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2))
```

**mupad** [B] time = 2.68, size = 60, normalized size = 0.82

$$\frac{2\,B\,\left(b\,x^3+a\right)^2-6\,B\,a^2+6\,A\,b\,\left(b\,x^3+a\right)-12\,B\,a\,\left(b\,x^3+a\right)+6\,A\,a\,b}{9\,b^3\,\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] (2*B*(a + b*x^3)^2 - 6*B*a^2 + 6*A*b*(a + b*x^3) - 12*B*a*(a + b*x^3) + 6*A
*a*b)/(9*b^3*(a + b*x^3)^(1/2))
```

**sympy** [A] time = 1.84, size = 124, normalized size = 1.70

$$\begin{cases} \frac{4Aa}{3b^2\sqrt{a+bx^3}} + \frac{2Ax^3}{3b\sqrt{a+bx^3}} - \frac{16Ba^2}{9b^3\sqrt{a+bx^3}} - \frac{8Bax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3)
) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x**
3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a*
*(3/2), True))
```

## 3.195 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=46

$$\frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(A*b - a*B))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)dx,x,x^3\right)\\ &= -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 33, normalized size = 0.72

$$\frac{2\left(2aB-Ab+bBx^3\right)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic** [A] time = 0.04, size = 33, normalized size = 0.72

$$\frac{2\left(2aB - Ab + bBx^3\right)}{3b^2\sqrt{a + bx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

**fricas** [A] time = 0.55, size = 41, normalized size = 0.89

$$\frac{2\left(Bbx^3 + 2\,Ba - Ab\right)\sqrt{bx^3 + a}}{3\left(b^3x^3 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

**giac** [A] time = 0.18, size = 38, normalized size = 0.83

$$\frac{2\sqrt{bx^3 + a}\,B}{3\,b^2} + \frac{2\,(Ba - Ab)}{3\sqrt{bx^3 + a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 2/3*sqrt(b*x^3 + a)*B/b^2 + 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*b^2)
```

**maple** [A] time = 0.05, size = 30, normalized size = 0.65

$$-\frac{2\left(-Bb\,x^3 + Ab - 2Ba\right)}{3\sqrt{b\,x^3 + a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x)
[Out] -2/3/(b*x^3+a)^(1/2)*(-B*b*x^3+A*b-2*B*a)/b^2
```

**maxima** [A] time = 0.56, size = 47, normalized size = 1.02

$$\frac{2}{3}\,B\left(\frac{\sqrt{bx^3 + a}}{b^2} + \frac{a}{\sqrt{bx^3 + a}\,b^2}\right) - \frac{2\,A}{3\sqrt{bx^3 + a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] 2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3 +
 a)*b)
```

**mupad** [B] time = 2.61, size = 33, normalized size = 0.72

$$\frac{2\,B\,a - 2\,A\,b + 2\,B\,\left(b\,x^3 + a\right)}{3\,b^2\,\sqrt{b\,x^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] (2*B*a - 2*A*b + 2*B*(a + b*x^3))/(3*b^2*(a + b*x^3)^(1/2))
```

**sympy** [A] time = 1.08, size = 75, normalized size = 1.63

$$\begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}} + \frac{4Ba}{3b^2\sqrt{a+bx^3}} + \frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) +
2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/2)
, True))
```

## 3.196 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=58

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

**Rubi** [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx,x,x^3\right)\\ &= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}+\frac{A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a}\\ &= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}+\frac{(2A)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab}\\ &= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 58, normalized size = 1.00

$$\frac{1}{3}\left(\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] ((2*(A*b - a*B))/(a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[
a]])/a^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.07, size = 58, normalized size = 1.00

$$-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{2(aB-Ab)}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*(-(A*b) + a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/
Sqrt[a]])/(3*a^(3/2))
```

**fricas** [A] time = 1.07, size = 170, normalized size = 2.93

$$\left[\frac{\left(Ab^2x^3+Aab\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}\left(Ba^2-Aab\right)}{3\left(a^2b^2x^3+a^3b\right)},\ \frac{2\left(\left(Ab^2x^3+Aab\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)-\sqrt{bx^3+a}\left(Ba^2-Aab\right)\right)}{3\left(a^2b^2x^3+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) +
2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3*(
(A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^
3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]
```

**giac** [A] time = 0.16, size = 53, normalized size = 0.91

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a}-\frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqrt
(b*x^3 + a)*a*b)
```

**maple** [A] time = 0.05, size = 57, normalized size = 0.98

$$\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}+\frac{2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a}\right)A-\frac{2B}{3\sqrt{b\,x^3+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*B/b/(b*x^3+a)^(1/2)+A*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3*arctanh((b*x^3+a)
^(1/2)/a^(1/2))/a^(3/2))
```

**maxima** [A] time = 1.35, size = 70, normalized size = 1.21

$$\frac{1}{3}\,A\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^3+a}\,a}\right)-\frac{2\,B}{3\,\sqrt{bx^3+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)
 + 2/(sqrt(b*x^3 + a)*a)) - 2/3*B/(sqrt(b*x^3 + a)*b)
```

**mupad** [B] time = 2.77, size = 65, normalized size = 1.12

$$\frac{\frac{2\,A}{3\,a}-\frac{2\,B}{3\,b}}{\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(3/2)),x)
```

```
[Out] ((2*A)/(3*a) - (2*B)/(3*b))/(a + b*x^3)^(1/2) + (A*log((((a + b*x^3)^(1/2)
- a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2))
```

**sympy** [A] time = 20.82, size = 56, normalized size = 0.97

$$\frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a\sqrt{-a}}-\frac{2\,(-Ab+Ba)}{3ab\sqrt{a+bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - 2*(-A*b + B*a)/(3*a*b*
sqrt(a + b*x**3))
```

## 3.197 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=86

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2aB-3Ab}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] -(3*A*b - 2*a*B)/(3*a^2*Sqrt[a + b*x^3]) - A/(3*a*x^3*Sqrt[a + b*x^3]) + ((
3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{A}{3ax^3\sqrt{a+bx^3}} + \frac{\left(-\frac{3Ab}{2}+aB\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)}{3a}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} - \frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{6a^2}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} - \frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3a^2b}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} + \frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.02, size = 57, normalized size = 0.66

$$
\frac{x^3(2aB-3Ab)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)-aA}{3a^2x^3\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a*A) + (-3*A*b + 2*a*B)*x^3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/
a])/(3*a^2*x^3*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 0.13, size = 77, normalized size = 0.90

$$
\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{-aA+2aBx^3-3Abx^3}{3a^2x^3\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a*A) - 3*A*b*x^3 + 2*a*B*x^3)/(3*a^2*x^3*Sqrt[a + b*x^3]) + ((3*A*b - 2*
a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

**fricas [A]** time = 0.96, size = 233, normalized size = 2.71

$$
\left[-\frac{\left(\left(2\,Bab-3\,Ab^2\right)x^6+\left(2\,Ba^2-3\,Aab\right)x^3\right)\sqrt{a}\,\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)-2\left(\left(2\,Ba^2-3\,Aab\right)x^3-Aa^2\right)\sqrt{bx^3+a}}{6\left(a^3bx^6+a^4x^3\right)},\ \frac{\left(\left(2\,Bab-3\,Ab^2\right)x^6+\left(2\,Ba^2-3\,Aab\right)x^3\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(\left(2\,Ba^2-3\,Aab\right)x^3-Aa^2\right)\sqrt{bx^3+a}}{3\left(a^3bx^6+a^4x^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(a)*log((b*x
^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*((2*B*a^2 - 3*A*a*b)*x^3 - A
```

```
*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), 1/3*(((2*B*a*b - 3*A*b^2)*x^6
 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (
(2*B*a^2 - 3*A*a*b)*x^3 - A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

**giac [A]** time = 0.17, size = 99, normalized size = 1.15

$$\frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^2}+\frac{2\left(bx^3+a\right)Ba-2\,Ba^2-3\left(bx^3+a\right)Ab+2\,Aab}{3\left(\left(bx^3+a\right)^{\frac{3}{2}}-\sqrt{bx^3+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 3*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(
2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^(3
/2) - sqrt(b*x^3 + a)*a)*a^2)
```

**maple [A]** time = 0.06, size = 100, normalized size = 1.16

$$\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2b}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^2}-\frac{\sqrt{b\,x^3+a}}{3a^2x^3}\right)A+\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}+\frac{2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x)
```

```
[Out] A*(-1/3*(b*x^3+a)^(1/2)/a^2/x^3-2/3/((x^3+a/b)*b)^(1/2)/a^2*b+b*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(5/2))+B*(2/3/((x^3+a/b)*b)^(1/2)/a-2/3*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(3/2))
```

**maxima [B]** time = 1.39, size = 144, normalized size = 1.67

$$-\frac{1}{6}A\left(\frac{2\left(3\left(bx^3+a\right)b-2\,ab\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)+\frac{1}{3}B\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^3+a}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a
)*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a
^(5/2)) + 1/3*B*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
))/a^(3/2) + 2/(sqrt(b*x^3 + a)*a))
```

**mupad [B]** time = 2.93, size = 131, normalized size = 1.52

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(3\,A\,b-2\,B\,a)}{6\,a^{5/2}}-\frac{\frac{2\,B\,a^2-3\,A\,a\,b}{2\,a^3}-\frac{a\left(\frac{A\,b^2}{3\,a^3}+\frac{5\,b\left(2\,B\,a^2-3\,A\,a\,b\right)}{6\,a^4}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
3*A*b - 2*B*a))/(6*a^(5/2)) - ((2*B*a^2 - 3*A*a*b)/(2*a^3) - (a*((A*b^2)/(3
```

```
*a^3) + (5*b*(2*B*a^2 - 3*A*a*b))/(6*a^4)))/b)/(a + b*x^3)^(1/2) - (A*(a +
b*x^3)^(1/2))/(3*a^2*x^3)
```

**sympy [B]** time = 79.79, size = 264, normalized size = 3.07

$$A\left(-\frac{1}{3a\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*
sqrt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)) + B*(
2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**
3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/
(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) +
3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2)
+ 3*a**(7/2)*b*x**3))
```

## 3.198 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=118

$$-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}}+\frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}\,(5Ab-4aB)}{4a^3x^3}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(6*a*x^6*Sqrt[a + b*x^3]) - (5*A*b - 4*a*B)/(6*a^2*x^3*Sqrt[a + b*x^3])
+ ((5*A*b - 4*a*B)*Sqrt[a + b*x^3])/(4*a^3*x^3) - (b*(5*A*b - 4*a*B)*ArcTan
h[Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx,x,x^3\right)\\
&= -\frac{A}{6ax^6\sqrt{a+bx^3}}+\frac{\left(-\frac{5Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^3\right)}{6a}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}-\frac{(5Ab-4aB)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)}{4a^2}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}+\frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3}+\frac{(b(5Ab-4aB))\,\text{Subst}\left(\int\right.}{8a^3}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}+\frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3}+\frac{(5Ab-4aB)\,\text{Subst}\left(\int \frac{}{-\frac{a}{b}}\right.}{4a^3}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^3}}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}+\frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3}-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{4a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.02, size = 60, normalized size = 0.51

$$
\frac{bx^6(5Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^3}{a}+1\right)-a^2A}{6a^3x^6\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a^2*A) + b*(5*A*b - 4*a*B)*x^6*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^
3)/a])/(6*a^3*x^6*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 0.16, size = 102, normalized size = 0.86

$$
\frac{\left(4abB-5Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{-2a^2A-4a^2Bx^3+5aAbx^3-12abBx^6+15Ab^2x^6}{12a^3x^6\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*a^2*A + 5*a*A*b*x^3 - 4*a^2*B*x^3 + 15*A*b^2*x^6 - 12*a*b*B*x^6)/(12*a^
3*x^6*Sqrt[a + b*x^3]) + ((-5*A*b^2 + 4*a*b*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(4*a^(7/2))
```

**fricas [A]** time = 0.93, size = 289, normalized size = 2.45

$$
\left[-\frac{3\left(\left(4Bab^2-5Ab^3\right)x^9+\left(4Ba^2b-5Aab^2\right)x^6\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\left(3\left(4Ba^2b-5Aab^2\right)x^6+2Aa^3+\left(4Ba^3-5Aa^2b\right)x^3\right)\sqrt{bx^3+a}}{24\left(a^4bx^9+a^5x^6\right)},-\frac{3\left(\left(4Bab^2-5Ab^3\right)x^9+\left(4Ba^2b-5Aab^2\right)x^6\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\left(3\left(4Ba^2b-5Aab^2\right)x^6+2Aa^3+\left(4Ba^3-5Aa^2b\right)x^3\right)\sqrt{bx^3+a}}{12\left(a^4bx^9+a^5x^6\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(a)
*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*(4*B*a^2*b - 5*A
*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*
x^9 + a^5*x^6), -1/12*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^
2)*x^6)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(4*B*a^2*b - 5*A*a
*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^
9 + a^5*x^6)]
```

**giac [A]** time = 0.18, size = 137, normalized size = 1.16

$$-\frac{\left(4\,Bab - 5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^3} - \frac{2\left(Bab - Ab^2\right)}{3\,\sqrt{bx^3+a}\,a^3} - \frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab - 4\,\sqrt{bx^3+a}\,Ba^2b - 7\left(bx^3+a\right)^{\frac{3}{2}}Ab^2 + 9\,\sqrt{bx^3+a}\,Aab^2}{12\,a^3b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
2/3*(B*a*b - A*b^2)/(sqrt(b*x^3 + a)*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a*b
 - 4*sqrt(b*x^3 + a)*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^3 + a
)*A*a*b^2)/(a^3*b^2*x^6)
```

**maple [A]** time = 0.05, size = 141, normalized size = 1.19

$$\left(-\frac{5b^2\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}} + \frac{2b^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^3} + \frac{7\sqrt{b\,x^3+a}\;b}{12a^3x^3} - \frac{\sqrt{b\,x^3+a}}{6a^2x^6}\right)A + \left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{2b}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^2} - \frac{\sqrt{b\,x^3+a}}{3a^2x^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-1/3*(b*x^3+a)^(1/2)/a^2/x^3-2/3/((x^3+a/b)*b)^(1/2)/a^2*b+b*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(5/2))+A*(-1/6*(b*x^3+a)^(1/2)/a^2/x^6+7/12*(b*x^3+
a)^(1/2)/a^3*b/x^3+2/3/((x^3+a/b)*b)^(1/2)/a^3*b^2-5/4*b^2*arctanh((b*x^3+a
)^(1/2)/a^(1/2))/a^(7/2))
```

**maxima [B]** time = 1.34, size = 215, normalized size = 1.82

$$\frac{1}{24}\,A\left(\frac{2\left(15\left(bx^3+a\right)^2b^2 - 25\left(bx^3+a\right)ab^2 + 8\,a^2b^2\right)}{\left(bx^3+a\right)^{\frac{5}{2}}a^3 - 2\left(bx^3+a\right)^{\frac{3}{2}}a^4 + \sqrt{bx^3+a}\,a^5} + \frac{15\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) - \frac{1}{6}\,B\left(\frac{2\left(3\left(bx^3+a\right)b - 2\,ab\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2 - \sqrt{bx^3+a}\,a^3} + \frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*A*(2*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^3
 + a)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 15*b^2*l
og((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) - 1/6*
B*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)*a^3
) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2
))
```

**mupad [B]** time = 3.18, size = 167, normalized size = 1.42

$$\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(5\,A\,b-4\,B\,a\right)}{8\,a^{7/2}} - \frac{\left(4\,B\,a^2-7\,A\,a\,b\right)\sqrt{b\,x^3+a}}{12\,a^4\,x^3} - \frac{A\,\sqrt{b\,x^3+a}}{6\,a^2\,x^6} - \frac{\frac{a\left(\frac{7\,A\,b^3-4\,B\,a\,b^2}{12\,a^4}-\frac{5\,b^2\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^4}\right)}{b}+\frac{3\,b\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^3}}{\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)
*(5*A*b - 4*B*a))/(8*a^(7/2)) - ((4*B*a^2 - 7*A*a*b)*(a + b*x^3)^(1/2))/(12
*a^4*x^3) - (A*(a + b*x^3)^(1/2))/(6*a^2*x^6) - ((a*((7*A*b^3 - 4*B*a*b^2)/
(12*a^4) - (5*b^2*(5*A*b - 4*B*a))/(8*a^4)))/b + (3*b*(5*A*b - 4*B*a))/(8*a
^3))/(a + b*x^3)^(1/2)
```

**sympy [A]** time = 179.01, size = 192, normalized size = 1.63

$$A\left(-\frac{1}{6a\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x**
(9/2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3) +
 1)) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))) + B*(-1/(3*a*
sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sqrt(a/(b*x
**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2))
```

## 3.199 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\,(Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}$$

**Rubi [A]** time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\,(Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(9*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(3*b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*B*(a +
 b*x^3)^(3/2))/(9*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}}+\frac{Ab-3aB}{b^3\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^3}\right)dx,x,x^3\right)\\
&= -\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 73, normalized size = 0.71

$$\frac{2\left(-16a^3B+8a^2b\left(A-3Bx^3\right)-6ab^2x^3\left(Bx^3-2A\right)+b^3x^6\left(3A+Bx^3\right)\right)}{9b^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x^
6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.06, size = 80, normalized size = 0.78

$$-\frac{2\left(16a^3B-8a^2Ab+24a^2bBx^3-12aAb^2x^3+6ab^2Bx^6-3Ab^3x^6-b^3Bx^9\right)}{9b^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(-8*a^2*A*b + 16*a^3*B - 12*a*A*b^2*x^3 + 24*a^2*b*B*x^3 - 3*A*b^3*x^6
+ 6*a*b^2*B*x^6 - b^3*B*x^9))/(9*b^4*(a + b*x^3)^(3/2))
```

**fricas [A]** time = 0.84, size = 98, normalized size = 0.95

$$\frac{2\left(Bb^3x^9-3\left(2Bab^2-Ab^3\right)x^6-16Ba^3+8Aa^2b-12\left(2Ba^2b-Aab^2\right)x^3\right)\sqrt{bx^3+a}}{9\left(b^6x^6+2ab^5x^3+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^3*x^9 - 3*(2*B*a*b^2 - A*b^3)*x^6 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

**giac [A]** time = 0.17, size = 104, normalized size = 1.01

$$-\frac{2\left(9\left(bx^3+a\right)Ba^2-Ba^3-6\left(bx^3+a\right)Aab+Aa^2b\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}b^4}+\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^8-9\sqrt{bx^3+a}\,Bab^8+3\sqrt{bx^3+a}\,Ab^9\right)}{9\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(9*(b*x^3 + a)*B*a^2 - B*a^3 - 6*(b*x^3 + a)*A*a*b + A*a^2*b)/((b*x^3
+ a)^(3/2)*b^4) + 2/9*((b*x^3 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^3 + a)*B*a*b^8
+ 3*sqrt(b*x^3 + a)*A*b^9)/b^12
```

**maple [A]** time = 0.05, size = 76, normalized size = 0.74

$$\frac{\frac{2}{9}B\,x^9b^3+\frac{2}{3}A\,b^3x^6-\frac{4}{3}Ba\,b^2x^6+\frac{8}{3}Aa\,b^2x^3-\frac{16}{3}B\,a^2b\,x^3+\frac{16}{9}A\,a^2b-\frac{32}{9}B\,a^3}{\left(b\,x^3+a\right)^{\frac{3}{2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] 2/9/(b*x^3+a)^(3/2)*(B*b^3*x^9+3*A*b^3*x^6-6*B*a*b^2*x^6+12*A*a*b^2*x^3-24*
B*a^2*b*x^3+8*A*a^2*b-16*B*a^3)/b^4
```

**maxima [A]** time = 0.59, size = 116, normalized size = 1.13

$$\frac{2}{9}B\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^4}-\frac{9\sqrt{bx^3+a}\,a}{b^4}-\frac{9\,a^2}{\sqrt{bx^3+a}\,b^4}+\frac{a^3}{\left(bx^3+a\right)^{\frac{3}{2}}b^4}\right)+\frac{2}{9}A\left(\frac{3\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}\,b^3}-\frac{a^2}{\left(bx^3+a\right)^{\frac{3}{2}}b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^4 - 9*sqrt(b*x^3 + a)*a/b^4 - 9*a^2/(sqrt(b*x^3
+ a)*b^4) + a^3/((b*x^3 + a)^(3/2)*b^4)) + 2/9*A*(3*sqrt(b*x^3 + a)/b^3 + 6
*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)^(3/2)*b^3))
```

**mupad [B]** time = 2.80, size = 145, normalized size = 1.41

$$\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(A\,b-2\,B\,a\right)}{b^3}-\frac{4\,B\,a}{3\,b^3}\right)}{3\,b}-\frac{\frac{2\,B\,a^2-2\,A\,a\,b}{3\,b^4}-\frac{a\,\left(\frac{2\,A\,b^2-2\,B\,a\,b}{3\,b^4}-\frac{2\,B\,a}{3\,b^3}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{a^2\,\left(\frac{2\,A}{9\,b}-\frac{2\,B\,a}{9\,b^2}\right)}{b^2\,{\left(b\,x^3+a\right)}^{3/2}}+\frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(A*b - 2*B*a))/b^3 - (4*B*a)/(3*b^3)))/(3*b) - ((2*B
*a^2 - 2*A*a*b)/(3*b^4) - (a*((2*A*b^2 - 2*B*a*b)/(3*b^4) - (2*B*a)/(3*b^3)
))/b)/(a + b*x^3)^(1/2) - (a^2*((2*A)/(9*b) - (2*B*a)/(9*b^2)))/(b^2*(a + b
*x^3)^(3/2)) + (2*B*x^3*(a + b*x^3)^(1/2))/(9*b^3)
```

**sympy [A]** time = 5.18, size = 338, normalized size = 3.28

$$\begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{32Ba^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{48Ba^2bx^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{12Bab^2x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{2Bb^3x^9}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} & \text{for}\: b \neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*
x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x
**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))
```

## 3.200 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

**Optimal**. Leaf size=73

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(9*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(3*b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}}+\frac{Ab-2aB}{b^2(a+bx)^{3/2}}+\frac{B}{b^2\sqrt{a+bx}}\right)dx,x,x^3\right)\\
&= \frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 54, normalized size = 0.74

$$\frac{16a^2B-4ab\left(A-6Bx^3\right)+6b^2x^3\left(Bx^3-A\right)}{9b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (16*a^2*B - 4*a*b*(A - 6*B*x^3) + 6*b^2*x^3*(-A + B*x^3))/(9*b^3*(a + b*x^3
)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.05, size = 56, normalized size = 0.77

$$\frac{2\left(8a^2B-2aAb+12abBx^3-3Ab^2x^3+3b^2Bx^6\right)}{9b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(-2*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 12*a*b*B*x^3 + 3*b^2*B*x^6))/(9*b^3*
(a + b*x^3)^(3/2))
```

**fricas** [A] time = 0.71, size = 75, normalized size = 1.03

$$\frac{2\left(3\,Bb^2x^6+3\left(4\,Bab-Ab^2\right)x^3+8\,Ba^2-2\,Aab\right)\sqrt{bx^3+a}}{9\left(b^5x^6+2\,ab^4x^3+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*B*b^2*x^6 + 3*(4*B*a*b - A*b^2)*x^3 + 8*B*a^2 - 2*A*a*b)*sqrt(b*x^3
+ a)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

**giac** [A] time = 0.20, size = 63, normalized size = 0.86

$$\frac{2\sqrt{bx^3+a}\,B}{3\,b^3}+\frac{2\left(6\left(bx^3+a\right)Ba-Ba^2-3\left(bx^3+a\right)Ab+Aab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*B/b^3 + 2/9*(6*(b*x^3 + a)*B*a - B*a^2 - 3*(b*x^3 + a)*
A*b + A*a*b)/((b*x^3 + a)^(3/2)*b^3)
```

**maple** [A] time = 0.05, size = 53, normalized size = 0.73

$$-\frac{2\left(-3B\,b^2x^6+3A\,b^2x^3-12Bab\,x^3+2Aab-8B\,a^2\right)}{9\left(b\,x^3+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9/(b*x^3+a)^(3/2)*(-3*B*b^2*x^6+3*A*b^2*x^3-12*B*a*b*x^3+2*A*a*b-8*B*a^2
)/b^3
```

**maxima** [A] time = 0.49, size = 84, normalized size = 1.15

$$\frac{2}{9}B\left(\frac{3\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}\,b^3}-\frac{a^2}{\left(bx^3+a\right)^{\frac{3}{2}}b^3}\right)-\frac{2}{9}A\left(\frac{3}{\sqrt{bx^3+a}\,b^2}-\frac{a}{\left(bx^3+a\right)^{\frac{3}{2}}b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*(3*sqrt(b*x^3 + a)/b^3 + 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)
^(3/2)*b^3)) - 2/9*A*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2))
```

**mupad [B]** time = 2.76, size = 60, normalized size = 0.82

$$\frac{6\,B\,{\left(b\,x^3+a\right)}^2-2\,B\,a^2-6\,A\,b\,\left(b\,x^3+a\right)+12\,B\,a\,\left(b\,x^3+a\right)+2\,A\,a\,b}{9\,b^3\,{\left(b\,x^3+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] (6*B*(a + b*x^3)^2 - 2*B*a^2 - 6*A*b*(a + b*x^3) + 12*B*a*(a + b*x^3) + 2*A
*a*b)/(9*b^3*(a + b*x^3)^(3/2))
```

**sympy [A]** time = 2.42, size = 240, normalized size = 3.29

$$\begin{cases} -\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} - \frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{24Babx^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{6Bb^2x^6}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)),
 Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))
```

## 3.201 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

**Optimal**. Leaf size=46

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b - a*B))/(9*b^2*(a + b*x^3)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^3])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)dx,x,x^3\right)\\ &= -\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 33, normalized size = 0.72

$$-\frac{2\left(2aB+Ab+3bBx^3\right)}{9b^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.04, size = 33, normalized size = 0.72

$$-\frac{2\left(2aB+Ab+3bBx^3\right)}{9b^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

**fricas** [A] time = 0.73, size = 52, normalized size = 1.13

$$-\frac{2\left(3\,Bbx^3+2\,Ba+Ab\right)\sqrt{bx^3+a}}{9\left(b^4x^6+2\,ab^3x^3+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2
*b^2)
```

**giac** [A] time = 0.16, size = 32, normalized size = 0.70

$$-\frac{2\left(3\left(bx^3+a\right)B-Ba+Ab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)
```

**maple** [A] time = 0.05, size = 30, normalized size = 0.65

$$-\frac{2\left(3Bb\,x^3+Ab+2Ba\right)}{9\left(b\,x^3+a\right)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9/(b*x^3+a)^(3/2)*(3*B*b*x^3+A*b+2*B*a)/b^2
```

**maxima** [A] time = 0.50, size = 49, normalized size = 1.07

$$-\frac{2}{9}\,B\left(\frac{3}{\sqrt{bx^3+a}\,b^2}-\frac{a}{\left(bx^3+a\right)^{\frac{3}{2}}b^2}\right)-\frac{2\,A}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x^
3 + a)^(3/2)*b)
```

**mupad [B]** time = 2.68, size = 33, normalized size = 0.72

$$-\frac{2\,A\,b-2\,B\,a+6\,B\,\left(b\,x^3+a\right)}{9\,b^2\,{\left(b\,x^3+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] -(2*A*b - 2*B*a + 6*B*(a + b*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

**sympy [A]** time = 1.34, size = 144, normalized size = 3.13

$$\begin{cases} -\frac{2Ab}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{4Ba}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{6Bbx^3}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)
) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*B*
b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b, 0)
), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))
```

## 3.202 $\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx$

**Optimal**. Leaf size=77

$$-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B))/(9*a*b*(a + b*x^3)^(3/2)) + (2*A)/(3*a^2*Sqrt[a + b*x^3]) -
 (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx,x,x^3\right)\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{A\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a^2}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{(2A)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3a^2b}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.05, size = 62, normalized size = 0.81

$$
\frac{2a(Ab-aB)+6Ab\left(a+bx^3\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)}{9a^2b\left(a+bx^3\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*a*(A*b - a*B) + 6*A*b*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b
*x^3)/a])/(9*a^2*b*(a + b*x^3)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 70, normalized size = 0.91

$$
-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}-\frac{2\left(a^2B-4aAb-3Ab^2x^3\right)}{9a^2b\left(a+bx^3\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*(-4*a*A*b + a^2*B - 3*A*b^2*x^3))/(9*a^2*b*(a + b*x^3)^(3/2)) - (2*A*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

**fricas [A]** time = 0.56, size = 243, normalized size = 3.16

$$
\left[\frac{3\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\sqrt{a}\,\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Aab^2x^3-Ba^3+4\,Aa^2b\right)\sqrt{bx^3+a}}{9\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)},\ \frac{2\left(3\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(3\,Aab^2x^3-Ba^3+4\,Aa^2b\right)\sqrt{bx^3+a}\right)}{9\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b
*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(
b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2*A*
a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*A*a*b
^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 +
 a^5*b)]
```

**giac [A]** time = 0.17, size = 67, normalized size = 0.87

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^2}-\frac{2\left(Ba^2-3\left(bx^3+a\right)Ab-Aab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/9*(B*a^2 - 3*(b*x
^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a^2*b)
```

**maple [A]** time = 0.09, size = 85, normalized size = 1.10

$$\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}+\frac{2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a^2}+\frac{2\sqrt{b\,x^3+a}}{9\left(x^3+\frac{a}{b}\right)^2 a\,b^2}\right)A-\frac{2B}{9\left(b\,x^3+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9*B/b/(b*x^3+a)^(3/2)+A*(2/9/a/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/3/a^2/(
(x^3+a/b)*b)^(1/2)-2/3/a^(5/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

**maxima [A]** time = 1.17, size = 81, normalized size = 1.05

$$\frac{1}{9}\,A\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^3+4\,a\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2}\right)-\frac{2\,B}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*A*(3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/
2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)^(3/2)*a^2)) - 2/9*B/((b*x^3 + a)^(3/2)*
b)
```

**mupad [B]** time = 2.78, size = 80, normalized size = 1.04

$$\frac{\frac{2\,A}{9\,a}-\frac{2\,B}{9\,b}}{\left(b\,x^3+a\right)^{3/2}}+\frac{2\,A}{3\,a^2\,\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(5/2)),x)
```

```
[Out] ((2*A)/(9*a) - (2*B)/(9*b))/(a + b*x^3)^(3/2) + (2*A)/(3*a^2*(a + b*x^3)^(1
/2)) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)
))/x^6))/(3*a^(5/2))
```

**sympy [A]** time = 38.43, size = 76, normalized size = 0.99

$$\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2A\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}}-\frac{2\left(-Ab+Ba\right)}{9ab\left(a+bx^3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

```
[Out] 2*A/(3*a**2*sqrt(a + b*x**3)) + 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a**2
*sqrt(-a)) - 2*(-A*b + B*a)/(9*a*b*(a + b*x**3)**(3/2))
```

## 3.203 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx$

**Optimal**. Leaf size=113

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}+\frac{2aB-5Ab}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

**Rubi [A]** time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(5*A*b - 2*a*B)/(9*a^2*(a + b*x^3)^(3/2)) - A/(3*a*x^3*(a + b*x^3)^(3/2))
 - (5*A*b - 2*a*B)/(3*a^3*Sqrt[a + b*x^3]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx,x,x^3\right)\\
&= -\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}+\frac{\left(-\frac{5Ab}{2}+aB\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^3\right)}{3a}\\
&= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{(5Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{6a^2}\\
&= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{(5Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{6a^3}\\
&= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{(5Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{3a^3b}\\
&= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.03, size = 57, normalized size = 0.50

$$\frac{x^3(2aB-5Ab)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^3}{a}+1\right)-3aA}{9a^2x^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-3*a*A + (-5*A*b + 2*a*B)*x^3*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^3)
/a])/(9*a^2*x^3*(a + b*x^3)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 99, normalized size = 0.88

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}+\frac{-3a^2A+8a^2Bx^3-20aAbx^3+6abBx^6-15Ab^2x^6}{9a^3x^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-3*a^2*A - 20*a*A*b*x^3 + 8*a^2*B*x^3 - 15*A*b^2*x^6 + 6*a*b*B*x^6)/(9*a^3
*x^3*(a + b*x^3)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
)/(3*a^(7/2))
```

**fricas** [A] time = 0.75, size = 351, normalized size = 3.11

$$\left[-\frac{3\left(\left(2\,Bab^{2}-5\,Ab^{3}\right)x^{9}+2\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{6}+\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{3}\right)\sqrt{a}\,\log\left(\frac{bx^{3}+2\,\sqrt{bx^{3}+a}\,\sqrt{a}+2\,a}{x^{3}}\right)-2\left(3\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{6}-3\,Aa^{3}+4\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{3}\right)\sqrt{bx^{3}+a}}{18\left(a^{4}b^{2}x^{9}+2\,a^{5}bx^{6}+a^{6}x^{3}\right)},\ \frac{3\left(\left(2\,Bab^{2}-5\,Ab^{3}\right)x^{9}+2\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{6}+\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{3}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^{3}+a}\,\sqrt{-a}}{a}\right)+\left(3\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{6}-3\,Aa^{3}+4\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{3}\right)\sqrt{bx^{3}+a}}{9\left(a^{4}b^{2}x^{9}+2\,a^{5}bx^{6}+a^{6}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B
*a^3 - 5*A*a^2*b)*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a
)/x^3) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*
b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*((2*
B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B*a^3 - 5*A*a^2
*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(2*B*a^2*b - 5*A*
a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]
```

**giac** [A] time = 0.19, size = 101, normalized size = 0.89

$$\frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^{3}+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^{3}}+\frac{2\left(3\left(bx^{3}+a\right)Ba+Ba^{2}-6\left(bx^{3}+a\right)Ab-Aab\right)}{9\left(bx^{3}+a\right)^{\frac{3}{2}}a^{3}}-\frac{\sqrt{bx^{3}+a}\,A}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 5*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/9*(
3*(b*x^3 + a)*B*a + B*a^2 - 6*(b*x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a
^3) - 1/3*sqrt(b*x^3 + a)*A/(a^3*x^3)
```

**maple** [A] time = 0.09, size = 157, normalized size = 1.39

$$\left(\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}-\frac{4b}{3\sqrt{\left(x^{3}+\frac{a}{b}\right)b}\;a^{3}}-\frac{2\sqrt{b\,x^{3}+a}}{9\left(x^{3}+\frac{a}{b}\right)^{2}a^{2}b}-\frac{\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}\right)A+\left(-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}+\frac{2}{3\sqrt{\left(x^{3}+\frac{a}{b}\right)b}\;a^{2}}+\frac{2\sqrt{b\,x^{3}+a}}{9\left(x^{3}+\frac{a}{b}\right)^{2}a\,b^{2}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x)
```

```
[Out] A*(-1/3/a^3*(b*x^3+a)^(1/2)/x^3-2/9/a^2/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-4/3*b
/a^3/((x^3+a/b)*b)^(1/2)+5/3*b/a^(7/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*
(2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2/a/b^2+2/3/((x^3+a/b)*b)^(1/2)/a^2-2/3/a^(5
/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

**maxima** [A] time = 1.25, size = 170, normalized size = 1.50

$$-\frac{1}{18}\,A\left(\frac{2\left(15\left(bx^{3}+a\right)^{2}b-10\left(bx^{3}+a\right)ab-2\,a^{2}b\right)}{\left(bx^{3}+a\right)^{\frac{5}{2}}a^{3}-\left(bx^{3}+a\right)^{\frac{3}{2}}a^{4}}+\frac{15\,b\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)+\frac{1}{9}\,B\left(\frac{3\,\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^{3}+4\,a\right)}{\left(bx^{3}+a\right)^{\frac{3}{2}}a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/18*A*(2*(15*(b*x^3 + a)^2*b - 10*(b*x^3 + a)*a*b - 2*a^2*b)/((b*x^3 + a)
^(5/2)*a^3 - (b*x^3 + a)^(3/2)*a^4) + 15*b*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) + 1/9*B*(3*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)
^(3/2)*a^2))
```

**mupad [B]** time = 2.97, size = 198, normalized size = 1.75

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(5\,A\,b-2\,B\,a)}{6\,a^{7/2}}-\frac{\frac{2\,B\,a^2-5\,A\,a\,b}{2\,a^4}-\frac{a\left(\frac{A\,b^2}{3\,a^4}+\frac{5\,b\left(2\,B\,a^2-5\,A\,a\,b\right)}{6\,a^5}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{\frac{2\,B\,a^3-5\,A\,a^2\,b}{4\,a^4}-\frac{a\left(\frac{13\,b\left(2\,B\,a^3-5\,A\,a^2\,b\right)}{36\,a^5}+\frac{A\,b^2}{3\,a^3}\right)}{b}}{\left(b\,x^3+a\right)^{3/2}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
5*A*b - 2*B*a))/(6*a^(7/2)) - ((2*B*a^2 - 5*A*a*b)/(2*a^4) - (a*((A*b^2)/(3
*a^4) + (5*b*(2*B*a^2 - 5*A*a*b))/(6*a^5)))/b)/(a + b*x^3)^(1/2) - ((2*B*a^
3 - 5*A*a^2*b)/(4*a^4) - (a*((13*b*(2*B*a^3 - 5*A*a^2*b))/(36*a^5) + (A*b^2
)/(3*a^3)))/b)/(a + b*x^3)^(3/2) - (A*(a + b*x^3)^(1/2))/(3*a^3*x^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.204 $\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$

**Optimal**. Leaf size=97

$$-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}+\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 88, 50, 63, 203}

$$\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (32*c^2*Sqrt[c + d*x^3])/(3*d^3) - (10*c*(c + d*x^3)^(3/2))/(9*d^3) + (2*(c
 + d*x^3)^(5/2))/(15*d^3) - (32*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])])/(Sqrt[3]*d^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{4c+dx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{5c\sqrt{c+dx}}{d^2}+\frac{(c+dx)^{3/2}}{d^2}+\frac{16c^2\sqrt{c+dx}}{d^2(4c+dx)}\right)dx,x,x^3\right)\\
&= -\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(16c^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx,x,x^3\right)}{3d^2}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{\left(16c^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx,x,x\right.}{d^2}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{\left(32c^3\right)\operatorname{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx}\right.}{d^3}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}
\end{aligned}
$$

**Mathematica [A]** time = 0.09, size = 77, normalized size = 0.79

$$
\frac{2\sqrt{c+dx^3}\left(218c^2-19cdx^3+3d^2x^6\right)-480\sqrt{3}\,c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{45d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6) - 480*Sqrt[3]*c^(5/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(45*d^3)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 78, normalized size = 0.80

$$
\frac{2\sqrt{c+dx^3}\left(218c^2-19cdx^3+3d^2x^6\right)}{45d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6))/(45*d^3) - (32*c^(5/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3)
```

**fricas [A]** time = 1.15, size = 156, normalized size = 1.61

$$
\left[\frac{2\left(120\sqrt{3}\sqrt{-c}\,c^2\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\left(3d^2x^6-19cdx^3+218c^2\right)\sqrt{dx^3+c}\right)}{45d^3},-\frac{2\left(240\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\left(3d^2x^6-19cdx^3+218c^2\right)\sqrt{dx^3+c}\right)}{45d^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [2/45*(120*sqrt(3)*sqrt(-c)*c^2*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt
(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3
+ c))/d^3, -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sq
rt(c)) - (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]
```

**giac [A]** time = 0.17, size = 82, normalized size = 0.85

$$-\frac{32\,\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{3\,d^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}-25\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+240\,\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/4
5*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^3
+ c)*c^2*d^12)/d^15
```

**maple [C]** time = 4.46, size = 506, normalized size = 5.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 1/d^2*(d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45*c/d*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d
*x^3+c)^(1/2)/d^2)-8/9*c/d*(d*x^3+c)^(3/2))+16*c^2/d^2*(2/3*(d*x^3+c)^(1/2)
/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d+4*c)))
```

**maxima [A]** time = 1.15, size = 69, normalized size = 0.71

$$-\frac{2\left(240\,\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-3\left(dx^3+c\right)^{\frac{5}{2}}+25\left(dx^3+c\right)^{\frac{3}{2}}c-240\,\sqrt{dx^3+c}\,c^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*
(d*x^3 + c)^(5/2) + 25*(d*x^3 + c)^(3/2)*c - 240*sqrt(d*x^3 + c)*c^2)/d^3
```

**mupad [B]** time = 4.54, size = 109, normalized size = 1.12

$$\frac{436\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}-\frac{38\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}+\frac{\sqrt{3}\,c^{5/2}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)16{}\mathrm{i}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (436*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/(15*d) - (
38*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (3^(1/2)*c^(5/2)*log((2*3^(1/2)*c +
c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*16i)/(3*d^3)
```

**sympy** [A] time = 38.27, size = 85, normalized size = 0.88

$$\frac{2\left(-\frac{16\sqrt{3}c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{16c^2\sqrt{c+dx^3}}{3}-\frac{5c\left(c+dx^3\right)^{\frac{3}{2}}}{9}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-16*sqrt(3)*c**(5/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + 16*c
**2*sqrt(c + d*x**3)/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x**3)**(5/2)/15
)/d**3
```

## 3.205 $\int \frac{x^5 \sqrt{c+dx^3}}{4c+dx^3} \, dx$

**Optimal**. Leaf size=76

$$\frac{8c^{3/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\, d^2} - \frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 50, 63, 203}

$$\frac{8c^{3/2} \tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\, d^2} - \frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (-8*c*Sqrt[c + d*x^3])/(3*d^2) + (2*(c + d*x^3)^(3/2))/(9*d^2) + (8*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{(4c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(4c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(8c^2\right)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 65, normalized size = 0.86

$$\frac{24\sqrt{3}\,c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)+2\left(dx^3-11c\right)\sqrt{c+dx^3}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
[Out] (2*(-11*c + d*x^3)*Sqrt[c + d*x^3] + 24*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x
^3]/(Sqrt[3]*Sqrt[c])])/(9*d^2)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 67, normalized size = 0.88

$$\frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d^2}-\frac{2\left(11c-dx^3\right)\sqrt{c+dx^3}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
[Out] (-2*(11*c - d*x^3)*Sqrt[c + d*x^3])/(9*d^2) + (8*c^(3/2)*ArcTan[Sqrt[c + d*
x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

**fricas [A]** time = 0.93, size = 129, normalized size = 1.70

$$\left[\frac{2\left(6\sqrt{3}\sqrt{-c}\,c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}\left(dx^3-11c\right)\right)}{9d^2}, \frac{2\left(12\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}\left(dx^3-11c\right)\right)}{9d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
[Out] [2/9*(6*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, 2/9*(12*sqrt(3
```

```
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 11*c))/d^2]
```

**giac [A]** time = 0.19, size = 64, normalized size = 0.84

$$\frac{8\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{3\,d^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4-12\,\sqrt{dx^3+c}\,cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 8/3*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^2 + 2/9*(
(d*x^3 + c)^(3/2)*d^4 - 12*sqrt(d*x^3 + c)*c*d^4)/d^6
```

**maple [C]** time = 0.17, size = 446, normalized size = 5.87

$$-\frac{4\left(\frac{2\sqrt{dx^3+c}}{3d}+\frac{i(-cd^2)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{(-cd^2)^{\frac{1}{3}}}{d}\right)d}{-3(-cd^2)^{\frac{1}{3}}+i\sqrt{3}(-cd^2)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{2(-cd^2)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}(d\_Z^3+4c)^2d^2+i(-cd^2)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}(d\_Z^3+4c)d-(-cd^2)^{\frac{1}{3}}\operatorname{RootOf}(d\_Z^3+4c)d-i\sqrt{3}(-cd^2)^{\frac{2}{3}}-(-cd^2)^{\frac{2}{3}}\right)\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}}{3},\frac{2i(-cd^2)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}(d\_Z^3+4c)^2d+i\sqrt{3}cd-3cd-i(-cd^2)^{\frac{2}{3}}\sqrt{3}\operatorname{RootOf}(d\_Z^3+4c)-3(-cd^2)^{\frac{2}{3}}\operatorname{RootOf}(d\_Z^3+4c)}{6cd},\sqrt{\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{\left(-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^3\sqrt{dx^3+c}}\right)c}{d}+\frac{2\left(dx^3+c\right)^{\frac{3}{2}}}{9d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^2-4*c/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c
*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

**maxima [A]** time = 1.30, size = 53, normalized size = 0.70

$$\frac{2\left(12\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}-12\,\sqrt{dx^3+c}\,c\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] 2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 12*sqrt(d*x^3 + c)*c)/d^2
```

**mupad [B]** time = 4.28, size = 88, normalized size = 1.16

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}-\frac{22\,c\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d) - (22*c*(c + d*x^3)^(1/2))/(9*d^2) + (3^(1/
2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)
/(4*c + d*x^3))*4i)/(3*d^2)
```

**sympy [A]** time = 16.96, size = 68, normalized size = 0.89

$$\frac{2\left(\frac{4\sqrt{3}\,c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}-\frac{4c\sqrt{c+dx^3}}{3}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(4*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 - 4*c*sq
rt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**2
```

## 3.206 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$

**Optimal**. Leaf size=57

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

**Rubi [A]** time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 50, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3d} - c\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3d} - \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}$$
$$= \frac{2\sqrt{c+dx^3}}{3d} - \frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d}$$

**Mathematica** [A] time = 0.02, size = 54, normalized size = 0.95

$$\frac{2\left(\sqrt{c+dx^3} - \sqrt{3}\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])]))/(3*d)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 57, normalized size = 1.00

$$\frac{2\sqrt{c+dx^3}}{3d} - \frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d)
```

**fricas** [A] time = 0.78, size = 110, normalized size = 1.93

$$\left[\frac{\sqrt{3}\sqrt{-c}\,\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+2\sqrt{dx^3+c}}{3d}, -\frac{2\left(\sqrt{3}\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c)) + 2*sqrt(d*x^3 + c))/d, -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*
sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*x^3 + c))/d]
```

**giac** [A] time = 0.16, size = 44, normalized size = 0.77

$$-\frac{2\sqrt{3}\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d} + \frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + 2/3*sq
rt(d*x^3 + c)/d
```

**maple** [C] time = 0.15, size = 425, normalized size = 7.46

$$\frac{2\sqrt{d x^3+c}}{3d}+\frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2 d^2+i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)d-\left(-c d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)d-i\sqrt{3}\left(-c d^2\right)^{\frac{2}{3}}-\left(-c d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c d^2\right)^{\frac{1}{3}}}}}{3},\frac{2i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2 d+i\sqrt{3}\,c d-3c d-i\left(-c d^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)-3\left(-c d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)}{6c d},\sqrt{\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^3\sqrt{d x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

**maxima** [A] time = 1.21, size = 42, normalized size = 0.74

$$-\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*
x^3 + c))/d
```

**mupad** [B] time = 3.87, size = 71, normalized size = 1.25

$$\frac{2\sqrt{d\,x^3+c}}{3\,d}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(
c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(3*d)
```

**sympy** [A] time = 6.10, size = 51, normalized size = 0.89

$$\frac{2\left(-\frac{\sqrt{3}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + sqrt(c +
 d*x**3)/3)/d
```

# 3.207 $\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx$

**Optimal**. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

**Rubi [A]** time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 83, 63, 208, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(4c+dx)}\,dx,x,x^3\right)$$
$$= \frac{1}{12}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)+\frac{1}{4}d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx,x,x^3\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{6d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

**Mathematica** [A] time = 0.02, size = 59, normalized size = 0.91

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)-\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(6*Sqrt[c])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 65, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])
```

**fricas** [A] time = 0.82, size = 147, normalized size = 2.26

$$\left[\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{12c},-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{12c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)*s
qrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c
)) - 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c]
```

**giac** [A] time = 0.16, size = 50, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/sqrt(c) + 1/6*arcta
n(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)
```

**maple** [C] time = 0.19, size = 468, normalized size = 7.20

$$-\frac{\left(\frac{2\sqrt{dx^3+c}}{3d}+\frac{i(-cd^2)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{(-cd^2)^{\frac{1}{3}}}{d}\right)d}{-3(-cd^2)^{\frac{1}{3}}+i\sqrt{3}(-cd^2)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{2(-cd^2)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}(d\_Z^3+4c)^2d^2+i(-cd^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}(d\_Z^3+4c)d-(-cd^2)^{\frac{1}{3}}\mathrm{RootOf}(d\_Z^3+4c)d-i\sqrt{3}(-cd^2)^{\frac{2}{3}}-(-cd^2)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}}{3},\frac{2i(-cd^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}(d\_Z^3+4c)^2d+i\sqrt{3}\,cd-3cd-i(-cd^2)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}(d\_Z^3+4c)-3(-cd^2)^{\frac{2}{3}}\mathrm{RootOf}(d\_Z^3+4c)}{6cd},\sqrt{\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{\left(-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^3\sqrt{dx^3+c}}\right)d}{4c}+\frac{-\frac{2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3}+\frac{2\sqrt{dx^3+c}}{3}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x)
```

```
[Out] -1/4/c*d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))+1/4/c*(2/3*(d*x^3+c)^(1/2)-2
/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)
```

**mupad** [B] time = 4.66, size = 93, normalized size = 1.43

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,\sqrt{c}}+\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{12\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(1/2)) + (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(12*c^(1/2))
```

**sympy** [A] time = 13.28, size = 66, normalized size = 1.02

$$\frac{2\left(\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}+\frac{\sqrt{3}\,d\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)
```

```
[Out] 2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*atan(sqrt(3)
*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d
```

## 3.208 $\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx$

**Optimal**. Leaf size=88

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 99, 156, 63, 208, 203}

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])
/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(4c+dx)}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3}+\frac{\text{Subst}\left(\int \frac{cd-\frac{d^2x}{2}}{x\sqrt{c+dx}\,(4c+dx)}\,dx,x,x^3\right)}{12c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3}+\frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{48c}-\frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx,x,x^3\right)}{16c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{24c}-\frac{d\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)}{8c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3}-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 88, normalized size = 1.00

$$
-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 88, normalized size = 1.00

$$
-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

**fricas [A]** time = 0.95, size = 194, normalized size = 2.20

$$
\left[-\frac{2\sqrt{3}\sqrt{c}\,dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+4\sqrt{dx^3+c}\,c}{48c^2x^3},\ -\frac{\sqrt{3}\sqrt{-c}\,dx^3\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-2\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+4\sqrt{dx^3+c}\,c}{48c^2x^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 - sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 4*sqr
t(d*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*sqr
t(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*arct
an(sqrt(d*x^3 + c)*sqrt(-c)/c) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

**giac [A]** time = 0.17, size = 72, normalized size = 0.82

$$-\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{24\,c^{\frac{3}{2}}}+\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{24\,\sqrt{-c}\,c}-\frac{\sqrt{dx^3+c}}{12\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/24*
d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/12*sqrt(d*x^3 + c)/(c*x
^3)
```

**maple [C]** time = 0.19, size = 511, normalized size = 5.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x)
```

```
[Out] 1/4/c*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1
/2))+1/16*d^2/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*
_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))-1/16/c^2*d*(2/3*(d*
x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

**mupad [B]** time = 4.86, size = 113, normalized size = 1.28

$$\frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{48\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{12\,c\,x^3}+\frac{\sqrt{3}\,d\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{48\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)
)/(48*c^(3/2)) - (c + d*x^3)^(1/2)/(12*c*x^3) + (3^(1/2)*d*log((2*3^(1/2)*c
 + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(48*c^(
3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^4\left(4c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)
```

## 3.209 $\int \frac{x^8}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

**Optimal**. Leaf size=78

$$\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

**Rubi [A]** time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 88, 63, 203}

$$\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-10*c*Sqrt[c + d*x^3])/(3*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (32*c^(3/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{5c}{d^2\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{d^2} + \frac{16c^2}{d^2\sqrt{c+dx}\,(4c+dx)}\right)dx, x, x^3\right)$$
$$= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{\left(16c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{3d^2}$$
$$= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{\left(32c^2\right)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}$$
$$= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}$$

**Mathematica** [**A**] time = 0.07, size = 65, normalized size = 0.83

$$\frac{32\sqrt{3}\,c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right) + 2\left(dx^3-14c\right)\sqrt{c+dx^3}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x
^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)
```

**IntegrateAlgebraic** [**A**] time = 0.06, size = 69, normalized size = 0.88

$$\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3} - \frac{2\left(14c-dx^3\right)\sqrt{c+dx^3}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-2*(14*c - d*x^3)*Sqrt[c + d*x^3])/(9*d^3) + (32*c^(3/2)*ArcTan[Sqrt[c + d
*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

**fricas** [**A**] time = 0.81, size = 129, normalized size = 1.65

$$\left[\frac{2\left(8\sqrt{3}\sqrt{-c}\,c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}\left(dx^3-14c\right)\right)}{9d^3}, \frac{2\left(16\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}\left(dx^3-14c\right)\right)}{9d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(8*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 14*c))/d^3, 2/9*(16*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 14*c))/d^3]
```

**giac [A]** time = 0.16, size = 64, normalized size = 0.82

$$\frac{32\,\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{9\,d^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6-15\,\sqrt{dx^3+c}\,cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 32/9*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/9*
((d*x^3 + c)^(3/2)*d^6 - 15*sqrt(d*x^3 + c)*c*d^6)/d^9
```

**maple [C]** time = 0.24, size = 467, normalized size = 5.99

$$\frac{\left(\frac{2\sqrt{dx^3+c}\,x^3}{9d}-\frac{4\sqrt{dx^3+c}\,c}{9d^2}\right)d-\frac{8\sqrt{dx^3+c}\,c}{3d}}{d^2}-\frac{16ic\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(d\_Z^3+4c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3+4c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\operatorname{RootOf}\left(d\_Z^3+4c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3+4c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3+4c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\operatorname{RootOf}\left(d\_Z^3+4c\right)}{6cd},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{9d^5\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/d^2*(d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)-8/3*c*(d*x^3
+c)^(1/2)/d)-16/9*I*c/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d+4*c))
```

**maxima [A]** time = 1.34, size = 53, normalized size = 0.68

$$\frac{2\left(16\,\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}-15\,\sqrt{dx^3+c}\,c\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 15*sqrt(d*x^3 + c)*c)/d^3
```

**mupad [B]** time = 5.38, size = 88, normalized size = 1.13

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}-\frac{28\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)16\mathrm{i}}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d^2) - (28*c*(c + d*x^3)^(1/2))/(9*d^3) + (3^(
1/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^
3)/(4*c + d*x^3))*16i)/(9*d^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{c+dx^3}\,\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

## 3.210 $\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

**Optimal**. Leaf size=59

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 80, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])])/(3*Sqrt[3]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{3d}$$
$$= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(8c)\,\mathrm{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}$$
$$= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

**Mathematica** [A] time = 0.02, size = 56, normalized size = 0.95

$$\frac{6\sqrt{c+dx^3} - 8\sqrt{3}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(9*d^2)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 59, normalized size = 1.00

$$\frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])])/(3*Sqrt[3]*d^2)
```

**fricas** [A] time = 0.85, size = 112, normalized size = 1.90

$$\left[\frac{2\left(2\sqrt{3}\,\sqrt{-c}\,\log\left(\frac{dx^3-2\sqrt{3}\,\sqrt{dx^3+c}\,\sqrt{-c}-2c}{dx^3+4c}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}, -\frac{2\left(4\sqrt{3}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [2/9*(2*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) -
2*c)/(d*x^3 + 4*c)) + 3*sqrt(d*x^3 + c))/d^2, -2/9*(4*sqrt(3)*sqrt(c)*arcta
n(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(d*x^3 + c))/d^2]
```

**giac** [A] time = 0.16, size = 49, normalized size = 0.83

$$-\frac{2\left(\frac{4\sqrt{3}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d} - \frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*s
qrt(d*x^3 + c)/d)/d
```

**maple [C]** time = 0.18, size = 425, normalized size = 7.20

$$\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{4i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2d^2+i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)d-\left(-c\,d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)d-i\sqrt{3}\left(-c\,d^2\right)^{\frac{2}{3}}-\left(-c\,d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3},\frac{2i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-c\,d^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)-3\left(-c\,d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)}{6cd},\sqrt{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{9d^4\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^2+4/9*I/d^4*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

**maxima [A]** time = 1.46, size = 43, normalized size = 0.73

$$-\frac{2\left(4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqr
t(d*x^3 + c))/d^2
```

**mupad [B]** time = 4.86, size = 71, normalized size = 1.20

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^2}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)4{}\mathrm{i}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)
*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*4i)/(9*d^2)
```

**sympy [A]** time = 15.58, size = 65, normalized size = 1.10

$$\begin{cases}\frac{2\left(-\frac{4\sqrt{3}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d}+\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for}\ d\neq 0\\ \frac{x^6}{24c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-4*sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))
/(9*d) + sqrt(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(24*c**(3/2)), True))
```

## 3.211 $\int \frac{x^2}{\sqrt{c+dx^3}\left(4c+dx^3\right)} \, dx$

**Optimal**. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

**Rubi [A]** time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {444, 63, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{c+dx^3}\left(4c+dx^3\right)} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)} \, dx, x, x^3\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{3c+x^2} \, dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

**fricas** [A] time = 0.72, size = 87, normalized size = 2.18

$$\left[-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)}{9cd}, \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c))/(c*d), 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqr
t(c))/(sqrt(c)*d)]
```

**giac** [A] time = 0.18, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

**maple** [C] time = 0.18, size = 413, normalized size = 10.32

$$-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3}, \frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)}{6cd}, \sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{9cd^3\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
[Out] -1/9*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-
```

```
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_
alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(
1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*d+4*c))
```

**maxima** [A] time = 1.24, size = 29, normalized size = 0.72

$$\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{9\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

**mupad** [B] time = 5.21, size = 56, normalized size = 1.40

$$\frac{\sqrt{3}\;\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{2\,d\,x^3+8\,c}\right)\,1\mathrm{i}}{9\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(
8*c + 2*d*x^3))*1i)/(9*c^(1/2)*d)
```

**sympy** [A] time = 10.09, size = 37, normalized size = 0.92

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)*d)
```

## 3.212 $\int \frac{1}{x\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

**Optimal**. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

**Rubi** [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 86, 63, 208, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(6*Sqrt[3]*c^(3/2)) - ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+dx^3}\left(4c+dx^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}(4c+dx)}dx, x, x^3\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}dx, x, x^3\right)}{12c} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}dx, x, x^3\right)}{12c}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{3c+x^2}dx, x, \sqrt{c+dx^3}\right)}{6c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}dx, x, \sqrt{c+dx^3}\right)}{6cd}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

**Mathematica** [A] time = 0.02, size = 59, normalized size = 0.91

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/18*(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] + 3*ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]])/c^(3/2)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 65, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/6*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(Sqrt[3]*c^(3/2)) - ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))
```

**fricas** [A] time = 0.84, size = 148, normalized size = 2.28

$$\left[-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{36c^2}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-6\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{36c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*s
qrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sqrt
(3)*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3
+ 4*c)) - 6*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]
```

**giac** [A] time = 0.17, size = 53, normalized size = 0.82

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18c^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*arc
tan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

**maple [C]** time = 0.19, size = 433, normalized size = 6.66

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{6c^{\frac{3}{2}}}+\frac{i(-cd^2)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{(-cd^2)^{\frac{1}{3}}}{d}\right)d}{-3(-cd^2)^{\frac{1}{3}}+i\sqrt{3}(-cd^2)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{2(-cd^2)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}(d\_Z^3+4c)^2d^2+i(-cd^2)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}(d\_Z^3+4c)d-(-cd^2)^{\frac{1}{3}}\operatorname{RootOf}(d\_Z^3+4c)d-i\sqrt{3}(-cd^2)^{\frac{2}{3}}-(-cd^2)^{\frac{2}{3}}\right)\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}d}{(-cd^2)^{\frac{1}{3}}}}}{3},\frac{2i(-cd^2)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}(d\_Z^3+4c)^2d+i\sqrt{3}cd-3cd-i(-cd^2)^{\frac{2}{3}}\sqrt{3}\operatorname{RootOf}(d\_Z^3+4c)-3(-cd^2)^{\frac{2}{3}}\operatorname{RootOf}(d\_Z^3+4c)}{6cd},\sqrt{\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{\left(-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)d}}\right)}{36c^2d^2\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/36*I/c^2/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*d+4*c))-1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)
```

**mupad [B]** time = 5.51, size = 94, normalized size = 1.45

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,c^{3/2}}+\frac{\sqrt{3}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{36\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(3/2)) + (3^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(36*c^(3/2))
```

**sympy [A]** time = 12.25, size = 63, normalized size = 0.97

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{6c\sqrt{-c}}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**3)/sqrt(-c))/(6*c*sqrt(-c)) - sqrt(3)*atan(sqrt(3)*sqrt(
c + d*x**3)/(3*sqrt(c)))/(18*c**(3/2))
```

## 3.213 $\int \frac{1}{x^4\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

**Optimal**. Leaf size=88

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{\sqrt{c+dx^3}}{12c^2x^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 103, 156, 63, 208, 203}

$$-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd+\frac{d^2x}{2}}{x\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{12c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{16c^2} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{48c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{8c^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{24c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} + \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.04, size = 88, normalized size = 1.00

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}} - \frac{\sqrt{c+dx^3}}{12c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2)
)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 88, normalized size = 1.00

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}} - \frac{\sqrt{c+dx^3}}{12c^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2)
)
```

**fricas [A]** time = 0.65, size = 194, normalized size = 2.20

$$\left[\frac{2\sqrt{3}\sqrt{c}\,dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) + 9\sqrt{c}\,dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 12\sqrt{dx^3+c}\,c}{144c^3x^3}, -\frac{\sqrt{3}\sqrt{-c}\,dx^3\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) + 18\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + 12\sqrt{dx^3+c}\,c}{144c^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 12*
sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 - 2
*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d*x^3
*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

**giac [A]** time = 0.16, size = 72, normalized size = 0.82

$$\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{72\, c^{\frac{5}{2}}} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}\, c^2} - \frac{\sqrt{dx^3+c}}{12\, c^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 1/8*d*
arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/12*sqrt(d*x^3 + c)/(c^2
*x^3)
```

**maple [C]** time = 0.23, size = 477, normalized size = 5.42

$$\frac{d \,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{24 c^{\frac{5}{2}}} + \frac{\frac{d \,\mathrm{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}} - \frac{\sqrt{d x^3+c}}{3 c x^3}}{4c} - \frac{i \left(-c d^2\right)^{\frac{1}{3}} \sqrt{\frac{i \left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{\left(-c d^2\right)^{\frac{1}{3}}}} \sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}} \sqrt{-\frac{i \left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{2\left(-c d^2\right)^{\frac{1}{3}}}} \left(2 \,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2 d^2 + i\left(-c d^2\right)^{\frac{1}{3}} \sqrt{3}\, \mathrm{RootOf}\left(d\_Z^3+4c\right) d - \left(-c d^2\right)^{\frac{1}{3}} \mathrm{RootOf}\left(d\_Z^3+4c\right) d - i\sqrt{3}\left(-c d^2\right)^{\frac{2}{3}} - \left(-c d^2\right)^{\frac{2}{3}}\right) \mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\, d}{\left(-c d^2\right)^{\frac{1}{3}}}}}{3}, \frac{2i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2 d + i\sqrt{3}\, c d - 3 c d - i\left(-c d^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right) - 3\left(-c d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)}{6 c d}, \sqrt{\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right) d}}\right)}{144 c^3 d \sqrt{d x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/4/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2))-1/144*I/d/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha
*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d+4*c))+1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 5.72, size = 112, normalized size = 1.27

$$\frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)}{16\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{12\,c^2\,x^3} + \frac{\sqrt{3}\,d\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{144\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)
)/(16*c^(5/2)) - (c + d*x^3)^(1/2)/(12*c^2*x^3) + (3^(1/2)*d*log((c^(1/2)*(
c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(144*
c^(5/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

## 3.214 $\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

**Optimal**. Leaf size=206

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3
*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c]
)]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3
)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+-$$

**Mathematica [C]** time = 0.03, size = 67, normalized size = 0.33

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(8*c*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [F]** time = 15.36, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[c + d*x^3]*(4*c + d*x^3)), x]
```

**fricas [B]** time = 2.37, size = 2274, normalized size = 11.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt
(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c
^5*d^4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^
5*d^4))^(5/6))*sqrt(d*x^3 + c) + (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*
d)*(-1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) + 3*(sqrt(3)*sqrt(1/3)*c^3*
d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(
1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5
/6))*sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3
)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/
2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3)
 + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*
d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c
*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c)
)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) + 1/9*s
qrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)
*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^
4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^5*d^4
))^(5/6))*sqrt(d*x^3 + c) - (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*d)*(-
1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) - 3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x
*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6)
- 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*
sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3)*(c^
4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1
/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) - 12
*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*d^4*x
^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*
x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) - 1/36*(1/43
2)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/
2)^(2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) +
 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4)
)^(1/3) + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3
)*(c^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(
1/6)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x
^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/36*(1/432)^(
1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(
2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*
(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1
/3) - 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c
^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)
*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 +
 c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/18*(1/432)^(1/6)
*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 +
```

48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + 2*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 1/18*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) - 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + 2*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(dx^3+4c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)

**maple [C]** time = 0.18, size = 416, normalized size = 2.02

$$-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3+4c\right)}{6cd},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{9cd^3\sqrt{dx^3+c}\,\mathrm{RootOf}\left(d\_Z^3+4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)

[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(dx^3+4c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)

**mupad [B]** time = 25.80, size = 453, normalized size = 2.20

$$\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(\sqrt{dx^3+c}+\sqrt{3}\sqrt{-c}-2^{1/3}\sqrt{3}\,(-c)^{1/6}d^{1/3}x\right)^3\left(54\sqrt{dx^3+c}-54\sqrt{3}\sqrt{-c}+54\,2^{1/3}\sqrt{3}\,(-c)^{1/6}d^{1/3}x\right)}{\left(d^{1/3}x-2^{2/3}(-c)^{1/3}\right)^6}\right)}{2916\,(-c)^{5/6}d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{3}\sqrt{-c}-2\sqrt{dx^3+c}+2^{1/3}\sqrt{3}\,(-c)^{1/6}d^{1/3}x+2^{1/3}(-c)^{1/6}d^{1/3}x\,3\mathrm{i}\right)^3\left(108\sqrt{dx^3+c}+108\sqrt{3}\sqrt{-c}+54\,2^{1/3}\sqrt{3}\,(-c)^{1/6}d^{1/3}x+2^{1/3}(-c)^{1/6}d^{1/3}x\,162\mathrm{i}\right)}{\left(2d^{1/3}x+2^{2/3}(-c)^{1/3}-2^{2/3}\sqrt{3}\,(-c)^{1/3}1\mathrm{i}\right)^6}\right)\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{2916\,(-c)^{5/6}d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{dx^3+c}+2\sqrt{3}\sqrt{-c}+2^{1/3}\sqrt{3}\,(-c)^{1/6}d^{1/3}x-2^{1/3}(-c)^{1/6}d^{1/3}x\,3\mathrm{i}\right)^3\left(108\sqrt{dx^3+c}-108\sqrt{3}\sqrt{-c}-54\,2^{1/3}\sqrt{3}\,(-c)^{1/6}d^{1/3}x+2^{1/3}(-c)^{1/6}d^{1/3}x\,162\mathrm{i}\right)}{\left(2d^{1/3}x+2^{2/3}(-c)^{1/3}+2^{2/3}\sqrt{3}\,(-c)^{1/3}1\mathrm{i}\right)^6}\right)\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{2916\,(-c)^{5/6}d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log((((c + d*x^3)^(1/2) + 3^(1/2)*(-c)^(1/2) - 2^(1/3
)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(54*(c + d*x^3)^(1/2) - 54*3^(1/2)*(-c)^(
1/2) + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(d^(1/3)*x - 2^(2/3)*(-c)^
(1/3))^6))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*3^(1/2
)*(-c)^(1/2) - 2*(c + d*x^3)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1
/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) + 108*3^(1/2)*(-
c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i + 54*2^(1/3)*3^(1/2)*(-c)^(1/6
)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) - 2^(2/3)*3^(1/2)*(-c)^(1/3
)*1i)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)
*314928^(1/3)*log(((2*(c + d*x^3)^(1/2) + 2*3^(1/2)*(-c)^(1/2) - 2^(1/3)*(-
c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d
*x^3)^(1/2) - 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i -
54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3)
 + 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916
*(-c)^(5/6)*d^(2/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+dx^3}\,\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

## 3.215 $\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx$

**Optimal**. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

**Rubi** [A] time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\,\left(4-x^3\right)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

**Mathematica** [C] time = 0.03, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

**IntegrateAlgebraic** [F] time = 20.88, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{1-x^3}\,\left(4-x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[1 - x^3]*(4 - x^3)), x]
```

**fricas [B]** time = 1.26, size = 1191, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

**maple [C]** time = 0.80, size = 164, normalized size = 1.29

$$\frac{i\sqrt{i\left(2x+1-i\sqrt{3}\right)}\,\sqrt{\frac{x-1}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(2x+1+i\sqrt{3}\right)}{2}}\,\left(-2\,\mathrm{RootOf}\left(\_Z^3-4\right)^2+\mathrm{RootOf}\left(\_Z^3-4\right)+1+i\sqrt{3}\,\left(-\mathrm{RootOf}\left(\_Z^3-4\right)+1\right)\right)\mathrm{RootOf}\left(\_Z^3-4\right)^2\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},-\frac{i\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3-4\right)^2}{3}+\frac{\mathrm{RootOf}\left(\_Z^3-4\right)}{2}+\frac{i\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3-4\right)}{6}-\frac{1}{2}+\frac{i\sqrt{3}}{6},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{36\sqrt{-x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((x-1)/(I*3^(1/
2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+_
alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/2
)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-
4))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

**mupad [B]** time = 3.42, size = 653, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

```
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

## 3.216 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$

**Optimal**. Leaf size=111

$$\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}$$

**Rubi [A]** time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}+\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-1024*c^3*Sqrt[c + d*x^3])/(3*d^4) - (38*c^2*(c + d*x^3)^(3/2))/(3*d^4) -
(4*c*(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(21*d^4) + (1024*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{57c^2\sqrt{c+dx}}{d^3}+\frac{512c^3\sqrt{c+dx}}{d^3(8c-dx)}-\frac{6c(c+dx)^{3/2}}{d^3}-\frac{(c+dx)^{5/2}}{d^3}\right)dx, x, x^3\right)\\
&= -\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(512c^3\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right.}{3d^3}\\
&= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(1536c^4\right)\text{Su}}{}\\
&= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(3072c^4\right)\text{Su}}{}\\
&= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{1024c^{7/2}\tanh}{d}
\end{aligned}
$$

**Mathematica [A]** time = 0.11, size = 81, normalized size = 0.73

$$
\frac{107520c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(18632c^3+764c^2dx^3+57cd^2x^6+5d^3x^9\right)}{105d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9)
+ 107520*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(105*d^4)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 82, normalized size = 0.74

$$
\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{2\sqrt{c+dx^3}\left(18632c^3+764c^2dx^3+57cd^2x^6+5d^3x^9\right)}{105d^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9))
/(105*d^4) + (1024*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

**fricas [A]** time = 0.90, size = 169, normalized size = 1.52

$$
\left[\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3\right)\sqrt{dx^3+c}\right)}{105\,d^4}, -\frac{2\left(53760\sqrt{-c}\,c^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3\right)\sqrt{dx^3+c}\right)}{105\,d^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^
```

```
3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c
))/d^4]
```

**giac** [A] time = 0.16, size = 100, normalized size = 0.90

$$-\frac{1024\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{2\left(5\left(dx^3+c\right)^{\frac{7}{2}}d^{24}+42\left(dx^3+c\right)^{\frac{5}{2}}cd^{24}+665\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{24}+17920\sqrt{dx^3+c}\,c^3d^{24}\right)}{105\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(d
*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2)*
c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28
```

**maple** [C] time = 0.35, size = 582, normalized size = 5.24

$$-\frac{512\left(\frac{2\sqrt{dx^3+c}}{3d}+\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{d}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^3\sqrt{dx^3+c}}\right)c^3}{d^3}-\frac{8\left(\frac{2\sqrt{dx^3+c}\,x^6}{15}+\frac{2\sqrt{dx^3+c}\,cx^3}{45d}-\frac{4\sqrt{dx^3+c}\,c^2}{45d^2}\right)c}{d^2}-\frac{\frac{2\sqrt{dx^3+c}\,x^9}{21}+\frac{2\sqrt{dx^3+c}\,cx^6}{105d}-\frac{8\sqrt{dx^3+c}\,c^2x^3}{315d^2}+\frac{16\sqrt{dx^3+c}\,c^3}{315d^3}}{d}-\frac{128\left(dx^3+c\right)^{\frac{3}{2}}c^2}{9d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/21*x^9*(d*x^3+c)^(1/2)+2/105*c/d*x^6*(d*x^3+c)^(1/2)-8/315*c^2/d^2*
x^3*(d*x^3+c)^(1/2)+16/315*c^3*(d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/15*(d*x^3+c)
^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3+c)^(1/2)*c^2/d^2)-128/9
*c^2*(d*x^3+c)^(3/2)/d^4-512*c^3/d^3*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

**maxima** [A] time = 1.31, size = 96, normalized size = 0.86

$$-\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+5\left(dx^3+c\right)^{\frac{7}{2}}+42\left(dx^3+c\right)^{\frac{5}{2}}c+665\left(dx^3+c\right)^{\frac{3}{2}}c^2+17920\sqrt{dx^3+c}\,c^3\right)}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/105*(26880*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 5*(d*x^3 + c)^(7/2) + 42*(d*x^3 + c)^(5/2)*c + 665*(d*x^3 + c
)^(3/2)*c^2 + 17920*sqrt(d*x^3 + c)*c^3)/d^4
```

**mupad** [B] time = 3.51, size = 118, normalized size = 1.06

$$\frac{512\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}-\frac{37264\,c^3\,\sqrt{d\,x^3+c}}{105\,d^4}-\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}-\frac{38\,c\,x^6\,\sqrt{d\,x^3+c}}{35\,d^2}-\frac{1528\,c^2\,x^3\,\sqrt{d\,x^3+c}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (512*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^4 - (37264*c^3*(c + d*x^3)^(1/2))/(105*d^4) - (2*x^9*(c + d*x^3)^(1/2)
)/(21*d) - (38*c*x^6*(c + d*x^3)^(1/2))/(35*d^2) - (1528*c^2*x^3*(c + d*x^3
)^(1/2))/(105*d^3)
```

**sympy [A]** time = 60.27, size = 99, normalized size = 0.89

$$\frac{2\left(-\frac{512c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{512c^3\sqrt{c+dx^3}}{3}-\frac{19c^2\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx^3\right)^{\frac{5}{2}}}{5}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-512*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 512*c**3*sqrt(c
 + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3)**(5/2)/5 -
(c + d*x**3)**(7/2)/21)/d**4
```

## 3.217 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$

**Optimal**. Leaf size=90

$$\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{128c^2\sqrt{c+dx^3}}{3d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-128*c^2*Sqrt[c + d*x^3])/(3*d^3) - (14*c*(c + d*x^3)^(3/2))/(9*d^3) - (2*
(c + d*x^3)^(5/2))/(15*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{7c\sqrt{c+dx}}{d^2}+\frac{64c^2\sqrt{c+dx}}{d^2(8c-dx)}-\frac{(c+dx)^{3/2}}{d^2}\right)dx, x, x^3\right)\\
&= -\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(64c^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(192c^3\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,\right.}{d^2}\\
&= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(384c^3\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c\right.}}{d^3}\\
&= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}
\end{aligned}
$$

**Mathematica [A]** time = 0.06, size = 70, normalized size = 0.78

$$\frac{5760c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(998c^2+41cdx^3+3d^2x^6\right)}{45d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6) + 5760*c^(5/2)*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^3)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 71, normalized size = 0.79

$$\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{2\sqrt{c+dx^3}\left(998c^2+41cdx^3+3d^2x^6\right)}{45d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6))/(45*d^3) + (128*c^(
5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

**fricas [A]** time = 0.75, size = 147, normalized size = 1.63

$$\left[\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3}, -\frac{2\left(2880\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(28
80*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 41*c*
d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]
```

**giac [A]** time = 0.17, size = 83, normalized size = 0.92

$$-\frac{128\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}+35\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+960\,\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^2*
d^12)/d^15
```

**maple [C]** time = 0.16, size = 507, normalized size = 5.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*((2/15*(d*x^3+c)^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3+
c)^(1/2)*c^2/d^2)*d+16/9*(d*x^3+c)^(3/2)*c/d)-64*c^2/d^2*(2/3*(d*x^3+c)^(1/
2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))
```

**maxima [A]** time = 1.21, size = 82, normalized size = 0.91

$$-\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+35\left(dx^3+c\right)^{\frac{3}{2}}c+960\,\sqrt{dx^3+c}\,c^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/45*(1440*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 35*(d*x^3 + c)^(3/2)*c + 960*sqrt(d*x^3 +
 c)*c^2)/d^3
```

**mupad [B]** time = 3.40, size = 98, normalized size = 1.09

$$\frac{64\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}-\frac{1996\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}-\frac{82\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (64*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^3 - (1996*c^2*(c + d*x^3)^(1/2))/(45*d^3) - (2*x^6*(c + d*x^3)^(1/2))/(
15*d) - (82*c*x^3*(c + d*x^3)^(1/2))/(45*d^2)
```

**sympy** [A] time = 30.23, size = 82, normalized size = 0.91

$$\frac{2\left(-\frac{64c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{64c^2\sqrt{c+dx^3}}{3}-\frac{7c\left(c+dx^3\right)^{\frac{3}{2}}}{9}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-64*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 64*c**2*sqrt(c +
 d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/15)/d**3
```

## 3.218 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$

**Optimal**. Leaf size=69

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 80, 50, 63, 206}

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-16*c*Sqrt[c + d*x^3])/(3*d^2) - (2*(c + d*x^3)^(3/2))/(9*d^2) + (16*c^(3/
2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)\\
&= -\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{(8c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{3d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{\left(24c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{\left(48c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}+\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 58, normalized size = 0.84

$$\frac{144c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(25c+dx^3\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3) + 144*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^2)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 59, normalized size = 0.86

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{2\sqrt{c+dx^3}\left(25c+dx^3\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3))/(9*d^2) + (16*c^(3/2)*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/d^2
```

**fricas** [A] time = 0.99, size = 121, normalized size = 1.75

$$\left[\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(dx^3+25\,c\right)\sqrt{dx^3+c}\right)}{9\,d^2},\,-\frac{2\left(72\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+\left(dx^3+25\,c\right)\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]
```

**giac [A]** time = 0.21, size = 65, normalized size = 0.94

$$-\frac{16\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^2}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4+24\sqrt{dx^3+c}\,cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -16*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/9*((d*x^3 +
 c)^(3/2)*d^4 + 24*sqrt(d*x^3 + c)*c*d^4)/d^6
```

**maple [C]** time = 0.18, size = 446, normalized size = 6.46

$$-\frac{8\left(\frac{2\sqrt{dx^3+c}}{3d}+\frac{i(-cd^2)^{\frac13}\sqrt{\frac{i\left(2x+\frac{-i\sqrt3(-cd^2)^{\frac13}+(-cd^2)^{\frac13}}{d}\right)d}{(-cd^2)^{\frac13}}}\sqrt{\frac{\left(x-\frac{(-cd^2)^{\frac13}}{d}\right)d}{-3(-cd^2)^{\frac13}+i\sqrt3(-cd^2)^{\frac13}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt3(-cd^2)^{\frac13}+(-cd^2)^{\frac13}}{d}\right)d}{2(-cd^2)^{\frac13}}}\left(2\,\mathrm{RootOf}(d\_Z^3-8c)^2d^2+i(-cd^2)^{\frac13}\sqrt3\,\mathrm{RootOf}(d\_Z^3-8c)d-(-cd^2)^{\frac13}\mathrm{RootOf}(d\_Z^3-8c)d-i\sqrt3(-cd^2)^{\frac23}-(-cd^2)^{\frac23}\right)\mathrm{EllipticPi}\left(\frac{\sqrt3\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac13}}{2d}-\frac{i\sqrt3(-cd^2)^{\frac13}}{2d}\right)\sqrt3\,d}{(-cd^2)^{\frac13}}}}{3},-\frac{2i(-cd^2)^{\frac13}\sqrt3\,\mathrm{RootOf}(d\_Z^3-8c)^2d+i\sqrt3\,cd-3cd-i(-cd^2)^{\frac23}\sqrt3\,\mathrm{RootOf}(d\_Z^3-8c)-3(-cd^2)^{\frac23}\mathrm{RootOf}(d\_Z^3-8c)}{18cd},\sqrt{\frac{i\sqrt3(-cd^2)^{\frac13}}{\left(-\frac{3(-cd^2)^{\frac13}}{2d}+\frac{i\sqrt3(-cd^2)^{\frac13}}{2d}\right)d}}\right)}{3d^3\sqrt{dx^3+c}}\right)c}{d}-\frac{2\left(dx^3+c\right)^{\frac32}}{9d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^2-8*c/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum
((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*
(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

**maxima [A]** time = 1.30, size = 66, normalized size = 0.96

$$-\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+24\sqrt{dx^3+c}\,c\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/9*(36*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 24*sqrt(d*x^3 + c)*c)/d^2
```

**mupad [B]** time = 3.51, size = 78, normalized size = 1.13

$$\frac{8\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}-\frac{50\,c\,\sqrt{d\,x^3+c}}{9\,d^2}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (8*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d^2 - (50*c*(c + d*x^3)^(1/2))/(9*d^2) - (2*x^3*(c + d*x^3)^(1/2))/(9*d)
```

**sympy [A]** time = 14.91, size = 65, normalized size = 0.94

$$\frac{2\left(-\frac{8c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{8c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-8*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 8*c*sqrt(c + d*x*
*3)/3 - (c + d*x**3)**(3/2)/9)/d**2
```

## 3.219 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3} \, dx$

**Optimal**. Leaf size=50

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

**Rubi [A]** time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 50, 63, 206}

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)\\ &= -\frac{2\sqrt{c+dx^3}}{3d}+(3c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\ &= -\frac{2\sqrt{c+dx^3}}{3d}+\frac{(6c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d}\\ &= -\frac{2\sqrt{c+dx^3}}{3d}+\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 47, normalized size = 0.94

$$-\frac{2\left(\sqrt{c+dx^3}-3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3*
d)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 50, normalized size = 1.00

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d
```

**fricas** [A] time = 0.82, size = 101, normalized size = 2.02

$$\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-2\sqrt{dx^3+c}}{3d},-\frac{2\left(3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\sqrt{dx^3+c}\right)}{3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 2*sqrt(d*x^3 + c))/d, -2/3*(3*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + sqrt(d*x^3 + c))/d]
```

**giac** [A] time = 0.16, size = 43, normalized size = 0.86

$$-\frac{2c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -2*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/3*sqrt(d*x^3 + c
)/d
```

**maple [C]** time = 0.15, size = 425, normalized size = 8.50

$$-\frac{2\sqrt{d x^3+c}}{3d} - \frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(d\_Z^3-8c\right)^2 d^2+i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3-8c\right)d-\left(-c d^2\right)^{\frac{1}{3}}\operatorname{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-c d^2\right)^{\frac{2}{3}}-\left(-c d^2\right)^{\frac{2}{3}}\right)\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c d^2\right)^{\frac{1}{3}}}}}{3}, -\frac{2i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3-8c\right)^2 d+i\sqrt{3}\,cd-3cd-i\left(-c d^2\right)^{\frac{2}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3-8c\right)-3\left(-c d^2\right)^{\frac{2}{3}}\operatorname{RootOf}\left(d\_Z^3-8c\right)}{18cd}, \sqrt{\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^3\sqrt{d x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/3*(d*x^3+c)^(1/2)/d-1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I
*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^
(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.15, size = 56, normalized size = 1.12

$$-\frac{3\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 2*sqrt(d*x^3 + c))/d
```

**mupad [B]** time = 3.50, size = 59, normalized size = 1.18

$$\frac{\sqrt{c}\ \ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d}-\frac{2\,\sqrt{d\,x^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d
 - (2*(c + d*x^3)^(1/2))/(3*d)
```

**sympy [A]** time = 5.11, size = 46, normalized size = 0.92

$$\frac{2\left(-\frac{c\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d*x**3)/3)/d
```

## 3.220 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)} \, dx$

**Optimal**. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

**Rubi** [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 83, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]/(12*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)}\,dx, x, x^3\right)\\ &= \frac{1}{24}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{8}(3d)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}\end{aligned}$$

**Mathematica** [A]     time = 0.02, size = 53, normalized size = 0.91

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(12*Sqrt[c])
```

**IntegrateAlgebraic** [A]     time = 0.04, size = 58, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]/(12*Sqrt[c])
```

**fricas** [A]     time = 0.91, size = 138, normalized size = 2.38

$$\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{24c}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{12c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, 1/12*(
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c))/c]
```

**giac** [A]     time = 0.16, size = 48, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3
+ c)/sqrt(-c))/sqrt(-c)
```

**maple [C]** time = 0.16, size = 468, normalized size = 8.07

$$-\frac{\left(\frac{2\sqrt{d x^3+c}}{3d}+\frac{i\sqrt{2}}{3d^3}\sum_{\alpha=\mathrm{RootOf}(\_Z^3 d-8c)}\frac{(-c d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac13}+(-cd^2)^{\frac13}}{d}\right)d}{2(-cd^2)^{\frac13}}}\sqrt{\frac{\left(x-\frac{(-cd^2)^{\frac13}}{d}\right)d}{-3(-cd^2)^{\frac13}+i\sqrt{3}(-cd^2)^{\frac13}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}(-cd^2)^{\frac13}+(-cd^2)^{\frac13}}{d}\right)d}{2(-cd^2)^{\frac13}}}}{\sqrt{d x^3+c}}\left(2\alpha^2 d^2+i(-cd^2)^{\frac13}\sqrt{3}\alpha d-(-cd^2)^{\frac13}\alpha d-i\sqrt{3}(-cd^2)^{\frac23}-(-cd^2)^{\frac23}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac13}}{2d}-\frac{i\sqrt{3}(-cd^2)^{\frac13}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac13}}},-\frac{2i(-cd^2)^{\frac13}\sqrt{3}\alpha^2 d+i\sqrt{3}cd-3cd-i(-cd^2)^{\frac23}\sqrt{3}\alpha-3(-cd^2)^{\frac23}\alpha}{18cd},\sqrt{\frac{i\sqrt{3}(-cd^2)^{\frac13}}{\left(-\frac{3(-cd^2)^{\frac13}}{2d}+\frac{i\sqrt{3}(-cd^2)^{\frac13}}{2d}\right)d}}\right)\right)d}{8c}+\frac{-\frac{2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3}+\frac{2\sqrt{dx^3+c}}{3}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x)
```

```
[Out] -1/8/c*d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I
*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)
/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(2/3*(d*x^3+c)^(1/2)
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x), x)
```

**mupad [B]** time = 4.69, size = 125, normalized size = 2.16

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(6\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^3\left(24\,c^2-24\,c^{3/2}\,\sqrt{d\,x^3+c}+d^2\,x^6-20\,c\,d\,x^3\right)^3}{x^{15}\left(8\,c-d\,x^3\right)^3\left(24\,c-d\,x^3\right)^3}\right)}{24\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))*(6*c + d
*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^3*(24*c^2 - 24*c^(3/2)*(c + d*x^3)^(1/2
) + d^2*x^6 - 20*c*d*x^3)^3)/(x^15*(8*c - d*x^3)^3*(24*c - d*x^3)^3))/(24*c
^(1/2))
```

**sympy [A]** time = 8.22, size = 60, normalized size = 1.03

$$\frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)
```

```
[Out] 2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan(sqrt(c + d*
x**3)/sqrt(-c))/(24*sqrt(-c)))/d
```

## 3.221 $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx$

**Optimal**. Leaf size=81

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

**Rubi** [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{\text{Subst}\left(\int \frac{5cd+\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{64c}\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c} + \frac{(3d)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{32c}\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 81, normalized size = 1.00

$$
\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3
2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 81, normalized size = 1.00

$$
\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3
2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

**fricas [A]** time = 0.76, size = 186, normalized size = 2.30

$$
\left[\frac{3\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+5\sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8\sqrt{dx^3+c}\,c}{192c^2x^3}, \frac{5\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-3\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-4\sqrt{dx^3+c}\,c}{96c^2x^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), 1/96*(5*sqrt(-c)*d*x^3*arctan(sqrt(d
*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c
)/c) - 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

**giac [A]** time = 0.17, size = 73, normalized size = 0.90

$$\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}\,c}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}\,c}-\frac{\sqrt{dx^3+c}}{24\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 5/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/32*d*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/(c*x^3)
```

**maple [C]** time = 0.18, size = 511, normalized size = 6.31

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}+(-c\,d^2)^{\frac{1}{3}}}{d}\right)d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{(-c\,d^2)^{\frac{1}{3}}}{d}\right)d}{-3(-c\,d^2)^{\frac{1}{3}}+i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}+(-c\,d^2)^{\frac{1}{3}}}{d}\right)d}{2(-c\,d^2)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}(d\_Z^3-8c)^2d^2+i(-c\,d^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}(d\_Z^3-8c)d-(-c\,d^2)^{\frac{1}{3}}\,\mathrm{RootOf}(d\_Z^3-8c)d-i\sqrt{3}\,(-c\,d^2)^{\frac{2}{3}}-(-c\,d^2)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}}{3},-\frac{2i(-c\,d^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}(d\_Z^3-8c)^2d+i\sqrt{3}\,cd-3cd-i(-c\,d^2)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}(d\_Z^3-8c)-3(-c\,d^2)^{\frac{2}{3}}\,\mathrm{RootOf}(d\_Z^3-8c)}{18cd},\sqrt{\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{\left(-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^3\sqrt{d\,x^3+c}}\right)d^2}{64c^2}+\frac{-\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{\sqrt{d\,x^3+c}}{3x^3}}{8c}+\frac{\left(-\frac{2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3}+\frac{2\sqrt{d\,x^3+c}}{3}\right)d}{64c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1
/2))-1/64*d^2/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3
)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*d*(2/3*(
d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)
```

**mupad [B]** time = 3.75, size = 69, normalized size = 0.85

$$\frac{d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{32\,\sqrt{c^3}}-\frac{5\,d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}}-\frac{\sqrt{d\,x^3+c}}{24\,c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(32*(c^3)^(1/2)) - (5*d*at
anh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(96*(c^3)^(1/2)) - (c + d*x^3)^(1/2
)/(24*c*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

## 3.222 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx$

**Optimal**. Leaf size=107

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

**Rubi [A]** time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c*x^6) - (d*Sqrt[c + d*x^3])/(64*c^2*x^3) + (d^2*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(256*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} + \frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\text{Subst}\left(\int \frac{6c^2d^2-3cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^2} + \frac{\left(3d^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx\right.}{512c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^2} + \frac{\left(3d^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,\right.}{256c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

**Mathematica [A]** time = 0.06, size = 96, normalized size = 0.90

$$\frac{3d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 3d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - 4\sqrt{c}\sqrt{c+dx^3}\left(4c+3dx^3\right)}{768c^{5/2}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 3*d*x^3) + 3*d^2*x^6*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])] + 3*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(5/2
)*x^6)
```

**IntegrateAlgebraic [A]** time = 0.14, size = 95, normalized size = 0.89

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}} + \frac{\left(-4c-3dx^3\right)\sqrt{c+dx^3}}{192c^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] ((-4*c - 3*d*x^3)*Sqrt[c + d*x^3])/(192*c^2*x^6) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(
256*c^(5/2))
```

**fricas [A]** time = 0.80, size = 188, normalized size = 1.76

$$\left[\frac{3\sqrt{c}\,d^2x^6\log\left(\frac{d^2x^6+24\,cdx^3+8\left(dx^3+4\,c\right)\sqrt{dx^3+c}\,\sqrt{c}+32\,c^2}{dx^6-8\,cx^3}\right)-8\left(3\,cdx^3+4\,c^2\right)\sqrt{dx^3+c}}{1536\,c^3x^6},\ -\frac{3\sqrt{-c}\,d^2x^6\arctan\left(\frac{\left(dx^3+4\,c\right)\sqrt{dx^3+c}\,\sqrt{-c}}{4\left(cdx^3+c^2\right)}\right)+4\left(3\,cdx^3+4\,c^2\right)\sqrt{dx^3+c}}{768\,c^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*sqrt(c)*d^2*x^6*log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c)*sqrt
(d*x^3 + c)*sqrt(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^3*x^6), -1/768*(3*sqrt(-c)*d^2*x^6*arctan(1/4*(d*x^3 + 4*
c)*sqrt(d*x^3 + c)*sqrt(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*sqrt(d
*x^3 + c))/(c^3*x^6)]
```

**giac [A]** time = 0.18, size = 100, normalized size = 0.93

$$-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\sqrt{-c}\,c^2}-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}\,c^2}-\frac{3\left(dx^3+c\right)^{\frac{3}{2}}d^2+\sqrt{dx^3+c}\,cd^2}{192\,c^2d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/256*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/192*(3*(d*x^3 + c)^(3/2
)*d^2 + sqrt(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)
```

**maple [C]** time = 0.19, size = 574, normalized size = 5.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*d*(d*x^3+c)^(1/2)/c/x^3+1/12
*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/512/c^3*d^3*(2/3*(d*x^3+c)
^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
```

```
lpha=RootOf(_Z^3*d-8*c))+1/512/c^3*d^2*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d
*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

**mupad [B]** time = 3.91, size = 83, normalized size = 0.78

$$\frac{d^2\,\mathrm{atanh}\left(\frac{d^4\,\sqrt{d\,x^3+c}}{2048\,c^{7/2}\,\left(\frac{d^4}{2048\,c^3}+\frac{d^5\,x^3}{8192\,c^4}\right)}\right)}{256\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{192\,c\,x^6}-\frac{{\left(d\,x^3+c\right)}^{3/2}}{64\,c^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((d^4*(c + d*x^3)^(1/2))/(2048*c^(7/2)*(d^4/(2048*c^3) + (d^5*x^3
)/(8192*c^4)))))/(256*c^(5/2)) - (c + d*x^3)^(1/2)/(192*c*x^6) - (c + d*x^3
)^(3/2)/(64*c^2*x^6)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

## 3.223 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

**Optimal**. Leaf size=130

$$\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+d\right.}{27}$$

**Rubi [A]** time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}+\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-3072*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(9*d^4) - (3
8*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (4*c*(c + d*x^3)^(7/2))/(7*d^4) - (2*(c
+ d*x^3)^(9/2))/(27*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{57c^2(c+dx)^{3/2}}{d^3}+\frac{512c^3(c+dx)^{3/2}}{d^3(8c-dx)}-\frac{6c(c+dx)^{5/2}}{d^3}-\frac{(c+dx)^{7/2}}{d^3}\right) dx, x, x\right.\\
&= -\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}+\frac{\left(512c^3\right)\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x\right.}{3d^3}\\
&= -\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}+\frac{\left(1536c^4\right)\mathrm{S}}{}\\
&= -\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+d\right.}{27d}\\
&= -\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+d\right.}{27d}\\
&= -\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+d\right.}{27d}
\end{aligned}
$$

**Mathematica** [A] time = 0.13, size = 93, normalized size = 0.72

$$\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{2\sqrt{c+dx^3}\left(1509176c^4+61892c^3dx^3+4611c^2d^2x^6+410cd^3x^9+35d^4x^{12}\right)}{945d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410
*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^4
```

**IntegrateAlgebraic** [A] time = 0.07, size = 93, normalized size = 0.72

$$\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{2\sqrt{c+dx^3}\left(1509176c^4+61892c^3dx^3+4611c^2d^2x^6+410cd^3x^9+35d^4x^{12}\right)}{945d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410
*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^4
```

**fricas** [A] time = 0.55, size = 191, normalized size = 1.47

$$\left[\frac{2\left(2177280c^{\frac{9}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-\left(35d^4x^{12}+410cd^3x^9+4611c^2d^2x^6+61892c^3dx^3+1509176c^4\right)\sqrt{dx^3+c}\right)}{945d^4}, -\frac{2\left(4354560\sqrt{-c}c^4\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(35d^4x^{12}+410cd^3x^9+4611c^2d^2x^6+61892c^3dx^3+1509176c^4\right)\sqrt{dx^3+c}\right)}{945d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]

**giac [A]** time = 0.17, size = 117, normalized size = 0.90

$$-\frac{9216\,c^5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{2\left(35\left(dx^3+c\right)^{\frac{9}{2}}d^{32}+270\left(dx^3+c\right)^{\frac{7}{2}}cd^{32}+3591\left(dx^3+c\right)^{\frac{5}{2}}c^2d^{32}+53760\left(dx^3+c\right)^{\frac{3}{2}}c^3d^{32}+1451520\sqrt{dx^3+c}\,c^4d^{32}\right)}{945\,d^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")

[Out] -9216*c^5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/945*(35*(d*x^3 + c)^(9/2)*d^32 + 270*(d*x^3 + c)^(7/2)*c*d^32 + 3591*(d*x^3 + c)^(5/2)*c^2*d^32 + 53760*(d*x^3 + c)^(3/2)*c^3*d^32 + 1451520*sqrt(d*x^3 + c)*c^4*d^32)/d^36

**maple [C]** time = 0.25, size = 634, normalized size = 4.88

$$-\frac{512\left(\frac{2\sqrt{dx^3+c}\,x^3}{9}+\frac{56\sqrt{dx^3+c}\,c}{9d}+\frac{3i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{\frac{i}{2}\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{\frac{i}{2}\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}}}\right)}{d^3\sqrt{dx^3+c}}\right)c^3}{d^3}-\frac{8\left(\frac{2\sqrt{dx^3+c}\,dx^9}{21}+\frac{16\sqrt{dx^3+c}\,cx^6}{105}+\frac{2\sqrt{dx^3+c}\,c^2x^3}{105d}-\frac{4\sqrt{dx^3+c}\,c^3}{105d^2}\right)c}{d^2}-\frac{\frac{2\sqrt{dx^3+c}\,dx^{12}}{27}+\frac{20\sqrt{dx^3+c}\,cx^9}{189}+\frac{2\sqrt{dx^3+c}\,c^2x^6}{315d}-\frac{8\sqrt{dx^3+c}\,c^3x^3}{945d^2}+\frac{16\sqrt{dx^3+c}\,c^4}{945d^3}}{d}-\frac{128\left(dx^3+c\right)^{\frac{5}{2}}c^2}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)

[Out] -1/d*(2/27*d*x^12*(d*x^3+c)^(1/2)+20/189*c*x^9*(d*x^3+c)^(1/2)+2/315*c^2/d*x^6*(d*x^3+c)^(1/2)-8/945*c^3/d^2*x^3*(d*x^3+c)^(1/2)+16/945*c^4/d^3*(d*x^3+c)^(1/2))-8*c/d^2*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105*c^2/d*x^3*(d*x^3+c)^(1/2)-4/105*c^3/d^2*(d*x^3+c)^(1/2))-128/15*c^2*(d*x^3+c)^(5/2)/d^4-512*c^3/d^3*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))

**maxima [A]** time = 1.29, size = 110, normalized size = 0.85

$$-\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+35\left(dx^3+c\right)^{\frac{9}{2}}+270\left(dx^3+c\right)^{\frac{7}{2}}c+3591\left(dx^3+c\right)^{\frac{5}{2}}c^2+53760\left(dx^3+c\right)^{\frac{3}{2}}c^3+1451520\sqrt{dx^3+c}\,c^4\right)}{945\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")

[Out] -2/945*(2177280*c^(9/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c))) + 35*(d*x^3 + c)^(9/2) + 270*(d*x^3 + c)^(7/2)*c + 3591*(d*x^3 + c)^(5/2)*c^2 + 53760*(d*x^3 + c)^(3/2)*c^3 + 1451520*sqrt(d*x^3 + c)*c^4)/d^4

**mupad [B]** time = 3.53, size = 135, normalized size = 1.04

$$\frac{4608\,c^{9/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}-\frac{2\,x^{12}\,\sqrt{d\,x^3+c}}{27}-\frac{3018352\,c^4\,\sqrt{d\,x^3+c}}{945\,d^4}-\frac{164\,c\,x^9\,\sqrt{d\,x^3+c}}{189\,d}-\frac{123784\,c^3\,x^3\,\sqrt{d\,x^3+c}}{945\,d^3}-\frac{3074\,c^2\,x^6\,\sqrt{d\,x^3+c}}{315\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (4608*c^(9/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 - (2*x^12*(c + d*x^3)^(1/2))/27 - (3018352*c^4*(c + d*x^3)^(1/2))/(
945*d^4) - (164*c*x^9*(c + d*x^3)^(1/2))/(189*d) - (123784*c^3*x^3*(c + d*x
^3)^(1/2))/(945*d^3) - (3074*c^2*x^6*(c + d*x^3)^(1/2))/(315*d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

## 3.224 $\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

**Optimal**. Leaf size=109

$$\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}$$

**Rubi** [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-384*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(9*d^3) - (14*
c*(c + d*x^3)^(5/2))/(15*d^3) - (2*(c + d*x^3)^(7/2))/(21*d^3) + (1152*c^(7
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{7c(c+dx)^{3/2}}{d^2} + \frac{64c^2(c+dx)^{3/2}}{d^2(8c-dx)} - \frac{(c+dx)^{5/2}}{d^2}\right) dx, x, x^3\right)$$
$$= -\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(64c^2\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d^2}$$
$$= -\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(192c^3\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x\right.}{d^2}$$
$$= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(1728c^4\right)\text{Sub}}{}$$
$$= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{\left(3456c^4\right)\text{Sul}}{}$$
$$= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3} - \frac{14c\left(c+dx^3\right)^{5/2}}{15d^3} - \frac{2\left(c+dx^3\right)^{7/2}}{21d^3} + \frac{1152c^{7/2}\,\text{tanl}}{d}$$

**Mathematica [A]** time = 0.09, size = 81, normalized size = 0.74

$$\frac{362880c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3}\left(62882c^3+2579c^2dx^3+192cd^2x^6+15d^3x^9\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^
9) + 362880*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(315*d^3)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 82, normalized size = 0.75

$$\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{2\sqrt{c+dx^3}\left(62882c^3+2579c^2dx^3+192cd^2x^6+15d^3x^9\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^
9))/(315*d^3) + (1152*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

**fricas [A]** time = 0.96, size = 169, normalized size = 1.55

$$\left[\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) - \left(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3\right)\sqrt{dx^3+c}\right)}{315\,d^3}, -\frac{2\left(181440\,\sqrt{-c}\,c^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right) + \left(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3\right)\sqrt{dx^3+c}\right)}{315\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d
*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*
x^3 + c))/d^3]
```

**giac [A]** time = 0.16, size = 100, normalized size = 0.92

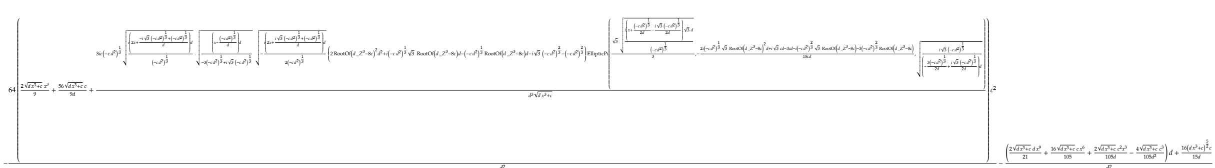
$$-\frac{1152\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{2\left(15\left(dx^3+c\right)^{\frac{7}{2}}d^{18}+147\left(dx^3+c\right)^{\frac{5}{2}}cd^{18}+2240\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{18}+60480\sqrt{dx^3+c}\,c^3d^{18}\right)}{315\,d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1152*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/315*(15*(
d*x^3 + c)^(7/2)*d^18 + 147*(d*x^3 + c)^(5/2)*c*d^18 + 2240*(d*x^3 + c)^(3/
2)*c^2*d^18 + 60480*sqrt(d*x^3 + c)*c^3*d^18)/d^21
```

**maple [C]** time = 0.16, size = 541, normalized size = 4.96



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/21*(d*x^3+c)^(1/2)*d*x^9+16/105*(d*x^3+c)^(1/2)*c*x^6+2/105*(d
*x^3+c)^(1/2)*c^2/d*x^3-4/105*(d*x^3+c)^(1/2)*c^3/d^2)+16/15*c/d*(d*x^3+c)^
(5/2))-64*c^2/d^2*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d
^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^
2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d
^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3
*d-8*c)))
```

**maxima [A]** time = 1.34, size = 96, normalized size = 0.88

$$-\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+15\left(dx^3+c\right)^{\frac{7}{2}}+147\left(dx^3+c\right)^{\frac{5}{2}}c+2240\left(dx^3+c\right)^{\frac{3}{2}}c^2+60480\sqrt{dx^3+c}\,c^3\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/315*(90720*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 15*(d*x^3 + c)^(7/2) + 147*(d*x^3 + c)^(5/2)*c + 2240*(d*x^3
+ c)^(3/2)*c^2 + 60480*sqrt(d*x^3 + c)*c^3)/d^3
```

**mupad [B]** time = 3.50, size = 115, normalized size = 1.06

$$\frac{576\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}-\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21}-\frac{125764\,c^3\,\sqrt{d\,x^3+c}}{315\,d^3}-\frac{128\,c\,x^6\,\sqrt{d\,x^3+c}}{105\,d}-\frac{5158\,c^2\,x^3\,\sqrt{d\,x^3+c}}{315\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (576*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 - (2*x^9*(c + d*x^3)^(1/2))/21 - (125764*c^3*(c + d*x^3)^(1/2))/(315
*d^3) - (128*c*x^6*(c + d*x^3)^(1/2))/(105*d) - (5158*c^2*x^3*(c + d*x^3)^(
1/2))/(315*d^2)
```

**sympy [A]** time = 108.79, size = 110, normalized size = 1.01

$$-\frac{1152c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^3\sqrt{-c}}-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{\frac{3}{2}}}{9d^3}-\frac{14c\left(c+dx^3\right)^{\frac{5}{2}}}{15d^3}-\frac{2\left(c+dx^3\right)^{\frac{7}{2}}}{21d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -1152*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**3*sqrt(-c)) - 384*c**3*s
qrt(c + d*x**3)/d**3 - 128*c**2*(c + d*x**3)**(3/2)/(9*d**3) - 14*c*(c + d*
x**3)**(5/2)/(15*d**3) - 2*(c + d*x**3)**(7/2)/(21*d**3)
```

# 3.225 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

**Optimal**. Leaf size=88

$$\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}$$

**Rubi** [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 80, 50, 63, 206}

$$-\frac{48c^2\sqrt{c+dx^3}}{d^2}+\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-48*c^2*Sqrt[c + d*x^3])/d^2 - (16*c*(c + d*x^3)^(3/2))/(9*d^2) - (2*(c +
d*x^3)^(5/2))/(15*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}+\frac{(8c)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}+\frac{\left(24c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d}\\
&= -\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}+\frac{\left(216c^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x\right.}{d}\\
&= -\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}+\frac{\left(432c^3\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+\right.}{d^2}\\
&= -\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}+\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 70, normalized size = 0.80

$$\frac{6480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(1123c^2+46cdx^3+3d^2x^6\right)}{45d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6) + 6480*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 71, normalized size = 0.81

$$\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{2\sqrt{c+dx^3}\left(1123c^2+46cdx^3+3d^2x^6\right)}{45d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6))/(45*d^2) + (144*c^
(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

**fricas [A]** time = 0.63, size = 147, normalized size = 1.67

$$\left[\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(3\,d^2x^6+46\,cdx^3+1123\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^2},\ -\frac{2\left(3240\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(3\,d^2x^6+46\,cdx^3+1123\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*(3
240*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 46*c
*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]
```

**giac** [A] time = 0.18, size = 83, normalized size = 0.94

$$-\frac{144\,c^{3}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^{2}}-\frac{2\left(3\left(dx^{3}+c\right)^{\frac{5}{2}}d^{8}+40\left(dx^{3}+c\right)^{\frac{3}{2}}cd^{8}+1080\sqrt{dx^{3}+c}\,c^{2}d^{8}\right)}{45\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2*d
^8)/d^10
```

**maple** [C] time = 0.18, size = 462, normalized size = 5.25

$$-\frac{8\left(\frac{2\sqrt{dx^{3}+c}\,x^{3}}{9}+\frac{56\sqrt{dx^{3}+c}\,c}{9d}+\frac{3ic\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)^{2}d^{2}+i\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)d-\left(-cd^{2}\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^{3}-8c\right)d-i\sqrt{3}\left(-cd^{2}\right)^{\frac{2}{3}}-\left(-cd^{2}\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^{2}\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)^{2}d+i\sqrt{3}\,cd-3cd-i\left(-cd^{2}\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)-3\left(-cd^{2}\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{d^{3}\sqrt{dx^{3}+c}}\right)c}{d}-\frac{2\left(dx^{3}+c\right)^{\frac{5}{2}}}{15d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/15*(d*x^3+c)^(5/2)/d^2-8*c/d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/
2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I
*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c)))
```

**maxima** [A] time = 1.44, size = 82, normalized size = 0.93

$$-\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^{3}+c}-3\sqrt{c}}{\sqrt{dx^{3}+c}+3\sqrt{c}}\right)+3\left(dx^{3}+c\right)^{\frac{5}{2}}+40\left(dx^{3}+c\right)^{\frac{3}{2}}c+1080\sqrt{dx^{3}+c}\,c^{2}\right)}{45\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/45*(1620*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 40*(d*x^3 + c)^(3/2)*c + 1080*sqrt(d*x^3
+ c)*c^2)/d^2
```

**mupad** [B] time = 3.52, size = 95, normalized size = 1.08

$$\frac{72\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15}-\frac{2246\,c^2\,\sqrt{d\,x^3+c}}{45\,d^2}-\frac{92\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (72*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^2 - (2*x^6*(c + d*x^3)^(1/2))/15 - (2246*c^2*(c + d*x^3)^(1/2))/(45*d^2
) - (92*c*x^3*(c + d*x^3)^(1/2))/(45*d)
```

**sympy** [A] time = 62.74, size = 90, normalized size = 1.02

$$-\frac{144c^3 \operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{\frac{3}{2}}}{9d^2}-\frac{2\left(c+dx^3\right)^{\frac{5}{2}}}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -144*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**2*sqrt(-c)) - 48*c**2*sqr
t(c + d*x**3)/d**2 - 16*c*(c + d*x**3)**(3/2)/(9*d**2) - 2*(c + d*x**3)**(5
/2)/(15*d**2)
```

## 3.226 $\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3} dx$

**Optimal**. Leaf size=67

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}$$

**Rubi** [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 50, 63, 206}

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-6*c*Sqrt[c + d*x^3])/d - (2*(c + d*x^3)^(3/2))/(9*d) + (18*c^(3/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$
$$= -\frac{2\left(c+dx^3\right)^{3/2}}{9d} + (3c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)$$
$$= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2\left(c+dx^3\right)^{3/2}}{9d} + \left(27c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2\left(c+dx^3\right)^{3/2}}{9d} + \frac{\left(54c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}$$
$$= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2\left(c+dx^3\right)^{3/2}}{9d} + \frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

**Mathematica** [A] time = 0.04, size = 58, normalized size = 0.87

$$\frac{162c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3}\left(28c+dx^3\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3) + 162*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c]])])/(9*d)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 59, normalized size = 0.88

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{2\sqrt{c+dx^3}\left(28c+dx^3\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3))/(9*d) + (18*c^(3/2)*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/d
```

**fricas** [A] time = 0.58, size = 121, normalized size = 1.81

$$\left[\frac{81\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) - 2\left(dx^3+28\,c\right)\sqrt{dx^3+c}}{9\,d}, -\frac{2\left(81\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right) + \left(dx^3+28\,c\right)\sqrt{dx^3+c}\right)}{9\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]
```

**giac** [A] time = 0.16, size = 65, normalized size = 0.97

$$-\frac{18\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d} - \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^2 + 27\,\sqrt{dx^3+c}\,cd^2\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 + c
)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3
```

**maple [C]** time = 0.16, size = 441, normalized size = 6.58

$$-\frac{2\sqrt{dx^3+c}\,x^3}{9}-\frac{56\sqrt{dx^3+c}\,c}{9d}-\frac{3ic\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{d^3\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/9*(d*x^3+c)^(1/2)*x^3-56/9*(d*x^3+c)^(1/2)*c/d-3*I*c/d^3*2^(1/2)*sum((-c
*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*
d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.39, size = 68, normalized size = 1.01

$$-\frac{81\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\left(dx^3+c\right)^{\frac{3}{2}}+54\,\sqrt{dx^3+c}\,c}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/9*(81*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 54*sqrt(d*x^3 + c)*c)/d
```

**mupad [B]** time = 3.45, size = 75, normalized size = 1.12

$$\frac{9\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d}-\frac{56\,c\,\sqrt{d\,x^3+c}}{9\,d}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (9*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d - (56*c*(c + d*x^3)^(1/2))/(9*d) - (2*x^3*(c + d*x^3)^(1/2))/9
```

**sympy [A]** time = 29.25, size = 65, normalized size = 0.97

$$-\frac{18c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d\sqrt{-c}}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{\frac{3}{2}}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -18*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) - 6*c*sqrt(c + d*
x**3)/d - 2*(c + d*x**3)**(3/2)/(9*d)
```

## 3.227 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)} dx$

**Optimal**. Leaf size=73

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

**Rubi [A]** time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 84, 156, 63, 208, 206}

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{2}{3}\sqrt{c+dx^3} - \frac{\text{Subst}\left(\int \frac{-c^2d-10cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{2}{3}\sqrt{c+dx^3} + \frac{1}{24}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{8}(27cd)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+d}}\right.$$

$$= -\frac{2}{3}\sqrt{c+dx^3} + \frac{1}{4}(27c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c}\right.}{12d}$$

$$= -\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

**Mathematica [A]** time = 0.05, size = 73, normalized size = 1.00

$$-\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

**IntegrateAlgebraic [A]** time = 0.05, size = 73, normalized size = 1.00

$$-\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

**fricas [A]** time = 0.85, size = 152, normalized size = 2.08

$$\left[\frac{9}{8}\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\frac{1}{24}\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-\frac{2}{3}\sqrt{dx^3+c},\ \frac{1}{12}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\frac{9}{4}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-\frac{2}{3}\sqrt{dx^3+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
[Out] [9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))
+ 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*sqr
t(d*x^3 + c), 1/12*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9/4*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 2/3*sqrt(d*x^3 + c)]
```

**giac [A]** time = 0.18, size = 61, normalized size = 0.84

$$\frac{c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{9\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}-\frac{2}{3}\sqrt{dx^3+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d*
x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)
```

**maple [C]** time = 0.19, size = 500, normalized size = 6.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x)
```

```
[Out] -1/8/c*d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
+1/8/c*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh
((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)
```

**mupad [B]** time = 5.89, size = 89, normalized size = 1.22

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^{27}}{x^6\left(8\,c-d\,x^3\right)^{27}}\right)}{24}-\frac{2\,\sqrt{d\,x^3+c}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
*(10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^27)/(x^6*(8*c - d*x^3)^27)))/
24 - (2*(c + d*x^3)^(1/2))/3
```

**sympy** [A] time = 24.74, size = 73, normalized size = 1.00

$$-\frac{9c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{2\sqrt{c+dx^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)
```

```
[Out] -9*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(4*sqrt(-c)) + c*atan(sqrt(c + d*x
**3)/sqrt(-c))/(12*sqrt(-c)) - 2*sqrt(c + d*x**3)/3
```

## 3.228 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx$

**Optimal**. Leaf size=78

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

**Rubi [A]** time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 98, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*x^3) + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} - \frac{\text{Subst}\left(\int \frac{-13c^2d-\frac{17}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{1}{192}(13d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right) + \frac{1}{64}\left(27d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\ldots}\right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{13}{96}\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right) + \frac{1}{32}(27d)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,\ldots\right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

**Mathematica [A]** time = 0.05, size = 78, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 78, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

**fricas [A]** time = 0.86, size = 186, normalized size = 2.38

$$\left[\frac{27\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+13\sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8\sqrt{dx^3+c}\,c}{192\,cx^3}, \frac{13\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-27\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-4\sqrt{dx^3+c}\,c}{96\,cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), 1/96*(13*sqrt(-c)*d*x^3*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(
-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c*x^3)]
```

**giac [A]** time = 0.17, size = 64, normalized size = 0.82

$$\frac{13\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}}-\frac{9\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 13/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/32*d*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/24*sqrt(d*x^3 + c)/x^3
```

**maple [C]** time = 0.26, size = 556, normalized size = 7.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((
d*x^3+c)^(1/2)/c^(1/2)))-1/64*d^2/c^2*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+
c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*d*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*
x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)
```

**mupad [B]** time = 3.52, size = 56, normalized size = 0.72

$$\frac{9\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{3\,\sqrt{c}}\right)}{32\,\sqrt{c}}-\frac{13\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96\,\sqrt{c}}-\frac{\sqrt{d\,x^3+c}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (9*d*atanh((c + d*x^3)^(1/2)/(3*c^(1/2))))/(32*c^(1/2)) - (13*d*atanh((c +
d*x^3)^(1/2)/c^(1/2)))/(96*c^(1/2)) - (c + d*x^3)^(1/2)/(24*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx-\int\frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

## 3.229 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx$

**Optimal**. Leaf size=104

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

**Rubi [A]** time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*x^6) - (11*d*Sqrt[c + d*x^3])/(192*c*x^3) + (9*d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(768*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{\text{Subst}\left(\int \frac{-22c^2d-\frac{35}{2}cd^2x}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{\text{Subst}\left(\int \frac{74c^3d^2+11c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{(37d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{1536c} + \frac{(27d^3)\,\text{Subst}\left(\int \frac{\cdots}{(8c-\cdots}\right.}{512\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{(37d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{768c} + \frac{(27d^2)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

**Mathematica [A]** time = 0.11, size = 96, normalized size = 0.92

$$\frac{27d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 37d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - 4\sqrt{c}\sqrt{c+dx^3}\left(4c+11dx^3\right)}{768c^{3/2}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 11*d*x^3) + 27*d^2*x^6*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])] - 37*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(
3/2)*x^6)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 95, normalized size = 0.91

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}} + \frac{\left(-4c-11dx^3\right)\sqrt{c+dx^3}}{192cx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] ((-4*c - 11*d*x^3)*Sqrt[c + d*x^3])/(192*c*x^6) + (9*d^2*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(768*c^(3/2))
```

**fricas [A]** time = 0.87, size = 218, normalized size = 2.10

$$\left[\frac{27\sqrt{c}\,d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+37\sqrt{c}\,d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8\left(11cdx^3+4c^2\right)\sqrt{dx^3+c}}{1536c^2x^6}, \frac{37\sqrt{-c}\,d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-27\sqrt{-c}\,d^2x^6\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-4\left(11cdx^3+4c^2\right)\sqrt{dx^3+c}}{768c^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), 1/768*(37*
sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d^2*x^6*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 +
 c))/(c^2*x^6)]
```

**giac [A]** time = 0.19, size = 101, normalized size = 0.97

$$\frac{37\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\sqrt{-c}\,c} - \frac{9\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}\,c} - \frac{11\left(dx^3+c\right)^{\frac{3}{2}}d^2-7\sqrt{dx^3+c}\,cd^2}{192\,cd^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 37/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9/256*d^2*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/192*(11*(d*x^3 + c)^(3/2)*d
^2 - 7*sqrt(d*x^3 + c)*c*d^2)/(c*d^2*x^6)
```

**maple [C]** time = 0.24, size = 617, normalized size = 5.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d-c^(1/2)*d*arct
anh((d*x^3+c)^(1/2)/c^(1/2)))+1/8/c*(-1/6*c*(d*x^3+c)^(1/2)/x^6-5/12*d*(d*x
^3+c)^(1/2)/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-1/512/c^3
*d^3*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*su
m((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
```

```
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/5
12/c^3*d^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arc
tanh((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

**mupad [B]** time = 3.74, size = 87, normalized size = 0.84

$$\frac{7\,\sqrt{d\,x^3+c}}{192\,x^6}-\frac{37\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{768\,\sqrt{c^3}}+\frac{9\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{256\,\sqrt{c^3}}-\frac{11\,\left(d\,x^3+c\right)^{3/2}}{192\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (7*(c + d*x^3)^(1/2))/(192*x^6) - (37*d^2*atanh((c*(c + d*x^3)^(1/2))/(c^3)
^(1/2)))/(768*(c^3)^(1/2)) + (9*d^2*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1
/2))))/(256*(c^3)^(1/2)) - (11*(c + d*x^3)^(3/2))/(192*c*x^6)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

### 3.230 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=90

$$\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}$$

**Rubi [A]** time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 88, 63, 206}

$$-\frac{38c^2\sqrt{c+dx^3}}{d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-38*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/(3*d^4) - (2*(c + d
*x^3)^(5/2))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(9*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{57c^2}{d^3\sqrt{c+dx}} + \frac{512c^3}{d^3(8c-dx)\sqrt{c+dx}} - \frac{6c\sqrt{c+dx}}{d^3} - \frac{(c+dx)^{3/2}}{d^3}\right) d$$

$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c\left(c+dx^3\right)^{3/2}}{3d^4} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^4} + \frac{\left(512c^3\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+}}\right.}{3d^3}$$

$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c\left(c+dx^3\right)^{3/2}}{3d^4} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^4} + \frac{\left(1024c^3\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx\right.}{3d^4}$$

$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c\left(c+dx^3\right)^{3/2}}{3d^4} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^4} + \frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

**Mathematica [A]** time = 0.11, size = 69, normalized size = 0.77

$$\frac{5120c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 6\sqrt{c+dx^3}\left(296c^2+12cdx^3+d^2x^6\right)}{45d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 72, normalized size = 0.80

$$\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4} - \frac{2\sqrt{c+dx^3}\left(296c^2+12cdx^3+d^2x^6\right)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6))/(15*d^4) + (1024*c^(5
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)
```

**fricas [A]** time = 0.92, size = 146, normalized size = 1.62

$$\left[\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) - 3\left(d^2x^6+12\,cdx^3+296\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^4}, -\frac{2\left(2560\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right) + 3\left(d^2x^6+12\,cdx^3+296\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(25
60*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^2*x^6 + 12*c*
d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]
```

**giac [A]** time = 0.16, size = 82, normalized size = 0.91

$$-\frac{1024\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^4} - \frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16} + 10\left(dx^3+c\right)^{\frac{3}{2}}cd^{16} + 285\sqrt{dx^3+c}\,c^2d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d*
x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c^2
*d^16)/d^20
```

**maple** [C] time = 0.28, size = 528, normalized size = 5.87

$$-\frac{8\left(\frac{2\sqrt{dx^3+c}\,x^3}{9d}-\frac{4\sqrt{dx^3+c}\,c}{9d^2}\right)c}{d^2}-\frac{\frac{2\sqrt{dx^3+c}\,x^6}{15d}-\frac{8\sqrt{dx^3+c}\,cx^3}{45d^2}+\frac{16\sqrt{dx^3+c}\,c^2}{45d^3}}{d}-\frac{128\sqrt{dx^3+c}\,c^2}{3d^4}-\frac{512ic^2\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}}}\right)}{27d^6\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d*(2/15/d*x^6*(d*x^3+c)^(1/2)-8/45*c/d^2*x^3*(d*x^3+c)^(1/2)+16/45*c^2*(
d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*
c/d^2)-128/3*c^2*(d*x^3+c)^(1/2)/d^4-512/27*I*c^2/d^6*2^(1/2)*sum((-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima** [A] time = 1.26, size = 82, normalized size = 0.91

$$-\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+30\left(dx^3+c\right)^{\frac{3}{2}}c+855\,\sqrt{dx^3+c}\,c^2\right)}{45\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(1280*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 30*(d*x^3 + c)^(3/2)*c + 855*sqrt(d*x^3 +
 c)*c^2)/d^4
```

**mupad** [B] time = 3.22, size = 98, normalized size = 1.09

$$\frac{512\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}-\frac{592\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}-\frac{8\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(9*d^4) - (592*c^2*(c + d*x^3)^(1/2))/(15*d^4) - (2*x^6*(c + d*x^3)^(1/2
))/(15*d^2) - (8*c*x^3*(c + d*x^3)^(1/2))/(5*d^3)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{11}}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**11/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

## 3.231 $\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=71

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

**Rubi [A]** time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 88, 63, 206}

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-14*c*Sqrt[c + d*x^3])/(3*d^3) - (2*(c + d*x^3)^(3/2))/(9*d^3) + (128*c^(3
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{7c}{d^2\sqrt{c+dx}} + \frac{64c^2}{d^2(8c-dx)\sqrt{c+dx}} - \frac{\sqrt{c+dx}}{d^2}\right)dx, x, x^3\right)$$
$$= -\frac{14c\sqrt{c+dx^3}}{3d^3} - \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{(64c^2)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^2}$$
$$= -\frac{14c\sqrt{c+dx^3}}{3d^3} - \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{(128c^2)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}$$
$$= -\frac{14c\sqrt{c+dx^3}}{3d^3} - \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

**Mathematica [A]** time = 0.07, size = 58, normalized size = 0.82

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3}\,(22c+dx^3)}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^3)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 61, normalized size = 0.86

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3} - \frac{2\sqrt{c+dx^3}\,(22c+dx^3)}{9d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(22*c + d*x^3))/(9*d^3) + (128*c^(3/2)*ArcTanh[Sqrt[c +
 d*x^3]/(3*Sqrt[c])])/(9*d^3)
```

**fricas [A]** time = 1.10, size = 121, normalized size = 1.70

$$\left[\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) - \left(dx^3+22\,c\right)\sqrt{dx^3+c}\right)}{9\,d^3}, -\frac{2\left(64\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right) + \left(dx^3+22\,c\right)\sqrt{dx^3+c}\right)}{9\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(32*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3, -2/9*(64*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3]
```

**giac [A]** time = 0.19, size = 65, normalized size = 0.92

$$-\frac{128\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}\,d^3} - \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6 + 21\,\sqrt{dx^3+c}\,cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -128/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/9*((d*x^
3 + c)^(3/2)*d^6 + 21*sqrt(d*x^3 + c)*c*d^6)/d^9
```

**maple [C]** time = 0.16, size = 468, normalized size = 6.59

$$-\frac{\left(\frac{2\sqrt{dx^3+c}\,x^3}{9d}-\frac{4\sqrt{dx^3+c}\,c}{9d^2}\right)d+\frac{16\sqrt{dx^3+c}\,c}{3d}}{d^2}-\frac{64ic(-cd^2)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{(-cd^2)^{\frac{1}{3}}}{d}\right)d}{-3(-cd^2)^{\frac{1}{3}}+i\sqrt{3}(-cd^2)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)d}{2(-cd^2)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i(-cd^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-(-cd^2)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}(-cd^2)^{\frac{2}{3}}-(-cd^2)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}}{3},-\frac{2i(-cd^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i(-cd^2)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3(-cd^2)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{\left(-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)d}}\right)}{27d^5\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d^2*((2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*c/d^2)*d+16/3*(d*x^3
+c)^(1/2)*c/d)-64/27*I*c/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.31, size = 66, normalized size = 0.93

$$-\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+21\sqrt{dx^3+c}\,c\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(32*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 21*sqrt(d*x^3 + c)*c)/d^3
```

**mupad [B]** time = 3.39, size = 78, normalized size = 1.10

$$\frac{64\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}-\frac{44\,c\,\sqrt{d\,x^3+c}}{9\,d^3}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(9*d^3) - (44*c*(c + d*x^3)^(1/2))/(9*d^3) - (2*x^3*(c + d*x^3)^(1/2))/(9
*d^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^8}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**8/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

## 3.232 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=52

$$\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

**Rubi** [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 80, 63, 206}

$$\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(9*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{(8c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$
$$= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{(16c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}$$
$$= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

**Mathematica** [A] time = 0.02, size = 49, normalized size = 0.94

$$-\frac{2\left(3\sqrt{c+dx^3}-8\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(
9*d^2)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 52, normalized size = 1.00

$$\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2} - \frac{2\sqrt{c+dx^3}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (-2*Sqrt[c + d*x^3])/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(9*d^2)
```

**fricas** [A] time = 0.65, size = 103, normalized size = 1.98

$$\left[\frac{2\left(4\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\sqrt{dx^3+c}\right)}{9d^2}, -\frac{2\left(8\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+c}\right)}{9d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [2/9*(4*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 3*sqrt(d*x^3 + c))/d^2, -2/9*(8*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sq
rt(-c)/c) + 3*sqrt(d*x^3 + c))/d^2]
```

**giac** [A] time = 0.16, size = 48, normalized size = 0.92

$$-\frac{2\left(\frac{8c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{3\sqrt{dx^3+c}}{d}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(8*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
+ c)/d)/d
```

**maple** [C] time = 0.17, size = 425, normalized size = 8.17

$$-\frac{2\sqrt{d x^3+c}}{3d^2}-\frac{8i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2 d^2+i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-c d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-c d^2\right)^{\frac{2}{3}}-\left(-c d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c d^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2 d+i\sqrt{3}\,cd-3cd-i\left(-c d^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-c d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{27d^4\sqrt{d x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*(d*x^3+c)^(1/2)/d^2-8/27*I/d^4*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(
-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/
2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^
2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/
2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima** [A] time = 1.40, size = 56, normalized size = 1.08

$$-\frac{2\left(4\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 3*sqrt(d*x^3 + c))/d^2
```

**mupad** [B] time = 3.27, size = 60, normalized size = 1.15

$$\frac{8\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}-\frac{2\sqrt{d\,x^3+c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (8*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/(9*d^2) - (2*(c + d*x^3)^(1/2))/(3*d^2)
```

**sympy** [A] time = 16.35, size = 61, normalized size = 1.17

$$\begin{cases} \frac{2\left(-\frac{8c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for}\ d \neq 0 \\ \frac{x^6}{48c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)) - sqr
t(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(48*c**(3/2)), True))
```

## 3.233 $\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

**Rubi [A]** time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 33, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

**IntegrateAlgebraic [A]** time = 0.03, size = 33, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

**fricas [A]** time = 0.84, size = 78, normalized size = 2.36

$$\left[\frac{\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)}{9\sqrt{c}\,d}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{9cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c)
*d), -2/9*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)/(c*d)]
```

**giac [A]** time = 0.16, size = 27, normalized size = 0.82

$$-\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] -2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)
```

**maple [C]** time = 0.20, size = 413, normalized size = 12.52

$$-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\operatorname{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3}, -\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\operatorname{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\operatorname{RootOf}\left(d\_Z^3-8c\right)}{18cd}, \sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{27cd^3\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
[Out] -1/27*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
```

```
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^
(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/
3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=
RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.21, size = 42, normalized size = 1.27

$$-\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/(sqrt
(c)*d)
```

**mupad [B]** time = 3.23, size = 45, normalized size = 1.36

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(9*c^(1/2)*
d)
```

**sympy [A]** time = 11.20, size = 32, normalized size = 0.97

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c))
```

## 3.234 $\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 86, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]]/(12*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{24c} + \frac{d\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{12c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{12cd}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(36*c^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.05, size = 58, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]]/(12*c^(3/2))
```

**fricas [A]** time = 0.89, size = 139, normalized size = 2.40

$$\left[\frac{\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{72c^2}, \frac{3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{36c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)
) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, 1/36
*(3*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c))/c^2]
```

**giac [A]** time = 0.17, size = 54, normalized size = 0.93

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}\,c} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{36\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

**maple** [C] time = 0.17, size = 433, normalized size = 7.47

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{12 c^{\frac{3}{2}}}-\frac{i\left(-c d^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}} \sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}} \sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}} \left(2 \operatorname{RootOf}\left(d\_Z^3-8c\right)^2 d^2+i\left(-c d^2\right)^{\frac{1}{3}} \sqrt{3}\, \operatorname{RootOf}\left(d\_Z^3-8c\right) d-\left(-c d^2\right)^{\frac{1}{3}} \operatorname{RootOf}\left(d\_Z^3-8c\right) d-i\sqrt{3}\left(-c d^2\right)^{\frac{2}{3}}-\left(-c d^2\right)^{\frac{2}{3}}\right) \operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\, d}{\left(-c d^2\right)^{\frac{1}{3}}}}}{3}, -\frac{2i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\, \operatorname{RootOf}\left(d\_Z^3-8c\right)^2 d+i\sqrt{3}\, c d-3cd-i\left(-c d^2\right)^{\frac{2}{3}}\sqrt{3}\, \operatorname{RootOf}\left(d\_Z^3-8c\right)-3\left(-c d^2\right)^{\frac{2}{3}}\operatorname{RootOf}\left(d\_Z^3-8c\right)}{18cd}, \sqrt{\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{216 c^2 d^2 \sqrt{d x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/216*I/c^2/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I
*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c))-1/12*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x), x)
```

**mupad** [B] time = 3.28, size = 47, normalized size = 0.81

$$-\frac{3\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)-\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{36\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] -(3*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)) - atanh((c*(c + d*x^3)^(1/2))/
(3*(c^3)^(1/2))))/(36*(c^3)^(1/2))
```

**sympy** [A] time = 12.22, size = 58, normalized size = 1.00

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{36c\sqrt{-c}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(36*c*sqrt(-c)) + atan(sqrt(c + d*x**3
)/sqrt(-c))/(12*c*sqrt(-c))
```

## 3.235 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=81

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}-\frac{\sqrt{c+dx^3}}{24c^2x^3}$$

**Rubi [A]** time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 103, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2
88*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}\\
&= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{64c^2} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2}\\
&= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{32c^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+d}\right.}{96c^2}\\
&= -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 81, normalized size = 1.00

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}} - \frac{\sqrt{c+dx^3}}{24c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 81, normalized size = 1.00

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}} - \frac{\sqrt{c+dx^3}}{24c^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

**fricas [A]** time = 0.80, size = 184, normalized size = 2.27

$$\left[\frac{\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10c}{dx^3-8c}\right) + 9\sqrt{c}\,dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\,\sqrt{c}+2c}{x^3}\right) - 24\sqrt{dx^3+c}\,c}{576\,c^3x^3}, -\frac{9\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right) + \sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3c}\right) + 12\sqrt{dx^3+c}\,c}{288\,c^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^
3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(9*sqrt(-c)*d*x^3*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) + sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

**giac [A]** time = 0.16, size = 73, normalized size = 0.90

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{32\sqrt{-c}\,c^2}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}\,c^2}-\frac{\sqrt{dx^3+c}}{24\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/32*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/288*d*arctan(1/
3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/24*sqrt(d*x^3 + c)/(c^2*x^3)
```

**maple [C]** time = 0.19, size = 477, normalized size = 5.89

$$-\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96c^{\frac{5}{2}}}+\frac{\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}}{8c}-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-c\,d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-c\,d^2\right)^{\frac{2}{3}}-\left(-c\,d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-c\,d^2\right)^{\frac{2}{3}}\sqrt{3}\;\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-c\,d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{1728c^3d\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2))-1/1728*I/d/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d-8*c))-1/96*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)
```

**mupad [B]** time = 3.42, size = 73, normalized size = 0.90

$$\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{32\,\sqrt{c^5}}+\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{288\,\sqrt{c^5}}-\frac{\sqrt{d\,x^3+c}}{24\,c^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2)))/(32*(c^5)^(1/2)) + (d*atanh(
(c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(288*(c^5)^(1/2)) - (c + d*x^3)^(
1/2)/(24*c^2*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^4\sqrt{c+dx^3}+dx^7\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)
```

## 3.236 $\int \frac{1}{x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=107

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

**Rubi [A]** time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c^2*x^6) + (5*d*Sqrt[c + d*x^3])/(192*c^3*x^3) + (d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(256*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} - \frac{\text{Subst}\left(\int \frac{10cd-\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c^2}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{\text{Subst}\left(\int \frac{42c^2d^2-5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^4}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^3} + \frac{d^3\,\text{Subst}\left(\int \frac{}{(8c}\right.}{15}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^3} + \frac{d^2\,\text{Subst}\left(\right.}{}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 95, normalized size = 0.89

$$
\frac{d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 63d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) + 12\sqrt{c}\sqrt{c+dx^3}\left(5dx^3-4c\right)}{2304c^{7/2}x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (12*Sqrt[c]*Sqrt[c + d*x^3]*(-4*c + 5*d*x^3) + d^2*x^6*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])] - 63*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2304*c^(7/
2)*x^6)
```

**IntegrateAlgebraic [A]** time = 0.14, size = 95, normalized size = 0.89

$$
\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}} + \frac{\sqrt{c+dx^3}\left(5dx^3-4c\right)}{192c^3x^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(192*c^3*x^6) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/(256*c^(7/2))
```

**fricas [A]** time = 0.81, size = 217, normalized size = 2.03

$$\left[\frac{\sqrt{c}\, d^2 x^6 \log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 63\,\sqrt{c}\, d^2 x^6 \log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right) + 24\left(5\,cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{4608\,c^4 x^6},\ \frac{63\,\sqrt{-c}\, d^2 x^6 \arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right) - \sqrt{-c}\, d^2 x^6 \arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right) + 12\left(5\,cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{2304\,c^4 x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), 1/2304*(63*sq
rt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*d^2*x^6*arctan
(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/
(c^4*x^6)]
```

**giac [A]** time = 0.18, size = 101, normalized size = 0.94

$$\frac{7\,d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\,\sqrt{-c}\,c^3} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2304\,\sqrt{-c}\,c^3} + \frac{5\left(dx^3+c\right)^{\frac{3}{2}} d^2 - 9\,\sqrt{dx^3+c}\,cd^2}{192\,c^3 d^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 7/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2304*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/192*(5*(d*x^3 + c)^(3/2
)*d^2 - 9*sqrt(d*x^3 + c)*c*d^2)/(c^3*d^2*x^6)
```

**maple [C]** time = 0.21, size = 540, normalized size = 5.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2
))/c^(3/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*d*(d*x^3+c)^(1/2)/c^2/x^3
-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/13824*I/c^4*2^(1/2)*su
m((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/76
8*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)
```

**mupad** [B] time = 3.50, size = 94, normalized size = 0.88

$$\frac{d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2304\,\sqrt{c^7}}-\frac{7\,d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{256\,\sqrt{c^7}}-\frac{3\,\sqrt{d\,x^3+c}}{64\,c^2\,x^6}+\frac{5\,\left(d\,x^3+c\right)^{3/2}}{192\,c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2))))/(2304*(c^7)^(1/2)) - (
7*d^2*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(256*(c^7)^(1/2)) - (3*(c
 + d*x^3)^(1/2))/(64*c^2*x^6) + (5*(c + d*x^3)^(3/2))/(192*c^3*x^6)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^7\sqrt{c+dx^3}+dx^{10}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**7*sqrt(c + d*x**3) + d*x**10*sqrt(c + d*x**3)), x)
```

## 3.237 $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

**Rubi [A]** time = 0.41, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(6*Sqrt[3]
*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*c^(5/
6)*d^(2/3))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
```

```
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{12cd}+\frac{\int \frac{1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{c}}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{6\sqrt[3]{c}\,d^{2/3}}-\frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{12\sqrt[3]{c}}+\frac{d^{4/3}\,\text{Subst}}{}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{6\sqrt[3]{c}\,d^{2/3}}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

**Mathematica [C]** time = 0.06, size = 67, normalized size = 0.48

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(16*c*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [F]** time = 15.45, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/((8*c - d*x^3)*Sqrt[c + d*x^3]), x]
```

**fricas [B]** time = 2.47, size = 2459, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*
d^4))^(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d
^4))^(5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqr
t(d*x^3 + c) + (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3)
+ 12*sqrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*
sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5
*x^6 + 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*
d^3*x^4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x
^2)*(1/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 -
1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(
2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6
) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^
3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^
8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 1/54*
sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*d^4))^
(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^(
5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqrt(d*x^
3 + c) - (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3) + 12*s
qrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*sqrt(3
)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5*x^6 +
 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*d^3*x^
4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x^2)*(1
/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c
^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) -
 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*
(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7
- 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20
*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 1/108*(1/(c
^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18
*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) + 6*sq
rt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*
c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7
+ 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38
*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 1/108*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4
+ 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 +
 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 6
4*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(
1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1
/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/
216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 108
8*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3
) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) -
 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x
^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 +
```

```
20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + 1/216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 2
76*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4
 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 +
 c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^
5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c
^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5
*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

**maple** [C] time = 0.16, size = 416, normalized size = 2.95

$$-\frac{i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)^{2}d^{2}+i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)d-\left(-c\,d^{2}\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)d-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{2}{3}}-\left(-c\,d^{2}\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)^{2}d+i\sqrt{3}\,cd-3cd-i\left(-c\,d^{2}\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)-3\left(-c\,d^{2}\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{27c\,d^{3}\sqrt{d\,x^{3}+c}\;\mathrm{RootOf}\left(d\,\_Z^{3}-8c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

**mupad** [B] time = 40.22, size = 272, normalized size = 1.93

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(\sqrt{d\,x^3+c}-\sqrt{c}+2\,c^{1/6}\,d^{1/3}\,x\right)^3}{x^3\,\left(d^{1/3}\,x-2\,c^{1/3}\right)^3}\right)}{54\,c^{5/6}\,d^{2/3}}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(-\sqrt{3}\,c^{1/6}\,d^{1/3}\,x+\sqrt{d\,x^3+c}\,1{}\mathrm{i}+\sqrt{c}\,1{}\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1{}\mathrm{i}\right)^3}{x^3\,\left(d^{1/3}\,x+c^{1/3}-\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)^3}\right)\,\sqrt{-1+\sqrt{3}\,1{}\mathrm{i}}}{108\,c^{5/6}\,d^{2/3}}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(\sqrt{3}\,c^{1/6}\,d^{1/3}\,x-\sqrt{d\,x^3+c}\,1{}\mathrm{i}+\sqrt{c}\,1{}\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1{}\mathrm{i}\right)^3}{x^3\,\left(d^{1/3}\,x+c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)^3}\right)\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}\,1{}\mathrm{i}}{108\,c^{5/6}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2) - c^(1/2) + 2*c^(1/6)
*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 2*c^(1/3))^3))/(54*c^(5/6)*d^(2/3)) + (2^(
```

```
1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2)*1i + c^(1/2)*1i
+ c^(1/6)*d^(1/3)*x*1i - 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 3^
(1/2)*c^(1/3)*1i + c^(1/3))^3))*(3^(1/2)*1i - 1)^(1/2))/(108*c^(5/6)*d^(2/3
)) + (2^(1/2)*log((((c + d*x^3)^(1/2) + c^(1/2))*(c^(1/2)*1i - (c + d*x^3)^
(1/2)*1i + c^(1/6)*d^(1/3)*x*1i + 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(3^(1/
2)*c^(1/3)*1i + d^(1/3)*x + c^(1/3))^3))*(3^(1/2)*1i + 1)^(1/2)*1i)/(108*c^
(5/6)*d^(2/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

## 3.238 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=90

$$\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{2c^2}{27d^4\sqrt{c+dx^3}}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^4}$$

**Rubi [A]** time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 87, 43, 63, 206}

$$\frac{2c^2}{27d^4\sqrt{c+dx^3}}+\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(27*d^4*Sqrt[c + d*x^3]) - (4*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*x^
3)^(3/2))/(9*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81
*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^3}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\mathrm{Subst}\left(\int\left(-\frac{c^2}{9d^3(c+dx)^{3/2}} - \frac{7c}{d^3\sqrt{c+dx}} - \frac{x}{d^2\sqrt{c+dx}} + \frac{512c^2}{9d^3(8c-dx)\sqrt{c+}}\right.\right.$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{\left(512c^2\right)\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^3} - \text{Su}$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{\left(1024c^2\right)\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^4} -$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2\left(c+dx^3\right)^{3/2}}{9d^4} + \frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

**Mathematica [C]** time = 0.06, size = 66, normalized size = 0.73

$$-\frac{2\left(512c^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right) - 456c^2 + 60cdx^3 + 3d^2x^6\right)}{27d^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(-456*c^2 + 60*c*d*x^3 + 3*d^2*x^6 + 512*c^2*Hypergeometric2F1[-1/2, 1,
 1/2, (c + d*x^3)/(9*c)]))/(27*d^4*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 73, normalized size = 0.81

$$\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} - \frac{2\left(56c^2 + 60cdx^3 + 3d^2x^6\right)}{27d^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(56*c^2 + 60*c*d*x^3 + 3*d^2*x^6))/(27*d^4*Sqrt[c + d*x^3]) + (1024*c^(
3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4)
```

**fricas [A]** time = 0.73, size = 189, normalized size = 2.10

$$\left[\frac{2\left(256\left(cdx^3+c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - 3\left(3d^2x^6+60cdx^3+56c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3+cd^4\right)}, -\frac{2\left(512\left(cdx^3+c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3\left(3d^2x^6+60cdx^3+56c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3+cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(256*(c*d*x^3 + c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c)
)/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c))
/(d^5*x^3 + c*d^4)]
```

**giac [A]** time = 0.20, size = 82, normalized size = 0.91

$$-\frac{1024\, c^{2} \arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}\, d^{4}}+\frac{2\, c^{2}}{27\,\sqrt{dx^{3}+c}\, d^{4}}-\frac{2\,\left(\left(dx^{3}+c\right)^{\frac{3}{2}} d^{8}+18\,\sqrt{dx^{3}+c}\, c d^{8}\right)}{9\, d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1024/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/27*c^2
/(sqrt(d*x^3 + c)*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*sqrt(d*x^3 + c)*c*
d^8)/d^12
```

**maple [C]** time = 0.30, size = 560, normalized size = 6.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d*(-2/3/d^3*c^2/((x^3+c/d)*d)^(1/2)+2/9/d^2*x^3*(d*x^3+c)^(1/2)-10/9*c*(
d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/3/d^2*c/((x^3+c/d)*d)^(1/2)+2/3*(d*x^3+c)^(1
/2)/d^2)+128/3*c^2/d^4/(d*x^3+c)^(1/2)-512*c^3/d^3*(2/27/d/c/((x^3+c/d)*d)^
(1/2)+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))
```

**maxima [A]** time = 1.25, size = 82, normalized size = 0.91

$$-\frac{2\,\left(256\, c^{\frac{3}{2}} \log\left(\frac{\sqrt{dx^{3}+c}-3\,\sqrt{c}}{\sqrt{dx^{3}+c}+3\,\sqrt{c}}\right)+9\,\left(dx^{3}+c\right)^{\frac{3}{2}}+162\,\sqrt{dx^{3}+c}\, c-\frac{3\, c^{2}}{\sqrt{dx^{3}+c}}\right)}{81\, d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(256*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 9*(d*x^3 + c)^(3/2) + 162*sqrt(d*x^3 + c)*c - 3*c^2/sqrt(d*x^3 +
 c))/d^4
```

**mupad [B]** time = 3.78, size = 95, normalized size = 1.06

$$\frac{512\, c^{3/2}\, \ln\left(\frac{10\, c+d\, x^{3}+6\,\sqrt{c}\,\sqrt{d\, x^{3}+c}}{8\, c-d\, x^{3}}\right)}{81\, d^{4}}-\frac{38\, c\,\sqrt{d\, x^{3}+c}}{9\, d^{4}}+\frac{2\, c^{2}}{27\, d^{4}\,\sqrt{d\, x^{3}+c}}-\frac{2\, x^{3}\,\sqrt{d\, x^{3}+c}}{9\, d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(81*d^4) - (38*c*(c + d*x^3)^(1/2))/(9*d^4) + (2*c^2)/(27*d^4*(c + d*x^3
)^(1/2)) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.239 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=71

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

**Rubi [A]** time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {446, 87, 63, 206}

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(27*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(3*d^3) + (128*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{c}{9d^2(c+dx)^{3/2}} - \frac{1}{d^2\sqrt{c+dx}} + \frac{64c}{9d^2(8c-dx)\sqrt{c+dx}}\right)dx, x, x^3\right)$$
$$= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(64c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^2}$$
$$= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(128c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^3}$$
$$= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

**Mathematica** [C] time = 0.05, size = 53, normalized size = 0.75

$$-\frac{2\left(64c\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-54c+9dx^3\right)}{27d^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(-54*c + 9*d*x^3 + 64*c*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*
c)]))/(27*d^3*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic** [A] time = 0.07, size = 62, normalized size = 0.87

$$\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3} - \frac{2\left(10c+9dx^3\right)}{27d^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(10*c + 9*d*x^3))/(27*d^3*Sqrt[c + d*x^3]) + (128*Sqrt[c]*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

**fricas** [A] time = 0.62, size = 161, normalized size = 2.27

$$\left[\frac{2\left(32\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\left(9dx^3+10c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3+cd^3\right)}, -\frac{2\left(64\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\left(9dx^3+10c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3+cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(32*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3), -
2/81*(64*(d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(9
*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3)]
```

**giac [A]** time = 0.17, size = 58, normalized size = 0.82

$$-\frac{128\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,d^3}-\frac{2\sqrt{dx^3+c}}{3\,d^3}-\frac{2\,c}{27\sqrt{dx^3+c}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -128/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/3*sqrt(d*
x^3 + c)/d^3 - 2/27*c/(sqrt(d*x^3 + c)*d^3)
```

**maple [C]** time = 0.17, size = 501, normalized size = 7.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d^2*(d*(2/3/((x^3+c/d)*d)^(1/2)*c/d^2+2/3*(d*x^3+c)^(1/2)/d^2)-16/3*c/d/
(d*x^3+c)^(1/2))-64*c^2/d^2*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2
^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^
(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8
*c)))
```

**maxima [A]** time = 1.12, size = 68, normalized size = 0.96

$$-\frac{2\left(32\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}+\frac{3\,c}{\sqrt{dx^3+c}}\right)}{81\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(32*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 27*sqrt(d*x^3 + c) + 3*c/sqrt(d*x^3 + c))/d^3
```

**mupad [B]** time = 3.71, size = 75, normalized size = 1.06

$$\frac{64\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3}-\frac{2\,c}{27\,d^3\sqrt{d\,x^3+c}}-\frac{2\sqrt{d\,x^3+c}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(81*d^3) - (2*c)/(27*d^3*(c + d*x^3)^(1/2)) - (2*(c + d*x^3)^(1/2))/(3*d^
3)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.240 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}} dx$

**Optimal**. Leaf size=52

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

**Rubi** [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 78, 63, 206}

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
Sqrt[c]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)$$
$$= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{8\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27d}$$
$$= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^2}$$
$$= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

**Mathematica** [A] time = 0.03, size = 49, normalized size = 0.94

$$\frac{2\left(\frac{3}{\sqrt{c+dx^3}}+\frac{8\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{81d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/
(81*d^2)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 52, normalized size = 1.00

$$\frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
Sqrt[c]*d^2)
```

**fricas** [A] time = 0.51, size = 149, normalized size = 2.87

$$\left[\frac{2\left(4\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\sqrt{dx^3+c}\,c\right)}{81\left(cd^3x^3+c^2d^2\right)}, -\frac{2\left(8\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-3\sqrt{dx^3+c}\,c\right)}{81\left(cd^3x^3+c^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(4*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 3*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d*x
^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)
*c)/(c*d^3*x^3 + c^2*d^2)]
```

**giac [A]** time = 0.17, size = 47, normalized size = 0.90

$$-\frac{2\left(\frac{8\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{3}{\sqrt{dx^3+c}\,d}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*(8*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3/(sqrt(d*x^3
+ c)*d))/d
```

**maple [C]** time = 0.18, size = 456, normalized size = 8.77

$$-\frac{8\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd}+\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{243c^2d^3\sqrt{dx^3+c}}\right)c}{d}+\frac{2}{3\sqrt{dx^3+c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3/d^2/(d*x^3+c)^(1/2)-8*c/d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3
*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))
```

**maxima [A]** time = 1.21, size = 56, normalized size = 1.08

$$-\frac{2\left(\frac{4\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{3}{\sqrt{dx^3+c}}\right)}{81\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(4*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3/sqrt(d*x^3 + c))/d^2
```

**mupad [B]** time = 3.68, size = 60, normalized size = 1.15

$$\frac{2}{27\,d^2\,\sqrt{d\,x^3+c}}+\frac{8\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*d^2*(c + d*x^3)^(1/2)) + (8*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)
^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2)
```

**sympy [A]** time = 27.57, size = 58, normalized size = 1.12

$$\begin{cases} \frac{2\left(\frac{1}{27d\sqrt{c+dx^3}} - \frac{8\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81d\sqrt{-c}}\right)}{d} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Piecewise((2*(1/(27*d*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-
c)))/(81*d*sqrt(-c)))/d, Ne(d, 0)), (x**6/(48*c**(5/2)), True))
```

## 3.241 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=55

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

**Rubi [A]** time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
c^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27c}$$
$$= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27cd}$$
$$= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

**Mathematica [C]** time = 0.01, size = 43, normalized size = 0.78

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(27*c*d*Sqrt[c + d*
x^3])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 55, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d} - \frac{2}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
c^(3/2)*d)
```

**fricas [A]** time = 0.72, size = 147, normalized size = 2.67

$$\left[\frac{(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) - 6\sqrt{dx^3+c}\,c}{81(c^2d^2x^3+c^3d)}, -\frac{2\left((dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right) + 3\sqrt{dx^3+c}\,c\right)}{81(c^2d^2x^3+c^3d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/81*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(
d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3 +
 c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*c)/
(c^2*d^2*x^3 + c^3*d)]
```

**giac [A]** time = 0.18, size = 48, normalized size = 0.87

$$-\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,cd} - \frac{2}{27\sqrt{dx^3+c}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 2/27/(sqrt(d*x^
3 + c)*c*d)
```

**maple [C]** time = 0.21, size = 435, normalized size = 7.91

$$-\frac{2}{27\sqrt{\left(x^{3}+\frac{c}{d}\right)d}\,cd}-\frac{i\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)^{2}d^{2}+i\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)d-\left(-cd^{2}\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^{3}-8c\right)d-i\sqrt{3}\left(-cd^{2}\right)^{\frac{2}{3}}-\left(-cd^{2}\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^{2}\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)^{2}d+i\sqrt{3}\,cd-3cd-i\left(-cd^{2}\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^{3}-8c\right)-3\left(-cd^{2}\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{243c^{2}d^{3}\sqrt{d\,x^{3}+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/27/((x^3+c/d)*d)^(1/2)/c/d-1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1
/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^
(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_al
pha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.23, size = 58, normalized size = 1.05

$$-\frac{\frac{\log\left(\frac{\sqrt{dx^{3}+c}-3\sqrt{c}}{\sqrt{dx^{3}+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{6}{\sqrt{dx^{3}+c}\,c}}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/81*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(
3/2) + 6/(sqrt(d*x^3 + c)*c))/d
```

**mupad [B]** time = 3.63, size = 63, normalized size = 1.15

$$\frac{\ln\left(\frac{10\,c+d\,x^{3}+6\,\sqrt{c}\,\sqrt{d\,x^{3}+c}}{8\,c-d\,x^{3}}\right)}{81\,c^{3/2}\,d}-\frac{2}{27\,c\,d\,\sqrt{d\,x^{3}+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(81*c^(3/2)
*d) - 2/(27*c*d*(c + d*x^3)^(1/2))
```

**sympy [A]** time = 25.53, size = 51, normalized size = 0.93

$$-\frac{2}{27cd\sqrt{c+dx^{3}}}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^{3}}}{3\sqrt{-c}}\right)}{81cd\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(27*c*d*sqrt(c + d*x**3)) - 2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(81*c*
d*sqrt(-c))
```

## 3.242 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}+\frac{2}{27c^2\sqrt{c+dx^3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 85, 156, 63, 208, 206}

$$\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5
/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd+d^2x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27c^2d}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{24c^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,\right.}{216c^2}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{108c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{\right.}}{12c^2d}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

**Mathematica [C]** time = 0.03, size = 63, normalized size = 0.83

$$\frac{9\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)-{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{108c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] + 9*Hypergeometric2F1[
-1/2, 1, 1/2, 1 + (d*x^3)/c])/(108*c^2*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.06, size = 76, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}} + \frac{2}{27c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5
/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))
```

**fricas [A]** time = 0.66, size = 213, normalized size = 2.80

$$\left[\frac{\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+27\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+48\sqrt{dx^3+c}\,c}{648\left(c^3dx^3+c^4\right)}, \frac{27\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+24\sqrt{dx^3+c}\,c}{324\left(c^3dx^3+c^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), 1/324*(27*(d*x^3 + c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4)]

**giac [A]** time = 0.16, size = 68, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}\,c^2} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}\,c^2} + \frac{2}{27\sqrt{dx^3+c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)

**maple [C]** time = 0.19, size = 485, normalized size = 6.38

$$-\frac{\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd} + \frac{i\sqrt{2}\sum_{\underline{\alpha}=\mathrm{RootOf}(\_Z^3 d-8c)}\left((-cd^2)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-cd^2)^{\frac{1}{3}}}{d}}{-3(-cd^2)^{\frac{1}{3}}+i\sqrt{3}(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{-i\left(2x+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)}{2(-cd^2)^{\frac{1}{3}}}}\left(2\underline{\alpha}^2d^2+i(-cd^2)^{\frac{1}{3}}\sqrt{3}\,\underline{\alpha}d-(-cd^2)^{\frac{1}{3}}\underline{\alpha}d-i\sqrt{3}(-cd^2)^{\frac{2}{3}}-(-cd^2)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}}{3}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}, -\frac{2i(-cd^2)^{\frac{1}{3}}\sqrt{3}\,\underline{\alpha}^2d+i\sqrt{3}\,cd-3cd-i(-cd^2)^{\frac{2}{3}}\sqrt{3}\,\underline{\alpha}-3(-cd^2)^{\frac{2}{3}}\underline{\alpha}}{18cd}, \sqrt{\frac{\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{d}}{-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{2d}}}\right)\right)}{243c^2d^3\sqrt{dx^3+c}}\right)d}{8c} + \frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}} + \frac{2}{3\sqrt{\left(x^3+\frac{c}{d}\right)d}\,c}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)

[Out] -1/8/c*d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(2/3/c/((x^3+c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)

**mupad [B]** time = 3.66, size = 68, normalized size = 0.89

$$\frac{2}{27\,c^2\sqrt{d\,x^3+c}} - \frac{\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{12\sqrt{c^5}} + \frac{\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{3\sqrt{c^5}}\right)}{324\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*c^2*(c + d*x^3)^(1/2)) - atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(
12*(c^5)^(1/2)) + atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))/(324*(c^5)
^(1/2))
```

**sympy [A]** time = 17.02, size = 78, normalized size = 1.03

$$\frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{324c^2\sqrt{-c}} + \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2/(27*c**2*sqrt(c + d*x**3)) - atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(324*c**
2*sqrt(-c)) + atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c**2*sqrt(-c))
```

**3.243** $$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$$

**Optimal**. Leaf size=100

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}$$

**Rubi** [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-25*d)/(216*c^3*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*Sqrt[c + d*x^3]) + (d*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(96*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{11cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2}\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{99c^2d^2}{2}-\frac{25}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{108c^4d}\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{(11d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^3} +\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{11\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c^3}\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\tanh^{-1}\left(\cdots\right)}{96c^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.04, size = 77, normalized size = 0.77

$$
\frac{-dx^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-99dx^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)-36c}{864c^3x^3\sqrt{c+dx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-36*c - d*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] - 99*d*x^
3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(864*c^3*x^3*Sqrt[c + d*x
^3])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 91, normalized size = 0.91

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}+\frac{-9c-25dx^3}{216c^3x^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-9*c - 25*d*x^3)/(216*c^3*x^3*Sqrt[c + d*x^3]) + (d*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(9
6*c^(7/2))
```

**fricas [A]** time = 0.66, size = 272, normalized size = 2.72

$$\left[\frac{\left(d^2x^6+cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+297\left(d^2x^6+cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-24\left(25cdx^3+9c^2\right)\sqrt{dx^3+c}}{5184\left(c^4dx^6+c^5x^3\right)},\ -\frac{297\left(d^2x^6+cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\left(d^2x^6+cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+12\left(25cdx^3+9c^2\right)\sqrt{dx^3+c}}{2592\left(c^4dx^6+c^5x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5184*((d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 2*sq
rt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c)
)/(c^4*d*x^6 + c^5*x^3), -1/2592*(297*(d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d
*x^3 + c)*sqrt(-c)/c) + 12*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6
 + c^5*x^3)]
```

**giac [A]** time = 0.18, size = 100, normalized size = 1.00

$$-\frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c^3}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}\,c^3}-\frac{25\left(dx^3+c\right)d-16\,cd}{216\left(\left(dx^3+c\right)^{\frac{3}{2}}-\sqrt{dx^3+c}\,c\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -11/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2592*d*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/216*(25*(d*x^3 + c)*d - 16
*c*d)/(((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)*c^3)
```

**maple [C]** time = 0.19, size = 549, normalized size = 5.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3*d/c^2/((x^3+c/d)*d)^(1/2)+d*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/64/c^2*d^2*(2/27/((x^3+c/d)*d)^(1/2)/c
/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
```

```
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))+1/64/c^2*d*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)
```

**mupad [B]** time = 3.80, size = 88, normalized size = 0.88

$$\frac{11\,d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{96\,\sqrt{c^7}}-\frac{25\,d}{216\,c^3\,\sqrt{d\,x^3+c}}+\frac{d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2592\,\sqrt{c^7}}-\frac{1}{24\,c^2\,x^3\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (11*d*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(96*(c^7)^(1/2)) - (25*d)
/(216*c^3*(c + d*x^3)^(1/2)) + (d*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1
/2))))/(2592*(c^7)^(1/2)) - 1/(24*c^2*x^3*(c + d*x^3)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^4\sqrt{c+dx^3}-7cdx^7\sqrt{c+dx^3}+d^2x^{10}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**4*sqrt(c + d*x**3) - 7*c*d*x**7*sqrt(c + d*x**3) +
d**2*x**10*sqrt(c + d*x**3)), x)
```

## 3.244 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=128

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{245d^2}{1728c^4\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{245d^2}{1728c^4\sqrt{c+dx^3}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (245*d^2)/(1728*c^4*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*Sqrt[c + d*x^3]) + (3*
d)/(64*c^3*x^3*Sqrt[c + d*x^3]) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2)
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^3(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{18cd-\frac{5d^2x}{2}}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{218c^2d^2-27cd^3x}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{384c^4}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{981c^3d^3 \ldots}{x(8c-d \ldots}\right.}{172\ldots}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\left(109d^2\right)\operatorname{Subst}\left( \ldots\right.}{15\ldots}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{(109d)\operatorname{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+d\ldots}}{3\sqrt{c\ldots}}\right.}{20736c^{9/2}}$$

**Mathematica [C]** time = 0.08, size = 91, normalized size = 0.71

$$\frac{-d^2x^6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+981d^2x^6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+36c\left(9dx^3-4c\right)}{6912c^4x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (36*c*(-4*c + 9*d*x^3) - d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3
)/(9*c)] + 981*d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(691
2*c^4*x^6*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.14, size = 106, normalized size = 0.83

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{-36c^2+81cdx^3+245d^2x^6}{1728c^4x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-36*c^2 + 81*c*d*x^3 + 245*d^2*x^6)/(1728*c^4*x^6*Sqrt[c + d*x^3]) + (d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2))
```

**fricas [A]** time = 0.79, size = 303, normalized size = 2.37

$$\left[\frac{\left(d^3x^9+cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2943\left(d^3x^9+cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+24\left(245cd^2x^6+81c^2dx^3-36c^3\right)\sqrt{dx^3+c}}{41472\left(c^5dx^9+c^6x^6\right)},\ \frac{2943\left(d^3x^9+cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\left(d^3x^9+cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+12\left(245cd^2x^6+81c^2dx^3-36c^3\right)\sqrt{dx^3+c}}{20736\left(c^5dx^9+c^6x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/41472*((d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3
- 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x^3
- 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), 1/20736*(2943*(d^3*x^9 +
c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d^3*x^9 + c*d^2*x
^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(245*c*d^2*x^6 + 8
1*c^2*d*x^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]
```

**giac [A]** time = 0.16, size = 118, normalized size = 0.92

$$\frac{109\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}\,c^4}-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{20736\,\sqrt{-c}\,c^4}+\frac{2\,d^2}{27\,\sqrt{dx^3+c}\,c^4}+\frac{13\left(dx^3+c\right)^{\frac{3}{2}}d^2-17\,\sqrt{dx^3+c}\,cd^2}{192\,c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 109/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/20736*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 2/27*d^2/(sqrt(d*x^3 +
 c)*c^4) + 1/192*(13*(d*x^3 + c)^(3/2)*d^2 - 17*sqrt(d*x^3 + c)*c*d^2)/(c^4
*d^2*x^6)
```

**maple [C]** time = 0.22, size = 636, normalized size = 4.97

$$-\frac{\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd}+\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{\frac{i}{2}\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{\frac{i}{2}\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{243c^2d^3\sqrt{dx^3+c}}\right)d^3}{512c^3}+\frac{-\frac{5d^2\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{4c^{\frac{7}{2}}}+\frac{2d^2}{3\sqrt{\left(x^3+\frac{c}{d}\right)d}\,c^3}+\frac{7\sqrt{dx^3+c}\,d}{12c^3x^3}-\frac{\sqrt{dx^3+c}}{6c^2x^6}}{8c}+\frac{\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{2d}{3\sqrt{\left(x^3+\frac{c}{d}\right)d}\,c^2}-\frac{\sqrt{dx^3+c}}{3c^2x^3}\right)d}{64c^2}+\frac{\left(-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}+\frac{2}{3\sqrt{\left(x^3+\frac{c}{d}\right)d}\,c}\right)d^2}{512c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*ar
ctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/c^2/x^6
+7/12*d*(d*x^3+c)^(1/2)/c^3/x^3+2/3*d^2/c^3/((x^3+c/d)*d)^(1/2)-5/4*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2))-1/512/c^3*d^3*(2/27/((x^3+c/d)*d)^(1
/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d-8*c))+1/512/c^3*d^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3
*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)
```

**mupad [B]** time = 4.03, size = 112, normalized size = 0.88

$$\frac{245\,d^2}{1728\,c^4\,\sqrt{d\,x^3+c}}-\frac{109\,d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)}{768\,\sqrt{c^9}}+\frac{d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)}{20736\,\sqrt{c^9}}-\frac{1}{48\,c^2\,x^6\,\sqrt{d\,x^3+c}}+\frac{3\,d}{64\,c^3\,x^3\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (245*d^2)/(1728*c^4*(c + d*x^3)^(1/2)) - (109*d^2*atanh((c^4*(c + d*x^3)^(1
/2))/(c^9)^(1/2)))/(768*(c^9)^(1/2)) + (d^2*atanh((c^4*(c + d*x^3)^(1/2))/(
3*(c^9)^(1/2))))/(20736*(c^9)^(1/2)) - 1/(48*c^2*x^6*(c + d*x^3)^(1/2)) + (
3*d)/(64*c^3*x^3*(c + d*x^3)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.245 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

**Optimal**. Leaf size=318

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\right.}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/
6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[
3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+}}{\sqrt{2}\,3^{3/4}\sqrt{}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

**Mathematica [C]** time = 0.10, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [F]** time = 31.27, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3))
, x]
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.39, size = 538, normalized size = 1.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/
3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/
b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*
b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(
1/2)*(4*3^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2-3*I*(-a*b^2)^(1/3)*3^(1/2)*_alp
ha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^
2)^(1/3)*_alpha*b+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(
-a*b^2)^(2/3)+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(
1/2)*a*b+2*3^(1/2)*a*b-2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha+2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a+10*a))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3 + a}\,\left(b\,x^3 + 2\,a\,\left(3\,\sqrt{3} + 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^3}\,\left(10a + 6\sqrt{3}\,a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5+3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)
```

## 3.246 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

**Optimal**. Leaf size=324

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{\ldots}\right.}{\sqrt{2}\sqrt{a-bx^3}}\right.}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 -
Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt
[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1
+ Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\ldots}{\sqrt{2}\,3^{3/4}\ldots}\right.}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

**Mathematica [C]** time = 0.08, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

**IntegrateAlgebraic [F]** time = 31.26, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3))
, x]
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.41, size = 509, normalized size = 1.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a*
b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-
3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*(
a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(1/2)*(4*3
^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2+3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-6*I*(
a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(1/3)*_alp
ha*b-3*I*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)+3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
2)^(1/3)*_alpha^2*b+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*I*a*b-2*3^
(1/2)*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha+2*3
^(1/2)*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b-6*3^(1/2)*a-10*a))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3 - 2a\left(3\sqrt{3} + 5\right)\right)\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\sqrt{a - b\,x^3}\,\left(b\,x^3 - 2\,a\,\left(3\,\sqrt{3} + 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] -int(x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6\sqrt{3}\,a\sqrt{a - bx^3} - 10a\sqrt{a - bx^3} + bx^3\sqrt{a - bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5+3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

## 3.247 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

**Optimal**. Leaf size=328

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\cdots}{\sqrt{2}\cdots}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

**Rubi** [A] time = 0.07, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left((1\cdots\right.}{\sqrt{2}\cdots}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}\cdots}$$

**Mathematica** [C] time = 0.12, size = 85, normalized size = 0.26

$$-\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))
```

**IntegrateAlgebraic [F]** time = 31.29, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3
)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.41, size = 510, normalized size = 1.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(b*x^3-a)^(1/2)*(4*3
^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2+3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-6*I*(
a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(1/3)*_alp
ha*b-3*I*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)+3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
2)^(1/3)*_alpha^2*b+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*I*a*b-2*3^
(1/2)*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha+2*3
^(1/2)*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b-6*3^(1/2)*a-10*a))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3 - a}\,\left(b\,x^3 - 2\,a\,\left(3\,\sqrt{3} + 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a + bx^3}\,\left(-6\sqrt{3}\,a - 10a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3-2*a*(5+3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)
```

## 3.248 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

**Optimal**. Leaf size=330

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\ldots\right.}{\ldots}\right.}{2\sqrt{2}\,3^{3/4}a^{5/6}\ldots}$$

**Rubi [A]** time = 0.07, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x
))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1\ldots\right.}{\ldots}\right.}{6\sqrt{2}\sqrt[4]{3}a\ldots}$$

**Mathematica [C]** time = 0.11, size = 87, normalized size = 0.26

$$-\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))
```

**IntegrateAlgebraic [F]** time = 31.17, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3
)), x]
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.36, size = 541, normalized size = 1.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(-b*x^3-a)^(
1/2)*(4*3^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2-3*I*(-a*b^2)^(1/3)*3^(1/2)*_alp
ha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^
2)^(1/3)*_alpha*b+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(
-a*b^2)^(2/3)+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(
1/2)*a*b+2*3^(1/2)*a*b-2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha+2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a+10*a))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}\,+5\right)\right)\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}\,+5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{10a\sqrt{-a-bx^3}\,+6\sqrt{3}\,a\sqrt{-a-bx^3}\,+bx^3\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3-2*a*(5+3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x**
3*sqrt(-a - b*x**3)), x)
```

## 3.249 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

**Optimal**. Leaf size=310

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right.}{\sqrt{2}}\right.}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*
x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])]])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sq
rt[a + b*x^3])]])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right.}{\sqrt{2}}\right.}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2}}$$

**Mathematica [C]** time = 0.15, size = 83, normalized size = 0.27

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [F]** time = 31.16, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3))
, x]
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.42, size = 538, normalized size = 1.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1
/2)*(4*3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2+3*I*(-a*b^2)^(1/3)*3^(1/2)*_alph
a*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2
)^(1/3)*_alpha*b-3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-
a*b^2)^(2/3)-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1
/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6*
(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b+4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(1
/2)*a*b-2*3^(1/2)*a*b+2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-2*3^(1/
2)*(-a*b^2)^(2/3)*_alpha-2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha)
/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a+10*a))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} - 5\right)\right)\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3 + a}\,\left(b\,x^3 - 2\,a\,\left(3\,\sqrt{3} - 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^3}\,\left(-6\sqrt{3}\,a + 10a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

## 3.250 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

**Optimal**. Leaf size=316

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\sqrt[4]{3}\left(1\cdots\right.\right.}{2\sqrt{2}\,3^{3/4}a^{5\cdots}}$$

**Rubi** [A] time = 0.05, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sq
rt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx=-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\sqrt[4]{3}\sqrt[6]{a}\left(\cdots\right.\right.}{3\sqrt{2}\sqrt[4]{3}a^{5/\cdots}}$$

**Mathematica** [C] time = 0.10, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

**IntegrateAlgebraic [F]** time = 31.17, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3))
, x]
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.37, size = 509, normalized size = 1.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(1/2)*(4*
3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2-3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^
(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(1/3)*_al
pha*b+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3
*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b
^2)^(1/3)*_alpha^2*b-4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*a
*b+2*I*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_al
pha+2*I*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2
)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alp
ha=RootOf(_Z^3*b+6*3^(1/2)*a-10*a))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3 + 2\,a\left(3\,\sqrt{3} - 5\right)\right)\sqrt{-bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{a - b\,x^3}\,\left(b\,x^3 + 2\,a\,\left(3\,\sqrt{3} - 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{a - bx^3} + 6\sqrt{3}\,a\sqrt{a - bx^3} + bx^3\sqrt{a - bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

## 3.251 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

**Optimal**. Leaf size=320

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.}{\sqrt{2}\sqrt{bx^3-a}}\right.}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

**Rubi** [A] time = 0.05, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {488}

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 +
 Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a}}{\sqrt{2}\,3^{3/4}\sqrt{}}\right.}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

**Mathematica** [C] time = 0.11, size = 84, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3])
```

**IntegrateAlgebraic [F]** time = 31.23, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]
), x]
```

**fricas [B]** time = 22.76, size = 5060, normalized size = 15.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(1/3*(3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3
 - sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b
^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sq
rt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(
-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 -
7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4
))^(1/3) - sqrt(3)*(b*x^4 - a*x) + 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(2
65*a^4*b^4*x^3 + 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x -
71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sq
rt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt(
(b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(
1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 211
2*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4
*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(
1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2
 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*
x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*
a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6
)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^
5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5
*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*
a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1
944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x -
3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(135
1*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2
*x^6 + 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 - a*x)) - 1/6*sqrt(3)*(1/1944)^(1/6
)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(1/3*(3*sqrt(b*x^3 - a)*(1
08*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^
3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41
*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) +
 (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^
5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 - 7*a^3*b - 4*sqrt(3)*(a^2*b
^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - sqrt(3)*(b*x^4
- a*x) - 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1448*a^5*
```

```
b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*
sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 - 100*a*b^3*x^9
+ 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x
^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(2
23*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(
1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 49
8*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^1
1 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 23
40)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3
 + 8*a^4) - 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896
*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x
^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/
6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - s
qrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2)
)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 -
 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*
a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4
))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*
a^4)))/(b*x^4 - a*x)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^
4))^(1/6)*log((b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*x^3 +
64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*a^6*b^
4*x^4 + 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^7 + 56
1*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) +
6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5
*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16
*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 64*sqrt(3)*(a*b^3
*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944
)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2
131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 23
40)/(a^5*b^4))^(5/6) + 4*sqrt(1/6)*(168*a^3*b^5*x^9 + 1845*a^4*b^4*x^6 + 13
68*a^5*b^3*x^3 + 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 + 1065*a^4*b^4*x^6 +
 792*a^5*b^3*x^3 + 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3
*(1/1944)^(1/6)*(5*a*b^4*x^10 + 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 + 64*a^4*
b*x - 3*sqrt(3)*(a*b^4*x^10 + 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqr
t(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6
+ 320*a^3*b*x^3 + 64*a^4)) - 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(1/6)*log((b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*x
^3 + 64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*a
^6*b^4*x^4 + 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^7
 + 561*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/
3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 6
4*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5
 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 64*sqrt(3)*(
a*b^3*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) - 2*sqrt(b*x^3 - a)*(1944*(1
/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(
3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3)
 + 2340)/(a^5*b^4))^(5/6) + 4*sqrt(1/6)*(168*a^3*b^5*x^9 + 1845*a^4*b^4*x^6
 + 1368*a^5*b^3*x^3 + 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 + 1065*a^4*b^4*
x^6 + 792*a^5*b^3*x^3 + 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)
) + 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 + 64
*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(135
1*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2
*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(1/6)*log((b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a
^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 -
 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^
5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5
```

```
*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b
^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a
^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32
*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(b*x^3
- a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^
4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 511
2*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 +
2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2
340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*
b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x
^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40
*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + 1/24*(1/1944)^(1/
6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 - 100*a*b^3*x^9 +
 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^
10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(22
3*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1
351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498
*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11
 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3
+ 8*a^4) - 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*
a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^
5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6
)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sq
rt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))
*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 -
12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a
^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4)
)^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a
^4))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.36, size = 510, normalized size = 1.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(b*x^3-a)^(1/2)*(4*3
^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2-3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^(
1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(1/3)*_alp
ha*b+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
2)^(1/3)*_alpha^2*b-4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*a*
```

```
b+2*I*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_alp
ha+2*I*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b+6*3^(1/2)*a-10*a))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3 + 2\,a\left(3\,\sqrt{3} - 5\right)\right)\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{b\,x^3 - a}\,\left(b\,x^3 + 2\,a\,\left(3\,\sqrt{3} - 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{-a + bx^3} + 6\sqrt{3}\,a\sqrt{-a + bx^3} + bx^3\sqrt{-a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x*
*3*sqrt(-a + b*x**3)), x)
```

## 3.252 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

**Optimal**. Leaf size=322

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}{\sqrt{2}\sqrt{-a-b\cdots}}\right.}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

**Rubi [A]** time = 0.05, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {488}

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\sqrt{2}\,3\cdots}\right.}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}\cdots}$$

**Mathematica [C]** time = 0.13, size = 86, normalized size = 0.27

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])
```

**IntegrateAlgebraic [F]** time = 31.15, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a-bx^3}\,\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)
), x]
```

**fricas [B]** time = 21.05, size = 5060, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(-1/3*(3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b
^3 - sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*
sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3
+ 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b
^4))^(1/3) + sqrt(3)*(b*x^4 + a*x) + 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)
*(265*a^4*b^4*x^3 - 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x
 - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5
*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sq
rt((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 -
6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 +
2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*
b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9
)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x
^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b
^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 -
 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^
(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(213
1*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340
)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960
*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2
280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*
(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*
x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2
*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/6*sqrt(3)*(1/1944)^
(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(-1/3*(3*sqrt(-b*x^3 -
 a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b^3 - sqrt(3)*(153*a^4*
b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/
6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b
^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 + 7*a^3*b - 4*sqrt(3)*
(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + sqrt(3)*(
b*x^4 + a*x) - 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 14
```

48*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log(-(b^4*x^12 - 68*a*b^3*x^9 + 168*a^2*b^2*x^6 + 544*a^3*b*x^3 + 64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 - 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 - 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 - 834*a^5*b^5*x^7 + 561*a^6*b^4*x^4 -

```
104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(2
6*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqr
t(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(
-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 64*sqrt(3)*(a*b^3*x^9 + 3*a^2*b^2
*x^6 + 3*a^3*b*x^3 + a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a
^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8
 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^
(5/6) - 4*sqrt(1/6)*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b^3*x^3
- 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5*b^3*x^
3 - 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3*(1/1944)^(1/6)
*(5*a*b^4*x^10 - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3*sqrt(3)
*(a*b^4*x^10 - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3
 + 64*a^4)) - 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*
log(-(b^4*x^12 - 68*a*b^3*x^9 + 168*a^2*b^2*x^6 + 544*a^3*b*x^3 + 64*a^4 +
6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 - 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 - 1
440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 - 834*a^5*b^5*x^7 + 561*a^6*b^4
*x^4 - 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(
1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2
- 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*
x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 64*sqrt(3)*(a*b^3*x^9 + 3*
a^2*b^2*x^6 + 3*a^3*b*x^3 + a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*
(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*
b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) - 4*sqrt(1/6)*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b
^3*x^3 - 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5
*b^3*x^3 - 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3*(1/1944
)^(1/6)*(5*a*b^4*x^10 - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3*
sqrt(3)*(a*b^4*x^10 - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqrt(3) + 2
340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^
3*b*x^3 + 64*a^4))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

**maple [C]** time = 0.39, size = 541, normalized size = 1.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(-b*x^3-a)^(
1/2)*(4*3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2+3*I*(-a*b^2)^(1/3)*3^(1/2)*_alp
ha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^
2)^(1/3)*_alpha*b-3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(
-a*b^2)^(2/3)-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b+4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(
```

```
1/2)*a*b-2*3^(1/2)*a*b+2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha-2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a+10*a))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3}\,-5\right)\right)\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 - a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a - bx^3}\,\left(-6\sqrt{3}\,a + 10a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

## 3.253 $\int \frac{x^8 \sqrt{c+dx^3}}{a+bx^3} \, dx$

**Optimal**. Leaf size=125

$$-\frac{2a^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}$$

**Rubi [A]** time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2a^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*a^2*Sqrt[c + d*x^3])/(3*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(9*b^2*
d^2) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt{c+dx}}{b^2d}+\frac{a^2\sqrt{c+dx}}{b^2(a+bx)}+\frac{(c+dx)^{3/2}}{bd}\right)dx, x, x^3\right)\\
&= -\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^2}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{\cdots}}\cdots\right)}{3b^3}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{\left(2a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\cdots}\cdots\right)}{3b^3d}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.31, size = 121, normalized size = 0.97

$$
\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd\left(c+dx^3\right)+b^2\left(-2c^2+cdx^3+3d^2x^6\right)\right)}{45b^3d^2}-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x^
3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 139, normalized size = 1.11

$$
\frac{2a^2\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{7/2}}+\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abcd-5abd^2x^3-2b^2c^2+b^2cdx^3+3b^2d^2x^6\right)}{45b^3d^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 + b^2*c*d*x^3 - 5*a
*b*d^2*x^3 + 3*b^2*d^2*x^6))/(45*b^3*d^2) + (2*a^2*Sqrt[-(b*c) + a*d]*ArcTa
n[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2))
```

**fricas [A]** time = 0.70, size = 280, normalized size = 2.24

$$
\left[\frac{15\,a^2d^2\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+\left(b^2cd-5\,abd^2\right)x^3\right)\sqrt{dx^3+c}}{45\,b^3d^2},\ -\frac{2\left(15\,a^2d^2\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+\left(b^2cd-5\,abd^2\right)x^3\right)\sqrt{dx^3+c}\right)}{45\,b^3d^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*c^
```

```
2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*s
qrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*c*d +
 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d^2)]
```

**giac [A]** time = 0.17, size = 139, normalized size = 1.11

$$\frac{2\left(a^2bc-a^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^8-5\left(dx^3+c\right)^{\frac{3}{2}}b^4cd^8-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^9+15\sqrt{dx^3+c}\,a^2b^2d^{10}\right)}{45\,b^5d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^8 - 5*(d*x^3 + c)^(3
/2)*b^4*c*d^8 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^3 + c)*a^2*b^2*
d^10)/(b^5*d^10)
```

**maple [C]** time = 0.42, size = 514, normalized size = 4.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 1/b^2*(b*(2/15*(d*x^3+c)^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3
+c)^(1/2)*c^2/d^2)-2/9*a/d*(d*x^3+c)^(3/2))+a^2/b^2*(2/3*(d*x^3+c)^(1/2)/b+
1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_a
lpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1
/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2
)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1
/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c),(I*3^(1/2)*(-c*d^
2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_
alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.17, size = 176, normalized size = 1.41

$$\frac{2a^2\sqrt{dx^3+c}}{3b^3}+\frac{2\left(dx^3+c\right)^{5/2}}{15bd^2}-\frac{2a\left(dx^3+c\right)^{3/2}}{9b^2d}-\frac{2c\left(dx^3+c\right)^{3/2}}{9bd^2}+\frac{a^2\ln\left(\frac{a^2d^2\,1i+b^2c^2\,2i-2\sqrt{b}\,\sqrt{dx^3+c}\,(ad-bc)^{3/2}-abd^2x^3\,1i+b^2cdx^3\,1i-abcd\,3i}{2bx^3+2a}\right)\sqrt{ad-bc}\,1i}{3b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*a^2*(c + d*x^3)^(1/2))/(3*b^3) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a
*(c + d*x^3)^(3/2))/(9*b^2*d) - (2*c*(c + d*x^3)^(3/2))/(9*b*d^2) + (a^2*lo
g((a^2*d^2*1i + b^2*c^2*2i - 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)
- a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c
)^(1/2)*1i)/(3*b^(7/2))
```

**sympy [A]** time = 30.27, size = 128, normalized size = 1.02

$$\frac{2\left(\frac{a^2d^3\sqrt{c+dx^3}}{3b^3}-\frac{a^2d^3(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}\left(-ad^2-bcd\right)}{9b^2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(a**2*d**3*sqrt(c + d*x**3)/(3*b**3) - a**2*d**3*(a*d - b*c)*atan(sqrt(c
+ d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + d*(c + d*x**3
)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b**2))/d**3
```

## 3.254 $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

**Optimal**. Leaf size=93

$$\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}$$

**Rubi [A]** time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (-2*a*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b*d) + (2*a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{2\left(c+dx^3\right)^{3/2}}{9bd} - \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9bd} - \frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9bd} - \frac{(2a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9bd} + \frac{2a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.09, size = 88, normalized size = 0.95

$$
\frac{2a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2\sqrt{c+dx^3}\left(b\left(c+dx^3\right)-3ad\right)}{9b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-3*a*d + b*(c + d*x^3)))/(9*b^2*d) + (2*a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 100, normalized size = 1.08

$$
-\frac{2a\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}} - \frac{2\sqrt{c+dx^3}\left(3ad-bc-bdx^3\right)}{9b^2d}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(-(b*c) + 3*a*d - b*d*x^3))/(9*b^2*d) - (2*a*Sqrt[-(b*c
) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/
(3*b^(5/2))
```

**fricas [A]** time = 0.55, size = 195, normalized size = 2.10

$$
\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(bdx^3+bc-3\,ad\right)\sqrt{dx^3+c}}{9\,b^2d}, \frac{2\left(3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\left(bdx^3+bc-3\,ad\right)\sqrt{dx^3+c}\right)}{9\,b^2d}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
 c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b*d*x^3 + b*c - 3*a*d)*sqrt(d*
x^3 + c))/(b^2*d), 2/9*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*
```

```
b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^3 +
c))/(b^2*d)]
```

**giac [A]** time = 0.16, size = 96, normalized size = 1.03

$$-\frac{2\left(abc - a^2 d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^2} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^3+c}\,abd^3\right)}{9\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^3 + c)*a*
b*d^3)/(b^3*d^3)
```

**maple [C]** time = 0.24, size = 458, normalized size = 4.92

$$-\frac{\left(\frac{2\sqrt{dx^3+c}}{3b} + \frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3}, \frac{\left(2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d}, \sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3bd^2\sqrt{dx^3+c}}\right)a}{b} + \frac{2\left(dx^3+c\right)^{\frac{3}{2}}}{9bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d-a/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c
*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.06, size = 136, normalized size = 1.46

$$\frac{2\left(d\,x^3+c\right)^{3/2}}{9\,b\,d} - \frac{2\,a\,\sqrt{d\,x^3+c}}{3\,b^2} + \frac{a\,\ln\left(\frac{a^2\,d^2\,1\mathrm{i}+b^2\,c^2\,2\mathrm{i}+2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1\mathrm{i}+b^2\,c\,d\,x^3\,1\mathrm{i}-a\,b\,c\,d\,3\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(3/2))/(9*b*d) - (2*a*(c + d*x^3)^(1/2))/(3*b^2) + (a*log((a
^2*d^2*1i + b^2*c^2*2i + 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2) - a*
b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)^(1
/2)*1i)/(3*b^(5/2))
```

**sympy** [A] time = 14.67, size = 95, normalized size = 1.02

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2}+\frac{ad^2(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{3}{2}}}{9b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**3)**(3
/2)/(9*b))/d**2
```

## 3.255 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$

**Optimal**. Leaf size=70

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{1}{3}\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}$$
$$= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(2(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}$$
$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

**Mathematica** [A] time = 0.04, size = 70, normalized size = 1.00

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b} - \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] ((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[b*c - a*d]])/b^(3/2))/3
```

**IntegrateAlgebraic** [A] time = 0.07, size = 80, normalized size = 1.14

$$\frac{2\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}} + \frac{2\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) + (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*
c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(3/2))
```

**fricas** [A] time = 0.64, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}}{3\,b}, -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^3+c}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c))/b, -2/3*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^3 + c))/b]
```

**giac [A]** time = 0.16, size = 66, normalized size = 0.94

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b}+\frac{2\,\sqrt{dx^3+c}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b
```

**maple [C]** time = 0.23, size = 434, normalized size = 6.20

$$\frac{2\sqrt{d\,x^3+c}}{3b}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-c\,d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\;\left(-c\,d^2\right)^{\frac{2}{3}}-\left(-c\,d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\;cd-3cd-i\left(-c\,d^2\right)^{\frac{2}{3}}\sqrt{3}\;\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-c\,d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3b\,d^2\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)
*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.16, size = 82, normalized size = 1.17

$$\frac{2\,\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)
*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^(3
/2))
```

**sympy** [A] time = 6.35, size = 68, normalized size = 0.97

$$\frac{2\left(\frac{d\sqrt{c+dx^3}}{3b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d
 - b*c)/b))/(3*b**2*sqrt((a*d - b*c)/b)))/d
```

## 3.256 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)} dx$

**Optimal**. Leaf size=85

$$\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

**Rubi [A]** time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a) + (2*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^3\right)\\
&= \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\\
&= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad}\\
&= -\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}
\end{aligned}
$$

**Mathematica** [A] time = 0.05, size = 81, normalized size = 0.95

$$
\frac{2\left(\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} - \sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (2*(-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*a)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 95, normalized size = 1.12

$$
-\frac{2\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a\sqrt{b}} - \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/
(b*c - a*d)])/(3*a*Sqrt[b]) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
(3*a)
```

**fricas** [A] time = 0.54, size = 383, normalized size = 4.51

$$
\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{3a}, \frac{2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{3a}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3a}, \frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)\right)}{3a}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*
sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
c)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
```

```
(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
3 + c)*sqrt(-c)/c))/a]
```

**giac [A]** time = 0.17, size = 79, normalized size = 0.93

$$-\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

**maple [C]** time = 0.26, size = 476, normalized size = 5.60

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}}{3b}+\frac{i(-c\,d^2)^{\frac{1}{3}}\sqrt{\frac{\frac{i}{2}\left(2x+\frac{-i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}+(-c\,d^2)^{\frac{1}{3}}}{d}\right)d}{(-c\,d^2)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{(-c\,d^2)^{\frac{1}{3}}}{d}\right)d}{-3(-c\,d^2)^{\frac{1}{3}}+i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}}\sqrt{-\frac{\frac{i}{2}\left(2x+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}+(-c\,d^2)^{\frac{1}{3}}}{d}\right)d}{2(-c\,d^2)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}(\_Z^3b+a)^2d^2+i(-c\,d^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}(\_Z^3b+a)d-(-c\,d^2)^{\frac{1}{3}}\,\mathrm{RootOf}(\_Z^3b+a)d-i\sqrt{3}\,(-c\,d^2)^{\frac{2}{3}}-(-c\,d^2)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{(-c\,d^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^2)^{\frac{1}{3}}}}}{3},\frac{\left(2i(-c\,d^2)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}(\_Z^3b+a)^2d+i\sqrt{3}\,cd-3cd-i(-c\,d^2)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}(\_Z^3b+a)-3(-c\,d^2)^{\frac{2}{3}}\,\mathrm{RootOf}(\_Z^3b+a)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{\left(-\frac{3(-c\,d^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^2)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3b\,d^2\sqrt{d\,x^3+c}}\right)b}{a}+\frac{-\frac{2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3}+\frac{2\sqrt{d\,x^3+c}}{3}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a),x)
```

```
[Out] -1/a*b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(
a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(2/3*(d*x^3+c)^(
1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)
```

**mupad [B]** time = 7.94, size = 114, normalized size = 1.34

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,a\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(
1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*a*b^(1/2))
```

**sympy** [A] time = 12.25, size = 85, normalized size = 1.00

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/b)))/d
```

## 3.257 $\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=115

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}-\frac{\sqrt{c+dx^3}}{3ax^3}$$

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}-\frac{\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*a*x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(3*a^2*Sqrt[c]) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.}{6a^2} \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{}{-\frac{c}{d}}\right.}{3a^2} \\
&= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 107, normalized size = 0.93

$$\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}} - 2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right) - \frac{a\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^3])/x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*a^2)
```

**IntegrateAlgebraic [A]** time = 0.26, size = 125, normalized size = 1.09

$$\frac{2\sqrt{b}\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*x^3) + (2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b
]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^2) + ((2*b*c - a*d
)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*Sqrt[c])
```

**fricas [A]** time = 0.52, size = 513, normalized size = 4.46

$$\left[\frac{2\sqrt{b^2c-abd}\,cx^3\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-(2bc-ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-2\sqrt{dx^3+c}\,ac}{6a^2cx^3}, \frac{4\sqrt{-b^2c+abd}\,cx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2bc-ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-2\sqrt{dx^3+c}\,ac}{6a^2cx^3}, -\frac{(2bc-ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}\,cx^3\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+\sqrt{dx^3+c}\,ac}{3a^2cx^3}, \frac{2\sqrt{-b^2c+abd}\,cx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2bc-ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{dx^3+c}\,ac}{3a^2cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3
 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c
*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), -1/3*(
(2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sqrt(-b^2*c
 + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(d*
x^3 + c)*a*c)/(a^2*c*x^3)]
```

**giac [A]** time = 0.19, size = 107, normalized size = 0.93

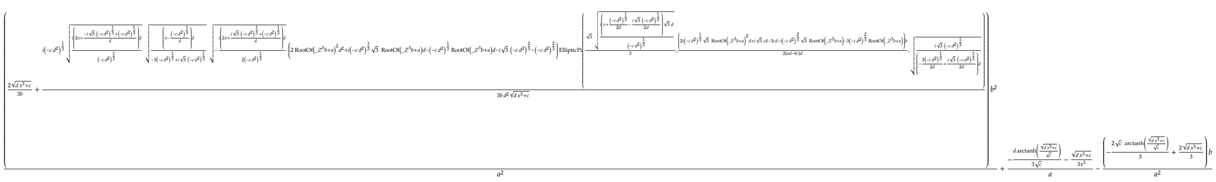
$$\frac{2\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c - a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/3*(2*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)/(a*x^3)
```

**maple [C]** time = 0.25, size = 518, normalized size = 4.50



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x)
```

```
[Out] 1/a*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2
))+1/a^2*b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*
(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d
)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_al
pha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-1/a^2*b*(2/3*(
d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)
```

**mupad [B]** time = 5.13, size = 137, normalized size = 1.19

$$\frac{\ln\left(\frac{a\,d-2\,b\,c+2\,\sqrt{d\,x^3+c}\,\sqrt{b^2\,c-a\,b\,d}-b\,d\,x^3}{b\,x^3+a}\right)\sqrt{b^2\,c-a\,b\,d}}{3\,a^2}-\frac{\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\left(a\,d-2\,b\,c\right)}{6\,a^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)),x)
```

```
[Out] (log((a*d - 2*b*c + 2*(c + d*x^3)^(1/2)*(b^2*c - a*b*d)^(1/2) - b*d*x^3)/(a
 + b*x^3))*(b^2*c - a*b*d)^(1/2))/(3*a^2) - (c + d*x^3)^(1/2)/(3*a*x^3) + (
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a
*d - 2*b*c))/(6*a^2*c^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)
```

## 3.258 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

**Optimal**. Leaf size=154

$$-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}+\frac{2a^2\sqrt{c+dx^3}\,(bc-ad)}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2\left(c+dx^3\right)^{5/2}(ad+bc)}{15b^2d^2}+\frac{2\left(c+d\right.}{21b}$$

**Rubi [A]** time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}+\frac{2a^2\sqrt{c+dx^3}\,(bc-ad)}{3b^4}-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}-\frac{2\left(c+dx^3\right)^{5/2}(ad+bc)}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^4) + (2*a^2*(c + d*x^3)^(3/2))/(9*
b^3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(15*b^2*d^2) + (2*(c + d*x^3)^(7/2
))/(21*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{a+bx}\,dx,x,x^3\right)$$

$$=\frac{1}{3}\,\text{Subst}\left(\int \left(\frac{(-bc-ad)(c+dx)^{3/2}}{b^2d}+\frac{a^2(c+dx)^{3/2}}{b^2(a+bx)}+\frac{(c+dx)^{5/2}}{bd}\right)dx,x,x^3\right)$$

$$=-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^3\right)}{3b^2}$$

$$=\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+\frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \ldots\right.}{3b^3}$$

$$=\frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+\ldots$$

$$=\frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+\ldots$$

$$=\frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}-\ldots$$

**Mathematica [A]** time = 0.24, size = 145, normalized size = 0.94

$$\frac{2\left(105a^2(bc-ad)\left(\frac{\sqrt{c+dx^3}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)+35a^2\left(c+dx^3\right)^{3/2}-\frac{21b\left(c+dx^3\right)^{5/2}(ad+bc)}{d^2}+\frac{15b^2\left(c+dx^3\right)^{7/2}}{d^2}\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(35*a^2*(c + d*x^3)^(3/2) - (21*b*(b*c + a*d)*(c + d*x^3)^(5/2))/d^2 + (
15*b^2*(c + d*x^3)^(7/2))/d^2 + 105*a^2*(b*c - a*d)*(Sqrt[c + d*x^3]/b - (S
qrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))
))/(315*b^3)
```

**IntegrateAlgebraic [A]** time = 0.29, size = 195, normalized size = 1.27

$$-\frac{2a^2(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{9/2}}-\frac{2\sqrt{c+dx^3}\left(105a^3d^3-140a^2bcd^2-35a^2bd^3x^3+21ab^2c^2d+42ab^2cd^2x^3+21ab^2d^3x^6+6b^3c^3-3b^3c^2dx^3-24b^3cd^2x^6-15b^3d^3x^9\right)}{315b^4d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(6*b^3*c^3 + 21*a*b^2*c^2*d - 140*a^2*b*c*d^2 + 105*a^3
*d^3 - 3*b^3*c^2*d*x^3 + 42*a*b^2*c*d^2*x^3 - 35*a^2*b*d^3*x^3 - 24*b^3*c*d
^2*x^6 + 21*a*b^2*d^3*x^6 - 15*b^3*d^3*x^9))/(315*b^4*d^2) - (2*a^2*(-(b*c)
 + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*
d)])/(3*b^(9/2))
```

**fricas [A]** time = 0.92, size = 410, normalized size = 2.66

$$\left[-\frac{105\left(a^2bcd^2-a^3d^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(15b^3d^3x^9+3\left(8b^3cd^2-7ab^2d^3\right)x^6-6b^3c^3-21ab^2c^2d+140a^2bcd^2-105a^3d^3+\left(3b^3c^2d-42ab^2cd^2+35a^2bd^3\right)x^3\right)\sqrt{dx^3+c}}{315b^4d^2},\;-\frac{2\left(105\left(a^2bcd^2-a^3d^3\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(15b^3d^3x^9+3\left(8b^3cd^2-7ab^2d^3\right)x^6-6b^3c^3-21ab^2c^2d+140a^2bcd^2-105a^3d^3+\left(3b^3c^2d-42ab^2cd^2+35a^2bd^3\right)x^3\right)\sqrt{dx^3+c}\right)}{315b^4d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b
*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*b^
3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2*d
+ 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^2*b*
d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d^3)*s
qrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a
*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21
*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^
2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2)]
```

**giac [A]** time = 0.17, size = 193, normalized size = 1.25

$$\frac{2\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^4}+\frac{2\left(15\left(dx^3+c\right)^{\frac{7}{2}}b^6d^{12}-21\left(dx^3+c\right)^{\frac{5}{2}}b^6cd^{12}-21\left(dx^3+c\right)^{\frac{5}{2}}ab^5d^{13}+35\left(dx^3+c\right)^{\frac{3}{2}}a^2b^4d^{14}+105\sqrt{dx^3+c}\,a^2b^4cd^{14}-105\sqrt{dx^3+c}\,a^3b^3d^{15}\right)}{315\,b^7d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 2/315*(15*(d*x^3 + c)^(7/2)*b^6*
d^12 - 21*(d*x^3 + c)^(5/2)*b^6*c*d^12 - 21*(d*x^3 + c)^(5/2)*a*b^5*d^13 +
35*(d*x^3 + c)^(3/2)*a^2*b^4*d^14 + 105*sqrt(d*x^3 + c)*a^2*b^4*c*d^14 - 10
5*sqrt(d*x^3 + c)*a^3*b^3*d^15)/(b^7*d^14)
```

**maple [C]** time = 0.36, size = 605, normalized size = 3.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 1/b^2*(b*(2/21*(d*x^3+c)^(1/2)*d*x^9+16/105*(d*x^3+c)^(1/2)*c*x^6+2/105*(d*
x^3+c)^(1/2)*c^2/d*x^3-4/105*(d*x^3+c)^(1/2)*c^3/d^2)-2/15*a/d*(d*x^3+c)^(5
/2))+a^2/b^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c
)/d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/
(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(
2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_a
lpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.12, size = 330, normalized size = 2.14

$$\frac{2\,d\,x^9\,\sqrt{d\,x^3+c}}{21\,b}-\frac{\left(\frac{2\,a\left(\frac{c^2}{b}+\frac{a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}+\frac{2\,c\left(\frac{2\,c^2}{b}+\frac{2\,a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{3\,d}\right)\sqrt{d\,x^3+c}}{3\,d}+\frac{x^3\,\sqrt{d\,x^3+c}\left(\frac{2\,c^2}{b}+\frac{2\,a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{9\,d}-\frac{x^6\,\sqrt{d\,x^3+c}\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{15\,d}+\frac{a^2\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (2*d*x^9*(c + d*x^3)^(1/2))/(21*b) - (((2*a*(c^2/b + (a*((a*d^2)/b^2 - (2*c
*d)/b))/b))/b + (2*c*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4*c*
((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(3*d))*(c + d*x^3)^(1/2))/(3*d) +
 (x^3*(c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4
*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(9*d) - (x^6*(c + d*x^3)^(1/2)
*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(15*d) + (a^2*log((a^2*d^2 + 2*b^2*c^2 -
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*b^(9/2))
```

**sympy [A]** time = 129.92, size = 153, normalized size = 0.99

$$\frac{2a^2\left(c+dx^3\right)^{\frac{3}{2}}}{9b^3}+\frac{2a^2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^5\sqrt{\frac{ad-bc}{b}}}+\frac{2\left(c+dx^3\right)^{\frac{7}{2}}}{21bd^2}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}\left(-2ad-2bc\right)}{15b^2d^2}+\frac{\sqrt{c+dx^3}\left(-2a^3d+2a^2bc\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*a**2*(c + d*x**3)**(3/2)/(9*b**3) + 2*a**2*(a*d - b*c)**2*atan(sqrt(c + d
*x**3)/sqrt((a*d - b*c)/b))/(3*b**5*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**
(7/2)/(21*b*d**2) + (c + d*x**3)**(5/2)*(-2*a*d - 2*b*c)/(15*b**2*d**2) + s
qrt(c + d*x**3)*(-2*a**3*d + 2*a**2*b*c)/(3*b**4)
```

## 3.259 $\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx$

**Optimal**. Leaf size=120

$$\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}-\frac{2a\sqrt{c+dx^3}\,(bc-ad)}{3b^3}-\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd}$$

**Rubi [A]** time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2}-\frac{2a\sqrt{c+dx^3}\,(bc-ad)}{3b^3}+\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (-2*a*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^3) - (2*a*(c + d*x^3)^(3/2))/(9*b^2
) + (2*(c + d*x^3)^(5/2))/(15*b*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{a\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{3b}\\
&= -\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{\left(a(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{\quad}{(a+\quad}\right.}{3b^3}\\
&= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{\left(2a(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{\quad}{a\cdot}\right.}{3b^3}\\
&= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} + \frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{\quad}}\right.}{3b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 111, normalized size = 0.92

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd\left(4c+dx^3\right)+3b^2\left(c+dx^3\right)^2\right)}{45b^3d}+\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^3
)))/(45*b^3*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 138, normalized size = 1.15

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2-20abcd-5abd^2x^3+3b^2c^2+6b^2cdx^3+3b^2d^2x^6\right)}{45b^3d}+\frac{2a(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + 6*b^2*c*d*x^3 - 5
*a*b*d^2*x^3 + 3*b^2*d^2*x^6))/(45*b^3*d) + (2*a*(-(b*c) + a*d)^(3/2)*ArcTa
n[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2))
```

**fricas [A]** time = 0.78, size = 297, normalized size = 2.48

$$\left[-\frac{15\left(abcd-a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(3\,b^2d^2x^6+3\,b^2c^2-20\,abcd+15\,a^2d^2+\left(6\,b^2cd-5\,abd^2\right)x^3\right)\sqrt{dx^3+c}}{45\,b^3d},\ \frac{2\left(15\left(abcd-a^2d^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\left(3\,b^2d^2x^6+3\,b^2c^2-20\,abcd+15\,a^2d^2+\left(6\,b^2cd-5\,abd^2\right)x^3\right)\sqrt{dx^3+c}\right)}{45\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a
*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(3*b^2*d^2*x
^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqr
t(d*x^3 + c))/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b)*ar
ctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (3*b^2*d^2*x^6
+ 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d
*x^3 + c))/(b^3*d)]
```

**giac** [A] time = 0.19, size = 151, normalized size = 1.26

$$-\frac{2\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\,\sqrt{-b^{2}c+abd}\,b^{3}}+\frac{2\left(3\left(dx^{3}+c\right)^{\frac{5}{2}}b^{4}d^{4}-5\left(dx^{3}+c\right)^{\frac{3}{2}}ab^{3}d^{5}-15\,\sqrt{dx^{3}+c}\,ab^{3}cd^{5}+15\,\sqrt{dx^{3}+c}\,a^{2}b^{2}d^{6}\right)}{45\,b^{5}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^4
 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*sqrt
(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

**maple** [C] time = 0.32, size = 531, normalized size = 4.42

$$-\frac{\left(\frac{2\sqrt{dx^{3}+c}\,dx^{3}}{9b}+\frac{2\left(-\frac{2cd}{3b}-\frac{(ad-2bc)d}{b^{2}}\right)\sqrt{dx^{3}+c}}{3d}+\frac{i\left(-a^{2}d^{2}+2abcd-b^{2}c^{2}\right)\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)^{2}d^{2}+i\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)d-\left(-cd^{2}\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^{3}b+a\right)d-i\sqrt{3}\left(-cd^{2}\right)^{\frac{2}{3}}-\left(-cd^{2}\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^{2}\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)^{2}d+i\sqrt{3}\,cd-3cd-i\left(-cd^{2}\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)-3\left(-cd^{2}\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^{3}b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3b^{2}d^{2}(ad-bc)\sqrt{dx^{3}+c}}\right)a}{b}+\frac{2\left(dx^{3}+c\right)^{\frac{5}{2}}}{15bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b/d-a/b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(
a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a
*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^
(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*b+a)))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.13, size = 215, normalized size = 1.79

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{3\,d}\right)}{3\,d}+\frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b}-\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{9\,d}+\frac{a\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (2*c*((
2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(3*d)))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))
/(15*b) - (x^3*(c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(9*d) +
(a*log((a^2*d^2 + 2*b^2*c^2 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2
i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1
i)/(3*b^(7/2))
```

**sympy [A]** time = 70.60, size = 116, normalized size = 0.97

$$-\frac{2a\left(c+dx^3\right)^{\frac{3}{2}}}{9b^2}-\frac{2a\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{2\left(c+dx^3\right)^{\frac{5}{2}}}{15bd}+\frac{\sqrt{c+dx^3}\left(2a^2d-2abc\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] -2*a*(c + d*x**3)**(3/2)/(9*b**2) - 2*a*(a*d - b*c)**2*atan(sqrt(c + d*x**3
)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**(5/2)
/(15*b*d) + sqrt(c + d*x**3)*(2*a**2*d - 2*a*b*c)/(3*b**3)
```

## 3.260 $\int \frac{x^2 (c+dx^3)^{3/2}}{a+bx^3} \, dx$

**Optimal**. Leaf size=96

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\sqrt{c+dx^3}\,(bc-ad)}{3b^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9b}$$

**Rubi [A]** time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}\,(bc-ad)}{3b^2}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\left(c+dx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b) - (2*
(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^
(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{a+bx^3} dx = \frac{1}{3} \operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx} dx, x, x^3\right)$$
$$= \frac{2\left(c+dx^3\right)^{3/2}}{9b} + \frac{(bc-ad) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3\right)}{3b}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9b} + \frac{(bc-ad)^2 \operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}} dx, x, x^3\right)}{3b^2}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9b} + \frac{\left(2(bc-ad)^2\right) \operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} dx, x, \sqrt{c+dx^3}\right)}{3b^2 d}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9b} - \frac{2(bc-ad)^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

**Mathematica** [A] time = 0.09, size = 85, normalized size = 0.89

$$\frac{2\sqrt{c+dx^3}\left(-3ad+4bc+bdx^3\right)}{9b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) - (2*(b*c - a*d)^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.17, size = 95, normalized size = 0.99

$$\frac{2\sqrt{c+dx^3}\left(-3ad+4bc+bdx^3\right)}{9b^2} - \frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) - (2*(-(b*c) + a*d)^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b
^(5/2))
```

**fricas** [A] time = 0.69, size = 204, normalized size = 2.12

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(bdx^3+4\,bc-3\,ad\right)\sqrt{dx^3+c}}{9\,b^2}, -\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(bdx^3+4\,bc-3\,ad\right)\sqrt{dx^3+c}\right)}{9\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqr
t(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3*a
*d)*sqrt(d*x^3 + c))/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-
sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c - 3*
a*d)*sqrt(d*x^3 + c))/b^2]
```

**giac [A]** time = 0.17, size = 113, normalized size = 1.18

$$\frac{2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\,\sqrt{-b^{2}c+abd}\,b^{2}}+\frac{2\left(\left(dx^{3}+c\right)^{\frac{3}{2}}b^{2}+3\,\sqrt{dx^{3}+c}\,b^{2}c-3\,\sqrt{dx^{3}+c}\,abd\right)}{9\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(d*
x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3
```

**maple [C]** time = 0.25, size = 507, normalized size = 5.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1
/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*
d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2
)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.91, size = 143, normalized size = 1.49

$$\frac{2\,d\,x^{3}\,\sqrt{d\,x^{3}+c}}{9\,b}-\frac{\sqrt{d\,x^{3}+c}\,\left(\frac{2\,a\,d^{2}}{b^{2}}-\frac{8\,c\,d}{3\,b}\right)}{3\,d}+\frac{\ln\left(\frac{a^{2}\,d^{2}+2\,b^{2}\,c^{2}-a\,b\,d^{2}\,x^{3}+b^{2}\,c\,d\,x^{3}-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^{3}+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^{3}+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i
- a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)
/(3*b^(5/2)) - ((c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (8*c*d)/(3*b)))/(3*d) +
(2*d*x^3*(c + d*x^3)^(1/2))/(9*b)
```

**sympy [A]** time = 35.29, size = 90, normalized size = 0.94

$$\frac{2\left(c+dx^3\right)^{\frac{3}{2}}}{9b}+\frac{\sqrt{c+dx^3}\,\left(-2ad+2bc\right)}{3b^2}+\frac{2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-2*a*d + 2*b*c)/(3*b**2) +
2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a
*d - b*c)/b))
```

## 3.261 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$

**Optimal**. Leaf size=104

$$\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

**Rubi** [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 84, 156, 63, 208}

$$\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*d*Sqrt[c + d*x^3])/(3*b) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
(3*a) + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a
*d]])/(3*a*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx, x, x^3\right)$$
$$= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{\text{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}$$
$$= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ab}$$
$$= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3abd}$$
$$= \frac{2d\sqrt{c+dx^3}}{3b} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}$$

**Mathematica [A]** time = 0.10, size = 105, normalized size = 1.01

$$\frac{2\left(a\sqrt{b}\,d\sqrt{c+dx^3}\;+(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)-b^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3ab^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*(a*Sqrt[b]*d*Sqrt[c + d*x^3] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]] + (b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*
d]]))/(3*a*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.23, size = 114, normalized size = 1.10

$$\frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3ab^{3/2}} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2d\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*d*Sqrt[c + d*x^3])/(3*b) + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[
-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a*b^(3/2)) - (2*c^(3/2)*Arc
Tanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a)
```

**fricas [A]** time = 1.00, size = 486, normalized size = 4.67

$$\left[\frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3ab}, \frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ad+2(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)}{3ab}, \frac{2b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+2\sqrt{dx^3+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3ab}, \frac{2\left(b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}\,ad+(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)\right)}{3ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(b*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/(a*b), 1/3*(b*c^
(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c
)*a*d + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-
```

```
(b*c - a*d)/b)/(b*c - a*d)))/(a*b), 1/3*(2*b*sqrt(-c)*c*arctan(sqrt(d*x^3 +
 c)*sqrt(-c)/c) + 2*sqrt(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3
 + a)))/(a*b), 2/3*(b*sqrt(-c)*c*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(
d*x^3 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b
*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

**giac [A]** time = 0.18, size = 112, normalized size = 1.08

$$\frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}+\frac{2\,\sqrt{dx^3+c}\,d}{3\,b}-\frac{2\,\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)) + 2/3*sqrt(d*x^3 + c)
*d/b - 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

**maple [C]** time = 0.26, size = 565, normalized size = 5.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a),x)
```

```
[Out] -1/a*b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x
^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b
*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+
a))+1/a*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arcta
nh((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)
```

**mupad [B]** time = 7.88, size = 155, normalized size = 1.49

$$\frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a}+\frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,a\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (2*d*(c + d*x^3)^(1/2))/(3*b) + (log((a^2*d^2 + 2*b^2*c^2 +
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a*b^(3/2))
```

**sympy [A]** time = 39.29, size = 102, normalized size = 0.98

$$\frac{2d\sqrt{c+dx^3}}{3b}+\frac{2c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}-\frac{2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)
```

```
[Out] 2*d*sqrt(c + d*x**3)/(3*b) + 2*c**2*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sq
rt(-c)) - 2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*
b**2*sqrt((a*d - b*c)/b))
```

## 3.262 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx$

**Optimal**. Leaf size=116

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

**Rubi [A]** time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] -(c*Sqrt[c + d*x^3])/(3*a*x^3) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(3*a^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+}}\right.}{3a^2}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int\right.}{3}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} + \frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

**Mathematica [A]** time = 0.13, size = 108, normalized size = 0.93

$$\frac{-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}+\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)-\frac{ac\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^3])/x^3) + Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]] - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/Sqrt[b])/(3*a^2)
```

**IntegrateAlgebraic [A]** time = 0.25, size = 130, normalized size = 1.12

$$\frac{\left(2bc^{3/2}-3a\sqrt{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}\,\sqrt{ad-bc}}{bc-ad}\right)}{3a^2\sqrt{b}}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*(c*Sqrt[c + d*x^3])/(a*x^3) - (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^2*Sqrt[b]) + ((2*b*c
^(3/2) - 3*a*Sqrt[c]*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

**fricas [A]** time = 0.88, size = 538, normalized size = 4.64

$$\left[-\frac{2(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+(2bc-3ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\,\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ac}{6a^2x^3}, -\frac{4(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2bc-3ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\,\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ac}{6a^2x^3}, -\frac{(2bc-3ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{dx^3+c}\,ac}{3a^2x^3}, -\frac{2(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2bc-3ad)\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}\,ac}{3a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2
*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*sqrt
(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 +
 c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sq
rt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(c)
*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c) + (b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(-c)*x
^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 + c)*a*c)/(a^2*x^3)]
```

**giac [A]** time = 0.17, size = 121, normalized size = 1.04

$$\frac{2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\,\sqrt{-b^{2}c+abd}\,a^{2}}-\frac{\left(2\,bc^{2}-3\,acd\right)\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{3\,a^{2}\sqrt{-c}}-\frac{\sqrt{dx^{3}+c}\,c}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/3*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(d*
x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)*c/(a*x^3)
```

**maple [C]** time = 0.28, size = 620, normalized size = 5.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x)
```

```
[Out] 1/a*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*
x^3+c)^(1/2)/c^(1/2)))+1/a^2*b^2*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c
*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2
+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-
I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2)),_alpha=RootOf(_Z^3*b+a)))-1/a^2*b*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^
3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{\frac{3}{2}}}{\left(bx^{3}+a\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)
```

**mupad [B]** time = 9.52, size = 167, normalized size = 1.44

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(3\,a\,d-2\,b\,c)}{6\,a^2}-\frac{c\,\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2\mathrm{i}}{b\,x^3+a}\right)(a\,d-b\,c)^{3/2}\,1\mathrm{i}}{3\,a^2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6)*(3*a*d - 2*b*c))/(6*a^2) - (c*(c + d*x^3)^(1/2))/(3*a*x^3) + (log((a
^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d
^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a^2
*b^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^3\right)^{\frac{3}{2}}}{x^4\left(a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)
```

## 3.263 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=104

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}\,(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}$$

**Rubi** [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}\,(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*(b*c + a*d)*Sqrt[c + d*x^3])/(3*b^2*d^2) + (2*(c + d*x^3)^(3/2))/(9*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)
*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^3\right)\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{a^2\operatorname{Subst}\left(\int\frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int\frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 91, normalized size = 0.88

$$\frac{2\sqrt{c+dx^3}\left(-3ad-2bc+bdx^3\right)}{9b^2d^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) - (2*a^2*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

**IntegrateAlgebraic [A]** time = 0.19, size = 102, normalized size = 0.98

$$-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}\sqrt{ad-bc}}-\frac{2\sqrt{c+dx^3}\left(3ad+2bc-bdx^3\right)}{9b^2d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(2*b*c + 3*a*d - b*d*x^3))/(9*b^2*d^2) - (2*a^2*ArcTan[
(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(5/2)*Sqrt[
-(b*c) + a*d])
```

**fricas [A]** time = 0.87, size = 289, normalized size = 2.78

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^3\right)\sqrt{dx^3+c}}{9\left(b^4cd^2-ab^3d^3\right)},\ \frac{2\left(3\,\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^3\right)\sqrt{dx^3+c}\right)}{9\left(b^4cd^2-ab^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x
^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^
3), 2/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^3 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d
 - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

**giac [A]** time = 0.17, size = 106, normalized size = 1.02

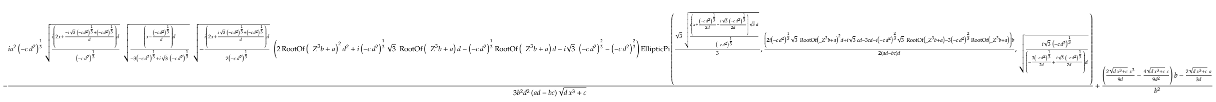
$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^3+c}\,b^2cd^4-3\,\sqrt{dx^3+c}\,abd^5\right)}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^3 + c)*b^2*c*d^4 - 3*s
qrt(d*x^3 + c)*a*b*d^5)/(b^3*d^6)
```

**maple [C]** time = 0.34, size = 488, normalized size = 4.69



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/b^2*(b*(2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*c/d^2)-2/3*a/d*(d*x
^3+c)^(1/2))-1/3*I*a^2/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*
I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/
2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d
-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*
3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/
(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.43, size = 121, normalized size = 1.16

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b\,d}-\frac{\left(\frac{2\,a}{b^2}+\frac{4\,c}{3\,b\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d}+\frac{a^2\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b*d) - (((2*a)/b^2 + (4*c)/(3*b*d))*(c + d*x^3
)^(1/2))/(3*d) + (a^2*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b
*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a + bx^3\right)\sqrt{c + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

## 3.264 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=74

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

**Rubi [A]** time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.12, size = 74, normalized size = 1.00

$$\frac{2\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\sqrt{b}\sqrt{c+dx^3}}{d}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*((Sqrt[b]*Sqrt[c + d*x^3])/d + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*b^(3/2))
```

**IntegrateAlgebraic** [A] time = 0.08, size = 84, normalized size = 1.14

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}\sqrt{ad-bc}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[
c + d*x^3])/(b*c - a*d)])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

**fricas** [A] time = 0.88, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{3\left(b^3cd-ab^2d^2\right)}, -\frac{2\left(\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\sqrt{dx^3+c}\left(b^2c-abd\right)\right)}{3\left(b^3cd-ab^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c
)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -2/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^3 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

**giac [A]** time = 0.17, size = 64, normalized size = 0.86

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^3+c}}{b}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^3 + c)/b)/d
```

**maple [C]** time = 0.23, size = 448, normalized size = 6.05

$$\frac{ia\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3bd^2\left(ad-bc\right)\sqrt{dx^3+c}}+\frac{2\sqrt{dx^3+c}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b/d+1/3*I*a/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3
)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.10, size = 86, normalized size = 1.16

$$\frac{2\sqrt{d\,x^3+c}}{3\,b\,d}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)1{}\mathrm{i}}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(
1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c
)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

## 3.265 $\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {444, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 61, normalized size = 1.20

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*Sq
rt[b]*Sqrt[-(b*c) + a*d])
```

**fricas** [A] time = 0.85, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)}{3\,\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)}{3\,\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b
*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]
```

**giac** [A] time = 0.16, size = 40, normalized size = 0.78

$$\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

**maple** [C] time = 0.28, size = 426, normalized size = 8.35

$$-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-c\,d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-c\,d^2\right)^{\frac{2}{3}}-\left(-c\,d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3}, \frac{\left(2i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-c\,d^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-c\,d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d}, \sqrt{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^2\,(ad-bc)\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.89, size = 70, normalized size = 1.37

$$\frac{\ln\left(\frac{a\,d\,1\mathrm{i}-b\,c\,2\mathrm{i}+2\,\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}-b\,d\,x^3\,1\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((a*d*1i - b*c*2i + 2*(c + d*x^3)^(1/2)*(a*b*d - b^2*c)^(1/2) - b*d*x^3
*1i)/(a + b*x^3))*1i)/(3*(a*b*d - b^2*c)^(1/2))
```

**sympy [A]** time = 10.26, size = 39, normalized size = 0.76

$$\frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

## 3.266 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=85

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

**Rubi [A]** time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}-\frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}-\frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 81, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*(-(ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*a)
```

**IntegrateAlgebraic** [A] time = 0.12, size = 95, normalized size = 1.12

$$\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a\sqrt{ad-bc}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)]
)/(3*a*Sqrt[-(b*c) + a*d]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt
[c])
```

**fricas** [A] time = 0.46, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{3ac}, \frac{2c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{3ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{3ac}, \frac{2\left(c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)\right)}{3ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-
c)/c))/(a*c), 2/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*
```

d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*c)]

**giac [A]** time = 0.17, size = 71, normalized size = 0.84

$$-\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

**maple [C]** time = 0.24, size = 453, normalized size = 5.33

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{i b\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-c\,d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{2}{3}}-\left(-c\,d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-c\,d^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-c\,d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2\left(ad-bc\right)d},\sqrt{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3a\,d^2\left(ad-bc\right)\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a*b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/
a/c^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

**mupad [B]** time = 7.31, size = 114, normalized size = 1.34

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,\sqrt{c}}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d -
b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(1/2))
```

**sympy [A]** time = 19.39, size = 70, normalized size = 0.82

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + 2
*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c))
```

## 3.267 $\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=117

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

**Rubi** [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*a*c*x^3) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a^2*c^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*
c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3ac}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{6a^2c}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{}{-\frac{c}{d}}\right.}{3a^2c}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.13, size = 151, normalized size = 1.29

$$\frac{2b^{3/2}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(ad-bc)} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*c*x^3) + (2*b*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*
a^2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*c^(3/2)) + (2*b^(3
/2)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*
a^2*(-(b*c) + a*d))
```

**IntegrateAlgebraic [A]** time = 0.23, size = 127, normalized size = 1.09

$$-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2\sqrt{ad-bc}} + \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*c*x^3) - (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a
*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^2*Sqrt[-(b*c) + a*d]) + ((2*b*c + a
*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c^(3/2))
```

**fricas [A]** time = 0.49, size = 565, normalized size = 4.83

$$\left[\frac{2bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+(2bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-2\sqrt{dx^3+c}\,ac}{6a^2c^2x^3}, -\frac{4bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-(2bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ac}{6a^2c^2x^3}, \frac{bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-(2bc+ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{dx^3+c}\,ac}{3a^2c^2x^3}, -\frac{2bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+(2bc+ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}\,ac}{3a^2c^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (2*b*c + a*d)*sqr
t(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3
+ c)*a*c)/(a^2*c^2*x^3), -1/6*(4*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*c)/(a^2*c^2*x^3), 1/3*(b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d
*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^3 + a)) - (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) -
 sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3), -1/3*(2*b*c^2*x^3*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^
3 + c)*a*c)/(a^2*c^2*x^3)]
```

**giac [A]** time = 0.19, size = 104, normalized size = 0.89

$$\frac{2\,b^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\,\sqrt{-b^{2}c+abd}\,a^{2}}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{3\,a^{2}\sqrt{-c}\,c}-\frac{\sqrt{dx^{3}+c}}{3\,acx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/3*(2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/3*sqrt(d*x^3 + c)/(a*c*x^3)
```

**maple [C]** time = 0.25, size = 498, normalized size = 4.26

$$\frac{2b\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a^{2}\sqrt{c}}+\frac{\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}}{a}-\frac{i b^{2}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)^{2}d^{2}+i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)d-\left(-c\,d^{2}\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^{3}b+a\right)d-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{2}{3}}-\left(-c\,d^{2}\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)^{2}d+i\sqrt{3}\,c d-3cd-i\left(-c\,d^{2}\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)-3\left(-c\,d^{2}\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^{3}b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\right)}{3a^{2}d^{2}\left(ad-bc\right)\sqrt{d\,x^{3}+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3
/2))-1/3*I/a^2*b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2
*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2
)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c
)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+2/3/a^2*b*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)\sqrt{dx^{3}+c}\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 8.42, size = 142, normalized size = 1.21

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)(a\,d+2\,b\,c)}{6\,a^2\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{3\,a\,c\,x^3}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)1\mathrm{i}}{3\,a^2\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
a*d + 2*b*c))/(6*a^2*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a*c*x^3) + (b^(3/2)*lo
g((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/
(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

## 3.268 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=107

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}\,(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

**Rubi [A]** time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 87, 63, 208}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}\,(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(3*d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(3*b*d^2)
 - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b
*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{c^2}{d(-bc+ad)(c+dx)^{3/2}}+\frac{1}{bd\sqrt{c+dx}}+\frac{a^2}{b(bc-ad)(a+bx)\sqrt{c+dx}}\right)\right.\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b(bc-ad)}\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd(bc-ad)}\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.08, size = 100, normalized size = 0.93

$$\frac{2\left(-a^2d^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+a^2d^2+abd\left(c+dx^3\right)+b^2(-c)\left(2c+dx^3\right)\right)}{3b^2d^2\sqrt{c+dx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(a^2*d^2 + a*b*d*(c + d*x^3) - b^2*c*(2*c + d*x^3) - a^2*d^2*Hypergeomet
ric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]))/(3*b^2*d^2*(-(b*c) + a*
d)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.21, size = 124, normalized size = 1.16

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}(ad-bc)^{3/2}}-\frac{2\left(-acd-ad^2x^3+2bc^2+bcdx^3\right)}{3bd^2\sqrt{c+dx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(2*b*c^2 - a*c*d + b*c*d*x^3 - a*d^2*x^3))/(3*b*d^2*(-(b*c) + a*d)*Sqrt
[c + d*x^3]) + (2*a^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(
b*c - a*d)])/(3*b^(3/2)*(-(b*c) + a*d)^(3/2))
```

**fricas [B]** time = 0.80, size = 440, normalized size = 4.11

$$\left[-\frac{\left(a^2d^3x^3+a^2cd^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^3-3\,ab^2c^2d+a^2bcd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^3\right)\sqrt{dx^3+c}}{3\left(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+\left(b^4c^2d^3-2\,ab^3cd^4+a^2b^2d^5\right)x^3\right)}, \frac{2\left(\left(a^2d^3x^3+a^2cd^2\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\left(2\,b^3c^3-3\,ab^2c^2d+a^2bcd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^3\right)\sqrt{dx^3+c}\right)}{3\left(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+\left(b^4c^2d^3-2\,ab^3cd^4+a^2b^2d^5\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c -
 a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3 -
 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3)
*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4*c^2
```

```
*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^2)*sq
rt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b
*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^
2 + a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b
^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]
```

**giac [A]** time = 0.18, size = 103, normalized size = 0.96

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^2c-abd\right)\sqrt{-b^2c+abd}}+\frac{2\,c^2}{3\left(bcd^2-ad^3\right)\sqrt{dx^3+c}}+\frac{2\sqrt{dx^3+c}}{3\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqr
t(-b^2*c + a*b*d)) + 2/3*c^2/((b*c*d^2 - a*d^3)*sqrt(d*x^3 + c)) + 2/3*sqrt
(d*x^3 + c)/(b*d^2)
```

**maple [C]** time = 0.34, size = 527, normalized size = 4.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b^2*(b*(2/3/((x^3+c/d)*d)^(1/2)*c/d^2+2/3*(d*x^3+c)^(1/2)/d^2)+2/3*a/d/(d
*x^3+c)^(1/2))+a^2/b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1
/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2)),_alpha=RootOf(_Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.46, size = 115, normalized size = 1.07

$$\frac{2\sqrt{d\,x^3+c}}{3\,b\,d^2}-\frac{2\,c^2}{3\,d^2\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{a^2\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d^2) - (2*c^2)/(3*d^2*(c + d*x^3)^(1/2)*(a*d - b
*c)) + (a^2*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*
2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

## 3.269 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=82

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}-\frac{2c}{3d\sqrt{c+dx^3}\,(bc-ad)}$$

**Rubi [A]** time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}-\frac{2c}{3d\sqrt{c+dx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(3*d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}$$

$$= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}$$

**Mathematica** [**A**] time = 0.12, size = 88, normalized size = 1.07

$$\frac{2\left(\frac{c(ad-bc)}{d\sqrt{c+dx^3}} + \frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}\right)}{3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((c*(-(b*c) + a*d))/(d*Sqrt[c + d*x^3]) + (a*Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*(b*c - a*d)^2)
```

**IntegrateAlgebraic** [**A**] time = 0.15, size = 92, normalized size = 1.12

$$\frac{2c}{3d\sqrt{c+dx^3}\,(ad-bc)} - \frac{2a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{b}\,(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c)/(3*d*(-(b*c) + a*d)*Sqrt[c + d*x^3]) - (2*a*ArcTan[(Sqrt[b]*Sqrt[-(b*
c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

**fricas** [**B**] time = 0.81, size = 326, normalized size = 3.98

$$\left[-\frac{\left(ad^2x^3+acd\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(b^2c^2-abcd\right)\sqrt{dx^3+c}}{3\left(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+\left(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4\right)x^3\right)}, -\frac{2\left(\left(ad^2x^3+acd\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\left(b^2c^2-abcd\right)\sqrt{dx^3+c}\right)}{3\left(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+\left(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a*d^2*x^3 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c*d
)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^
2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (b^
2*c^2 - a*b*c*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^
3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]
```

**giac** [A] time = 0.19, size = 78, normalized size = 0.95

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{c}{\sqrt{dx^3+c}\,(bc-ad)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*(b*c - a*d)) + c/(sqrt(d*x^3 + c)*(b*c - a*d)))/d
```

**maple** [C] time = 0.27, size = 487, normalized size = 5.94

$$-\frac{\left(-\frac{i b\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-c d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-c d^2\right)^{\frac{2}{3}}-\left(-c d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c d^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-c d^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-c d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^2(-ad+bc)(ad-bc)\sqrt{d x^3+c}}-\frac{2}{3(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}\right)a}{b}-\frac{2}{3\sqrt{d x^3+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/b/d/(d*x^3+c)^(1/2)-a/b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^
2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 5.99, size = 94, normalized size = 1.15

$$\frac{2\,c}{3\,d\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*c)/(3*d*(c + d*x^3)^(1/2)*(a*d - b*c)) + (a*log((2*b*c - a*d + b^(1/2)*(
c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(1/2
)*(a*d - b*c)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

## 3.270 $\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}} dx$

**Optimal**. Leaf size=77

$$\frac{2}{3\sqrt{c+dx^3}(bc-ad)} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

**Rubi** [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{2}{3\sqrt{c+dx^3}(bc-ad)} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}$$
$$= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}$$
$$= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

**Mathematica [C]** time = 0.03, size = 52, normalized size = 0.68

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)}{3\sqrt{c+dx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)])/(3*(-(b*c
) + a*d)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 109, normalized size = 1.42

$$\frac{\frac{2}{3}-\frac{2\sqrt{b}\sqrt{c+dx^3}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{ad-bc}}}{bc\sqrt{c+dx^3}-ad\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2/3 - (2*Sqrt[b]*Sqrt[c + d*x^3]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c
 + d*x^3])/(b*c - a*d)])/(3*Sqrt[-(b*c) + a*d]))/(b*c*Sqrt[c + d*x^3] - a*d
*Sqrt[c + d*x^3])
```

**fricas [A]** time = 0.70, size = 236, normalized size = 3.06

$$\left[-\frac{\left(dx^3+c\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}}{3\left(\left(bcd-ad^2\right)x^3+bc^2-acd\right)}, -\frac{2\left(\left(dx^3+c\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-\sqrt{dx^3+c}\right)}{3\left(\left(bcd-ad^2\right)x^3+bc^2-acd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c
))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), -2/3*((d*x^3 + c)*sqrt(-b/(b*c -
a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b
*c)) - sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]
```

**giac [A]** time = 0.19, size = 73, normalized size = 0.95

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{2}{3\,\sqrt{dx^3+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))
```

**maple [C]** time = 0.23, size = 463, normalized size = 6.01

$$-\frac{ib\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{3d^2\left(-ad+bc\right)\left(ad-bc\right)\sqrt{dx^3+c}}-\frac{2}{3\left(ad-bc\right)\sqrt{\left(x^3+\frac{c}{d}\right)d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*
d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3
*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.85, size = 89, normalized size = 1.16

$$-\frac{2}{3\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
 - b*d*x^3)/(a + b*x^3))*1i)/(3*(a*d - b*c)^(3/2)) - 2/(3*(c + d*x^3)^(1/2)*
(a*d - b*c))
```

**sympy [A]** time = 32.78, size = 66, normalized size = 0.86

$$-\frac{2}{3\sqrt{c+dx^3}\,(ad-bc)}-\frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(3*sqrt(c + d*x**3)*(a*d - b*c)) - 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b
*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c))
```

## 3.271 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=114

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}-\frac{2d}{3c\sqrt{c+dx^3}\,(bc-ad)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

**Rubi [A]** time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 85, 156, 63, 208}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}-\frac{2d}{3c\sqrt{c+dx^3}\,(bc-ad)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a*c^(3/2)) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*a*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3c(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,d\right.}{3a(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3acd} - \frac{\left(2b^2\right)\operatorname{Subst}\left(\int -\right.}{3ac}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

**Mathematica [C]** time = 0.05, size = 89, normalized size = 0.78

$$-\frac{2\left(bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+(ad-bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)\right)}{3ac\sqrt{c+dx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(b*c*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)] + (-(
b*c) + a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(3*a*c*(b*c -
a*d)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.29, size = 124, normalized size = 1.09

$$-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a(ad-bc)^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}\,(bc-ad)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[
-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a*(-(b*c) + a*d)^(3/2)) - (
2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*c^(3/2))
```

**fricas [B]** time = 0.58, size = 790, normalized size = 6.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
```

```
d)))/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d*x^
3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*
d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*(b*c^2*d*x^3 + b*c
^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c -
 a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log
((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*
b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*((b*c*d - a*
d^2)*x^3 + b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (b*
c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a*b*c^4 - a^2*c^
3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -2/3*(sqrt(d*x^3 + c)*a*c*d - (b*c^2*
d*x^3 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqr
t(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*b*c^4 - a^2*c^3*d + (a*b*c^
3*d - a^2*c^2*d^2)*x^3)]
```

**giac [A]** time = 0.17, size = 111, normalized size = 0.97

$$-\frac{2\,b^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\left(abc-a^{2}d\right)\sqrt{-b^{2}c+abd}}-\frac{2\,d}{3\,\sqrt{dx^{3}+c}\left(bc^{2}-acd\right)}+\frac{2\,\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*sq
rt(-b^2*c + a*b*d)) - 2/3*d/(sqrt(d*x^3 + c)*(b*c^2 - a*c*d)) + 2/3*arctan(
sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

**maple [C]** time = 0.29, size = 512, normalized size = 4.49

$$-\frac{\left(-\frac{i b\left(-c d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^{2}\right)^{\frac{1}{3}}+\left(-c d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^{2}\right)^{\frac{1}{3}}+\left(-c d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^{2}\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(\_Z^{3}b+a\right)^{2}d^{2}+i\left(-c d^{2}\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(\_Z^{3}b+a\right)d-\left(-c d^{2}\right)^{\frac{1}{3}}\operatorname{RootOf}\left(\_Z^{3}b+a\right)d-i\sqrt{3}\left(-c d^{2}\right)^{\frac{2}{3}}-\left(-c d^{2}\right)^{\frac{2}{3}}\right)\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c d^{2}\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-c d^{2}\right)^{\frac{1}{3}}\sqrt{3}\operatorname{RootOf}\left(\_Z^{3}b+a\right)^{2}d+i\sqrt{3}\,c d-3cd-i\left(-c d^{2}\right)^{\frac{2}{3}}\sqrt{3}\operatorname{RootOf}\left(\_Z^{3}b+a\right)-3\left(-c d^{2}\right)^{\frac{2}{3}}\operatorname{RootOf}\left(\_Z^{3}b+a\right)\right)b}{2\left(ad-bc\right)d},\sqrt{\frac{i\sqrt{3}\left(-c d^{2}\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c d^{2}\right)^{\frac{1}{3}}}{2d}\right)d}}\right)\right)}{3d^{2}\left(-ad+bc\right)\left(ad-bc\right)\sqrt{d x^{3}+c}}-\frac{2}{3\left(ad-bc\right)\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}\right)b}{a}+\frac{-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}+\frac{2}{3\sqrt{\left(x^{3}+\frac{c}{d}\right)d}\,c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a*b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+
b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^
2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d
^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*b+a))+1/a*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)\left(dx^{3}+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)
```

**mupad [B]** time = 8.44, size = 139, normalized size = 1.22

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,c^{3/2}}+\frac{2\,d}{3\,c\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,a\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(3/2)) + (2*d)/(3*c*(c + d*x^3)^(1/2)*(a*d - b*c)) + (b^(3/2)*log((2*b
*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b
*x^3))*1i)/(3*a*(a*d - b*c)^(3/2))
```

**sympy [A]** time = 24.63, size = 104, normalized size = 0.91

$$\frac{2d}{3c\sqrt{c+dx^3}\,(ad-bc)}+\frac{2b\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}+\frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2*d/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + 2*b*atan(sqrt(c + d*x**3)/sqrt((a*
d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + 2*atan(sqrt(c + d*x**3
)/sqrt(-c))/(3*a*c*sqrt(-c))
```

## 3.272 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=158

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}\,(bc-ad)}-\frac{1}{3acx^3\sqrt{c+dx^3}}$$

**Rubi [A]** time = 0.22, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}\,(bc-ad)}-\frac{1}{3acx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d*(b*c - 3*a*d))/(3*a*c^2*(b*c - a*d)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*Sqr
t[c + d*x^3]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c
^(5/2)) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3
*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{3ac}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad)-\frac{1}{4}bd(b}{x(a+bx)\sqrt{c+dx}}\right.}{3ac^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c\right.}}{3a^2d(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{2}{-}$$

**Mathematica [C]** time = 0.06, size = 117, normalized size = 0.74

$$\frac{2b^2c^2x^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+(ad-bc)\left(x^3(3ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+ac\right)}{3a^2c^2x^3\sqrt{c+dx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b^2*c^2*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]
 + (-(b*c) + a*d)*(a*c + (2*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/2, 1, 1/2
, 1 + (d*x^3)/c]))/(3*a^2*c^2*(b*c - a*d)*x^3*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.39, size = 165, normalized size = 1.04

$$\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2(ad-bc)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}+\frac{-acd-3ad^2x^3+bc^2+bcdx^3}{3ac^2x^3\sqrt{c+dx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b*c^2 - a*c*d + b*c*d*x^3 - 3*a*d^2*x^3)/(3*a*c^2*(-(b*c) + a*d)*x^3*Sqrt[
c + d*x^3]) + (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3]
)/(b*c - a*d)])/(3*a^2*(-(b*c) + a*d)^(3/2)) + ((2*b*c + 3*a*d)*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(3*a^2*c^(5/2))
```

**fricas [B]** time = 0.87, size = 1120, normalized size = 7.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2
*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)
) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3
*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3)
 + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))
/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/6*(4*(
b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^
2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(c)*log
((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^2*c^2*d +
(a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2
)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*(((2*b^2*c^2*d + a*b*c*d^2 - 3*a
^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d
))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c -
 a*d)))/(b*x^3 + a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3
)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*
d)*x^3), -1/3*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(
-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + ((2*b^
2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b*c^3 - a^2*c^2*d +
 (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^
2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3)]
```

**giac [A]** time = 0.17, size = 173, normalized size = 1.09

$$\frac{2\,b^3 \arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}} - \frac{\left(dx^3+c\right)bcd - 3\left(dx^3+c\right)ad^2 + 2\,acd^2}{3\left(abc^3-a^2c^2d\right)\left(\left(dx^3+c\right)^{\frac{3}{2}} - \sqrt{dx^3+c}\,c\right)} - \frac{(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/3*((d*x^3 + c)*b*c*d - 3*(d*x^3 + c)*a*d^2 + 2*a*c
*d^2)/((a*b*c^3 - a^2*c^2*d)*((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)) - 1/3
*(2*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

**maple [C]** time = 0.41, size = 575, normalized size = 3.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/a^2*b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1
/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2
*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((
x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(
-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c
*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1
/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d
-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-1/a^2*b*(2/3/((x^3+c/d)
*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)
```

**mupad** [B] time = 10.47, size = 597, normalized size = 3.78

$$\frac{\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)\left(\sqrt{dx^3+c}+\sqrt{c}\right)^3}{x^6}\right)(3ad+2bc)}{6a^2c^{5/2}}-\frac{\sqrt{dx^3+c}}{3ac^2x^3}-\frac{\frac{c\left(\frac{c\left(\frac{c\left(\frac{3a^2d^4+24abcd^3+15b^2c^2d^2}{8a^3c^5}+\frac{c\left(\frac{c\left(\frac{3b^2d^4}{8a^3c^5}+\frac{b^2d^4(5ad-3bc)}{8a^3c^4(bc^2-acd)}\right)-\frac{bd^4(ad+2bc)(5ad-3bc)}{4a^3c^5(bc^2-acd)}}{d}-\frac{3bd^3(ad+2bc)}{4a^3c^5}+\frac{d(5ad-3bc)(3a^2d^4+24abcd^3+15b^2c^2d^2)}{24a^3c^5(bc^2-acd)}\right)}{d}-\frac{d^2(5ad-3bc)(6a^2d^2+14abcd+3b^2c^2)}{12a^3c^4(bc^2-acd)}\right)}{d}-\frac{d(6a^2d^2+14abcd+3b^2c^2)}{4a^3c^4}+\frac{d^2(5ad-3bc)(13ad+18bc)}{24a^2c^3(bc^2-acd)}\right)}{d}+\frac{d(13ad+18bc)}{8a^2c^3}-\frac{d(3ad+2bc)(5ad-3bc)}{6a^2c^2(bc^2-acd)}\right)}{d}-\frac{3ad+2bc}{2a^2c^2}}{\sqrt{dx^3+c}}+\frac{b^{5/2}\ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}\,2i}{bx^3+a}\right)1i}{3a^2(ad-bc)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
3*a*d + 2*b*c))/(6*a^2*c^(5/2)) - (c + d*x^3)^(1/2)/(3*a*c^2*x^3) - ((c*((c
*((c*((3*a^2*d^4 + 15*b^2*c^2*d^2 + 24*a*b*c*d^3)/(8*a^3*c^5) + (c*((c*((3*
b^2*d^4)/(8*a^3*c^5) + (b^2*d^4*(5*a*d - 3*b*c))/(8*a^3*c^4*(b*c^2 - a*c*d)
) - (b*d^4*(a*d + 2*b*c)*(5*a*d - 3*b*c))/(4*a^3*c^5*(b*c^2 - a*c*d))))/d -
 (3*b*d^3*(a*d + 2*b*c))/(4*a^3*c^5) + (d*(5*a*d - 3*b*c)*(3*a^2*d^4 + 15*b
^2*c^2*d^2 + 24*a*b*c*d^3))/(24*a^3*c^5*(b*c^2 - a*c*d))))/d - (d^2*(5*a*d
- 3*b*c)*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(12*a^3*c^4*(b*c^2 - a*c*d))
))/d - (d*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(4*a^3*c^4) + (d^2*(5*a*d -
 3*b*c)*(13*a*d + 18*b*c))/(24*a^2*c^3*(b*c^2 - a*c*d))))/d + (d*(13*a*d +
18*b*c))/(8*a^2*c^3) - (d*(3*a*d + 2*b*c)*(5*a*d - 3*b*c))/(6*a^2*c^2*(b*c^
2 - a*c*d))))/d - (3*a*d + 2*b*c)/(2*a^2*c^2))/(c + d*x^3)^(1/2) + (b^(5/2)
*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^
3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(3/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right) \left(c + dx^3\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

## 3.273 $\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

**Optimal**. Leaf size=117

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{2c\sqrt{c+dx^3}\,(1146c+47dx^3)}{15d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)}$$

**Rubi [A]** time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 97, 153, 147, 63, 206}

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{2c\sqrt{c+dx^3}\,(1146c+47dx^3)}{15d^4}+\frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*x^6*Sqrt[c + d*x^3])/(15*d^2) + (x^9*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)
) + (2*c*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/(15*d^4) - (3968*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3c+\frac{7dx}{2}\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{x\left(-56c^2d-\frac{141}{2}cd^2x\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{15d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\,\left(1146c+47dx^3\right)}{15d^4} - \frac{\left(1984c^3\right)\text{Subst}\left(\int \frac{\ldots}{(8c-\ldots}\ldots\right)}{3d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\,\left(1146c+47dx^3\right)}{15d^4} - \frac{\left(3968c^3\right)\text{Subst}\left(\int \frac{1}{9c-\ldots}\ldots\right)}{3d^4}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\,\left(1146c+47dx^3\right)}{15d^4} - \frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+d\ldots}}{3\sqrt{c}}\ldots\right)}{9d^4}$$

**Mathematica [A]** time = 0.11, size = 101, normalized size = 0.86

$$\frac{19840c^{5/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+6\sqrt{c+dx^3}\,\left(-9168c^3+770c^2dx^3+19cd^2x^6+d^3x^9\right)}{45d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (6*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9) + 1
9840*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4*(-
8*c + d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 95, normalized size = 0.81

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{2\sqrt{c+dx^3}\,\left(9168c^3-770c^2dx^3-19cd^2x^6-d^3x^9\right)}{15d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(9168*c^3 - 770*c^2*d*x^3 - 19*c*d^2*x^6 - d^3*x^9))/(1
5*d^4*(-8*c + d*x^3)) - (3968*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(9*d^4)
```

**fricas [A]** time = 0.64, size = 219, normalized size = 1.87

$$\left[\frac{2\left(4960\left(c^2dx^3-8c^3\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(d^3x^9+19cd^2x^6+770c^2dx^3-9168c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^5x^3-8cd^4\right)},\frac{2\left(9920\left(c^2dx^3-8c^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\left(d^3x^9+19cd^2x^6+770c^2dx^3-9168c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^5x^3-8cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 - 91
68*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*c^3
)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^3*x^9 + 19*c*d^2*x
^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

**giac [A]** time = 0.16, size = 110, normalized size = 0.94

$$\frac{3968\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^4}-\frac{512\sqrt{dx^3+c}\,c^3}{3\left(dx^3-8c\right)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16}+25\left(dx^3+c\right)^{\frac{3}{2}}cd^{16}+960\sqrt{dx^3+c}\,c^2d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqrt
(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(d*
x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20
```

**maple [C]** time = 0.28, size = 952, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^3*(d*(2/15*(d*x^3+c)^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3
+c)^(1/2)*c^2/d^2)+32/9*c/d*(d*x^3+c)^(3/2))+512*c^3/d^3*(-1/3/d*(d*x^3+c)^
(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(
2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1
/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*
d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
```

```
d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+192*c^2/d^3*(2/3*(d*x^3+c)^(1/2)/d+1/
3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha
^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*
(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1
/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf
(_Z^3*d-8*c)))
```

**maxima** [A] time = 1.24, size = 107, normalized size = 0.91

$$\frac{2\left(4960\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+75\left(dx^3+c\right)^{\frac{3}{2}}c+2880\,\sqrt{dx^3+c}\,c^2-\frac{3840\,\sqrt{dx^3+c}\,c^3}{dx^3-8\,c}\right)}{45\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(4960*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 75*(d*x^3 + c)^(3/2)*c + 2880*sqrt(d*x^3 +
 c)*c^2 - 3840*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^4
```

**mupad** [B] time = 4.09, size = 127, normalized size = 1.09

$$\frac{1984\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}+\frac{1972\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}+\frac{18\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}+\frac{512\,c^3\,\sqrt{d\,x^3+c}}{3\,d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (1984*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(9*d^4) + (1972*c^2*(c + d*x^3)^(1/2))/(15*d^4) + (2*x^6*(c + d*x^3)^(1
/2))/(15*d^2) + (18*c*x^3*(c + d*x^3)^(1/2))/(5*d^3) + (512*c^3*(c + d*x^3)
^(1/2))/(3*d^4*(8*c - d*x^3))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.274 $\int \frac{x^8 \sqrt{c+dx^3}}{\left(8c-dx^3\right)^2} \, dx$

**Optimal**. Leaf size=102

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 89, 80, 50, 63, 206}

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (352*c*Sqrt[c + d*x^3])/(27*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (64*c*(c
 + d*x^3)^(3/2))/(27*d^3*(8*c - d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(8c-dx)^2}\,dx,x,x^3\right)$$

$$= \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(104c^2d+9cd^2x\right)}{8c-dx}\,dx,x,x^3\right)}{27cd^3}$$

$$= \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{(176c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{27d^2}$$

$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(176c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,\right.}{3d^2}$$

$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(352c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+}\right.}{3d^3}$$

$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

**Mathematica [A]** time = 0.09, size = 90, normalized size = 0.88

$$\frac{352c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+2\sqrt{c+dx^3}\left(-488c^2+41cdx^3+d^2x^6\right)}{9d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6) + 352*c^(3/2)*(8*c - d
*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3*(-8*c + d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 84, normalized size = 0.82

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{2\sqrt{c+dx^3}\left(488c^2-41cdx^3-d^2x^6\right)}{9d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(488*c^2 - 41*c*d*x^3 - d^2*x^6))/(9*d^3*(-8*c + d*x^3)
) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)
```

**fricas [A]** time = 0.68, size = 191, normalized size = 1.87

$$\left[\frac{2\left(88\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(d^2x^6+41\,cdx^3-488\,c^2\right)\sqrt{dx^3+c}\right)}{9\left(d^4x^3-8\,cd^3\right)},\ \frac{2\left(176\left(cdx^3-8c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(d^2x^6+41\,cdx^3-488\,c^2\right)\sqrt{dx^3+c}\right)}{9\left(d^4x^3-8\,cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(
d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x
^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^
4*x^3 - 8*c*d^3)]
```

**giac [A]** time = 0.18, size = 93, normalized size = 0.91

$$\frac{352\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^3}-\frac{64\sqrt{dx^3+c}\,c^2}{3\left(dx^3-8\,c\right)d^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6+48\sqrt{dx^3+c}\,cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(d
*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(d*
x^3 + c)*c*d^6)/d^9
```

**maple [C]** time = 0.21, size = 892, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+64*c^2/d^2*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54
*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alph
a^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)
*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/1
8*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootO
f(_Z^3*d-8*c)))+16*c/d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d
^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)
/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1
```

```
/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^
2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

**maxima [A]** time = 1.22, size = 91, normalized size = 0.89

$$\frac{2\left(88\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+48\,\sqrt{dx^3+c}\,c-\frac{96\,\sqrt{dx^3+c}\,c^2}{dx^3-8\,c}\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*(88*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + (d*x^3 + c)^(3/2) + 48*sqrt(d*x^3 + c)*c - 96*sqrt(d*x^3 + c)*c^2/(
d*x^3 - 8*c))/d^3
```

**mupad [B]** time = 4.01, size = 107, normalized size = 1.05

$$\frac{98\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{176\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{64\,c^2\,\sqrt{d\,x^3+c}}{3\,d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (98*c*(c + d*x^3)^(1/2))/(9*d^3) + (176*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) + (2*x^3*(c + d*x^3)^(1/2))/(
9*d^2) + (64*c^2*(c + d*x^3)^(1/2))/(3*d^3*(8*c - d*x^3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^8\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**8*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

## 3.275 $\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=82

$$\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}+\frac{26\sqrt{c+dx^3}}{27d^2}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

**Rubi** [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 50, 63, 206}

$$\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}+\frac{26\sqrt{c+dx^3}}{27d^2}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (26*Sqrt[c + d*x^3])/(27*d^2) + (8*(c + d*x^3)^(3/2))/(27*d^2*(8*c - d*x^3)
) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x\sqrt{c+dx}}{(8c-dx)^2}\,dx,x,x^3\right)\\
&= \frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{13\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{27d}\\
&= \frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{(13c)\,\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d}\\
&= \frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{(26c)\,\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^2}\\
&= \frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 79, normalized size = 0.96

$$\frac{6\sqrt{c+dx^3}\left(dx^3-12c\right)+26\sqrt{c}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (6*(-12*c + d*x^3)*Sqrt[c + d*x^3] + 26*Sqrt[c]*(8*c - d*x^3)*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.07, size = 73, normalized size = 0.89

$$-\frac{2\sqrt{c+dx^3}\left(12c-dx^3\right)}{3d^2\left(dx^3-8c\right)}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (-2*(12*c - d*x^3)*Sqrt[c + d*x^3])/(3*d^2*(-8*c + d*x^3)) - (26*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)
```

**fricas [A]** time = 0.47, size = 165, normalized size = 2.01

$$\left[\frac{13\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+6\sqrt{dx^3+c}\left(dx^3-12c\right)}{9\left(d^3x^3-8cd^2\right)},\ \frac{2\left(13\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+c}\left(dx^3-12c\right)\right)}{9\left(d^3x^3-8cd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2),
2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*s
qrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]
```

**giac [A]** time = 0.16, size = 69, normalized size = 0.84

$$\frac{26\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^2}+\frac{2\sqrt{dx^3+c}}{3\,d^2}-\frac{8\sqrt{dx^3+c}\,c}{3\left(dx^3-8\,c\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 26/9*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) + 2/3*sqrt(d*x^3
 + c)/d^2 - 8/3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^2)
```

**maple [C]** time = 0.17, size = 874, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 8*c/d*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)
^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/d*(2/3*(d*x
^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d-8*c)))
```

**maxima [A]** time = 1.25, size = 79, normalized size = 0.96

$$\frac{13\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+6\sqrt{dx^3+c}-\frac{24\sqrt{dx^3+c}\,c}{dx^3-8\,c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(13*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 6*sqrt(d*x^3 + c) - 24*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^2
```

**mupad [B]** time = 3.99, size = 87, normalized size = 1.06

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^2}+\frac{13\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}+\frac{8\,c\,\sqrt{d\,x^3+c}}{3\,d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (13*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(
c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^2) + (8*c*(c + d*x^3)^(1/2))/(3*d^2*
(8*c - d*x^3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

## 3.276 $\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=64

$$\frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 47, 63, 206}

$$\frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(3*d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(9*Sqrt[c]*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)\\ &= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\ &= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 61, normalized size = 0.95

$$\frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] ((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/S
qrt[c])/(9*d)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 63, normalized size = 0.98

$$-\frac{\sqrt{c+dx^3}}{3d\left(dx^3-8c\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(d*(-8*c + d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])]/(9*Sqrt[c]*d)
```

**fricas** [A] time = 0.49, size = 149, normalized size = 2.33

$$\left[\frac{\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}\,c}{18\left(cd^2x^3-8\,c^2d\right)}, \frac{\left(dx^3-8\,c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-3\,\sqrt{dx^3+c}\,c}{9\left(cd^2x^3-8\,c^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3 -
 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)*c
)/(c*d^2*x^3 - 8*c^2*d)]
```

**giac [A]** time = 0.16, size = 53, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d} - \frac{\sqrt{dx^3+c}}{3\left(dx^3-8\,c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c)
/((d*x^3 - 8*c)*d)
```

**maple [C]** time = 0.18, size = 439, normalized size = 6.86

$$-\frac{\sqrt{dx^3+c}}{3\left(dx^3-8c\right)d}+\frac{i\sqrt{2}\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{i\left(-cd^2\right)^{\frac13}\sqrt{\frac{\frac{i}{2}\left(2x+\frac{-i\sqrt3\left(-cd^2\right)^{\frac13}+\left(-cd^2\right)^{\frac13}}{d}\right)d}{\left(-cd^2\right)^{\frac13}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac13}}{d}\right)d}{-3\left(-cd^2\right)^{\frac13}+i\sqrt3\left(-cd^2\right)^{\frac13}}}\sqrt{-\frac{\frac{i}{2}\left(2x+\frac{i\sqrt3\left(-cd^2\right)^{\frac13}+\left(-cd^2\right)^{\frac13}}{d}\right)d}{\left(-cd^2\right)^{\frac13}}}\left(2\_\alpha^2d^2+i\left(-cd^2\right)^{\frac13}\sqrt3\,\_\alpha d-\left(-cd^2\right)^{\frac13}\_\alpha d-i\sqrt3\left(-cd^2\right)^{\frac23}-\left(-cd^2\right)^{\frac23}\right)\mathrm{EllipticPi}\left(\frac{\sqrt3}{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac13}}{2d}-\frac{i\sqrt3\left(-cd^2\right)^{\frac13}}{2d}\right)\sqrt3\,d}{\left(-cd^2\right)^{\frac13}}},-\frac{2i\left(-cd^2\right)^{\frac13}\sqrt3\,\_\alpha^2d+i\sqrt3\,cd-3cd-i\left(-cd^2\right)^{\frac23}\sqrt3\,\_\alpha-3\left(-cd^2\right)^{\frac23}\_\alpha}{18cd},\sqrt{\frac{i\sqrt3\left(-cd^2\right)^{\frac13}}{d\left(-\frac{3\left(-cd^2\right)^{\frac13}}{2d}+\frac{i\sqrt3\left(-cd^2\right)^{\frac13}}{2d}\right)}}\right)}{\sqrt{dx^3+c}}}{54cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*
(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d
)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.39, size = 66, normalized size = 1.03

$$\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{6\sqrt{dx^3+c}}{dx^3-8c}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqrt
(c) - 6*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d
```

**mupad [B]** time = 3.93, size = 72, normalized size = 1.12

$$\frac{\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{18\,\sqrt{c}\,d} + \frac{\sqrt{d\,x^3+c}}{3\,d\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(18*c^(1/2)
*d) + (c + d*x^3)^(1/2)/(3*d*(8*c - d*x^3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

## 3.277 $\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=88

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

**Rubi [A]** time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)^2}\,dx,x,x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}-\frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{192c}+\frac{(5d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{192c}\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}+\frac{5\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{96c}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{96cd}\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 102, normalized size = 1.16

$$\frac{12\sqrt{c}\sqrt{c+dx^3}+5\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+3\left(dx^3-8c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (12*Sqrt[c]*Sqrt[c + d*x^3] + 5*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])] + 3*(-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(288*c^(3/2)*(
8*c - d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 88, normalized size = 1.00

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))
```

**fricas [A]** time = 0.53, size = 226, normalized size = 2.57

$$\left[\frac{5\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-24\sqrt{dx^3+c}\,c}{576\left(c^2dx^3-8c^3\right)},\frac{3\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-5\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-12\sqrt{dx^3+c}\,c}{288\left(c^2dx^3-8c^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)
*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), 1/288*(3*
(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 5*(d*x^3 - 8*c)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*c)/(c
^2*d*x^3 - 8*c^3)]
```

**giac [A]** time = 0.16, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c} - \frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}\,c} - \frac{\sqrt{dx^3+c}}{24\left(dx^3-8c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c)
```

**maple [C]** time = 0.18, size = 912, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/8/c*d*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^
2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2
)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))-1/64/c^2*d*
(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I
*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^
(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(2/3*(d*x^3+c)^(1/2)-2/3*a
rctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)
```

**mupad [B]** time = 3.98, size = 76, normalized size = 0.86

$$\frac{5\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{288\,\sqrt{c^3}}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}}+\frac{\sqrt{d\,x^3+c}}{8\,c\,\left(24\,c-3\,d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (5*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(288*(c^3)^(1/2)) - atanh(
(c*(c + d*x^3)^(1/2))/(c^3)^(1/2))/(96*(c^3)^(1/2)) + (c + d*x^3)^(1/2)/(8*
c*(24*c - 3*d*x^3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)
```

## 3.278 $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=124

$$\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}+\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}$$

**Rubi [A]** time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}+\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (d*Sqrt[c + d*x^3])/(96*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c*x^3*(8*c
 - d*x^3)) + (7*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(128*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-54c^2d^2-9cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^3d}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{256c^2} + \frac{\left(7d^2\right)\text{Subst}\left(\int \cdots\right.}{7\cdots}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{128c^2} + \frac{(7d)\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}$$

**Mathematica [A]** time = 0.15, size = 97, normalized size = 0.78

$$\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) + \frac{12\sqrt{c}\sqrt{c+dx^3}\left(4c-dx^3\right)}{dx^6-8cx^3}}{1152c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(115
2*c^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 102, normalized size = 0.82

$$\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}+\frac{\sqrt{c+dx^3}\,(dx^3-4c)}{96c^2x^3\,(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((-4*c + d*x^3)*Sqrt[c + d*x^3])/(96*c^2*x^3*(8*c - d*x^3)) + (7*d*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]])/(128*c^(5/2))
```

**fricas [A]** time = 0.52, size = 278, normalized size = 2.24

$$\left[\frac{7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3-2\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\left(cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{2304\left(c^3dx^6-8\,c^4x^3\right)},\ \frac{9\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-7\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-12\left(cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{1152\left(c^3dx^6-8\,c^4x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 -
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^3*d*x^6 - 8*c^4*x^3), 1/1152*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan
(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*
sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^3*d*
x^6 - 8*c^4*x^3)]
```

**giac [A]** time = 0.17, size = 113, normalized size = 0.91

$$\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{128\sqrt{-c}\,c^2}-\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{1152\sqrt{-c}\,c^2}-\frac{(dx^3+c)^{\frac{3}{2}}d-5\sqrt{dx^3+c}\,cd}{96\left((dx^3+c)^2-10\,(dx^3+c)c+9\,c^2\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/128*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 7/1152*d*arctan(1
/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/96*((d*x^3 + c)^(3/2)*d - 5
*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2)
```

**maple [C]** time = 0.21, size = 957, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/64/c^2*d^2*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((
-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
```

```
c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c
^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
)-1/256/c^3*d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)
*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*
d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*
_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(2/3*(d
*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)
```

**mupad [B]** time = 4.21, size = 117, normalized size = 0.94

$$\frac{\frac{5\,d\,\sqrt{d\,x^3+c}}{32\,c}-\frac{d\,{\left(d\,x^3+c\right)}^{3/2}}{32\,c^2}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}+\frac{d\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,7{}\mathrm{i}}{9}\right)\,1{}\mathrm{i}}{128\,\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((5*d*(c + d*x^3)^(1/2))/(32*c) - (d*(c + d*x^3)^(3/2))/(32*c^2))/(3*(c + d
*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c^2*(c + d*x^3)^(1/2))/(c
^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*7i)/9)*1i)/
(128*(c^5)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.279 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=164

$$\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}+\frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}$$

**Rubi [A]** time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}+\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d^2*Sqrt[c + d*x^3])/(1536*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c*x^
6*(8*c - d*x^3)) - (7*d*Sqrt[c + d*x^3])/(384*c^2*x^3*(8*c - d*x^3)) + (23*
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(2048*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)^2}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{7cd+\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-6c^2d^2-\frac{21}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{384c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{54c^3d^3+45c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,d\right.}{27648c^5d}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{4096c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\right.}{2048c^3}$$

$$= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} -$$

**Mathematica [A]** time = 0.21, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(32c^2+28cdx^3-5d^2x^6\right)}{dx^9-8cx^6}+23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18432c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 118, normalized size = 0.72

$$\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}+\frac{\sqrt{c+dx^3}\left(-32c^2-28cdx^3+5d^2x^6\right)}{1536c^3x^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-32*c^2 - 28*c*d*x^3 + 5*d^2*x^6))/(1536*c^3*x^6*(8*c - d
*x^3)) + (23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d
^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2048*c^(7/2))
```

**fricas [A]** time = 0.47, size = 310, normalized size = 1.89

$$\left[\frac{23\left(d^3x^9-8cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+9\left(d^3x^9-8cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-24\left(5cd^2x^6-28c^2dx^3-32c^3\right)\sqrt{dx^3+c}}{36864\left(c^4dx^9-8c^5x^6\right)}, \frac{9\left(d^3x^9-8cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-23\left(d^3x^9-8cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-12\left(5cd^2x^6-28c^2dx^3-32c^3\right)\sqrt{dx^3+c}}{18432\left(c^4dx^9-8c^5x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d*x^
3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), 1/18432*(9*(d^3*x^9 -
 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 23*(d^3*x^9 - 8
*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(5*c*d^2*x
^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)]
```

**giac [A]** time = 0.18, size = 105, normalized size = 0.64

$$\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}\,c^3}-\frac{23\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{18432\sqrt{-c}\,c^3}-\frac{\sqrt{dx^3+c}\,d^2}{1536\left(dx^3-8\,c\right)c^3}-\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{384\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23/18432*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/1536*sqrt(d*x^3 + c)
*d^2/((d*x^3 - 8*c)*c^3) - 1/384*(d*x^3 + c)^(3/2)/(c^3*x^6)
```

**maple [C]** time = 0.19, size = 1020, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/512/c^3*d^3*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
```

```
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(
-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256
/c^3*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
1/2))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*(d*x^3+c)^(1/2)/c*d/x^3+1/12*
d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-3/4096/c^4*d^3*(2/3*(d*x^3+c)
^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c)))+3/4096/c^4*d^2*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((
d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2 x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

**mupad [B]** time = 4.45, size = 154, normalized size = 0.94

$$\frac{\frac{d^2\sqrt{d\,x^3+c}}{512\,c}-\frac{19\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{256\,c^2}+\frac{5\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{512\,c^3}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,23{}\mathrm{i}}{9}\right)\,1{}\mathrm{i}}{2048\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)^2),x)
[Out] ((d^2*(c + d*x^3)^(1/2))/(512*c) - (19*d^2*(c + d*x^3)^(3/2))/(256*c^2) + (
5*d^2*(c + d*x^3)^(5/2))/(512*c^3))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3
) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(
1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*23i)/9)*1i)/(204
8*(c^7)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

## 3.280 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=134

$$-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}+\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

**Rubi [A]** time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 97, 153, 147, 50, 63, 206}

$$\frac{1664c^3\sqrt{c+dx^3}}{d^4}-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (1664*c^3*Sqrt[c + d*x^3])/d^4 + (3*x^6*(c + d*x^3)^(3/2))/(7*d^2) + (x^9*(
c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (2*c*(c + d*x^3)^(3/2)*(694*c + 51*
d*x^3))/(21*d^4) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
```

```
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{(8c-dx)^2}\,dx,x,x^3\right)$$

$$= \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}\left(3c+\frac{9dx}{2}\right)}{8c-dx}\,dx,x,x^3\right)}{3d}$$

$$= \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}\left(-72c^2d-\frac{255}{2}cd^2x\right)}{8c-dx}\,dx,x,x^3\right)}{21d^3}$$

$$= \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4} - \frac{\left(832c^3\right)\text{Subst}\,(}{}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right.}{21d^4}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right.}{21d^4}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right.}{21d^4}$$

**Mathematica [A]** time = 0.08, size = 111, normalized size = 0.83

$$\frac{2\left(52416c^{7/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+\sqrt{c+dx^3}\left(-145328c^4+12206c^3dx^3+301c^2d^2x^6+16cd^3x^9+d^4x^{12}\right)\right)}{21d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*(Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c
*d^3*x^9 + d^4*x^12) + 52416*c^(7/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c]])]))/(21*d^4*(-8*c + d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.10, size = 104, normalized size = 0.78

$$-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{2\sqrt{c+dx^3}\left(145328c^4-12206c^3dx^3-301c^2d^2x^6-16cd^3x^9-d^4x^{12}\right)}{21d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(145328*c^4 - 12206*c^3*d*x^3 - 301*c^2*d^2*x^6 - 16*c*
d^3*x^9 - d^4*x^12))/(21*d^4*(-8*c + d*x^3)) - (4992*c^(7/2)*ArcTanh[Sqrt[c
 + d*x^3]/(3*Sqrt[c])])/d^4
```

**fricas [A]** time = 0.44, size = 239, normalized size = 1.78

$$\left[\frac{2\left(26208\left(c^3dx^3-8c^4\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(d^4x^{12}+16cd^3x^9+301c^2d^2x^6+12206c^3dx^3-145328c^4\right)\sqrt{dx^3+c}\right)}{21\left(d^5x^3-8cd^4\right)},\frac{2\left(52416\left(c^3dx^3-8c^4\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(d^4x^{12}+16cd^3x^9+301c^2d^2x^6+12206c^3dx^3-145328c^4\right)\sqrt{dx^3+c}\right)}{21\left(d^5x^3-8cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 +
12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21*(5
2416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) +
(d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)*
sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

**giac [A]** time = 0.17, size = 127, normalized size = 0.95

$$\frac{4992\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{1536\sqrt{dx^3+c}\,c^4}{\left(dx^3-8\,c\right)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{7}{2}}d^{24}+21\left(dx^3+c\right)^{\frac{5}{2}}cd^{24}+448\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{24}+15680\sqrt{dx^3+c}\,c^3d^{24}\right)}{21\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d*
x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x^3
 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 + c)
*c^3*d^24)/d^28
```

**maple [C]** time = 0.28, size = 998, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^3*(d*(2/21*(d*x^3+c)^(1/2)*d*x^9+16/105*(d*x^3+c)^(1/2)*c*x^6+2/105*(d*
x^3+c)^(1/2)*c^2/d*x^3-4/105*(d*x^3+c)^(1/2)*c^3/d^2)+32/15*c/d*(d*x^3+c)^(
5/2))+512*c^3/d^3*(-3*c/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+c)^(1/2)/d
+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_al
pha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/
2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1
/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c)))+192*c^2/d^3*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)
*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))
```

**maxima** [A] time = 1.18, size = 119, normalized size = 0.89

$$\frac{2\left(26208\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{7}{2}}+21\left(dx^3+c\right)^{\frac{5}{2}}c+448\left(dx^3+c\right)^{\frac{3}{2}}c^2+15680\,\sqrt{dx^3+c}\,c^3-\frac{16128\,\sqrt{dx^3+c}\,c^4}{dx^3-8\,c}\right)}{21\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/21*(26208*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + (d*x^3 + c)^(7/2) + 21*(d*x^3 + c)^(5/2)*c + 448*(d*x^3 + c)^(3
/2)*c^2 + 15680*sqrt(d*x^3 + c)*c^3 - 16128*sqrt(d*x^3 + c)*c^4/(d*x^3 - 8*
c))/d^4
```

**mupad** [B] time = 4.10, size = 147, normalized size = 1.10

$$\frac{2496\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}+\frac{32300\,c^3\,\sqrt{d\,x^3+c}}{21\,d^4}+\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}+\frac{16\,c\,x^6\,\sqrt{d\,x^3+c}}{7\,d^2}+\frac{986\,c^2\,x^3\,\sqrt{d\,x^3+c}}{21\,d^3}+\frac{1536\,c^4\,\sqrt{d\,x^3+c}}{d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2496*c^(7/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 + (32300*c^3*(c + d*x^3)^(1/2))/(21*d^4) + (2*x^9*(c + d*x^3)^(1/2)
)/(21*d) + (16*c*x^6*(c + d*x^3)^(1/2))/(7*d^2) + (986*c^2*x^3*(c + d*x^3)^
(1/2))/(21*d^3) + (1536*c^4*(c + d*x^3)^(1/2))/(d^4*(8*c - d*x^3))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.281 $\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

**Optimal**. Leaf size=119

$$-\frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}+\frac{160c^2\sqrt{c+dx^3}}{d^3}+\frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)}+\frac{160c\left(c+dx^3\right)^{3/2}}{27d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 89, 80, 50, 63, 206}

$$\frac{160c^2\sqrt{c+dx^3}}{d^3}-\frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}+\frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)}+\frac{160c\left(c+dx^3\right)^{3/2}}{27d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (160*c^2*Sqrt[c + d*x^3])/d^3 + (160*c*(c + d*x^3)^(3/2))/(27*d^3) + (2*(c
+ d*x^3)^(5/2))/(15*d^3) + (64*c*(c + d*x^3)^(5/2))/(27*d^3*(8*c - d*x^3))
- (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(8c-dx)^2}\,dx,x,x^3\right)$$

$$= \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(168c^2d+9cd^2x\right)}{8c-dx}\,dx,x,x^3\right)}{27cd^3}$$

$$= \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{(80c)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx,x,x^3\right)}{9d^2}$$

$$= \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(80c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{d^2}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(720c^3\right)\text{Subst}}{}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(1440c^3\right)\text{Subs}}{}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{480c^{5/2}\tanh^{-1}}{d^3}$$

**Mathematica [A]** time = 0.13, size = 102, normalized size = 0.86

$$\frac{21600c^{5/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+2\sqrt{c+dx^3}\,\left(-29944c^3+2515c^2dx^3+62cd^2x^6+3d^3x^9\right)}{45d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9)
 + 21600*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^
3*(-8*c + d*x^3))
```

**IntegrateAlgebraic** [A] time = 0.09, size = 93, normalized size = 0.78

$$-\frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{2\sqrt{c+dx^3}\left(29944c^3-2515c^2dx^3-62cd^2x^6-3d^3x^9\right)}{45d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(29944*c^3 - 2515*c^2*d*x^3 - 62*c*d^2*x^6 - 3*d^3*x^9)
)/(45*d^3*(-8*c + d*x^3)) - (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^3
```

**fricas** [A] time = 0.44, size = 219, normalized size = 1.84

$$\left[\frac{2\left(5400\left(c^2dx^3-8c^3\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(3d^3x^9+62cd^2x^6+2515c^2dx^3-29944c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^4x^3-8cd^3\right)},\frac{2\left(10800\left(c^2dx^3-8c^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(3d^3x^9+62cd^2x^6+2515c^2dx^3-29944c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^4x^3-8cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 - 2
9944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 - 8*
c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^3*x^9 + 62*c*d^
2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

**giac** [A] time = 0.20, size = 111, normalized size = 0.93

$$\frac{480\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{192\sqrt{dx^3+c}\,c^3}{\left(dx^3-8\,c\right)d^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}+80\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+3120\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x^
3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x^3
 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15
```

**maple** [C] time = 0.19, size = 920, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/d^3+64*c^2/d^2*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3
*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+16*c/d^2*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*
```

```
(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

**maxima [A]** time = 1.39, size = 107, normalized size = 0.90

$$\frac{2\left(5400\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+80\left(dx^3+c\right)^{\frac{3}{2}}c+3120\,\sqrt{dx^3+c}\,c^2-\frac{4320\,\sqrt{dx^3+c}\,c^3}{dx^3-8\,c}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(5400*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 80*(d*x^3 + c)^(3/2)*c + 3120*sqrt(d*x^3 +
 c)*c^2 - 4320*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^3
```

**mupad [B]** time = 4.05, size = 127, normalized size = 1.07

$$\frac{240\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}+\frac{6406\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}+\frac{172\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}+\frac{192\,c^3\,\sqrt{d\,x^3+c}}{d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (240*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 + (6406*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/
(15*d) + (172*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (192*c^3*(c + d*x^3)^(1/2
))/(d^3*(8*c - d*x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.282 $\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

**Optimal**. Leaf size=97

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}+\frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)}+\frac{14\left(c+dx^3\right)^{3/2}}{27d^2}+\frac{14c\sqrt{c+dx^3}}{d^2}$$

**Rubi** [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 50, 63, 206}

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}+\frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)}+\frac{14\left(c+dx^3\right)^{3/2}}{27d^2}+\frac{14c\sqrt{c+dx^3}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (14*c*Sqrt[c + d*x^3])/d^2 + (14*(c + d*x^3)^(3/2))/(27*d^2) + (8*(c + d*x^
3)^(5/2))/(27*d^2*(8*c - d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{7\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d}$$

$$= \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{(7c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{\left(63c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x\right.}{d}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{\left(126c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+d}\right.}{d^2}$$

$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

**Mathematica [A]** time = 0.06, size = 90, normalized size = 0.93

$$\frac{378c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+2\sqrt{c+dx^3}\left(-524c^2+44cdx^3+d^2x^6\right)}{9d^2\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6) + 378*c^(3/2)*(8*c - d
*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 82, normalized size = 0.85

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{2\sqrt{c+dx^3}\left(524c^2-44cdx^3-d^2x^6\right)}{9d^2\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(524*c^2 - 44*c*d*x^3 - d^2*x^6))/(9*d^2*(-8*c + d*x^3)
) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

**fricas [A]** time = 0.45, size = 192, normalized size = 1.98

$$\left[\frac{189\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2\left(d^2x^6+44cdx^3-524c^2\right)\sqrt{dx^3+c}}{9\left(d^3x^3-8cd^2\right)}, \frac{2\left(189\left(cdx^3-8c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(d^2x^6+44cdx^3-524c^2\right)\sqrt{dx^3+c}\right)}{9\left(d^3x^3-8cd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(189*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c)
)/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c))/
(d^3*x^3 - 8*c*d^2)]
```

**giac [A]** time = 0.17, size = 93, normalized size = 0.96

$$\frac{42\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^2}-\frac{24\,\sqrt{dx^3+c}\,c^2}{\left(dx^3-8\,c\right)d^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4+51\,\sqrt{dx^3+c}\,cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 42*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 24*sqrt(d*x^3
+ c)*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*sqrt(d*x^3 +
 c)*c*d^4)/d^6
```

**maple [C]** time = 0.17, size = 902, normalized size = 9.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 8*c/d*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2
^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^
(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8
*c)))+1/d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

**maxima [A]** time = 1.33, size = 93, normalized size = 0.96

$$\frac{189\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\left(dx^3+c\right)^{\frac{3}{2}}+102\,\sqrt{dx^3+c}\,c-\frac{216\,\sqrt{dx^3+c}\,c^2}{dx^3-8\,c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(189*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 102*sqrt(d*x^3 + c)*c - 216*sqrt(d*x^3 + c)*
c^2/(d*x^3 - 8*c))/d^2
```

**mupad [B]** time = 4.04, size = 107, normalized size = 1.10

$$\frac{104\,c\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{21\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}+\frac{24\,c^2\,\sqrt{d\,x^3+c}}{d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (104*c*(c + d*x^3)^(1/2))/(9*d^2) + (21*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^2 + (2*x^3*(c + d*x^3)^(1/2))/(9*d)
 + (24*c^2*(c + d*x^3)^(1/2))/(d^2*(8*c - d*x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.283 $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=77

$$\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{\sqrt{c+dx^3}}{d}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

**Rubi [A]** time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {444, 47, 50, 63, 206}

$$\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{\sqrt{c+dx^3}}{d}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/d + (c + d*x^3)^(3/2)/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{d}+\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{1}{2}(9c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{d}+\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{(9c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\
&= \frac{\sqrt{c+dx^3}}{d}+\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 43, normalized size = 0.56

$$\frac{2\left(c+dx^3\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{dx^3+c}{9c}\right)}{1215c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (c + d*x^3)/(9*c)])/(12
15*c^2*d)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 71, normalized size = 0.92

$$-\frac{\sqrt{c+dx^3}\left(25c-2dx^3\right)}{3d\left(dx^3-8c\right)}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] -1/3*((25*c - 2*d*x^3)*Sqrt[c + d*x^3])/(d*(-8*c + d*x^3)) - (3*Sqrt[c]*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

**fricas [A]** time = 0.44, size = 162, normalized size = 2.10

$$\left[\frac{9\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2\left(2dx^3-25c\right)\sqrt{dx^3+c}}{6\left(d^2x^3-8cd\right)},\ \frac{9\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\left(2dx^3-25c\right)\sqrt{dx^3+c}}{3\left(d^2x^3-8cd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d), 1
/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*d*
x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]
```

**giac** [A] time = 0.17, size = 69, normalized size = 0.90

$$\frac{3\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d}+\frac{2\,\sqrt{dx^3+c}}{3\,d}-\frac{3\,\sqrt{dx^3+c}\,c}{\left(dx^3-8\,c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 2/3*sqrt(d*x^3 + c)
/d - 3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d)
```

**maple** [C] time = 0.18, size = 451, normalized size = 5.86

$$-\frac{3\sqrt{d\,x^3+c}\;c}{\left(d\,x^3-8c\right)d}+\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-c\,d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-c\,d^2\right)^{\frac{2}{3}}-\left(-c\,d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-c\,d^2\right)^{\frac{2}{3}}\sqrt{3}\;\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-c\,d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{2d^3\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*
sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima** [A] time = 1.31, size = 79, normalized size = 1.03

$$\frac{9\,\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+4\,\sqrt{dx^3+c}-\frac{18\,\sqrt{dx^3+c}\,c}{dx^3-8\,c}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(9*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(
c))) + 4*sqrt(d*x^3 + c) - 18*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d
```

**mupad** [B] time = 3.99, size = 87, normalized size = 1.13

$$\frac{2\,\sqrt{d\,x^3+c}}{3\,d}+\frac{3\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{2\,d}+\frac{3\,c\,\sqrt{d\,x^3+c}}{d\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c +
 d*x^3)^(1/2))/(8*c - d*x^3)))/(2*d) + (3*c*(c + d*x^3)^(1/2))/(d*(8*c - d*
x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.284 $\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=85

$$\frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

**Rubi** [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 98, 156, 63, 208, 206}

$$\frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{-c^2d+\frac{7}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24cd}\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{1}{192}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) - \frac{1}{64}(9d)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,d\right.\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{9}{32}\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96d}\\
&= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 100, normalized size = 1.18

$$\frac{36\sqrt{c}\sqrt{c+dx^3} + \left(9dx^3-72c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + \left(dx^3-8c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (36*Sqrt[c]*Sqrt[c + d*x^3] + (-72*c + 9*d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])] + (-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c]*(8
*c - d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.08, size = 85, normalized size = 1.00

$$\frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

**fricas [A]** time = 0.46, size = 220, normalized size = 2.59

$$\left[\frac{9\left(dx^{3}-8c\right)\sqrt{c}\log\left(\frac{dx^{3}-6\sqrt{dx^{3}+c}\sqrt{c}+10c}{dx^{3}-8c}\right)+\left(dx^{3}-8c\right)\sqrt{c}\log\left(\frac{dx^{3}-2\sqrt{dx^{3}+c}\sqrt{c}+2c}{x^{3}}\right)-72\sqrt{dx^{3}+c}\,c}{192\left(cdx^{3}-8c^{2}\right)},\ \frac{\left(dx^{3}-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^{3}+c}\sqrt{-c}}{c}\right)+9\left(dx^{3}-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^{3}+c}\sqrt{-c}}{3c}\right)-36\sqrt{dx^{3}+c}\,c}{96\left(cdx^{3}-8c^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/192*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + (d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*s
qrt(c) + 2*c)/x^3) - 72*sqrt(d*x^3 + c)*c)/(c*d*x^3 - 8*c^2), 1/96*((d*x^3
- 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 9*(d*x^3 - 8*c)*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 36*sqrt(d*x^3 + c)*c)/(c*d*x^3
- 8*c^2)]
```

**giac [A]** time = 0.17, size = 70, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}}+\frac{3\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}}-\frac{3\sqrt{dx^{3}+c}}{8\left(dx^{3}-8c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) + 3/32*arctan(1/3*sqrt(d*x^3
 + c)/sqrt(-c))/sqrt(-c) - 3/8*sqrt(d*x^3 + c)/(d*x^3 - 8*c)
```

**maple [C]** time = 0.17, size = 956, normalized size = 11.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/8/c*d*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3
*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))-1/64/c^2*d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/
d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d
^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*
d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^
3*d-8*c)))+1/64/c^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^
(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)
```

**mupad** [B] time = 4.72, size = 101, normalized size = 1.19

$$\frac{3\sqrt{d\,x^3+c}}{8\left(8\,c-d\,x^3\right)}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^9}{x^6\left(8\,c-d\,x^3\right)^9}\right)}{192\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (3*(c + d*x^3)^(1/2))/(8*(8*c - d*x^3)) + log((((c + d*x^3)^(1/2) - c^(1/2)
)^3*((c + d*x^3)^(1/2) + c^(1/2))*(10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/
2))^9)/(x^6*(8*c - d*x^3)^9))/(192*c^(1/2))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.285 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}\,dx$

**Optimal**. Leaf size=121

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}$$

**Rubi [A]** time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(96*c*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*x^3*(8*c -
 d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)^2}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-14c^2d-\frac{19}{2}cd^2x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c} \\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{126c^3d^2+45c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^3d} \\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{(7d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c} + \frac{\left(9d^2\right)\text{Subst}\left(\int \frac{}{(8c-} \right.}{25} \\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c} + \frac{(9d)\,\text{Subst}\left(\int \frac{}{9} \right.}{} \\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.16, size = 97, normalized size = 0.80

$$
\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) + \frac{4\sqrt{c}\sqrt{c+dx^3}\left(4c-5dx^3\right)}{dx^6-8cx^3}}{384c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*(4*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(38
4*c^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 103, normalized size = 0.85

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{\sqrt{c+dx^3}\left(5dx^3-4c\right)}{96cx^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(96*c*x^3*(8*c - d*x^3)) + (3*d*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]])/(384*c^(3/2))
```

**fricas [A]** time = 0.45, size = 280, normalized size = 2.31

$$\left[\frac{9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-8\left(5\,cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{768\left(c^2dx^6-8\,c^3x^3\right)},\frac{7\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-9\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-4\left(5\,cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{384\left(c^2dx^6-8\,c^3x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 - 2
*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^2*d*x^6 - 8*c^3*x^3), 1/384*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(
sqrt(d*x^3 + c)*sqrt(-c)/c) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) - 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d*
x^6 - 8*c^3*x^3)]
```

**giac [A]** time = 0.19, size = 114, normalized size = 0.94

$$\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c}-\frac{3\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{128\,\sqrt{-c}\,c}-\frac{5\left(dx^3+c\right)^{\frac{3}{2}}d-9\,\sqrt{dx^3+c}\,cd}{96\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 7/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3/128*d*arctan(1/3*
sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/96*(5*(d*x^3 + c)^(3/2)*d - 9*sq
rt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c)
```

**maple [C]** time = 0.19, size = 1014, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/64/c^2*d^2*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*
I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
```

```
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))+1/64/c^2*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d-c^(
1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))-1/256/c^3*d^2*(2/9*(d*x^3+c)^(1/2)
*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(
2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*(
(x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*
(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-
c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(2/9*(d*x^3+c)^(
1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2
)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)
```

**mupad [B]** time = 4.24, size = 110, normalized size = 0.91

$$\frac{\frac{9\,d\,\sqrt{d\,x^3+c}}{32}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{32\,c}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}+\frac{d\,\left(\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)\,9{}\mathrm{i}}{7}\right)\,7{}\mathrm{i}}{384\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((9*d*(c + d*x^3)^(1/2))/32 - (5*d*(c + d*x^3)^(3/2))/(32*c))/(3*(c + d*x^3
)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1
/2))*1i - (atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2)))*9i)/7)*7i)/(384*(c^
3)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.286 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx$

**Optimal**. Leaf size=161

$$\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}+\frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)}-\frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

**Rubi [A]** time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)}+\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}-\frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)}-\frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (7*d^2*Sqrt[c + d*x^3])/(512*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*x^6*(
8*c - d*x^3)) - (23*d*Sqrt[c + d*x^3])/(384*c*x^3*(8*c - d*x^3)) + (15*d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(2048*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)^2}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{-23c^2d-\frac{37}{2}cd^2x}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}\\
&= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{102c^3d^2+\frac{69}{2}c^2d^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}\\
&= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{-918c^4d^3-189c^3d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx, x\right.}{27648c^5d}\\
&= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\left(17d^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x,\right.}{4096c^2}\\
&= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{(17d)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{\right.}{2048c^2}\\
&= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2}{}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 112, normalized size = 0.70

$$\frac{\frac{4\sqrt{c}\sqrt{c+dx^3}\left(32c^2+92cdx^3-21d^2x^6\right)}{dx^9-8cx^6}+45d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-51d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.14, size = 118, normalized size = 0.73

$$\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}+\frac{\sqrt{c+dx^3}\left(-32c^2-92cdx^3+21d^2x^6\right)}{1536c^2x^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-32*c^2 - 92*c*d*x^3 + 21*d^2*x^6))/(1536*c^2*x^6*(8*c -
d*x^3)) + (15*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (1
7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2048*c^(5/2))
```

**fricas [A]** time = 0.47, size = 310, normalized size = 1.93

$$\left[\frac{45\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+51\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-8\left(21\,cd^2x^6-92\,c^2dx^3-32\,c^3\right)\sqrt{dx^3+c}}{12288\left(c^3dx^9-8\,c^4x^6\right)},\ \frac{51\left(d^3x^9-8\,cd^2x^6\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-45\left(d^3x^9-8\,cd^2x^6\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-4\left(21\,cd^2x^6-92\,c^2dx^3-32\,c^3\right)\sqrt{dx^3+c}}{6144\left(c^3dx^9-8\,c^4x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d
*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*d*x
^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), 1/6144*(51*(d^3*x^9
- 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 45*(d^3*x^9 -
8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(21*c*d^2*
x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6)]
```

**giac [A]** time = 0.18, size = 129, normalized size = 0.80

$$\frac{17\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^2}-\frac{15\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^2}-\frac{3\,\sqrt{dx^3+c}\,d^2}{512\left(dx^3-8\,c\right)c^2}-\frac{3\left(dx^3+c\right)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}\,cd^2}{384\,c^2d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 17/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 15/2048*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 3/512*sqrt(d*x^3 + c)*
d^2/((d*x^3 - 8*c)*c^2) - 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 + c
)*c*d^2)/(c^2*d^2*x^6)
```

**maple [C]** time = 0.19, size = 1075, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/512/c^3*d^3*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2
*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
```

```
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d
-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/64/c^2*(-1/6*c*(d*x^3+c)^(1/
2)/x^6-5/12*d*(d*x^3+c)^(1/2)/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/
c^(1/2))-3/4096/c^4*d^3*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3
*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alph
a^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)
*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/1
8*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootO
f(_Z^3*d-8*c)))+3/4096/c^4*d^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/
2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2 x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)
```

**mupad [B]** time = 4.62, size = 151, normalized size = 0.94

$$\frac{\frac{81\,d^2\,\sqrt{d\,x^3+c}}{512}-\frac{67\,d^2\,\left(d\,x^3+c\right)^{3/2}}{256\,c}+\frac{21\,d^2\,\left(d\,x^3+c\right)^{5/2}}{512\,c^2}}{33\,c\,\left(d\,x^3+c\right)^2-57\,c^2\,\left(d\,x^3+c\right)-3\,\left(d\,x^3+c\right)^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,15{}\mathrm{i}}{17}\right)\,17{}\mathrm{i}}{2048\,\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x)
```

```
[Out] ((81*d^2*(c + d*x^3)^(1/2))/512 - (67*d^2*(c + d*x^3)^(3/2))/(256*c) + (21*
d^2*(c + d*x^3)^(5/2))/(512*c^2))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3)
- 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/
2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*15i)/17)*17i)/(204
8*(c^5)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

## 3.287 $\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=95

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{2\sqrt{c+dx^3}\,(170c+7dx^3)}{27d^4}+\frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)}$$

**Rubi [A]** time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 98, 147, 63, 206}

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)}+\frac{2\sqrt{c+dx^3}\,(170c+7dx^3)}{27d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*x^6*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (2*Sqrt[c + d*x^3]*(170*c
+ 7*d*x^3))/(27*d^4) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(81*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x\left(16c^2+21cdx\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^2}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(1472c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,\right.}{27d^3}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(2944c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c}\right.}{27d^4}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

**Mathematica [A]** time = 0.05, size = 91, normalized size = 0.96

$$\frac{2944c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 6\sqrt{c+dx^3}\left(-1360c^2+114cdx^3+3d^2x^6\right)}{81d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (6*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6) + 2944*c^(3/2)*(8*
c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4*(-8*c + d*x^3))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 84, normalized size = 0.88

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} - \frac{2\sqrt{c+dx^3}\left(1360c^2-114cdx^3-3d^2x^6\right)}{27d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1360*c^2 - 114*c*d*x^3 - 3*d^2*x^6))/(27*d^4*(-8*c + d
*x^3)) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4)
```

**fricas [A]** time = 0.44, size = 195, normalized size = 2.05

$$\left[\frac{2\left(736\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(3d^2x^6+114cdx^3-1360c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3-8cd^4\right)}, \frac{2\left(1472\left(cdx^3-8c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\left(3d^2x^6+114cdx^3-1360c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3-8cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(736*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt(d*x^3
 + c))/(d^5*x^3 - 8*c*d^4), 2/81*(1472*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/
3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt
(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

**giac [A]** time = 0.19, size = 93, normalized size = 0.98

$$\frac{2944\, c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}\, d^4} - \frac{512\,\sqrt{dx^3+c}\, c^2}{27\left(dx^3-8\,c\right)d^4} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}} d^8 + 45\,\sqrt{dx^3+c}\, cd^8\right)}{9\, d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2944/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/27*sq
rt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^4) + 2/9*((d*x^3 + c)^(3/2)*d^8 + 45*sqr
t(d*x^3 + c)*c*d^8)/d^12
```

**maple [C]** time = 0.30, size = 916, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/d^3*((2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*c/d^2)*d+32/3*(d*x^3+
c)^(1/2)*c/d)+512*c^3/d^3*(-1/27/c/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/486*I/c^
2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))+64/9*I*c/d^6*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.34, size = 93, normalized size = 0.98

$$\frac{2\left(736\, c^{\frac{3}{2}} \log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right) + 9\left(dx^3+c\right)^{\frac{3}{2}} + 405\,\sqrt{dx^3+c}\, c - \frac{768\,\sqrt{dx^3+c}\, c^2}{dx^3-8\,c}\right)}{81\, d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(736*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 9*(d*x^3 + c)^(3/2) + 405*sqrt(d*x^3 + c)*c - 768*sqrt(d*x^3 + c)
*c^2/(d*x^3 - 8*c))/d^4
```

**mupad [B]** time = 4.06, size = 107, normalized size = 1.13

$$\frac{92\,c\,\sqrt{d\,x^3+c}}{9\,d^4}+\frac{1472\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{512\,c^2\,\sqrt{d\,x^3+c}}{27\,d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (92*c*(c + d*x^3)^(1/2))/(9*d^4) + (1472*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) + (2*x^3*(c + d*x^3)^(1/2))
/(9*d^3) + (512*c^2*(c + d*x^3)^(1/2))/(27*d^4*(8*c - d*x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.288 $\int \frac{x^8}{\left(8c-dx^3\right)^2 \sqrt{c+dx^3}} \, dx$

**Optimal**. Leaf size=83

$$\frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)}+\frac{2\sqrt{c+dx^3}}{3d^3}-\frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

**Rubi** [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 89, 80, 63, 206}

$$\frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)}+\frac{2\sqrt{c+dx^3}}{3d^3}-\frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) + (64*c*Sqrt[c + d*x^3])/(27*d^3*(8*c - d*x^3))
 - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{40c^2d+9cd^2x}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^3}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{(112c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^2}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{(224c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^3}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{224\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 82, normalized size = 0.99

$$-\frac{64c\sqrt{c+dx^3}}{27d^3\left(dx^3-8c\right)} + \frac{2\sqrt{c+dx^3}}{3d^3} - \frac{224\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) - (64*c*Sqrt[c + d*x^3])/(27*d^3*(-8*c + d*x^3)
) - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 73, normalized size = 0.88

$$-\frac{2\sqrt{c+dx^3}\left(104c-9dx^3\right)}{27d^3\left(dx^3-8c\right)} - \frac{224\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*(104*c - 9*d*x^3)*Sqrt[c + d*x^3])/(27*d^3*(-8*c + d*x^3)) - (224*Sqrt[
c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

**fricas [A]** time = 0.43, size = 167, normalized size = 2.01

$$\left[\frac{2\left(56\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(9dx^3-104c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3-8cd^3\right)}, \frac{2\left(112\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\left(9dx^3-104c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3-8cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^
3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)
 + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

**giac [A]** time = 0.17, size = 69, normalized size = 0.83

$$\frac{224\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,d^3}+\frac{2\sqrt{dx^3+c}}{3\,d^3}-\frac{64\sqrt{dx^3+c}\,c}{27\left(dx^3-8\,c\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 224/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) + 2/3*sqrt(d*x
^3 + c)/d^3 - 64/27*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^3)
```

**maple [C]** time = 0.20, size = 874, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^3+64*c^2/d^2*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1
/486*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))+16/27*I/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)
*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.56, size = 79, normalized size = 0.95

$$\frac{2\left(56\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}-\frac{96\sqrt{dx^3+c}\,c}{dx^3-8\,c}\right)}{81\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(56*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 27*sqrt(d*x^3 + c) - 96*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^3
```

**mupad** [B] time = 4.00, size = 87, normalized size = 1.05

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^3}+\frac{112\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3}+\frac{64\,c\,\sqrt{d\,x^3+c}}{27\,d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^3) + (112*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^3) + (64*c*(c + d*x^3)^(1/2))/(27*
d^3*(8*c - d*x^3))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^8}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

## 3.289 $\int \frac{x^5}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=64

$$\frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 78, 63, 206}

$$\frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{5\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d}$$
$$= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{10\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^2}$$
$$= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

**Mathematica [A]** time = 0.05, size = 63, normalized size = 0.98

$$-\frac{8\sqrt{c+dx^3}}{27d^2\left(dx^3-8c\right)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 63, normalized size = 0.98

$$-\frac{8\sqrt{c+dx^3}}{27d^2\left(dx^3-8c\right)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

**fricas [A]** time = 0.47, size = 155, normalized size = 2.42

$$\left[\frac{5\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-24\,\sqrt{dx^3+c}\,c}{81\left(cd^3x^3-8\,c^2d^2\right)}, \frac{2\left(5\left(dx^3-8\,c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-12\,\sqrt{dx^3+c}\,c\right)}{81\left(cd^3x^3-8\,c^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(5*
(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x
^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]
```

**giac** [A] time = 0.16, size = 58, normalized size = 0.91

$$\frac{2\left(\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{12\sqrt{dx^3+c}}{(dx^3-8c)d}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3 +
 c)/((d*x^3 - 8*c)*d))/d
```

**maple** [C] time = 0.17, size = 861, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 8*c/d*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum((-
c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*
d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c
*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/27*I/
d^4/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))
```

**maxima** [A] time = 1.32, size = 67, normalized size = 1.05

$$\frac{\frac{5\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{24\sqrt{dx^3+c}}{dx^3-8c}}{81\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/81*(5*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sq
rt(c) - 24*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d^2
```

**mupad** [B] time = 4.01, size = 72, normalized size = 1.12

$$\frac{5\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}+\frac{8\,\sqrt{d\,x^3+c}}{27\,d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (5*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^(
1/2)*d^2) + (8*(c + d*x^3)^(1/2))/(27*d^2*(8*c - d*x^3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

## 3.290 $\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}$$

**Rubi** [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)]/(81*c^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{54c}$$
$$= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27cd}$$
$$= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

**Mathematica** [A] time = 0.04, size = 64, normalized size = 0.96

$$\frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])])/(81*c^(3/2)*d)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 67, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d} + \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)]/(81*c^(3/2)*d)
```

**fricas** [A] time = 0.44, size = 153, normalized size = 2.28

$$\left[\frac{\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}\,c}{162\,\left(c^2d^2x^3-8\,c^3d\right)}, -\frac{\left(dx^3-8\,c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,\sqrt{dx^3+c}\,c}{81\,\left(c^2d^2x^3-8\,c^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((d*
x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 +
 c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]
```

**giac [A]** time = 0.16, size = 59, normalized size = 0.88

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,cd}-\frac{\sqrt{dx^3+c}}{27\left(dx^3-8c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^3
 + c)/((d*x^3 - 8*c)*c*d)
```

**maple [C]** time = 0.19, size = 442, normalized size = 6.60

$$-\frac{\sqrt{dx^3+c}}{27\left(dx^3-8c\right)cd}-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{486c^2d^3\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.39, size = 72, normalized size = 1.07

$$-\frac{\frac{6\sqrt{dx^3+c}}{\left(dx^3+c\right)c-9\,c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{162\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/162*(6*sqrt(d*x^3 + c)/((d*x^3 + c)*c - 9*c^2) + log((sqrt(d*x^3 + c) -
3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d
```

**mupad [B]** time = 3.98, size = 75, normalized size = 1.12

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{162\,c^{3/2}\,d}+\frac{\sqrt{d\,x^3+c}}{27\,c\,d\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(162*c^(3/2
)*d) + (c + d*x^3)^(1/2)/(27*c*d*(8*c - d*x^3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

## 3.291 $\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=88

$$\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}+\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}$$

**Rubi [A]** time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 103, 156, 63, 208, 206}

$$\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{-9cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2d}\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2} + \frac{(13d)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,\right.}{1728c^2}\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{864c^2} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x,\right.}{96c^2d}\\
&= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 83, normalized size = 0.94

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3}+13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-27\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 88, normalized size = 1.00

$$\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}+\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))
```

**fricas [A]** time = 0.47, size = 226, normalized size = 2.57

$$\left[\frac{13\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+27\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-24\sqrt{dx^3+c}\,c}{5184\left(c^3dx^3-8c^4\right)}, \frac{27\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-13\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-12\sqrt{dx^3+c}\,c}{2592\left(c^3dx^3-8c^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5184*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
 c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), 1/2592
*(27*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 13*(d*x^3
- 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)
*c)/(c^3*d*x^3 - 8*c^4)]
```

**giac [A]** time = 0.19, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c^2} - \frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}\,c^2} - \frac{\sqrt{dx^3+c}}{216\left(dx^3-8\,c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 13/2592*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/216*sqrt(d*x^3 + c)/((d*x^3 - 8
*c)*c^2)
```

**maple [C]** time = 0.19, size = 880, normalized size = 10.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*d*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(
-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))-1/172
8*I/c^3/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_a
lpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1
/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2
)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-
1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)
```

**mupad [B]** time = 4.01, size = 80, normalized size = 0.91

$$\frac{13\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{2592\,\sqrt{c^5}}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{96\,\sqrt{c^5}}+\frac{\sqrt{d\,x^3+c}}{72\,c^2\,\left(24\,c-3\,d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (13*atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(2592*(c^5)^(1/2)) - at
anh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(96*(c^5)^(1/2)) + (c + d*x^3)^(1/
2)/(72*c^2*(24*c - 3*d*x^3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

## 3.292 $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=124

$$\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}$$

**Rubi** [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(864*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c^2*x^3
*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2
)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{2cd-\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{-18c^2d^2+5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^4d}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{d\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c^3} + \frac{\left(11d^2\right)\operatorname{S}}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c^3} + \frac{(11}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c}}\right.}{384c^{7/2}}$$

**Mathematica [A]** time = 0.15, size = 97, normalized size = 0.78

$$\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+27d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)+\frac{12\sqrt{c}\sqrt{c+dx^3}\left(36c-5dx^3\right)}{dx^6-8cx^3}}{10368c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(10368*c^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.13, size = 103, normalized size = 0.83

$$\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}+\frac{\sqrt{c+dx^3}\left(5dx^3-36c\right)}{864c^3x^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-36*c + 5*d*x^3))/(864*c^3*x^3*(8*c - d*x^3)) + (11*d*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2)) + (d*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]])/(384*c^(7/2))
```

**fricas [A]** time = 0.45, size = 280, normalized size = 2.26

$$\left[\frac{11\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\left(5\,cdx^3-36\,c^2\right)\sqrt{dx^3+c}}{20736\left(c^4dx^6-8\,c^5x^3\right)},-\frac{27\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+11\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+12\left(5\,cdx^3-36\,c^2\right)\sqrt{dx^3+c}}{10368\left(c^4dx^6-8\,c^5x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*s
qrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3
 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(d*x
^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(27*(d^2*x^6 - 8*c*d*x^3)*sqrt(-
c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 11*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3
+ c))/(c^4*d*x^6 - 8*c^5*x^3)]
```

**giac [A]** time = 0.16, size = 114, normalized size = 0.92

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c^3}-\frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{10368\,\sqrt{-c}\,c^3}-\frac{5\left(dx^3+c\right)^{\frac{3}{2}}d-41\,\sqrt{dx^3+c}\,cd}{864\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 11/10368*d*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/864*(5*(d*x^3 + c)^(3/2)
*d - 41*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^
3)
```

**maple [C]** time = 0.20, size = 926, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64/c^2*d^2*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)
*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^
```

```
(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+
1/64/c^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(3/2))-1/6912*I/c^4/d*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(
-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))-1/384*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
7/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\,(dx^3-8\,c)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^4), x)
```

**mupad [B]** time = 4.11, size = 117, normalized size = 0.94

$$\frac{\frac{41\,d\,\sqrt{d\,x^3+c}}{288\,c^2}-\frac{5\,d\,\left(d\,x^3+c\right)^{3/2}}{288\,c^3}}{3\,\left(d\,x^3+c\right)^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}-\frac{d\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,11\mathrm{i}}{27}\right)\,1\mathrm{i}}{384\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((41*d*(c + d*x^3)^(1/2))/(288*c^2) - (5*d*(c + d*x^3)^(3/2))/(288*c^3))/(3
*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) - (d*(atanh((c^3*(c + d*x^3)^(1
/2))/(c^7)^(1/2))*1i + (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*11i)
/27)*1i)/(384*(c^7)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

## 3.293 $\int \frac{1}{x^7\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=164

$$\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}-\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)}$$

**Rubi [A]** time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$-\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)}+\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-35*d^2*Sqrt[c + d*x^3])/(13824*c^4*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c
^2*x^6*(8*c - d*x^3)) + (3*d*Sqrt[c + d*x^3])/(128*c^3*x^3*(8*c - d*x^3)) +
 (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{9cd-\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{38c^2d^2-\frac{27}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^4}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-34\ldots}{x(8\ldots}\right.}{2\ldots}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{\left(19d^2\right)\text{Subst}\ldots}{\ldots}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{(19d)\,\text{Subst}\ldots}{\ldots}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{31d^2\tanh^{-1}\ldots}{165888\ldots}$$

**Mathematica** [A] time = 0.21, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(288c^2-324cdx^3+35d^2x^6\right)}{dx^9-8cx^6}+31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-513d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{165888c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x^
6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))
```

**IntegrateAlgebraic [A]** time = 0.15, size = 118, normalized size = 0.72

$$\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}+\frac{\sqrt{c+dx^3}\left(-288c^2+324cdx^3-35d^2x^6\right)}{13824c^4x^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-288*c^2 + 324*c*d*x^3 - 35*d^2*x^6))/(13824*c^4*x^6*(8*c
 - d*x^3)) + (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2))
 - (19*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))
```

**fricas [A]** time = 0.65, size = 310, normalized size = 1.89

$$\left[\frac{31\left(d^3x^9-8cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+513\left(d^3x^9-8cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+24\left(35cd^2x^6-324c^2dx^3+288c^3\right)\sqrt{dx^3+c}}{331776\left(c^5dx^9-8c^6x^6\right)}, \frac{513\left(d^3x^9-8cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-31\left(d^3x^9-8cd^2x^6\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+12\left(35cd^2x^6-324c^2dx^3+288c^3\right)\sqrt{dx^3+c}}{165888\left(c^5dx^9-8c^6x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log(
(d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*c^2
*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6*x^6), 1/165888*(513*(
d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 31*(d^
3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(
35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6
*x^6)]
```

**giac [A]** time = 0.18, size = 128, normalized size = 0.78

$$\frac{19\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6144\,\sqrt{-c}\,c^4}-\frac{31\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{165888\,\sqrt{-c}\,c^4}-\frac{\sqrt{dx^3+c}\,d^2}{13824\,\left(dx^3-8\,c\right)c^4}+\frac{\left(dx^3+c\right)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}\,cd^2}{384\,c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 19/6144*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31/165888*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/13824*sqrt(d*x^3 +
 c)*d^2/((d*x^3 - 8*c)*c^4) + 1/384*((d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 +
 c)*c*d^2)/(c^4*d^2*x^6)
```

**maple [C]** time = 0.20, size = 989, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/512/c^3*d^3*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
```

```
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1
/2))/c^(3/2))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*(d*x^3+c)^(1/2)/c^2*
d/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/36864*I/c^5*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
-1/2048*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2 x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)
```

**mupad [B]** time = 4.37, size = 155, normalized size = 0.95

$$-\frac{\frac{647\,d^2\,\sqrt{d\,x^3+c}}{4608\,c^2}-\frac{197\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{2304\,c^3}+\frac{35\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{4608\,c^4}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,31{}\mathrm{i}}{513}\right)\,19{}\mathrm{i}}{6144\,\sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] (d^2*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i - (atanh((c^4*(c + d*x^
3)^(1/2))/(3*(c^9)^(1/2)))*31i)/513)*19i)/(6144*(c^9)^(1/2)) - ((647*d^2*(c
 + d*x^3)^(1/2))/(4608*c^2) - (197*d^2*(c + d*x^3)^(3/2))/(2304*c^3) + (35*
d^2*(c + d*x^3)^(5/2))/(4608*c^4))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3)
 - 3*(c + d*x^3)^3 + 27*c^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

## 3.294 $\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=95

$$\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}+\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

**Rubi** [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 98, 146, 63, 206}

$$\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (8*x^6)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*(38*c + 39*d*x^3))/(81*
d^4*Sqrt[c + d*x^3]) - (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
243*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{x\left(16c^2+13cdx\right)}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{27cd^2}$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}} - \frac{(320c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,\right.}{81d^3}$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}} - \frac{(640c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c\ \right.}{81d^4}$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}} - \frac{640\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

**Mathematica [C]** time = 0.05, size = 88, normalized size = 0.93

$$\frac{6\left(752c^2-198cdx^3+9d^2x^6\right)-640c\left(8c-dx^3\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{81d^4\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (6*(752*c^2 - 198*c*d*x^3 + 9*d^2*x^6) - 640*c*(8*c - d*x^3)*Hypergeometric
2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(81*d^4*(-8*c + d*x^3)*Sqrt[c + d*x^3
])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 127, normalized size = 1.34

$$\frac{\left(\frac{5120c^{3/2}}{243d^4}-\frac{640\sqrt{c}\,x^3}{243d^3}\right)\sqrt{c+dx^3}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{608c^2}{81d^4}-\frac{548cx^3}{81d^3}+\frac{2x^6}{3d^2}}{dx^3\sqrt{c+dx^3}-8c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((-608*c^2)/(81*d^4) - (548*c*x^3)/(81*d^3) + (2*x^6)/(3*d^2) + ((5120*c^(3
/2))/(243*d^4) - (640*Sqrt[c]*x^3)/(243*d^3))*Sqrt[c + d*x^3]*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(-8*c*Sqrt[c + d*x^3] + d*x^3*Sqrt[c + d*x^3])
```

**fricas [A]** time = 0.76, size = 233, normalized size = 2.45

$$\left[\frac{2\left(160\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(27\,d^2x^6-274\,cdx^3-304\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(d^6x^6-7\,cd^5x^3-8\,c^2d^4\right)}, \frac{2\left(320\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\left(27\,d^2x^6-274\,cdx^3-304\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(d^6x^6-7\,cd^5x^3-8\,c^2d^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(160*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*c^
2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^3 - 8*c^2*d^4), 2/243*(320*(d^2*x^
6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*
(27*d^2*x^6 - 274*c*d*x^3 - 304*c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^
3 - 8*c^2*d^4)]
```

**giac [A]** time = 0.19, size = 88, normalized size = 0.93

$$\frac{640\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}\,d^4}+\frac{2\sqrt{dx^3+c}}{3\,d^4}-\frac{2\left(85\left(dx^3+c\right)c+3\,c^2\right)}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 640/243*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/3*sqrt(d*
x^3 + c)/d^4 - 2/81*(85*(d*x^3 + c)*c + 3*c^2)/(((d*x^3 + c)^(3/2) - 9*sqrt
(d*x^3 + c)*c)*d^4)
```

**maple [C]** time = 0.29, size = 970, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d^3*((2/3/((x^3+c/d)*d)^(1/2)*c/d^2+2/3*(d*x^3+c)^(1/2)/d^2)*d-32/3/(d*x^
3+c)^(1/2)*c/d)+512*c^3/d^3*(-1/243/d/c^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-2/243
/d/c^2/((x^3+c/d)*d)^(1/2)-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2
*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1
/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*
d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alph
a)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+192*c^2/d^3*(2/27/((x^3+
c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)
*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

**maxima** [A] time = 1.21, size = 98, normalized size = 1.03

$$\frac{2\left(160\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+81\sqrt{dx^3+c}-\frac{3\left(85\left(dx^3+c\right)c+3c^2\right)}{\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c}\right)}{243\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/243*(160*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 81*sqrt(d*x^3 + c) - 3*(85*(d*x^3 + c)*c + 3*c^2)/((d*x^3 + c)^(
3/2) - 9*sqrt(d*x^3 + c)*c))/d^4
```

**mupad** [B] time = 4.38, size = 111, normalized size = 1.17

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^4}+\frac{320\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,d^4}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{176\,c^2}{81\,d^4}+\frac{170\,c\,x^3}{81\,d^3}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^4) + (320*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*d^4) + ((c + d*x^3)^(1/2)*((176*c^2
)/(81*d^4) + (170*c*x^3)/(81*d^3)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.295 $\int \frac{x^8}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=83

$$\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

**Rubi** [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 89, 78, 63, 206}

$$\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -22/(81*d^3*Sqrt[c + d*x^3]) + (64*c)/(27*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]
) - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*Sqrt[c]*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{-24c^2d+9cd^2x}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{27cd^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{16\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x\right.}{81d^2}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{32\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx}\right.}{81d^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

**Mathematica [A]** time = 0.11, size = 71, normalized size = 0.86

$$\frac{2\left(\frac{3\left(8c+11dx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 115, normalized size = 1.39

$$\frac{\frac{16c}{81d^3} + \left(\frac{32x^3}{243\sqrt{c}\,d^2} - \frac{256\sqrt{c}}{243d^3}\right)\sqrt{c+dx^3}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + \frac{22x^3}{81d^2}}{8c\sqrt{c+dx^3} - dx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((16*c)/(81*d^3) + (22*x^3)/(81*d^2) + ((-256*Sqrt[c])/(243*d^3) + (32*x^3)
/(243*Sqrt[c]*d^2))*Sqrt[c + d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
8*c*Sqrt[c + d*x^3] - d*x^3*Sqrt[c + d*x^3])
```

**fricas [A]** time = 0.71, size = 223, normalized size = 2.69

$$\left[\frac{2\left(8\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(11\,cdx^3+8\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}, \frac{2\left(16\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-3\left(11\,cdx^3+8\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 +
 c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 + c)
)/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^3 -
 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*(11*c*d*x^3 + 8
*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]
```

**giac** [A] time = 0.17, size = 67, normalized size = 0.81

$$\frac{32\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}\,d^3}-\frac{2\left(11\,dx^3+8\,c\right)}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 32/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/81*(11*d*x^3
 + 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d^3)
```

**maple** [C] time = 0.18, size = 926, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/d^3/(d*x^3+c)^(1/2)+64*c^2/d^2*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2
/d-2/243/((x^3+c/d)*d)^(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1
/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+16*c/d^2*(2/27/(
(x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2
*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((
x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(
-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c
*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^
(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/
d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

**maxima** [A] time = 1.41, size = 81, normalized size = 0.98

$$\frac{2\left(\frac{8\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{3\left(11\,dx^3+8\,c\right)}{\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c}\right)}{243\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/243*(8*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3*(11*d*x^3 + 8*c)/((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^3
```

**mupad [B]** time = 4.30, size = 94, normalized size = 1.13

$$\frac{16\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,\sqrt{c}\,d^3}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{16\,c}{81\,d^3}+\frac{22\,x^3}{81\,d^2}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (16*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*c
^(1/2)*d^3) + ((c + d*x^3)^(1/2)*((16*c)/(81*d^3) + (22*x^3)/(81*d^2)))/(8*
c^2 - d^2*x^6 + 7*c*d*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.296 $\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2}{81cd^2\sqrt{c+dx^3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 51, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2}{81cd^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(81*c*d^2*Sqrt[c + d*x^3]) + 8/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(3/2)*d^2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{9d}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{81cd}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx}\right.}{81cd^2}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

**Mathematica [C]** time = 0.03, size = 68, normalized size = 0.80

$$-\frac{2\left(\left(dx^3-8c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+12c\right)}{81cd^2\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(12*c + (-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c
)]))/(81*c*d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.08, size = 117, normalized size = 1.38

$$\frac{\left(\frac{16}{243\sqrt{c}\,d^2}-\frac{2x^3}{243c^{3/2}d}\right)\sqrt{c+dx^3}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+\frac{2x^3}{81cd}+\frac{8}{81d^2}}{8c\sqrt{c+dx^3}-dx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (8/(81*d^2) + (2*x^3)/(81*c*d) + (16/(243*Sqrt[c]*d^2) - (2*x^3)/(243*c^(3/
2)*d))*Sqrt[c + d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(8*c*Sqrt[c +
d*x^3] - d*x^3*Sqrt[c + d*x^3])
```

**fricas [A]** time = 0.71, size = 223, normalized size = 2.62

$$\left[\frac{\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\left(cdx^3+4\,c^2\right)\sqrt{dx^3+c}}{243\left(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2\right)}, -\frac{2\left(\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\left(cdx^3+4\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^
2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^
2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]
```

**giac [A]** time = 0.19, size = 76, normalized size = 0.89

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}cd}+\frac{3\left(dx^3+4c\right)}{\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)cd}\right)}{243d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/243*(arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) + 3*(d*x^3 + 4*
c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c*d))/d
```

**maple [C]** time = 0.18, size = 908, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 8*c/d*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)/c
^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))+1/d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/
d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d
^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*
d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^
3*d-8*c)))
```

**maxima [A]** time = 1.23, size = 83, normalized size = 0.98

$$-\frac{\frac{6\left(dx^3+4c\right)}{\left(dx^3+c\right)^{\frac{3}{2}}c-9\sqrt{dx^3+c}c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{243d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/243*(6*(d*x^3 + 4*c)/((d*x^3 + c)^(3/2)*c - 9*sqrt(d*x^3 + c)*c^2) + log
((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d^2
```

**mupad [B]** time = 4.26, size = 96, normalized size = 1.13

$$\frac{\left(\frac{8}{81\,d^2}+\frac{2\,x^3}{81\,c\,d}\right)\,\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}+\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,c^{3/2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((8/(81*d^2) + (2*x^3)/(81*c*d))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7*c*
d*x^3) + log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(2
43*c^(3/2)*d^2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.297 $\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

**Optimal**. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

**Rubi** [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}+\frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/(81*c^2*d*Sqrt[c + d*x^3]) + 1/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{18c}$$
$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{162c^2}$$
$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81c^2d}$$
$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

**Mathematica [C]** time = 0.01, size = 43, normalized size = 0.49

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{243c^2d\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (c + d*x^3)/(9*c)])/(243*c^2*d*Sqrt[c +
 d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.10, size = 76, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d} + \frac{dx^3-5c}{81c^2d\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-5*c + d*x^3)/(81*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)
```

**fricas [A]** time = 0.59, size = 219, normalized size = 2.49

$$\left[\frac{\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\left(cdx^3-5\,c^2\right)\sqrt{dx^3+c}}{486\left(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d\right)}, -\frac{\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\left(cdx^3-5\,c^2\right)\sqrt{dx^3+c}}{243\left(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(c^
3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 - 5*c^2)*sqrt
(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]
```

**giac [A]** time = 0.16, size = 72, normalized size = 0.82

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}\,c^2d}-\frac{dx^3-5\,c}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3
 - 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)
```

**maple [C]** time = 0.19, size = 463, normalized size = 5.26

$$-\frac{\sqrt{dx^3+c}}{243\left(dx^3-8c\right)c^2d}-\frac{2}{243\sqrt{\left(x^3+\frac{c}{d}\right)d}\,c^2d}-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},-\frac{2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(d\_Z^3-8c\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(d\_Z^3-8c\right)}{18cd},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{1458c^3d^3\sqrt{dx^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)/c^2/d-1/
1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))
```

**maxima [A]** time = 1.27, size = 85, normalized size = 0.97

$$-\frac{\frac{6\left(dx^3-5\,c\right)}{\left(dx^3+c\right)^{\frac{3}{2}}c^2-9\sqrt{dx^3+c}\,c^3}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{486\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/486*(6*(d*x^3 - 5*c)/((d*x^3 + c)^(3/2)*c^2 - 9*sqrt(d*x^3 + c)*c^3) + l
og((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(5/2))/d
```

**mupad [B]** time = 4.26, size = 97, normalized size = 1.10

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{486\,c^{5/2}\,d}-\frac{\left(\frac{5}{81\,c\,d}-\frac{x^3}{81\,c^2}\right)\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(486*c^(5/2
)*d) - ((5/(81*c*d) - x^3/(81*c^2))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7
*c*d*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**2/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.298 $\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=106

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}+\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$\frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*Sqrt[c + d*x^3]) + 1/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(7776*c^(7/2)) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(96*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{216c^2d}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-\frac{81}{2}c^2d^2+\frac{15}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,\right.}{972c^4d^2}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{192c^3} +$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+d}\right.}{2592c^3}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\tanh^{-1}\left(\right.}{96}$$

**Mathematica [C]** time = 0.05, size = 97, normalized size = 0.92

$$\frac{\left(7dx^3-56c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+27\left(8c-dx^3\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+12c}{2592c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (12*c + (-56*c + 7*d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)
] + 27*(8*c - d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(2592*
c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic** [A] time = 0.10, size = 163, normalized size = 1.54

$$\frac{\sqrt{c+dx^3}\left(\frac{7}{972c^{5/2}}-\frac{7dx^3}{7776c^{7/2}}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+\sqrt{c+dx^3}\left(\frac{dx^3}{96c^{7/2}}-\frac{1}{12c^{5/2}}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)-\frac{5dx^3}{648c^3}+\frac{43}{648c^2}}{8c\sqrt{c+dx^3}-dx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (43/(648*c^2) - (5*d*x^3)/(648*c^3) + Sqrt[c + d*x^3]*(7/(972*c^(5/2)) - (7
*d*x^3)/(7776*c^(7/2)))*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + Sqrt[c + d*x
^3]*(-1/12*1/c^(5/2) + (d*x^3)/(96*c^(7/2)))*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]])/(8*c*Sqrt[c + d*x^3] - d*x^3*Sqrt[c + d*x^3])
```

**fricas** [A] time = 0.66, size = 316, normalized size = 2.98

$$\left[\frac{7\left(d^2x^6-7cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+81\left(d^2x^6-7cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+24\left(5cdx^3-43c^2\right)\sqrt{dx^3+c}}{15552\left(c^4d^2x^6-7c^5dx^3-8c^6\right)}, \frac{81\left(d^2x^6-7cdx^3-8c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-7\left(d^2x^6-7cdx^3-8c^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+12\left(5cdx^3-43c^2\right)\sqrt{dx^3+c}}{7776\left(c^4d^2x^6-7c^5dx^3-8c^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/15552*(7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqr
t(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3 - 4
3*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 - 8*c^6), 1/7776*(81*(d^
2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*
(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/
c) + 12*(5*c*d*x^3 - 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 -
8*c^6)]
```

**giac** [A] time = 0.17, size = 93, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c^3}-\frac{7\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{7776\sqrt{-c}\,c^3}+\frac{5\,dx^3-43\,c}{648\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 7/7776*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/648*(5*d*x^3 - 43*c)/(((d*x^3 +
c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^3)
```

**maple** [C] time = 0.17, size = 953, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*d*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)
/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
```

```
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d-8*c))-1/64/c^2*d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/2
43*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_
alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(
1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*d-8*c))+1/64/c^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

**mupad [B]** time = 4.33, size = 101, normalized size = 0.95

$$-\frac{\frac{5\left(d\,x^3+c\right)}{216\,c^3}-\frac{2}{9\,c^2}}{27\,c\,\sqrt{d\,x^3+c}-3\left(d\,x^3+c\right)^{3/2}}+\frac{\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)7\mathrm{i}}{81}\right)1\mathrm{i}}{96\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^(
1/2))/(3*(c^7)^(1/2)))*7i)/81)*1i)/(96*(c^7)^(1/2)) - ((5*(c + d*x^3))/(216
*c^3) - 2/(9*c^2))/(27*c*(c + d*x^3)^(1/2) - 3*(c + d*x^3)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.299 $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=143

$$\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}-\frac{35d}{2592c^4\sqrt{c+dx^3}}+\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{35d}{2592c^4\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-35*d)/(2592*c^4*Sqrt[c + d*x^3]) + (5*d)/(864*c^3*(8*c - d*x^3)*Sqrt[c +
d*x^3]) - 1/(24*c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*d*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(384*c^(9/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{10cd-\frac{5d^2x}{2}}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2}$$

$$= \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{-90}{x(8c\cdots}\right.}{1\cdots}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+\cdots}}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+\cdots}}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+\cdots}}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+\cdots}}$$

**Mathematica [C]** time = 0.06, size = 117, normalized size = 0.82

$$\frac{5dx^3\left(dx^3-8c\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+135dx^3\left(dx^3-8c\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+12c\left(5dx^3-36c\right)}{10368c^4x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (12*c*(-36*c + 5*d*x^3) + 5*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1,
 1/2, (c + d*x^3)/(9*c)] + 135*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + (d*x^3)/c])/(10368*c^4*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 114, normalized size = 0.80

$$\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}+\frac{-108c^2-265cdx^3+35d^2x^6}{2592c^4x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-108*c^2 - 265*c*d*x^3 + 35*d^2*x^6)/(2592*c^4*x^3*(8*c - d*x^3)*Sqrt[c +
d*x^3]) + (5*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(9/2))
```

**fricas [A]** time = 0.78, size = 368, normalized size = 2.57

$$\left[\frac{5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+405\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\left(35\,cd^2x^6-265\,c^2dx^3-108\,c^3\right)\sqrt{dx^3+c}}{62208\left(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3\right)}, -\frac{405\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)+5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+12\left(35\,cd^2x^6-265\,c^2dx^3-108\,c^3\right)\sqrt{dx^3+c}}{31104\left(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 6*sq
rt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6 -
 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) -
24*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 -
 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x
^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 5*(d^3*x^9 - 7*c*d^2*x^6
- 8*c^2*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(35*c*d
^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7*c^6*d*x
^6 - 8*c^7*x^3)]
```

**giac [A]** time = 0.17, size = 129, normalized size = 0.90

$$-\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c^4}-\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{31104\,\sqrt{-c}\,c^4}-\frac{35\left(dx^3+c\right)^2d-335\left(dx^3+c\right)cd+192\,c^2d}{2592\left(\left(dx^3+c\right)^{\frac{5}{2}}-10\left(dx^3+c\right)^{\frac{3}{2}}c+9\,\sqrt{dx^3+c}\,c^2\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -5/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 5/31104*d*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/2592*(35*(d*x^3 + c)^2*d
- 335*(d*x^3 + c)*c*d + 192*c^2*d)/(((d*x^3 + c)^(5/2) - 10*(d*x^3 + c)^(3/
2)*c + 9*sqrt(d*x^3 + c)*c^2)*c^4)
```

**maple [C]** time = 0.20, size = 1019, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*d^2*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^
(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-
2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1
/256/c^3*d^2*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*
d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d
^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/256/c^3
*d*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)
```

**mupad [B]** time = 4.56, size = 133, normalized size = 0.93

$$-\frac{\frac{2\,d}{9\,c^2}+\frac{35\,d\,\left(d\,x^3+c\right)^2}{864\,c^4}-\frac{335\,d\,\left(d\,x^3+c\right)}{864\,c^3}}{3\,\left(d\,x^3+c\right)^{5/2}-30\,c\,\left(d\,x^3+c\right)^{3/2}+27\,c^2\,\sqrt{d\,x^3+c}}-\frac{d\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1{}\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,1{}\mathrm{i}}{81}\right)\,5{}\mathrm{i}}{384\,\sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] - ((2*d)/(9*c^2) + (35*d*(c + d*x^3)^2)/(864*c^4) - (335*d*(c + d*x^3))/(86
4*c^3))/(3*(c + d*x^3)^(5/2) - 30*c*(c + d*x^3)^(3/2) + 27*c^2*(c + d*x^3)^
(1/2)) - (d*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i + (atanh((c^4*(c
 + d*x^3)^(1/2))/(3*(c^9)^(1/2)))*1i)/81)*5i)/(384*(c^9)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.300 $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

**Optimal**. Leaf size=185

$$\frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}}-\frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}+\frac{665d^2}{41472c^5\sqrt{c+dx^3}}-\frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}}+\frac{17}{384c^3x^3(8c-}$$

**Rubi [A]** time = 0.16, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$-\frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}}+\frac{665d^2}{41472c^5\sqrt{c+dx^3}}+\frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}}-\frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}+\frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}}-\frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (665*d^2)/(41472*c^5*Sqrt[c + d*x^3]) - (71*d^2)/(13824*c^4*(8*c - d*x^3)*S
qrt[c + d*x^3]) - 1/(48*c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (17*d)/(38
4*c^3*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (13*d^2*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(497664*c^(11/2)) - (33*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(2048*c^(11/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{17cd-\frac{7d^2x}{2}}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{19}{x(8}\right.}{}$$

$$= -\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{}{384c^3x^3\left(8c\right.}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+}}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+}}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+}}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+}}$$

**Mathematica** [C] time = 0.06, size = 135, normalized size = 0.73

$$\frac{13d^2x^6\left(dx^3-8c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-3\left(4c\left(288c^2-612cdx^3+71d^2x^6\right)+891d^2x^6\left(dx^3-8c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)\right)}{165888c^5x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (13*d^2*x^6*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c
)] - 3*(4*c*(288*c^2 - 612*c*d*x^3 + 71*d^2*x^6) + 891*d^2*x^6*(-8*c + d*x^
3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(165888*c^5*x^6*(8*c -
d*x^3)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic** [A] time = 0.19, size = 208, normalized size = 1.12

$$\frac{x^6\sqrt{c+dx^3}\left(\frac{13d^2}{62208c^{9/2}}-\frac{13d^3x^3}{497664c^{11/2}}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+x^6\sqrt{c+dx^3}\left(\frac{33d^3x^3}{2048c^{11/2}}-\frac{33d^2}{256c^{9/2}}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)-\frac{665d^3x^9}{41472c^5}+\frac{5107d^2x^6}{41472c^4}+\frac{17dx^3}{384c^3}-\frac{1}{48c^2}}{x^6\left(8c\sqrt{c+dx^3}-dx^3\sqrt{c+dx^3}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-1/48*1/c^2 + (17*d*x^3)/(384*c^3) + (5107*d^2*x^6)/(41472*c^4) - (665*d^3
*x^9)/(41472*c^5) + x^6*Sqrt[c + d*x^3]*((13*d^2)/(62208*c^(9/2)) - (13*d^3
```

```
*x^3)/(497664*c^(11/2)))*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + x^6*Sqrt[c
+ d*x^3]*((-33*d^2)/(256*c^(9/2)) + (33*d^3*x^3)/(2048*c^(11/2)))*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(x^6*(8*c*Sqrt[c + d*x^3] - d*x^3*Sqrt[c + d*x^3])
)
```

**fricas [A]** time = 0.90, size = 398, normalized size = 2.15

$$\left[\frac{13\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+8019\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)+24\left(665\,cd^3x^9-5107\,c^2d^2x^6-1836\,c^3dx^3+864\,c^4\right)\sqrt{dx^3+c}}{995328\left(c^6d^2x^{12}-7\,c^7dx^9-8\,c^8x^6\right)},\ \frac{8019\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right)-13\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+12\left(665\,cd^3x^9-5107\,c^2d^2x^6-1836\,c^3dx^3+864\,c^4\right)\sqrt{dx^3+c}}{497664\left(c^6d^2x^{12}-7\,c^7dx^9-8\,c^8x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/995328*(13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 +
 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^12 - 7*c*d^
3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d*x^3 + 864*c^4)*s
qrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6), 1/497664*(8019*(d
^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt
(-c)/c) - 13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^
3*d*x^3 + 864*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6
)]
```

**giac [A]** time = 0.18, size = 149, normalized size = 0.81

$$\frac{33\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^5}-\frac{13\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{497664\,\sqrt{-c}\,c^5}+\frac{341\left(dx^3+c\right)d^2-3072\,cd^2}{41472\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c\right)c^5}+\frac{3\left(dx^3+c\right)^{\frac{3}{2}}d^2-4\,\sqrt{dx^3+c}\,cd^2}{384\,c^5d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 33/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13/497664*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 1/41472*(341*(d*x^3
+ c)*d^2 - 3072*c*d^2)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) + 1/
384*(3*(d*x^3 + c)^(3/2)*d^2 - 4*sqrt(d*x^3 + c)*c*d^2)/(c^5*d^2*x^6)
```

**maple [C]** time = 0.21, size = 1106, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/512/c^3*d^3*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)
^(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/c^2/
x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2
))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c^2/x^6+7/12*(d*x^3+c)^(1/2)/c^3*d/x^3+2/
3/((x^3+c/d)*d)^(1/2)/c^3*d^2-5/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7
/2))-3/4096/c^4*d^3*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*s
```

```
um((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-
3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+3/
4096/c^4*d^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2)
)/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)
```

**mupad [B]** time = 4.76, size = 171, normalized size = 0.92

$$\frac{\frac{2\,d^2}{9\,c^2}-\frac{10373\,d^2\,\left(d\,x^3+c\right)}{13824\,c^3}+\frac{3551\,d^2\,{\left(d\,x^3+c\right)}^2}{6912\,c^4}-\frac{665\,d^2\,{\left(d\,x^3+c\right)}^3}{13824\,c^5}}{33\,c\,{\left(d\,x^3+c\right)}^{5/2}-3\,{\left(d\,x^3+c\right)}^{7/2}+27\,c^3\,\sqrt{d\,x^3+c}-57\,c^2\,{\left(d\,x^3+c\right)}^{3/2}}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{\sqrt{c^{11}}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^{11}}}\right)\,13{}\mathrm{i}}{8019}\right)\,33{}\mathrm{i}}{2048\,\sqrt{c^{11}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((2*d^2)/(9*c^2) - (10373*d^2*(c + d*x^3))/(13824*c^3) + (3551*d^2*(c + d*x
^3)^2)/(6912*c^4) - (665*d^2*(c + d*x^3)^3)/(13824*c^5))/(33*c*(c + d*x^3)^
(5/2) - 3*(c + d*x^3)^(7/2) + 27*c^3*(c + d*x^3)^(1/2) - 57*c^2*(c + d*x^3)
^(3/2)) + (d^2*(atanh((c^5*(c + d*x^3)^(1/2))/(c^11)^(1/2))*1i - (atanh((c^
5*(c + d*x^3)^(1/2))/(3*(c^11)^(1/2)))*13i)/8019)*33i)/(2048*(c^11)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.301 $\int \frac{x^8 \sqrt{c+dx^3}}{\left(a+bx^3\right)^2} \, dx$

**Optimal**. Leaf size=161

$$-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}-\frac{a\sqrt{c+dx^3}\,(4bc-5ad)}{3b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

**Rubi** [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}-\frac{a\sqrt{c+dx^3}\,(4bc-5ad)}{3b^3(bc-ad)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -(a*(4*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(
3/2))/(9*b^2*d) - (a^2*(c + d*x^3)^(3/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) +
 (a*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*
b^(7/2)*Sqrt[b*c - a*d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
```

```
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(-\frac{1}{2}a(2bc-3ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \cdots\right.}{6b\cdots}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subst}\left(\cdots\right.}{3\cdots}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{a(4bc-5ad)\tanh^{-1}\left(\cdots\right.}{3b^{7/2}\sqrt{bc-a\cdots}}$$

**Mathematica** [A] time = 0.29, size = 147, normalized size = 0.91

$$\frac{-\frac{a^2\left(c+dx^3\right)^{3/2}}{a+bx^3} + \frac{a(5ad-4bc)\left(\sqrt{b}\sqrt{c+dx^3}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}} + \frac{2\left(c+dx^3\right)^{3/2}(bc-ad)}{3d}}{3b^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*(b*c - a*d)*(c + d*x^3)^(3/2))/(3*d) - (a^2*(c + d*x^3)^(3/2))/(a + b*x
^3) + (a*(-4*b*c + 5*a*d)*(Sqrt[b]*Sqrt[c + d*x^3] - Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]))/b^(3/2))/(3*b^2*(b*c - a*d))
```

**IntegrateAlgebraic [A]** time = 0.27, size = 142, normalized size = 0.88

$$\frac{\left(4abc-5a^2d\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}\,\sqrt{ad-bc}}{bc-ad}\right)}{3b^{7/2}\sqrt{ad-bc}}+\frac{\sqrt{c+dx^3}\,\left(-15a^2d+2abc-10abdx^3+2b^2cx^3+2b^2dx^6\right)}{9b^3d\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(2*a*b*c - 15*a^2*d + 2*b^2*c*x^3 - 10*a*b*d*x^3 + 2*b^2*d
*x^6))/(9*b^3*d*(a + b*x^3)) + ((4*a*b*c - 5*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-(
b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2)*Sqrt[-(b*c) + a*d])
```

**fricas [A]** time = 0.92, size = 469, normalized size = 2.91

$$\left[-\frac{3\left(4a^2bcd-5a^3d^2+\left(4ab^2cd-5a^2bd^2\right)x^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\left(b^4cd-ab^3d^2\right)x^6+2ab^3c^2-17a^2b^2cd+15a^3bd^2+2\left(b^4c^2-6ab^3cd+5a^2b^2d^2\right)x^3\right)\sqrt{dx^3+c}}{18\left(ab^5cd-a^2b^4d^2+\left(b^6cd-ab^5d^2\right)x^3\right)},\ -\frac{3\left(4a^2bcd-5a^3d^2+\left(4ab^2cd-5a^2bd^2\right)x^3\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\left(2\left(b^4cd-ab^3d^2\right)x^6+2ab^3c^2-17a^2b^2cd+15a^3bd^2+2\left(b^4c^2-6ab^3cd+5a^2b^2d^2\right)x^3\right)\sqrt{dx^3+c}}{9\left(ab^5cd-a^2b^4d^2+\left(b^6cd-ab^5d^2\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(
b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^
2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*s
qrt(d*x^3 + c))/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3), -1/9
*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*b*d^2 +
2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c*d
- a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]
```

**giac [A]** time = 0.18, size = 136, normalized size = 0.84

$$-\frac{\sqrt{dx^3+c}\,a^2d}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^3}-\frac{\left(4\,abc-5\,a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^4d^2-6\,\sqrt{dx^3+c}\,ab^3d^3\right)}{9\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/(((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*c
 - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^3)/
(b^6*d^3)
```

**maple [C]** time = 0.36, size = 917, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b^2/d-2*a/b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
```

```
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*
b+a)))+a^2/b^2*(-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/d/b*2^(1/2)*sum(1/(a
*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/
3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^
3*b+a)))
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [**B**] time = 6.84, size = 202, normalized size = 1.25

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{4\,c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}+\frac{2\,a\,d}{b^3}\right)}{3\,d}+\frac{a^2\,\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)}+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{7/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((4*c)/(3*b^2) - (2*
b^2*c - 2*a*b*d)/b^4 + (2*a*d)/b^3))/(3*d) + (a*log((2*b*c - a*d + b^(1/2)*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b
*c)*1i)/(6*b^(7/2)*(a*d - b*c)^(1/2)) + (a^2*((2*a*d)/(3*(2*b^2*c - 2*a*b*d
)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b^2*(a + b*x^3))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.302 $\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=136

$$-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^3}\,(2bc-3ad)}{3b^2(bc-ad)}+\frac{a\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

**Rubi [A]** time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{c+dx^3}\,(2bc-3ad)}{3b^2(bc-ad)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}+\frac{a\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(3/2
))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)\\
&= \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}\\
&= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b^2}\\
&= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+d}\right.}{3b^2d}\\
&= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

**Mathematica [A]** time = 0.15, size = 117, normalized size = 0.86

$$
\frac{\frac{(2bc-3ad)\left(\sqrt{b}\sqrt{c+dx^3}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}}+\frac{a\left(c+dx^3\right)^{3/2}}{a+bx^3}}{3b(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((a*(c + d*x^3)^(3/2))/(a + b*x^3) + ((2*b*c - 3*a*d)*(Sqrt[b]*Sqrt[c + d*x
^3] - Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]))/
b^(3/2))/(3*b*(b*c - a*d))
```

**IntegrateAlgebraic [A]** time = 0.20, size = 108, normalized size = 0.79

$$
\frac{(3ad-2bc)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}\sqrt{ad-bc}}+\frac{\left(3a+2bx^3\right)\sqrt{c+dx^3}}{3b^2\left(a+bx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((3*a + 2*b*x^3)*Sqrt[c + d*x^3])/(3*b^2*(a + b*x^3)) + ((-2*b*c + 3*a*d)*A
rcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(5/2)
*Sqrt[-(b*c) + a*d])
```

**fricas [A]** time = 0.63, size = 334, normalized size = 2.46

$$
\left[-\frac{\left(\left(2\,b^2c-3\,abd\right)x^3+2\,abc-3\,a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(3\,ab^2c-3\,a^2bd+2\left(b^3c-ab^2d\right)x^3\right)\sqrt{dx^3+c}}{6\left(ab^4c-a^2b^3d+\left(b^5c-ab^4d\right)x^3\right)},\ \frac{\left(\left(2\,b^2c-3\,abd\right)x^3+2\,abc-3\,a^2d\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\left(3\,ab^2c-3\,a^2bd+2\left(b^3c-ab^2d\right)x^3\right)\sqrt{dx^3+c}}{3\left(ab^4c-a^2b^3d+\left(b^5c-ab^4d\right)x^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*lo
g((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 +
a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c))/
(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d)*x^
3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^
2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d
)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3)]
```

**giac [A]** time = 0.17, size = 102, normalized size = 0.75

$$\frac{\sqrt{dx^3+c}\,ad}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^2}+\frac{\left(2\,bc-3\,ad\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2}+\frac{2\,\sqrt{dx^3+c}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*a*d/(((d*x^3 + c)*b - b*c + a*d)*b^2) + 1/3*(2*b*c - 3*
a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b
^2) + 2/3*sqrt(d*x^3 + c)/b^2
```

**maple [C]** time = 0.26, size = 897, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2
*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((
x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(
-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c
*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1
/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d
-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(-1/3*(d*x^3+c)^(1/
2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b
*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.09, size = 152, normalized size = 1.12

$$\frac{2\sqrt{d\,x^3+c}}{3\,b^2}-\frac{a\,\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b^2) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/
2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*b^(5
/2)*(a*d - b*c)^(1/2)) - (a*((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(
2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.303 $\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=80

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

**Rubi [A]** time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 47, 63, 208}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*b*(a + b*x^3)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx,x,x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b}$$
$$= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b}$$
$$= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.11, size = 80, normalized size = 1.00

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{ad-bc}}\right)}{3b^{3/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

**IntegrateAlgebraic** [A] time = 0.18, size = 90, normalized size = 1.12

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

**fricas** [A] time = 0.81, size = 255, normalized size = 3.19

$$\left[\frac{\left(bdx^3+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{6\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x^3\right)}, \frac{\left(bdx^3+ad\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\sqrt{dx^3+c}\left(b^2c-abd\right)}{3\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sq
rt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 + a*
d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^
3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c
- a*b^3*d)*x^3)]
```

**giac [A]** time = 0.20, size = 79, normalized size = 0.99

$$\frac{d\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^3+c}\,d}{3\left(\left(dx^3+c\right)b-bc+ad\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*b)
```

**maple [C]** time = 0.27, size = 453, normalized size = 5.66

$$-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{\frac{i}{2}\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{\frac{i}{2}\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{6bd\,(ad-bc)\sqrt{dx^3+c}}-\frac{\sqrt{dx^3+c}}{3\left(bx^3+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)
^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2
/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.69, size = 125, normalized size = 1.56

$$\frac{\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d
*x^3)^(1/2))/(a + b*x^3) + (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(6*b^(3/2)*(a*d - b*c)^(1/
2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

## 3.304 $\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=121

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)}$$

**Rubi** [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt
[c]])/(3*a^2) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(3*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{\mathrm{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\\
&= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2}\\
&= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \ldots\right)}{3a^2d}\\
&= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}
\end{aligned}
$$

**Mathematica [A]** time = 0.29, size = 112, normalized size = 0.93

$$
\frac{\frac{a\sqrt{c+dx^3}}{a+bx^3} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}} - 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3])/(a + b*x^3) - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]] + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(Sq
rt[b]*Sqrt[b*c - a*d]))/(3*a^2)
```

**IntegrateAlgebraic [A]** time = 0.51, size = 131, normalized size = 1.08

$$
\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2\sqrt{b}\sqrt{ad-bc}} - \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) + ((2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b
*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^2*Sqrt[b]*Sqrt[-(b*c) + a*d]
) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

**fricas [B]** time = 0.73, size = 856, normalized size = 7.07

$$
\left[-\frac{\left(\left(2b^2c-abd\right)x^3+2abc-a^2d\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^3\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-2\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{6\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^3\right)}, -\frac{\left(\left(2b^2c-abd\right)x^3+2abc-a^2d\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^3\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{3\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^3\right)}, \frac{4\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\left(\left(2b^2c-abd\right)x^3+2abc-a^2d\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{6\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^3\right)}, -\frac{\left(\left(2b^2c-abd\right)x^3+2abc-a^2d\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-2\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{3\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))
- 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3
)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b^2*c - a
^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3
), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2
*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b
*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4
*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c
 - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/
(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*a
rctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]
```

**giac [A]** time = 0.17, size = 114, normalized size = 0.94

$$\frac{\sqrt{dx^3+c}\,d}{3\left(\left(dx^3+c\right)b-bc+ad\right)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*a) - 1/3*(2*b*c - a*d)*a
rctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) +
2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

**maple [C]** time = 0.28, size = 934, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2
*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1
/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*
d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I
*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)
/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-1/a*b*(-1/3*(d*x^3
+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2
*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1
/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*
d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I
```

```
*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)
/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(2/3*(d*x^3+
c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)
```

**mupad [B]** time = 8.28, size = 182, normalized size = 1.50

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}-\frac{\left(\frac{b\,d}{3\left(b^2\,c-a\,b\,d\right)}-\frac{b^2\,c}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\left(a\,d-2\,b\,c\right)1\mathrm{i}}{6\,a^2\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) - (((b*d)/(3*(b^2*c - a*b*d)) - (b^2*c)/(3*a*(b^2*c - a*b*d
)))*(c + d*x^3)^(1/2))/(a + b*x^3) + (log((2*b*c - a*d + (c + d*x^3)^(1/2)*
(a*b*d - b^2*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*a^2*(
a*b*d - b^2*c)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x \left(a + bx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)
```

## 3.305 $\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=161

$$-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

**Rubi** [A] time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 99, 151, 156, 63, 208}

$$-\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-2*b*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*x^3*(a +
b*x^3)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*Sqrt[c])
- (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d
]])/(3*a^3*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3} - \frac{(4bc-$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d} - \frac{(4$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\right.}{3a^3\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.28, size = 190, normalized size = 1.18

$$\frac{\sqrt{c}\left(a\left(a+2bx^3\right)\sqrt{c+dx^3}\,(bc-ad)+\sqrt{b}\,x^3\left(a+bx^3\right)(4bc-3ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)-x^3\left(a+bx^3\right)\left(a^2d^2-5abcd+4b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}\,x^3\left(a+bx^3\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*x^3*(a + b*x^3)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]]) + Sqrt[c]*(a*(b*c - a*d)*(a + 2*b*x^3)*Sqrt[c + d*x^3] + Sqrt[b
]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*x^3*(a + b*x^3)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]]))/(3*a^3*Sqrt[c]*(-(b*c) + a*d)*x^3*(a + b*x^3))
```

**IntegrateAlgebraic [A]** time = 0.54, size = 157, normalized size = 0.98

$$\frac{\left(3a\sqrt{b}\,d-4b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}\,\sqrt{ad-bc}}{bc-ad}\right)}{3a^3\sqrt{ad-bc}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}+\frac{\left(-a-2bx^3\right)\sqrt{c+dx^3}}{3a^2x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((-a - 2*b*x^3)*Sqrt[c + d*x^3])/(3*a^2*x^3*(a + b*x^3)) + ((-4*b^(3/2)*c +
 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c -
a*d)])/(3*a^3*Sqrt[-(b*c) + a*d]) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]])/(3*a^3*Sqrt[c])
```

**fricas [A]** time = 1.11, size = 870, normalized size = 5.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(
b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt
(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*
x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*
c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*
c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + (
(4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(
a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c^2 - 3*a*b*c*
d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a))
 + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/3
*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c
*x^6 + a^4*c*x^3)]
```

**giac [A]** time = 0.22, size = 183, normalized size = 1.14

$$\frac{\left(4\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^3}-\frac{(4\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\left(dx^3+c\right)^{\frac{3}{2}}bd-2\,\sqrt{dx^3+c}\,bcd+\sqrt{dx^3+c}\,ad^2}{3\left(\left(dx^3+c\right)^2b-2\left(dx^3+c\right)bc+bc^2+\left(dx^3+c\right)ad-acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqr
t(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))
/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*d - 2*sqrt(d*x^3 + c)*b*c*d +
sqrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x
^3 + c)*a*d - a*c*d)*a^2)
```

**maple [C]** time = 0.26, size = 978, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1
/2))+2/a^3*b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3
)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*b^2*(-
1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2*b/a^3*
(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)
```

**mupad [B]** time = 9.69, size = 438, normalized size = 2.72

$$\frac{\left(\frac{a\left(\frac{a\left(\frac{a\left(\frac{b^2 d^2}{2a^3c^2}-\frac{b^2d^2(3ad-4bc)}{6a^2c^2(a^2d-abc)}+\frac{b^2d(2ad-bc)(3ad-4bc)}{6a^3c^2(a^2d-abc)}\right)}{b}-\frac{bd(2ad-bc)}{2a^3c^2}+\frac{b(3ad-4bc)(-a^2d^2+2abcd+2b^2c^2)}{6a^3c^2(a^2d-abc)}\right)}{b}-\frac{-a^2d^2+2abcd+2b^2c^2}{2a^3c^2}+\frac{b(ad-4bc)(3ad-4bc)}{6a^2c(a^2d-abc)}\right)}{b}-\frac{ad-4bc}{2a^2c}\right)\sqrt{dx^3+c}}{bx^3+a}-\frac{\sqrt{dx^3+c}}{3a^2x^3}+\frac{\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)^3\left(\sqrt{dx^3+c}+\sqrt{c}\right)}{x^6}\right)(ad-4bc)}{6a^3\sqrt{c}}+\frac{\sqrt{b}\,\ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\,\sqrt{dx^3+c}\,\sqrt{ad-bc}\,2\mathrm{i}}{bx^3+a}\right)(3ad-4bc)\,1\mathrm{i}}{6a^3\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (((a*((a*((a*((b^2*d^2)/(2*a^3*c^2) - (b^2*d^2*(3*a*d - 4*b*c))/(6*a^2*c^2*
(a^2*d - a*b*c)) + (b^2*d*(2*a*d - b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d
- a*b*c))))/b - (b*d*(2*a*d - b*c))/(2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*b^2
*c^2 - a^2*d^2 + 2*a*b*c*d))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c^2 -
 a^2*d^2 + 2*a*b*c*d)/(2*a^3*c^2) + (b*(a*d - 4*b*c)*(3*a*d - 4*b*c))/(6*a^
2*c*(a^2*d - a*b*c))))/b - (a*d - 4*b*c)/(2*a^2*c))*(c + d*x^3)^(1/2))/(a +
 b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*x^3) + (log((((c + d*x^3)^(1/2) - c^(1/2
))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a*d - 4*b*c))/(6*a^3*c^(1/2)) + (
```

```
b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i -
 b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(1/2))
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)
[Out] Timed out
```

## 3.306 $\int \frac{x^8 (c+dx^3)^{3/2}}{(a+bx^3)^2} \, dx$

<u>**Optimal**</u>. Leaf size=189

$$-\frac{a^2 \left(c+dx^3\right)^{5/2}}{3b^2 \left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-7ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}-\frac{a\sqrt{c+dx^3}\,(4bc-7ad)}{3b^4}-\frac{a\left(c+dx^3\right)^{3/2}(4bc}{9b^3(bc-ad)}$$

<u>**Rubi**</u> [A] time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2 \left(c+dx^3\right)^{5/2}}{3b^2 \left(a+bx^3\right)(bc-ad)}-\frac{a\left(c+dx^3\right)^{3/2}(4bc-7ad)}{9b^3(bc-ad)}-\frac{a\sqrt{c+dx^3}\,(4bc-7ad)}{3b^4}+\frac{a(4bc-7ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}+\frac{2\left(c+dx^3\right)^{5/2}}{15b^2 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(4*b*c - 7*a*d)*Sqrt[c + d*x^3])/(3*b^4) - (a*(4*b*c - 7*a*d)*(c + d*x^
3)^(3/2))/(9*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(5/2))/(15*b^2*d) - (a^2*(c
+ d*x^3)^(5/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

<u>Rule 50</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 63</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 80</u>

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

<u>Rule 89</u>

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(-\frac{1}{2}a(2bc-5ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-7ad))\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-7ad))}{}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right.}{3b^2(bc-ad)\left(a\right.}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right.}{3b^2(bc-ad)\left(a\right.}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right.}{3b^2(bc-ad)\left(a\right.}$$

**Mathematica [A]** time = 0.19, size = 162, normalized size = 0.86

$$\frac{\sqrt{c+dx^3}\left(105a^3d^2+5a^2bd\left(14dx^3-19c\right)+2ab^2\left(3c^2-34cdx^3-7d^2x^6\right)+6b^3x^3\left(c+dx^3\right)^2\right)}{45b^4d\left(a+bx^3\right)} + \frac{a(4bc-7ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c
+ 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b*x
^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

**IntegrateAlgebraic [A]** time = 0.29, size = 211, normalized size = 1.12

$$\frac{\left(7a^3d^2-11a^2bcd+4ab^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{9/2}\sqrt{ad-bc}}+\frac{\sqrt{c+dx^3}\left(105a^3d^2-95a^2bcd+70a^2bd^2x^3+6ab^2c^2-68ab^2cdx^3-14ab^2d^2x^6+6b^3c^2x^3+12b^3cdx^6+6b^3d^2x^9\right)}{45b^4d\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 6*b^3*c^2*x^3
- 68*a*b^2*c*d*x^3 + 70*a^2*b*d^2*x^3 + 12*b^3*c*d*x^6 - 14*a*b^2*d^2*x^6 +
 6*b^3*d^2*x^9))/(45*b^4*d*(a + b*x^3)) + ((4*a*b^2*c^2 - 11*a^2*b*c*d + 7*
a^3*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/
(3*b^(9/2)*Sqrt[-(b*c) + a*d])
```

**fricas [A]** time = 0.69, size = 443, normalized size = 2.34

$$\left[-\frac{15\left(4a^2bcd-7a^3d^2+\left(4ab^2cd-7a^2bd^2\right)x^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(6b^3d^2x^9+2\left(6b^3cd-7ab^2d^2\right)x^6+6ab^2c^2-95a^2bcd+105a^3d^2+2\left(3b^3c^2-34ab^2cd+35a^2bd^2\right)x^3\right)\sqrt{dx^3+c}}{90\left(b^5dx^3+ab^4d\right)},\ \frac{15\left(4a^2bcd-7a^3d^2+\left(4ab^2cd-7a^2bd^2\right)x^3\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\left(6b^3d^2x^9+2\left(6b^3cd-7ab^2d^2\right)x^6+6ab^2c^2-95a^2bcd+105a^3d^2+2\left(3b^3c^2-34ab^2cd+35a^2bd^2\right)x^3\right)\sqrt{dx^3+c}}{45\left(b^5dx^3+ab^4d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt
((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c
- a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^
6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d
+ 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*(4*a^
2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a*d)/b)
*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^3*d^2*x
^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3
*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^5*d*x^3 + a*b^4*d)]
```

**giac [A]** time = 0.18, size = 211, normalized size = 1.12

$$-\frac{\left(4ab^2c^2-11a^2bcd+7a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^4}-\frac{\sqrt{dx^3+c}\,a^2bcd-\sqrt{dx^3+c}\,a^3d^2}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^4}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^8d^4-10\left(dx^3+c\right)^{\frac{3}{2}}ab^7d^5-30\sqrt{dx^3+c}\,ab^7cd^5+45\sqrt{dx^3+c}\,a^2b^6d^6\right)}{45\,b^{10}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt
(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b*c
*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(3*(
d*x^3 + c)^(5/2)*b^8*d^4 - 10*(d*x^3 + c)^(3/2)*a*b^7*d^5 - 30*sqrt(d*x^3 +
 c)*a*b^7*c*d^5 + 45*sqrt(d*x^3 + c)*a^2*b^6*d^6)/(b^10*d^5)
```

**maple [C]** time = 0.37, size = 1003, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b^2/d-2*a/b^2*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b
*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
```

```
2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*
d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2)),_alpha=RootOf(_Z^3*b+a))+a^2/b^2*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)
/(b*x^3+a)+2/3*d*(d*x^3+c)^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)
*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*
d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^
2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_a
lpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 7.75, size = 331, normalized size = 1.75

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,{\left(a\,d-b\,c\right)}^2}{b^4}+\frac{2\,c\,\left(\frac{2\,d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{3\,d}+\frac{2\,a\,\left(\frac{d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{a\,d^2}{b^3}\right)}{b}\right)}{3\,d}+\frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b^2}-\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{9\,d}-\frac{a^2\,\left(\frac{2\,b\,c^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,\left(7\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*(a*d - b*c)^2)/b^4 + (2*c*((2*d*(a*d - 2*b*c))/b^3 +
 (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(3*d) + (2*a*((d*(a*d - 2*b*c))/b^3 + (a
*d^2)/b^3))/b))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))/(15*b^2) - (x^3*(c + d*
x^3)^(1/2)*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(9*
d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i -
 b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(7*a*d - 4*b*c)*1i)/(6*b^(9/2)) -
(a^2*((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b
*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b^2*(a +
b*x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.307 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=163

$$-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{\sqrt{c+dx^3}\,(2bc-5ad)}{3b^3}+\frac{\left(c+dx^3\right)^{3/2}(2bc-5ad)}{9b^2(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{3b\left(a+bx^3\right)(bc-a$$

**Rubi** [A] time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(c+dx^3\right)^{3/2}(2bc-5ad)}{9b^2(bc-ad)}+\frac{\sqrt{c+dx^3}\,(2bc-5ad)}{3b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{a\left(c+dx^3\right)^{5/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3) + ((2*b*c - 5*a*d)*(c + d*x^3)^(3
/2))/(9*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(5/2))/(3*b*(b*c - a*d)*(a + b*x^
3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(3*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x\right.}{6b^2}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{((2bc-5a}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{((2bc-5a}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-5ad}{}$$

**Mathematica [A]** time = 0.14, size = 125, normalized size = 0.77

$$\frac{\sqrt{c+dx^3}\left(-15a^2d+ab\left(11c-10dx^3\right)+2b^2x^3\left(4c+dx^3\right)\right)}{9b^3\left(a+bx^3\right)} - \frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x^
3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

**IntegrateAlgebraic [A]** time = 0.29, size = 150, normalized size = 0.92

$$\frac{\left(-5a^2d^2+7abcd-2b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{7/2}\sqrt{ad-bc}} + \frac{\sqrt{c+dx^3}\left(-15a^2d+11abc-10abdx^3+8b^2cx^3+2b^2dx^6\right)}{9b^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(11*a*b*c - 15*a^2*d + 8*b^2*c*x^3 - 10*a*b*d*x^3 + 2*b^2*
d*x^6))/(9*b^3*(a + b*x^3)) + ((-2*b^2*c^2 + 7*a*b*c*d - 5*a^2*d^2)*ArcTan[
(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(7/2)*Sqrt[
-(b*c) + a*d])
```

**fricas** [A] time = 1.10, size = 314, normalized size = 1.93

$$\left[-\frac{3\left(\left(2\,b^{2}c-5\,abd\right)x^{3}+2\,abc-5\,a^{2}d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^{3}+2\,bc-ad+2\,\sqrt{dx^{3}+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^{3}+a}\right)-2\left(2\,b^{2}dx^{6}+2\left(4\,b^{2}c-5\,abd\right)x^{3}+11\,abc-15\,a^{2}d\right)\sqrt{dx^{3}+c}}{18\left(b^{4}x^{3}+ab^{3}\right)},\ -\frac{3\left(\left(2\,b^{2}c-5\,abd\right)x^{3}+2\,abc-5\,a^{2}d\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^{3}+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2\,b^{2}dx^{6}+2\left(4\,b^{2}c-5\,abd\right)x^{3}+11\,abc-15\,a^{2}d\right)\sqrt{dx^{3}+c}}{9\left(b^{4}x^{3}+ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b)
*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x
^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^2*d
)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*
a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c
- a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*
c - 15*a^2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3)]
```

**giac** [A] time = 0.19, size = 173, normalized size = 1.06

$$\frac{\left(2\,b^{2}c^{2}-7\,abcd+5\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\,\sqrt{-b^{2}c+abd}\,b^{3}}+\frac{\sqrt{dx^{3}+c}\,abcd-\sqrt{dx^{3}+c}\,a^{2}d^{2}}{3\left(\left(dx^{3}+c\right)b-bc+ad\right)b^{3}}+\frac{2\left(\left(dx^{3}+c\right)^{\frac{3}{2}}b^{4}+3\,\sqrt{dx^{3}+c}\,b^{4}c-6\,\sqrt{dx^{3}+c}\,ab^{3}d\right)}{9\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - sqr
t(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 + c)^
(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6
```

**maple** [C] time = 0.27, size = 983, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+
c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)
*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
)-a/b*(1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/2)/b^2*
d+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^
(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 7.38, size = 229, normalized size = 1.40

$$\frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{4\,c\,d}{3\,b^2}\right)}{3\,d}+\frac{a\,\left(\frac{2\,b\,c^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,\left(5\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*c
))/b^3 + (2*a*d^2)/b^3 + (4*c*d)/(3*b^2)))/(3*d) + (log((2*b*c - a*d + b^(1
/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b
*c)^(1/2)*(5*a*d - 2*b*c)*1i)/(6*b^(7/2)) + (a*((2*b*c^2)/(3*(2*b^2*c - 2*a
*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*
a*b*d))))/b)*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.308 $\int \frac{x^2 (c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

**Optimal**. Leaf size=94

$$-\frac{d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx^3)^{3/2}}{3b(a+bx^3)}+\frac{d\sqrt{c+dx^3}}{b^2}$$

**Rubi [A]** time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 47, 50, 63, 208}

$$-\frac{d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx^3)^{3/2}}{3b(a+bx^3)}+\frac{d\sqrt{c+dx^3}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (d*Sqrt[c + d*x^3])/b^2 - (c + d*x^3)^(3/2)/(3*b*(a + b*x^3)) - (d*Sqrt[b*c
 - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)\\
&= -\frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{2b}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{(d(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2b^2}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{b^2}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} - \frac{d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.02, size = 54, normalized size = 0.57

$$
\frac{2d\left(c+dx^3\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b\left(dx^3+c\right)}{ad-bc}\right)}{15(ad-bc)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (2*d*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x^3))/(-(
b*c) + a*d))])/(15*(-(b*c) + a*d)^2)
```

**IntegrateAlgebraic [A]** time = 0.19, size = 103, normalized size = 1.10

$$
\frac{d\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}} + \frac{\sqrt{c+dx^3}\left(3ad-bc+2bdx^3\right)}{3b^2\left(a+bx^3\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-(b*c) + 3*a*d + 2*b*d*x^3))/(3*b^2*(a + b*x^3)) + (d*Sqr
t[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c -
a*d)])/b^(5/2)
```

**fricas [A]** time = 0.90, size = 234, normalized size = 2.49

$$
\left[\frac{3\left(bdx^3+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(2\,bdx^3-bc+3\,ad\right)\sqrt{dx^3+c}}{6\left(b^3x^3+ab^2\right)}, -\frac{3\left(bdx^3+ad\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2\,bdx^3-bc+3\,ad\right)\sqrt{dx^3+c}}{3\left(b^3x^3+ab^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c +
3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(-(b
*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
(2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2)]
```

**giac [A]** time = 0.17, size = 122, normalized size = 1.30

$$\frac{2\,\sqrt{dx^3+c}\,d}{3\,b^2}+\frac{\left(bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx^3+c}\,bcd-\sqrt{dx^3+c}\,ad^2}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*x^3 + c)*d/b^2 + (b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-
b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/3*(sqrt(d*x^3 + c)*b*c*d - s
qrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^2)
```

**maple [C]** time = 0.27, size = 466, normalized size = 4.96

$$\frac{2\sqrt{d\,x^3+c}\;d}{3b^2}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{\frac{i}{2}\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{\frac{i}{2}\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-c\,d^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-c\,d^2\right)^{\frac{2}{3}}-\left(-c\,d^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{3}\;\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-c\,d^2\right)^{\frac{2}{3}}\sqrt{3}\;\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-c\,d^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{2b^2d\sqrt{d\,x^3+c}}+\frac{(ad-bc)\sqrt{d\,x^3+c}}{3\left(b\,x^3+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/2)/b^2*d+1/2*I
/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 7.35, size = 170, normalized size = 1.81

$$\frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b^2}-\frac{\left(\frac{2\,b\,c^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*(c + d*x^3)^(1/2))/(3*b^2) - (((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*
((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)
*(c + d*x^3)^(1/2))/(a + b*x^3) + (d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(
2*b^(5/2))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.309 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx$

**Optimal**. Leaf size=131

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}\,(bc-ad)}{3ab\left(a+bx^3\right)}$$

**Rubi [A]** time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}\,(bc-ad)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) - (2*c^(3/2)*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(3*a^2) + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx=\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx,x,x^3\right)$$

$$=\frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)}+\frac{\mathrm{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3ab}$$

$$=\frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)}+\frac{c^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2}-\frac{((bc-ad)(2bc+ad))\,\mathrm{Subst}\left(\int \frac{}{(a+b \cdots}\right.}{6a^2b}$$

$$=\frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)}+\frac{\left(2c^2\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2d}-\frac{((bc-ad)(2bc+ad))\,\mathrm{Sub}\cdots}{}$$

$$=\frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

**Mathematica [A]** time = 0.19, size = 122, normalized size = 0.93

$$\frac{\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}+\frac{a\sqrt{c+dx^3}\,(bc-ad)}{b\left(a+bx^3\right)}-2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) - 2*c^(3/2)*ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))/(3*a^2)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 156, normalized size = 1.19

$$\frac{\left(-a^2d^2-abcd+2b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2b^{3/2}\sqrt{ad-bc}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}\,(bc-ad)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) + ((2*b^2*c^2 - a*b*c*d -
 a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3]])/(b*c - a*d)])
/(3*a^2*b^(3/2)*Sqrt[-(b*c) + a*d]) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/(3*a^2)
```

**fricas [A]** time = 1.07, size = 686, normalized size = 5.24

$$\left[\frac{\left((2b^2c+abd)x^3+2abc+a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(b^2cx^3+abc\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\left(abc-a^2d\right)}{6\left(a^2b^2x^3+a^3b\right)},\ \frac{\left((2b^2c+abd)x^3+2abc+a^2d\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\left(b^2cx^3+abc\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+\sqrt{dx^3+c}\left(abc-a^2d\right)}{3\left(a^2b^2x^3+a^3b\right)},\ \frac{4\left(b^2cx^3+abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\left((2b^2c+abd)x^3+2abc+a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\sqrt{dx^3+c}\left(abc-a^2d\right)}{6\left(a^2b^2x^3+a^3b\right)},\ \frac{\left((2b^2c+abd)x^3+2abc+a^2d\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+2\left(b^2cx^3+abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}\left(abc-a^2d\right)}{3\left(a^2b^2x^3+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b*
d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a))
 + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/3*((
(2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*sqrt(c
)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 + c)*(a*b
*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arc
tan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)
*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt(
(b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*
x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a
*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + 2*(b^2
*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 +
c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b)]
```

**giac [A]** time = 0.17, size = 155, normalized size = 1.18

$$\frac{2\,c^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}} - \frac{\left(2\,b^2c^2 - abcd - a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2b} + \frac{\sqrt{dx^3+c}\,bcd - \sqrt{dx^3+c}\,ad^2}{3\left(\left(dx^3+c\right)b - bc + ad\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*(2*b^2*c^2 -
a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^
2*c + a*b*d)*a^2*b) + 1/3*(sqrt(d*x^3 + c)*b*c*d - sqrt(d*x^3 + c)*a*d^2)/(
((d*x^3 + c)*b - b*c + a*d)*a*b)
```

**maple [C]** time = 0.27, size = 1036, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d
*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d
-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*
b+a))-1/a*b*(1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/
2)/b^2*d+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha
*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
```

```
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x
^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)
```

**mupad [B]** time = 9.14, size = 214, normalized size = 1.63

$$\frac{c^{3/2}\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{a\left(\frac{b\,d^2}{3\left(b^2\,c-a\,b\,d\right)}-\frac{2\,b^2\,c\,d}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)}{b}+\frac{b^2\,c^2}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) + ((c + d*x^3)^(1/2)*((a*((b*d^2)/(3*(b^2*c - a*b*d)) - (2*
b^2*c*d)/(3*a*(b^2*c - a*b*d))))/b + (b^2*c^2)/(3*a*(b^2*c - a*b*d))))/(a +
 b*x^3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d + 2*b*c)*1i)/(6*a^2*b^(3/2
))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.310 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=170

$$\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}-\frac{\sqrt{c+dx^3}\,(2bc-ad)}{3a^2\left(a+bx^3\right)}-\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

**Rubi [A]** time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 98, 151, 156, 63, 208}

$$-\frac{\sqrt{c+dx^3}\,(2bc-ad)}{3a^2\left(a+bx^3\right)}+\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}-\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - (c*Sqrt[c + d*x^3])/
(3*a*x^3*(a + b*x^3)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/(3*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,\right.}{3a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3} + \ldots$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}\,(\ldots}{}$$

**Mathematica [A]** time = 0.33, size = 142, normalized size = 0.84

$$\frac{\frac{a\sqrt{c+dx^3}\left(-ac+adx^3-2bcx^3\right)}{x^3\left(a+bx^3\right)} + \sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + Sqr
t[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]] - (Sqrt[b*c - a*d]*(4
*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b])/(3
*a^3)
```

**IntegrateAlgebraic** [A] time = 0.68, size = 179, normalized size = 1.05

$$\frac{\left(4bc^{3/2}-3a\sqrt{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}+\frac{\sqrt{c+dx^3}\left(-ac+adx^3-2bcx^3\right)}{3a^2x^3\left(a+bx^3\right)}+\frac{\left(-a^2d^2+5abcd-4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^3\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(3*a^2*x^3*(a + b*x^3)) +
((-4*b^2*c^2 + 5*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt
[c + d*x^3])/(b*c - a*d)]])/(3*a^3*Sqrt[b]*Sqrt[-(b*c) + a*d]) + ((4*b*c^(3/
2) - 3*a*Sqrt[c]*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3)
```

**fricas** [A] time = 1.18, size = 838, normalized size = 4.93

$$\left[-\frac{\left((4b^2c-abd)x^6+(4abc-a^2d)x^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\left((4b^2c-3abd)x^6+(4abc-3a^2d)x^3\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\left((2abc-a^2d)x^3+a^2c\right)\sqrt{dx^3+c}}{6\left(a^3bx^6+a^4x^3\right)},\ -\frac{2\left((4b^2c-abd)x^6+(4abc-a^2d)x^3\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\left((4b^2c-3abd)x^6+(4abc-3a^2d)x^3\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\left((2abc-a^2d)x^3+a^2c\right)\sqrt{dx^3+c}}{6\left(a^3bx^6+a^4x^3\right)},\ -\frac{2\left((4b^2c-3abd)x^6+(4abc-3a^2d)x^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\left((4b^2c-abd)x^6+(4abc-a^2d)x^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left((2abc-a^2d)x^3+a^2c\right)\sqrt{dx^3+c}}{6\left(a^3bx^6+a^4x^3\right)},\ -\frac{\left((4b^2c-abd)x^6+(4abc-a^2d)x^3\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\left((4b^2c-3abd)x^6+(4abc-3a^2d)x^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\left((2abc-a^2d)x^3+a^2c\right)\sqrt{dx^3+c}}{3\left(a^3bx^6+a^4x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*
log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^
3 + a)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 +
a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x
^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*
sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3
*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2
*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1
/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3
)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt
((b*c - a*d)/b))/(b*x^3 + a)) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^
3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^
2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/
b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(
-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*a*b*c - a^2*d)*x^3 + a^2*c)*sq
rt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]
```

**giac** [A] time = 0.22, size = 216, normalized size = 1.27

$$\frac{\left(4\,b^2c^2-5\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^3}-\frac{\left(4\,bc^2-3\,acd\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\left(dx^3+c\right)^{\frac{3}{2}}bcd-2\,\sqrt{dx^3+c}\,bc^2d-\left(dx^3+c\right)^{\frac{3}{2}}ad^2+2\,\sqrt{dx^3+c}\,acd^2}{3\left(\left(dx^3+c\right)^2b-2\left(dx^3+c\right)bc+bc^2+\left(dx^3+c\right)ad-acd\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c^2 - 3*a*c*d)*arctan(sqrt(
d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*c*d - 2*sq
rt(d*x^3 + c)*b*c^2*d - (d*x^3 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^3 + c)*a*c*d^2
)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)*
a^2)
```

**maple** [C] time = 0.25, size = 1093, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d-c^(1/2)*d*arctanh((
d*x^3+c)^(1/2)/c^(1/2)))+2/a^3*b^2*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b
*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*
d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*b^2*(1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(
b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/2)/b^2*d+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*
_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2*b/a^3*(2/
9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)
^(1/2)/c^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)
```

**mupad [B]** time = 10.82, size = 531, normalized size = 3.12

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)^3\left(\sqrt{dx^3+c}+\sqrt{c}\right)}{x^6}\right)(3ad-4bc)}{6a^3} - \frac{c\sqrt{dx^3+c}}{3a^2x^3} - \frac{\sqrt{dx^3+c}\left(\frac{3ad-4bc}{2a^2} - \frac{a\left(\frac{a\left(\frac{a\left(\frac{bd^2(ad+bc)}{a^3c^2} - \frac{a\left(\frac{b^2d^3}{2a^3c^2} - \frac{b^2d^3(3ad-4bc)}{6a^2c^2(a^2d-abc)} + \frac{b^2d^2(ad+bc)(3ad-4bc)}{3a^3c^2(a^2d-abc)}\right)}{b} + \frac{b(3ad-4bc)(a^2d^3+4abcd^2-b^2c^2d)}{6a^3c^2(a^2d-abc)}\right)}{b} - \frac{a^2d^3+4abcd^2-b^2c^2d}{2a^3c^2} + \frac{b(3ad-4bc)(-4a^2cd^2+2abc^2d+2b^2c^3)}{6a^3c^2(a^2d-abc)}\right)}{b} - \frac{-4a^2cd^2+2abc^2d+2b^2c^3}{2a^3c^2} + \frac{b(3ad-4bc)^2}{6a^2(a^2d-abc)}\right)}{b}\right)}{bx^3+a} + \frac{\ln\left(\frac{2bc-ad+bdx^3+\sqrt{b}\,\sqrt{dx^3+c}\,\sqrt{ad-bc}\,2i}{bx^3+a}\right)\sqrt{ad-bc}\,(ad-4bc)\,1i}{6a^3\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6)*(3*a*d - 4*b*c))/(6*a^3) - (c*(c + d*x^3)^(1/2))/(3*a^2*x^3) - ((c +
 d*x^3)^(1/2)*((3*a*d - 4*b*c)/(2*a^2) - (a*((a*((a*((b*d^2*(a*d + b*c))/(a
^3*c^2) - (a*((b^2*d^3)/(2*a^3*c^2) - (b^2*d^3*(3*a*d - 4*b*c))/(6*a^2*c^2*
```

```
(a^2*d - a*b*c)) + (b^2*d^2*(a*d + b*c)*(3*a*d - 4*b*c))/(3*a^3*c^2*(a^2*d
- a*b*c))))/b + (b*(3*a*d - 4*b*c)*(a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2))/(6*
a^3*c^2*(a^2*d - a*b*c))))/b - (a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2)/(2*a^3*c
^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d))/(6*a^3*c^
2*(a^2*d - a*b*c))))/b - (2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d)/(2*a^3*c^2
) + (b*(3*a*d - 4*b*c)^2)/(6*a^2*(a^2*d - a*b*c))))/b))/(a + b*x^3) + (log(
(2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
 + b*x^3))*(a*d - b*c)^(1/2)*(a*d - 4*b*c)*1i)/(6*a^3*b^(1/2))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

## 3.311 $\int \frac{x^8}{(a+bx^3)^2 \sqrt{c+dx^3}} \, dx$

**Optimal**. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^3}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}+\frac{2\sqrt{c+dx^3}}{3b^2d}$$

**Rubi [A]** time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^3}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}+\frac{2\sqrt{c+dx^3}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b^2*d) - (a^2*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)*(a
 + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right.}{6b^2(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+\right.}{3b^2d(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.28, size = 107, normalized size = 0.87

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}\left(\frac{a^2}{\left(a+bx^3\right)(ad-bc)}+\frac{2}{d}\right)}{b^2} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((Sqrt[c + d*x^3]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^3))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/3
```

**IntegrateAlgebraic [A]** time = 0.31, size = 143, normalized size = 1.16

$$\frac{\left(3a^2d-4abc\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx^3}\left(3a^2d-2abc+2abdx^3-2b^2cx^3\right)}{3b^2d\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(Sqrt[c + d*x^3]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^3 + 2*a*b*d*x^3))/(b^
2*d*(b*c - a*d)*(a + b*x^3)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-
(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(5/2)*(-(b*c) + a*d)^(3/2)
)
```

**fricas [B]** time = 1.14, size = 475, normalized size = 3.86

$$\left[\frac{\left(4\,a^{2}bcd-3\,a^{3}d^{2}+\left(4\,ab^{2}cd-3\,a^{2}bd^{2}\right)x^{3}\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx^{3}+2\,bc-ad+2\,\sqrt{dx^{3}+c}\,\sqrt{b^{2}c-abd}}{bx^{3}+a}\right)+2\left(2\,ab^{3}c^{2}-5\,a^{2}b^{2}cd+3\,a^{3}bd^{2}+2\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{3}\right)\sqrt{dx^{3}+c}}{6\left(ab^{5}c^{2}d-2\,a^{2}b^{4}cd^{2}+a^{3}b^{3}d^{3}+\left(b^{6}c^{2}d-2\,ab^{5}cd^{2}+a^{2}b^{4}d^{3}\right)x^{3}\right)},-\frac{\left(4\,a^{2}bcd-3\,a^{3}d^{2}+\left(4\,ab^{2}cd-3\,a^{2}bd^{2}\right)x^{3}\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{dx^{3}+c}\,\sqrt{-b^{2}c+abd}}{bdx^{3}+bc}\right)-\left(2\,ab^{3}c^{2}-5\,a^{2}b^{2}cd+3\,a^{3}bd^{2}+2\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{3}\right)\sqrt{dx^{3}+c}}{3\left(ab^{5}c^{2}d-2\,a^{2}b^{4}cd^{2}+a^{3}b^{3}d^{3}+\left(b^{6}c^{2}d-2\,ab^{5}cd^{2}+a^{2}b^{4}d^{3}\right)x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2*
c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*
d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3), -1
/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3)]
```

**giac [A]** time = 0.23, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^{3}+c}\,a^{2}d}{3\left(b^{3}c-ab^{2}d\right)\left(\left(dx^{3}+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^{2}d\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\left(b^{3}c-ab^{2}d\right)\sqrt{-b^{2}c+abd}}+\frac{2\,\sqrt{dx^{3}+c}}{3\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^3 + c)*b - b*c + a*d))
- 1/3*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 2/3*sqrt(d*x^3 + c)/(b^2*d)
```

**maple [C]** time = 0.42, size = 911, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b^2/d+2/3*I*a/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+a^2/b^2*(
1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(
-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*
```

```
d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 7.29, size = 160, normalized size = 1.30

$$\frac{2\sqrt{d\,x^3+c}\,\left(2\,b^2\,c-2\,a\,b\,d\right)}{3\,d\,\left(2\,b^4\,c-2\,a\,b^3\,d\right)}-\frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2)*(2*b^2*c - 2*a*b*d))/(3*d*(2*b^4*c - 2*a*b^3*d)) - (2*
a^2*(c + d*x^3)^(1/2))/(3*b*(a + b*x^3)*(2*b^2*c - 2*a*b*d)) + (a*log((a*d
 - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*
x^3))*(3*a*d - 4*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.312 $\int \frac{x^5}{\left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=99

$$\frac{a\sqrt{c+dx^3}}{3b\left(a+bx^3\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

**Rubi** [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^3}}{3b\left(a+bx^3\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.10, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{\left(a+bx^3\right)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*b^(
3/2))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 109, normalized size = 1.10

$$\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}(ad-bc)^{3/2}} + \frac{a\sqrt{c+dx^3}}{3b\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) + ((2*b*c - a*d)*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(3/2)*(-(b*c
) + a*d)^(3/2))
```

**fricas [A]** time = 1.11, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^3+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{6\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^3\right)}, \frac{\left(\left(2\,b^2c-abd\right)x^3+2\,abc-a^2d\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{3\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) +
 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
```

```
^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^3)]
```

**giac [A]** time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^3+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^3+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

**maple [C]** time = 0.27, size = 892, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*
x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*
c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,
(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.85, size = 111, normalized size = 1.12

$$\frac{2\,a\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(3/2)) + (2*a*(c +
 d*x^3)^(1/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.313 $\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)}$$

**Rubi** [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*(b*c - a*d)*(a + b*x^3)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}$$

**Mathematica** [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)(ad-bc)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]/((-(b*c) + a*d)*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3
```

**IntegrateAlgebraic** [A] time = 0.11, size = 97, normalized size = 1.11

$$-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)} - \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{b}\,(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(
b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

**fricas** [B] time = 0.89, size = 302, normalized size = 3.47

$$\left[-\frac{\left(bdx^3+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{6\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^3\right)},-\frac{\left(bdx^3+ad\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\sqrt{dx^3+c}\left(b^2c-abd\right)}{3\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b^2*c
```

```
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^3)]
```

**giac [A]** time = 0.16, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^3+c}\,d}{3\left(\left(dx^3+c\right)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*(b*c - a
*d))
```

**maple [C]** time = 0.26, size = 457, normalized size = 5.25

$$-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{d}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)}}\right)}{6d\,(ad-bc)^2\sqrt{dx^3+c}}+\frac{\sqrt{dx^3+c}}{3\,(ad-bc)\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(
-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*
d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 6.37, size = 104, normalized size = 1.20

$$-\frac{2\,b\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*
x^3)/(a + b*x^3))*1i)/(6*b^(1/2)*(a*d - b*c)^(3/2)) - (2*b*(c + d*x^3)^(1/2
))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.314 $\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(bc-ad)}$$

**Rubi [A]** time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*Sqrt[c + d*x^3])/(3*a*(b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(b(2bc-3ad))\operatorname{Subst}\left(\int \frac{}{(a+}\right.}{6a^2(bc-ad}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(b(2bc-3ad))\operatorname{Subs}}{3}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.26, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^3}}{\left(a+bx^3\right)(bc-ad)} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*a^2)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 146, normalized size = 1.11

$$\frac{\left(3a\sqrt{b}\,d-2b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2(ad-bc)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(b*Sqrt[c + d*x^3])/(a*(-(b*c) + a*d)*(a + b*x^3)) + ((-2*b^(3/2)*c +
3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a
*d)])/(3*a^2*(-(b*c) + a*d)^(3/2)) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(
3*a^2*Sqrt[c])
```

**fricas [A]** time = 0.93, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^3+c}\,abc + \left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right) + 2\left(\left(b^2c-abd\right)x^3+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{6\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^3\right)}, \frac{\sqrt{dx^3+c}\,abc + \left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) + \left(\left(b^2c-abd\right)x^3+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{3\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^3\right)}, \frac{2\sqrt{dx^3+c}\,abc + 4\left(\left(b^2c-abd\right)x^3+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + \left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)}{6\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^3\right)}, \frac{\sqrt{dx^3+c}\,abc + \left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) + 2\left(\left(b^2c-abd\right)x^3+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{3\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c
)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3
 + b*c)) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqr
t(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^3), 1/6*(2*sqrt(d*x^3 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^3 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c^2 - 3*a^2*
c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c
 - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c)*
a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

**giac [A]** time = 0.16, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^3+c}\,bd}{3\left(abc-a^2d\right)\left(\left(dx^3+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*b*d/((a*b*c - a^2*d)*((d*x^3 + c)*b - b*c + a*d)) - 1/3
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

**maple [C]** time = 0.28, size = 915, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a^2*b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^
(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x
+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)
*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-1/a*b*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2
)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
```

```
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)
*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2/3*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/a^2/c^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2 \sqrt{dx^3+c}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)
```

**mupad [B]** time = 9.65, size = 162, normalized size = 1.23

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,\sqrt{c}}+\frac{b^2\,\sqrt{d\,x^3+c}}{3\,a\,\left(b\,x^3+a\right)\,\left(b^2\,c-a\,b\,d\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)\,1\mathrm{i}}{6\,a^2\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(1/2)) + (b^2*(c + d*x^3)^(1/2))/(3*a*(a + b*x^3)*(b^2*c - a*b*d)) +
 (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
 - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^2 \sqrt{c+dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

## 3.315 $\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b\sqrt{c+dx^3}\,(2bc-ad)}{3a^2c\left(a+bx^3\right)(bc-ad)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}$$

**Rubi** [A] time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b\sqrt{c+dx^3}\,(2bc-ad)}{3a^2c\left(a+bx^3\right)(bc-ad)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*c*(b*c - a*d)*(a + b*x^3)) - Sqrt
[c + d*x^3]/(3*a*c*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(3*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{3a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c}}\right.}{6a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b^{3/2}}{}$$

**Mathematica [A]** time = 0.61, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^3}\left(a^2d+ab\left(dx^3-c\right)-2b^2cx^3\right)}{x^3\left(a+bx^3\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(a^2*d - 2*b^2*c*x^3 + a*b*(-c + d*x^3)))/((b*c - a*d)*
x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(3*a^3*c)
```

**IntegrateAlgebraic** [A] time = 0.64, size = 187, normalized size = 1.01

$$\frac{\left(4b^{5/2}c-5ab^{3/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^3(ad-bc)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}+\frac{\sqrt{c+dx^3}\left(-a^2d+abc-abdx^3+2b^2cx^3\right)}{3a^2cx^3\left(a+bx^3\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]*(a*b*c - a^2*d + 2*b^2*c*x^3 - a*b*d*x^3))/(3*a^2*c*(-(b*c
) + a*d)*x^3*(a + b*x^3)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^3*(-(b*c) + a*d)^(3/
2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(3/2))
```

**fricas** [A] time = 0.91, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)
*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 +
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a
^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*l
og((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arcta
n(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/3*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3)]
```

**giac** [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^3+c}\,b^2c^2d-\left(dx^3+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^3+c}\,abcd^2-\sqrt{dx^3+c}\,a^2d^3}{3\left(a^2bc^2-a^3cd\right)\left(\left(dx^3+c\right)^2b-2\left(dx^3+c\right)bc+bc^2+\left(dx^3+c\right)ad-acd\right)}-\frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^3 + c)*b^2*c^2*d - (d*x^3 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^3 + c
)*a*b*c*d^2 - sqrt(d*x^3 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^3 + c)^
```

```
2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)) - 1/3*(4*b*c +
a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

**maple [C]** time = 0.29, size = 961, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2))-2/3*I/a^3*b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b
*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*b^2*(1/3/(a*d-b*c)*(
d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(
1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)
^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alp
ha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+4/3*b/a^3*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^2 \sqrt{dx^3 + c}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 11.55, size = 355, normalized size = 1.92

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{d\,a^2+4\,b\,c\,a}{2\,a^3\,c^2}-\frac{a\,\left(\frac{2\,c\,b^2+2\,a\,d\,b}{2\,a^3\,c^2}-\frac{a\,\left(\frac{b^2\,d}{2\,a^3\,c^2}+\frac{b\,\left(2\,c\,b^2+2\,a\,d\,b\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}-\frac{b^2\,d\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}+\frac{b\,\left(d\,a^2+4\,b\,c\,a\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}\right)}{b\,x^3+a}-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,c\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\,{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}^3}{x^6}\right)\,\left(a\,d+4\,b\,c\right)}{6\,a^3\,c^{3/2}}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,a^3\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((a^2*d + 4*a*b*c)/(2*a^3*c^2) - (a*((2*b^2*c + 2*a*b*d)
/(2*a^3*c^2) - (a*((b^2*d)/(2*a^3*c^2) + (b*(2*b^2*c + 2*a*b*d)*(3*a*d - 4*
b*c))/(6*a^3*c^2*(a^2*d - a*b*c)) - (b^2*d*(3*a*d - 4*b*c))/(6*a^2*c^2*(a^2
*d - a*b*c))))/b + (b*(a^2*d + 4*a*b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d
- a*b*c))))/b))/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*c*x^3) + (log((((c +
```

```
 d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d + 4*b*c
))/(6*a^3*c^(3/2)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b*c)*1i)/(6*a^3*(a*
d - b*c)^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

### 3.316 $\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=150

$$\frac{-a^2d^2-2b^2c^2}{3b^2d\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{a^2}{3b^2\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

**Rubi [A]** time = 0.19, antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 78, 63, 208}

$$-\frac{a^2d^2+2b^2c^2}{3b^2d\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{a^2}{3b^2\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(2*b^2*c^2 + a^2*d^2)/(3*b^2*d*(b*c - a*d)^2*Sqrt[c + d*x^3]) - a^2/(3*b^2
*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc+ad)+b(bc-ad)x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\,\ldots}{\ldots}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\,\ldots}{\ldots}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{a(4bc-ad)\,\text{ta}\ldots}{3b^{3/2}(\ldots}$$

**Mathematica [A]** time = 0.30, size = 134, normalized size = 0.89

$$\frac{\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{\sqrt{b}\left(a^2d\left(c+dx^3\right)+2abc^2+2b^2c^2x^3\right)}{d\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d)
^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2))/(3*b^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.25, size = 151, normalized size = 1.01

$$\frac{\left(4abc-a^2d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}(ad-bc)^{5/2}} + \frac{-a^2cd-a^2d^2x^3-2abc^2-2b^2c^2x^3}{3bd\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*a*b*c^2 - a^2*c*d - 2*b^2*c^2*x^3 - a^2*d^2*x^3)/(3*b*d*(b*c - a*d)^2*(
a + b*x^3)*Sqrt[c + d*x^3]) + ((4*a*b*c - a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c
) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*b^(3/2)*(-(b*c) + a*d)^(5/2))
```

**fricas [B]** time = 0.92, size = 746, normalized size = 4.97

$$\left[-\frac{\left(\left(4ab^2cd^2-a^2bd^3\right)x^6+4a^2bc^2d-a^3cd^2+\left(4ab^2c^2d+3a^2bcd^2-a^3d^3\right)x^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(2ab^3c^3-a^2b^2c^2d-a^3bcd^2+\left(2b^4c^3-2ab^3c^2d+a^2b^2cd^2-a^3bd^3\right)x^3\right)\sqrt{dx^3+c}}{6\left(ab^5c^4d-3a^2b^4c^3d^2+3a^3b^3c^2d^3-a^4b^2cd^4+\left(b^6c^3d^2-3ab^5c^2d^3+3a^2b^4cd^4-a^3b^3d^5\right)x^6+\left(b^6c^4d-2ab^5c^3d^2+2a^3b^3cd^4-a^4b^2d^5\right)x^3\right)}, -\frac{\left(\left(4ab^2cd^2-a^2bd^3\right)x^6+4a^2bc^2d-a^3cd^2+\left(4ab^2c^2d+3a^2bcd^2-a^3d^3\right)x^3\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\left(2ab^3c^3-a^2b^2c^2d-a^3bcd^2+\left(2b^4c^3-2ab^3c^2d+a^2b^2cd^2-a^3bd^3\right)x^3\right)\sqrt{dx^3+c}}{3\left(ab^5c^4d-3a^2b^4c^3d^2+3a^3b^3c^2d^3-a^4b^2cd^4+\left(b^6c^3d^2-3ab^5c^2d^3+3a^2b^4cd^4-a^3b^3d^5\right)x^6+\left(b^6c^4d-2ab^5c^3d^2+2a^3b^3cd^4-a^4b^2d^5\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3
+ 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(2*
a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d + a^2*
b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*c^3*d
^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d^3 + 3
*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2*a^3*b^
3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2
*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d
 + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^
4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*
d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2
*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3)]
```

**giac [A]** time = 0.19, size = 195, normalized size = 1.30

$$-\frac{\left(4\,abc-a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)b^2c^2-2\,b^2c^3+2\,abc^2d+\left(dx^3+c\right)a^2d^2}{3\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)*
b^2*c^2 - 2*b^2*c^3 + 2*a*b*c^2*d + (d*x^3 + c)*a^2*d^2)/((b^3*c^2*d - 2*a*
b^2*c*d^2 + a^2*b*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*
x^3 + c)*a*d))
```

**maple [C]** time = 0.36, size = 978, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/b^2/d/(d*x^3+c)^(1/2)-2*a/b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*
I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(
2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1
/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/
d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+a^2/b^2*(-1/3*b/(a*d-b*c)^2*(d*
x^3+c)^(1/2)/(b*x^3+a)-2/3*d/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/2*I*b/d*2^(1
/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alp
ha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2
```

```
)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^
(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)
^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_al
pha=RootOf(_Z^3*b+a)))
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [**B**] time = 7.90, size = 367, normalized size = 2.45

$$\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\,\left(a\,d+b\,c\right)}{b\,d}+\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)+\frac{a\,c\,\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d}\right)}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*(x^3*((((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2))*(a*d + b*c))/(b*d) + (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d
 - 2*a*b^2*c*d^2)) + (a*c*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2)))/(b*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (a*log((
2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
+ b*x^3))*(a*d - 4*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(5/2))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.317 $\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=134

$$\frac{a}{3b\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{5/2}}$$

**Rubi [A]** time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{3b\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(3*b*(b*c - a*d)^2*Sqrt[c + d*x^3]) + a/(3*b*(b*c - a*d)*(a +
 b*x^3)*Sqrt[c + d*x^3]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\text{Subst}\left(\right.}{6(b}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\text{Subst}\left(\right.}{3a}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(2bc+ad)\tanh^{-1}}{3\sqrt{b}\,(bc-a}$$

**Mathematica [C]** time = 0.03, size = 91, normalized size = 0.68

$$\frac{\left(a+bx^3\right)(ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+a(bc-ad)}{3b\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (a*(b*c - a*d) + (2*b*c + a*d)*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2,
(b*(c + d*x^3))/(b*c - a*d)])/(3*b*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3
])
```

**IntegrateAlgebraic [A]** time = 0.26, size = 123, normalized size = 0.92

$$\frac{3ac+adx^3+2bcx^3}{3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}+\frac{(-ad-2bc)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}\,\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{b}\,(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (3*a*c + 2*b*c*x^3 + a*d*x^3)/(3*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
 + ((-2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3]])/(b*c
 - a*d)])/(3*Sqrt[b]*(-(b*c) + a*d)^(5/2))
```

**fricas [B]** time = 1.42, size = 630, normalized size = 4.70

$$\left[\frac{\left(\left(2b^2cd+abd^2\right)x^6+2abc^2+a^2cd+\left(2b^2c^2+3abcd+a^2d^2\right)x^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(3ab^2c^2-3a^2bcd+\left(2b^3c^2-ab^2cd-a^2bd^2\right)x^3\right)\sqrt{dx^3+c}}{6\left(ab^4c^4-3a^2b^3c^3d+3a^3b^2c^2d^2-a^4bcd^3+\left(b^5c^3d-3ab^4c^2d^2+3a^2b^3cd^3-a^3b^2d^4\right)x^6+\left(b^5c^4-2ab^4c^3d+2a^3b^2cd^3-a^4bd^4\right)x^3\right)}, \frac{\left(\left(2b^2cd+abd^2\right)x^6+2abc^2+a^2cd+\left(2b^2c^2+3abcd+a^2d^2\right)x^3\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\left(3ab^2c^2-3a^2bcd+\left(2b^3c^2-ab^2cd-a^2bd^2\right)x^3\right)\sqrt{dx^3+c}}{3\left(ab^4c^4-3a^2b^3c^3d+3a^3b^2c^2d^2-a^4bcd^3+\left(b^5c^3d-3ab^4c^2d^2+3a^2b^3cd^3-a^3b^2d^4\right)x^6+\left(b^5c^4-2ab^4c^3d+2a^3b^2cd^3-a^4bd^4\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b
*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqr
t(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*b*c
*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c^4 -
 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*b^4*c
^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*d + 2*
a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^
2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*a
rctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c^2
- 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(
a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4
*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3)]
```

**giac [A]** time = 0.20, size = 181, normalized size = 1.35

$$\frac{\frac{\left(2\,bcd+ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{2\left(dx^3+c\right)bcd-2\,bc^2d+\left(dx^3+c\right)ad^2+2\,acd^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3 + c)*b*c*d -
2*b*c^2*d + (d*x^3 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d)))/d
```

**maple [C]** time = 0.27, size = 958, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c
)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)
^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf
(_Z^3*b+a))-a/b*(-1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(a*d-b*c
)^2/((x^3+c/d)*d)^(1/2)*d+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3
)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 7.70, size = 247, normalized size = 1.84

$$-\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)-\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d + 2*b*c)*1i)/(6*b^(1/2)*(a*d - b*c)^(5/2)) - ((c + d*x
^3)^(1/2)*(x^3*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3
*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2*a*b
^2*c*d^2)) - (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)))/(a*c + x^3
*(a*d + b*c) + b*d*x^6)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.318 $\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=108

$$-\frac{d}{\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{1}{3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

**Rubi** [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$-\frac{d}{\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{1}{3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d/((b*c - a*d)^2*Sqrt[c + d*x^3])) - 1/(3*(b*c - a*d)*(a + b*x^3)*Sqrt[c
+ d*x^3]) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(b*c - a*d)^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{2(bc-ad)}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(bd)\,\text{Subst}\left(\int \frac{1}{(a+bx)\cdots}\cdots\right)}{2(bc-a\cdots}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\cdots\right)}{(bc-a\cdots}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+\cdots}}{\sqrt{bc-a\cdots}}\right)}{(bc-ad)^{5/2}}$$

**Mathematica [C]** time = 0.02, size = 54, normalized size = 0.50

$$-\frac{2d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b\left(dx^3+c\right)}{ad-bc}\right)}{3\sqrt{c+dx^3}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x^3))/(-(b*c) + a*d))])/(
3*(-(b*c) + a*d)^2*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.24, size = 110, normalized size = 1.02

$$\frac{-2ad-bc-3bdx^3}{3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2} + \frac{\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-(b*c) - 2*a*d - 3*b*d*x^3)/(3*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
+ (Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d
)]])/(-(b*c) + a*d)^(5/2)
```

**fricas [B]** time = 1.47, size = 450, normalized size = 4.17

$$\left[\frac{3\left(bd^2x^6+\left(bcd+ad^2\right)x^3+acd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\left(3bdx^3+bc+2ad\right)\sqrt{dx^3+c}}{6\left(\left(b^3c^2d-2ab^2cd^2+a^2bd^3\right)x^6+ab^2c^3-2a^2bc^2d+a^3cd^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^3\right)}, \frac{3\left(bd^2x^6+\left(bcd+ad^2\right)x^3+acd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-\left(3bdx^3+bc+2ad\right)\sqrt{dx^3+c}}{3\left(\left(b^3c^2d-2ab^2cd^2+a^2bd^3\right)x^6+ab^2c^3-2a^2bc^2d+a^3cd^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(b/(b*c - a*d))*log((
b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/
```

```
(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), 1/3*(3*(b*d^2*x^6 + (b*c
*d + a*d^2)*x^3 + a*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c
- a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (3*b*d*x^3 + b*c + 2*a*d)*sq
rt(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
x^3)]
```

**giac [A]** time = 0.21, size = 153, normalized size = 1.42

$$-\frac{bd\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}-\frac{3\left(dx^3+c\right)bd-2\,bcd+2\,ad^2}{3\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(3*(d*x^3 + c)*b*d - 2*b*c*d + 2*a*d^
2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*
b*c + sqrt(d*x^3 + c)*a*d))
```

**maple [C]** time = 0.25, size = 485, normalized size = 4.49

$$\frac{ib\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)d-\left(-cd^2\right)^{\frac{1}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)d-i\sqrt{3}\left(-cd^2\right)^{\frac{2}{3}}-\left(-cd^2\right)^{\frac{2}{3}}\right)\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-cd^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-cd^2\right)^{\frac{1}{3}}}}}{3},\frac{\left(2i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d+i\sqrt{3}\,cd-3cd-i\left(-cd^2\right)^{\frac{2}{3}}\sqrt{3}\,\mathrm{RootOf}\left(\_Z^3b+a\right)-3\left(-cd^2\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^3b+a\right)\right)b}{2(ad-bc)d},\sqrt{\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{\left(-\frac{3\left(-cd^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}{2d}\right)d}}\right)}{2d\left(ad-bc\right)^3\sqrt{dx^3+c}}-\frac{\sqrt{dx^3+c}\,b}{3\left(ad-bc\right)^2\left(bx^3+a\right)}-\frac{2d}{3\left(ad-bc\right)^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(a*d-b*c)^2/((x^3+c/d)*d)^
(1/2)*d+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 7.43, size = 199, normalized size = 1.84

$$-\frac{\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}+\frac{b^2\,d^2\,x^3}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\,\sqrt{d\,x^3+c}}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+\frac{\sqrt{b}\,d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{2\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i - b*d*x^3)/(a + b*x^3))*1i)/(2*(a*d - b*c)^(5/2)) - (((3*b*d*(a*d + b*c)
- b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (b^2*d^2
*x^3)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(c + d*x^3)^(1/2))/(a*c + x^
3*(a*d + b*c) + b*d*x^6)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

## 3.319 $\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=172

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b}{3a\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}\,(bc-ad)^2}$$

**Rubi [A]** time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b}{3a\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(3*a*c*(b*c - a*d)^2*Sqrt[c + d*x^3]) + b/(3*a*(b*c - a*d
)*(a + b*x^3)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^
2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(3*a^2*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}(b}{\ldots}\right.}{3\ldots}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+d\ldots}}\right.}{3a^2c\ldots}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\ldots}\right.}{3\ldots}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx\ldots}}{\sqrt{c}}\right.}{3a^2c^{3/2}\ldots}$$

**Mathematica [C]** time = 0.13, size = 123, normalized size = 0.72

$$\frac{\frac{b(2bc-5ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)}{a(ad-bc)} + \left(\frac{2b}{a}-\frac{2d}{c}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right) + \frac{b}{a+bx^3}}{3a\sqrt{c+dx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b/(a + b*x^3) + (b*(2*b*c - 5*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c +
 d*x^3))/(b*c - a*d)])/(a*(-(b*c) + a*d)) + ((2*b)/a - (2*d)/c)*Hypergeomet
ric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(3*a*(b*c - a*d)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.72, size = 183, normalized size = 1.06

$$\frac{\left(2b^{5/2}c-5ab^{3/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2(ad-bc)^{5/2}}+\frac{2a^2d^2+2abd^2x^3+b^2c^2+b^2cdx^3}{3ac\left(a+bx^3\right)\sqrt{c+dx^3}\,(ad-bc)^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b^2*c^2 + 2*a^2*d^2 + b^2*c*d*x^3 + 2*a*b*d^2*x^3)/(3*a*c*(-(b*c) + a*d)^2
*(a + b*x^3)*Sqrt[c + d*x^3]) + ((2*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt
[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^3])/(b*c - a*d)])/(3*a^2*(-(b*c) + a*d)
^(5/2)) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c^(3/2))
```

**fricas [B]** time = 1.66, size = 1819, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log
((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*
b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))
/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^
3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
 a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*
b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b
*d*x^3 + b*c)) + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 -
 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + (a*b^2*
c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^
3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^
2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5
*c^2*d^3)*x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (2*a*b^2*c^4 - 5
*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c
^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d
 - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a*b
^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3
*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b
^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^3 + b*c)) + 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^
3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b^2*c^3 + 2*a^3*c*
d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*
d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3)
]
```

**giac [A]** time = 0.19, size = 226, normalized size = 1.31

$$-\frac{\left(2\,b^{3}c-5\,ab^{2}d\right)\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\sqrt{-b^{2}c+abd}}+\frac{\left(dx^{3}+c\right)b^{2}cd+2\left(dx^{3}+c\right)abd^{2}-2\,abcd^{2}+2\,a^{2}d^{3}}{3\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}\right)\left(\left(dx^{3}+c\right)^{\frac{3}{2}}b-\sqrt{dx^{3}+c}\,bc+\sqrt{dx^{3}+c}\,ad\right)}+\frac{2\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{3\,a^{2}\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*((d*x^3 +
 c)*b^2*c*d + 2*(d*x^3 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + s
qrt(d*x^3 + c)*a*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
)
```

**maple [C]** time = 0.27, size = 1002, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a^2*b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*
d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*
d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c
*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*b+a))-1/a*b*(-1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(
a*d-b*c)^2/((x^3+c/d)*d)^(1/2)*d+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^
2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2
*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{2}\left(dx^{3}+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)
```

**mupad [B]** time = 12.18, size = 288, normalized size = 1.67

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,c^{3/2}}+\frac{\left(\frac{{\left(2\,a\,d+b\,c\right)}^4+{\left(2\,a\,d+b\,c\right)}^2\,\left(\left(a\,d+2\,b\,c\right)\,\left(2\,a\,d+b\,c\right)-9\,a\,b\,c\,d\right)}{9\,a\,c\,{\left(2\,a\,d+b\,c\right)}^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x^3\,\left(2\,a\,d+b\,c\right)}{3\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,\sqrt{d\,x^3+c}}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(3/2)) + ((((2*a*d + b*c)^4 + (2*a*d + b*c)^2*((a*d + 2*b*c)*(2*a*d
+ b*c) - 9*a*b*c*d))/(9*a*c*(2*a*d + b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
) + (b*d*x^3*(2*a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(c + d
*x^3)^(1/2))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (b^(3/2)*log((2*b*c - a*d
+ b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5
*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(5/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a + bx^{3}\right)^{2} \left(c + dx^{3}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.320 $\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=241

$$-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{3a^2c^2\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{b(}{3a^2c\left(a+bx^3\right.}$$

**Rubi** [A] time = 0.36, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{3a^2c^2\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}-\frac{b(2bc-ad)}{3a^2c\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}-\frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*Sqrt[c +
d*x^3]) - (b*(2*b*c - a*d))/(3*a^2*c*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3
]) - 1/(3*a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}}{}\right.}{}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

**Mathematica [C]** time = 0.16, size = 189, normalized size = 0.78

$$\frac{b^2c^2x^3\left(a+bx^3\right)(4bc-7ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)-(ad-bc)\left(x^3\left(a+bx^3\right)\left(3a^2d^2+abcd-4b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+ac\left(a^2d+ab\left(dx^3-c\right)-2b^2cx^3\right)\right)}{3a^3c^2x^3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b^2*c^2*(4*b*c - 7*a*d)*x^3*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (b
*(c + d*x^3))/(b*c - a*d)] - (-(b*c) + a*d)*(a*c*(a^2*d - 2*b^2*c*x^3 + a*b
*(-c + d*x^3)) + (-4*b^2*c^2 + a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Hyperge
ometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(3*a^3*c^2*(b*c - a*d)^2*x^3*(a +
 b*x^3)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 0.95, size = 272, normalized size = 1.13

$$\frac{\left(7ab^{5/2}d-4b^{7/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}\sqrt{ad-bc}}{bc-ad}\right)}{3a^3(ad-bc)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}+\frac{-a^3cd^2-3a^3d^3x^3+2a^2bc^2d+a^2bcd^2x^3-3a^2bd^3x^6-ab^2c^3+ab^2c^2dx^3+2ab^2cd^2x^6-2b^3c^3x^3-2b^3c^2dx^6}{3a^2c^2x^3\left(a+bx^3\right)\sqrt{c+dx^3}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-(a*b^2*c^3) + 2*a^2*b*c^2*d - a^3*c*d^2 - 2*b^3*c^3*x^3 + a*b^2*c^2*d*x^3
 + a^2*b*c*d^2*x^3 - 3*a^3*d^3*x^3 - 2*b^3*c^2*d*x^6 + 2*a*b^2*c*d^2*x^6 -
3*a^2*b*d^3*x^6)/(3*a^2*c^2*(-(b*c) + a*d)^2*x^3*(a + b*x^3)*Sqrt[c + d*x^3
]) + ((-4*b^(7/2)*c + 7*a*b^(5/2)*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqr
t[c + d*x^3])/(b*c - a*d)])/(3*a^3*(-(b*c) + a*d)^(5/2)) + ((4*b*c + 3*a*d)
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2))
```

**fricas [B]** time = 2.00, size = 2384, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(b/(b*c -
 a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b
*c - a*d)))/(b*x^3 + a)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^
3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b
*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2
+ 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x
^3) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^
2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*
d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a
^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2
*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*
b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (
4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6
+ (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt
(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2
*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c
*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3
)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9
 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b
^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2*((4*b^4*c^3*d - 5*a*b^3
```

```
*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d -
7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2
*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c
)*sqrt(-c)/c) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3
*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt
(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*
sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^
2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*
c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((
a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4
*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*
d + a^6*c^4*d^2)*x^3), -1/3*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*
c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4
*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(
b*c - a*d))/(b*d*x^3 + b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*
c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a
^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*
d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*
b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4
*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c
^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3)]
```

**giac [A]** time = 0.22, size = 367, normalized size = 1.52

$$\frac{\left(4\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)^2b^3c^2d-2\left(dx^3+c\right)b^3c^3d-2\left(dx^3+c\right)^2ab^2cd^2+3\left(dx^3+c\right)ab^2c^2d^2+3\left(dx^3+c\right)^2a^2bd^3-7\left(dx^3+c\right)a^2bcd^3+2\,a^2bc^2d^3+3\left(dx^3+c\right)a^3d^4-2\,a^3cd^4}{3\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(\left(dx^3+c\right)^{\frac{5}{2}}b-2\left(dx^3+c\right)^{\frac{3}{2}}bc+\sqrt{dx^3+c}\,bc^2+\left(dx^3+c\right)^{\frac{3}{2}}ad-\sqrt{dx^3+c}\,acd\right)}-\frac{\left(4\,bc+3\,ad\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3
+ c)^2*b^3*c^2*d - 2*(d*x^3 + c)*b^3*c^3*d - 2*(d*x^3 + c)^2*a*b^2*c*d^2 +
3*(d*x^3 + c)*a*b^2*c^2*d^2 + 3*(d*x^3 + c)^2*a^2*b*d^3 - 7*(d*x^3 + c)*a^2
*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^3 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^3 + c)^(5/2)*b - 2*(d*x^3 + c)^(3
/2)*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^(3/2)*a*d - sqrt(d*x^3 + c)*a
*c*d)) - 1/3*(4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)
*c^2)
```

**maple [C]** time = 0.25, size = 1067, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+2/a^3*b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^
(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*
(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*
((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)
*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d
)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(
-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^
(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a
*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
```

```
(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*b^2*(-1/3/(a*d-
b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)*d+1/
2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2*b/a^3*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*a
rctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)
```

**mupad [B]** time = 19.63, size = 18847, normalized size = 78.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*b*log(1/x^6))/(3*a^3*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a^2*c^2*x^3) + (d*l
og(1/x^6))/(2*a^2*c^(5/2)) + (2*b*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(
c + d*x^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2))
)/(3*a^3*c^(3/2)) + (d*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(c + d*x^3)^
(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2)))/(2*a^2*c^
(5/2)) - (b^7*c^9*x^4*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^
6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 +
 a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*
c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) - (5*a^9*d^7*x^4*(c +
 d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4
- a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a
^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^
8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (3*a^2*d^2*x*(c + d*x^3)^(1/2))/(a^2*b^2*c^
4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4
 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) + (2*b^2*c^2*x*(c + d*x^3)^(1/2
))/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a
^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) - (b^(7/2)*c*log(
(a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (
a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) +
(a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2))
- (a^11*b^(5/2)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2
)) + (a^10*b^(7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(
1/2)) - (a^9*b^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)
^(1/2)) + (a^8*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b
*c)^(1/2)) + (a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d
 - b*c)^(1/2)) - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^
3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2)
+ b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^
```

(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (360*a^8*b^5*c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^10*b^3*c^5*d^4*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8*d*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)))*2i)/(3*a^3*(a*d - b*c)^(5/2)) + (b^(5/2)*d*log((a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (360*a^8*b^5*c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^10*b^3*c^5*d^4*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8*d*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)))*7i)/(6*a^2*(a*d - b*c)^(5/2)) + (5*a^4*d^4*x^4*(c + d*x^3)^(1/2))/(2*(a^4*b^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x^7 + 3*a^4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (65*a^3*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) - (8*b^3*c^3*x^4*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (14*b^3*c^4*x^4*(c + d*x^3)^(1/2))/(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7) - (5*a^7*c^2*d^5*x*(c + d*x^3)^(1/2))/(2*(a^5*b^4*c^9*x + a^4*b^5*c^9*x^4 - 3*a^6*b^3*c^7*d^2*x^4 - a^7*b^2*c^6*d^3*x^4 - 3*a^6*b^3*c^6*d^3*x^7 + 2*a^7*b^2*c^5*d^4*x^7 + 2*a^8*b*c^6*d^3*x - 3*a^7*b^2*c^7*d^2*x + a^5*b^4*c^8*d*x^4 + 2*a^8*b*c^5*d^4*x^4 + a^4*b^5*c^8*d*x^7)) - (5*a^7*c*d^6*x^4*(c + d*x^3)^(1/2))/(2*(a^5*b^4*c^9*x + a^4*b^5*c^9*x^4 - 3*a^6*b^3*c^7*d^2*x^4 - a^7*b^2*c^6*d^3*x^4 - 3*a^6*b^3*c^6*d^3*x^7 + 2*a^7*b^2*c^5*d^4*x^7 + 2*a^8*b*c^6*d^3*x - 3*a^7*b^2*c^7*d^2*x + a^5*b^4*c^8*d*x^4 + 2*a^8*b*c^5*d^4*x^4 + a^4*b^5*c^8*d*x^7)) - (3*a^8*c^2*d^5*x*(c + d*x^3)^(1/2))/(8*(a^6*b^4*c^9*x + a^5*b^5*c^9*x^4 - 3*a^7*b^3*c^7*d^2*x^4 - a^8*b^2*c^6*d^3*x^4 - 3*a^7*b^3*c^6*d^3*x^7 + 2*a^8*b^2*c^5*d^4*x^7 + 2*a^9*b*c^6*d^3*x - 3*a^8*b^2*c^7*d^2*x + a^6*b^4*c^8*d*x^4 + 2*a^9*b*c^5*d^4*x^4 + a^5*b^5*c^8*d*x^7)) - (3*a^8*c*d^6*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^4*c^9*x + a^5*b^5*c^9*x^4 - 3*a^7*b^3*c^7*d^2*x^4 - a^8*b^2*c^6*d^3*x^4 - 3*a^7*b^3*c^6*d^3*x^7 + 2*a^8*b^2*c^5*d^4*x^7 + 2*a^9*b*c^6*d^3*x - 3*a^8*b^2*c^7*d^2*x + a^6*b^4*c^8*d*x^4 + 2*a^9*b*c^5*d^4*x^4 + a^5*b^5*c^8*d*x^7)) + (23*a^9*c^3*d^5*x*(c + d*x^3)^(1/2))/(8*(a^7*b^4*c^10*x +

a^6*b^5*c^10*x^4 - 3*a^8*b^3*c^8*d^2*x^4 - a^9*b^2*c^7*d^3*x^4 - 3*a^8*b^3*
c^7*d^3*x^7 + 2*a^9*b^2*c^6*d^4*x^7 + 2*a^10*b*c^7*d^3*x - 3*a^9*b^2*c^8*d^
2*x + a^7*b^4*c^9*d*x^4 + 2*a^10*b*c^6*d^4*x^4 + a^6*b^5*c^9*d*x^7)) - (a*b
^6*c^9*x*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^6*x^4 + a^5*b
^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 + a^5*b^4*c^8*
d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*c^9*d^2*x - a
^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) + (3*a^2*b^6*c^9*x^4*(c + d*x^3)^(
1/2))/(4*(2*a^10*c^7*d^4*x + 2*a^10*c^6*d^5*x^4 + a^7*b^3*c^9*d^2*x^4 - 3*a
^8*b^2*c^8*d^3*x^4 + a^6*b^4*c^9*d^2*x^7 - 3*a^8*b^2*c^7*d^4*x^7 + a^7*b^3*
c^10*d*x - 3*a^9*b*c^8*d^3*x + a^6*b^4*c^10*d*x^4 - a^9*b*c^7*d^4*x^4 + 2*a
^9*b*c^6*d^5*x^7)) - (5*a^6*c^2*d^3*x*(c + d*x^3)^(1/2))/(2*(a^6*b^2*c^7*x
+ a^5*b^3*c^7*x^4 + 2*a^7*b*c^6*d*x + 2*a^6*b^2*c^5*d^2*x^7 + 3*a^6*b^2*c^6
*d*x^4 + 2*a^7*b*c^5*d^2*x^4 + a^5*b^3*c^6*d*x^7)) - (5*a^6*c*d^4*x^4*(c +
d*x^3)^(1/2))/(2*(a^6*b^2*c^7*x + a^5*b^3*c^7*x^4 + 2*a^7*b*c^6*d*x + 2*a^6
*b^2*c^5*d^2*x^7 + 3*a^6*b^2*c^6*d*x^4 + 2*a^7*b*c^5*d^2*x^4 + a^5*b^3*c^6*
d*x^7)) + (4*a^2*b^4*c^6*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^8*x + 2*a^8*c^5*d^
3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b
^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4
+ a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7) + (8*a*b^5*c^6*x^4*(c + d*x^3)^(
1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4
*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x +
a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4
*x^7)) - (14*a^2*b^4*c^7*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^9*x + 2*a^8*c^6*d^
3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b
^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4
+ a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7) - (14*a*b^5*c^7*x^4*(c + d*x^3)^
(1/2))/(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x
^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^
5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x
^7) + (a^3*b^4*c^7*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x
 + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*
c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a
^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (269*a^4*b^4*c^8*x*(c + d*x^3)^(
1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^
6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2
*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^
6*d^4*x^7)) + (65*a^6*c^2*d^4*x*(c + d*x^3)^(1/2))/(8*(a^5*b^3*c^8*x + 2*a^
8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 -
 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*
d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (65*a^6*c*d^5*x^4*(c
+ d*x^3)^(1/2))/(24*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*
a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c
^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^
7*b*c^4*d^4*x^7)) - (41*a^6*c^3*d^4*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x
+ 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2
*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*
b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (5*a^7*c^3*d^4*
x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4
+ 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8
*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 +
2*a^8*b*c^5*d^4*x^7)) + (47*a^8*c^4*d^4*x*(c + d*x^3)^(1/2))/(6*(a^7*b^3*c^
10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2
*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^
4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (34*a^3
*b^2*c^5*x*(c + d*x^3)^(1/2))/(3*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6
*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*
c^5*d^2*x^7)) + (47*a^3*c^2*d^2*x*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^6*x^4 +
2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b
^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) + (38*a^3*c*d^3*x^4*(c + d*x^3)^(1/2))

/(3*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) - (257*a^5*c^3*d^2*x*(c + d*x^3)^(1/2))/(24*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (5*a^9*c*d^6*x*(c + d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4 - a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (23*a^9*c^2*d^6*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^4*c^10*x + a^6*b^5*c^10*x^4 - 3*a^8*b^3*c^8*d^2*x^4 - a^9*b^2*c^7*d^3*x^4 - 3*a^8*b^3*c^7*d^3*x^7 + 2*a^9*b^2*c^6*d^4*x^7 + 2*a^10*b*c^7*d^3*x - 3*a^9*b^2*c^8*d^2*x + a^7*b^4*c^9*d*x^4 + 2*a^10*b*c^6*d^4*x^4 + a^6*b^5*c^9*d*x^7)) + (a^2*b^6*c^10*x*(c + d*x^3)^(1/2))/(2*(2*a^10*c^7*d^5*x + 2*a^10*c^6*d^6*x^4 + a^6*b^4*c^10*d^2*x^4 + a^7*b^3*c^9*d^3*x^4 - 3*a^8*b^2*c^8*d^4*x^4 + a^6*b^4*c^9*d^3*x^7 - 3*a^8*b^2*c^7*d^5*x^7 - 3*a^9*b*c^8*d^4*x + a^7*b^3*c^10*d^2*x - a^9*b*c^7*d^5*x^4 + 2*a^9*b*c^6*d^6*x^7)) + (a*b^7*c^10*x^4*(c + d*x^3)^(1/2))/(2*(2*a^10*c^7*d^5*x + 2*a^10*c^6*d^6*x^4 + a^6*b^4*c^10*d^2*x^4 + a^7*b^3*c^9*d^3*x^4 - 3*a^8*b^2*c^8*d^4*x^4 + a^6*b^4*c^9*d^3*x^7 - 3*a^8*b^2*c^7*d^5*x^7 - 3*a^9*b*c^8*d^4*x + a^7*b^3*c^10*d^2*x - a^9*b*c^7*d^5*x^4 + 2*a^9*b*c^6*d^6*x^7)) + (a^2*b^5*c^7*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (269*a^3*b^5*c^8*x^4*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (26*a^6*c^2*d^5*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (5*a^7*c^2*d^5*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (79*a^8*c^3*d^5*x^4*(c + d*x^3)^(1/2))/(12*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (34*a^2*b^3*c^5*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (239*a^5*c^2*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (3*a^2*b^5*c^8*x*(c + d*x^3)^(1/2))/(4*(2*a^9*c^6*d^4*x + 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^7*b^2*c^7*d^3*x^4 + a^5*b^4*c^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c^9*d*x - 3*a^8*b*c^7*d^3*x + a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8*b*c^5*d^5*x^7)) - (3*a*b^6*c^8*x^4*(c + d*x^3)^(1/2))/(4*(2*a^9*c^6*d^4*x + 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^7*b^2*c^7*d^3*x^4 + a^5*b^4*c^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c^9*d*x - 3*a^8*b*c^7*d^3*x + a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8*b*c^5*d^5*x^7)) + (3*a^3*b^5*c^9*x*(c + d*x^3)^(1/2))/(4*(2*a^10*c^7*d^4*x + 2*a^10*c^6*d^5*x^4 + a^7*b^3*c^9*d^2*x^4 - 3*a^8*b^2*c^8*d^3*x^4 + a^6*b^4*c^9*d^2*x^7 - 3*a^8*b^2*c^7*d^4*x^7 + a^7*b^3*c^10*d*x - 3*a^9*b*c^8*d^3*x + a^6*b^4*c^10*d*x^4 - a^9*b*c^7*d^4*x^4 + 2*a^9*b*c^6*d^5*x^7)) + (5*a^4*c*d^3*x*(c + d*x^3)^(1/2))/(2*(a^4*b^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x^7 + 3*a^4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (8*a*b^4*c^5*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (5*a^5*c*d^4*x*(c + d*x^3)^(1/2))/(a^4*b^3

```
*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*
c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4
 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) + (5*a^10*c
^2*d^6*x*(c + d*x^3)^(1/2))/(4*(a^7*b^5*c^10*x + a^6*b^6*c^10*x^4 - 3*a^8*b
^4*c^8*d^2*x^4 - a^9*b^3*c^7*d^3*x^4 + 2*a^10*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c
^7*d^3*x^7 + 2*a^9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^10*b^2*c^7*d
^3*x + a^7*b^5*c^9*d*x^4 + a^6*b^6*c^9*d*x^7)) + (5*a^10*c*d^7*x^4*(c + d*x
^3)^(1/2))/(4*(a^7*b^5*c^10*x + a^6*b^6*c^10*x^4 - 3*a^8*b^4*c^8*d^2*x^4 -
a^9*b^3*c^7*d^3*x^4 + 2*a^10*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c^7*d^3*x^7 + 2*a^
9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^10*b^2*c^7*d^3*x + a^7*b^5*c^
9*d*x^4 + a^6*b^6*c^9*d*x^7)) - (11*a*b^2*c^3*x*(c + d*x^3)^(1/2))/(3*(a^3*
b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4
*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (14*a*b^2*c^4*x*(c + d
*x^3)^(1/2))/(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5
*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7) + (11*a
*b*d^2*x^4*(c + d*x^3)^(1/2))/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3
*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*
c^2*d^2*x^7)) - (143*a^3*c*d^2*x*(c + d*x^3)^(1/2))/(24*(a^3*b^2*c^5*x^4 +
2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b
^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (22*b^2*c*d*x^4*(c + d*x^3)^(1/2))/(
3*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^
3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) + (11*a^3*b^2*c^3
*d^2*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7
*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 -
3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x
^7 + 2*a^6*b*c^3*d^4*x^7)) - (31*a^4*b^2*c^4*d^2*x*(c + d*x^3)^(1/2))/(2*(a
^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^
6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7
*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (2
8*a^2*b^4*c^5*d*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x
+ a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c
^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^
4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (29*a^5*b*c^2*d^4*x^4*(c + d*x^3)
^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d
^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x
+ a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d
^4*x^7)) + (23*a^4*b^2*c^5*d^2*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a
^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4
- 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6
*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) + (7*a^2*b^4*c^6*d*x^4
*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 +
 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*
b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2
*a^7*b*c^5*d^4*x^7)) - (7*a^5*b*c^3*d^4*x^4*(c + d*x^3)^(1/2))/(4*(a^5*b^3*
c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c
^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4
- a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (9*a^5*b^
2*c^5*d^2*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^
4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x
^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^
8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^3*b^4*c^6*d*x^4*(c + d*x^3)^(1/2))/(
8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 -
3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3
*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7))
- (9*a^6*b*c^3*d^4*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3
*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^
2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 +
 a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (23*a^6*b^2*c^6*d^2*x*(c + d*x
^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^
```

10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^
8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9
*b*c^6*d^4*x^7)) - (31*a^4*b^4*c^7*d*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^1
0*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*
c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4
 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (21*a^7*
b*c^4*d^4*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^
6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7
*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*
b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (209*a^2*b*c^2*d^2*x^4*(c + d*x^3)^
(1/2))/(12*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d
*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) - (89*a^
3*b^2*c^4*d*x^4*(c + d*x^3)^(1/2))/(6*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4
+ a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a
^6*b*c^5*d^2*x^7)) - (109*a^4*b*c^3*d^2*x^4*(c + d*x^3)^(1/2))/(12*(a^5*b^2
*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*
x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (33*a^2*b*c^2*d*x*(c + d*
x^3)^(1/2))/(2*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c
^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (3
29*a^2*b*c^3*d*x*(c + d*x^3)^(1/2))/(12*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^
4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2
*a^4*b*c^4*d^2*x^7)) - (205*a^4*b*c^4*d*x*(c + d*x^3)^(1/2))/(12*(a^5*b^2*c
^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^
4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) + (6*a^2*b^3*c^3*d^2*x^4*(c +
 d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c
^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^
2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c
^3*d^4*x^7) - (a^3*b^2*c^2*d^3*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^
7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 -
 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*
d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (19*a^3*b^3*c^4*d^2*x^
4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4
+ 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7
*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 +
2*a^7*b*c^4*d^4*x^7)) + (10*a^4*b^2*c^3*d^3*x^4*(c + d*x^3)^(1/2))/(3*(a^5*
b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b
^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*
x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) - (a^3*
b^3*c^5*d^2*x^4*(c + d*x^3)^(1/2))/(2*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^
4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d
^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^
4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (97*a^4*b^2*c^4*d^3*x^4*(c + d*x^3)^(
1/2))/(6*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4
*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x +
a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4
*x^7)) - (9*a^4*b^3*c^5*d^2*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^
9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 -
 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*
d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^5*b^2*c^4*d^3*x^
4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4
+ 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8
*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 +
2*a^8*b*c^5*d^4*x^7)) + (31*a^5*b^3*c^6*d^2*x^4*(c + d*x^3)^(1/2))/(24*(a^7
*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*
a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c
^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) +
(415*a^6*b^2*c^5*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^
7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3
*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^

```
3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (11*a*b*c*d*x*(c + d*x^
3)^(1/2))/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3
*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) - (10*
a^2*b^3*c^4*d*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^
3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d
^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^
4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (19*a^4*b*c^2*d^3*x*(c + d*x^3)^(1/2)
)/(6*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4
 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*
b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7
)) - (8*a*b^4*c^4*d*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x
 + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*
c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a
^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (15*a^4*b*c*d^4*x^4*(c + d*x^3)^(
1/2))/(2*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4
*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x +
a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4
*x^7)) + (12*a^3*b^3*c^5*d*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^8*x + 2*a^8*c^5*
d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6
*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^
4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7) + (51*a^5*b*c^3*d^3*x*(c + d*x
^3)^(1/2))/(4*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^
4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2
*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^
4*d^4*x^7)) - (14*a^3*b^3*c^6*d*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*
a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4
 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^
6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (169*a^5*b*c^4*d^3*
x*(c + d*x^3)^(1/2))/(12*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4
 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^
7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 +
 2*a^7*b*c^5*d^4*x^7)) - (9*a^4*b^3*c^6*d*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*
c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c
^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4
- a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^6*b*
c^4*d^3*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*
c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7
 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*
d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (7*a^5*b^3*c^7*d*x*(c + d*x^3)^(1/2))/(8*(a
^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 -
3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3
*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7))
+ (77*a^7*b*c^5*d^3*x*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^
3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8
*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^
4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (41*a*b^2*c^2*d*x^4*(c + d*
x^3)^(1/2))/(3*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c
^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) - (7
1*a^2*b*c*d^2*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^
4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2
*a^4*b*c^3*d^2*x^7)) + (133*a*b^2*c^3*d*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*
c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x
^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + b x^3\right)^2 \left(c + d x^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**4*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

## 3.321 $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=88

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

**Rubi [A]** time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*b^(3/2)*d^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^3\right)}{6bd}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^3}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.27, size = 123, normalized size = 1.40

$$
\frac{b\sqrt{d}\sqrt{a+bx^3}\left(c+dx^3\right)-\sqrt{bc-ad}\,(ad+bc)\sqrt{\frac{b\left(c+dx^3\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{3b^2d^{3/2}\sqrt{c+dx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^3]*(c + d*x^3) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^3))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^3])/Sqrt[b*c - a*
d]])/(3*b^2*d^(3/2)*Sqrt[c + d*x^3])
```

**IntegrateAlgebraic [A]** time = 1.30, size = 122, normalized size = 1.39

$$
\frac{(-ad-bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3b^{3/2}d^{3/2}}-\frac{\sqrt{c+dx^3}\,(ad-bc)}{3bd\sqrt{a+bx^3}\left(\frac{b\left(c+dx^3\right)}{a+bx^3}-d\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*((-(b*c) + a*d)*Sqrt[c + d*x^3])/(b*d*Sqrt[a + b*x^3]*(-d + (b*(c + d*
x^3))/(a + b*x^3))) + ((-(b*c) - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/(Sq
rt[d]*Sqrt[a + b*x^3])])/(3*b^(3/2)*d^(3/2))
```

**fricas [A]** time = 0.58, size = 256, normalized size = 2.91

$$
\left[\frac{4\sqrt{bx^3+a}\sqrt{dx^3+c}\,bd+(bc+ad)\sqrt{bd}\log\left(8b^2d^2x^6+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^3-4\left(2bdx^3+bc+ad\right)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{bd}\right)}{12b^2d^2},\ \frac{2\sqrt{bx^3+a}\sqrt{dx^3+c}\,bd+(bc+ad)\sqrt{-bd}\arctan\left(\frac{\left(2bdx^3+bc+ad\right)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-bd}}{2\left(b^2d^2x^6+abcd+\left(b^2cd+abd^2\right)x^3\right)}\right)}{6b^2d^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 - 4
```

```
*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d)))/(b^2*d
^2), 1/6*(2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*ar
ctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)
/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)))/(b^2*d^2)]
```

**giac** [A] time = 0.21, size = 104, normalized size = 1.18

$$\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^3+a}\,\sqrt{bd}+\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{bx^3+a}\,\sqrt{b^2c+\left(bx^3+a\right)bd-abd}}{bd}}{3\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*c + a*d)*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b*x^3 + a)*sqrt(b^2*c + (b*x^3 + a)
*b*d - a*b*d)/(b*d))/abs(b)
```

**maple** [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad** [B] time = 9.25, size = 283, normalized size = 3.22

$$\frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{d^3\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{b\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}-\frac{8\,\sqrt{a}\,\sqrt{c}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{3\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}\right)(a\,d+b\,c)}{3\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((2*a*d)/3 + (2*b*c)/3))/(d^3*((c + d*x^3)^
(1/2) - c^(1/2))) + (((a + b*x^3)^(1/2) - a^(1/2))^3*((2*a*d)/3 + (2*b*c)/3
))/(b*d^2*((c + d*x^3)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x^3
)^(1/2) - a^(1/2))^2)/(3*d^2*((c + d*x^3)^(1/2) - c^(1/2))^2))/(((a + b*x^3
)^(1/2) - a^(1/2))^4/((c + d*x^3)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a +
 b*x^3)^(1/2) - a^(1/2))^2)/(d*((c + d*x^3)^(1/2) - c^(1/2))^2)) - (2*atanh
```

```
((d^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^3)^(1/2) - c^(1
/2))))*(a*d + b*c))/(3*b^(3/2)*d^(3/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

## 3.322 $\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

**Rubi [A]** time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^3}}{\sqrt{b}\,\sqrt{c+dx^3}}\right)}{3\sqrt{b}\,\sqrt{d}}$$

**Mathematica** [A] time = 0.10, size = 85, normalized size = 1.77

$$\frac{2\sqrt{c+dx^3}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{d}\,\sqrt{bc-ad}\,\sqrt{\frac{b\left(c+dx^3\right)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^3])/Sqrt[b*c - a*d]])/(3*S
qrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^3))/(b*c - a*d)])
```

**IntegrateAlgebraic** [A] time = 0.85, size = 48, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{d}\,\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

**fricas** [B] time = 0.64, size = 194, normalized size = 4.04

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^6+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^3+4\left(2\,bdx^3+bc+ad\right)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{bd}\right)}{6\,bd}, -\frac{\sqrt{-bd}\arctan\left(\frac{\left(2\,bdx^3+bc+ad\right)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{-bd}}{2\left(b^2d^2x^6+abcd+\left(b^2cd+abd^2\right)x^3\right)}\right)}{3\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqr
t(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x^3))/(b*d)]
```

**giac [A]** time = 0.18, size = 54, normalized size = 1.12

$$-\frac{2\,b\log\left(\left|-\sqrt{bx^3+a}\,\sqrt{bd}+\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right|\right)}{3\,\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d -
a*b*d)))/(sqrt(b*d)*abs(b))
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 5.04, size = 49, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}\right)}{3\,\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(4*atan((b*((c + d*x^3)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^3)^(1/2)
 - a^(1/2)))))/(3*(-b*d)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

## 3.323 $\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

**Optimal**. Leaf size=48

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

**Rubi [A]** time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {446, 93, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 48, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

**IntegrateAlgebraic [A]** time = 0.88, size = 48, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^3}}{\sqrt{c}\sqrt{a+bx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^3])/(Sqrt[c]*Sqrt[a + b*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

**fricas [B]** time = 0.65, size = 204, normalized size = 4.25

$$\left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^3-4\left((bc+ad)x^3+2\,ac\right)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{ac}}{x^6}\right)}{6\,ac}, \frac{\sqrt{-ac}\arctan\left(\frac{\left((bc+ad)x^3+2\,ac\right)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{-ac}}{2\left(abcdx^6+a^2c^2+\left(abc^2+a^2cd\right)x^3\right)}\right)}{3\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x
^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^3 +
 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]
```

**giac [B]** time = 0.19, size = 89, normalized size = 1.85

$$-\frac{2\sqrt{bd}\,b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{3\sqrt{-abcd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*
d)*abs(b))
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 7.57, size = 136, normalized size = 2.83

$$-\frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{3\,\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^3)^(1/2) - a^(1/2))/((c + d*x^3)^(1/2) - c^(1/2))) - log(((
c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*c^(1/2) - (a^(1/2
)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^(1/2))))/((c + d*
x^3)^(1/2) - c^(1/2))))/(3*a^(1/2)*c^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

## 3.324 $\int \frac{1}{x^4 \sqrt{a+bx^3} \sqrt{c+dx^3}} dx$

**Optimal**. Leaf size=91

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 96, 93, 208}

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)}{6ac}$$
$$= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3ac}$$
$$= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

**Mathematica** [A] time = 0.07, size = 91, normalized size = 1.00

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}} - \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

**IntegrateAlgebraic** [A] time = 1.65, size = 119, normalized size = 1.31

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}} - \frac{\sqrt{a+bx^3}\,(ad-bc)}{3ac\sqrt{c+dx^3}\left(a-\frac{c(a+bx^3)}{c+dx^3}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*((-(b*c) + a*d)*Sqrt[a + b*x^3])/(a*c*Sqrt[c + d*x^3]*(a - (c*(a + b*x
^3))/(c + d*x^3))) + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a
]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

**fricas** [A] time = 0.79, size = 278, normalized size = 3.05

$$\left[\frac{\sqrt{ac}\,(bc+ad)x^3\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3+4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)-4\sqrt{bx^3+a}\sqrt{dx^3+c}\,ac}{12\,a^2c^2x^3}, -\frac{\sqrt{-ac}\,(bc+ad)x^3\arctan\left(\frac{((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-ac}}{2\,(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)}\right)+2\sqrt{bx^3+a}\sqrt{dx^3+c}\,ac}{6\,a^2c^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a*c)*(b*c + a*d)*x^3*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 + 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b
*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6) - 4*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*a*c)/(a^2*c^2*x^3), -1/6*(sqrt(-a*c)*(b*c + a*d)*x^3*arctan(1/2*((b*c +
a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 +
 a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3)) + 2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*a*
c)/(a^2*c^2*x^3)]
```

**giac [B]** time = 0.24, size = 413, normalized size = 4.54

$$\frac{\sqrt{bd}\, b^{4} d \left(\frac{(bc+ad)\arctan\left(-\frac{b^{2}c+abd-\left(\sqrt{bx^{3}+a}\,\sqrt{bd}-\sqrt{b^{2}c+(bx^{3}+a)bd-abd}\right)^{2}}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,ab^{3}cd}-\frac{2\left(b^{3}c^{2}-2ab^{2}cd+a^{2}bd^{2}-\left(\sqrt{bx^{3}+a}\,\sqrt{bd}-\sqrt{b^{2}c+(bx^{3}+a)bd-abd}\right)^{2}bc-\left(\sqrt{bx^{3}+a}\,\sqrt{bd}-\sqrt{b^{2}c+(bx^{3}+a)bd-abd}\right)^{2}ad\right)}{\left(b^{4}c^{2}-2ab^{3}cd+a^{2}b^{2}d^{2}-2\left(\sqrt{bx^{3}+a}\,\sqrt{bd}-\sqrt{b^{2}c+(bx^{3}+a)bd-abd}\right)^{2}b^{2}c-2\left(\sqrt{bx^{3}+a}\,\sqrt{bd}-\sqrt{b^{2}c+(bx^{3}+a)bd-abd}\right)^{2}abd+\left(\sqrt{bx^{3}+a}\,\sqrt{bd}-\sqrt{b^{2}c+(bx^{3}+a)bd-abd}\right)^{4}\right)ab^{2}cd}\right)}{3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^3 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^3 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

**maple [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

**mupad [B]** time = 10.77, size = 481, normalized size = 5.29

$$\frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{12}+\frac{a\,d\,b}{12}\right)}{a^{3/2}\,c^{3/2}\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{b^2}{12\,a\,c\,d}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{12}-\frac{a\,b\,c\,d}{4}+\frac{b^2\,c^2}{12}\right)}{a^2\,c^2\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2\left(a\,d+b\,c\right)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}+\frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2}-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2}-\frac{d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{12\,a\,c\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((b^2*c)/12 + (a*b*d)/12))/(a^(3/2)*c^(3/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))) - b^2/(12*a*c*d) + (((a + b*x^3)^(1/2) -
a^(1/2))^2*((a^2*d^2)/12 + (b^2*c^2)/12 - (a*b*c*d)/4))/(a^2*c^2*d*((c + d*
x^3)^(1/2) - c^(1/2))^2))/((((a + b*x^3)^(1/2) - a^(1/2))^3/((c + d*x^3)^(1/
2) - c^(1/2))^3 + (b*((a + b*x^3)^(1/2) - a^(1/2)))/(d*((c + d*x^3)^(1/2) -
 c^(1/2))) - (((a + b*x^3)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^3)^(1/2) - a^(1/2))/(
(c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(6*a
```

```
^2*c^2) - (log(((c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*
c^(1/2) - (a^(1/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^
(1/2))))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2
)*d))/(6*a^2*c^2) - (d*((a + b*x^3)^(1/2) - a^(1/2)))/(12*a*c*((c + d*x^3)^
(1/2) - c^(1/2)))
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

## 3.325 $\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

**Optimal**. Leaf size=161

$$-\frac{a^2e^{7/2}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}\,(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}\,(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}}{}$$

**Rubi [A]** time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$-\frac{a^2e^{7/2}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}\,(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}\,(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (a*(2*A*b - a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b^2) + ((2*A*b - a*B)
*(e*x)^(9/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/
(9*b*e) - (a^2*(2*A*b - a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)
*Sqrt[a + b*x^3])])/(24*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(-9Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\sqrt{a+bx^3}\,dx}{9b}$$

$$= \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} + \frac{(a(2Ab-aB))\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}}{8b}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+\right.}{9be}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+\right.}{9be}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+\right.}{9be}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+\right.}{9be}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+\right.}{9be}$$

**Mathematica [A]** time = 0.24, size = 145, normalized size = 0.90

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(3a^{3/2}(aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(-3a^2B+2ab\left(3A+Bx^3\right)+4b^2x^3\left(3A+2Bx^3\right)\right)\right)}{72b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-3*a^2
*B + 2*a*b*(3*A + B*x^3) + 4*b^2*x^3*(3*A + 2*B*x^3)) + 3*a^(3/2)*(-2*A*b +
 a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*
x^3)/a])
```

**IntegrateAlgebraic [A]** time = 0.60, size = 162, normalized size = 1.01

$$\frac{\sqrt{a+bx^3}\left(-3a^2Be^6(ex)^{3/2}+6aAbe^6(ex)^{3/2}+2abBe^3(ex)^{9/2}+12Ab^2e^3(ex)^{9/2}+8b^2B(ex)^{15/2}\right)}{72b^2e^4}+\frac{e^5\sqrt{\frac{b}{e^3}}\left(2a^2Ab-a^3B\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (Sqrt[a + b*x^3]*(6*a*A*b*e^6*(e*x)^(3/2) - 3*a^2*B*e^6*(e*x)^(3/2) + 12*A*
b^2*e^3*(e*x)^(9/2) + 2*a*b*B*e^3*(e*x)^(9/2) + 8*b^2*B*(e*x)^(15/2)))/(72*
b^2*e^4) + ((2*a^2*A*b - a^3*B)*Sqrt[b/e^3]*e^5*Log[-(Sqrt[b/e^3]*(e*x)^(3/
2)) + Sqrt[a + b*x^3]])/(24*b^3)
```

**fricas [A]** time = 1.26, size = 295, normalized size = 1.83

$$\left[-\frac{3\left(Ba^{3}-2\,Aa^{2}b\right)e^{3}\sqrt{\frac{e}{b}}\log\left(-8\,b^{2}ex^{6}-8\,abex^{3}-a^{2}e+4\left(2\,b^{2}x^{4}+abx\right)\sqrt{bx^{3}+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(8\,Bb^{2}e^{3}x^{7}+2\left(Bab+6\,Ab^{2}\right)e^{3}x^{4}-3\left(Ba^{2}-2\,Aab\right)e^{3}x\right)\sqrt{bx^{3}+a}\sqrt{ex}}{288\,b^{2}},-\frac{3\left(Ba^{3}-2\,Aa^{2}b\right)e^{3}\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^{3}+a}\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2\,bex^{3}+ae}\right)-2\left(8\,Bb^{2}e^{3}x^{7}+2\left(Bab+6\,Ab^{2}\right)e^{3}x^{4}-3\left(Ba^{2}-2\,Aab\right)e^{3}x\right)\sqrt{bx^{3}+a}\sqrt{ex}}{144\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(
8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)*
sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/144*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(-e/b
)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*
(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)
*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

**giac [A]** time = 0.75, size = 251, normalized size = 1.56

$$\frac{1}{12}\sqrt{bx^{3}e^{4}+ae^{4}}\left(2\,x^{3}e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Ax^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}\sqrt{bx^{3}e^{4}+ae^{4}}\left(2\left(4\,x^{3}e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^{3}e^{3}-\frac{3\,a^{2}e^{(-1)}}{b^{2}}\right)Bx^{\frac{3}{2}}e^{\frac{5}{2}}-\frac{\left(B^{2}a^{6}e^{7}-4\,ABa^{5}be^{7}+4\,A^{2}a^{4}b^{2}e^{7}\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\left(Ba^{3}x^{\frac{3}{2}}e^{\frac{11}{2}}-2\,Aa^{2}bx^{\frac{3}{2}}e^{\frac{11}{2}}\right)\sqrt{b}\,e^{\frac{1}{2}}+\sqrt{B^{2}a^{7}e^{12}-4\,ABa^{6}be^{12}+4\,A^{2}a^{5}b^{2}e^{12}+\left(Ba^{3}x^{\frac{3}{2}}e^{\frac{11}{2}}-2\,Aa^{2}bx^{\frac{3}{2}}e^{\frac{11}{2}}\right)^{2}be}\right|\right)}{24\,b^{\frac{5}{2}}\left|-Ba^{3}e^{3}+2\,Aa^{2}be^{3}\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*A*x^(3/2)*e^(5/2)
+ 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3*a
^2*e^(-1)/b^2)*B*x^(3/2)*e^(5/2) - 1/24*(B^2*a^6*e^7 - 4*A*B*a^5*b*e^7 + 4*
A^2*a^4*b^2*e^7)*e^(-1/2)*log(abs(-(B*a^3*x^(3/2)*e^(11/2) - 2*A*a^2*b*x^(3
/2)*e^(11/2))*sqrt(b)*e^(1/2) + sqrt(B^2*a^7*e^12 - 4*A*B*a^6*b*e^12 + 4*A^
2*a^5*b^2*e^12 + (B*a^3*x^(3/2)*e^(11/2) - 2*A*a^2*b*x^(3/2)*e^(11/2))^2*b*
e)))/(b^(5/2)*abs(-B*a^3*e^3 + 2*A*a^2*b*e^3))
```

**maple [C]** time = 8.02, size = 7293, normalized size = 45.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^{3}+A\right)\sqrt{bx^{3}+a}\,(ex)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,\sqrt{b\,x^3+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2), x)
```

**sympy [B]** time = 130.86, size = 292, normalized size = 1.81

$$\frac{A a^{\frac{3}{2}} e^{\frac{7}{2}} x^{\frac{3}{2}}}{12 b \sqrt{1+\frac{b x^{3}}{a}}}+\frac{A \sqrt{a}\, e^{\frac{7}{2}} x^{\frac{9}{2}}}{4 \sqrt{1+\frac{b x^{3}}{a}}}-\frac{A a^{2} e^{\frac{7}{2}} \operatorname{asinh}{\left(\frac{\sqrt{b} x^{\frac{3}{2}}}{\sqrt{a}} \right)}}{12 b^{\frac{3}{2}}}+\frac{A b e^{\frac{7}{2}} x^{\frac{15}{2}}}{6 \sqrt{a} \sqrt{1+\frac{b x^{3}}{a}}}-\frac{B a^{\frac{5}{2}} e^{\frac{7}{2}} x^{\frac{3}{2}}}{24 b^{2} \sqrt{1+\frac{b x^{3}}{a}}}-\frac{B a^{\frac{3}{2}} e^{\frac{7}{2}} x^{\frac{9}{2}}}{72 b \sqrt{1+\frac{b x^{3}}{a}}}+\frac{5 B \sqrt{a}\, e^{\frac{7}{2}} x^{\frac{15}{2}}}{36 \sqrt{1+\frac{b x^{3}}{a}}}+\frac{B a^{3} e^{\frac{7}{2}} \operatorname{asinh}{\left(\frac{\sqrt{b} x^{\frac{3}{2}}}{\sqrt{a}} \right)}}{24 b^{\frac{5}{2}}}+\frac{B b e^{\frac{7}{2}} x^{\frac{21}{2}}}{9 \sqrt{a} \sqrt{1+\frac{b x^{3}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*e**(7/2)*x**(3/2)/(12*b*sqrt(1 + b*x**3/a)) + A*sqrt(a)*e**(7/2)
*x**(9/2)/(4*sqrt(1 + b*x**3/a)) - A*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/s
qrt(a))/(12*b**(3/2)) + A*b*e**(7/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a
)) - B*a**(5/2)*e**(7/2)*x**(3/2)/(24*b**2*sqrt(1 + b*x**3/a)) - B*a**(3/2)
*e**(7/2)*x**(9/2)/(72*b*sqrt(1 + b*x**3/a)) + 5*B*sqrt(a)*e**(7/2)*x**(15/
2)/(36*sqrt(1 + b*x**3/a)) + B*a**3*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a)
)/(24*b**(5/2)) + B*b*e**(7/2)*x**(21/2)/(9*sqrt(a)*sqrt(1 + b*x**3/a))
```

## 3.326 $\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

**Optimal**. Leaf size=121

$$\frac{a\sqrt{e}\,(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

**Rubi [A]** time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a\sqrt{e}\,(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((4*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(3/2)*(a +
b*x^3)^(3/2))/(6*b*e) + (a*(4*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/
2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} - \frac{\left(-6Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{6b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}}{8b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\,\text{Subst}\left(\int\right.}{4be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\,\text{Subst}\left(\int\right.}{12be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\,\text{Subst}\left(\int\right.}{12be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{a(4Ab-aB)\sqrt{e}\tanh^{-1}\left(\right.}{12b^{3/2}}$$

**Mathematica [A]** time = 0.15, size = 119, normalized size = 0.98

$$\frac{\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}\,x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(B\left(a+2bx^3\right)+4Ab\right)-\sqrt{a}\,(aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)\right)}{12b^{3/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(4*A*b + B*
(a + 2*b*x^3)) - Sqrt[a]*(-4*A*b + a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])
)/(12*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])
```

**IntegrateAlgebraic [A]** time = 0.52, size = 123, normalized size = 1.02

$$\frac{\sqrt{a+bx^3}\left(aBe^3(ex)^{3/2}+4Abe^3(ex)^{3/2}+2bB(ex)^{9/2}\right)}{12be^4} - \frac{e^2\sqrt{\frac{b}{e^3}}\left(4aAb-a^2B\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{12b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (Sqrt[a + b*x^3]*(4*A*b*e^3*(e*x)^(3/2) + a*B*e^3*(e*x)^(3/2) + 2*b*B*(e*x)
^(9/2)))/(12*b*e^4) - ((4*a*A*b - a^2*B)*Sqrt[b/e^3]*e^2*Log[-(Sqrt[b/e^3]*
(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(12*b^2)
```

**fricas [A]** time = 1.16, size = 221, normalized size = 1.83

$$\left[-\frac{\left(Ba^2-4\,Aab\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(2\,Bbx^4+(Ba+4\,Ab)x\right)\sqrt{bx^3+a}\,\sqrt{ex}}{48\,b},\ \frac{\left(Ba^2-4\,Aab\right)\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\,\sqrt{bx^3+a}\,\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)+2\left(2\,Bbx^4+(Ba+4\,Ab)x\right)\sqrt{bx^3+a}\,\sqrt{ex}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((B*a^2 - 4*A*a*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
- 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*x^4
 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/24*((B*a^2 - 4*A*a*b)*s
qrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*
e)) + 2*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]
```

**giac [A]** time = 0.42, size = 137, normalized size = 1.13

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(2\,x^3e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Bx^{\frac{3}{2}}e^{\left(-\frac{1}{2}\right)}+\frac{Ba^2e^{\frac{1}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{12\,b^{\frac{3}{2}}}+\frac{1}{3}\left(\sqrt{bx^3e^4+ae^4}\,x^{\frac{3}{2}}e^{\frac{3}{2}}-\frac{ae^{\frac{7}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{\sqrt{b}}\right)Ae^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*B*x^(3/2)*e^(-1/2)
 + 1/12*B*a^2*e^(1/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4
)))/b^(3/2) + 1/3*(sqrt(b*x^3*e^4 + a*e^4)*x^(3/2)*e^(3/2) - a*e^(7/2)*log(
abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/sqrt(b))*A*e^(-3)
```

**maple [C]** time = 1.09, size = 6858, normalized size = 56.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\left(B\,x^3+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2), x)
```

**sympy [A]** time = 9.67, size = 201, normalized size = 1.66

$$\frac{A\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3\sqrt{b}}+\frac{Ba^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}{12be\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}}-\frac{Ba^2\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{12b^{\frac{3}{2}}}+\frac{Bb\,(ex)^{\frac{15}{2}}}{6\sqrt{a}\,e^7\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*a**(3/2)*(e*x)**(3/2)/(12
*b*e*sqrt(1 + b*x**3/a)) + B*sqrt(a)*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**3/a
)) - B*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(12*b**(
3/2)) + B*b*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a))
```

## 3.327 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx$

**Optimal**. Leaf size=118

$$\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+2Ab)}{3ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

**Rubi** [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+2Ab)}{3ae^4}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((2*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*e^4) - (2*A*(a + b*x^3)^(3
/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^
(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}+\frac{(2Ab+aB)\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}+\frac{(2Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2e^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}+\frac{(2Ab+aB)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x\right.}{e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}+\frac{(2Ab+aB)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x\right.}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}+\frac{(2Ab+aB)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x,\right.}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}+\frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}$$

**Mathematica** [A] time = 0.21, size = 87, normalized size = 0.74

$$\frac{x\sqrt{a+bx^3}\left(\frac{x^{3/2}(aB+2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{bx^3}{a}+1}}-2A+Bx^3\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(-2*A + B*x^3 + ((2*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*
x^(3/2))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x^3)/a])))/(3*(e*x)^(5/2))
```

**IntegrateAlgebraic** [A] time = 0.56, size = 99, normalized size = 0.84

$$\frac{\sqrt{a+bx^3}\left(Be^3x^3-2Ae^3\right)}{3e^4(ex)^{3/2}}-\frac{\sqrt{\frac{b}{e^3}}\,(aB+2Ab)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}\,(ex)^{3/2}\right)}{3be}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*e^3 + B*e^3*x^3))/(3*e^4*(e*x)^(3/2)) - ((2*A*b + a*
B)*Sqrt[b/e^3]*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b*e)
```

**fricas [A]** time = 1.55, size = 207, normalized size = 1.75

$$\left[\frac{(Ba+2\,Ab)\sqrt{be}\,x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\,\sqrt{be}\,\sqrt{ex}\right)+4\,\left(Bbx^3-2\,Ab\right)\sqrt{bx^3+a}\,\sqrt{ex}}{12\,be^3x^2},\,-\frac{(Ba+2\,Ab)\sqrt{-be}\,x^2\arctan\left(\frac{2\,\sqrt{bx^3+a}\,\sqrt{-be}\,\sqrt{ex}\,x}{2\,bex^3+ae}\right)-2\,\left(Bbx^3-2\,Ab\right)\sqrt{bx^3+a}\,\sqrt{ex}}{6\,be^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((B*a + 2*A*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e -
 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(B*b*x^3 - 2*A*
b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b*e)*x
^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(
B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

**maple [C]** time = 1.04, size = 6668, normalized size = 56.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{{\left(e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2), x)
```

**sympy [A]** time = 9.72, size = 160, normalized size = 1.36

$$-\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2A\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}}-\frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}\,e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] -2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2)*
sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) +
B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

## 3.328 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}\,dx$

**Optimal**. Leaf size=79

$$-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}+\frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {451, 277, 329, 275, 217, 206}

$$-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}+\frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (-2*B*Sqrt[a + b*x^3])/(3*x^(3/2)) - (2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2))
+ (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
```

```
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}\,dx &= -\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}+B\int \frac{\sqrt{a+bx^3}}{x^{5/2}}\,dx\\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}+(bB)\int \frac{\sqrt{x}}{\sqrt{a+bx^3}}\,dx\\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}+(2bB)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}+\frac{1}{3}(2bB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,x^{3/2}\right)\\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}+\frac{1}{3}(2bB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^{3/2}}{\sqrt{a+bx^3}}\right)\\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}+\frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 87, normalized size = 1.10

$$\frac{2\sqrt{a+bx^3}\left(\frac{3\sqrt{a}\,\sqrt{b}\,B\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\frac{a\left(A+3Bx^3\right)+Abx^3}{x^{9/2}}\right)}{9a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((A*b*x^3 + a*(A + 3*B*x^3))/x^(9/2)) + (3*Sqrt[a]*Sqr
t[b]*B*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[1 + (b*x^3)/a]))/(9*a)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 75, normalized size = 0.95

$$\frac{2}{3}\sqrt{b}\,B\log\left(\sqrt{a+bx^3}+\sqrt{b}\,x^{3/2}\right)-\frac{2\sqrt{a+bx^3}\left(aA+3aBx^3+Abx^3\right)}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^3]*(a*A + A*b*x^3 + 3*a*B*x^3))/(9*a*x^(9/2)) + (2*Sqrt[b]
*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/3
```

**fricas [A]** time = 0.95, size = 180, normalized size = 2.28

$$\left[\frac{3\,Ba\sqrt{b}\,x^5\log\left(-8\,b^2x^6-8\,abx^3-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x}-a^2\right)-4\left((3\,Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}\,\sqrt{x}}{18\,ax^5},\ -\frac{3\,Ba\sqrt{-b}\,x^5\arctan\left(\frac{2\,\sqrt{bx^3+a}\,\sqrt{-b}\,x^{\frac{3}{2}}}{2\,bx^3+a}\right)+2\left((3\,Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}\,\sqrt{x}}{9\,ax^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqr
t(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^
3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 + a)*
sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 +
a)*sqrt(x))/(a*x^5)]
```

**giac [A]** time = 0.25, size = 109, normalized size = 1.38

$$-\frac{2\,Bb\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}+\frac{2\left(3\,Bab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,Ba\sqrt{-b}\,\sqrt{b}+A\sqrt{-b}\,b^{\frac{3}{2}}\right)}{9\,a\sqrt{-b}}-\frac{2\left(3\,Ba^3\sqrt{b+\frac{a}{x^3}}+Aa^2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -2/3*B*b*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/9*(3*B*a*b*arctan(sq
rt(b)/sqrt(-b)) + 3*B*a*sqrt(-b)*sqrt(b) + A*sqrt(-b)*b^(3/2))/(a*sqrt(-b))
 - 2/9*(3*B*a^3*sqrt(b + a/x^3) + A*a^2*(b + a/x^3)^(3/2))/a^3
```

**maple [C]** time = 1.00, size = 3759, normalized size = 47.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x)
```

```
[Out] -2/9*(b*x^3+a)^(1/2)/x^(9/2)/b*(18*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5
*a-3*A*(x*(b*x^3+a))^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3)))^(1/2)*x^3*b^2+I*A*(x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*b^2-3*A*(x*(b*x^3+a))^(1/2)*(1/b^2*x
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b-36*I*B*3^(1/2)*(-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*(-a*b^2)^(1/3)*x^6*a*b-18*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi
((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-
1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*(-a*b^2)^(2/3)*x^5*a+I*A*(x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b^2*x*(-b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b-18*I*B*3^(1/2)*(-(I*3^(1/2)
-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
```

```
/2))/(I*3^(1/2)-3))^(1/2))*x^7*a*b^2+3*I*B*(x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b
^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*a*b+36*I*B*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^6*a*b-18*B*(-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*x^7*a*b^2+18*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^7*a*b^2+36*B*(
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(1/3)*x^6*a*b-36*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*
3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*(-a*b^2)^(1/3)*x^6*a*b-18*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5*a+18*B*(-(I*3^(1/
2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5*a+18*I*B*3^(1/2)*(-(I*3^(1/
2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
x^7*a*b^2-9*B*(x*(b*x^3+a))^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*x^3*a*b)/(x*(b*x^3+a))^(1/2)/a/(I*3^(1/2)-3)/(1/b^2*x*(-b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

**maxima** [A] time = 1.22, size = 81, normalized size = 1.03

$$-\frac{1}{3}\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}}\right)B-\frac{2\left(bx^3+a\right)^{\frac{3}{2}}A}{9\,ax^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^
3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*B - 2/9*(b*x^3 + a)^(3/2)*A/(
a*x^(9/2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2), x)
```

**sympy [A]** time = 61.86, size = 131, normalized size = 1.66

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9x^3}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a}-\frac{2B\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3}-\frac{2Bbx^{\frac{3}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(11/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*x**3) - 2*A*b**(3/2)*sqrt(a/(b*x**3) +
 1)/(9*a) - 2*B*sqrt(a)/(3*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*B*sqrt(b)*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/3 - 2*B*b*x**(3/2)/(3*sqrt(a)*sqrt(1 + b*x**3/a)
)
```

## 3.329 $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

**Optimal**. Leaf size=201

$$-\frac{a^3 e^{7/2} (8Ab-3aB) \tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192 b^{5/2}}+\frac{a^2 e^2 (ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-3aB)}{192 b^2}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72 b e}+ \frac{a(e\cdots}{}$$

**Rubi [A]** time = 0.14, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^2 e^2 (ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-3aB)}{192 b^2}-\frac{a^3 e^{7/2} (8Ab-3aB) \tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192 b^{5/2}}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72 b e}+\frac{a (ex)^{9/2}\sqrt{a+bx^3}\,(8Ab-3aB)}{96 b e}+\frac{B (ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12 b e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b^2) + (a*(8*A*b
 - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(96*b*e) + ((8*A*b - 3*a*B)*(e*x)^(9
/2)*(a + b*x^3)^(3/2))/(72*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e
) - (a^3*(8*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqr
t[a + b*x^3])])/(192*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{\left(-12Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}dx}{12b}$$

$$= \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} + \frac{(a(8Ab-3aB}{}$$

$$= \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8.}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8.}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8.}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8.}{}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8.}{}$$

**Mathematica [A]** time = 0.33, size = 167, normalized size = 0.83

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(3a^{5/2}(3aB-8Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(-9a^3B+6a^2b\left(4A+Bx^3\right)+8ab^2x^3\left(14A+9Bx^3\right)+16b^3x^6\left(4A+3Bx^3\right)\right)\right)}{576b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-9*a^3
*B + 6*a^2*b*(4*A + B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A
 + 9*B*x^3)) + 3*a^(5/2)*(-8*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]
]))/(576*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])
```

**IntegrateAlgebraic [A]** time = 0.84, size = 200, normalized size = 1.00

$$\frac{e^5\sqrt{\frac{b}{e^3}}\left(8a^3Ab-3a^4B\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{192b^3}+\frac{\sqrt{a+bx^3}\left(-9a^3Be^9(ex)^{3/2}+24a^2Abe^9(ex)^{3/2}+6a^2bBe^6(ex)^{9/2}+112aAb^2e^6(ex)^{9/2}+72ab^2Be^3(ex)^{15/2}+64Ab^3e^3(ex)^{15/2}+48b^3B(ex)^{21/2}\right)}{576b^2e^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[a + b*x^3]*(24*a^2*A*b*e^9*(e*x)^(3/2) - 9*a^3*B*e^9*(e*x)^(3/2) + 11
2*a*A*b^2*e^6*(e*x)^(9/2) + 6*a^2*b*B*e^6*(e*x)^(9/2) + 64*A*b^3*e^3*(e*x)^
(15/2) + 72*a*b^2*B*e^3*(e*x)^(15/2) + 48*b^3*B*(e*x)^(21/2)))/(576*b^2*e^7
) + ((8*a^3*A*b - 3*a^4*B)*Sqrt[b/e^3]*e^5*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) +
 Sqrt[a + b*x^3]])/(192*b^3)
```

**fricas [A]** time = 1.20, size = 355, normalized size = 1.77

$$\left[-\frac{3\left(3Ba^4-8Aa^3b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8b^2ex^6-8abex^3-a^2e+4\left(2b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(48Bb^3e^3x^{10}+8\left(9Bab^2+8Ab^3\right)e^3x^7+2\left(3Ba^2b+56Aab^2\right)e^3x^4-3\left(3Ba^3-8Aa^2b\right)e^3x\right)\sqrt{bx^3+a}\sqrt{ex}}{2304b^2},\ -\frac{3\left(3Ba^4-8Aa^3b\right)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2bex^3+ae}\right)-2\left(48Bb^3e^3x^{10}+8\left(9Bab^2+8Ab^3\right)e^3x^7+2\left(3Ba^2b+56Aab^2\right)e^3x^4-3\left(3Ba^3-8Aa^2b\right)e^3x\right)\sqrt{bx^3+a}\sqrt{ex}}{1152b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/2304*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*
x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) -
4*(48*B*b^3*e^3*x^10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*
A*a*b^2)*e^3*x^4 - 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x)
)/b^2, -1/1152*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3
+ a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(48*B*b^3*e^3*x^10 + 8
*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*A*a*b^2)*e^3*x^4 - 3*(3*
B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

**giac [B]** time = 1.45, size = 387, normalized size = 1.93

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(2x^3e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Aax^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}\sqrt{bx^3e^4+ae^4}\left(2\left(4x^3e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^3e^3-\frac{3a^2e^{(-1)}}{b^2}\right)Bax^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}\sqrt{bx^3e^4+ae^4}\left(2\left(4x^3e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^3e^3-\frac{3a^2e^{(-1)}}{b^2}\right)Abx^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{576}\sqrt{bx^3e^4+ae^4}\left(2\left(4\left(6x^3e^{(-7)}+\frac{ae^{(-7)}}{b}\right)x^3e^3-\frac{5a^2e^{(-4)}}{b^2}\right)x^3e^3+\frac{15a^3e^{(-1)}}{b^3}\right)Bbx^{\frac{3}{2}}e^{\frac{5}{2}}-\frac{\left(9B^2a^8e^7-48ABa^7be^7+64A^2a^6b^2e^7\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\left(3Ba^4x^{\frac{3}{2}}e^{\frac{11}{2}}-8Aa^3bx^{\frac{3}{2}}e^{\frac{11}{2}}\right)\sqrt{b}e^{\frac{1}{2}}+\sqrt{9B^2a^9e^{12}-48ABa^8be^{12}+64A^2a^7b^2e^{12}+\left(3Ba^4x^{\frac{3}{2}}e^{\frac{11}{2}}-8Aa^3bx^{\frac{3}{2}}e^{\frac{11}{2}}\right)^2be}\right|\right)}{192b^{\frac{5}{2}}\left|-3Ba^4e^3+8Aa^3be^3\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*A*a*x^(3/2)*e^(5/2
) + 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3
*a^2*e^(-1)/b^2)*B*a*x^(3/2)*e^(5/2) + 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x
^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3*a^2*e^(-1)/b^2)*A*b*x^(3/2)*e^(5/2) + 1
/576*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*x^3*e^(-7) + a*e^(-7)/b)*x^3*e^3 - 5*
a^2*e^(-4)/b^2)*x^3*e^3 + 15*a^3*e^(-1)/b^3)*B*b*x^(3/2)*e^(5/2) - 1/192*(9
*B^2*a^8*e^7 - 48*A*B*a^7*b*e^7 + 64*A^2*a^6*b^2*e^7)*e^(-1/2)*log(abs(-(3*
B*a^4*x^(3/2)*e^(11/2) - 8*A*a^3*b*x^(3/2)*e^(11/2))*sqrt(b)*e^(1/2) + sqrt
(9*B^2*a^9*e^12 - 48*A*B*a^8*b*e^12 + 64*A^2*a^7*b^2*e^12 + (3*B*a^4*x^(3/2
)*e^(11/2) - 8*A*a^3*b*x^(3/2)*e^(11/2))^2*b*e)))/(b^(5/2)*abs(-3*B*a^4*e^3
 + 8*A*a^3*b*e^3))
```

**maple [C]** time = 1.14, size = 7705, normalized size = 38.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,(e\,x)^{7/2}\,\left(b\,x^3 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
[Out] Timed out
```

## 3.330 $\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=161

$$\frac{a^2\sqrt{e}\,(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}\,(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a\right.}{9b}$$

**Rubi [A]** time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a^2\sqrt{e}\,(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}\,(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a*(6*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b*e) + ((6*A*b - a*B)*(e*
x)^(3/2)*(a + b*x^3)^(3/2))/(36*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9
*b*e) + (a^2*(6*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*S
qrt[a + b*x^3])])/(24*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} - \frac{\left(-9Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{3/2}dx}{9b}$$

$$= \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{(a(6Ab-aB))\int}{8}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$

**Mathematica [A]** time = 0.18, size = 143, normalized size = 0.89

$$\frac{\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(3a^2B+2ab\left(15A+7Bx^3\right)+4b^2x^3\left(3A+2Bx^3\right)\right)-3a^{3/2}(aB-6Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)\right)}{72b^{3/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(3*a^2*B +
4*b^2*x^3*(3*A + 2*B*x^3) + 2*a*b*(15*A + 7*B*x^3)) - 3*a^(3/2)*(-6*A*b + a
*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^
3)/a])
```

**IntegrateAlgebraic [A]** time = 0.71, size = 162, normalized size = 1.01

$$\frac{\sqrt{a+bx^3}\left(3a^2Be^6(ex)^{3/2}+30aAbe^6(ex)^{3/2}+14abBe^3(ex)^{9/2}+12Ab^2e^3(ex)^{9/2}+8b^2B(ex)^{15/2}\right)}{72be^7}-\frac{e^2\sqrt{\frac{b}{e^3}}\left(6a^2Ab-a^3B\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{24b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[a + b*x^3]*(30*a*A*b*e^6*(e*x)^(3/2) + 3*a^2*B*e^6*(e*x)^(3/2) + 12*A
*b^2*e^3*(e*x)^(9/2) + 14*a*b*B*e^3*(e*x)^(9/2) + 8*b^2*B*(e*x)^(15/2)))/(7
```

```
2*b*e^7) - ((6*a^2*A*b - a^3*B)*Sqrt[b/e^3]*e^2*Log[-(Sqrt[b/e^3]*(e*x)^(3/
2)) + Sqrt[a + b*x^3]])/(24*b^2)
```

**fricas [A]** time = 1.56, size = 273, normalized size = 1.70

$$\left[-\frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(8\,Bb^2x^7+2\left(7\,Bab+6\,Ab^2\right)x^4+3\left(Ba^2+10\,Aab\right)x\right)\sqrt{bx^3+a}\sqrt{ex}}{288\,b},\ \frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)+2\left(8\,Bb^2x^7+2\left(7\,Bab+6\,Ab^2\right)x^4+3\left(Ba^2+10\,Aab\right)x\right)\sqrt{bx^3+a}\sqrt{ex}}{144\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*B*
b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 +
a)*sqrt(e*x))/b, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^
3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(8*B*b^2*x^7 + 2*(7*
B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b
]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,
[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%
}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at paramet
ers values [91,88.2886286299,-21,88]Warning, choosing root of [1,0,%%%{-2,[
1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%
}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2
,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at parameters values [66,6.82230772497,-23,7
9]Warning, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+
%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,
0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at
parameters values [6,94.9264369817,-8,31]2*B*b*exp(1)/exp(3)*2*((17740800*b
^10*exp(1)^4/638668800/b^10/exp(1)^11*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*
exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+4435200*b^9*exp(1)^7*a
/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqr
t(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-6652800*b^8*exp(1)^10*a^2/6386688
00/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(
1)^4+b*(x*exp(1))^3*exp(1))+2*A*b*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/
exp(1)^5*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x
*exp(1))*sqrt(x*exp(1))+120*b^3*exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1)
)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*B*
a*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*e
xp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*
exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*
sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*A*a*exp(1)/exp(3)/exp(1)/3*(1/2*sq
rt(x*exp(1))*x*exp(1)*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-2*a*exp(1)^4/4
/sqrt(b*exp(1))*ln(abs(sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-sqrt(b*exp(1)
)*sqrt(x*exp(1))*x*exp(1))))-1/2*exp(1)/4/exp(1)*exp(3)/b/exp(3)/abs(2*b*a^
2*A+B*a^3)/(4*exp(1)*b^2*a^4*A^2+4*exp(1)*b*B*a^5*A+exp(1)*B^2*a^6)^-1/3/sq
rt(b*exp(1))*ln(abs(sqrt(4*A^2*a^5*b^2*exp(1)^6+4*A*B*a^6*b*exp(1)^6+B^2*a^
7*exp(1)^6+9*b*(-B*a^3*exp(1)*sqrt(x*exp(1))*x*exp(1)/3-2*A*a^2*b*exp(1)*sq
rt(x*exp(1))*x*exp(1)/3)^2*exp(1))-sqrt(9*b*exp(1))*(-B*a^3*exp(1)*sqrt(x*e
xp(1))*x*exp(1)/3-2*A*a^2*b*exp(1)*sqrt(x*exp(1))*x*exp(1)/3)))
```

**maple** [C]   time = 1.06, size = 7290, normalized size = 45.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x)
[Out] result too large to display
```

**maxima** [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} \sqrt{ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*sqrt(e*x), x)
```

**mupad** [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3 + A\right)\,\sqrt{e\,x}\,\left(b\,x^3 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)
```

**sympy** [B]   time = 25.46, size = 335, normalized size = 2.08

$$\frac{Aa^{\frac{3}{2}} (ex)^{\frac{3}{2}} \sqrt{1 + \frac{bx^3}{a}}}{3e} + \frac{Aa^{\frac{3}{2}} (ex)^{\frac{3}{2}}}{12e\sqrt{1 + \frac{bx^3}{a}}} + \frac{A\sqrt{a}\, b\, (ex)^{\frac{9}{2}}}{4e^4\sqrt{1 + \frac{bx^3}{a}}} + \frac{Aa^2\sqrt{e} \operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\, e^{\frac{3}{2}}}\right)}{4\sqrt{b}} + \frac{Ab^2 (ex)^{\frac{15}{2}}}{6\sqrt{a}\, e^7\sqrt{1 + \frac{bx^3}{a}}} + \frac{Ba^{\frac{5}{2}} (ex)^{\frac{3}{2}}}{24be\sqrt{1 + \frac{bx^3}{a}}} + \frac{17Ba^{\frac{3}{2}} (ex)^{\frac{9}{2}}}{72e^4\sqrt{1 + \frac{bx^3}{a}}} + \frac{11B\sqrt{a}\, b\, (ex)^{\frac{15}{2}}}{36e^7\sqrt{1 + \frac{bx^3}{a}}} - \frac{Ba^3\sqrt{e} \operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\, e^{\frac{3}{2}}}\right)}{24b^{\frac{3}{2}}} + \frac{Bb^2 (ex)^{\frac{21}{2}}}{9\sqrt{a}\, e^{10}\sqrt{1 + \frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)*(e*x)**(1/2),x)
[Out] A*a**(3/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(3/2)*(e*x)**(3/2)/
(12*e*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**
3/a)) + A*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(4*sq
rt(b)) + A*b**2*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a)) + B*a**(5
/2)*(e*x)**(3/2)/(24*b*e*sqrt(1 + b*x**3/a)) + 17*B*a**(3/2)*(e*x)**(9/2)/(
72*e**4*sqrt(1 + b*x**3/a)) + 11*B*sqrt(a)*b*(e*x)**(15/2)/(36*e**7*sqrt(1
+ b*x**3/a)) - B*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2))
)/(24*b**(3/2)) + B*b**2*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a))
```

## 3.331 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

**Optimal**. Leaf size=152

$$\frac{a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+4Ab)}{6ae^4}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+4Ab)}{4e^4}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

**Rubi [A]** time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+4Ab)}{6ae^4}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+4Ab)}{4e^4}+\frac{a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}\,e^{5/2}}-\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((4*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(4*e^4) + ((4*A*b + a*B)*(e*x)^
(3/2)*(a + b*x^3)^(3/2))/(6*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(
3/2)) + (a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*
x^3])])/(4*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(4Ab+aB)\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\,dx}{ae^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3(4Ab+aB))\int \sqrt{ex}\sqrt{a-}}{4e^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5}}{3ae(ex)^{3/2}}$$

**Mathematica [A]** time = 0.15, size = 126, normalized size = 0.83

$$\frac{x\sqrt{a+bx^3}\left(3\sqrt{a}\,x^{3/2}(aB+4Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{\frac{bx^3}{a}+1}\left(-8aA+5aBx^3+4Abx^3+2bBx^6\right)\right)}{12\sqrt{b}\,(ex)^{5/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(Sqrt[b]*Sqrt[1 + (b*x^3)/a]*(-8*a*A + 4*A*b*x^3 + 5*a*B
*x^3 + 2*b*B*x^6) + 3*Sqrt[a]*(4*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2
))/Sqrt[a]]))/(12*Sqrt[b]*(e*x)^(5/2)*Sqrt[1 + (b*x^3)/a])
```

**IntegrateAlgebraic [A]** time = 0.76, size = 125, normalized size = 0.82

$$\frac{\sqrt{a+bx^3}\left(-8aAe^6+5aBe^6x^3+4Abe^6x^3+2bBe^6x^6\right)}{12e^7(ex)^{3/2}} - \frac{\sqrt{\frac{b}{e^3}}\left(a^2B+4aAb\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{4be}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-8*a*A*e^6 + 4*A*b*e^6*x^3 + 5*a*B*e^6*x^3 + 2*b*B*e^6*x^
6))/(12*e^7*(e*x)^(3/2)) - ((4*a*A*b + a^2*B)*Sqrt[b/e^3]*Log[-(Sqrt[b/e^3]
*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(4*b*e)
```

**fricas [A]** time = 1.45, size = 255, normalized size = 1.68

$$\left[\frac{3 \left(Ba^2 + 4 \, Aab\right)\sqrt{be}\, x^2 \log\left(-8 \, b^2 e x^6 - 8 \, abex^3 - a^2 e - 4 \left(2 \, bx^4 + ax\right)\sqrt{bx^3 + a}\, \sqrt{be}\, \sqrt{ex}\right) + 4 \left(2 \, Bb^2 x^6 + \left(5 \, Bab + 4 \, Ab^2\right)x^3 - 8 \, Aab\right)\sqrt{bx^3 + a}\, \sqrt{ex}}{48 \, be^3 x^2}, -\frac{3 \left(Ba^2 + 4 \, Aab\right)\sqrt{-be}\, x^2 \arctan\left(\frac{2 \, \sqrt{bx^3+a}\, \sqrt{-be}\, \sqrt{ex}\, x}{2 \, bex^3+ae}\right) - 2 \left(2 \, Bb^2 x^6 + \left(5 \, Bab + 4 \, Ab^2\right)x^3 - 8 \, Aab\right)\sqrt{bx^3 + a}\, \sqrt{ex}}{24 \, be^3 x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(2*B*b^2*
x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*
x^2), -1/24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sq
rt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(2*B*b^2*x^6 + (5*B*a*b + 4*A*b
^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{\left(ex\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

**maple [C]** time = 1.18, size = 7108, normalized size = 46.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{\left(ex\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{{\left(e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2), x)
```

**sympy [B]** time = 24.32, size = 289, normalized size = 1.90

$$-\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}-\frac{2A\sqrt{a}\,bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,bx^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}\,e^{\frac{5}{2}}}+\frac{Bb^2x^{\frac{15}{2}}}{6\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3/
2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*sqr
t(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2) + B
*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2)/(1
2*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt(1 +
b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/2)) +
B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

## 3.332 $\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

**Optimal**. Leaf size=241

$$-\frac{a^4 e^{7/2} (10Ab-3aB) \tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384 b^{5/2}}+\frac{a^3 e^2 (ex)^{3/2} \sqrt{a+bx^3}\,(10Ab-3aB)}{384 b^2}+\frac{a^2 (ex)^{9/2} \sqrt{a+bx^3}\,(10Ab-3aB)}{192 b e}+$$

**Rubi [A]** time = 0.16, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^3 e^2 (ex)^{3/2} \sqrt{a+bx^3}\,(10Ab-3aB)}{384 b^2}-\frac{a^4 e^{7/2} (10Ab-3aB) \tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384 b^{5/2}}+\frac{a^2 (ex)^{9/2} \sqrt{a+bx^3}\,(10Ab-3aB)}{192 b e}+\frac{(ex)^{9/2} \left(a+bx^3\right)^{5/2} (10Ab-3aB)}{120 b e}+\frac{a (ex)^{9/2} \left(a+bx^3\right)^{3/2} (10Ab-3aB)}{144 b e}+\frac{B (ex)^{9/2} \left(a+bx^3\right)^{7/2}}{15 b e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (a^3*(10*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(384*b^2) + (a^2*(10
*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(192*b*e) + (a*(10*A*b - 3*a*B)*
(e*x)^(9/2)*(a + b*x^3)^(3/2))/(144*b*e) + ((10*A*b - 3*a*B)*(e*x)^(9/2)*(a
 + b*x^3)^(5/2))/(120*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) - (
a^4*(10*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ b*x^3])])/(384*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx &= \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{\left(-15Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}dx}{15b}\\
&= \frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}+\frac{(a(10Ab-3a}{}\\
&= \frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\\
&= \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\\
&= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\\
&= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\\
&= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\\
&= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\\
&= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 188, normalized size = 0.78

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(15a^{7/2}(3aB-10Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(-45a^4B+30a^3b\left(5A+Bx^3\right)+4a^2b^2x^3\left(295A+186Bx^3\right)+16ab^3x^6\left(85A+63Bx^3\right)+96b^4x^9\left(5A+4Bx^3\right)\right)\right)}{5760b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-45*a^
4*B + 30*a^3*b*(5*A + B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(8
```

```
5*A + 63*B*x^3) + 4*a^2*b^2*x^3*(295*A + 186*B*x^3)) + 15*a^(7/2)*(-10*A*b
+ 3*a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(5760*b^(5/2)*Sqrt[x]*Sqrt[1
+ (b*x^3)/a])
```

**IntegrateAlgebraic** [A] time = 0.95, size = 238, normalized size = 0.99

$$\frac{e^5\sqrt{\frac{b}{e^3}}\left(10a^4Ab-3a^5B\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{384b^3}+\frac{\sqrt{a+bx^3}\left(-45a^4Be^{12}(ex)^{3/2}+150a^3Abe^{12}(ex)^{3/2}+30a^3bBe^9(ex)^{9/2}+1180a^2Ab^2e^9(ex)^{9/2}+744a^2b^2Be^6(ex)^{15/2}+1360aAb^3e^6(ex)^{15/2}+1008ab^3Be^3(ex)^{21/2}+480Ab^4e^3(ex)^{21/2}+384b^4B(ex)^{27/2}\right)}{5760b^2e^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[a + b*x^3]*(150*a^3*A*b*e^12*(e*x)^(3/2) - 45*a^4*B*e^12*(e*x)^(3/2)
+ 1180*a^2*A*b^2*e^9*(e*x)^(9/2) + 30*a^3*b*B*e^9*(e*x)^(9/2) + 1360*a*A*b^
3*e^6*(e*x)^(15/2) + 744*a^2*b^2*B*e^6*(e*x)^(15/2) + 480*A*b^4*e^3*(e*x)^(
21/2) + 1008*a*b^3*B*e^3*(e*x)^(21/2) + 384*b^4*B*(e*x)^(27/2)))/(5760*b^2*
e^10) + ((10*a^4*A*b - 3*a^5*B)*Sqrt[b/e^3]*e^5*Log[-(Sqrt[b/e^3]*(e*x)^(3/
2)) + Sqrt[a + b*x^3]])/(384*b^3)
```

**fricas** [A] time = 1.33, size = 409, normalized size = 1.70

$$\left[-\frac{15\left(3Ba^5-10Aa^4b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8b^2ex^6-8abex^3-a^2e+4\left(2b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(384Bb^4e^3x^{13}+48\left(21Bab^3+10Ab^4\right)e^3x^{10}+8\left(93Ba^2b^2+170Aab^3\right)e^3x^7+10\left(3Ba^3b+118Aa^2b^2\right)e^3x^4-15\left(3Ba^4-10Aa^3b\right)e^3x\right)\sqrt{bx^3+a}\sqrt{ex}}{23040b^2},-\frac{15\left(3Ba^5-10Aa^4b\right)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2bex^3+ae}\right)-2\left(384Bb^4e^3x^{13}+48\left(21Bab^3+10Ab^4\right)e^3x^{10}+8\left(93Ba^2b^2+170Aab^3\right)e^3x^7+10\left(3Ba^3b+118Aa^2b^2\right)e^3x^4-15\left(3Ba^4-10Aa^3b\right)e^3x\right)\sqrt{bx^3+a}\sqrt{ex}}{11520b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b
*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b))
 - 4*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B*a^
2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*
(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/11520*(15*
(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*
x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 +
10*A*b^4)*e^3*x^10 + 8*(93*B*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b
 + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a
)*sqrt(e*x))/b^2]
```

**giac** [B] time = 1.49, size = 563, normalized size = 2.34

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(2x^3e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Aa^2x^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}\sqrt{bx^3e^4+ae^4}\left(2\left(4x^3e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^3e^3-\frac{3a^2e^{(-1)}}{b^2}\right)Ba^2x^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{36}\sqrt{bx^3e^4+ae^4}\left(2\left(4x^3e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^3e^3-\frac{3a^2e^{(-1)}}{b^2}\right)Aabx^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{288}\sqrt{bx^3e^4+ae^4}\left(2\left(4\left(6x^3e^{(-7)}+\frac{ae^{(-7)}}{b}\right)x^3e^3-\frac{5a^2e^{(-4)}}{b^2}\right)x^3e^3+\frac{15a^3e^{(-1)}}{b^3}\right)Babx^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{576}\sqrt{bx^3e^4+ae^4}\left(2\left(4\left(6x^3e^{(-7)}+\frac{ae^{(-7)}}{b}\right)x^3e^3-\frac{5a^2e^{(-4)}}{b^2}\right)x^3e^3+\frac{15a^3e^{(-1)}}{b^3}\right)Ab^2x^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{5760}\sqrt{bx^3e^4+ae^4}\left(2\left(4\left(6\left(8x^3e^{(-10)}+\frac{ae^{(-10)}}{b}\right)x^3e^3-\frac{7a^2e^{(-7)}}{b^2}\right)x^3e^3+\frac{35a^3e^{(-4)}}{b^3}\right)x^3e^3-\frac{105a^4e^{(-1)}}{b^4}\right)Bb^2x^{\frac{3}{2}}e^{\frac{5}{2}}-\frac{\left(9B^2a^{10}e^7-60ABa^9be^7+100A^2a^8b^2e^7\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\left(3Ba^5x^{\frac{3}{2}}e^{\frac{11}{2}}-10Aa^4bx^{\frac{3}{2}}e^{\frac{11}{2}}\right)\sqrt{b}e^{\frac{1}{2}}+\sqrt{9B^2a^{11}e^{12}-60ABa^{10}be^{12}+100A^2a^9b^2e^{12}+\left(3Ba^5x^{\frac{3}{2}}e^{\frac{11}{2}}-10Aa^4bx^{\frac{3}{2}}e^{\frac{11}{2}}\right)^2be}\right|\right)}{384b^{\frac{5}{2}}\left|-3Ba^5e^3+10Aa^4be^3\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*A*a^2*x^(3/2)*e^(5
/2) + 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 -
 3*a^2*e^(-1)/b^2)*B*a^2*x^(3/2)*e^(5/2) + 1/36*sqrt(b*x^3*e^4 + a*e^4)*(2*
(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3*a^2*e^(-1)/b^2)*A*a*b*x^(3/2)*e^(5/
2) + 1/288*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*x^3*e^(-7) + a*e^(-7)/b)*x^3*e^
3 - 5*a^2*e^(-4)/b^2)*x^3*e^3 + 15*a^3*e^(-1)/b^3)*B*a*b*x^(3/2)*e^(5/2) +
1/576*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*x^3*e^(-7) + a*e^(-7)/b)*x^3*e^3 - 5
*a^2*e^(-4)/b^2)*x^3*e^3 + 15*a^3*e^(-1)/b^3)*A*b^2*x^(3/2)*e^(5/2) + 1/576
0*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*(8*x^3*e^(-10) + a*e^(-10)/b)*x^3*e^3 -
7*a^2*e^(-7)/b^2)*x^3*e^3 + 35*a^3*e^(-4)/b^3)*x^3*e^3 - 105*a^4*e^(-1)/b^4
)*B*b^2*x^(3/2)*e^(5/2) - 1/384*(9*B^2*a^10*e^7 - 60*A*B*a^9*b*e^7 + 100*A^
2*a^8*b^2*e^7)*e^(-1/2)*log(abs(-(3*B*a^5*x^(3/2)*e^(11/2) - 10*A*a^4*b*x^(
3/2)*e^(11/2))*sqrt(b)*e^(1/2) + sqrt(9*B^2*a^11*e^12 - 60*A*B*a^10*b*e^12
+ 100*A^2*a^9*b^2*e^12 + (3*B*a^5*x^(3/2)*e^(11/2) - 10*A*a^4*b*x^(3/2)*e^(
11/2))^2*b*e)))/(b^(5/2)*abs(-3*B*a^5*e^3 + 10*A*a^4*b*e^3))
```

**maple** [C] time = 1.06, size = 8117, normalized size = 33.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3 + a\right)}^{5/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

## 3.333 $\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx$

**Optimal**. Leaf size=201

$$\frac{5a^3\sqrt{e}\,(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/2}\cdots}{}$$

**Rubi [A]** time = 0.13, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{5a^3\sqrt{e}\,(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/2}\left(a+bx^3\right)^{3/2}(8Ab-aB)}{288be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b*e) + (5*a*(8*A*b - a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(288*b*e) + ((8*A*b - a*B)*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(72*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) + (5*a^3*(8*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(192*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} - \frac{\left(-12Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{5/2}dx}{12b}$$

$$= \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} + \frac{(5a(8Ab-aB))}{}$$

$$= \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

**Mathematica [A]** time = 0.43, size = 146, normalized size = 0.73

$$\frac{x\sqrt{ex}\sqrt{a+bx^3}\left(\frac{(8Ab-aB)\left(15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(33a^2+26abx^3+8b^2x^6\right)\right)}{48\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}}+B\left(a+bx^3\right)^3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 + ((8*A*b - a*B)*(Sqrt[b]*x^(
3/2)*Sqrt[1 + (b*x^3)/a]*(33*a^2 + 26*a*b*x^3 + 8*b^2*x^6) + 15*a^(5/2)*Arc
Sinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(48*Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]))
)/(12*b)
```

**IntegrateAlgebraic [A]** time = 0.77, size = 200, normalized size = 1.00

$$\frac{\sqrt{a+bx^3}\left(15a^3Be^9(ex)^{3/2}+264a^2Abe^9(ex)^{3/2}+118a^2bBe^6(ex)^{9/2}+208aAb^2e^6(ex)^{9/2}+136ab^2Be^3(ex)^{15/2}+64Ab^3e^3(ex)^{15/2}+48b^3B(ex)^{21/2}\right)}{576be^{10}} - \frac{5e^2\sqrt{\frac{b}{e^3}}\left(8a^3Ab-a^4B\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{192b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[a + b*x^3]*(264*a^2*A*b*e^9*(e*x)^(3/2) + 15*a^3*B*e^9*(e*x)^(3/2) +
208*a*A*b^2*e^6*(e*x)^(9/2) + 118*a^2*b*B*e^6*(e*x)^(9/2) + 64*A*b^3*e^3*(e
*x)^(15/2) + 136*a*b^2*B*e^3*(e*x)^(15/2) + 48*b^3*B*(e*x)^(21/2)))/(576*b*
e^10) - (5*(8*a^3*A*b - a^4*B)*Sqrt[b/e^3]*e^2*Log[-(Sqrt[b/e^3]*(e*x)^(3/2
)) + Sqrt[a + b*x^3]])/(192*b^2)
```

**fricas [A]** time = 1.39, size = 323, normalized size = 1.61

$$\left[-\frac{15\left(Ba^4-8\,Aa^3b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(48\,Bb^3x^{10}+8\left(17\,Bab^2+8\,Ab^3\right)x^7+2\left(59\,Ba^2b+104\,Aab^2\right)x^4+3\left(5\,Ba^3+88\,Aa^2b\right)x\right)\sqrt{bx^3+a}\,\sqrt{ex}}{2304\,b},\ \frac{15\left(Ba^4-8\,Aa^3b\right)\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\,\sqrt{bx^3+a}\,\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)+2\left(48\,Bb^3x^{10}+8\left(17\,Bab^2+8\,Ab^3\right)x^7+2\left(59\,Ba^2b+104\,Aab^2\right)x^4+3\left(5\,Ba^3+88\,Aa^2b\right)x\right)\sqrt{bx^3+a}\,\sqrt{ex}}{1152\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2304*(15*(B*a^4 - 8*A*a^3*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
 a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(48
*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2)*x
^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/1152*(15*(
B*a^4 - 8*A*a^3*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-
e/b)/(2*b*e*x^3 + a*e)) + 2*(48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 +
 2*(59*B*a^2*b + 104*A*a*b^2)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3
+ a)*sqrt(e*x))/b]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,
[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%
}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at paramet
ers values [91,88.2886286299,-21,88]Warning, choosing root of [1,0,%%%{-2,[
1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%
}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2
,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at parameters values [66,6.82230772497,-23,7
9]Warning, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+
%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,
0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at
parameters values [6,94.9264369817,-8,31]2*B*b^2*exp(1)/exp(3)*2*(((2634721
6896000*b^16*exp(1)^7/1264666411008000/b^16/exp(1)^17*sqrt(x*exp(1))*sqrt(x
*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+439120
2816000*b^15*exp(1)^10*a/1264666411008000/b^16/exp(1)^17)*sqrt(x*exp(1))*sq
rt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-54
89003520000*b^14*exp(1)^13*a^2/1264666411008000/b^16/exp(1)^17)*sqrt(x*exp(
1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(
1))+8233505280000*b^13*exp(1)^16*a^3/1264666411008000/b^16/exp(1)^17)*sqrt(
x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(
1))+2*A*b^2*exp(1)/exp(3)*2*((17740800*b^10*exp(1)^4/638668800/b^10/exp(1)^
11*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1
))*sqrt(x*exp(1))+4435200*b^9*exp(1)^7*a/638668800/b^10/exp(1)^11)*sqrt(x*e
xp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*e
xp(1))-6652800*b^8*exp(1)^10*a^2/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*s
qrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*B*a^2
*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*ex
p(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*e
xp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*s
qrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+4*A*a*b*exp(1)/exp(3)*2*(240*b^4*exp(
```

```
1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*ex
p(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*exp(1)^4*a/5760/b^4/exp(1)^5)*s
qrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*
exp(1))+2*A*a^2*exp(1)/exp(3)/exp(1)/3*(1/2*sqrt(x*exp(1))*x*exp(1)*sqrt(a*
exp(1)^4+b*(x*exp(1))^3*exp(1))-2*a*exp(1)^4/4/sqrt(b*exp(1))*ln(abs(sqrt(a
*exp(1)^4+b*(x*exp(1))^3*exp(1))-sqrt(b*exp(1))*sqrt(x*exp(1))*x*exp(1))))+
4*B*a*b*exp(1)/exp(3)*2*((17740800*b^10*exp(1)^4/638668800/b^10/exp(1)^11*s
qrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*s
qrt(x*exp(1))+4435200*b^9*exp(1)^7*a/638668800/b^10/exp(1)^11)*sqrt(x*exp(1
))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1
))-6652800*b^8*exp(1)^10*a^2/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(
x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-1/2*exp(1)/
32/exp(1)*exp(3)/b/exp(3)/abs(24*b*a^3*A+5*B*a^4)/(576*exp(1)*b^2*a^6*A^2+2
40*exp(1)*b*B*a^7*A+25*exp(1)*B^2*a^8)^-1/3/sqrt(b*exp(1))*ln(abs(sqrt(576*
A^2*a^7*b^2*exp(1)^6+240*A*B*a^8*b*exp(1)^6+25*B^2*a^9*exp(1)^6+9*b*(-5*B*a
^4*exp(1)*sqrt(x*exp(1))*x*exp(1)/3-24*A*a^3*b*exp(1)*sqrt(x*exp(1))*x*exp(
1)/3)^2*exp(1))-sqrt(9*b*exp(1))*(-5*B*a^4*exp(1)*sqrt(x*exp(1))*x*exp(1)/3
-24*A*a^3*b*exp(1)*sqrt(x*exp(1))*x*exp(1)/3)))
```

**maple [C]** time = 1.10, size = 7702, normalized size = 38.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x)
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*sqrt(e*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,\sqrt{e\,x}\,\left(b\,x^3 + a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2), x)
```

**sympy [B]** time = 56.28, size = 413, normalized size = 2.05

$$\frac{Aa^{\frac{5}{2}}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e} + \frac{Aa^{\frac{5}{2}}(ex)^{\frac{3}{2}}}{8e\sqrt{1+\frac{bx^3}{a}}} + \frac{35Aa^{\frac{3}{2}}b(ex)^{\frac{9}{2}}}{72e^4\sqrt{1+\frac{bx^3}{a}}} + \frac{17A\sqrt{a}b^2(ex)^{\frac{15}{2}}}{36e^7\sqrt{1+\frac{bx^3}{a}}} + \frac{5Aa^3\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{24\sqrt{b}} + \frac{Ab^3(ex)^{\frac{21}{2}}}{9\sqrt{a}e^{10}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Ba^{\frac{7}{2}}(ex)^{\frac{3}{2}}}{192be\sqrt{1+\frac{bx^3}{a}}} + \frac{133Ba^{\frac{5}{2}}(ex)^{\frac{9}{2}}}{576e^4\sqrt{1+\frac{bx^3}{a}}} + \frac{127Ba^{\frac{3}{2}}b(ex)^{\frac{15}{2}}}{288e^7\sqrt{1+\frac{bx^3}{a}}} + \frac{23B\sqrt{a}b^2(ex)^{\frac{21}{2}}}{72e^{10}\sqrt{1+\frac{bx^3}{a}}} - \frac{5Ba^4\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}(ex)^{\frac{3}{2}}}{\sqrt{a}e^{\frac{3}{2}}}\right)}{192b^{\frac{3}{2}}} + \frac{Bb^3(ex)^{\frac{27}{2}}}{12\sqrt{a}e^{13}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)*(e*x)**(1/2),x)
[Out] A*a**(5/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(5/2)*(e*x)**(3/2)/
(8*e*sqrt(1 + b*x**3/a)) + 35*A*a**(3/2)*b*(e*x)**(9/2)/(72*e**4*sqrt(1 + b
*x**3/a)) + 17*A*sqrt(a)*b**2*(e*x)**(15/2)/(36*e**7*sqrt(1 + b*x**3/a)) +
5*A*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(24*sqrt(b)
```

```
) + A*b**3*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a)) + 5*B*a**(7/2
)*(e*x)**(3/2)/(192*b*e*sqrt(1 + b*x**3/a)) + 133*B*a**(5/2)*(e*x)**(9/2)/(
576*e**4*sqrt(1 + b*x**3/a)) + 127*B*a**(3/2)*b*(e*x)**(15/2)/(288*e**7*sqr
t(1 + b*x**3/a)) + 23*B*sqrt(a)*b**2*(e*x)**(21/2)/(72*e**10*sqrt(1 + b*x**
3/a)) - 5*B*a**4*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(19
2*b**(3/2)) + B*b**3*(e*x)**(27/2)/(12*sqrt(a)*e**13*sqrt(1 + b*x**3/a))
```

## 3.334 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

**Optimal**. Leaf size=188

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(aB+6Ab)}{9ae^4}+\frac{5(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+6Ab)}{36e^4}+\frac{5a(ex)}{}$$

**Rubi [A]** time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(aB+6Ab)}{9ae^4}+\frac{5(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+6Ab)}{36e^4}+\frac{5a(ex)^{3/2}\sqrt{a+bx^3}\,(aB+6Ab)}{24e^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (5*a*(6*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*e^4) + (5*(6*A*b + a*B)
*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*e^4) + ((6*A*b + a*B)*(e*x)^(3/2)*(a +
b*x^3)^(5/2))/(9*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + (5*
a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])
/(24*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}+\frac{(6Ab+aB)\int\sqrt{ex}\left(a+bx^3\right)^{5/2}\,dx}{ae^3}$$

$$=\frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}+\frac{(5(6Ab+aB))\int\sqrt{ex}\left(a+bx\right.}{6e^3}$$

$$=\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4}-\frac{2A\left(a+bx^3\right)}{3ae(ex)^{3/2}}$$

$$=\frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(e}{}$$

$$=\frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(e}{}$$

$$=\frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(e}{}$$

$$=\frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(e}{}$$

$$=\frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(e}{}$$

**Mathematica [A]** time = 0.21, size = 150, normalized size = 0.80

$$\frac{x\sqrt{a+bx^3}\left(15a^{3/2}x^{3/2}(aB+6Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{\frac{bx^3}{a}+1}\left(a^2\left(33Bx^3-48A\right)+a\left(54Abx^3+26bBx^6\right)+4b^2x^6\left(3A+2Bx^3\right)\right)\right)}{72\sqrt{b}(ex)^{5/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(Sqrt[b]*Sqrt[1 + (b*x^3)/a]*(4*b^2*x^6*(3*A + 2*B*x^3)
+ a^2*(-48*A + 33*B*x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^(3/2)*(6*A*b
 + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*Sqrt[b]*(e*x)^(5/2
)*Sqrt[1 + (b*x^3)/a])
```

**IntegrateAlgebraic [A]** time = 0.85, size = 157, normalized size = 0.84

$$\frac{\sqrt{a+bx^3}\left(-48a^2Ae^9+33a^2Be^9x^3+54aAbe^9x^3+26abBe^9x^6+12Ab^2e^9x^6+8b^2Be^9x^9\right)}{72e^{10}(ex)^{3/2}}-\frac{5\sqrt{\frac{b}{e^3}}\left(a^3B+6a^2Ab\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{24be}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-48*a^2*A*e^9 + 54*a*A*b*e^9*x^3 + 33*a^2*B*e^9*x^3 + 12*
A*b^2*e^9*x^6 + 26*a*b*B*e^9*x^6 + 8*b^2*B*e^9*x^9))/(72*e^10*(e*x)^(3/2))
- (5*(6*a^2*A*b + a^3*B)*Sqrt[b/e^3]*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[
a + b*x^3]])/(24*b*e)
```

**fricas [A]** time = 1.28, size = 309, normalized size = 1.64

$$\left[\frac{15\left(Ba^3+6\,Aa^2b\right)\sqrt{be}\,x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\,\sqrt{be}\,\sqrt{ex}\right)+4\left(8\,Bb^3x^9+2\left(13\,Bab^2+6\,Ab^3\right)x^6-48\,Aa^2b+3\left(11\,Ba^2b+18\,Aab^2\right)x^3\right)\sqrt{bx^3+a}\,\sqrt{ex}}{288\,be^3x^2},\ -\frac{15\left(Ba^3+6\,Aa^2b\right)\sqrt{-be}\,x^2\arctan\left(\frac{2\,\sqrt{bx^3+a}\,\sqrt{-be}\,\sqrt{ex}\,x}{2\,bex^3+ae}\right)-2\left(8\,Bb^3x^9+2\left(13\,Bab^2+6\,Ab^3\right)x^6-48\,Aa^2b+3\left(11\,Ba^2b+18\,Aab^2\right)x^3\right)\sqrt{bx^3+a}\,\sqrt{ex}}{144\,be^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(8*B*
b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*
a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/144*(15*(B*a^3 + 6*A
*a^2*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b
*e*x^3 + a*e)) - 2*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b
 + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

**maple [C]** time = 1.16, size = 7544, normalized size = 40.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{5/2}}{{\left(e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2), x)
```

**sympy [B]**     time = 63.94, size = 403, normalized size = 2.14

$$-\frac{2Aa^{\frac{5}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}-\frac{7Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,b^2x^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{5Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4e^{\frac{5}{2}}}+\frac{Ab^3x^{\frac{15}{2}}}{6\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{8e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{35Ba^{\frac{3}{2}}bx^{\frac{9}{2}}}{72e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{17B\sqrt{a}\,b^2x^{\frac{15}{2}}}{36e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{24\sqrt{b}\,e^{\frac{5}{2}}}+\frac{Bb^3x^{\frac{21}{2}}}{9\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(5/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*a**(3/2)*b*x**
(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 7*A*a**(3/2)*b*x**(3/2)/(12*e**(5/2
)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b**2*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x**3
/a)) + 5*A*a**2*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*e**(5/2)) + A*b*
*3*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**(5/2)*x**(3/2)*
sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(5/2)*x**(3/2)/(8*e**(5/2)*sqrt(1 +
b*x**3/a)) + 35*B*a**(3/2)*b*x**(9/2)/(72*e**(5/2)*sqrt(1 + b*x**3/a)) + 17
*B*sqrt(a)*b**2*x**(15/2)/(36*e**(5/2)*sqrt(1 + b*x**3/a)) + 5*B*a**3*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/(24*sqrt(b)*e**(5/2)) + B*b**3*x**(21/2)/(9*sqrt
(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

## 3.335 $\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=121

$$-\frac{ae^{7/2}(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-3aB)}{12b^2}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

**Rubi [A]** time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 321, 329, 275, 217, 206}

$$\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-3aB)}{12b^2}-\frac{ae^{7/2}(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((4*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b^2) + (B*(e*x)^(9/2)
*Sqrt[a + b*x^3])/(6*b*e) - (a*(4*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(-6Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{6b}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{8b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{a+\ldots}}\ldots\right)}{4b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\ldots}}\ldots\right)}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{bx\ldots}{e\ldots}}\ldots\right)}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{a(4Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(e\ldots}{e^{3/2}\sqrt{a\ldots}}\right)}{12b^{5/2}}$$

**Mathematica [A]** time = 0.17, size = 97, normalized size = 0.80

$$\frac{e^3\sqrt{ex}\left(\sqrt{b}\,x^{3/2}\sqrt{a+bx^3}\left(-3aB+4Ab+2bBx^3\right)+a(3aB-4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)\right)}{12b^{5/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3)
 + a*(-4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]]))/(12*b^(5
/2)*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 1.06, size = 124, normalized size = 1.02

$$\frac{e^5\sqrt{\frac{b}{e^3}}\left(4aAb-3a^2B\right)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}(ex)^{3/2}\right)}{12b^3}+\frac{\sqrt{a+bx^3}\left(-3aBe^3(ex)^{3/2}+4Abe^3(ex)^{3/2}+2bB(ex)^{9/2}\right)}{12b^2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (Sqrt[a + b*x^3]*(4*A*b*e^3*(e*x)^(3/2) - 3*a*B*e^3*(e*x)^(3/2) + 2*b*B*(e*
x)^(9/2)))/(12*b^2*e) + ((4*a*A*b - 3*a^2*B)*Sqrt[b/e^3]*e^5*Log[-(Sqrt[b/e
^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(12*b^3)
```

**fricas [A]** time = 1.44, size = 245, normalized size = 2.02

$$\left[-\frac{\left(3Ba^2-4Aab\right)e^3\sqrt{\frac{e}{b}}\log\left(-8b^2ex^6-8abex^3-a^2e+4\left(2b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(2Bbe^3x^4-(3Ba-4Ab)e^3x\right)\sqrt{bx^3+a}\sqrt{ex}}{48b^2},\ -\frac{\left(3Ba^2-4Aab\right)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2bex^3+ae}\right)-2\left(2Bbe^3x^4-(3Ba-4Ab)e^3x\right)\sqrt{bx^3+a}\sqrt{ex}}{24b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B
*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/24*(
(3*B*a^2 - 4*A*a*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*s
qrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sq
rt(b*x^3 + a)*sqrt(e*x))/b^2]
```

**giac [A]** time = 0.36, size = 114, normalized size = 0.94

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(\frac{2\,Bx^3e^{(-2)}}{b}-\frac{\left(3\,Bab^3e^5-4\,Ab^4e^5\right)e^{(-7)}}{b^5}\right)x^{\frac{3}{2}}e^{\frac{7}{2}}-\frac{\left(3\,Ba^2b^3e^9-4\,Aab^4e^9\right)e^{\left(-\frac{11}{2}\right)}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{12\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*B*x^3*e^(-2)/b - (3*B*a*b^3*e^5 - 4*A*b^4*e
^5)*e^(-7)/b^5)*x^(3/2)*e^(7/2) - 1/12*(3*B*a^2*b^3*e^9 - 4*A*a*b^4*e^9)*e^
(-11/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(11/2)
```

**maple [C]** time = 1.01, size = 6861, normalized size = 56.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{7}{2}}}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/sqrt(b*x^3 + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2), x)
```

**sympy [A]** time = 95.30, size = 194, normalized size = 1.60

$$\frac{A\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3b}-\frac{Aae^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{Ba^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx^3}{a}}}-\frac{B\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{9}{2}}}{12b\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^2e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{Be^{\frac{7}{2}}x^{\frac{15}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(7/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*b) - A*a*e**(7/2)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - B*a**(3/2)*e**(7/2)*x**(3/2)/(4*b**
2*sqrt(1 + b*x**3/a)) - B*sqrt(a)*e**(7/2)*x**(9/2)/(12*b*sqrt(1 + b*x**3/a
)) + B*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*b**(5/2)) + B*e**(7
/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a))
```

## 3.336 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\sqrt{a+bx^3}} \, dx$

**Optimal**. Leaf size=83

$$\frac{\sqrt{e}\,(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

**Rubi [A]** time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {459, 329, 275, 217, 206}

$$\frac{\sqrt{e}\,(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(S
qrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ex}\,(A+Bx^3)}{\sqrt{a+bx^3}}\,dx &= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} - \frac{\left(-3Ab+\frac{3aB}{2}\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{3b} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.06, size = 78, normalized size = 0.94

$$
\frac{\sqrt{ex}\left((2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)+\sqrt{b}\,Bx^{3/2}\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (Sqrt[e*x]*(Sqrt[b]*B*x^(3/2)*Sqrt[a + b*x^3] + (2*A*b - a*B)*ArcTanh[(Sqrt
[b]*x^(3/2))/Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

**IntegrateAlgebraic [A]** time = 0.53, size = 89, normalized size = 1.07

$$
\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} - \frac{e^2\sqrt{\frac{b}{e^3}}\,(2Ab-aB)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}\,(ex)^{3/2}\right)}{3b^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) - ((2*A*b - a*B)*Sqrt[b/e^3]*e^2*Lo
g[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b^2)
```

**fricas [A]** time = 1.28, size = 184, normalized size = 2.22

$$
\left[\frac{4\sqrt{bx^3+a}\sqrt{ex}\,Bx-(Ba-2\,Ab)\sqrt{\frac{e}{b}}\,\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)}{12\,b}, \frac{2\sqrt{bx^3+a}\sqrt{ex}\,Bx+(Ba-2\,Ab)\sqrt{-\frac{e}{b}}\,\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)}{6\,b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(e*x)*B*x - (B*a - 2*A*b)*sqrt(e/b)*log(-8*b^2
*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e
*x)*sqrt(e/b)))/b, 1/6*(2*sqrt(b*x^3 + a)*sqrt(e*x)*B*x + (B*a - 2*A*b)*sqr
```

```
t(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)
))/b]
```

**giac** [A] time = 0.36, size = 72, normalized size = 0.87

$$\frac{\sqrt{bx^3e^4+ae^4}\,Bx^{\frac{3}{2}}e^{\left(-\frac{3}{2}\right)}}{3\,b}+\frac{\left(Bae^5-2\,Abe^5\right)e^{\left(-\frac{9}{2}\right)}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{3\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3*e^4 + a*e^4)*B*x^(3/2)*e^(-3/2)/b + 1/3*(B*a*e^5 - 2*A*b*e^5
)*e^(-9/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(3/2)
```

**maple** [C] time = 1.15, size = 6424, normalized size = 77.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{ex}}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{e\,x}}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2), x)
```

**sympy** [A] time = 6.81, size = 107, normalized size = 1.29

$$\frac{2A\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3\sqrt{b}}+\frac{B\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3be}-\frac{Ba\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*
sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*b*e) - B*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*b**(3/2))
```

## 3.337 $\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx$

**Optimal**. Leaf size=75

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {451, 329, 275, 217, 206}

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(3
/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{e^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{(2B)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{(2B)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{(2B)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} + \frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}$$

**Mathematica** [**A**] time = 0.06, size = 65, normalized size = 0.87

$$\frac{2x\left(\frac{Bx^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{\sqrt{b}} - \frac{A\sqrt{a+bx^3}}{a}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[
a + b*x^3]])/Sqrt[b]))/(3*(e*x)^(5/2))
```

**IntegrateAlgebraic** [**A**] time = 0.79, size = 81, normalized size = 1.08

$$-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}} - \frac{2B\sqrt{\frac{b}{e^3}}\log\left(\sqrt{a+bx^3} - \sqrt{\frac{b}{e^3}}\,(ex)^{3/2}\right)}{3be}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) - (2*B*Sqrt[b/e^3]*Log[-(Sqrt[b/
e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b*e)
```

**fricas** [**A**] time = 1.19, size = 183, normalized size = 2.44

$$\left[\frac{\sqrt{be}\,Bax^2\log\left(-8\,b^2ex^6 - 8\,abex^3 - a^2e - 4\left(2\,bx^4 + ax\right)\sqrt{bx^3+a}\,\sqrt{be}\,\sqrt{ex}\right) - 4\,\sqrt{bx^3+a}\,\sqrt{ex}\,Ab}{6\,abe^3x^2}, -\frac{\sqrt{-be}\,Bax^2\arctan\left(\frac{2\,\sqrt{bx^3+a}\,\sqrt{-be}\,\sqrt{ex}\,x}{2\,bex^3+ae}\right) + 2\,\sqrt{bx^3+a}\,\sqrt{ex}\,Ab}{3\,abe^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b*e)*B*a*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4
 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) - 4*sqrt(b*x^3 + a)*sqrt(e*x)*
```

```
A*b)/(a*b*e^3*x^2), -1/3*(sqrt(-b*e)*B*a*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(
-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) + 2*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)/(a*
b*e^3*x^2)]
```

**giac [A]** time = 0.26, size = 110, normalized size = 1.47

$$-\frac{2}{3}\left(\left(\frac{B\arctan\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{\sqrt{-be}}\right)}{\sqrt{-be}}+\frac{\sqrt{be+\frac{ae}{x^3}}\,Ae^{(-1)}}{a}\right)e^{(-1)}-\frac{\left(Ba\arctan\left(\frac{\sqrt{b}e^{\frac{1}{2}}}{\sqrt{-be}}\right)e+\sqrt{-be}\,A\sqrt{b}e^{\frac{1}{2}}\right)e^{(-2)}}{\sqrt{-be}\,a}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*((B*arctan(sqrt(b*e + a*e/x^3)/sqrt(-b*e))/sqrt(-b*e) + sqrt(b*e + a*e
/x^3)*A*e^(-1)/a)*e^(-1) - (B*a*arctan(sqrt(b)*e^(1/2)/sqrt(-b*e))*e + sqrt
(-b*e)*A*sqrt(b)*e^(1/2))*e^(-2)/(sqrt(-b*e)*a))*e^(-1)
```

**maple [C]** time = 1.00, size = 3397, normalized size = 45.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a)^(1/2)/x/b^2*(6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b^2*e-6*I*B*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b^2*e-12*I*B*3^(1/2)*(-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-a*b^2)^(1/3)*x^3*a*b*e+12*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticP
i((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)
-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*(-a*b^2)^(1/3)*x^3*a*b*e+6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^2
*a*e-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*
```

```
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^
2*a*e-6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*x^4*a*b^2*e+6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3
^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
x^4*a*b^2*e+12*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^3*a*b*e-12*B*(-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^3*a*b*e-6*B*(-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*
x^2*a*e+6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^2*a*e+I
*A*(e*x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3)))^(1/2)*b^2-3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*b^2)/e^2/(e*x)^(1/2)/(e*x*(b*x^3+a))^(1/
2)/a/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(
1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{B\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{3\sqrt{b}}}{e^{\frac{5}{2}}} - \frac{2\left(b\sqrt{e}x^4 + a\sqrt{e}x\right)A}{3\sqrt{bx^3+a}\,ae^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*integrate(sqrt(x)/sqrt(b*x^3 + a), x)/e^(5/2) - 2/3*(b*sqrt(e)*x^4 + a*sq
rt(e)*x)*A/(sqrt(b*x^3 + a)*a*e^3*x^(5/2))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^3+A}{(e\,x)^{5/2}\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)), x)
```

**sympy** [A] time = 10.11, size = 60, normalized size = 0.80

$$-\frac{2A\sqrt{b}\,\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}}+\frac{2B\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}\,e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3/
2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

## 3.338 $\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=120

$$\frac{e^{7/2}(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}-\frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 288, 329, 275, 217, 206}

$$-\frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}}+\frac{e^{7/2}(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -((2*A*b - 3*a*B)*e^2*(e*x)^(3/2))/(3*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(9/2)
)/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)
^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} - \frac{\left(-3Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{(2Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

**Mathematica [A]** time = 0.18, size = 109, normalized size = 0.91

$$\frac{e^3\sqrt{ex}\left(\sqrt{b}\,x^{3/2}\left(3aB-2Ab+bBx^3\right)-\sqrt{a}\sqrt{\frac{bx^3}{a}+1}\,(3aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{x}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3) - Sqrt[a]*(-2*A*
b + 3*a*B)*Sqrt[1 + (b*x^3)/a]*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(3*b^(5
/2)*Sqrt[x]*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 1.76, size = 136, normalized size = 1.13

$$\frac{\sqrt{a+bx^3}\left(3aBe^5(ex)^{3/2}-2Abe^5(ex)^{3/2}+bBe^2(ex)^{9/2}\right)}{3b^2\left(ae^3+be^3x^3\right)} - \frac{e^5\sqrt{\frac{b}{e^3}}\,(2Ab-3aB)\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}\,(ex)^{3/2}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*b*e^5*(e*x)^(3/2) + 3*a*B*e^5*(e*x)^(3/2) + b*B*e^2*
(e*x)^(9/2)))/(3*b^2*(a*e^3 + b*e^3*x^3)) - ((2*A*b - 3*a*B)*Sqrt[b/e^3]*e^
5*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b^3)
```

**fricas** [A]  time = 1.92, size = 307, normalized size = 2.56

$$\left[-\frac{\left((3\,Bab-2\,Ab^{2})e^{3}x^{3}+(3\,Ba^{2}-2\,Aab)e^{3}\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^{2}ex^{6}-8\,abex^{3}-a^{2}e-4\,(2\,b^{2}x^{4}+abx)\sqrt{bx^{3}+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\,(Bbe^{3}x^{4}+(3\,Ba-2\,Ab)e^{3}x)\sqrt{bx^{3}+a}\sqrt{ex}}{12\,(b^{3}x^{3}+ab^{2})},\ \frac{\left((3\,Bab-2\,Ab^{2})e^{3}x^{3}+(3\,Ba^{2}-2\,Aab)e^{3}\right)\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\,\sqrt{bx^{3}+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^{3}+ae}\right)+2\,(Bbe^{3}x^{4}+(3\,Ba-2\,Ab)e^{3}x)\sqrt{bx^{3}+a}\sqrt{ex}}{6\,(b^{3}x^{3}+ab^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(e/b)*l
og(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 +
a)*sqrt(e*x)*sqrt(e/b)) - 4*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b*x^
3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2), 1/6*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3
*B*a^2 - 2*A*a*b)*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sq
rt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2)]
```

**giac** [A]  time = 0.37, size = 107, normalized size = 0.89

$$\frac{\left(\frac{Bx^{3}e^{4}}{b}+\frac{3\,Bab^{3}e^{4}-2\,Ab^{4}e^{4}}{b^{5}}\right)x^{\frac{3}{2}}e^{\frac{3}{2}}}{3\,\sqrt{bx^{3}e^{4}+ae^{4}}}+\frac{\left(3\,Bab^{3}e^{4}-2\,Ab^{4}e^{4}\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^{2}+\sqrt{bx^{3}e^{4}+ae^{4}}\right|\right)}{3\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3*e^4/b + (3*B*a*b^3*e^4 - 2*A*b^4*e^4)/b^5)*x^(3/2)*e^(3/2)/sqrt(
b*x^3*e^4 + a*e^4) + 1/3*(3*B*a*b^3*e^4 - 2*A*b^4*e^4)*e^(-1/2)*log(abs(-sq
rt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(11/2)
```

**maple** [C]  time = 1.13, size = 7016, normalized size = 58.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] result too large to display
```

**maxima** [F]  time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)(ex)^{\frac{7}{2}}}{\left(bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(3/2), x)
```

**mupad** [F]  time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^{3}+A\right)\,{(e\,x)}^{7/2}}{{\left(b\,x^{3}+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

## 3.339 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=85

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

**Rubi** [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {452, 329, 275, 217, 206}

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTan
h[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{B\int\frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{be}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3be}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\text{Subst}\left(\int\frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

**Mathematica [A]** time = 0.10, size = 93, normalized size = 1.09

$$\frac{2\sqrt{ex}\left(a^{3/2}B\sqrt{\frac{bx^3}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\,x^{3/2}(Ab-aB)\right)}{3ab^{3/2}\sqrt{x}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[e*x]*(Sqrt[b]*(A*b - a*B)*x^(3/2) + a^(3/2)*B*Sqrt[1 + (b*x^3)/a]*A
rcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(3*a*b^(3/2)*Sqrt[x]*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic [A]** time = 1.01, size = 107, normalized size = 1.26

$$-\frac{2e^2(ex)^{3/2}\sqrt{a+bx^3}\,(aB-Ab)}{3ab\left(ae^3+be^3x^3\right)}-\frac{2Be^2\sqrt{\frac{b}{e^3}}\,\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}\,(ex)^{3/2}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(-(A*b) + a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*b*(a*e^3 + b*e^3*x
^3)) - (2*B*Sqrt[b/e^3]*e^2*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3
]])/(3*b^2)
```

**fricas [A]** time = 0.86, size = 234, normalized size = 2.75

$$\left[-\frac{4\sqrt{bx^3+a}\,(Ba-Ab)\sqrt{ex}\,x-\left(Babx^3+Ba^2\right)\sqrt{\frac{e}{b}}\,\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)}{6\left(ab^2x^3+a^2b\right)},-\frac{2\sqrt{bx^3+a}\,(Ba-Ab)\sqrt{ex}\,x+\left(Babx^3+Ba^2\right)\sqrt{-\frac{e}{b}}\,\arctan\left(\frac{2\sqrt{bx^3+a}\,\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)}{3\left(ab^2x^3+a^2b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x - (B*a*b*x^3 + B*a^2)*sqrt
(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b
*x^3 + a)*sqrt(e*x)*sqrt(e/b)))/(a*b^2*x^3 + a^2*b), -1/3*(2*sqrt(b*x^3 + a
)*(B*a - A*b)*sqrt(e*x)*x + (B*a*b*x^3 + B*a^2)*sqrt(-e/b)*arctan(2*sqrt(b*
x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)))/(a*b^2*x^3 + a^2*b)]
```

**giac [A]** time = 0.62, size = 75, normalized size = 0.88

$$-\frac{2\,Be^{\frac{1}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^{2}+\sqrt{bx^{3}e^{4}+ae^{4}}\right|\right)}{3\,b^{\frac{3}{2}}}-\frac{2\,(Bae-Abe)x^{\frac{3}{2}}e^{\frac{3}{2}}}{3\,\sqrt{bx^{3}e^{4}+ae^{4}}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*B*e^(1/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(
3/2) - 2/3*(B*a*e - A*b*e)*x^(3/2)*e^(3/2)/(sqrt(b*x^3*e^4 + a*e^4)*a*b)
```

**maple [C]** time = 1.00, size = 3654, normalized size = 42.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3*(e*x)^(1/2)/(b*x^3+a)^(1/2)/b^3*(-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a
))^(1/2)*x^2*a*b^2-12*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(
1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e
*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x*a*b+12*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x
^3+a))^(1/2)*(-a*b^2)^(1/3)*x*a*b-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(
1/2)*(-a*b^2)^(2/3)*a+6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3
^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(e*x*(b*x^3+a))^(1/2)*x^2*a*b^2+I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^2*b^3+6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
```

```
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x
^2*a*b^2-6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x
^2*a*b^2-12*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*a*b+12*B*(-a
*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*
a*b-I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*x^2*a*b^2+6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^
3+a))^(1/2)*(-a*b^2)^(2/3)*a+6*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(
1/2)*a-6*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^
3+a))^(1/2)*a-3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1
/2)*x^2*b^3+3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*x^2*a*b^2)/x/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3)))^(1/2)/a
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{ex}}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{e\,x}}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2), x)
```

**sympy [A]** time = 33.62, size = 95, normalized size = 1.12

$$\frac{2A\sqrt{e}\,x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + B\left(\frac{2\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}} - \frac{2\sqrt{e}\,x^{\frac{3}{2}}}{3\sqrt{a}\,b\sqrt{1+\frac{bx^3}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*sqr
t(1 + b*x**3/a)))
```

## 3.340 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=67

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

**Rubi [A]** time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))/
(3*a^2*e^4*Sqrt[a + b*x^3])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx &= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{(2Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{ae^3}\\ &= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 45, normalized size = 0.67

$$\frac{x\left(-2aA+2aBx^3-4Abx^3\right)}{3a^2(ex)^{5/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-2*a*A - 4*A*b*x^3 + 2*a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])
```

**IntegrateAlgebraic** [A] time = 1.04, size = 71, normalized size = 1.06

$$\frac{2\sqrt{a+bx^3}\left(-aAe^3+aBe^3x^3-2Abe^3x^3\right)}{3a^2e(ex)^{3/2}\left(ae^3+be^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-(a*A*e^3) - 2*A*b*e^3*x^3 + a*B*e^3*x^3))/(3*a^2*e*(e*
x)^(3/2)*(a*e^3 + b*e^3*x^3))
```

**fricas** [A] time = 0.65, size = 57, normalized size = 0.85

$$\frac{2\left((Ba-2\,Ab)x^3-Aa\right)\sqrt{bx^3+a}\,\sqrt{ex}}{3\left(a^2be^3x^5+a^3e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b*e^3*x^5 + a^
3*e^3*x^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

**maple** [A] time = 0.05, size = 39, normalized size = 0.58

$$-\frac{2\left(2A\,x^3b-Ba\,x^3+Aa\right)x}{3\sqrt{b\,x^3+a}\,\;(ex)^{\frac{5}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/(e*x)^(5/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

**mupad [B]** time = 4.75, size = 70, normalized size = 1.04

$$-\frac{\left(\frac{2A}{3abe^2}+\frac{x^3(4Ab-2Ba)}{3a^2be^2}\right)\sqrt{bx^3+a}}{x^4\sqrt{ex}+\frac{ax\sqrt{ex}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x)
```

```
[Out] -(((2*A)/(3*a*b*e^2) + (x^3*(4*A*b - 2*B*a))/(3*a^2*b*e^2))*(a + b*x^3)^(1/
2))/(x^4*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

## 3.341 $\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

**Optimal**. Leaf size=114

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}$$

**Rubi** [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {452, 288, 329, 275, 217, 206}

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*B*e^2*(e*x)^(3
/2))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(
e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{B\int \frac{(ex)^{7/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{b}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(Be^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(2Be^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(2Be^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(2Be^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 119, normalized size = 1.04

$$\frac{2e^3\sqrt{ex}\left(3a^{3/2}B\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\,x^{3/2}\left(-3a^2B-4abBx^3+Ab^2x^3\right)\right)}{9ab^{5/2}\sqrt{x}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3) + 3*
a^(3/2)*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]
]))/(9*a*b^(5/2)*Sqrt[x]*(a + b*x^3)^(3/2))
```

**IntegrateAlgebraic [A]** time = 1.91, size = 137, normalized size = 1.20

$$-\frac{2\sqrt{a+bx^3}\left(3a^2Be^8(ex)^{3/2}+4abBe^5(ex)^{9/2}-Ab^2e^5(ex)^{9/2}\right)}{9ab^2\left(ae^3+be^3x^3\right)^2}-\frac{2Be^5\sqrt{\frac{b}{e^3}}\log\left(\sqrt{a+bx^3}-\sqrt{\frac{b}{e^3}}\,(ex)^{3/2}\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^3]*(3*a^2*B*e^8*(e*x)^(3/2) - A*b^2*e^5*(e*x)^(9/2) + 4*a*
b*B*e^5*(e*x)^(9/2)))/(9*a*b^2*(a*e^3 + b*e^3*x^3)^2) - (2*B*Sqrt[b/e^3]*e^
5*Log[-(Sqrt[b/e^3]*(e*x)^(3/2)) + Sqrt[a + b*x^3]])/(3*b^3)
```

**fricas [A]** time = 0.84, size = 345, normalized size = 3.03

$$\left[\frac{3\left(Bab^{2}e^{3}x^{6}+2\,Ba^{2}be^{3}x^{3}+Ba^{3}e^{3}\right)\sqrt{\frac{e}{b}}\,\log\left(-8\,b^{2}ex^{6}-8\,abex^{3}-a^{2}e-4\left(2\,b^{2}x^{4}+abx\right)\sqrt{bx^{3}+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(\left(4\,Bab-Ab^{2}\right)e^{3}x^{4}+3\,Ba^{2}e^{3}x\right)\sqrt{bx^{3}+a}\,\sqrt{ex}}{18\left(ab^{4}x^{6}+2\,a^{2}b^{3}x^{3}+a^{3}b^{2}\right)},-\frac{3\left(Bab^{2}e^{3}x^{6}+2\,Ba^{2}be^{3}x^{3}+Ba^{3}e^{3}\right)\sqrt{-\frac{e}{b}}\,\arctan\left(\frac{2\,\sqrt{bx^{3}+a}\,\sqrt{ex}\,bx\sqrt{-\frac{e}{b}}}{2\,bex^{3}+ae}\right)+2\left(\left(4\,Bab-Ab^{2}\right)e^{3}x^{4}+3\,Ba^{2}e^{3}x\right)\sqrt{bx^{3}+a}\,\sqrt{ex}}{9\left(ab^{4}x^{6}+2\,a^{2}b^{3}x^{3}+a^{3}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(e/b)*log(-8
*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sq
rt(e*x)*sqrt(e/b)) - 4*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x
^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2), -1/9*(3*(B*a*b^2*
e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a
)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*((4*B*a*b - A*b^2)*e^3*x^
4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 +
a^3*b^2)]
```

**giac [A]** time = 0.48, size = 102, normalized size = 0.89

$$-\frac{2\,x^{\frac{3}{2}}\left(\frac{3\,Bae^{8}}{b^{2}}+\frac{\left(4\,Ba^{5}b^{6}e^{24}-Aa^{4}b^{7}e^{24}\right)x^{3}e^{(-16)}}{a^{5}b^{7}}\right)e^{\frac{3}{2}}}{9\left(bx^{3}e^{4}+ae^{4}\right)^{\frac{3}{2}}}-\frac{2\,Be^{\frac{7}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^{2}+\sqrt{bx^{3}e^{4}+ae^{4}}\right|\right)}{3\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*x^(3/2)*(3*B*a*e^8/b^2 + (4*B*a^5*b^6*e^24 - A*a^4*b^7*e^24)*x^3*e^(-1
6)/(a^5*b^7))*e^(3/2)/(b*x^3*e^4 + a*e^4)^(3/2) - 2/3*B*e^(7/2)*log(abs(-sq
rt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(5/2)
```

**maple [C]** time = 1.05, size = 7081, normalized size = 62.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)\left(ex\right)^{\frac{7}{2}}}{\left(bx^{3}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^{3}+A\right)\,{\left(e\,x\right)}^{7/2}}{{\left(b\,x^{3}+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

# 3.342 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

**Optimal**. Leaf size=79

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 264}

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)*
(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{\left(2\left(3Ab+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}\\ &= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 44, normalized size = 0.56

$$\frac{2x\sqrt{ex}\left(3aA+aBx^3+2Abx^3\right)}{9a^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))
```

**IntegrateAlgebraic [A]** time = 1.26, size = 76, normalized size = 0.96

$$\frac{2\sqrt{a+bx^3}\left(3aAe^5(ex)^{3/2}+aBe^2(ex)^{9/2}+2Abe^2(ex)^{9/2}\right)}{9a^2\left(ae^3+be^3x^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*a*A*e^5*(e*x)^(3/2) + 2*A*b*e^2*(e*x)^(9/2) + a*B*e^2
*(e*x)^(9/2)))/(9*a^2*(a*e^3 + b*e^3*x^3)^2)
```

**fricas [A]** time = 0.61, size = 59, normalized size = 0.75

$$\frac{2\left((Ba+2\,Ab)x^4+3\,Aax\right)\sqrt{bx^3+a}\,\sqrt{ex}}{9\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b^2*x^6 +
2*a^3*b*x^3 + a^4)
```

**giac [A]** time = 0.32, size = 64, normalized size = 0.81

$$\frac{2\,x^{\frac{3}{2}}\left(\frac{3\,Ae^5}{a}+\frac{\left(Ba^5b^5e^{21}+2\,Aa^4b^6e^{21}\right)x^3e^{(-16)}}{a^6b^5}\right)e^{\frac{3}{2}}}{9\left(bx^3e^4+ae^4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*x^(3/2)*(3*A*e^5/a + (B*a^5*b^5*e^21 + 2*A*a^4*b^6*e^21)*x^3*e^(-16)/(a
^6*b^5))*e^(3/2)/(b*x^3*e^4 + a*e^4)^(3/2)
```

**maple [A]** time = 0.05, size = 39, normalized size = 0.49

$$\frac{2\left(2A\,x^3b+Ba\,x^3+3Aa\right)\sqrt{ex}\;x}{9\left(b\,x^3+a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] 2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)\sqrt{ex}}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(5/2), x)
```

**mupad [B]** time = 4.63, size = 73, normalized size = 0.92

$$\frac{\left(\frac{2\,A\,x\,\sqrt{e\,x}}{3\,a\,b^2}+\frac{x^4\,\sqrt{e\,x}\,\left(4\,A\,b+2\,B\,a\right)}{9\,a^2\,b^2}\right)\,\sqrt{b\,x^3+a}}{x^6+\frac{a^2}{b^2}+\frac{2\,a\,x^3}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] (((2*A*x*(e*x)^(1/2))/(3*a*b^2) + (x^4*(e*x)^(1/2)*(4*A*b + 2*B*a))/(9*a^2*
b^2))*(a + b*x^3)^(1/2))/(x^6 + a^2/b^2 + (2*a*x^3)/b)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.343 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}} \, dx$

**Optimal**. Leaf size=104

$$-\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

**Rubi [A]** time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - (2*(4*A*b - a*B)*(e*x)^(3/2)
)/(9*a^2*e^4*(a + b*x^3)^(3/2)) - (4*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^3*e^4*
Sqrt[a + b*x^3])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx &= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{(4Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{5/2}}\,dx}{ae^3}\\
&= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{(2(4Ab-aB))\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{3a^2e^3}\\
&= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 65, normalized size = 0.62

$$\frac{x\left(-6a^2\left(A-Bx^3\right)+4abx^3\left(Bx^3-6A\right)-16Ab^2x^6\right)}{9a^3(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (x*(-16*A*b^2*x^6 - 6*a^2*(A - B*x^3) + 4*a*b*x^3*(-6*A + B*x^3)))/(9*a^3*(
e*x)^(5/2)*(a + b*x^3)^(3/2))
```

**IntegrateAlgebraic [A]** time = 1.11, size = 100, normalized size = 0.96

$$\frac{2\sqrt{a+bx^3}\left(-3a^2Ae^6+3a^2Be^6x^3-12aAbe^6x^3+2abBe^6x^6-8Ab^2e^6x^6\right)}{9a^3e(ex)^{3/2}\left(ae^3+be^3x^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-3*a^2*A*e^6 - 12*a*A*b*e^6*x^3 + 3*a^2*B*e^6*x^3 - 8*A
*b^2*e^6*x^6 + 2*a*b*B*e^6*x^6))/(9*a^3*e*(e*x)^(3/2)*(a*e^3 + b*e^3*x^3)^2
)
```

**fricas [A]** time = 0.63, size = 93, normalized size = 0.89

$$\frac{2\left(2\left(Bab-4\,Ab^2\right)x^6+3\left(Ba^2-4\,Aab\right)x^3-3\,Aa^2\right)\sqrt{bx^3+a}\sqrt{ex}}{9\left(a^3b^2e^3x^8+2\,a^4be^3x^5+a^5e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x^
3 + a)*sqrt(e*x)/(a^3*b^2*e^3*x^8 + 2*a^4*b*e^3*x^5 + a^5*e^3*x^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

**maple [A]** time = 0.05, size = 62, normalized size = 0.60

$$-\frac{2\left(8Ab^2x^6-2Bab\,x^6+12Aab\,x^3-3B\,a^2x^3+3A\,a^2\right)x}{9\left(b\,x^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9*x*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(b*x^3+a)
^(3/2)/a^3/(e*x)^(5/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

**mupad** [B] time = 4.80, size = 115, normalized size = 1.11

$$-\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,A}{3\,a\,b^2\,e^2}-\frac{x^3\,\left(6\,B\,a^2-24\,A\,a\,b\right)}{9\,a^3\,b^2\,e^2}+\frac{x^6\,\left(16\,A\,b^2-4\,B\,a\,b\right)}{9\,a^3\,b^2\,e^2}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^4\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x)
```

```
[Out] -((a + b*x^3)^(1/2)*((2*A)/(3*a*b^2*e^2) - (x^3*(6*B*a^2 - 24*A*a*b))/(9*a^
3*b^2*e^2) + (x^6*(16*A*b^2 - 4*B*a*b))/(9*a^3*b^2*e^2)))/(x^7*(e*x)^(1/2)
+ (a^2*x*(e*x)^(1/2))/b^2 + (2*a*x^4*(e*x)^(1/2))/b)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.344 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=220

$$\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}+\frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx\right.}{4b^4d}$$

**Rubi [A]** time = 0.24, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}+\frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}+\frac{a\left(a+bx^3\right)^{7/3}}{7b^4d}-\frac{\left(a+bx^3\right)^{10/3}}{10b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^3*(a + b*x^3)^(1/3))/(b^4*d)) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) + (a
*(a + b*x^3)^(7/3))/(7*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) + (2^(1/3)*a^
(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(10/3)*Log[a - b*x^3])/(3*2^(2/3)*b^4*d) - (a^(10/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2\sqrt[3]{a+bx}}{b^3d} + \frac{a(a+bx)^{4/3}}{b^3d} - \frac{(a+bx)^{7/3}}{b^3d} + \frac{a^3\sqrt[3]{a+bx}}{b^3(ad-bdx)}\right)dx, x, x^3\right)$$

$$= -\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{a^3\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{\left(2a^4\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(\cdots}\right.}{3b^3}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d} + \frac{a^{1\cdots}}{\cdots}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d} - \frac{a^{1\cdots}}{\cdots}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d} + \frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a\cdots}}{\sqrt[3]{a}}}{\sqrt{3}}\cdots\right.}{\sqrt{3}\,b^4d}$$

**Mathematica [A]** time = 0.26, size = 230, normalized size = 1.05

$$-\frac{140\sqrt[3]{2}a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-70\sqrt[3]{2}a^{10/3}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-140\sqrt[3]{2}\sqrt{3}a^{10/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+507a^3\sqrt[3]{a+bx^3}+111a^2bx^3\sqrt[3]{a+bx^3}+42b^3x^9\sqrt[3]{a+bx^3}+66ab^2x^6\sqrt[3]{a+bx^3}}{420b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/420*(507*a^3*(a + b*x^3)^(1/3) + 111*a^2*b*x^3*(a + b*x^3)^(1/3) + 66*a*
b^2*x^6*(a + b*x^3)^(1/3) + 42*b^3*x^9*(a + b*x^3)^(1/3) - 140*2^(1/3)*Sqrt
[3]*a^(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 14
0*2^(1/3)*a^(10/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 70*2^(1/3)*a^
(10/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3
)^(2/3)])/(b^4*d)
```

**IntegrateAlgebraic [A]** time = 0.25, size = 224, normalized size = 1.02

$$-\frac{\sqrt[3]{2}\,a^{10/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3b^4d}+\frac{a^{10/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}b^4d}+\frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^4d}+\frac{\sqrt[3]{a+bx^3}\left(-169a^3-37a^2bx^3-22ab^2x^6-14b^3x^9\right)}{140b^4d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-169*a^3 - 37*a^2*b*x^3 - 22*a*b^2*x^6 - 14*b^3*x^9))/(
140*b^4*d) + (2^(1/3)*a^(10/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3
))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^4*d) - (2^(1/3)*a^(10/3)*Log[-2*a^(1/3) +
 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^4*d) + (a^(10/3)*Log[2*a^(2/3) + 2^(2/3)*
a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*b^4*d)
```

**fricas [A]** time = 0.74, size = 186, normalized size = 0.85

$$-\frac{140\,\sqrt{3}\,2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^3\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+70\cdot 2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^3\log\left(2^{\frac{2}{3}}\left(-a\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-140\cdot 2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^3\log\left(2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)+3\left(14\,b^3x^9+22\,ab^2x^6+37\,a^2bx^3+169\,a^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{420\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(2^
(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(1/3))
/(b^4*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
```

```
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofa^3*(2*a)^(1/3)*1/6/b^4/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^
3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+a^3*(2*a)^(1/3)/sqrt(3)/b^4/d*atan(((a+b*
x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a^4*b^40*d^10*(2*a)^(1
/3)*1/6/a/b^44/d^11*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/10*(a+b*x^3)^(1
/3)*(a+b*x^3)^3*b^36*d^9-1/7*(a+b*x^3)^(1/3)*(a+b*x^3)^2*a*b^36*d^9+1/4*(a+
b*x^3)^(1/3)*(a+b*x^3)*a^2*b^36*d^9+(a+b*x^3)^(1/3)*a^3*b^36*d^9)/b^40/d^10
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{11}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima [A]** time = 1.55, size = 183, normalized size = 0.83

$$\frac{\frac{140\,\sqrt{3}\,2^{\frac{1}{3}} a^{\frac{10}{3}} \arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}} a^{\frac{1}{3}}+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{70\cdot 2^{\frac{1}{3}} a^{\frac{10}{3}} \log\left(2^{\frac{2}{3}} a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}} a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}-\frac{140\cdot 2^{\frac{1}{3}} a^{\frac{10}{3}} \log\left(-2^{\frac{1}{3}} a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}-\frac{3\left(14\left(bx^{3}+a\right)^{\frac{10}{3}}-20\left(bx^{3}+a\right)^{\frac{7}{3}} a+35\left(bx^{3}+a\right)^{\frac{4}{3}} a^{2}+140\left(bx^{3}+a\right)^{\frac{1}{3}} a^{3}\right)}{d}}{420\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/420*(140*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 70*2^(1/3)*a^(10/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 140*2^(1
/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*(14*(b*x^3 + a
)^(10/3) - 20*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/3)*a^2 + 140*(b*x^3 +
 a)^(1/3)*a^3)/d)/b^4
```

**mupad [B]** time = 4.81, size = 240, normalized size = 1.09

$$\frac{a\left(b\,x^3+a\right)^{7/3}}{7\,b^4\,d}-\frac{a^3\left(b\,x^3+a\right)^{1/3}}{b^4\,d}-\frac{a^2\left(b\,x^3+a\right)^{4/3}}{4\,b^4\,d}-\frac{\left(b\,x^3+a\right)^{10/3}}{10\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\left(b\,x^3+a\right)^{1/3}}{b^4\,d}-\frac{6\,2^{1/3}\,a^{13/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^4\,d}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^4\,d}+\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\left(b\,x^3+a\right)^{1/3}}{b^4\,d}+\frac{18\,2^{1/3}\,a^{13/3}\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a^3*(a + b*x^3)^(1/3))/(b^4*d) - (a^2*(a
 + b*x^3)^(4/3))/(4*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) - (2^(1/3)*a^(10
/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b^4*d) - (2^(1/3)*a^(10/3)
*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) - (6*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/
2 - 1/2))/(b^4*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b^4*d) + (2^(1/3)*a^(10/3)*lo
g((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) + (18*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/6
+ 1/6))/(b^4*d))*((3^(1/2)*1i)/6 + 1/6))/(b^4*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{11} \sqrt[3]{a+bx^{3}}}{-a+bx^{3}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**11*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

## 3.345 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

**Optimal**. Leaf size=174

$$\frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}$$

**Rubi** [A] time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^2*(a + b*x^3)^(1/3))/(b^3*d)) - (a + b*x^3)^(7/3)/(7*b^3*d) + (2^(1/3)
*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(7/3)*Log[a - b*x^3])/(3*2^(2/3)*b^3*d) - (a^(7/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2 \sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{(a+bx)^{4/3}}{b^2 d} + \frac{a^2 \sqrt[3]{a+bx}}{b^2(ad-bdx)}\right) dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{7b^3 d} + \frac{a^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3 d} + \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3 d} + \frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3} b^3 d} + \frac{a^{7/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\,\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3} b^3 d}$$

$$= -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3 d} + \frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3} b^3 d} - \frac{a^{7/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3} b^3 d} - \frac{\left(\sqrt[3]{2}\right.}{}$$

$$= -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{\left(a+bx^3\right)^{7/3}}{7b^3 d} + \frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3 d} + \frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3} b^3 d} - \frac{a^{7/3}\,\text{lo}}{}$$

**Mathematica [A]** time = 0.14, size = 207, normalized size = 1.19

$$\frac{1}{3}\left(-\frac{\frac{2\sqrt[3]{2}\,a^{7/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{bd} - \frac{\sqrt[3]{2}\,a^{7/3}\left(\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\,\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)\right)}{bd}}{2b^2} - \frac{3a^2\sqrt[3]{a+bx^3}}{b^3 d} - \frac{3\left(a+bx^3\right)^{7/3}}{7b^3 d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-3*a^2*(a + b*x^3)^(1/3))/(b^3*d) - (3*(a + b*x^3)^(7/3))/(7*b^3*d) - ((2
*2^(1/3)*a^(7/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(b*d) - (2^(1/3)
*a^(7/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a
^(1/3))] + Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)]))/(b*d))/(2*b^2))/3
```

**IntegrateAlgebraic [A]** time = 0.19, size = 213, normalized size = 1.22

$$-\frac{\sqrt[3]{2}\,a^{7/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3b^3d}+\frac{a^{7/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^3d}+\frac{\sqrt[3]{a+bx^3}\left(-8a^2-2abx^3-b^2x^6\right)}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-8*a^2 - 2*a*b*x^3 - b^2*x^6))/(7*b^3*d) + (2^(1/3)*a^(
7/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^3*d) - (2^(1/3)*a^(7/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)]
)/(3*b^3*d) + (a^(7/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) +
2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*b^3*d)
```

**fricas [A]** time = 0.49, size = 174, normalized size = 1.00

$$-\frac{14\,\sqrt{3}\,2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^2\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+7\cdot 2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^2\log\left(2^{\frac{2}{3}}\left(-a\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-14\cdot 2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^2\log\left(2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)+6\left(b^2x^6+2\,abx^3+8\,a^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{42\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/42*(14*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^2*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3
 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 7*2^(1/3)*(-a)^(1/3)*a^2*log(2^(2/
3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) -
 14*2^(1/3)*(-a)^(1/3)*a^2*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*
(b^2*x^6 + 2*a*b*x^3 + 8*a^2)*(b*x^3 + a)^(1/3))/(b^3*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
```

```
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofa^2*(2*a)^(1/3)*1/6/b^3/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^
3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+a^2*(2*a)^(1/3)/sqrt(3)/b^3/d*atan(((a+b*
x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a^3*b^21*d^7*(2*a)^(1/
3)*1/6/a/b^24/d^8*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/7*(a+b*x^3)^(1/3)
*(a+b*x^3)^2*b^18*d^6+(a+b*x^3)^(1/3)*a^2*b^18*d^6)/b^21/d^7
```

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{8}}{-bd\,x^{3}+ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima [A]** time = 1.41, size = 155, normalized size = 0.89

$$\frac{\frac{14\,\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{7\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}-\frac{14\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}-\frac{6\left(\left(bx^{3}+a\right)^{\frac{7}{3}}+7\left(bx^{3}+a\right)^{\frac{1}{3}}a^{2}\right)}{d}}{42\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/42*(14*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 7*2^(1/3)*a^(7/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 14*2^(1/3)*a^
(7/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*((b*x^3 + a)^(7/3) +
7*(b*x^3 + a)^(1/3)*a^2)/d)/b^3
```

**mupad [B]** time = 4.65, size = 219, normalized size = 1.26

$$\frac{2^{1/3}\,{\left(-a\right)}^{7/3}\,\ln\left(6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}-6\,2^{1/3}\,{\left(-a\right)}^{10/3}\right)}{3\,b^3\,d}-\frac{a^2\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}-\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d}-\frac{2^{1/3}\,{\left(-a\right)}^{7/3}\,\ln\left(\frac{6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}+\frac{6\,2^{1/3}\,{\left(-a\right)}^{10/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^3\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^3\,d}+\frac{2^{1/3}\,{\left(-a\right)}^{7/3}\,\ln\left(\frac{6\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^3\,d}-\frac{18\,2^{1/3}\,{\left(-a\right)}^{10/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*(-a)^(7/3)*log(6*a^3*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(10/3)))/(
3*b^3*d) - (a^2*(a + b*x^3)^(1/3))/(b^3*d) - (a + b*x^3)^(7/3)/(7*b^3*d) -
(2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) + (6*2^(1/3)*(-a)
^(10/3)*((3^(1/2)*1i)/2 + 1/2))/(b^3*d))*((3^(1/2)*1i)/2 + 1/2))/(3*b^3*d)
+ (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) - (18*2^(1/3)*(
-a)^(10/3)*((3^(1/2)*1i)/6 - 1/6))/(b^3*d))*((3^(1/2)*1i)/6 - 1/6))/(b^3*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{8}\sqrt[3]{a+bx^{3}}}{-a+bx^{3}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

## 3.346 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

**Optimal**. Leaf size=172

$$\frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}+\frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}$$

**Rubi [A]** time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 50, 57, 617, 204, 31}

$$\frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}+\frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(b^2*d)) - (a + b*x^3)^(4/3)/(4*b^2*d) + (2^(1/3)*a
^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(4/3)*Log[a - b*x^3])/(3*2^(2/3)*b^2*d) - (a^(4/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$
$$= -\frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d} + \frac{a^{4/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right.}{2^{2/3}b^2d}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} - \frac{\left(\sqrt[3]{2}\right.}{}$$
$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{\sqrt[3]{2}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d} - \frac{a^{4/3}\log}{}$$

**Mathematica [A]** time = 0.10, size = 186, normalized size = 1.08

$$-\frac{4\sqrt[3]{2}\,a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-2\sqrt[3]{2}\,a^{4/3}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-4\sqrt[3]{2}\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+3bx^3\sqrt[3]{a+bx^3}+15a\sqrt[3]{a+bx^3}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/12*(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt[
3]*a^(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*2^
(1/3)*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 2*2^(1/3)*a^(4/3)*
Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/(b^2*d)
```

**IntegrateAlgebraic [A]** time = 0.18, size = 202, normalized size = 1.17

$$-\frac{\sqrt[3]{2}\,a^{4/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3b^2d}+\frac{a^{4/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}b^2d}+\frac{\sqrt[3]{2}\,a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^2d}+\frac{\left(-5a-bx^3\right)\sqrt[3]{a+bx^3}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-5*a - b*x^3)*(a + b*x^3)^(1/3))/(4*b^2*d) + (2^(1/3)*a^(4/3)*ArcTan[1/Sq
rt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^2*d) - (
2^(1/3)*a^(4/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^2*d) + (a
^(4/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x
^3)^(2/3)])/(3*2^(2/3)*b^2*d)
```

**fricas [A]** time = 0.78, size = 157, normalized size = 0.91

$$-\frac{4\sqrt{3}\,2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\,(bx^3+a)^{\frac{1}{3}}\,(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2\cdot 2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\,a\log\left(2^{\frac{2}{3}}\,(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-4\cdot 2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\,a\log\left(2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)+3\left(bx^3+5\,a\right)\left(bx^3+a\right)^{\frac{1}{3}}}{12\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*2^(1/3)*(-a)^(1/3)*a*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 +
a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2*2^(1/3)*(-a)^(1/3)*a*log(2^(2/3)*(-
a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 4*2^
(1/3)*(-a)^(1/3)*a*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 3*(b*x^3 +
 5*a)*(b*x^3 + a)^(1/3))/(b^2*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
```

```
rootofa*(2*a)^(1/3)*1/6/b^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)
^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+a*(2*a)^(1/3)/sqrt(3)/b^2/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a^2*b^8*d^4*(2*a)^(1/3)*1/
6/a/b^10/d^5*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/4*(a+b*x^3)^(1/3)*(a+b
*x^3)*b^6*d^3+(a+b*x^3)^(1/3)*a*b^6*d^3)/b^8/d^4
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^5}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima [A]** time = 1.43, size = 153, normalized size = 0.89

$$\frac{\frac{4\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{4\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{3\left(\left(bx^3+a\right)^{\frac{4}{3}}+4\left(bx^3+a\right)^{\frac{1}{3}}a\right)}{d}}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/12*(4*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2*2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2/3)
+ 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 4*2^(1/3)*a^(4
/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*((b*x^3 + a)^(4/3) + 4*
(b*x^3 + a)^(1/3)*a)/d)/b^2
```

**mupad [B]** time = 4.66, size = 200, normalized size = 1.16

$$-\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b^2\,d}-\frac{a\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^2\,d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}-\frac{6\,2^{1/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^2\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^2\,d}+\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}+\frac{18\,2^{1/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d) + (18*2^(1/3)*a^(7/3
)*((3^(1/2)*1i)/6 + 1/6))/(b^2*d))*((3^(1/2)*1i)/6 + 1/6))/(b^2*d) - (a*(a
+ b*x^3)^(1/3))/(b^2*d) - (2^(1/3)*a^(4/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*
a^(1/3)))/(3*b^2*d) - (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d
) - (6*2^(1/3)*a^(7/3)*((3^(1/2)*1i)/2 - 1/2))/(b^2*d))*((3^(1/2)*1i)/2 - 1
/2))/(3*b^2*d) - (a + b*x^3)^(4/3)/(4*b^2*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^5\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

## 3.347 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=150

$$-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}$$

**Rubi [A]** time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {444, 50, 57, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(1/3)*Log[a - b*x^3]
)/(3*2^(2/3)*b*d) - (a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
2/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd} + \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} + \frac{a^{2/3}\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} - \frac{\left(\sqrt[3]{2}\sqrt[3]{a}\right)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a\right.}}{2^{2/3}bd}$$

**Mathematica [A]** time = 0.05, size = 167, normalized size = 1.11

$$\frac{\sqrt[3]{2}\sqrt[3]{a}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-6\sqrt[3]{a+bx^3}-2\sqrt[3]{2}\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a +
 b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[2^(1/3)*a^(1/3) -
(a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*b*d)
```

**IntegrateAlgebraic [A]** time = 0.17, size = 190, normalized size = 1.27

$$\frac{\sqrt[3]{a}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}bd} - \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{\sqrt[3]{2}\sqrt[3]{a}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3bd} + \frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) - (2^(1/3)*a^(1/3)*Log[
-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b*d) + (a^(1/3)*Log[2*a^(2/3) +
 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)
*b*d)
```

**fricas [A]** time = 0.70, size = 144, normalized size = 0.96

$$-\frac{2\sqrt{3}\,2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3a}\right)+2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)+6\,(bx^3+a)^{\frac{1}{3}}}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^
(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2/3
) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3)*(
-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(1/3)
)/(b*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootof(2*a)^(1/3)*1/6/b/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(1/
3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)/sqrt(3)/b/d*atan(((a+b*x^3)^(1/3)+1
/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a*b*d*(2*a)^(1/3)*1/6/a/b^2/d^2*ln
(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(a+b*x^3)^(1/3)/b/d
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^2}{-bd\,x^3+ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima [A]** time = 1.27, size = 139, normalized size = 0.93

$$\frac{\frac{2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{6\left(bx^3+a\right)^{\frac{1}{3}}}{d}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3) + 2
^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 2*2^(1/3)*a^(1/3)
*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*(b*x^3 + a)^(1/3)/d)/b
```

**mupad [B]** time = 4.64, size = 194, normalized size = 1.29

$$\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(6\,a\,\left(b\,x^3+a\right)^{1/3}-6\,2^{1/3}\,(-a)^{4/3}\right)}{3\,b\,d}-\frac{\left(b\,x^3+a\right)^{1/3}}{b\,d}+\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(\frac{6\,a\,\left(b\,x^3+a\right)^{1/3}}{b\,d}-\frac{6\,2^{1/3}\,(-a)^{4/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b\,d}-\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(\frac{6\,a\,\left(b\,x^3+a\right)^{1/3}}{b\,d}+\frac{6\,2^{1/3}\,(-a)^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*(-a)^(1/3)*log(6*a*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(4/3)))/(3*b
*d) - (a + b*x^3)^(1/3)/(b*d) + (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1
/3))/(b*d) - (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 - 1/2))/(b*d))*((3^(1/2)
*1i)/2 - 1/2))/(3*b*d) - (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1/3))/(b
*d) + (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/(b*d))*((3^(1/2)*1i)/2
+ 1/2))/(3*b*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

# 3.348 $\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=214

$$\frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}$$

**Rubi** [A] time = 0.17, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {446, 83, 57, 617, 204, 31}

$$\frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}-\frac{\log(x)}{2a^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*d)) + (2^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(2/3)*d) - Log[x]/(2*a^(2/3)*d) + Log[a - b*x^3]/(3*2^(2/
3)*a^(2/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*d) - Log[2^(1/3
)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(ad-bdx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right) + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x,\right.}{2^{2/3}a^{2/3}d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d} + \frac{\text{Subs}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d} + \frac{\sqrt[3]{2}\,\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d} - \frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\right.}{2a^{2/}}$$

**Mathematica [A]** time = 0.11, size = 233, normalized size = 1.09

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-\sqrt[3]{2}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6a^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(
1/3)*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*
Log[a^(1/3) - (a + b*x^3)^(1/3)] + 2*2^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x
^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] -
 2^(1/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x
^3)^(2/3)])/(a^(2/3)*d)
```

**IntegrateAlgebraic [A]** time = 0.35, size = 286, normalized size = 1.34

$$\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3a^{2/3}d}-\frac{\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3a^{2/3}d}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6a^{2/3}d}+\frac{\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}a^{2/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2
/3)*d)) + (2^(1/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(2/3)*d) + Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(3*a^(2/
3)*d) - (2^(1/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^(2/3)*d)
 - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*a^(2/3)*
```

```
d) + Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3
)^(2/3)]/(3*2^(2/3)*a^(2/3)*d)
```

**fricas [B]** time = 0.72, size = 1046, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/
3)*arctan(1/12*(1/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)) - 3*sqrt(3)*a
*d^2)*sqrt(-4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^
4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/2)^(2/3)*a^2*d^2*(-(
3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 4*(b*x^3 + a)^(2/3))*(-
(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) - 1/6*(1/2)^(2/3)*(sqrt(
3)*a^3*d^5*sqrt(1/(a^4*d^6)) - 3*sqrt(3)*a*d^2)*(b*x^3 + a)^(1/3)*(-(3*a^2*
d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 1/3*sqrt(3)) + 2/3*sqrt(3)*(1
/2)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*arctan(1/6*(1
/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)) + 3*sqrt(3)*a*d^2)*sqrt((1/2)^
(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3
))^(1/3)*sqrt(1/(a^4*d^6)) + (1/2)^(2/3)*a^2*d^2*((3*a^2*d^3*sqrt(1/(a^4*d^
6)) - 1)/(a^2*d^3))^(2/3) + (b*x^3 + a)^(2/3))*((3*a^2*d^3*sqrt(1/(a^4*d^6)
) - 1)/(a^2*d^3))^(2/3) - 1/6*(1/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)
) + 3*sqrt(3)*a*d^2)*(b*x^3 + a)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(
a^2*d^3))^(2/3) - 1/3*sqrt(3)) - 1/6*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d
^6)) + 1)/(a^2*d^3))^(1/3)*log(-4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*(-(
3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/
2)^(2/3)*a^2*d^2*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 4*(
b*x^3 + a)^(2/3)) - 1/6*(1/2)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2
*d^3))^(1/3)*log(4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(1
/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/2)^(2/3)*a^2*d^2
*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(2/3) + 4*(b*x^3 + a)^(2/3))
 + 1/3*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*log
((1/2)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*s
qrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3)) + 1/3*(1/2)^(1/3)*((3*a^2*d^3*sqrt(1/
(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(
1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3))
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
```

```
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootof1/3/((2*a)^(1/3))^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(
1/3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)/sqrt(3)/a/d*atan(((a+b*x^3)^(1/3)
+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*1/6/a/d*ln(abs((a+b*
x^3)^(1/3)-(2*a)^(1/3)))-a^(1/3)*1/6/a/d*ln(((a+b*x^3)^(1/3))^2+a^(1/3)*(a+
b*x^3)^(1/3)+a^(1/3)*a^(1/3))-a^(1/3)/sqrt(3)/a/d*atan(((a+b*x^3)^(1/3)+1/2
*a^(1/3))/sqrt(3)*2/a^(1/3))+a^(1/3)*1/3/a/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)
))
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(-bd\,x^{3}+ad\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(bdx^{3}-ad\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x), x)
```

**mupad [B]** time = 5.10, size = 345, normalized size = 1.61

$$\ln\left(\left(b\,x^3+a\right)^{1/3}-a\,d\,{\left(\frac{1}{a^2\,d^3}\right)}^{1/3}\right)\,{\left(\frac{1}{27\,a^2\,d^3}\right)}^{1/3}+\ln\left(\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a\,d\,{\left(-\frac{1}{a^2\,d^3}\right)}^{1/3}\right)\,{\left(-\frac{2}{27\,a^2\,d^3}\right)}^{1/3}-\ln\left(2^{1/3}\,a\,d\,{\left(-\frac{1}{a^2\,d^3}\right)}^{1/3}-2\,\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,\sqrt{3}\,a\,d\,{\left(-\frac{1}{a^2\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2}{27\,a^2\,d^3}\right)}^{1/3}+\ln\left(2\,\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a\,d\,{\left(-\frac{1}{a^2\,d^3}\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,a\,d\,{\left(-\frac{1}{a^2\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2}{27\,a^2\,d^3}\right)}^{1/3}+\ln\left(2\,\left(b\,x^3+a\right)^{1/3}+a\,d\,{\left(\frac{1}{a^2\,d^3}\right)}^{1/3}-\sqrt{3}\,a\,d\,{\left(\frac{1}{a^2\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{1}{27\,a^2\,d^3}\right)}^{1/3}-\ln\left(2\,\left(b\,x^3+a\right)^{1/3}+a\,d\,{\left(\frac{1}{a^2\,d^3}\right)}^{1/3}+\sqrt{3}\,a\,d\,{\left(\frac{1}{a^2\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{1}{27\,a^2\,d^3}\right)}^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - a*d*(1/(a^2*d^3))^(1/3))*(1/(27*a^2*d^3))^(1/3) + l
og((a + b*x^3)^(1/3) + 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3))*(-2/(27*a^2*d^3))^
(1/3) - log(2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) - 2*(a + b*x^3)^(1/3) + 2^(1/3
)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-2/(27*a^2*d
^3))^(1/3) + log(2*(a + b*x^3)^(1/3) - 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) + 2
^(1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-2/(27*
a^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) - 3^(1/
```

```
2)*a*d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*d^3))^(1/3
) - log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) + 3^(1/2)*a*d*(1/(a^2
*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*d^3))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax+bx^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d
```

## 3.349 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$

**Optimal**. Leaf size=268

$$\frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}d}+\frac{\sqrt[3]{2}\,b\tan^{-1}\left(\ldots\right.}{\sqrt{\ldots}}$$

**Rubi [A]** time = 0.25, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 156, 50, 57, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}+\frac{b\sqrt[3]{a+bx^3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}d}+\frac{\sqrt[3]{2}\,b\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}d}-\frac{2b\log(x)}{3a^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/(3*a^2*d) - (a + b*x^3)^(4/3)/(3*a^2*d*x^3) - (4*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*d) + (2^(1/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(5/3)*d) - (2*b*Log[x])/(3*a^(5/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(2/3)*a^(5/3)*d) + (2*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(ad-bdx)}\,dx,x,x^3\right)$$

$$=-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}-\frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd+\frac{1}{3}b^2dx\right)}{x(ad-bdx)}\,dx,x,x^3\right)}{3a^2d}$$

$$=-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}+\frac{b^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx,x,x^3\right)}{3a^2}+\frac{(4b)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx,x,x^3\right)}{9a^2d}$$

$$=\frac{b\sqrt[3]{a+bx^3}}{3a^2d}-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}+\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx,x,x^3\right)}{3a}+\frac{(4b)\,\text{Subst}\left(\int \frac{}{x(a\cdots}\right.}{9a\cdots}$$

$$=\frac{b\sqrt[3]{a+bx^3}}{3a^2d}-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}-\frac{2b\log(x)}{3a^{5/3}d}+\frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a}\cdots\right.}{3a^{5/3}d}$$

$$=\frac{b\sqrt[3]{a+bx^3}}{3a^2d}-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}-\frac{2b\log(x)}{3a^{5/3}d}+\frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\cdots}{\cdots}$$

$$=\frac{b\sqrt[3]{a+bx^3}}{3a^2d}-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}-\frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}d}+\frac{\sqrt[3]{2}\,b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}d}-\frac{2b\log(x\cdots}{3a^{5/3}d}$$

**Mathematica [A]** time = 0.12, size = 280, normalized size = 1.04

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^3}+4bx^3\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-3\sqrt[3]{2}bx^3\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-8bx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+6\sqrt[3]{2}bx^3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+8\sqrt{3}bx^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-6\sqrt[3]{2}\sqrt{3}bx^3\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{18a^{5/3}dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] -1/18*(6*a^(2/3)*(a + b*x^3)^(1/3) + 8*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*
x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3
)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 8*b*x^3*Log[a^(1/3) - (a + b*x^3)^
(1/3)] + 6*2^(1/3)*b*x^3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] + 4*b*x^3
*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 3*2^(1/3)*b
*x^3*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)])/(a^(5/3)*d*x^3)
```

**IntegrateAlgebraic** [**A**]   time = 0.57, size = 318, normalized size = 1.19

$$\frac{4b\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9a^{5/3}d}-\frac{\sqrt[3]{2}\,b\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3a^{5/3}d}-\frac{2b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{9a^{5/3}d}+\frac{b\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}a^{5/3}d}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}d}+\frac{\sqrt[3]{2}\,b\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}d}-\frac{\sqrt[3]{a+bx^3}}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(1/3)/(a*d*x^3) - (4*b*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*d) + (2^(1/3)*b*ArcTan[1/Sqrt[
3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*d) +
(4*b*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(9*a^(5/3)*d) - (2^(1/3)*b*Log[-2*a
^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^(5/3)*d) - (2*b*Log[a^(2/3) + a^(
1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(9*a^(5/3)*d) + (b*Log[2*a^(2/
3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(
2/3)*a^(5/3)*d)
```

**fricas** [**A**]   time = 0.74, size = 321, normalized size = 1.20

$$-\frac{6\sqrt{3}\,2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+3\cdot 2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-6\cdot 2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)+8\sqrt{3}\left(a^2\right)^{\frac{1}{6}}abx^3\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+4\left(a^2\right)^{\frac{2}{3}}bx^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-8\left(a^2\right)^{\frac{2}{3}}bx^3\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+6\left(bx^3+a\right)^{\frac{1}{3}}a^2}{18\,a^3dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 3*2^(1/3)*a^2*b*x^3*(
-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*a*
(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 6*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*lo
g(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 8*sqrt(3)*(a^2)^(1/6)*a*b
*x^3*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^
(1/3)*(a^2)^(2/3))/a^2) + 4*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a + (a^
2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 8*(a^2)^(2/3)*b*x^3*log((b*x^
3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2)/(a^3*d*x^3)
```

**giac** [**F(-2)**]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
```

```
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofb/3/a/((2*a)^(1/3))^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)
^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)*b/sqrt(3)/a^2/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*b*(2*a)^(1/3)*1/6/a^2/d*ln
(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-2*a^(1/3)*b*1/9/a^2/d*ln(((a+b*x^3)^(1/3
))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))-4*a^(1/3)*b/3/sqrt(3)/a^2/d*a
tan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+4*b*a^(1/3)*1/9/a^2/d*
ln(abs((a+b*x^3)^(1/3)-a^(1/3)))-(a+b*x^3)^(1/3)*b/3/a/d/(a+b*x^3-a)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(-bd\,x^{3}+ad\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(bdx^{3}-ad\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)
```

**mupad [B]** time = 5.35, size = 455, normalized size = 1.70

$$\frac{4\ln\left(b\left(b\,x^3+a\right)^{1/3}-a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}}{9}+\ln\left(b\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a^2\,d\,\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\,\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)^{1/3}+\ln\left(2\,b\left(b\,x^3+a\right)^{1/3}+a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}-\sqrt{3}\,a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{64\,b^3}{729\,a^5\,d^3}\right)^{1/3}-\ln\left(2\,b\left(b\,x^3+a\right)^{1/3}+a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}+\sqrt{3}\,a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{64\,b^3}{729\,a^5\,d^3}\right)^{1/3}-\ln\left(2^{1/3}\,a^2\,d\,\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}-2\,b\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,\sqrt{3}\,a^2\,d\,\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)^{1/3}+\ln\left(2\,b\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^2\,d\,\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}+2^{1/3}\,\sqrt{3}\,a^2\,d\,\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)^{1/3}-\frac{b\left(b\,x^3+a\right)^{1/3}}{3\,a\,\left(d\,\left(b\,x^3+a\right)-a\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x)
```

```
[Out] (4*log(b*(a + b*x^3)^(1/3) - a^2*d*(b^3/(a^5*d^3))^(1/3))*(b^3/(a^5*d^3))^(
1/3))/9 + log(b*(a + b*x^3)^(1/3) + 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3))*(
-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a^5*
d^3))^(1/3) - 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2
)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(
a^5*d^3))^(1/3) + 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(
1/3) - 2*b*(a + b*x^3)^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)
*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*
x^3)^(1/3) - 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(
-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1
/3) - (b*(a + b*x^3)^(1/3))/(3*a*(d*(a + b*x^3) - a*d))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^4+bx^7}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d
```

## 3.350 $\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=283

$$\frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}-\frac{11b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}d}+\frac{\sqrt[3]{2}\,b^2\tan \cdots}{\cdots}$$

**Rubi [A]** time = 0.26, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 149, 156, 57, 617, 204, 31}

$$\frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}-\frac{11b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}d}+\frac{\sqrt[3]{2}\,b^2\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}d}-\frac{11b^2\log(x)}{18a^{8/3}d}-\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3}-\frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-2*b*(a + b*x^3)^(1/3))/(9*a^2*d*x^3) - (a + b*x^3)^(4/3)/(6*a^2*d*x^6) -
(11*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(8/3)*d) + (2^(1/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*d) - (11*b^2*Log[x])/(18*a^(8/3)*d) + (
b^2*Log[a - b*x^3])/(3*2^(2/3)*a^(8/3)*d) + (11*b^2*Log[a^(1/3) - (a + b*x^
3)^(1/3)])/(18*a^(8/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/
(2^(2/3)*a^(8/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd-\frac{2}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{22}{9}a^2b^2d^2-\frac{14}{9}ab^3d^2x}{x(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{\left(11b^2\right)\text{Subs}\cdots}{\cdots}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d} - \frac{\left(11b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\cdots}\cdots\right)}{18a^{8/\cdots}}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d} + \frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right.}{18a^{8/3}d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{8/3}d} + \frac{\sqrt[3]{2}\,b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}d} - \cdots$$

**Mathematica [A]** time = 0.14, size = 314, normalized size = 1.11

$$-\frac{11b^2x^6\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-9\sqrt[3]{2}\,b^2x^6\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+21a^{2/3}bx^3\sqrt[3]{a+bx^3}+9a^{5/3}\sqrt[3]{a+bx^3}-22b^2x^6\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+18\sqrt[3]{2}\,b^2x^6\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+22\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-18\sqrt[3]{2}\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{54a^{8/3}dx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] -1/54*(9*a^(5/3)*(a + b*x^3)^(1/3) + 21*a^(2/3)*b*x^3*(a + b*x^3)^(1/3) + 2
2*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*
2^(1/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]] - 22*b^2*x^6*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 18*2^(1/3)*b^2*x^6*L
og[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] + 11*b^2*x^6*Log[a^(2/3) + a^(1/3)*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*b^2*x^6*Log[2^(2/3)*a^(2
/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(a^(8/3)*d*x^
6)
```

**IntegrateAlgebraic [A]** time = 0.72, size = 340, normalized size = 1.20

$$\frac{11b^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{27a^{8/3}d}-\frac{\sqrt[3]{2}\,b^2\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3a^{8/3}d}-\frac{11b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54a^{8/3}d}+\frac{b^2\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\,2^{2/3}a^{8/3}d}-\frac{11b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{8/3}d}+\frac{\sqrt[3]{2}\,b^2\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}d}+\frac{\left(-3a-7bx^3\right)\sqrt[3]{a+bx^3}}{18a^2dx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*a - 7*b*x^3)*(a + b*x^3)^(1/3))/(18*a^2*d*x^6) - (11*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*d) + (2^
(1/3)*b^2*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(8/3)*d) + (11*b^2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(27*a^(8
/3)*d) - (2^(1/3)*b^2*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^(8/
3)*d) - (11*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/(54*a^(8/3)*d) + (b^2*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3)
+ 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^(8/3)*d)
```

**fricas [A]** time = 0.64, size = 345, normalized size = 1.22

$$-\frac{18\sqrt{3}\,2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+9\cdot 2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)-18\cdot 2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)+22\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^6\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+11\left(a^2\right)^{\frac{2}{3}}b^2x^6\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-22\left(a^2\right)^{\frac{2}{3}}b^2x^6\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+3\left(7a^2bx^3+3a^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{54a^4dx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(
2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 9*2^(1/3)*a^2*b^2*
x^6*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/
3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 18*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^
(1/3)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 22*sqrt(3)*(a^2)^
(1/6)*a*b^2*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + 11*(a^2)^(2/3)*b^2*x^6*log((b*x^3 + a)
^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 22*(a^2)^(2/3)*
b^2*x^6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x^3 + 3*a^3)*(b
*x^3 + a)^(1/3))/(a^4*d*x^6)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
```

```
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofb^2/3/a^2/((2*a)^(1/3))^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*
x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)*b^2/sqrt(3)/a^3/d*atan(((a+
b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*b^2*(2*a)^(1/3)*1/6/
a^3/d*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-11*a^(1/3)*b^2*1/54/a^3/d*ln(((a
+b*x^3)^(1/3))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))-11*a^(1/3)*b^2/9/
sqrt(3)/a^3/d*atan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+11*b^2*
a^(1/3)*1/27/a^3/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)))-(7*(a+b*x^3)^(1/3)*(a+b
*x^3)*b^2-4*(a+b*x^3)^(1/3)*a*b^2)/18/a^2/d/(a+b*x^3-a)^2
```

**maple** [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(-bd\,x^{3}+ad\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(bdx^{3}-ad\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^7), x)
```

**mupad** [B] time = 5.44, size = 490, normalized size = 1.73

$$\frac{\frac{2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}{9\,a}-\frac{7\,b^2\,{\left(b\,x^3+a\right)}^{4/3}}{18\,a^2}}{d\,{\left(b\,x^3+a\right)}^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}+\frac{11\,\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}-a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{27}+\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}-\ln\left(2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}-2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}+\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}+2^{1/3}\,\sqrt{3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}+\frac{11\,\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}-\sqrt{3}\,a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{54}-\frac{11\,\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}+\sqrt{3}\,a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\,1{}\mathrm{i}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x)
```

```
[Out] ((2*b^2*(a + b*x^3)^(1/3))/(9*a) - (7*b^2*(a + b*x^3)^(4/3))/(18*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + (11*log(b^2*(a + b*x^3)^(1/3) -
 a^3*d*(b^6/(a^8*d^3))^(1/3))*(b^6/(a^8*d^3))^(1/3))/27 + log(b^2*(a + b*x^
3)^(1/3) + 2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3))*(-(2*b^6)/(27*a^8*d^3))^(1
/3) - log(2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3) - 2*b^2*(a + b*x^3)^(1/3) +
2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(
2*b^6)/(27*a^8*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) - 2^(1/3)*a^3*d*(-
b^6/(a^8*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3
^(1/2)*1i)/2 - 1/2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + (11*log(2*b^2*(a + b*x^
3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) - 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3
)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^8*d^3))^(1/3))/54 - (11*log(2*b^2*(a + b*x^3
)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) + 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3)
*1i)*(3^(1/2)*1i + 1)*(b^6/(a^8*d^3))^(1/3))/54
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^7+bx^{10}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**7 + b*x**10), x)/d
```

## 3.351 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

**Optimal**. Leaf size=268

$$\frac{a^2 \log \left(ad-bdx^3\right)}{3\ 2^{2/3} b^{8/3} d}+\frac{11 a^2 \log \left(\sqrt[3]{b} x-\sqrt[3]{a+bx^3}\right)}{18 b^{8/3} d}-\frac{a^2 \log \left(\sqrt[3]{2} \sqrt[3]{b} x-\sqrt[3]{a+bx^3}\right)}{2^{2/3} b^{8/3} d}+\frac{11 a^2 \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9 \sqrt{3} b^{8/3} d}-\frac{\sqrt[3]{2} \ldots}{\ldots}$$

**Rubi** [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^8 \sqrt[3]{a+bx^3} F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad \sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), (b*x^3)/a
])/(8*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{x^7 \sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3} \, dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^8 \sqrt[3]{a+bx^3} F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad \sqrt[3]{1+\frac{bx^3}{a}}} \end{aligned}$$

**Mathematica** [C] time = 0.21, size = 177, normalized size = 0.66

$$\frac{22abx^5 \left(1-\frac{b^2x^6}{a^2}\right)^{2/3} F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5x^2 \left(\left(1-\frac{bx^3}{a}\right)^{2/3} \left(7a^2+10abx^3+3b^2x^6\right)-7a^2 \left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)\right)}{90b^2d \left(a+bx^3\right)^{2/3} \left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (22*a*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/
a), (b*x^3)/a] - 5*x^2*((1 - (b*x^3)/a)^(2/3)*(7*a^2 + 10*a*b*x^3 + 3*b^2*x
^6) - 7*a^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^
3)/(a - b*x^3)]))/(90*b^2*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

**IntegrateAlgebraic [A]** time = 1.21, size = 364, normalized size = 1.36

$$\frac{11a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27b^{8/3}d}-\frac{\sqrt[3]{2}a^2\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3b^{8/3}d}+\frac{11a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}b^{8/3}d}-\frac{\sqrt[3]{2}a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}b^{8/3}d}-\frac{11a^2\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54b^{8/3}d}+\frac{a^2\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\,2^{2/3}b^{8/3}d}+\frac{\sqrt[3]{a+bx^3}\left(-7ax^2-3bx^5\right)}{18b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-7*a*x^2 - 3*b*x^5))/(18*b^2*d) + (11*a^2*ArcTan[(Sqrt[
3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(8/3)*d) - (
2^(1/3)*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/
3))])/(Sqrt[3]*b^(8/3)*d) + (11*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/
(27*b^(8/3)*d) - (2^(1/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)]
)/(3*b^(8/3)*d) - (11*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/(54*b^(8/3)*d) + (a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3
)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*b^(8/3)*d)
```

**fricas [A]** time = 0.64, size = 362, normalized size = 1.35

$$-\frac{18\sqrt{3}\,2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-18\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+9\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}b^2x^2\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+22\sqrt{3}a^2\left(b^2\right)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\left(b^2\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{\frac{1}{6}}}{3b^2x}\right)-22a^2\left(b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+11a^2\left(b^2\right)^{\frac{2}{3}}\log\left(\frac{\left(b^2\right)^{\frac{1}{3}}bx^2+\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)+3\left(3b^3x^5+7ab^2x^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{54b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 18*2^(1/3)*a^2*b^2*(
-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 9*2
^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)
*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 22*sqrt(3
)*a^2*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 22*a^2*(b^2)^(2/3)*log(-((b^
2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 11*a^2*(b^2)^(2/3)*log(((b^2)^(1/3)*
b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*
b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^4*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^7}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^7}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}} x^7}{bdx^3 - ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7 \left(b\, x^3 + a\right)^{1/3}}{a\, d - b\, d\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^7 \sqrt[3]{a+bx^3}}{-a+bx^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

## 3.352 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

**Optimal**. Leaf size=233

$$\frac{a \log \left(ad-bdx^3\right)}{3\ 2^{2/3} b^{5/3} d}+\frac{2 a \log \left(\sqrt[3]{b} x-\sqrt[3]{a+bx^3}\right)}{3 b^{5/3} d}-\frac{a \log \left(\sqrt[3]{2} \sqrt[3]{b} x-\sqrt[3]{a+bx^3}\right)}{2^{2/3} b^{5/3} d}+\frac{4 a \tan^{-1}\left(\frac{\frac{2 \sqrt[3]{b} x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3 \sqrt{3} b^{5/3} d}-\frac{\sqrt[3]{2} a \tan^{-1}\left(\cdot\right.}{\sqrt{3} b^{5}}$$

**Rubi [C]** time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.28, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{511, 510\}$

$$\frac{x^5 \sqrt[3]{a+bx^3} F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]
)/(5*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{x^4 \sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3} \, dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^5 \sqrt[3]{a+bx^3} F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.17, size = 160, normalized size = 0.69

$$\frac{4bx^5\left(1-\frac{b^2x^6}{a^2}\right)^{2/3} F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5x^2\left(\left(a+bx^3\right)\left(1-\frac{bx^3}{a}\right)^{2/3}-a\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)\right)}{15bd\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (4*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a),
 (b*x^3)/a] - 5*x^2*((a + b*x^3)*(1 - (b*x^3)/a)^(2/3) - a*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a - b*x^3)]))/(15*b*d*(a
 + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

**IntegrateAlgebraic** [A]   time = 0.87, size = 342, normalized size = 1.47

$$\frac{4a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{9b^{5/3}d}-\frac{\sqrt[3]{2}a\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3b^{5/3}d}+\frac{4a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{3\sqrt{3}b^{5/3}d}-\frac{\sqrt[3]{2}a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}b^{5/3}d}-\frac{2a\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{9b^{5/3}d}+\frac{a\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\,2^{2/3}b^{5/3}d}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/3*(x^2*(a + b*x^3)^(1/3))/(b*d) + (4*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/
3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(5/3)*d) - (2^(1/3)*a*ArcTan[(Sq
rt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*b^(5/3)
*d) + (4*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(5/3)*d) - (2^(1/3)*
a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^(5/3)*d) - (2*a*Log[b
^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(9*b^(5/3)*d
) + (a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a
 + b*x^3)^(2/3)])/(3*2^(2/3)*b^(5/3)*d)
```

**fricas** [A]   time = 0.63, size = 338, normalized size = 1.45

$$-\frac{6\sqrt{3}\,2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-6\cdot2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+3\cdot2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}b^2x^2\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+6\left(bx^3+a\right)^{\frac{1}{3}}b^2x^2+8\sqrt{3}a\left(b^2\right)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\left(b^2\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{\frac{1}{6}}}{3b^2x}\right)-8a\left(b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+4a\left(b^2\right)^{\frac{2}{3}}\log\left(\frac{\left(b^2\right)^{\frac{1}{3}}bx^2+\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)}{18\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(
b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 6*2^(1/3)*a*b^2*(-1/b^2
)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 3*2^(1/3)
*a*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3
+ a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a)^(1/
3)*b^2*x^2 + 8*sqrt(3)*a*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x
+ 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 8*a*(b^2)
^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 4*a*(b^2)^(2/3)*log(
((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)
/x^2))/(b^3*d)
```

**giac** [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

**maple** [F]   time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^4}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}} x^4}{bdx^3 - ad}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(b\, x^3 + a\right)^{1/3}}{a\, d - b\, d\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^4 \sqrt[3]{a+bx^3}}{-a+bx^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

## 3.353 $\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=201

$$\frac{\log\left(ad-bdx^3\right)}{3\,2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{\sqrt[3]{2}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}$$

**Rubi [C]** time = 0.04, antiderivative size = 66, normalized size of antiderivative = 0.33, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a]
)/(2*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int\frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx&=\frac{\sqrt[3]{a+bx^3}\int\frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\&=\frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 63, normalized size = 0.31

$$\frac{x^2\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3
)/a])/(2*d*(a + b*x^3)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.60, size = 309, normalized size = 1.54

$$\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{2/3}d}-\frac{\sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}\,x\right)}{3b^{2/3}d}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6b^{2/3}d}+\frac{\log\left(2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\,2^{2/3}b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(2
/3)*d) - (2^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^
3)^(1/3))])/(Sqrt[3]*b^(2/3)*d) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*
b^(2/3)*d) - (2^(1/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^(
2/3)*d) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
]/(6*b^(2/3)*d) + Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) +
 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(2/3)*b^(2/3)*d)
```

**fricas [B]** time = 0.62, size = 313, normalized size = 1.56

$$-\frac{2\sqrt{3}\,2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}\,x}{3\,x}\right)-2\cdot2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}b^2x^2\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+2\sqrt{3}\left(b^2\right)^{\frac{1}{6}}b\arctan\left(\frac{\left(\sqrt{3}\left(b^2\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{\frac{1}{6}}}{3\,b^2x}\right)-2\left(b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(b^2\right)^{\frac{2}{3}}\log\left(\frac{\left(b^2\right)^{\frac{1}{3}}bx^2+\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)}{6\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 2*2^(1/3)*b^2*(-1/b^2)^(1/
3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 2^(1/3)*b^2*(-
1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3
)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 2*sqrt(3)*(b^2)^(1/6)*b*ar
ctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3)
)*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1
/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2
/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

**maple [F]** time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

## 3.354 $\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=156

$$-\frac{\sqrt[3]{a+bx^3}}{adx}+\frac{\sqrt[3]{b}\log\left(ad-bdx^3\right)}{3\,2^{2/3}ad}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}-\frac{\sqrt[3]{2}\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,ad}$$

**Rubi [C]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 0.49, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1-\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)}{adx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 - (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, (-2*b*x^3)/(a - b*x^3)])/(a*d*x*(1 + (b*x^3)/a)^(1/3)))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1-\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)}{adx\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 45, normalized size = 0.29

$$-\frac{\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{2bx^3}{bx^3+a}\right)}{adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1, 2/3, (2*b*x^3)/(a + b*x^3)]
)/(a*d*x))
```

**IntegrateAlgebraic [A]** time = 0.43, size = 205, normalized size = 1.31

$$\frac{\sqrt[3]{b}\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\,2^{2/3}ad}-\frac{\sqrt[3]{a+bx^3}}{adx}-\frac{\sqrt[3]{2}\sqrt[3]{b}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3ad}-\frac{\sqrt[3]{2}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}ad}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*d*x)) - (2^(1/3)*b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/
(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a*d) - (2^(1/3)*b^(1/3)*
Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a*d) + (b^(1/3)*Log[2*b^(
2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]
)/(3*2^(2/3)*a*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

**maple [F]** time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

## 3.355 $\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=183

$$\frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\,2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}-\frac{\sqrt[3]{2}\,b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

**Rubi [C]** time = 0.42, antiderivative size = 117, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{a^2-bx^3\left(a+3bx^3\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+3bx^3\left(a-bx^3\right){}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+4abx^3+3b^2x^6}{4a^2dx^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -(a^2 + 4*a*b*x^3 + 3*b^2*x^6 - b*x^3*(a + 3*b*x^3)*Hypergeometric2F1[2/3,
1, 5/3, (2*b*x^3)/(a + b*x^3)] + 3*b*x^3*(a - b*x^3)*Hypergeometric2F1[2/3,
 2, 5/3, (2*b*x^3)/(a + b*x^3)])/(4*a^2*d*x^4*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{a^2+4abx^3+3b^2x^6-bx^3\left(a+3bx^3\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)+3bx^3\left(a-bx^3\right){}_2F_1\left(\frac{2}{3},2;\frac{5}{3}\right.}{4a^2dx^4\left(a+bx^3\right)^{2/3}}\end{aligned}$$

**Mathematica [C]** time = 5.10, size = 125, normalized size = 0.68

$$\frac{b^2x^2\left(\frac{a+bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a\left(1-\frac{bx^3}{a}\right)}\right)}{a^2d\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}-\frac{\left(\frac{5b}{4a^2x}+\frac{1}{4ax^4}\right)\sqrt[3]{a+bx^3}}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -(((1/(4*a*x^4) + (5*b)/(4*a^2*x))*(a + b*x^3)^(1/3))/d) + (b^2*x^2*((a + b
*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a*(1 - (b*x^3)/
a))])/(a^2*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.48, size = 217, normalized size = 1.19

$$-\frac{\sqrt[3]{2}\,b^{4/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}\,x\right)}{3a^2d}-\frac{\sqrt[3]{2}\,b^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,a^2d}+\frac{b^{4/3}\log\left(2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\,2^{2/3}a^2d}+\frac{\left(-a-5bx^3\right)\sqrt[3]{a+bx^3}}{4a^2dx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] ((-a - 5*b*x^3)*(a + b*x^3)^(1/3))/(4*a^2*d*x^4) - (2^(1/3)*b^(4/3)*ArcTan[
(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a^2*
d) - (2^(1/3)*b^(4/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^2
*d) + (b^(4/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^
(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^2*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^3+a)^{1/3}}{x^5\,(a\,d-b\,d\,x^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^5+bx^8}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d
```

## 3.356 $\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=210

$$\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\,2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}-\frac{\sqrt[3]{2}\,b^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^3d}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\sqrt[3]{}$$

**Rubi [C]** time = 19.78, antiderivative size = 244, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-9bx^3\left(a-bx^3\right)^2{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-2bx^3\left(2a^2+3abx^3+9b^2x^6\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+15a^2bx^3{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+10a^2bx^3+4a^3-27b^3x^9{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+12ab^2x^6{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+24ab^2x^6+18b^3x^9}{28a^3dx^7\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] -(4*a^3 + 10*a^2*b*x^3 + 24*a*b^2*x^6 + 18*b^3*x^9 - 2*b*x^3*(2*a^2 + 3*a*b
*x^3 + 9*b^2*x^6)*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] + 1
5*a^2*b*x^3*Hypergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 12*a*b^
2*x^6*Hypergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 27*b^3*x^9*Hy
pergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 9*b*x^3*(a - b*x^3)^2
*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, (2*b*x^3)/(a + b*x^3)])/(28*a^3*d
*x^7*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int\frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=\frac{\sqrt[3]{a+bx^3}\int\frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$=-\frac{4a^3+10a^2bx^3+24ab^2x^6+18b^3x^9-2bx^3\left(2a^2+3abx^3+9b^2x^6\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)+1}{}$$

**Mathematica [C]** time = 5.12, size = 135, normalized size = 0.64

$$\frac{7b^3x^9\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)-\left(1-\frac{bx^3}{a}\right)^{2/3}\left(a^3+3a^2bx^3+10ab^2x^6+8b^3x^9\right)}{7a^3dx^7\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] (-((1 - (b*x^3)/a)^(2/3)*(a^3 + 3*a^2*b*x^3 + 10*a*b^2*x^6 + 8*b^3*x^9)) +
7*b^3*x^9*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)
/(a - b*x^3)])/(7*a^3*d*x^7*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.52, size = 228, normalized size = 1.09

$$-\frac{\sqrt[3]{2}\,b^{7/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}\,x\right)}{3a^3d}-\frac{\sqrt[3]{2}\,b^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,a^3d}+\frac{b^{7/3}\log\left(2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\,2^{2/3}a^3d}+\frac{\sqrt[3]{a+bx^3}\left(-a^2-2abx^3-8b^2x^6\right)}{7a^3dx^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-a^2 - 2*a*b*x^3 - 8*b^2*x^6))/(7*a^3*d*x^7) - (2^(1/3)
*b^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))
])/(Sqrt[3]*a^3*d) - (2^(1/3)*b^(7/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3
)^(1/3)])/(3*a^3*d) + (b^(7/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b
*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^3*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(bdx^{3}-ad\right)x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{1/3}}{x^{8}\,\left(a\,d-b\,d\,x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^{3}}}{-ax^{8}+bx^{11}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**8 + b*x**11), x)/d
```

## 3.357 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

**Optimal**. Leaf size=237

$$\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\,2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}-\frac{\sqrt[3]{2}\,b^{10/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{37b^2\sqrt[3]{a}}{140a^{}}$$

**Rubi [C]** time = 29.21, antiderivative size = 423, normalized size of antiderivative = 1.78, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-54bx^3\left(a-bx^3\right)^2\left(2a+3bx^3\right)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+27bx^3\left(a-bx^3\right)^3\,{}_4F_3\left(\frac{2}{3},2,2,2;1,1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-36a^2b^2x^6\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+99a^2b^2x^6\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+90a^2b^2x^6-28a^3bx^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+117a^3bx^3\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+64a^3bx^3+28a^4-162b^4x^{12}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-297b^4x^{12}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-54ab^3x^9\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+81ab^3x^9\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+216ab^3x^9+162b^4x^{12}}{280a^4dx^{10}\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] -(28*a^4 + 64*a^3*b*x^3 + 90*a^2*b^2*x^6 + 216*a*b^3*x^9 + 162*b^4*x^12 - 2
8*a^3*b*x^3*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 36*a^2*
b^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 54*a*b^3*x^
9*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 162*b^4*x^12*Hype
rgeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] + 117*a^3*b*x^3*Hypergeom
etric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 99*a^2*b^2*x^6*Hypergeometri
c2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 81*a*b^3*x^9*Hypergeometric2F1[2
/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 297*b^4*x^12*Hypergeometric2F1[2/3, 2,
 5/3, (2*b*x^3)/(a + b*x^3)] - 54*b*x^3*(a - b*x^3)^2*(2*a + 3*b*x^3)*Hyper
geometricPFQ[{2/3, 2, 2}, {1, 5/3}, (2*b*x^3)/(a + b*x^3)] + 27*b*x^3*(a -
b*x^3)^3*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, (2*b*x^3)/(a + b*x^
3)])/(280*a^4*d*x^10*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$=-\frac{28a^4+64a^3bx^3+90a^2b^2x^6+216ab^3x^9+162b^4x^{12}-28a^3bx^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)-}{}$$

**Mathematica** [C] time = 5.20, size = 130, normalized size = 0.55

$$\frac{-14a^4-36a^3bx^3-59a^2b^2x^6+\frac{140b^4x^{12}\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)}{\left(1-\frac{bx^3}{a}\right)^{2/3}}-206ab^3x^9-169b^4x^{12}}{140a^4dx^{10}\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] (-14*a^4 - 36*a^3*b*x^3 - 59*a^2*b^2*x^6 - 206*a*b^3*x^9 - 169*b^4*x^12 + (
140*b^4*x^12*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x
^3)/(a - b*x^3)])/(1 - (b*x^3)/a)^(2/3))/(140*a^4*d*x^10*(a + b*x^3)^(2/3))
```

**IntegrateAlgebraic** [A] time = 0.57, size = 239, normalized size = 1.01

$$-\frac{\sqrt[3]{2}\,b^{10/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}\,x\right)}{3a^4d}-\frac{\sqrt[3]{2}\,b^{10/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,a^4d}+\frac{b^{10/3}\log\left(2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\,2^{2/3}a^4d}+\frac{\sqrt[3]{a+bx^3}\left(-14a^3-22a^2bx^3-37ab^2x^6-169b^3x^9\right)}{140a^4dx^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-14*a^3 - 22*a^2*b*x^3 - 37*a*b^2*x^6 - 169*b^3*x^9))/(
140*a^4*d*x^10) - (2^(1/3)*b^(10/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +
 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a^4*d) - (2^(1/3)*b^(10/3)*Log[-2*b^
(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^4*d) + (b^(10/3)*Log[2*b^(2/3)*x
^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2
^(2/3)*a^4*d)
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)
[Out] Timed out
```

## 3.358 $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=416

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\cdots\right)}{6\,2^{2/3}\cdots}$$

**Rubi [C]** time = 0.03, antiderivative size = 61, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/
(a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.15, size = 154, normalized size = 0.37

$$\frac{4ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{d\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/(d*(a - b*x^3)*(4*a*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + AppellF
1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

**IntegrateAlgebraic [A]** time = 3.09, size = 533, normalized size = 1.28

$$\frac{\log \left(2^{2/3}a^{2/3}-\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+2\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}x+2^{2/3}b^{2/3}x^2\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\log \left(2^{2/3}a^{2/3}+2\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+4\left(a+bx^3\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+2\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}x+2^{2/3}b^{2/3}x^2\right)}{6\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\sqrt[3]{2}\log \left(\sqrt[3]{a+bx^3}+\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\log \left(2\sqrt[3]{a+bx^3}-\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a+bx^3}-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a+bx^3}+\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (2^(1/3)*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1/3)
*b^(1/3)*x + (a + b*x^3)^(1/3))])/(Sqrt[3]*a^(1/3)*b^(1/3)*d) + ArcTan[(Sqr
t[3]*(a + b*x^3)^(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^
(1/3))]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)*d) - (2^(1/3)*Log[2^(1/3)*a^(1/3)
+ 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)])/(3*a^(1/3)*b^(1/3)*d) - Log[-(2^(
1/3)*a^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)
*b^(1/3)*d) + Log[2^(2/3)*a^(2/3) + 2*2^(2/3)*a^(1/3)*b^(1/3)*x + 2^(2/3)*b
^(2/3)*x^2 - 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) - 2^(1/3)*b^(1/3)*x*(a + b*x
^3)^(1/3) + (a + b*x^3)^(2/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)*d) + Log[2^(2/3)*
a^(2/3) + 2*2^(2/3)*a^(1/3)*b^(1/3)*x + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*a^(
1/3)*(a + b*x^3)^(1/3) + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a + b*x
^3)^(2/3)]/(6*2^(2/3)*a^(1/3)*b^(1/3)*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

## 3.359 $\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=223

$$\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+\right.}{5b}$$

**Rubi [A]** time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) + (a
*(a + b*x^3)^(8/3))/(8*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) - (2^(2/3)*a^
(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(11/3)*Log[a - b*x^3])/(3*2^(1/3)*b^4*d) - (a^(11/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a^2(a+bx)^{2/3}}{b^3d}+\frac{a(a+bx)^{5/3}}{b^3d}-\frac{(a+bx)^{8/3}}{b^3d}+\frac{a^3(a+bx)^{2/3}}{b^3(ad-bdx)}\right)dx, x, x^3\right)$$

$$= -\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{a^3\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{\left(2a^4\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt[3]{a}\ldots}\ldots\right.}{3\ldots}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}\ldots$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}\ldots$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{1+\ldots}{\ldots}\right)}{\sqrt{3}\,b^4d}\ldots$$

**Mathematica [A]** time = 0.28, size = 163, normalized size = 0.73

$$-\frac{-220\;2^{2/3}a^{11/3}\log\left(a-bx^3\right)+660\;2^{2/3}a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+440\;2^{2/3}\sqrt{3}\,a^{11/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+3\left(a+bx^3\right)^{2/3}\left(293a^3+98a^2bx^3+65ab^2x^6+40b^3x^9\right)}{1320b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/1320*(3*(a + b*x^3)^(2/3)*(293*a^3 + 98*a^2*b*x^3 + 65*a*b^2*x^6 + 40*b^
3*x^9) + 440*2^(2/3)*Sqrt[3]*a^(11/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 220*2^(2/3)*a^(11/3)*Log[a - b*x^3] + 660*2^(2/3)*a^
(11/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(b^4*d)
```

**IntegrateAlgebraic** [A] time = 0.27, size = 225, normalized size = 1.01

$$-\frac{2^{2/3}a^{11/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3b^4d}+\frac{a^{11/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\sqrt[3]{2}\,b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^4d}+\frac{\left(a+bx^3\right)^{2/3}\left(-293a^3-98a^2bx^3-65ab^2x^6-40b^3x^9\right)}{440b^4d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-293*a^3 - 98*a^2*b*x^3 - 65*a*b^2*x^6 - 40*b^3*x^9))/(
440*b^4*d) - (2^(2/3)*a^(11/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3
))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^4*d) - (2^(2/3)*a^(11/3)*Log[-2*a^(1/3) +
 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^4*d) + (a^(11/3)*Log[2*a^(2/3) + 2^(2/3)*
a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*b^4*d)
```

**fricas** [A] time = 0.65, size = 209, normalized size = 0.94

$$-\frac{440\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a^3\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+220\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a^3\log\left(4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{2}{3}}a-2\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\right)-440\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a^3\log\left(-4^{\frac{2}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}a\right)+3\left(40b^3x^9+65ab^2x^6+98a^2bx^3+293a^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{1320\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*a^
3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^
(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2)^(2
/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b*x^3
+ 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
```

lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofa^3*((2*a)^(1/3))^2*1/6/b^4/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+
b*x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-a^3*((2*a)^(1/3))^2/sqrt(3)/b^4/d*ata
n(((a+b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a^
4*b^44*d^11*(2*a)^(1/3)*1/6/a/b^48/d^12*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3))
)-(1/11*((a+b*x^3)^(1/3))^2*(a+b*x^3)^3*b^40*d^10-1/8*((a+b*x^3)^(1/3))^2*(
a+b*x^3)^2*a*b^40*d^10+1/5*((a+b*x^3)^(1/3))^2*(a+b*x^3)*a^2*b^40*d^10+1/2*
((a+b*x^3)^(1/3))^2*a^3*b^40*d^10)/b^44/d^11

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{11}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)

[Out] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)

**maxima [A]** time = 1.25, size = 183, normalized size = 0.82

$$-\frac{\frac{440\,\sqrt{3}\,2^{\frac{2}{3}} a^{\frac{11}{3}} \arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}} a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{220\cdot 2^{\frac{2}{3}} a^{\frac{11}{3}} \log\left(2^{\frac{2}{3}} a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{d}+\frac{440\cdot 2^{\frac{2}{3}} a^{\frac{11}{3}} \log\left(-2^{\frac{1}{3}} a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right)}{d}+\frac{3\left(40\,(bx^3+a)^{\frac{11}{3}}-55\,(bx^3+a)^{\frac{8}{3}} a+88\,(bx^3+a)^{\frac{5}{3}} a^2+220\,(bx^3+a)^{\frac{2}{3}} a^3\right)}{d}}{1320\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -1/1320*(440*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a
^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 220*2^(2/3)*a^(11/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 440*2
^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(40*(b*x^3
+ a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 88*(b*x^3 + a)^(5/3)*a^2 + 220*(b*x^
3 + a)^(2/3)*a^3)/d)/b^4

**mupad [B]** time = 4.85, size = 261, normalized size = 1.17

$$\frac{a\,(b\,x^3+a)^{8/3}}{8\,b^4\,d}-\frac{a^3\,(b\,x^3+a)^{2/3}}{2\,b^4\,d}-\frac{a^2\,(b\,x^3+a)^{5/3}}{5\,b^4\,d}-\frac{(b\,x^3+a)^{11/3}}{11\,b^4\,d}+\frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(4\,a^8\,(b\,x^3+a)^{1/3}+4\,2^{1/3}\,(-a)^{25/3}\right)}{3\,b^4\,d}-\frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{2\,4^{2/3}\,(-a)^{25/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2}{b^8\,d^2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3\,b^4\,d}+\frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{18\,4^{2/3}\,(-a)^{25/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2}{b^8\,d^2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)

[Out] (a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*
(a + b*x^3)^(5/3))/(5*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) + (4^(1/3)*(-a
)^(11/3)*log(4*a^8*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(25/3)))/(3*b^4*d) -
(4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (2*4^(2/3)*(
-a)^(25/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^8*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*
b^4*d) + (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (18
*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^8*d^2))*((3^(1/2)*1i)/6 -
 1/6))/(b^4*d)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

## 3.360 $\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=177

$$\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}$$

**Rubi** [A] time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$-\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d) - (2^(2/3)
*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(8/3)*Log[a - b*x^3])/(3*2^(1/3)*b^3*d) - (a^(8/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(-\frac{(a+bx)^{5/3}}{b^2d}+\frac{a^2(a+bx)^{2/3}}{b^2(ad-bdx)}\right)dx, x, x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{\left(2a^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}+\frac{a^{8/3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{2}\,\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a}\right.}{\sqrt[3]{2}\,b^3d}\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^3d}+\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 153, normalized size = 0.86

$$\frac{4\,2^{2/3}a^{8/3}\log\left(a-bx^3\right)-8\,2^{2/3}\sqrt{3}\,a^{8/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\left(4\,2^{2/3}a^{8/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\left(a+bx^3\right)^{2/3}\left(5a^2+2abx^3+b^2x^6\right)\right)}{24b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3)
)/Sqrt[3]] + 4*2^(2/3)*a^(8/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(5*a^2
 + 2*a*b*x^3 + b^2*x^6) + 4*2^(2/3)*a^(8/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^
3)^(1/3)]))/(24*b^3*d)
```

**IntegrateAlgebraic [A]** time = 0.22, size = 214, normalized size = 1.21

$$-\frac{2^{2/3}a^{8/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3b^3d}+\frac{a^{8/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\sqrt[3]{2}\,b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^3d}+\frac{\left(a+bx^3\right)^{2/3}\left(-5a^2-2abx^3-b^2x^6\right)}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*a^2 - 2*a*b*x^3 - b^2*x^6))/(8*b^3*d) - (2^(2/3)*a^(
8/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^3*d) - (2^(2/3)*a^(8/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)]
)/(3*b^3*d) + (a^(8/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) +
2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*b^3*d)
```

**fricas [A]** time = 0.72, size = 197, normalized size = 1.11

$$-\frac{8 \cdot 4^{\frac{1}{3}} \sqrt{3} \left(-a^{2}\right)^{\frac{1}{3}} a^{2} \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} - \sqrt{3} a}{3 a}\right) + 4 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a^{2} \log\left(4^{\frac{2}{3}} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{2}{3}} + 2 \left(bx^{3}+a\right)^{\frac{2}{3}} a - 2 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a\right) - 8 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a^{2} \log\left(-4^{\frac{2}{3}} \left(-a^{2}\right)^{\frac{2}{3}} + 2 \left(bx^{3}+a\right)^{\frac{1}{3}} a\right) + 3 \left(b^{2} x^{6} + 2 \, a b x^{3} + 5 \, a^{2}\right) \left(bx^{3}+a\right)^{\frac{2}{3}}}{24 \, b^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/24*(8*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^2*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4*4^(1/3)*(-a^2)^(1/3)*a^2*log(
4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*
(-a^2)^(1/3)*a) - 8*4^(1/3)*(-a^2)^(1/3)*a^2*log(-4^(2/3)*(-a^2)^(2/3) + 2*
(b*x^3 + a)^(1/3)*a) + 3*(b^2*x^6 + 2*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/3))/(
b^3*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
```

```
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofa^2*((2*a)^(1/3))^2*1/6/b^3/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+
b*x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-a^2*((2*a)^(1/3))^2/sqrt(3)/b^3/d*ata
n(((a+b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a^
3*b^24*d^8*(2*a)^(1/3)*1/6/a/b^27/d^9*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-
(1/8*((a+b*x^3)^(1/3))^2*(a+b*x^3)^2*b^21*d^7+1/2*((a+b*x^3)^(1/3))^2*a^2*b
^21*d^7)/b^24/d^8
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{8}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

**maxima [A]** time = 1.17, size = 155, normalized size = 0.88

$$-\frac{\frac{8\,\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{4\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}+\frac{8\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\left(\left(bx^{3}+a\right)^{\frac{8}{3}}+4\left(bx^{3}+a\right)^{\frac{2}{3}}a^{2}\right)}{d}}{24\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/24*(8*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 4*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 8*2^(2/3)*a^(
8/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*((b*x^3 + a)^(8/3) + 4
*(b*x^3 + a)^(2/3)*a^2)/d)/b^3
```

**mupad [B]** time = 4.91, size = 206, normalized size = 1.16

$$-\frac{\left(b\,x^3+a\right)^{8/3}}{8\,b^3\,d}-\frac{a^2\,\left(b\,x^3+a\right)^{2/3}}{2\,b^3\,d}-\frac{4^{1/3}\,a^{8/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^3\,d}-\frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,\left(b\,x^3+a\right)^{1/3}}{b^6\,d^2}-\frac{2\,4^{2/3}\,a^{19/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{b^6\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^3\,d}+\frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,\left(b\,x^3+a\right)^{1/3}}{b^6\,d^2}-\frac{18\,4^{2/3}\,a^{19/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)^2}{b^6\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1/3))/(b^6*d^2) - (18*4^(2/3)*a^(1
9/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^6*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b^3*d) -
 (a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (4^(1/3)*a^(8/3)*log((a + b*x^3)^(1/3)
 - 2^(1/3)*a^(1/3)))/(3*b^3*d) - (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1
/3))/(b^6*d^2) - (2*4^(2/3)*a^(19/3)*((3^(1/2)*1i)/2 - 1/2)^2)/(b^6*d^2))*(
(3^(1/2)*1i)/2 - 1/2))/(3*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{8}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

## 3.361 $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=175

$$\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}$$

**Rubi [A]** time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 50, 55, 617, 204, 31}

$$\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) - (2^(2/3)*a
^(5/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(5/3)*Log[a - b*x^3])/(3*2^(1/3)*b^2*d) - (a^(5/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{a\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d} + \frac{a^{5/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\,\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a}\right.}{\sqrt[3]{2}\,b^2d}$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^2d} +$$

$$= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} - \frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^2d} + \frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d} - \frac{a^5}{}$$

**Mathematica [A]** time = 0.15, size = 143, normalized size = 0.82

$$\frac{5\,2^{2/3}a^{5/3}\log\left(a-bx^3\right)-3\left(5\,2^{2/3}a^{5/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\left(a+bx^3\right)^{2/3}\left(7a+2bx^3\right)\right)-10\,2^{2/3}\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{30b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-10*2^(2/3)*Sqrt[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3
))/Sqrt[3]] + 5*2^(2/3)*a^(5/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(7*a
+ 2*b*x^3) + 5*2^(2/3)*a^(5/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(
30*b^2*d)
```

**IntegrateAlgebraic [A]** time = 0.21, size = 203, normalized size = 1.16

$$-\frac{2^{2/3}a^{5/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3b^2d}+\frac{a^{5/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\sqrt[3]{2}b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}b^2d}+\frac{\left(-7a-2bx^3\right)\left(a+bx^3\right)^{2/3}}{10b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-7*a - 2*b*x^3)*(a + b*x^3)^(2/3))/(10*b^2*d) - (2^(2/3)*a^(5/3)*ArcTan[1
/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^2*d)
- (2^(2/3)*a^(5/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^2*d) +
 (a^(5/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a +
b*x^3)^(2/3)])/(3*2^(1/3)*b^2*d)
```

**fricas [A]** time = 0.94, size = 181, normalized size = 1.03

$$-\frac{10\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+5\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\log\left(4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{2}{3}}a-2\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\right)-10\cdot 4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}a\log\left(-4^{\frac{2}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}a\right)+3\left(2bx^3+7a\right)\left(bx^3+a\right)^{\frac{2}{3}}}{30b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/30*(10*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3
 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 5*4^(1/3)*(-a^2)^(1/3)*a*log(4^(
2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a
^2)^(1/3)*a) - 10*4^(1/3)*(-a^2)^(1/3)*a*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x
^3 + a)^(1/3)*a) + 3*(2*b*x^3 + 7*a)*(b*x^3 + a)^(2/3))/(b^2*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
```

```
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofa*((2*a)^(1/3))^2*1/6/b^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*
x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-a*((2*a)^(1/3))^2/sqrt(3)/b^2/d*atan(((
a+b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a^2*b^
10*d^5*(2*a)^(1/3)*1/6/a/b^12/d^6*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/5
*((a+b*x^3)^(1/3))^2*(a+b*x^3)*b^8*d^4+1/2*((a+b*x^3)^(1/3))^2*a*b^8*d^4)/b
^10/d^5
```

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} x^5}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

**maxima** [A] time = 1.12, size = 155, normalized size = 0.89

$$-\frac{\frac{10\,\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{5\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{10\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\left(2\,\left(bx^3+a\right)^{\frac{5}{3}}+5\,\left(bx^3+a\right)^{\frac{2}{3}}a\right)}{d}}{30\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/30*(10*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 5*2^(2/3)*a^(5/3)*log(2^(2/3)*a^(2/3
) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 10*2^(2/3)*a
^(5/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(2*(b*x^3 + a)^(5/3)
 + 5*(b*x^3 + a)^(2/3)*a)/d)/b^2
```

**mupad** [B] time = 4.84, size = 221, normalized size = 1.26

$$\frac{4^{1/3}\,{\left(-a\right)}^{5/3}\,\ln\left(4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}+4\,2^{1/3}\,{\left(-a\right)}^{13/3}\right)}{3\,b^2\,d}-\frac{a\,{\left(b\,x^3+a\right)}^{2/3}}{2\,b^2\,d}-\frac{{\left(b\,x^3+a\right)}^{5/3}}{5\,b^2\,d}-\frac{4^{1/3}\,{\left(-a\right)}^{5/3}\,\ln\left(\frac{4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d^2}+\frac{2\,4^{2/3}\,{\left(-a\right)}^{13/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^4\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^2\,d}+\frac{4^{1/3}\,{\left(-a\right)}^{5/3}\,\ln\left(\frac{4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d^2}+\frac{18\,4^{2/3}\,{\left(-a\right)}^{13/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{b^4\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*(-a)^(5/3)*log(4*a^4*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(13/3)))/(
3*b^2*d) - (a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) -
(4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (2*4^(2/3)*(-
a)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^4*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*b
^2*d) + (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (18*4
^(2/3)*(-a)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^4*d^2))*((3^(1/2)*1i)/6 - 1
/6))/(b^2*d)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^5\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

## 3.362 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=153

$$\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}-\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

**Rubi [A]** time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {444, 50, 55, 617, 204, 31}

$$\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}-\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(2*b*d) - (2^(2/3)*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(2/3)*Log[a - b*x^3]
)/(3*2^(1/3)*b*d) - (a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
1/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{1}{3}(2a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)\\ &= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}+\frac{a^{2/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-\frac{a\operatorname{Subst}\left(\int\right.}{}\\ &= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}+\frac{\left(2^{2/3}a^{2/3}\right)\operatorname{Subst}\left(\int\right.}{}\\ &= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a}\right.}{\sqrt[3]{2}\,bd}\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 130, normalized size = 0.85

$$\frac{2^{2/3}a^{2/3}\log\left(a-bx^3\right)-3\left(2^{2/3}a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\left(a+bx^3\right)^{2/3}\right)-2\,2^{2/3}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3)
)/Sqrt[3]] + 2^(2/3)*a^(2/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3) + 2^(2/3
)*a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(6*b*d)
```

**IntegrateAlgebraic [A]** time = 0.19, size = 193, normalized size = 1.26

$$-\frac{2^{2/3}a^{2/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3bd}+\frac{a^{2/3}\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\sqrt[3]{2}\,bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,bd}-\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(b*d) - (2^(2/3)*a^(2/3)*ArcTan[1/Sqrt[3] + (2^(2/3)
*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) - (2^(2/3)*a^(2/3)*Lo
g[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b*d) + (a^(2/3)*Log[2*a^(2/3)
```

```
 + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/
3)*b*d)
```

**fricas [A]** time = 1.17, size = 167, normalized size = 1.09

$$-\frac{2 \cdot 4^{\frac{1}{3}} \sqrt{3} \left(-a^{2}\right)^{\frac{1}{3}} \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} - \sqrt{3} a}{3 a}\right) + 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} \log\left(4^{\frac{2}{3}} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{2}{3}} + 2 \left(bx^{3}+a\right)^{\frac{2}{3}} a - 2 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a\right) - 2 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} \log\left(-4^{\frac{2}{3}} \left(-a^{2}\right)^{\frac{2}{3}} + 2 \left(bx^{3}+a\right)^{\frac{1}{3}} a\right) + 3 \left(bx^{3}+a\right)^{\frac{2}{3}}}{6 \, bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 + a
)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(1/3
)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(1/
3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootof((2*a)^(1/3))^2*1/6/b/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)
^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2/sqrt(3)/b/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a*b^2*d^2*(2*a
)^(1/3)*1/6/a/b^3/d^3*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-1/2*((a+b*x^3)^(
1/3))^2*b*d/b^2/d^2
```

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{2}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

**maxima** [A] time = 1.31, size = 140, normalized size = 0.92

$$-\frac{\frac{2\,\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}+\frac{2\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\,\left(bx^{3}+a\right)^{\frac{2}{3}}}{d}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 2^(2/3)*a^(2/3)*log(2^(2/3)*a^(2/3) +
2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 2*2^(2/3)*a^(2/3
)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(b*x^3 + a)^(2/3)/d)/b
```

**mupad** [B] time = 4.83, size = 186, normalized size = 1.22

$$-\frac{\left(b\,x^{3}+a\right)^{2/3}}{2\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left(\left(b\,x^{3}+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^{2}\,\left(b\,x^{3}+a\right)^{1/3}}{b^{2}\,d^{2}}-\frac{2\,4^{2/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}}{b^{2}\,d^{2}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d}+\frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^{2}\,\left(b\,x^{3}+a\right)^{1/3}}{b^{2}\,d^{2}}-\frac{18\,4^{2/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^{2}}{b^{2}\,d^{2}}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (18*4^(2/3)*a^(7
/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^2*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b*d) - (4
^(1/3)*a^(2/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b*d) - (4^(1/3)
*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (2*4^(2/3)*a^(7/3)*((3^(
1/2)*1i)/2 - 1/2)^2)/(b^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(3*b*d) - (a + b*x^
3)^(2/3)/(2*b*d)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{2}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

## 3.363 $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

**Optimal**. Leaf size=214

$$\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}$$

**Rubi** [A] time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {446, 83, 55, 617, 204, 31}

$$\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{\log(x)}{2\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
d) - (2^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(1/3)*d) - Log[x]/(2*a^(1/3)*d) + Log[a - b*x^3]/(3*2^(1/3)*
a^(1/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*d) - Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)]/(2^(1/3)*a^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(ad-bdx)}\,dx,x,x^3\right)$$

$$=\frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^3\right)}{3d}$$

$$=-\frac{\log(x)}{2\sqrt[3]{a}\,d}+\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,\sqrt[3]{a}\,d}+\frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d}-\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\,\sqrt[3]{\ldots}}\right.}{\ldots}$$

$$=-\frac{\log(x)}{2\sqrt[3]{a}\,d}+\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d}-\frac{\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,\sqrt[3]{a}\,d}-\frac{\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$=\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,d}-\frac{\log(x)}{2\sqrt[3]{a}\,d}+\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+\ldots}\right)}{2\sqrt[3]{a}\,d}$$

**Mathematica** [A] time = 0.08, size = 164, normalized size = 0.77

$$\frac{2^{2/3}\log\left(a-bx^3\right)+3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-3\,2^{2/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)}{6\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(2/3)*
Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*Log[x
] + 2^(2/3)*Log[a - b*x^3] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)] - 3*2^(2/3)
*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*d)
```

**IntegrateAlgebraic** [A] time = 0.35, size = 286, normalized size = 1.34

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6\sqrt[3]{a}\,d}+\frac{\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\sqrt[3]{2}\,\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3\sqrt[3]{a}\,d}-\frac{2^{2/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3\sqrt[3]{a}\,d}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3
)*d) - (2^(2/3)*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(1/3)*d) + Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(3*a^(1/3)*
d) - (2^(2/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^(1/3)*d) -
Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*a^(1/3)*d)
```

```
+ Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(
2/3)]/(3*2^(1/3)*a^(1/3)*d)
```

**fricas [A]** time = 0.74, size = 530, normalized size = 2.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 +
 a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log(
(2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a -
 a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 4^(1
/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)
*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2
/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a
^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*s
qrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*(-1/a)^(1/
3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3)
+ 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/
3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3
+ a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a
*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
```

```
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootof1/3/(2*a)^(1/3)/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(1/3)
+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2/sqrt(3)/a/d*atan(((a+b*x^3)^(1/3)
+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*(2*a)^(1/3)*1/6/a/d*
ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(a^(1/3))^2*1/6/a/d*ln(((a+b*x^3)^(1/3
))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+1/sqrt(3)/a^(1/3)/d*atan(((a+
b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+a^(1/3)*a^(1/3)*1/3/a/d*ln(abs
((a+b*x^3)^(1/3)-a^(1/3)))
```

**maple [F]** time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(-bd\,x^{3}+ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{\left(bdx^{3}-ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)
```

**mupad [B]** time = 5.86, size = 369, normalized size = 1.72

$$\ln\left(2\left(bx^3+a\right)^{1/3}-2\,2^{1/3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}\right)\left(-\frac{4}{27\,a\,d^3}\right)^{1/3}+\ln\left(\left(bx^3+a\right)^{1/3}-a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}\right)\left(\frac{1}{27\,a\,d^3}\right)^{1/3}-\ln\left(4\left(bx^3+a\right)^{1/3}+2\,2^{1/3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}-2^{1/3}\sqrt{3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}2\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{4}{27\,a\,d^3}\right)^{1/3}+\ln\left(4\left(bx^3+a\right)^{1/3}+2\,2^{1/3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}+2^{1/3}\sqrt{3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}2\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{4}{27\,a\,d^3}\right)^{1/3}-\ln\left(2\left(bx^3+a\right)^{1/3}+a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}-\sqrt{3}\,a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{27\,a\,d^3}\right)^{1/3}+\ln\left(2\left(bx^3+a\right)^{1/3}+a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}+\sqrt{3}\,a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{27\,a\,d^3}\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log(2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3))*(-4/(27*a*d^3
))^(1/3) + log((a + b*x^3)^(1/3) - a*d^2*(1/(a*d^3))^(2/3))*(1/(27*a*d^3))^
(1/3) - log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) - 2^(1
/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-4/(27*a*d
^3))^(1/3) + log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) +
 2^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-4/(2
7*a*d^3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) - 3^(1/
2)*a*d^2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*d^3))^(1/3)
+ log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) + 3^(1/2)*a*d^2*(1/(a*d
^3))^(2/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*d^3))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-ax+bx^{4}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d
```

## 3.364 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=269

$$\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}d}-\frac{2^{2/3}b\tan^{-1}}{\sqrt{}}$$

**Rubi** [A] time = 0.26, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 156, 50, 55, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3}+\frac{b\left(a+bx^3\right)^{2/3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}d}-\frac{2^{2/3}b\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}d}-\frac{5b\log(x)}{6a^{4/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(2/3))/(3*a^2*d) - (a + b*x^3)^(5/3)/(3*a^2*d*x^3) + (5*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)
*d) - (2^(2/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(4/3)*d) - (5*b*Log[x])/(6*a^(4/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(1/3)*a^(4/3)*d) + (5*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd+\frac{2}{3}b^2dx\right)}{x(ad-bdx)}\,dx, x, x^3\right)}{3a^2d}$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3a^2} + \frac{(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^3\right)}{9a^2d}$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{3a} + \frac{(5b)\,\text{Subst}\left(\int \frac{}{x}\right.}{9a}$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{}\right.}{6a^{4/3}d}$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} -$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{5b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}d} - \frac{2^{2/3}b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d} - \frac{5b\log}{6a^{4/}}$$

**Mathematica [A]** time = 0.09, size = 213, normalized size = 0.79

$$\frac{10\sqrt{3}\,bx^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 3\left(2\sqrt[3]{a}\left(a+bx^3\right)^{2/3} - 2^{2/3}bx^3\log\left(a-bx^3\right) - 5bx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + 3\,2^{2/3}bx^3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + 2\,2^{2/3}\sqrt{3}\,bx^3\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) + 5bx^3\log(x)\right)}{18a^{4/3}dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*(
2*a^(1/3)*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 5*b*x^3*Log[x] - 2^(2/3)*b*x^3*Log[a
 - b*x^3] - 5*b*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 3*2^(2/3)*b*x^3*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(18*a^(4/3)*d*x^3)
```

**IntegrateAlgebraic [A]** time = 0.55, size = 319, normalized size = 1.19

$$\frac{5b\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9a^{4/3}d}-\frac{2^{2/3}b\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3a^{4/3}d}-\frac{5b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{18a^{4/3}d}+\frac{b\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\sqrt[3]{2}a^{4/3}d}+\frac{5b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}a^{4/3}d}-\frac{2^{2/3}b\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d}-\frac{\left(a+bx^3\right)^{2/3}}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(2/3)/(a*d*x^3) + (5*b*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))]])/(3*Sqrt[3]*a^(4/3)*d) - (2^(2/3)*b*ArcTan[1/Sqrt[
3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]])/(Sqrt[3]*a^(4/3)*d) +
(5*b*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(9*a^(4/3)*d) - (2^(2/3)*b*Log[-2*a
^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^(4/3)*d) - (5*b*Log[a^(2/3) + a^(
1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(4/3)*d) + (b*Log[2*a^(2
/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^
(1/3)*a^(4/3)*d)
```

**fricas [A]** time = 0.51, size = 612, normalized size = 2.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 15*sqrt(1/3)*a*b*x^3*sqrt(-1
/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3
+ a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*
a)/x^3) + 3*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-
1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*
b*x^3*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + 5*a
^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) -
 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)
/(a^2*d*x^3), -1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 3*4^(1/3)*a*b*x
^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*
(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(-4
^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 30*sqrt(1/3)*a^(2/3)*b*x^3*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 5*a^(2/3)*b*x^3*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 10*a^(2/3)*b
*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)/(a^2*d*x^3)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
```

```
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofb/3/a/(2*a)^(1/3)/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(1/
3)+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2*b/sqrt(3)/a^2/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*b*(2*a)^(1/3)*
1/6/a^2/d*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-5*(a^(1/3))^2*b*1/18/a^2/d*l
n(((a+b*x^3)^(1/3))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+5*b/3/sqrt(3
)/a/a^(1/3)/d*atan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+5*a^(1/
3)*b*a^(1/3)*1/9/a^2/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)))-((a+b*x^3)^(1/3))^2
*b/3/a/d/(a+b*x^3-a)
```

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(-bd\,x^{3}+ad\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{\left(bdx^{3}-ad\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)
```

**mupad** [B] time = 5.56, size = 490, normalized size = 1.82

$$\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}-2\,2^{1/3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)}^{1/3}+\frac{5\,\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}-a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}\right)\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{1/3}}{9}-\ln\left(4\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}-2^{1/3}\,\sqrt{3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)}^{1/3}+\ln\left(4\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}+2^{1/3}\,\sqrt{3}\,a^3\,d^2\,{\left(-\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)}^{1/3}-\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}-\sqrt{3}\,a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{125\,b^3}{729\,a^4\,d^3}\right)}^{1/3}+\ln\left(2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}+a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}+\sqrt{3}\,a^3\,d^2\,{\left(\frac{b^3}{a^4\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(\frac{125\,b^3}{729\,a^4\,d^3}\right)}^{1/3}-\frac{b\,{\left(b\,x^3+a\right)}^{2/3}}{3\,a\,\left(d\,\left(b\,x^3+a\right)-a\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x)
```

```
[Out] log(2*b^2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3))*(-(
4*b^3)/(27*a^4*d^3))^(1/3) + (5*log(b^2*(a + b*x^3)^(1/3) - a^3*d^2*(b^3/(a
^4*d^3))^(2/3))*(b^3/(a^4*d^3))^(1/3))/9 - log(4*b^2*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3) + log
(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) + 2^(1/
3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b
^3)/(27*a^4*d^3))^(1/3) - log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^3/(a^4*d
^3))^(2/3) - 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/
2)*((125*b^3)/(729*a^4*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*
(b^3/(a^4*d^3))^(2/3) + 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)
*1i)/2 - 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) - (b*(a + b*x^3)^(2/3))/(3*a*
(d*(a + b*x^3) - a*d))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^4+bx^7}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d
```

## 3.365 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=284

$$\frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{7/3}d}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}d}-\frac{2^{2/3}b^2\tan}{}$$

**Rubi** [A] time = 0.28, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 149, 156, 55, 617, 204, 31}

$$\frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{7/3}d}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}d}-\frac{2^{2/3}b^2\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}d}-\frac{7b^2\log(x)}{9a^{7/3}d}-\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3}-\frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*b*(a + b*x^3)^(2/3))/(18*a^2*d*x^3) - (a + b*x^3)^(5/3)/(6*a^2*d*x^6) +
 (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt
[3]*a^(7/3)*d) - (2^(2/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*d) - (7*b^2*Log[x])/(9*a^(7/3)*d) + (b
^2*Log[a - b*x^3])/(3*2^(1/3)*a^(7/3)*d) + (7*b^2*Log[a^(1/3) - (a + b*x^3)
^(1/3)])/(9*a^(7/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^
(1/3)*a^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd-\frac{1}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{28}{9}a^2b^2d^2-\frac{8}{9}ab^3d^2x}{x\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{\left(14b^2\right)\text{Sul}\cdots}{\cdots}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{7/3}d} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\cdots}\right.}{9a^{7\cdots}}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a}\cdots\right.}{9a^{7/3}d}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{7/3}d} - \frac{2^{2/3}b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\cdots\right.}{\sqrt{3}\,a^{7/3}d}$$

**Mathematica** [A] time = 0.16, size = 247, normalized size = 0.87

$$\frac{28\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\left(3a^{4/3}\left(a+bx^3\right)^{2/3}-3\,2^{2/3}b^2x^6\log\left(a-bx^3\right)-14b^2x^6\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+9\,2^{2/3}b^2x^6\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+6\,2^{2/3}\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+8\sqrt[3]{a}\,bx^3\left(a+bx^3\right)^{2/3}+14b^2x^6\log(x)\right)}{54a^{7/3}dx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (28*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3
*(3*a^(4/3)*(a + b*x^3)^(2/3) + 8*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 6*2^(2/
3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]
] + 14*b^2*x^6*Log[x] - 3*2^(2/3)*b^2*x^6*Log[a - b*x^3] - 14*b^2*x^6*Log[a
^(1/3) - (a + b*x^3)^(1/3)] + 9*2^(2/3)*b^2*x^6*Log[2^(1/3)*a^(1/3) - (a +
b*x^3)^(1/3)]))/(54*a^(7/3)*d*x^6)
```

**IntegrateAlgebraic** [A] time = 0.68, size = 341, normalized size = 1.20

$$\frac{14b^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{27a^{7/3}d}-\frac{2^{2/3}b^2\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{a}\right)}{3a^{7/3}d}-\frac{7b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{27a^{7/3}d}+\frac{b^2\log\left(2a^{2/3}+2^{2/3}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}\right)}{3\sqrt[3]{2}\,a^{7/3}d}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{7/3}d}-\frac{2^{2/3}b^2\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}d}+\frac{\left(-3a-8bx^3\right)\left(a+bx^3\right)^{2/3}}{18a^2dx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*a - 8*b*x^3)*(a + b*x^3)^(2/3))/(18*a^2*d*x^6) + (14*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*d) - (2^
(2/3)*b^2*ArcTan[1/Sqrt[3] + (2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(7/3)*d) + (14*b^2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(27*a^(7
/3)*d) - (2^(2/3)*b^2*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^(7/
3)*d) - (7*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]
)/(27*a^(7/3)*d) + (b^2*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) +
 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^(7/3)*d)
```

**fricas** [A] time = 0.50, size = 660, normalized size = 2.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3
)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*sq
rt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b
*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3)
 + 3*a)/x^3) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/
3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 18*4^
(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1
/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(8*a*
b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6), -1/54*(18*4^(1/3)*sqrt(3)*a*
b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1
/3) - 1/3*sqrt(3)) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 +
a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) -
 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 +
 a)^(1/3)) - 84*sqrt(1/3)*a^(2/3)*b^2*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^
3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3)
- a^(1/3)) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
raic extensions not allowed in a rootofAlgebraic extensions not allowed in
a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions no
t allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
extensions not allowed in a rootofAlgebraic extensions not allowed in a roo
tofAlgebraic extensions not allowed in a rootofAlgebraic extensions not all
owed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exten
sions not allowed in a rootofAlgebraic extensions not allowed in a rootofAl
gebraic extensions not allowed in a rootofAlgebraic extensions not allowed
in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions
 not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebra
ic extensions not allowed in a rootofAlgebraic extensions not allowed in a
rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions not
allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic ex
tensions not allowed in a rootofAlgebraic extensions not allowed in a rooto
fAlgebraic extensions not allowed in a rootofAlgebraic extensions not allow
ed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensi
ons not allowed in a rootofAlgebraic extensions not allowed in a rootofAlge
braic extensions not allowed in a rootofAlgebraic extensions not allowed in
 a rootofAlgebraic extensions not allowed in a rootofAlgebraic extensions n
ot allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic
 extensions not allowed in a rootofAlgebraic extensions not allowed in a ro
otofAlgebraic extensions not allowed in a rootofAlgebraic extensions not al
lowed in a rootofAlgebraic extensions not allowed in a rootofAlgebraic exte
nsions not allowed in a rootofAlgebraic extensions not allowed in a rootofA
lgebraic extensions not allowed in a rootofAlgebraic extensions not allowed
 in a rootofAlgebraic extensions not allowed in a rootofAlgebraic extension
s not allowed in a rootofAlgebraic extensions not allowed in a rootofAlgebr
aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofb^2/3/a^2/(2*a)^(1/3)/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)
^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2*b^2/sqrt(3)/a^3/d*atan(((a+
b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*b^2*(2*a
)^(1/3)*1/6/a^3/d*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-7*(a^(1/3))^2*b^2*1/
27/a^3/d*ln(((a+b*x^3)^(1/3))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+14
*b^2/9/sqrt(3)/a^2/a^(1/3)/d*atan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a
^(1/3))+14*a^(1/3)*b^2*a^(1/3)*1/27/a^3/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)))+
(-8*((a+b*x^3)^(1/3))^2*(a+b*x^3)*b^2+5*((a+b*x^3)^(1/3))^2*a*b^2)/18/a^2/d
/(a+b*x^3-a)^2
```

**maple [F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(-bd\,x^{3}+ad\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{\left(bdx^{3}-ad\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)
```

**mupad [B]** time = 5.45, size = 513, normalized size = 1.81

$$\frac{\frac{5\,b^2\,{\left(b\,x^3+a\right)}^{2/3}}{18\,a}-\frac{4\,b^2\,{\left(b\,x^3+a\right)}^{5/3}}{9\,a^2}}{d\,{\left(b\,x^3+a\right)}^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}+\ln\left(2\,b^4\,{\left(b\,x^3+a\right)}^{1/3}-2\,2^{1/3}\,a^5\,d^2\,{\left(-\frac{b^6}{a^7\,d^3}\right)}^{2/3}\right)\,{\left(-\frac{4\,b^6}{27\,a^7\,d^3}\right)}^{1/3}+\frac{14\,\ln\left(b^4\,{\left(b\,x^3+a\right)}^{1/3}-a^5\,d^2\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{2/3}\right)\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{1/3}}{27}-\ln\left(4\,b^4\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a^5\,d^2\,{\left(-\frac{b^6}{a^7\,d^3}\right)}^{2/3}-2^{1/3}\,\sqrt{3}\,a^5\,d^2\,{\left(-\frac{b^6}{a^7\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4\,b^6}{27\,a^7\,d^3}\right)}^{1/3}+\ln\left(4\,b^4\,{\left(b\,x^3+a\right)}^{1/3}+2\,2^{1/3}\,a^5\,d^2\,{\left(-\frac{b^6}{a^7\,d^3}\right)}^{2/3}+2^{1/3}\,\sqrt{3}\,a^5\,d^2\,{\left(-\frac{b^6}{a^7\,d^3}\right)}^{2/3}\,2{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(-\frac{4\,b^6}{27\,a^7\,d^3}\right)}^{1/3}-\frac{7\,\ln\left(2\,b^4\,{\left(b\,x^3+a\right)}^{1/3}+a^5\,d^2\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{2/3}-\sqrt{3}\,a^5\,d^2\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{1/3}}{27}+\frac{7\,\ln\left(2\,b^4\,{\left(b\,x^3+a\right)}^{1/3}+a^5\,d^2\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{2/3}+\sqrt{3}\,a^5\,d^2\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{2/3}\,1{}\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(\frac{b^6}{a^7\,d^3}\right)}^{1/3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x)
```

```
[Out] ((5*b^2*(a + b*x^3)^(2/3))/(18*a) - (4*b^2*(a + b*x^3)^(5/3))/(9*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + log(2*b^4*(a + b*x^3)^(1/3) - 2
*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3))*(-(4*b^6)/(27*a^7*d^3))^(1/3) + (1
4*log(b^4*(a + b*x^3)^(1/3) - a^5*d^2*(b^6/(a^7*d^3))^(2/3))*(b^6/(a^7*d^3)
)^(1/3))/27 - log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^
3))^(2/3) - 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i
)/2 + 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) + log(4*b^4*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) - (7*
log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) - 3^(1/2)*a^5*d
^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^7*d^3))^(1/3))/27 + (
7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) + 3^(1/2)*a^5
*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^7*d^3))^(1/3))/27
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^7+bx^{10}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**7 + b*x**10), x)/d
```

## 3.366 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=264

$$\frac{a^2\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,b^{7/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^{7/3}d}+\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{7/3}d}-\frac{14a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}d}+\frac{2^{2/3}a\cdots}{\cdots}$$

**Rubi [C]** time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), (b*x^3)/a
])/(7*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int\frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx&=\frac{\left(a+bx^3\right)^{2/3}\int\frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\&=\frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica [C]** time = 0.30, size = 244, normalized size = 0.92

$$\frac{2\,2^{2/3}a^2\sqrt[3]{a+bx^3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)+21ab^{4/3}x^4\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-3\sqrt[3]{b}\left(a+bx^3\right)\left(8ax+3bx^4\right)}{54b^{7/3}d\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-3*b^(1/3)*(a + b*x^3)*(8*a*x + 3*b*x^4) + 21*a*b^(4/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + 2*2^(2/3)*a^2
*(a + b*x^3)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)
^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1
 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3
)^(1/3)]))/(54*b^(7/3)*d*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 1.14, size = 361, normalized size = 1.37

$$\frac{14a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27b^{7/3}d}-\frac{2^{2/3}a^2\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3b^{7/3}d}-\frac{14a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}b^{7/3}d}+\frac{2^{2/3}a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}b^{7/3}d}-\frac{7a^2\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{27b^{7/3}d}+\frac{a^2\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\sqrt[3]{2}b^{7/3}d}+\frac{\left(a+bx^3\right)^{2/3}\left(-8ax-3bx^4\right)}{18b^2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-8*a*x - 3*b*x^4))/(18*b^2*d) - (14*a^2*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(7/3)*d) + (2^
(2/3)*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)
)])/(Sqrt[3]*b^(7/3)*d) + (14*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2
7*b^(7/3)*d) - (2^(2/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/
(3*b^(7/3)*d) - (7*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a +
 b*x^3)^(2/3)])/(27*b^(7/3)*d) + (a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x
*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*b^(7/3)*d)
```

**fricas [A]** time = 0.52, size = 701, normalized size = 2.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 42*sqrt(1/3)*a^2*b*sqrt(-1
/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4
/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
b^(2/3)) + 2*a) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2
/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*4^(1/3
)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^
3 + a)^(2/3))/x^2) - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
 + 14*a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d), -1/54
*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*lo
g(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-
1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b
*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 84*sqrt(1/3)*a^2*b^(2/3)*arct
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 28*a^2*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 14*a^2*b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8
*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{6}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}} x^{6}}{bdx^{3}-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{6}\left(b\,x^{3}+a\right)^{2/3}}{a\,d-b\,d\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{6}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

## 3.367 $\int \frac{x^3\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=229

$$\frac{a\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,b^{4/3}d}-\frac{a\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^{4/3}d}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}d}-\frac{5a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d}+\frac{2^{2/3}a\tan^{-1}\left(\text{[illegible]}\right)}{\sqrt{3}\,b^{4\text{[illegible]}}}$$

**Rubi [C]** time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.29, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]
)/(4*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica [C]** time = 0.22, size = 216, normalized size = 0.94

$$\frac{\frac{15x^4\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{2^{2/3}a\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{b^{4/3}}-\frac{12x\left(a+bx^3\right)^{2/3}}{b}}{36d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-12*x*(a + b*x^3)^(2/3))/b + (15*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3,
1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/(a + b*x^3)^(1/3) + (2^(2/3)*a*(2*Sq
rt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log
[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)
/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/b^(4/3))/(36*
d)
```

**IntegrateAlgebraic [A]** time = 0.84, size = 339, normalized size = 1.48

$$\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{9b^{4/3}d}-\frac{2^{2/3}a\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3b^{4/3}d}-\frac{5a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{3\sqrt{3}b^{4/3}d}+\frac{2^{2/3}a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}b^{4/3}d}-\frac{5a\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18b^{4/3}d}+\frac{a\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\sqrt[3]{2}b^{4/3}d}-\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/3*(x*(a + b*x^3)^(2/3))/(b*d) - (5*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)
*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d) + (2^(2/3)*a*ArcTan[(Sqrt
[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*b^(4/3)*d
) + (5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(4/3)*d) - (2^(2/3)*a*
Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*b^(4/3)*d) - (5*a*Log[b^(
2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3)*d)
 + (a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a
+ b*x^3)^(2/3)])/(3*2^(1/3)*b^(4/3)*d)
```

**fricas [A]** time = 0.51, size = 653, normalized size = 2.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)
*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^(2
/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x
^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/
3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*
(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-
1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/
3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
+ a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1
/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3)
)/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3
 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(
1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^
2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/
3))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^3}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(b\,x^3+a\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

## 3.368 $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

**Optimal**. Leaf size=200

$$\frac{\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\sqrt[3]{b}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d}+\frac{2^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\cdots}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d}$$

**Rubi [C]** time = 0.03, antiderivative size = 61, normalized size of antiderivative = 0.30, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/
(a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx&=\frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\&=\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica [C]** time = 0.16, size = 156, normalized size = 0.78

$$\frac{4ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{d\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/(d*(a - b*x^3)*(4*a*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + 2*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

**IntegrateAlgebraic [A]** time = 0.60, size = 309, normalized size = 1.54

$$-\frac{\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6\sqrt[3]{b}\,d}+\frac{\log\left(2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}\,d}-\frac{2^{2/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\sqrt[3]{b}\,d}+\frac{2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^
(1/3)*d)) + (2^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b
*x^3)^(1/3))])/(Sqrt[3]*b^(1/3)*d) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/
(3*b^(1/3)*d) - (2^(2/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*
b^(1/3)*d) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2
/3)]/(6*b^(1/3)*d) + Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + 2^(1/3)*(a + b*x^3)^(2/3)]/(3*2^(1/3)*b^(1/3)*d)
```

**fricas [A]** time = 0.54, size = 611, normalized size = 3.06

$$\left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-3\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)-2\cdot 4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{2\cdot 4^{\frac{1}{3}}bx^2\left(-\frac{1}{b}\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-2b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6bd},\ -\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-2\cdot 4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{2\cdot 4^{\frac{1}{3}}bx^2\left(-\frac{1}{b}\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-6\sqrt{\frac{1}{3}}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)-2b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 3*sqrt(1/3)*b*sqrt(-1/b^(2/3))*l
og(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (
b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) +
2*a) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 +
 a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) -
 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 2
*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(2*4^
(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-1/b)^(1/3))/x) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*
(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(
1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b
*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2
*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

**maple** [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{bdx^{3}-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

## 3.369 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$

**Optimal**. Leaf size=157

$$\frac{b^{2/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,ad}+\frac{2^{2/3}b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,ad}-\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}$$

**Rubi** [C] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 0.50, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)}{2adx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3
, (-2*b*x^3)/(a - b*x^3)])/(2*a*d*x^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx&=\frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\&=-\frac{\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)}{2adx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica** [C] time = 0.04, size = 47, normalized size = 0.30

$$-\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};\frac{2bx^3}{bx^3+a}\right)}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 1, 1/3, (2*b*x^3)/(a + b*x^
3)])/(a*d*x^2)
```

**IntegrateAlgebraic [A]** time = 0.46, size = 206, normalized size = 1.31

$$-\frac{2^{2/3}b^{2/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}\,x\right)}{3ad}+\frac{2^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,ad}+\frac{b^{2/3}\log\left(2^{2/3}\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\sqrt[3]{2}\,ad}-\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*d*x^2) + (2^(2/3)*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)
*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a*d) - (2^(2/3)*b^(2
/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a*d) + (b^(2/3)*Log[2
*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2
/3)])/(3*2^(1/3)*a*d)
```

**fricas [B]** time = 144.75, size = 434, normalized size = 2.76

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}x^2\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}\left(5b^2x^7-4abx^4-a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}\left(-b^2\right)^{\frac{2}{3}}+6\cdot 4^{\frac{1}{3}}\sqrt{3}\left(19b^3x^8+16ab^2x^5+a^2bx^2\right)\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}-\sqrt{3}\left(71b^4x^9+111ab^3x^6+33a^2b^2x^3+a^3b\right)}{3\left(109b^4x^9+105ab^3x^6+3a^2b^2x^3-a^3b\right)}\right)-2\cdot 4^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}x^2\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x^2-6\left(bx^3+a\right)^{\frac{2}{3}}bx+4^{\frac{1}{3}}\left(bx^3-a\right)\left(-b^2\right)^{\frac{1}{3}}}{bx^3-a}\right)+4^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}x^2\log\left(-\frac{6\cdot 4^{\frac{1}{3}}\left(5b^2x^4+abx\right)\left(bx^3+a\right)^{\frac{2}{3}}\left(-b^2\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\left(19b^2x^6+16abx^3+a^2\right)\left(-b^2\right)^{\frac{2}{3}}-24\left(2b^3x^5+ab^2x^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{b^2x^6-2abx^3+a^2}\right)+9\left(bx^3+a\right)^{\frac{2}{3}}}{18adx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(2*4^(1/3)*sqrt(3)*(-b^2)^(1/3)*x^2*arctan(1/3*(3*4^(2/3)*sqrt(3)*(5*
b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b^2)^(2/3) + 6*4^(1/3)*sqr
t(3)*(19*b^3*x^8 + 16*a*b^2*x^5 + a^2*b*x^2)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
 - sqrt(3)*(71*b^4*x^9 + 111*a*b^3*x^6 + 33*a^2*b^2*x^3 + a^3*b))/(109*b^4*
x^9 + 105*a*b^3*x^6 + 3*a^2*b^2*x^3 - a^3*b)) - 2*4^(1/3)*(-b^2)^(1/3)*x^2*
log((3*4^(2/3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x^2 - 6*(b*x^3 + a)^(2/3)*b*x
 + 4^(1/3)*(b*x^3 - a)*(-b^2)^(1/3))/(b*x^3 - a)) + 4^(1/3)*(-b^2)^(1/3)*x^
2*log(-(6*4^(1/3)*(5*b^2*x^4 + a*b*x)*(b*x^3 + a)^(2/3)*(-b^2)^(1/3) - 4^(2
/3)*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*(-b^2)^(2/3) - 24*(2*b^3*x^5 + a*b^2*x^
2)*(b*x^3 + a)^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2)) + 9*(b*x^3 + a)^(2/3))/(
a*d*x^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^3+bx^6}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d
```

## 3.370 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

**Optimal**. Leaf size=182

$$\frac{b^{5/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,a^2d}-\frac{b^{5/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^2d}+\frac{2^{2/3}b^{5/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}-\frac{7b\left(a+bx^3\right)^{2/3}}{10a^2dx^2}-\frac{\left(a+bx^3\right)^{2/3}}{5adx^5}$$

**Rubi [C]** time = 0.43, antiderivative size = 121, normalized size of antiderivative = 0.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{2a^2-4bx^3\left(2a+3bx^3\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+12bx^3\left(a-bx^3\right){}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+5abx^3+3b^2x^6}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -(2*a^2 + 5*a*b*x^3 + 3*b^2*x^6 - 4*b*x^3*(2*a + 3*b*x^3)*Hypergeometric2F1
[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*b*x^3*(a - b*x^3)*Hypergeometric2
F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)])/(10*a^2*d*x^5*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx=\frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$=-\frac{2a^2+5abx^3+3b^2x^6-4bx^3\left(2a+3bx^3\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)+12bx^3\left(a-bx^3\right){}_2F_1\left(\frac{1}{3}\right.}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

**Mathematica [A]** time = 5.20, size = 170, normalized size = 0.93

$$\frac{5\,2^{2/3}b^{5/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)-\frac{3\left(a+bx^3\right)^{2/3}\left(2a+7bx^3\right)}{x^5}}{30a^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(2*a + 7*b*x^3))/x^5 + 5*2^(2/3)*b^(5/3)*(2*Sqrt[3]*
ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (
2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b +
a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(30*a^2*d)
```

**IntegrateAlgebraic [A]** time = 0.49, size = 216, normalized size = 1.19

$$-\frac{2^{2/3}b^{5/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3a^2d}+\frac{2^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^2d}+\frac{b^{5/3}\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\sqrt[3]{2}a^2d}+\frac{\left(-2a-7bx^3\right)\left(a+bx^3\right)^{2/3}}{10a^2dx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-2*a - 7*b*x^3)*(a + b*x^3)^(2/3))/(10*a^2*d*x^5) + (2^(2/3)*b^(5/3)*ArcT
an[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a
^2*d) - (2^(2/3)*b^(5/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*
a^2*d) + (b^(5/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) +
 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^2*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^6\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^6+bx^9}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d
```

## 3.371 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx$

**Optimal**. Leaf size=209

$$\frac{b^{8/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,a^3d}-\frac{b^{8/3}\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^3d}+\frac{2^{2/3}b^{8/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^3d}-\frac{5b^2\left(a+bx^3\right)^{2/3}}{8a^3dx^2}-\frac{b\left(a+bx^3\right)^{2/3}}{4a^2dx^5}$$

**Rubi [C]** time = 10.68, antiderivative size = 244, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-18bx^3\left(a-bx^3\right)^2\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-4bx^3\left(5a^2+6abx^3+9b^2x^6\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+42a^2bx^3\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+11a^2bx^3+5a^3-54b^3x^9\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+12ab^2x^6\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+15ab^2x^6+9b^3x^9}{40a^3dx^8\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] -(5*a^3 + 11*a^2*b*x^3 + 15*a*b^2*x^6 + 9*b^3*x^9 - 4*b*x^3*(5*a^2 + 6*a*b*
x^3 + 9*b^2*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 42
*a^2*b*x^3*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*a*b^2
*x^6*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b^3*x^9*Hyp
ergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 18*b*x^3*(a - b*x^3)^2
*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)])/(40*a^3*d
*x^8*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{5a^3+11a^2bx^3+15ab^2x^6+9b^3x^9-4bx^3\left(5a^2+6abx^3+9b^2x^6\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)+42}{}\end{aligned}$$

**Mathematica** [A] time = 5.27, size = 179, normalized size = 0.86

$$\frac{4\,2^{2/3}b^{8/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)-\frac{3\left(a+bx^3\right)^{2/3}\left(a^2+2abx^3+5b^2x^6\right)}{x^8}}{24a^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 4*2^(2/3)*b^(8/
3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]]
 - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(24*a
^3*d)
```

**IntegrateAlgebraic** [A] time = 0.55, size = 227, normalized size = 1.09

$$-\frac{2^{2/3}b^{8/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3a^3d}+\frac{2^{2/3}b^{8/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^3d}+\frac{b^{8/3}\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\sqrt[3]{2}a^3d}+\frac{\left(a+bx^3\right)^{2/3}\left(-a^2-2abx^3-5b^2x^6\right)}{8a^3dx^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-a^2 - 2*a*b*x^3 - 5*b^2*x^6))/(8*a^3*d*x^8) + (2^(2/3)
*b^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))
])/(Sqrt[3]*a^3*d) - (2^(2/3)*b^(8/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3
)^(1/3)])/(3*a^3*d) + (b^(8/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b
*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^3*d)
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

**maple** [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^9\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d),x)
[Out] Timed out
```

## 3.372 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx$

**Optimal**. Leaf size=236

$$\frac{b^{11/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,a^4d}-\frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^4d}+\frac{2^{2/3}b^{11/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}-\frac{293b^3\left(a+bx^3\right)^{2/3}}{440a^4dx^2}-\frac{49b^2}{2\cdots}$$

**Rubi [C]** time = 17.60, antiderivative size = 391, normalized size of antiderivative = 1.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-54bx^3\left(a-bx^3\right)^2\left(5a+6bx^3\right){}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+54bx^3\left(a-bx^3\right)^3{}_4F_3\left(\frac{1}{3},2,2,2;1,1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-180a^2b^2x^6{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+198a^2b^2x^6{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+99a^2b^2x^6-160a^3bx^3{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+396a^3bx^3{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+85a^3bx^3+40a^4-324b^4x^{12}{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-594b^4x^{12}{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-216ab^3x^9{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+135ab^3x^9+81b^4x^{12}}{440a^4dx^{11}\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] -(40*a^4 + 85*a^3*b*x^3 + 99*a^2*b^2*x^6 + 135*a*b^3*x^9 + 81*b^4*x^12 - 16
0*a^3*b*x^3*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 180*a^2
*b^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 216*a*b^3*
x^9*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 324*b^4*x^12*Hy
pergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 396*a^3*b*x^3*Hyperge
ometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 198*a^2*b^2*x^6*Hypergeome
tric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 594*b^4*x^12*Hypergeometric2F
1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b*x^3*(a - b*x^3)^2*(5*a + 6*b*x
^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)] + 54*b*
x^3*(a - b*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2}, {1, 1, 4/3}, (2*b*x^3)/
(a + b*x^3)])/(440*a^4*d*x^11*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx=\frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$=-\frac{40a^4+85a^3bx^3+99a^2b^2x^6+135ab^3x^9+81b^4x^{12}-160a^3bx^3{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)-\cdots}{\cdots}$$

**Mathematica** [A]   time = 5.35, size = 196, normalized size = 0.83

$$\frac{\frac{2^{2/3}b^{11/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{a^4}-\frac{3\left(a+bx^3\right)^{2/3}\left(40a^3+65a^2bx^3+98ab^2x^6+293b^3x^9\right)}{220a^4x^{11}}}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9)
)/(220*a^4*x^11) + (2^(2/3)*b^(11/3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1
/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x
^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(
1/3)*x)/(b + a*x^3)^(1/3)]))/a^4)/(6*d)
```

**IntegrateAlgebraic** [A]   time = 0.62, size = 238, normalized size = 1.01

$$-\frac{2^{2/3}b^{11/3}\log\left(2^{2/3}\sqrt[3]{a+bx^3}-2\sqrt[3]{b}x\right)}{3a^4d}+\frac{2^{2/3}b^{11/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}a^4d}+\frac{b^{11/3}\log\left(2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}\left(a+bx^3\right)^{2/3}+2b^{2/3}x^2\right)}{3\sqrt[3]{2}a^4d}+\frac{\left(a+bx^3\right)^{2/3}\left(-40a^3-65a^2bx^3-98ab^2x^6-293b^3x^9\right)}{440a^4dx^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-40*a^3 - 65*a^2*b*x^3 - 98*a*b^2*x^6 - 293*b^3*x^9))/(
440*a^4*d*x^11) + (2^(2/3)*b^(11/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +
 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a^4*d) - (2^(2/3)*b^(11/3)*Log[-2*b^
(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a^4*d) + (b^(11/3)*Log[2*b^(2/3)*x
^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2
^(1/3)*a^4*d)
```

**fricas** [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

**maple** [F]   time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^{12}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)
[Out] Timed out
```

**3.373** $\int \frac{x^{14}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=127

$$\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi** [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*(1 - x^3)^(5/3))/5 - (1 - x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + ArcTan[(1
 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^
(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-2(1-x)^{2/3}+2(1-x)^{5/3}-(1-x)^{8/3}+\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right) dx, x, x^3\right)\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right.\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{}{2^{2/3}+\sqrt[3]{}}\right.\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 113, normalized size = 0.89

$$\frac{1}{660}\left(-55\,2^{2/3}\log\left(x^3+1\right)+165\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+110\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+3\left(1-x^3\right)^{2/3}\left(-20x^9+5x^6-38x^3+53\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9) + 110*2^(2/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 55*2^(2/3)*Log[1 + x^3] + 165*
2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/660
```

**IntegrateAlgebraic [A]** time = 0.15, size = 148, normalized size = 1.17

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{1}{220}\left(1-x^3\right)^{2/3}\left(-20x^9+5x^6-38x^3+53\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9))/220 + ArcTan[1/Sqrt[3] + (
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 -
 x^3)^(1/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x
^3)^(2/3)]/(6*2^(1/3))
```

**fricas [A]** time = 0.93, size = 118, normalized size = 0.93

$$-\frac{1}{220}\left(20x^9-5x^6+38x^3-53\right)\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6
)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12
*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2
^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**giac [A]** time = 0.17, size = 134, normalized size = 1.06

$$-\frac{1}{11}\left(x^3-1\right)^3\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{4}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/11*(x^3 - 1)^3*(-x^3 + 1)^(2/3) - 1/4*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/6
*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3)))
 + 2/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/
3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

**maple [C]** time = 9.32, size = 792, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/220*(20*x^9-5*x^6+38*x^3-53)*(x^3-1)/(-x^3+1)^(1/3)+1/6*RootOf(_Z^3-4)*ln
((-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*
x^3-90*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4
)^2*x^3-3*RootOf(_Z^3-4)*x^3-90*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-126*RootOf(_Z^3-4)*(-x^3+1
)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2
/3)+7*RootOf(_Z^3-4)+210*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2))/(x+1)/(x^2-x+1))-1/6*ln((-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+RootOf(_Z^3-4)*x^3-24*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-1
26*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)+42*(-x^3+1)^(2/3)-7*RootOf(_Z^3-4)+168*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln((-3*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+Root
Of(_Z^3-4)*x^3-24*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-
21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-126*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2/3)-7*RootOf(_
Z^3-4)+168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2
-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima [A]** time = 1.24, size = 119, normalized size = 0.94

$$\frac{1}{11}\left(-x^3+1\right)^{\frac{11}{3}}-\frac{1}{4}\left(-x^3+1\right)^{\frac{8}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/11*(-x^3 + 1)^(11/3) - 1/4*(-x^3 + 1)^(8/3) + 1/6*sqrt(3)*2^(2/3)*arctan(
1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 2/5*(-x^3 + 1)^(5/3)
- 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) +
 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**mupad [B]** time = 5.03, size = 133, normalized size = 1.05

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2\left(1-x^3\right)^{5/3}}{5}-\frac{\left(1-x^3\right)^{8/3}}{4}+\frac{\left(1-x^3\right)^{11/3}}{11}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2*(1 - x^3)^(5/3))/5 - (1 - x
^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)
*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3)
 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

**3.374** $\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=128

$$-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi** [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/2 + (1 - x^3)^(5/3)/5 - (1 - x^3)^(8/3)/8 - ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)
) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}}-(1-x)^{2/3}+(1-x)^{5/3}-\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right)dx,x,x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{\ldots}}\right.\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 128, normalized size = 1.00

$$\frac{1}{120}\left(6\left(1-x^3\right)^{2/3}x^3-51\left(1-x^3\right)^{2/3}+10\,2^{2/3}\log\left(x^3+1\right)-30\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-20\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-15\left(1-x^3\right)^{2/3}x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-51*(1 - x^3)^(2/3) + 6*x^3*(1 - x^3)^(2/3) - 15*x^6*(1 - x^3)^(2/3) - 20*
2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*
Log[1 + x^3] - 30*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/120
```

**IntegrateAlgebraic [A]** time = 0.14, size = 144, normalized size = 1.12

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{1}{40}\left(1-x^3\right)^{2/3}\left(-5x^6+2x^3-17\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-17 + 2*x^3 - 5*x^6))/40 - ArcTan[1/Sqrt[3] + (2^(2/3)*(1
 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3
)]/(3*2^(1/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]
/(6*2^(1/3))
```

**fricas [A]** time = 0.87, size = 137, normalized size = 1.07

$$-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\left(-1\right)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{40}\left(5x^6-2x^3+17\right)\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/40*(5*x^6
- 2*x^3 + 17)*(-x^3 + 1)^(2/3)
```

**giac [A]** time = 0.18, size = 127, normalized size = 0.99

$$-\frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2/3)
```

**maple [C]** time = 8.41, size = 683, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/40*(5*x^6-2*x^3+17)*(x^3-1)/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3+4)^2+6*_Z*R
ootOf(_Z^3+4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+3
6*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+5*RootOf(_Z^3+4)*x^3+6*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42
*(-x^3+1)^(2/3)-35*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z
^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*ln((-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*R
ootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z
*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*RootOf(_Z^3+4)*x^3-18*Ro
otOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*
(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-28*RootOf(_Z^3+4)+42*RootOf(RootOf(_Z^3+4)
^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-ln((-12*Ro
otOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*
RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3
+12*RootOf(_Z^3+4)*x^3-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-28*RootOf(_Z^3
+4)+42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1
))*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

**maxima [A]** time = 1.25, size = 119, normalized size = 0.93

$$-\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}}-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^3 + 1)^(8/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

**mupad [B]** time = 4.73, size = 133, normalized size = 1.04

$$\frac{{\left(1-x^3\right)}^{5/3}}{5}-\frac{{\left(1-x^3\right)}^{2/3}}{2}-\frac{2^{2/3}\,\ln\left({\left(1-x^3\right)}^{1/3}-2^{1/3}\right)}{6}-\frac{{\left(1-x^3\right)}^{8/3}}{8}-\frac{2^{2/3}\,\ln\left({\left(1-x^3\right)}^{1/3}-\frac{2^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}+\frac{2^{2/3}\,\ln\left({\left(1-x^3\right)}^{1/3}-\frac{2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(5/3)/5 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1
/3)))/6 - (1 - x^3)^(8/3)/8 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1
/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^
(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

## 3.375 $\int \frac{x^8}{\sqrt[3]{1-x^3}\left(1+x^3\right)} \, dx$

**Optimal**. Leaf size=97

$$\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(5/3)/5 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-(1-x)^{2/3}+\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right)dx, x, x^3\right)\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subs}}{}\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\right.}{\sqrt[3]{2}}\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.07, size = 96, normalized size = 0.99

$$
\frac{1}{60}\left(12\left(1-x^3\right)^{5/3}-5\,2^{2/3}\log\left(x^3+1\right)+15\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 5*2^(2/3)*Log[1 + x^3] + 15*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(
1/3)])/60
```

**IntegrateAlgebraic [A]** time = 0.11, size = 131, normalized size = 1.35

$$
\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(5/3)/5 + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(
2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas [A]** time = 1.16, size = 106, normalized size = 1.09

$$
-\frac{1}{5}\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(s
qrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) +
2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (
-x^3 + 1)^(1/3))
```

**giac** [A] time = 0.20, size = 98, normalized size = 1.01

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)
))
```

**maple** [C] time = 6.73, size = 673, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/5*(x^3-1)^2/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf
(_Z^3-4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Root
Of(_Z^3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3
-5*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+
42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/
(x+1)/(x^2-x+1))-1/6*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42
*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*
RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln((18*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima** [A] time = 1.19, size = 97, normalized size = 1.00

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**mupad** [B] time = 4.65, size = 111, normalized size = 1.14

$$\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{\left(1-x^3\right)^{5/3}}{5}+\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (1 - x^3)^(5/3)/5 + (2^(2/3)*l
og((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 -
 (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
 + 1))/12
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.376 $\int \frac{x^5}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=98

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 55, 617, 204, 31}

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/2 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)
*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^
(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Sub}}{}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2}\right.}{\sqrt[3]{2}}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

**Mathematica** [A] time = 0.03, size = 95, normalized size = 0.97

$$
\frac{1}{12}\left(-6\left(1-x^3\right)^{2/3} + 2^{2/3}\log\left(x^3+1\right) - 3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) - 2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
])/12
```

**IntegrateAlgebraic** [A] time = 0.12, size = 132, normalized size = 1.35

$$
-\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3) - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]
]/(2^(1/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[2
 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas** [A] time = 1.01, size = 125, normalized size = 1.28

$$
-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\left(-1\right)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 +
1)^(2/3)
```

**giac [A]** time = 0.18, size = 98, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2
/3)
```

**maple [C]** time = 3.82, size = 671, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/2*(x^3-1)/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*ln((15*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^
3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf
(_Z^3+4)^2*x^3+5*RootOf(_Z^3+4)*x^3+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-35
*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x
+1)/(x^2-x+1))-1/6*ln((-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36
*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*RootOf(_Z^3+4)*x^3-18*RootOf(RootOf(_Z^3+4
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42*
(-x^3+1)^(2/3)-28*RootOf(_Z^3+4)+42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-ln((-12*RootOf(RootOf(_Z^3+4
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*RootOf(_Z^3+4)
*x^3-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(
_Z^3+4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-28*RootOf(_Z^3+4)+42*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_
Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

**maxima [A]** time = 1.21, size = 97, normalized size = 0.99

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

**mupad [B]** time = 4.69, size = 111, normalized size = 1.13

$$-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{2/3}}{2}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2}{4}\right)\left(-1+\sqrt{3}\,1i\right)}{12}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1i\right)^2}{4}\right)\left(1+\sqrt{3}\,1i\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
+ 1))/12 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^
(1/3)))/6
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.377 $\int \frac{x^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)} \, dx$

**Optimal**. Leaf size=82

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi** [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}dx, x, x^3\right)$$
$$= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}dx,\right.}{2\sqrt[3]{2}}$$
$$= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$
$$= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

**Mathematica [A]** time = 0.02, size = 73, normalized size = 0.89

$$\frac{-\log\left(x^3+1\right)+3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6*2^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.09, size = 116, normalized size = 1.41

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + L
og[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1
/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas [A]** time = 0.58, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**giac [A]** time = 0.18, size = 87, normalized size = 1.06

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

**maple [C]** time = 5.59, size = 476, normalized size = 5.80

$$\mathrm{RootOf}\left(36\_Z^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+\mathrm{RootOf}\left(\_Z^3-4\right)^2\right)\ln\left(\frac{72x^3\,\mathrm{RootOf}\left(36\_Z^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+\mathrm{RootOf}\left(\_Z^3-4\right)^2\right)^2 \mathrm{RootOf}\left(\_Z^3-4\right)^2+\cdots}{(x+1)\left(x^2-x+1\right)}\right) \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(_Z
^3-4)*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+126*R
ootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)+42*(-x^3+1)^(2/3)-35*RootOf(_Z^3-4)-168*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-4)*ln(-(6*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+45*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-2*R
ootOf(_Z^3-4)*x^3-15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x
^3-63*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)-21*(-x^3+1)^(2/3)+14*RootOf(_Z^3-4)+105*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

**maxima [A]** time = 1.30, size = 86, normalized size = 1.05

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**mupad [B]** time = 4.89, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.378 $\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=137

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}$$

**Rubi [A]** time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 55, 618, 204, 31, 617}

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 -
x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(1/3)
) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right) - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2 \ldots}\right.\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\operatorname{Subst}\left(\int \frac{\ldots}{-3\ldots}\right.}{\ldots}\\
&= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x\ldots}\right.
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 133, normalized size = 0.97

$$\frac{1}{12}\left(2^{2/3}\log\left(x^3+1\right)+6\log\left(1-\sqrt[3]{1-x^3}\right)-3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-6\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcT
an[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 6*Log[x] + 2^(2/3)*Log[1 + x^3]
 + 6*Log[1 - (1 - x^3)^(1/3)] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/1
2
```

**IntegrateAlgebraic [A]** time = 0.20, size = 195, normalized size = 1.42

$$\frac{1}{3}\log\left(\sqrt[3]{1-x^3}-1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}}-\frac{1}{6}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)+\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[1/Sqrt[3]
+ (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-1 + (1 - x^3)
^(1/3)]/3 - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[1 + (1 - x^
3)^(1/3) + (1 - x^3)^(2/3)]/6 + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(
1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas [C]** time = 1.48, size = 410, normalized size = 2.99

$$\frac{1}{12}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(\frac{1}{8}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}-\frac{3}{4}\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+3\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(\frac{3}{8}\cdot 2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+\frac{3}{8}\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+3\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(-\frac{3}{8}\cdot 2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+\frac{3}{8}\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+3\left(-x^{3}+1\right)^{\frac{1}{3}}\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{3}\log\left(-\frac{1}{24}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}+\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{4}{3}\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3
*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))
- 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log(3/8
*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)
^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)
)*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/
3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3)*
(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/3*log(-1/24*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3))^3 + (-x^3 + 1)^(1/3) - 4/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1
)^(1/3) + 1)
```

**giac [A]** time = 0.20, size = 149, normalized size = 1.09

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3
+ 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

**maple [F]** time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{3}+1\right)^{\frac{1}{3}}\left(x^{3}+1\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)
```

**mupad [B]** time = 4.80, size = 256, normalized size = 1.87

$$\frac{\ln\left(6-6\left(1-x^{3}\right)^{1/3}\right)}{3}+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^{3}\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^{2}-135\left(1-x^{3}\right)^{1/3}\right)-\left(1-x^{3}\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(-\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^{3}\left(1458\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^{2}-135\left(1-x^{3}\right)^{1/3}\right)-\left(1-x^{3}\right)^{1/3}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\frac{2^{2/3}\ln\left(\frac{3\left(1-x^{3}\right)^{1/3}}{2}-\frac{3\,2^{1/3}}{2}\right)}{6}+\frac{(-1)^{1/3}\,2^{2/3}\ln\left(\frac{3\left(1-x^{3}\right)^{1/3}}{2}-\frac{3\,(-1)^{2/3}\,2^{1/3}}{2}\right)}{6}-\frac{(-1)^{1/3}\,2^{2/3}\ln\left(-\frac{\left(1+\sqrt{3}\,1i\right)^{3}\left(135\left(1-x^{3}\right)^{1/3}-\frac{81\,(-1)^{2/3}\,2^{1/3}\left(1+\sqrt{3}\,1i\right)^{2}}{4}\right)}{432}-\left(1-x^{3}\right)^{1/3}\right)\left(1+\sqrt{3}\,1i\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] log(6 - 6*(1 - x^3)^(1/3))/3 + log(((3^(1/2)*1i)/6 - 1/6)^3*(1458*((3^(1/2)
*1i)/6 - 1/6)^2 - 135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 -
 1/6) - log(- ((3^(1/2)*1i)/6 + 1/6)^3*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - 135
*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log(
(3*(1 - x^3)^(1/3))/2 - (3*2^(1/3))/2))/6 + ((-1)^(1/3)*2^(2/3)*log((3*(1 -
 x^3)^(1/3))/2 - (3*(-1)^(2/3)*2^(1/3))/2))/6 - ((-1)^(1/3)*2^(2/3)*log(- (
(3^(1/2)*1i + 1)^3*(135*(1 - x^3)^(1/3) - (81*(-1)^(2/3)*2^(1/3)*(3^(1/2)*1
i + 1)^2)/4))/432 - (1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.379 $\int \frac{1}{x^4 \sqrt[3]{1-x^3} \left(1+x^3\right)} \, dx$

**Optimal**. Leaf size=157

$$-\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right.}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(3*x^3) - (2*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[x]/3 - Log[1 + x^3]/(6*2^(1/3)) - Log[1 - (1 - x^3)^(1/3)]/3 + Log[2^(
1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^2(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{1}{3}\text{Subst}\left(\int \frac{\frac{2}{3}-\frac{x}{3}}{\sqrt[3]{1-x}\,x(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3}+\frac{\log(x)}{3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{1}{3}\text{Subst}\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3}+\frac{\log(x)}{3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(x)}{3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-
\end{aligned}
$$

**Mathematica [A]** time = 0.22, size = 153, normalized size = 0.97

$$
\frac{1}{36}\left(3\left(-\frac{4\left(1-x^3\right)^{2/3}}{x^3}-2^{2/3}\log\left(x^3+1\right)-4\log\left(1-\sqrt[3]{1-x^3}\right)+3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+4\log(x)\right)-8\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 3*((-4*(1 - x^3)^(2/3
))/x^3 + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] +
4*Log[x] - 2^(2/3)*Log[1 + x^3] - 4*Log[1 - (1 - x^3)^(1/3)] + 3*2^(2/3)*Lo
g[2^(1/3) - (1 - x^3)^(1/3)]))/36
```

**IntegrateAlgebraic** [A] time = 0.27, size = 215, normalized size = 1.37

$$-\frac{(1-x^3)^{2/3}}{3x^3}-\frac{2}{9}\log\left(\sqrt[3]{1-x^3}-1\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\sqrt[3]{2}}+\frac{1}{9}\log\left((1-x^3)^{2/3}+\sqrt[3]{1-x^3}+1\right)-\frac{\log\left(\sqrt[3]{2}\,(1-x^3)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(1 - x^3)^(2/3)/x^3 - (2*ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3
]])/(3*Sqrt[3]) + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^
(1/3)*Sqrt[3]) - (2*Log[-1 + (1 - x^3)^(1/3)])/9 + Log[-2 + 2^(2/3)*(1 - x^
3)^(1/3)]/(3*2^(1/3)) + Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/9 - Log[
2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas** [A] time = 0.48, size = 187, normalized size = 1.19

$$\frac{6\sqrt{6}\,2^{\frac{1}{6}}x^3\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}x^3\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+6\cdot 2^{\frac{2}{3}}x^3\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-8\sqrt{3}\,x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+4x^3\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-8x^3\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)-12\left(-x^3+1\right)^{\frac{2}{3}}}{36x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*(6*sqrt(6)*2^(1/6)*x^3*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)
*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*x^3*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3)
+ (-x^3 + 1)^(2/3)) + 6*2^(2/3)*x^3*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 8*sq
rt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 4*x^3*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*x^3*log((-x^3 + 1)^(1/3) - 1) -
12*(-x^3 + 1)^(2/3))/x^3
```

**giac** [A] time = 0.19, size = 163, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3x^3}+\frac{1}{9}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 2/9*sqrt(3)*arctan
(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/3*(-x^3 + 1)^(2/3)/x^3 + 1/9*log
((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 2/9*log(abs((-x^3 + 1)^(1/3) -
1))
```

**maple** [F] time = 2.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{3}+1\right)^{\frac{1}{3}} \left(x^{3}+1\right) x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)
```

**mupad [B]** time = 4.86, size = 382, normalized size = 2.43

$$\frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(81\,2^{1/3}-75\,(1-x^3)^{1/3}\right)}{6}-\frac{38}{3}\right)}{18}+\frac{16\,(1-x^3)^{1/3}}{27}\right)}{6}-\frac{(1-x^3)^{2/3}}{3\,x^3}-\frac{2\ln\left(\frac{344\,(1-x^3)^{1/3}}{243}-\frac{344}{243}\right)}{9}+\ln\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)\left(1458\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2-75\,(1-x^3)^{1/3}\right)-\frac{38}{3}\right)+\frac{16\,(1-x^3)^{1/3}}{27}\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)-\ln\left(\frac{16\,(1-x^3)^{1/3}}{27}-\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2\left(\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)\left(1458\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)^2-75\,(1-x^3)^{1/3}\right)+\frac{38}{3}\right)\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1i}{9}\right)+\frac{2^{2/3}\ln\left(\frac{16\,(1-x^3)^{1/3}}{27}+\frac{2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2\left(\frac{2^{2/3}\left(-1+\sqrt{3}\,1i\right)\left(\frac{81\,2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2}{4}-75\,(1-x^3)^{1/3}\right)}{12}-\frac{38}{3}\right)}{72}\right)\left(-1+\sqrt{3}\,1i\right)}{12}-\frac{2^{2/3}\ln\left(\frac{16\,(1-x^3)^{1/3}}{27}-\frac{2^{1/3}\left(1+\sqrt{3}\,1i\right)^2\left(\frac{2^{2/3}\left(1+\sqrt{3}\,1i\right)\left(\frac{81\,2^{1/3}\left(1+\sqrt{3}\,1i\right)^2}{4}-75\,(1-x^3)^{1/3}\right)}{12}+\frac{38}{3}\right)}{72}\right)\left(1+\sqrt{3}\,1i\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((2^(1/3)*((2^(2/3)*(81*2^(1/3) - 75*(1 - x^3)^(1/3)))/6 - 38/3
))/18 + (16*(1 - x^3)^(1/3))/27))/6 - (1 - x^3)^(2/3)/(3*x^3) - (2*log((344
*(1 - x^3)^(1/3))/243 - 344/243))/9 + log(((3^(1/2)*1i)/9 + 1/9)^2*(((3^(1/
2)*1i)/9 + 1/9)*(1458*((3^(1/2)*1i)/9 + 1/9)^2 - 75*(1 - x^3)^(1/3)) - 38/3
) + (16*(1 - x^3)^(1/3))/27)*((3^(1/2)*1i)/9 + 1/9) - log((16*(1 - x^3)^(1/
3))/27 - ((3^(1/2)*1i)/9 - 1/9)^2*(((3^(1/2)*1i)/9 - 1/9)*(1458*((3^(1/2)*1
i)/9 - 1/9)^2 - 75*(1 - x^3)^(1/3)) + 38/3))*((3^(1/2)*1i)/9 - 1/9) + (2^(2
/3)*log((16*(1 - x^3)^(1/3))/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^
(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2)*1i - 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12
- 38/3))/72)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((16*(1 - x^3)^(1/3))/27 -
(2^(1/3)*(3^(1/2)*1i + 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2
)*1i + 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12 + 38/3))/72)*(3^(1/2)*1i + 1))/12
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.380 $\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=154

$$-\frac{1}{3}\left(1-x^3\right)^{2/3}x-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\cdots}{3\cdots}}{\cdots}\right)}{\sqrt[3]{2}\sqrt{3\cdots}}$$

**Rubi [A]** time = 0.15, antiderivative size = 226, normalized size of antiderivative = 1.47, number of steps used = 15, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {494, 470, 522, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{9}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{2}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(x*(1 - x^3)^(2/3))/3 + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3
*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sq
rt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/9 - (2*Log[1 + x/
(1 - x^3)^(1/3)])/9 - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(
1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^6}{\left(1+x^3\right)^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1-x^3}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1\ldots}}\right.$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x\ldots\right.$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{9}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2\ldots}\right.$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{1}{9}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(\ldots\right.}{\ldots}$$

$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{9}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \ldots\right.$$

**Mathematica** [C] time = 0.16, size = 144, normalized size = 0.94

$$\frac{1}{36}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)-12\left(1-x^3\right)^{2/3}x+2^{2/3}\left(2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-12*x*(1 - x^3)^(2/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^(2
/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - Log[
1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] + 2*Log[
1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/36
```

**IntegrateAlgebraic** [A] time = 0.54, size = 230, normalized size = 1.49

$$-\frac{1}{3}\left(1-x^3\right)^{2/3}x-\frac{2}{9}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{1}{9}\log\left(-\sqrt[3]{1-x^3}x+\left(1-x^3\right)^{2/3}+x^2\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(x*(1 - x^3)^(2/3)) - (2*ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))])
/(3*Sqrt[3]) + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(1/3)*
Sqrt[3]) - (2*Log[x + (1 - x^3)^(1/3)])/9 + Log[2*x + 2^(2/3)*(1 - x^3)^(1/
3)]/(3*2^(1/3)) + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/9 - Log[-2
*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas** [A] time = 0.47, size = 201, normalized size = 1.31

$$-\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x-2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)+\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)-\frac{2}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{9}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(2/3)*x - 1/6*sqrt(6)*2^(1/6)*arctan(-1/6*2^(1/6)*(sqrt(6)*
2^(1/3)*x - 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) + 1/6*2^(2/3)*log((2^(1/3)*x + (
-x^3 + 1)^(1/3))/x) - 1/12*2^(2/3)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1
/3)*x + (-x^3 + 1)^(2/3))/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqr
t(3)*(-x^3 + 1)^(1/3))/x) - 2/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/9*log((x^
2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**maple [F]** time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.381 $\int \frac{x^3}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=135

$$\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.53, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {494, 481, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^2/(1 - x^3)
^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3 + Log[1 + (2^(
2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[
1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{x^3}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Sub}\\
&= \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) -\\
&= -\frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}} - \right.}{6\sqrt[3]{2}}\\
&= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\left(\right.
\end{aligned}
$$

**Mathematica [C]** time = 0.02, size = 26, normalized size = 0.19

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])/4
```

**IntegrateAlgebraic [A]** time = 0.37, size = 212, normalized size = 1.57

$$\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}x+\left(1-x^3\right)^{2/3}+x^2\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))]/Sqrt[3] - ArcTan[(Sqrt[3]*x)/(
-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]
/3 - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[x^2 - x*(1 - x^3)
^(1/3) + (1 - x^3)^(2/3)]/6 + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1
/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas [C]** time = 1.43, size = 452, normalized size = 3.35

$$\frac{1}{12}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(-\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}-6\cdot 2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+8x-24\left(-x^3+1\right)^{\frac{1}{3}}}{8x}\right)-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(-\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}-8\left(-x^3+1\right)^{\frac{1}{3}}\right)}{8x}\right)-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}\right)\log\left(\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{2}+8\left(-x^3+1\right)^{\frac{1}{3}}\right)}{8x}\right)-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)+\frac{1}{3}\log\left(\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^{3}+32x+24\left(-x^3+1\right)^{\frac{1}{3}}}{24x}\right)-\frac{1}{6}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-1
)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-
1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(
1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)*(-1)^
(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(
-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(
1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(1/3)*x*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/3*sqrt(3)*a
rctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*log(1/24*(x*(I
*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/3))/x) - 1/6*
log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**maple [F]** time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(1-x^3\right)^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.382 $\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=88

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-\ldots}}\right.\\
&= \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}\ldots}{1-\sqrt[3]{2}\ldots}\right.}{\ldots}\\
&= -\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}\\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 112, normalized size = 1.27

$$\frac{2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}+1\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + (
2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^
(1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.27, size = 126, normalized size = 1.43

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) + Log[
2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-2*x^2 + 2^(2/3)*x*(1 - x^
3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas [B]** time = 2.60, size = 253, normalized size = 2.88

$$-\frac{1}{18}\,\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\,\sqrt{6}\,2^{\frac{2}{3}}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71\,x^{9}-111\,x^{6}+33\,x^{3}-1\right)+12\,\sqrt{6}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right)+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}+2^{\frac{2}{3}}\left(x^{3}+1\right)+6\left(-x^{3}+1\right)^{\frac{2}{3}}x}{x^{3}+1}\right)-\frac{1}{36}\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(5\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(19\,x^{6}-16\,x^{3}+1\right)-12\left(2\,x^{5}-x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{6}+2\,x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**maple [C]** time = 4.94, size = 931, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(
2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*
x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3
+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^
(2/3)-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2))/(x+1)/(x^2-x+1))-1/6*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)
^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^
3-4)-ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf
(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+
1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*
```

```
_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

**3.383** $\int \frac{1}{x^3 \sqrt[3]{1-x^3}\left(1+x^3\right)} dx$

**Optimal**. Leaf size=105

$$\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{2x^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{2/3}}{2x^2}+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(2*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
```

```
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{1+x^3}{x^3\left(1+2x^3\right)}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,d
\end{aligned}$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

**Mathematica [C]** time = 0.60, size = 82, normalized size = 0.78

$$-\frac{\left(-6x^6+4x^3+2\right)\,_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2x^3}{x^3-1}\right)-3x^3\left(x^3+1\right)\,_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{2x^3}{x^3-1}\right)}{4x^2\left(1-x^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/4*((2 + 4*x^3 - 6*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)
] - 3*x^3*(1 + x^3)*Hypergeometric2F1[4/3, 2, 7/3, (2*x^3)/(-1 + x^3)])/(x^
2*(1 - x^3)^(4/3))
```

**IntegrateAlgebraic [A]** time = 0.29, size = 145, normalized size = 1.38

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{2x^2}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3)/x^2 - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3)
)]/(2^(1/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log
[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(1/3))
```

**fricas [B]** time = 2.58, size = 307, normalized size = 2.92

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^2\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^2\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3+1\right)+6\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^2\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(5x^4-x)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(19x^6-16x^3+1\right)+12\left(2x^5-x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+18\left(-x^3+1\right)^{\frac{2}{3}}}{36x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^2*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/
3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3)*
(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^
6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1/3)*x^
2*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x^3
+ 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*x^2*log(-(3*2^
(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6
- 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 18*
(-x^3 + 1)^(2/3))/x^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

**maple [C]** time = 4.49, size = 929, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/2*(x^3-1)/x^2/(-x^3+1)^(1/3)-1/6*ln(-(-36*RootOf(RootOf(_Z^3+4)^2+6*_Z*Ro
otOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+3*RootOf(RootOf(_Z^3+4)^2+6*_Z
*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-30*(-x^3+1)^(1
/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2
-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
```

```
(_Z^3+4)+36*_Z^2)*x^3-3*RootOf(_Z^3+4)*x^3-10*(-x^3+1)^(2/3)*x-12*RootOf(Ro
otOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(_Z^3+4))/(x+1)/(x^2-x+1)
)*RootOf(_Z^3+4)-ln(-(-36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)^2*RootOf(_Z^3+4)^2*x^3+3*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36
*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*
RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-30*(-x^3+1)^(1/3)*RootOf(_Z^3+4)
*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2-(-x^3+1)^(1/3)*Ro
otOf(_Z^3+4)^2*x^2+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*
x^3-3*RootOf(_Z^3+4)*x^3-10*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z
^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+1/6*RootOf(_Z^3+4)*ln((-54*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-3*RootOf(R
ootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+
1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4
)^2*x+6*(-x^3+1)^(1/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*x^2+5*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2-18*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-RootOf(_Z^3+4)*x^3+2*(-x^3+1)^(2
/3)*x+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(_Z^3+4
))/(x+1)/(x^2-x+1))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.384 $\int \frac{1}{x^6 \sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=124

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{5x^5}+\frac{\left(1-x^3\right)^{2/3}}{5x^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(5/3)/(5*x^5) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^3\right)^2}{x^6\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^6}+\frac{1}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}+\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2 \cdots}\right.\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,d \cdots\right.}{\sqrt[3]{2} \cdots}\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}
\end{aligned}
$$

**Mathematica [A]** time = 5.25, size = 123, normalized size = 0.99

$$
\frac{2\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)}{6\sqrt[3]{2}}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/5*(1 - x^3)^(5/3)/x^5 + (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)
^(1/3))/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1
 + x^3)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/(6*2^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.31, size = 149, normalized size = 1.20

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}\left(x^3-1\right)}{5x^5}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-1 + x^3))/(5*x^5) + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1
- x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^
(1/3)) - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/
(6*2^(1/3))
```

**fricas [B]** time = 2.55, size = 283, normalized size = 2.28

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}x^5\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-10\cdot 2^{\frac{2}{3}}x^5\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}\left(x^3+1\right)+6\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\right)+5\cdot 2^{\frac{2}{3}}x^5\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(5x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(19x^6-16x^3+1\right)-12\left(2x^5-x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-36\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{180x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/180*(10*sqrt(6)*2^(1/6)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7
 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3
 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105
*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2
^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*x^5*log((3*
2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(
2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(x^3 - 1)*(-x^3 + 1)
^(2/3))/x^5
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

**maple [C]** time = 2.75, size = 955, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] -1/5*(x^6-2*x^3+1)/x^5/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
```

```
tOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x
^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*R
ootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*x^3+10*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-6*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*ln(-(6*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12
*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf
(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z
^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^
3+1)^(2/3)*x+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln(-(6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x+2*RootO
f(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^
2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.385 $\int \frac{1}{x^9 \sqrt[3]{1-x^3} (1+x^3)} dx$

**Optimal**. Leaf size=141

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3} - \sqrt[3]{2}x\right)}{2\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{(1-x^3)^{2/3}}{8x^8} + \frac{(1-x^3)^{2/3}}{20x^5} - \frac{17(1-x^3)^{2/3}}{40x^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{(1-x^3)^{8/3}}{8x^8} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{2/3}}{2x^2} + \frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(2*x^2) - (1 - x^3)^(5/3)/(5*x^5) - (1 - x^3)^(8/3)/(8*x^8
) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^
(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
```

```
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{\left(1+x^3\right)^3}{x^9\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{x^9}+\frac{1}{x^6}+\frac{1}{x^3}+\frac{1}{-1-2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{\left(1-x^3\right)^{5/3}}{5x^5} - \frac{\left(1-x^3\right)^{8/3}}{8x^8} + \text{Subst}\left(\int \frac{1}{-1-2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{\left(1-x^3\right)^{5/3}}{5x^5} - \frac{\left(1-x^3\right)^{8/3}}{8x^8} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{\left(1-x^3\right)^{5/3}}{5x^5} - \frac{\left(1-x^3\right)^{8/3}}{8x^8} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{}{1-\sqrt[3]{2}}\right.$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{\left(1-x^3\right)^{5/3}}{5x^5} - \frac{\left(1-x^3\right)^{8/3}}{8x^8} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\right.}{3\sqrt[3]{2}}$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{\left(1-x^3\right)^{5/3}}{5x^5} - \frac{\left(1-x^3\right)^{8/3}}{8x^8} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\right.}{6\sqrt[3]{2}}$$

**Mathematica [A]** time = 5.11, size = 133, normalized size = 0.94

$$\frac{-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)+\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)}{6\sqrt[3]{2}} - \frac{\left(1-x^3\right)^{2/3}\left(17x^6-2x^3+5\right)}{40x^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/40*((1 - x^3)^(2/3)*(5 - 2*x^3 + 17*x^6))/x^8 + (-2*Sqrt[3]*ArcTan[(-1 +
 (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Log[1 + (2^(2/3)*x^2)/(-1 + x^3
)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^
(1/3)])/(6*2^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.30, size = 157, normalized size = 1.11

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\sqrt[3]{2}}+\frac{\left(1-x^3\right)^{2/3}\left(-17x^6+2x^3-5\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-5 + 2*x^3 - 17*x^6))/(40*x^8) - ArcTan[(Sqrt[3]*x)/(-x +
 2^(2/3)*(1 - x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^
(1/3)]/(3*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 -
x^3)^(2/3)]/(6*2^(1/3))
```

**fricas [B]** time = 2.61, size = 320, normalized size = 2.27

$$-\frac{20\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^8\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-20\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^8\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3+1\right)+6\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\right)+10\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^8\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(5x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(19x^6-16x^3+1\right)+12\left(2x^5-x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+9\left(17x^6-2x^3+5\right)\left(-x^3+1\right)^{\frac{2}{3}}}{360x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(20*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^8*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(
2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3
)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*
x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 20*2^(2/3)*(-1)^(1/3)
*x^8*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x
^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 10*2^(2/3)*(-1)^(1/3)*x^8*log(
-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(1
9*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
 + 9*(17*x^6 - 2*x^3 + 5)*(-x^3 + 1)^(2/3))/x^8
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

**maple [C]** time = 4.07, size = 963, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/40*(17*x^9-19*x^6+7*x^3-5)/x^8/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)*ln(-(-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3
+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+24*(-x^3+1)^(1/3)*RootO
```

```
f(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2+5*(-x^3+
1)^(1/3)*RootOf(_Z^3+4)^2*x^2+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4
)+36*_Z^2)*x^3+9*RootOf(_Z^3+4)*x^3-10*(-x^3+1)^(2/3)*x-6*RootOf(RootOf(_Z^
3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-3*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))-1/6*
ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4
)^2*x^3-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+
4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*RootOf(_Z^3+4)^2*x+24*(-x^3+1)^(1/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+
6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)
*x^3+2*(-x^3+1)^(2/3)*x-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)+2*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-ln((18*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-6*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^
(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2
*x+24*(-x^3+1)^(1/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2)*x^2-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+6*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^3+2*(-x^3+1)^(2/3)
*x-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+2*RootOf(_Z^3+4))
/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9\,{\left(1-x^3\right)}^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9}\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.386 $\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi** [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx=\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

**Mathematica** [C] time = 0.02, size = 26, normalized size = 0.11

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

**IntegrateAlgebraic** [A] time = 1.08, size = 340, normalized size = 1.46

$$-\frac{\log\left(-\sqrt[3]{1-x^3}+\sqrt[3]{2}x-\sqrt[3]{2}\right)}{3\sqrt[3]{2}}-\frac{\log\left(2\sqrt[3]{1-x^3}+\sqrt[3]{2}x-\sqrt[3]{2}\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{1-x^3}-\sqrt[3]{2}x+\sqrt[3]{2}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{1-x^3}+2\sqrt[3]{2}x-2\sqrt[3]{2}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(\left(1-x^3\right)^{2/3}+\left(\sqrt[3]{2}x-\sqrt[3]{2}\right)\sqrt[3]{1-x^3}+2^{2/3}x^2-2\,2^{2/3}x+2^{2/3}\right)}{6\sqrt[3]{2}}+\frac{\log\left(4\left(1-x^3\right)^{2/3}+\left(2\sqrt[3]{2}-2\sqrt[3]{2}x\right)\sqrt[3]{1-x^3}+2^{2/3}x^2-2\,2^{2/3}x+2^{2/3}\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^3)^(1/3
))]/(2^(1/3)*Sqrt[3]) - ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^
```

```
(1/3)*x + (1 - x^3)^(1/3))]/(2^(1/3)*Sqrt[3]) - Log[-2^(1/3) + 2^(1/3)*x -
(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)
]/(6*2^(1/3)) + Log[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-2^(1/3) + 2^(1/
3)*x)*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/(6*2^(1/3)) + Log[2^(2/3) - 2*2^(2
/3)*x + 2^(2/3)*x^2 + (2*2^(1/3) - 2*2^(1/3)*x)*(1 - x^3)^(1/3) + 4*(1 - x^
3)^(2/3)]/(12*2^(1/3))
```

**fricas [B]** time = 2.32, size = 373, normalized size = 1.60

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+\sqrt{6}\,2^{\frac{1}{3}}\left(x^{18}+42x^{15}-417x^{12}+812x^{9}-417x^{6}+42x^{3}+1\right)\right)}{6\left(x^{18}-102x^{15}+447x^{12}-628x^{9}+447x^{6}-102x^{3}+1\right)}\right)-\frac{1}{72}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{8}-4x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{12}-32x^{9}+78x^{6}-32x^{3}+1\right)-6\left(x^{10}-11x^{7}+11x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{12}+4x^{9}+6x^{6}+4x^{3}+1}\right)+\frac{1}{36}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\left(-x^{3}+1\right)^{\frac{2}{3}}x^{2}-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{6}+2x^{3}+1\right)}{x^{6}+2x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**maple [F]** time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^{3}+1\right)^{\frac{1}{3}}\left(x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.387 $\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=125

$$-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 57, 617, 204, 31}

$$-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + (1 - x^3)^(4/3)/4 - (1 - x^3)^(7/3)/7 + ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3))
- Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{(1-x)^{2/3}} - \sqrt[3]{1-x} + (1-x)^{4/3} - \frac{1}{(1-x)^{2/3}(1+x)}\right)dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-\right.}{2\ 2^{2/3}}$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Su}}{}$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log}{}$$

**Mathematica [A]** time = 0.11, size = 151, normalized size = 1.21

$$\frac{1}{84}\left(3\sqrt[3]{1-x^3}\,x^3 - 75\sqrt[3]{1-x^3} - 14\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + 7\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 14\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right) - 12\sqrt[3]{1-x^3}\,x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-75*(1 - x^3)^(1/3) + 3*x^3*(1 - x^3)^(1/3) - 12*x^6*(1 - x^3)^(1/3) + 14*
2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 14*2^(1/3)*
Log[2^(1/3) - (1 - x^3)^(1/3)] + 7*2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3)
+ (1 - x^3)^(2/3)])/84
```

**IntegrateAlgebraic [A]** time = 0.14, size = 141, normalized size = 1.13

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\ 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{28}\sqrt[3]{1-x^3}\left(-4x^6+x^3-25\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(1/3)*(-25 + x^3 - 4*x^6))/28 + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 -
 x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]
/(3*2^(2/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(
6*2^(2/3))
```

**fricas [A]** time = 0.45, size = 142, normalized size = 1.14

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + 2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}} + 2\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}} + 2\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{28}\left(4x^6-x^3+25\right)\left(-x^3+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/28*(4*x^6 - x^3 + 25)*(-x^3 + 1)^(1/3)
```

**giac [A]** time = 0.18, size = 127, normalized size = 1.02

$$-\frac{1}{7}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/7*(x^3 - 1)^2*(-x^3 + 1)^(1/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

**maple [C]** time = 7.45, size = 1579, normalized size = 12.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/28*(4*x^6-x^3+25)*(x^3-1)/(-x^3+1)^(2/3)+(1/6*RootOf(_Z^3+2)*ln((36*RootO
f(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6-24*R
ootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-36
*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^
3+24*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*
x^3+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6-4*RootOf(_Z^
3+2)*x^6-144*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2
)+36*_Z^2)*RootOf(_Z^3+2)*x^3-9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90
*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
Z^3+2)+36*_Z^2)-48*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^3
+32*RootOf(_Z^3+2)*x^3+144*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z
*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1
/3)-48*(x^6-2*x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+3
6*_Z^2)-28*RootOf(_Z^3+2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))-1/6*ln(-(90*Roo
tOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+12
*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-
90*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*
x^3-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^
3*x^3-45*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6-6*RootOf(
_Z^3+2)*x^6+27*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3
+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+12*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3
+45*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootO
f(_Z^3+2)+36*_Z^2)+150*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)
*x^3+20*RootOf(_Z^3+2)*x^3-27*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6
*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-12*RootOf(_Z^3+2)^2*(x^6-2*x^3+1
)^(1/3)+9*(x^6-2*x^3+1)^(2/3)-105*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+
2)+36*_Z^2)-14*RootOf(_Z^3+2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(_Z^3
+2)-ln(-(90*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_
Z^3+2)^2*x^6+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf
(_Z^3+2)^3*x^6-90*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*Ro
otOf(_Z^3+2)^2*x^3-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*
RootOf(_Z^3+2)^3*x^3-45*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2
)*x^6-6*RootOf(_Z^3+2)*x^6+27*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6
```

```
*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+12*RootOf(_Z^3+2)^2*(x^6-2*x
^3+1)^(1/3)*x^3+45*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+150*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
Z^3+2)+36*_Z^2)*x^3+20*RootOf(_Z^3+2)*x^3-27*(x^6-2*x^3+1)^(1/3)*RootOf(Roo
tOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-12*RootOf(_Z^3+2)
^2*(x^6-2*x^3+1)^(1/3)+9*(x^6-2*x^3+1)^(2/3)-105*RootOf(RootOf(_Z^3+2)^2+6*
_Z*RootOf(_Z^3+2)+36*_Z^2)-14*RootOf(_Z^3+2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+
1))*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2))/(-x^3+1)^(2/3)*((
x^3-1)^2)^(1/3)
```

**maxima** [A] time = 1.52, size = 119, normalized size = 0.95

$$-\frac{1}{7}\left(-x^3+1\right)^{\frac{7}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/7*(-x^3 + 1)^(7/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

**mupad** [B] time = 5.18, size = 135, normalized size = 1.08

$$\frac{\left(1-x^3\right)^{4/3}}{4}-\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\ln\left(3\,2^{1/3}-3\left(1-x^3\right)^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{7/3}}{7}-\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}+\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(4/3)/4 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^
(1/3)))/6 - (1 - x^3)^(7/3)/7 - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)
*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 + (2^(1/3)*log((3*2^(1/3)*(3^(1/
2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

## 3.388 $\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=98

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 57, 617, 204, 31}

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(4/3)/4 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(1-x)^{2/3}(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\sqrt[3]{1-x}+\frac{1}{(1-x)^{2/3}(1+x)}\right)dx,x,x^3\right)\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x}\right.}{2\sqrt[3]{2}}\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\right.}{2^{2/3}}\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}
\end{aligned}
$$

**Mathematica [A]** time = 0.09, size = 135, normalized size = 1.38

$$\frac{1}{12}\left(-3\sqrt[3]{1-x^3}\,x^3+3\sqrt[3]{1-x^3}+2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(1/3) - 3*x^3*(1 - x^3)^(1/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
] - 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12
```

**IntegrateAlgebraic [A]** time = 0.12, size = 132, normalized size = 1.35

$$\frac{1}{4}\left(1-x^3\right)^{4/3}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(4/3)/4 - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(
2^(2/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas [A]** time = 0.48, size = 114, normalized size = 1.16

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{4}\left(x^3-1\right)\left(-x^3+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1
/4*(x^3 - 1)*(-x^3 + 1)^(1/3)
```

**giac [A]** time = 0.18, size = 98, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^{3}+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3
)))
```

**maple [C]** time = 7.46, size = 1584, normalized size = 16.16

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/4*(x^3-1)^2/(-x^3+1)^(2/3)+(RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+3
6*_Z^2)*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Root
Of(_Z^3-2)^2*x^6-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*Roo
tOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^
2*RootOf(_Z^3-2)^2*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^
2)*RootOf(_Z^3-2)^3*x^3+72*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_
Z^2)*x^6-3*RootOf(_Z^3-2)*x^6-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^
2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+9*RootOf(_Z^3-2)^2*(x^6-2
*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^
3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-240*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)*x^3+10*RootOf(_Z^3-2)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(R
ootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)-9*RootOf(_Z^3-2
)^2*(x^6-2*x^3+1)^(1/3)-18*(x^6-2*x^3+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+
6*_Z*RootOf(_Z^3-2)+36*_Z^2)-7*RootOf(_Z^3-2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x
+1))-1/6*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Roo
tOf(_Z^3-2)^2*x^6+30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*R
ootOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2
)^2*RootOf(_Z^3-2)^2*x^3-30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2)*RootOf(_Z^3-2)^3*x^3-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+3
6*_Z^2)*x^6-5*RootOf(_Z^3-2)*x^6+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+24*RootOf(_Z^3-2)^2*(x
^6-2*x^3+1)^(1/3)*x^3-90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+192*RootOf(RootOf(_Z^3-2)^2+6*_Z*Ro
otOf(_Z^3-2)+36*_Z^2)*x^3+40*RootOf(_Z^3-2)*x^3-90*(x^6-2*x^3+1)^(1/3)*Root
Of(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)-24*RootOf(_
Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-48*(x^6-2*x^3+1)^(2/3)-168*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-35*RootOf(_Z^3-2))/(x-1)/(x+1)/(x^2+x+1)/
(x^2-x+1))*RootOf(_Z^3-2)-ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-
2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_
Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootO
f(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-30*RootOf(RootOf(_Z^3-2)^2+6*_Z*R
ootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3-24*RootOf(RootOf(_Z^3-2)^2+6*_Z
*RootOf(_Z^3-2)+36*_Z^2)*x^6-5*RootOf(_Z^3-2)*x^6+90*(x^6-2*x^3+1)^(1/3)*Ro
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+24*Ro
otOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^3-90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2
```

```
)^2*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+192*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+40*RootOf(_Z^3-2)*x^3-90*(x^6-2*
x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)-24*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-48*(x^6-2*x^3+1)^(2/3)-168*Ro
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-35*RootOf(_Z^3-2))/(x-1)
/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

**maxima [A]** time = 1.19, size = 97, normalized size = 0.99

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**mupad [B]** time = 4.98, size = 113, normalized size = 1.15

$$\frac{2^{1/3}\ln\left(\frac{(1-x^3)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}+\frac{\left(1-x^3\right)^{4/3}}{4}+\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((1 - x^3)^(1/3)/2 - 2^(1/3)/2))/6 + (1 - x^3)^(4/3)/4 + (2^(1/
3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/12 - (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^
(1/2)*1i + 1))/12
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.389 $\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=95

$$-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi** [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 57, 617, 204, 31}

$$-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*S
qrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\sqrt[3]{1-x^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\sqrt[3]{1-x^3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,d\right.}{2\sqrt[3]{2}}\\
&= -\sqrt[3]{1-x^3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-\right.}{2^{2/3}}\\
&= -\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}
\end{aligned}
$$

**Mathematica** [A] time = 0.05, size = 118, normalized size = 1.24

$$
\frac{1}{12}\left(-12\sqrt[3]{1-x^3} - 2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + \sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3
) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12
```

**IntegrateAlgebraic** [A] time = 0.10, size = 129, normalized size = 1.36

$$
-\sqrt[3]{1-x^3} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3} + 2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\ 2^{2/3}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2
^(2/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[2 + 2
^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas** [A] time = 0.46, size = 130, normalized size = 1.37

$$
-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + 2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}} + 2\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}} + 2\left(-x^3+1\right)^{\frac{1}{3}}\right) - \left(-x^3+1\right)^{\frac{1}{3}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 (-x^3 + 1)^(1/3)
```

**giac [A]** time = 0.23, size = 98, normalized size = 1.03

$$\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right|\right)-\left(-x^{3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

**maple [C]** time = 5.10, size = 1582, normalized size = 16.65

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] (x^3-1)/(-x^3+1)^(2/3)+(-1/6*ln((30*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6+144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(
_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6-30*RootOf(RootOf(_Z^3+2)^2+6*_Z*Roo
tOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-144*RootOf(RootOf(_Z^3+2)^2+6*_Z*
RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+5*RootOf(_Z^3+2)*x^6+24*Root
Of(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6+24*RootOf(_Z^3+2)^2*(x
^6-2*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*R
ootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^
3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)-40*RootOf(_Z^3+
2)*x^3-192*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^3-24*Root
Of(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-48*(x^6-2*x^3+1)^(2/3)+35*
RootOf(_Z^3+2)+168*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2))/(x
-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(_Z^3+2)-ln((30*RootOf(RootOf(_Z^3+2)^
2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6+144*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6-30*RootOf(RootOf(_
Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-144*RootOf(RootO
f(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+5*RootOf(_Z
^3+2)*x^6+24*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6+24*Ro
otOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf
(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+90*(x^6-2*x^3+1)
^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2
)-40*RootOf(_Z^3+2)*x^3-192*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*
_Z^2)*x^3-24*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)-90*(x^6-2*x^3+1)^(1/3)*Ro
otOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-48*(x^6-2
*x^3+1)^(2/3)+35*RootOf(_Z^3+2)+168*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+2)^2+6*_
Z*RootOf(_Z^3+2)+36*_Z^2)+RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z
^2)*ln(-(6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3
+2)^3*x^6-144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf
(_Z^3+2)^2*x^6-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootO
f(_Z^3+2)^3*x^3+144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*
RootOf(_Z^3+2)^2*x^3-3*RootOf(_Z^3+2)*x^6+72*RootOf(RootOf(_Z^3+2)^2+6*_Z*R
ootOf(_Z^3+2)+36*_Z^2)*x^6-9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90*(x
^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*Root
```

```
Of(_Z^3+2)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2
)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+10*RootOf(_Z^3+2)*x^3-240*RootOf(RootOf(_Z
^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^3+9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(
1/3)-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*
_Z^2)*RootOf(_Z^3+2)+18*(x^6-2*x^3+1)^(2/3)-7*RootOf(_Z^3+2)+168*RootOf(Roo
tOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1)
))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

**maxima [A]** time = 1.38, size = 97, normalized size = 1.02

$$\frac{1}{6}\,\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)-\left(-x^{3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

**mupad [B]** time = 4.89, size = 113, normalized size = 1.19

$$-\frac{2^{1/3}\,\ln\left(\frac{{\left(1-x^3\right)}^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}-{\left(1-x^3\right)}^{1/3}-\frac{2^{1/3}\,\ln\left(3\,{\left(1-x^3\right)}^{1/3}-\frac{3\,2^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}+\frac{2^{1/3}\,\ln\left(\frac{3\,2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+3\,{\left(1-x^3\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1
i + 1))/12 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*
(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log((1 - x^3)^(1/3)/2
- 2^(1/3)/2))/6
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(- \left(x - 1\right) \left(x^{2} + x + 1\right)\right)^{\frac{2}{3}} \left(x + 1\right) \left(x^{2} - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.390 $\int \frac{x^2}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=83

$$-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 57, 617, 204, 31}

$$-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) - Log[1
+ x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}
\end{aligned}
$$

**Mathematica** [A] time = 0.03, size = 94, normalized size = 1.13

$$
-\frac{-2\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[2^(1/
3) - (1 - x^3)^(1/3)] + Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])
/2^(2/3)
```

**IntegrateAlgebraic** [A] time = 0.10, size = 117, normalized size = 1.41

$$
\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\ 2^{2/3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]))
+ Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[2 + 2^(2/3)*(1 - x^3)
^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas** [A] time = 0.49, size = 98, normalized size = 1.18

$$
-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
```

**giac** [A] time = 0.20, size = 87, normalized size = 1.05

$$
-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

**maple** [C] time = 6.28, size = 529, normalized size = 6.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*ln((144*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3-6*RootOf(Root
Of(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3-24*RootOf(Ro
otOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+RootOf(_Z^3-
2)^2*x^3+168*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)-252*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(-x^3+1)^(1/
3)-7*RootOf(_Z^3-2)^2-42*RootOf(_Z^3-2)*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3))/(
x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-2)*ln(-(180*RootOf(RootOf(_Z^3-2)^2+6*_Z*Ro
otOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z
*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3+90*RootOf(RootOf(_Z^3-2)^2+6*
_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+3*RootOf(_Z^3-2)^2*x^3-210*Ro
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)+252*RootO
f(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(-x^3+1)^(1/3)-7*RootOf(_Z^
3-2)^2+42*RootOf(_Z^3-2)*(-x^3+1)^(1/3)-42*(-x^3+1)^(2/3))/(x+1)/(x^2-x+1))
```

**maxima** [A] time = 1.18, size = 86, normalized size = 1.04

$$-\frac{1}{6}\,\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**mupad** [B] time = 5.05, size = 102, normalized size = 1.23

$$\frac{2^{1/3}\ln\left(3\,2^{1/3}-3\left(1-x^3\right)^{1/3}\right)}{6}+\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^(1/3)))/6 + (2^(1/3)*log(3*(1 - x^3)^(
1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(
(3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.391 $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=137

$$\frac{\log(x^3+1)}{6\,2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}$$

**Rubi [A]** time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 57, 618, 204, 31, 617}

$$\frac{\log(x^3+1)}{6\,2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1
 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(2
/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+\right.\\
&= -\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\text{Subst}\left(\int\right.}{}\\
&= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{\right.}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 179, normalized size = 1.31

$$\frac{1}{12}\left(4\log\left(1-\sqrt[3]{1-x^3}\right)-2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)-2\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)-4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[1 - (1 - x^3)^(1/3)] - 2
*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)
^(1/3) + (1 - x^3)^(2/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/1
2
```

**IntegrateAlgebraic [A]** time = 0.19, size = 195, normalized size = 1.42

$$\frac{1}{3}\log\left(\sqrt[3]{1-x^3}-1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\ 2^{2/3}}-\frac{1}{6}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)+\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[1/Sqrt[
3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[-1 + (1 - x
^3)^(1/3)]/3 - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (1 -
 x^3)^(1/3) + (1 - x^3)^(2/3)]/6 + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3
)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas** [A] time = 0.48, size = 182, normalized size = 1.33

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((
-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

**giac** [A] time = 0.24, size = 149, normalized size = 1.09

$$\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 +
 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

**maple** [F] time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^3+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)
```

**mupad** [B] time = 4.92, size = 344, normalized size = 2.51

$$\frac{\ln\left(5-5\left(1-x^3\right)^{1/3}\right)}{3}-\frac{2^{1/3}\ln\left(6\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+243\left(1-x^3\right)^{1/3}\right)}{36}+9\right)}{6}\right)}{6}+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\left(243\left(1-x^3\right)^{1/3}+243-\sqrt{3}\,243\mathrm{i}\right)+9\right)+6\left(1-x^3\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(6\left(1-x^3\right)^{1/3}-\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\left(243\left(1-x^3\right)^{1/3}+243+\sqrt{3}\,243\mathrm{i}\right)+9\right)\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{(-1)^{1/3}\,2^{1/3}\ln\left(6\left(1-x^3\right)^{1/3}-\frac{(-1)^{1/3}\,2^{1/3}\left(\frac{(-1)^{2/3}\,2^{2/3}\left(243\,(-1)^{1/3}\,2^{1/3}-243\left(1-x^3\right)^{1/3}\right)}{36}-9\right)}{6}\right)}{6}-\frac{(-1)^{1/3}\,2^{1/3}\ln\left(6\left(1-x^3\right)^{1/3}-\frac{(-1)^{1/3}\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{(-1)^{2/3}\,2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\left(243\left(1-x^3\right)^{1/3}+\frac{243\,(-1)^{1/3}\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)}{144}+9\right)}{12}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] log(5 - 5*(1 - x^3)^(1/3))/3 - (2^(1/3)*log(6*(1 - x^3)^(1/3) - (2^(1/3)*((
2^(2/3)*(243*2^(1/3) + 243*(1 - x^3)^(1/3)))/36 + 9))/6))/6 + log(((3^(1/2)
```

```
*1i)/6 - 1/6)*(((3^(1/2)*1i)/6 - 1/6)^2*(243*(1 - x^3)^(1/3) - 3^(1/2)*243i
 + 243) + 9) + 6*(1 - x^3)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(6*(1 - x^3)^
(1/3) - ((3^(1/2)*1i)/6 + 1/6)*(((3^(1/2)*1i)/6 + 1/6)^2*(3^(1/2)*243i + 24
3*(1 - x^3)^(1/3) + 243) + 9))*((3^(1/2)*1i)/6 + 1/6) + ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(((-1)^(2/3)*2^(2/3)*(243*(-1)
^(1/3)*2^(1/3) - 243*(1 - x^3)^(1/3)))/36 - 9))/6))/6 - ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(((-1)^(2/3)*
2^(2/3)*(3^(1/2)*1i + 1)^2*(243*(1 - x^3)^(1/3) + (243*(-1)^(1/3)*2^(1/3)*(
3^(1/2)*1i + 1))/2))/144 + 9))/12)*(3^(1/2)*1i + 1))/12
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.392 $\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=158

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 57, 618, 204, 31, 617}

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3)/(3*x^3) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[
3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log
[x]/6 - Log[1 + x^3]/(6*2^(2/3)) - Log[1 - (1 - x^3)^(1/3)]/6 + Log[2^(1/3)
 - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}x^2(1+x)}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{\frac{1}{3}-\frac{2x}{3}}{(1-x)^{2/3}x(1+x)}\,dx,x,x^3\right)\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx,x,x^3\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,\right.\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\log(x)}{6}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^3}\right)+\frac{1}{6}\operatorname{Subs}\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\log(x)}{6}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x)}{6}-\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-
\end{aligned}
$$

**Mathematica [A]** time = 0.10, size = 196, normalized size = 1.24

$$\frac{1}{36}\left(-\frac{12\sqrt[3]{1-x^3}}{x^3}-4\log\left(1-\sqrt[3]{1-x^3}\right)+6\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-3\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)+2\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-6\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[
3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 4*L
og[1 - (1 - x^3)^(1/3)] + 6*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] - 3*2^(1
/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + 2*Log[1 + (1 - x^3
)^(1/3) + (1 - x^3)^(2/3)])/36
```

**IntegrateAlgebraic** [A] time = 0.25, size = 216, normalized size = 1.37

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{1}{9}\log\left(\sqrt[3]{1-x^3}-1\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{3\,2^{2/3}}+\frac{1}{18}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)-\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{6\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(1 - x^3)^(1/3)/x^3 + ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/
(3*Sqrt[3]) - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3
)*Sqrt[3]) - Log[-1 + (1 - x^3)^(1/3)]/9 + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)
]/(3*2^(2/3)) + Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/18 - Log[2 + 2^(
2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas** [A] time = 0.49, size = 195, normalized size = 1.23

$$-\frac{12\cdot 4^{\frac{1}{6}}\sqrt{3}x^3\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)+3\cdot 4^{\frac{2}{3}}x^3\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)-6\cdot 4^{\frac{2}{3}}x^3\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-8\sqrt{3}x^3\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-4x^3\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+8x^3\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)+24\left(-x^3+1\right)^{\frac{1}{3}}}{72x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(12*4^(1/6)*sqrt(3)*x^3*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1
)^(1/3) + 4^(1/3)*sqrt(3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3) +
2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 + 1)
^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3))
- 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3 + 1)
^(1/3) - 1) + 24*(-x^3 + 1)^(1/3))/x^3
```

**giac** [A] time = 0.22, size = 163, normalized size = 1.03

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3x^3}+\frac{1}{18}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{9}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/12*2^(1
/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/3*(-x^3 + 1)^(1/3)/x^3 + 1/18*l
og((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 1/9*log(abs((-x^3 + 1)^(1/3)
- 1))
```

**maple** [F] time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^3+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)

**mupad** [B] time = 5.07, size = 368, normalized size = 2.33

$$\frac{2^{1/3}\ln\left(\frac{10\left(1-x^3\right)^{1/3}}{9}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+27\left(1-x^3\right)^{1/3}\right)}{36}-\frac{25}{3}\right)}{6}\right)}{6}-\frac{\left(1-x^3\right)^{1/3}}{3\,x^3}-\frac{\ln\left(\frac{31\left(1-x^3\right)^{1/3}}{243}-\frac{31}{243}\right)}{9}-\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(27\left(1-x^3\right)^{1/3}+81-\sqrt{3}\,81\mathrm{i}\right)-\frac{25}{3}\right)+\frac{10\left(1-x^3\right)^{1/3}}{9}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)+\ln\left(\frac{10\left(1-x^3\right)^{1/3}}{9}-\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(27\left(1-x^3\right)^{1/3}+81+\sqrt{3}\,81\mathrm{i}\right)-\frac{25}{3}\right)\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)+\frac{2^{1/3}\ln\left(\frac{10\left(1-x^3\right)^{1/3}}{9}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\left(\frac{243\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}+27\left(1-x^3\right)^{1/3}\right)}{144}-\frac{25}{3}\right)}{12}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(\frac{10\left(1-x^3\right)^{1/3}}{9}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\left(\frac{243\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}-27\left(1-x^3\right)^{1/3}\right)}{144}+\frac{25}{3}\right)}{12}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*((2^(2/3)*(243*2^(1/3) + 27*(1 - x^3)^(1/3)))/36 - 25/3))/6))/6 - (1 - x^3)^(1/3)/(3*x^3) - log((31*(1 - x^3)^(1/3))/243 - 31/243)/9 - log(((3^(1/2)*1i)/18 - 1/18)*(((3^(1/2)*1i)/18 - 1/18)^2*(27*(1 - x^3)^(1/3) - 3^(1/2)*81i + 81) - 25/3) + (10*(1 - x^3)^(1/3))/9)*((3^(1/2)*1i)/18 - 1/18) + log((10*(1 - x^3)^(1/3))/9 - ((3^(1/2)*1i)/18 + 1/18)*(((3^(1/2)*1i)/18 + 1/18)^2*(3^(1/2)*81i + 27*(1 - x^3)^(1/3) + 81) - 25/3))*((3^(1/2)*1i)/18 + 1/18) + (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i - 1)^2*((243*2^(1/3)*(3^(1/2)*1i - 1))/2 + 27*(1 - x^3)^(1/3)))/144 - 25/3))/12)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i + 1)^2*((243*2^(1/3)*(3^(1/2)*1i + 1))/2 - 27*(1 - x^3)^(1/3)))/144 + 25/3))/12)*(3^(1/2)*1i + 1))/12

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(-\left(x-1\right) \left(x^{2}+x+1\right)\right)^{\frac{2}{3}} \left(x+1\right) \left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)

[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)

## 3.393 $\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=160

$$\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{1}{3}\sqrt[3]{1-x^3}$$

**Rubi [A]** time = 0.16, antiderivative size = 228, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {494, 470, 584, 634, 618, 204, 628, 292, 31, 617}

$$-\frac{1}{3}\sqrt[3]{1-x^3}\,x^2-\frac{1}{18}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(x^2*(1 - x^3)^(1/3))/3 + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*S
qrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt
[3]) - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/18 + Log[1 + x/(1 -
 x^3)^(1/3)]/9 + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x
^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^7}{\left(1+x^3\right)^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{x\left(2+x^3\right)}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int\left(\frac{1}{3(1+x)} + \frac{-1-x}{3\left(1-x+x^2\right)} + \frac{3x}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right.$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{-1-x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) +$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log}{}$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\log\Big($$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \right.$$

**Mathematica [C]** time = 0.09, size = 78, normalized size = 0.49

$$\frac{1}{15}x^2\left(x^3\left(-F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};x^3,-x^3\right)\right) + \frac{5\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{2x^3}{x^3+1}\right)}{\left(x^3+1\right)^{2/3}} - 5\sqrt[3]{1-x^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(-5*(1 - x^3)^(1/3) - x^3*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3] + (5*H
ypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)])/(1 + x^3)^(2/3)))/15
```

**IntegrateAlgebraic [A]** time = 0.54, size = 232, normalized size = 1.45

$$\frac{1}{9}\log\left(\sqrt[3]{1-x^3}+x\right) - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\,2^{2/3}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{3}\sqrt[3]{1-x^3}x^2 - \frac{1}{18}\log\left(-\sqrt[3]{1-x^3}x+\left(1-x^3\right)^{2/3}+x^2\right) + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(x^2*(1 - x^3)^(1/3)) - ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))]/(
3*Sqrt[3]) + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(2/3)*Sq
rt[3]) + Log[x + (1 - x^3)^(1/3)]/9 - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3
*2^(2/3)) - Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/18 + Log[-2*x^2
+ 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas [A]** time = 0.46, size = 232, normalized size = 1.45

$$-\frac{1}{3}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}+\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\,x\right)}{6\,x}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}x-2\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}x^{2}+4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+2\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)+\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{9}\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{18}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(1/3)*x^2 + 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/
6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 4^(1/3)*sqrt(3)*x)/x) + 1
/12*4^(2/3)*(-1)^(1/3)*log(-(4^(2/3)*(-1)^(1/3)*x - 2*(-x^3 + 1)^(1/3))/x)
- 1/24*4^(2/3)*(-1)^(1/3)*log((2*4^(1/3)*(-1)^(2/3)*x^2 + 4^(2/3)*(-1)^(1/3
)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-1/3*(
sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^(1/3))
/x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**maple [F]** time = 2.04, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(-x^{3}+1\right)^{\frac{2}{3}}\left(x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.394 $\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)} dx$

**Optimal**. Leaf size=139

$$-\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi [A]** time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.49, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {494, 481, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{6}\log\left(\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^2/(1 - x^3)
^(2/3) - x/(1 - x^3)^(1/3)]/6 - Log[1 + x/(1 - x^3)^(1/3)]/3 - Log[1 + (2^(
2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[
1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
```

```
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right.$$

$$= \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\right.}{6\ 2^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log$$

**Mathematica** [C] time = 0.05, size = 26, normalized size = 0.19

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3])/5
```

**IntegrateAlgebraic** [A] time = 0.38, size = 212, normalized size = 1.53

$$-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}}+\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}x+\left(1-x^3\right)^{2/3}+x^2\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^(1/3))]/Sqrt[3] - ArcTan[(Sqrt[3]*x)/(
-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(2/3)*Sqrt[3]) - Log[x + (1 - x^3)^(1/3)]
/3 + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[x^2 - x*(1 - x^3)
^(1/3) + (1 - x^3)^(2/3)]/6 - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1
/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas** [A] time = 0.46, size = 197, normalized size = 1.42

$$\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}x-4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+2\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)-\frac{1}{3}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(4^(1/3)*sqrt(3)*x - 4^(2/3)*sqrt(3
)*(-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/
x) - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3
 + 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 +
1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1
)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**maple** [F] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-(x-1)\,(x^2+x+1)\right)^{\frac{2}{3}}\,(x+1)\,(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.395 $\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=88

$$\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi [A]** time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= -\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\right.}{2\sqrt[3]{2}}$$

$$= \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.43

$$\frac{x^2\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{2x^3}{x^3+1}\right)}{2\left(x^3+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^2*Hypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)])/(2*(1 + x^3)^(2/3
))
```

**IntegrateAlgebraic [A]** time = 0.29, size = 126, normalized size = 1.43

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\ 2^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2^(2/3)*Sqrt[3]) - Log[
2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^
3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas [B]** time = 1.92, size = 283, normalized size = 3.22

$$-\frac{1}{18}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-12\sqrt{3}\,(-1)^{\frac{1}{3}}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\left(71\,x^{9}-111\,x^{6}+33\,x^{3}-1\right)\right)}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right)+\frac{1}{36}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{3}+1\right)-6\left(-x^{3}+1\right)^{\frac{2}{3}}x}{x^{3}+1}\right)-\frac{1}{72}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(5\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(19\,x^{6}-16\,x^{3}+1\right)-24\left(2\,x^{5}-x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{6}+2\,x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1
)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/36*4^(2/3)*(-1)^(1/3)*log(-
(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 + 1) -
 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/72*4^(2/3)*(-1)^(1/3)*log((6*4^(1/3)*
(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6 - 16*x
^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**maple [C]** time = 4.24, size = 938, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/6*RootOf(_Z^3+2)*ln((54*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z
^2)^2*RootOf(_Z^3+2)^3*x^3+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36
*_Z^2)*RootOf(_Z^3+2)^4*x^3-12*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootO
f(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x-27*RootOf(RootOf(_Z^3+2)^2+6*_Z*
RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3-3*x^3*RootOf(_Z^3+2)^2+6*RootOf(
RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-4*RootOf(_
Z^3+2)*(-x^3+1)^(1/3)*x^2+5*(-x^3+1)^(2/3)*x+9*RootOf(RootOf(_Z^3+2)^2+6*_Z
*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))-
1/6*ln(-(72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_
Z^3+2)^3*x^3+18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf
(_Z^3+2)^4*x^3-24*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6
*_Z*RootOf(_Z^3+2)+36*_Z^2)*x+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2
)+36*_Z^2)*RootOf(_Z^3+2)*x^3+3*x^3*RootOf(_Z^3+2)^2-60*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-8*RootOf(_Z^3+2)*(-x^3
+1)^(1/3)*x^2-2*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2)*RootOf(_Z^3+2)-3*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(_Z
^3+2)-ln(-(72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf
(_Z^3+2)^3*x^3+18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*Root
Of(_Z^3+2)^4*x^3-24*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2
+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3
+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+3*x^3*RootOf(_Z^3+2)^2-60*RootOf(RootOf(_Z^
```

```
3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-8*RootOf(_Z^3+2)*(-x
^3+1)^(1/3)*x^2-2*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-3*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(
RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.396 $\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=103

$$-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
(2^(2/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1
- x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
```

```
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1+x^3}{x^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x}}\right.}{3\sqrt[3]{2}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\,2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} - \text{Subst}\Big($$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{2^{2}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}}$$

**Mathematica [C]** time = 0.55, size = 81, normalized size = 0.79

$$-\frac{5\left(-3x^6+x^3+2\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2x^3}{x^3-1}\right)-12x^3\left(x^3+1\right)\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{2x^3}{x^3-1}\right)}{10x\left(1-x^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/10*(5*(2 + x^3 - 3*x^6)*Hypergeometric2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3
)] - 12*x^3*(1 + x^3)*Hypergeometric2F1[5/3, 2, 8/3, (2*x^3)/(-1 + x^3)])/(
x*(1 - x^3)^(5/3))
```

**IntegrateAlgebraic [A]** time = 0.30, size = 143, normalized size = 1.39

$$-\frac{\sqrt[3]{1-x^3}}{x}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(
2^(2/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[-2*
x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

**fricas [B]** time = 1.94, size = 272, normalized size = 2.64

$$\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}x\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}+12\sqrt{3}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\left(71x^9-111x^6+33x^3-1\right)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)+2\cdot 4^{\frac{2}{3}}x\log\left(\frac{3\cdot 4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2+6\left(-x^3+1\right)^{\frac{2}{3}}x+4^{\frac{1}{3}}\left(x^3+1\right)}{x^3+1}\right)-4^{\frac{2}{3}}x\log\left(\frac{6\cdot 4^{\frac{1}{3}}\left(5x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}+4^{\frac{2}{3}}\left(19x^6-16x^3+1\right)-24\left(2x^5-x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-72\left(-x^3+1\right)^{\frac{1}{3}}}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(4*4^(1/6)*sqrt(3)*x*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(19*x^8 - 1
6*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*sqrt(3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(
2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6
+ 3*x^3 + 1)) + 2*4^(2/3)*x*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 +
 1)^(2/3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x*log((6*4^(1/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 72*(-x^3 + 1)^(1/3))/x
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

**maple [C]** time = 4.06, size = 1387, normalized size = 13.47

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] (x^3-1)/x/(-x^3+1)^(2/3)+(-1/6*ln((18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_
Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootO
f(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2-9*(x^6-2*x^3+1
)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)
*x^4-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-
2)^2*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3
```

```
-2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-RootOf
(_Z^3-2)*x^6+3*(x^6-2*x^3+1)^(2/3)*x^2+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)
*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x+6*RootOf(RootOf(_Z^
3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*RootOf(_Z^3-2)*x^3-3*RootOf(RootO
f(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-RootOf(_Z^3-2))/(x-1)/(x^2+x+1)/(x
+1)/(x^2-x+1))*RootOf(_Z^3-2)-ln((18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z
^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2-9*(x^6-2*x^3+1)
^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
x^4-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2
)^2*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-
2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-RootOf(
_Z^3-2)*x^6+3*(x^6-2*x^3+1)^(2/3)*x^2+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)*
RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x+6*RootOf(RootOf(_Z^3
-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*RootOf(_Z^3-2)*x^3-3*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-RootOf(_Z^3-2))/(x-1)/(x^2+x+1)/(x+
1)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+1/6*Root
Of(_Z^3-2)*ln(-(-36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*
RootOf(_Z^3-2)^2*x^6-12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2
)*RootOf(_Z^3-2)^3*x^6+18*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*
RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2+3*(x^6-2*x^3+1)^(1/3)*RootOf(_
Z^3-2)^2*x^4+36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Root
Of(_Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*Ro
otOf(_Z^3-2)^3*x^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
x^6-6*RootOf(_Z^3-2)*x^6-3*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x+24*RootOf
(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+8*RootOf(_Z^3-2)*x^3-6*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-2*RootOf(_Z^3-2))/(x-1)
/(x^2+x+1)/(x+1)/(x^2-x+1)))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(1-x^3\right)^{2/3} \left(x^3+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}} (x+1)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.397 $\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=124

$$\frac{\sqrt[3]{1-x^3}}{4x}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

**Rubi** [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{4/3}}{4x^4}+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(4/3)/(4*x^4) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(2/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
```

```
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{\left(1+x^3\right)^2}{x^5\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{x^5}+\frac{x}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} + \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right.}{3\sqrt[3]{2}}$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{2^{2/3}}$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4x^4} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

**Mathematica** [C] time = 13.90, size = 145, normalized size = 1.17

$$-\frac{81\left(x^3+1\right)^2 x^3\,{}_4F_3\left(\frac{2}{3},2,2,2;1,1,\frac{8}{3};\frac{2x^3}{x^3-1}\right)+216\left(x^9+x^6\right)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{8}{3};\frac{2x^3}{x^3-1}\right)+5\left(\left(9x^9-20x^6-13x^3+4\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2x^3}{x^3-1}\right)+9x^9+x^6-9x^3-1\right)}{60x^4\left(1-x^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/60*(5*(-1 - 9*x^3 + x^6 + 9*x^9 + (4 - 13*x^3 - 20*x^6 + 9*x^9)*Hypergeo
metric2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3)]) + 216*(x^6 + x^9)*Hypergeometri
cPFQ[{2/3, 2, 2}, {1, 8/3}, (2*x^3)/(-1 + x^3)] + 81*x^3*(1 + x^3)^2*Hyperg
eometricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, (2*x^3)/(-1 + x^3)])/(x^4*(1 - x^3
)^(5/3))
```

**IntegrateAlgebraic [A]** time = 0.33, size = 149, normalized size = 1.20

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{3\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt[3]{1-x^3}\left(x^3-1\right)}{4x^4}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(1/3)*(-1 + x^3))/(4*x^4) + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1
- x^3)^(1/3))]/(2^(2/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^
(2/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/
(6*2^(2/3))
```

**fricas [B]** time = 1.86, size = 312, normalized size = 2.52

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}x^4\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-12\sqrt{3}\left(-1\right)^{\frac{1}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\left(71x^9-111x^6+33x^3-1\right)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-2\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}x^4\log\left(-\frac{3\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2-4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(x^3+1\right)-6\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\right)+4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}x^4\log\left(\frac{6\cdot 4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(5x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(19x^6-16x^3+1\right)-24\left(2x^5-x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)-18\left(x^3-1\right)\left(-x^3+1\right)^{\frac{1}{3}}}{72x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(
1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x
^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x
^4*log(-(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^
3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^4*log((6*4
^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6
 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18
*(x^3 - 1)*(-x^3 + 1)^(1/3))/x^4
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

**maple [C]** time = 3.51, size = 1386, normalized size = 11.18

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] -1/4*(x^6-2*x^3+1)/x^4/(-x^3+1)^(2/3)+(-1/6*ln((18*RootOf(RootOf(_Z^3+2)^2+
6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+6*RootOf(RootOf(_Z^3+2)
```

```
^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-18*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-6*RootOf(RootOf(_Z
^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-9*(x^6-2*x^3+1)^(
2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*
x^2-9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootO
f(_Z^3+2)+36*_Z^2)*x^4+3*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^
2)*x^6+RootOf(_Z^3+2)*x^6+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootO
f(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*R
ootOf(_Z^3+2)+36*_Z^2)*x^3-2*RootOf(_Z^3+2)*x^3+3*(x^6-2*x^3+1)^(2/3)*x^2+3
*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+RootOf(_Z^3+2))/(x-1)
/(x^2+x+1)/(x+1)/(x^2-x+1))*RootOf(_Z^3+2)-ln((18*RootOf(RootOf(_Z^3+2)^2+6
*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+6*RootOf(RootOf(_Z^3+2)^
2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-18*RootOf(RootOf(_Z^3+2
)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-6*RootOf(RootOf(_Z^
3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-9*(x^6-2*x^3+1)^(2
/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x
^2-9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf
(_Z^3+2)+36*_Z^2)*x^4+3*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2
)*x^6+RootOf(_Z^3+2)*x^6+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf
(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*Ro
otOf(_Z^3+2)+36*_Z^2)*x^3-2*RootOf(_Z^3+2)*x^3+3*(x^6-2*x^3+1)^(2/3)*x^2+3*
RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+RootOf(_Z^3+2))/(x-1)/
(x^2+x+1)/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_
Z^2)+1/6*RootOf(_Z^3+2)*ln((36*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+
36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3
+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-36*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z
^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootO
f(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3+18*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+
2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^2-18*RootOf(Roo
tOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6-3*(x^6-2*x^3+1)^(1/3)*RootOf
(_Z^3+2)^2*x^4-6*RootOf(_Z^3+2)*x^6+24*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(
_Z^3+2)+36*_Z^2)*x^3+3*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)^2*x+8*RootOf(_Z^3
+2)*x^3-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)-2*RootOf(_Z^
3+2))/(x-1)/(x^2+x+1)/(x+1)/(x^2-x+1)))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.398 $\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=291

$$-\frac{1}{2}\sqrt[3]{1-x^3}\,x+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\right.}{12\ 2^{2/3}}$$

**Rubi [C]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^7*AppellF1[7/3, 2/3, 1, 10/3, x^3, -x^3])/7
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};x^3,-x^3\right)$$

**Mathematica [C]** time = 0.16, size = 115, normalized size = 0.40

$$\frac{1}{2}x\sqrt[3]{1-x^3}\left(-\frac{4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x
^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3,
 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/2
```

**IntegrateAlgebraic [F]** time = 11.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^6/((1 - x^3)^(2/3)*(1 + x^3)), x]
```

**fricas** [A] time = 2.52, size = 356, normalized size = 1.22

$$\frac{1}{36} \cdot 4^{\frac{1}{6}} \sqrt{3} \arctan\left(-\frac{4^{\frac{1}{6}} \left(6 \cdot 4^{\frac{2}{3}} \sqrt{3} \left(x^{16} - 33 x^{13} + 110 x^{10} - 110 x^{7} + 33 x^{4} - x\right) \left(-x^{3} + 1\right)^{\frac{1}{3}} - 48 \sqrt{3} \left(x^{14} - 2 x^{11} - 6 x^{8} - 2 x^{5} + x^{2}\right) \left(-x^{3} + 1\right)^{\frac{2}{3}} - 4^{\frac{1}{3}} \sqrt{3} \left(x^{18} + 42 x^{15} - 417 x^{12} + 812 x^{9} - 417 x^{6} + 42 x^{3} + 1\right)\right)}{6 \left(x^{18} - 102 x^{15} + 447 x^{12} - 628 x^{9} + 447 x^{6} - 102 x^{3} + 1\right)}\right) + \frac{1}{72} \cdot 4^{\frac{2}{3}} \log\left(-\frac{12 \left(-x^{3} + 1\right)^{\frac{2}{3}} x^{2} - 3 \cdot 4^{\frac{2}{3}} \left(x^{4} - x\right) \left(-x^{3} + 1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}} \left(x^{6} + 2 x^{3} + 1\right)}{x^{6} + 2 x^{3} + 1}\right) - \frac{1}{144} \cdot 4^{\frac{2}{3}} \log\left(\frac{24 \cdot 4^{\frac{1}{3}} \left(x^{8} - 4 x^{5} + x^{2}\right) \left(-x^{3} + 1\right)^{\frac{2}{3}} + 4^{\frac{2}{3}} \left(x^{12} - 32 x^{9} + 78 x^{6} - 32 x^{3} + 1\right) + 12 \left(x^{10} - 11 x^{7} + 11 x^{4} - x\right) \left(-x^{3} + 1\right)^{\frac{1}{3}}}{x^{12} + 4 x^{9} + 6 x^{6} + 4 x^{3} + 1}\right) - \frac{1}{2} \left(-x^{3} + 1\right)^{\frac{1}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(x^16 - 33*x^13
 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*sqrt(3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*
x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x
^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3 + 1)^
(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^3 + 1
))/(x^6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 + x^2)*(
-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10
- 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)
) - 1/2*(-x^3 + 1)^(1/3)*x
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(x^{3} + 1\right) \left(-x^{3} + 1\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**maple** [C] time = 14.51, size = 696, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/2*x*(x^3-1)/(-x^3+1)^(2/3)+(1/12*RootOf(_Z^3-2)*ln((6*RootOf(RootOf(_Z^3-
2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^3*x^3+36*RootOf(RootOf(_Z^3
-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-RootOf(_Z^3-2)*x^6
-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^6-9*(x^6-2*x^3+1)^
(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*
x-6*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2+6*RootOf(_Z^3-2)*x^3+36*RootOf
(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x-R
ootOf(_Z^3-2)-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2))/(x+1)^
2/(x^2-x+1)^2)+1/4*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*ln(-
(12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^3*x^
3+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^2
*x^3+2*RootOf(_Z^3-2)*x^6+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)*x^6-9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+
9*_Z^2)*RootOf(_Z^3-2)^2*x-6*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2-18*Ro
otOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2
)+9*_Z^2)*x^2-4*RootOf(_Z^3-2)*x^3-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z
^3-2)+9*_Z^2)*x^3+2*RootOf(_Z^3-2)+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z
^3-2)+9*_Z^2))/(x+1)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.399 $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)} dx$

**Optimal**. Leaf size=294

$$-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}-\ \tan$$

**Rubi** [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};x^3,-x^3\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -AppellF1[-2/3, 2/3, 1, 1/3, x^3, -x^3]/(2*x^2)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)} dx = -\frac{F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};x^3,-x^3\right)}{2x^2}$$

**Mathematica** [C] time = 0.10, size = 120, normalized size = 0.41

$$\frac{\sqrt[3]{1-x^3}\left(\frac{4x^3F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-1\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 +
 x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/
3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/(2*x^2)
```

**IntegrateAlgebraic** [F] time = 34.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)), x]
```

**fricas [A]** time = 2.42, size = 396, normalized size = 1.35

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}x^2\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x\right)\left(-x^3+1\right)^{\frac{1}{3}}+48\sqrt{3}(-1)^{\frac{1}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\left(x^{18}+42x^{15}-417x^{12}+812x^{9}-417x^{6}+42x^{3}+1\right)\right)}{6\left(x^{18}-102x^{15}+447x^{12}-628x^{9}+447x^{6}-102x^{3}+1\right)}\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^2\log\left(\frac{24\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^8-4x^5+x^2\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{12}-32x^9+78x^6-32x^3+1\right)+12\left(x^{10}-11x^7+11x^4-x\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^{12}+4x^9+6x^6+4x^3+1}\right)-2\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^2\log\left(-\frac{12\left(-x^3+1\right)^{\frac{2}{3}}x^2+3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^4-x\right)\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^6+2x^3+1\right)}{x^6+2x^3+1}\right)+72\left(-x^3+1\right)^{\frac{1}{3}}}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/144*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^2*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1
)^(1/3) + 48*sqrt(3)*(-1)^(1/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3
 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^
6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3
+ 1)) + 4^(2/3)*(-1)^(1/3)*x^2*log((24*4^(1/3)*(-1)^(2/3)*(x^8 - 4*x^5 + x^
2)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 +
 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x
^6 + 4*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x^2*log(-(12*(-x^3 + 1)^(2/3)*x^2 +
 3*4^(2/3)*(-1)^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(-1)^(2/3)*(x^6
+ 2*x^3 + 1))/(x^6 + 2*x^3 + 1)) + 72*(-x^3 + 1)^(1/3))/x^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

**maple [C]** time = 14.43, size = 991, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/2*(x^3-1)/x^2/(-x^3+1)^(2/3)+(-1/12*ln((6*RootOf(RootOf(_Z^3+2)^2+3*_Z*Ro
otOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^3-18*RootOf(RootOf(_Z^3+2)^2+3*_Z*R
ootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+RootOf(_Z^3+2)*x^6-3*RootOf(Ro
otOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf
(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x-18*RootOf(
RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)*(x^6-2*x^3+1)^(
1/3)*x^2-6*RootOf(_Z^3+2)*x^3+18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2
)+9*_Z^2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x+RootOf(_Z^3+2)-3*RootOf(RootOf(_Z^3+2
)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2))/(x+1)^2/(x^2-x+1)^2)*RootOf(_Z^3+2)-1/4*ln
((6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^
3-18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2
*x^3+RootOf(_Z^3+2)*x^6-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^
2)*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*
RootOf(_Z^3+2)+9*_Z^2)*x-18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_
Z^2)*RootOf(_Z^3+2)*(x^6-2*x^3+1)^(1/3)*x^2-6*RootOf(_Z^3+2)*x^3+18*RootOf(
RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x+Ro
otOf(_Z^3+2)-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2))/(x+1)^2
/(x^2-x+1)^2)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)+1/12*Root
```

```
Of(_Z^3+2)*ln((6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf
(_Z^3+2)^3*x^3+36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*Roo
tOf(_Z^3+2)^2*x^3-RootOf(_Z^3+2)*x^6-6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(
_Z^3+2)+9*_Z^2)*x^6-9*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z
^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x-6*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3
)*x^2+2*RootOf(_Z^3+2)*x^3+12*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9
*_Z^2)*x^3-RootOf(_Z^3+2)-6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_
Z^2))/(x+1)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.400 $\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx$

**Optimal**. Leaf size=141

$$\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 87, 43, 627, 51, 55, 617, 204, 31}

$$\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)
^(8/3)/8 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]
) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}}-\frac{x^2}{\sqrt[3]{1-x}}+\frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\right)dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}}\,dx, x, x^3\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\,dx, x\right.\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}}-2(1-x)^{2/3}+(1-x)^{5/3}\right)dx, x, x^3\right)+\frac{1}{3}\operatorname{Sub}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+}\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{1}{4}\operatorname{Subst}\left(\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}\right.}{4}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\text{lo}}{}
\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 53, normalized size = 0.38

$$\frac{20\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-5x^9-x^6-23x^3+29}{40\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (29 - 23*x^3 - x^6 - 5*x^9 + 20*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2
])/(40*(1 - x^3)^(1/3))
```

**IntegrateAlgebraic** [A] time = 0.18, size = 156, normalized size = 1.11

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}\left(5x^9+x^6+23x^3-49\right)}{40\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-49 + 23*x^3 + x^6 + 5*x^9))/(40*(-1 + x^3)) + ArcTan[1/S
qrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[-2 +
2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^
(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))
```

**fricas** [A] time = 0.47, size = 140, normalized size = 0.99

$$\frac{10\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+10\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+3\left(5x^9+x^6+23x^3-49\right)\left(-x^3+1\right)^{\frac{2}{3}}}{120\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/120*(10*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x
^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x
^3 + 1)^(1/3)) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

**giac** [A] time = 0.19, size = 136, normalized size = 0.96

$$\frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)
^(1/3)
```

**maple** [C] time = 3.78, size = 682, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] -1/40*(5*x^9+x^6+23*x^3-49)/(-x^3+1)^(1/3)-1/12*ln((18*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-
1/2*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z
```

```
^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(
_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-1
2*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-4
2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(
x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+1/2*Ro
otOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((18*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-6*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-5*RootOf(_Z^3-4)*x^3+21*RootOf
(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))
```

**maxima [A]** time = 1.07, size = 128, normalized size = 0.91

$$\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}}+\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/8*(-x^3 + 1)^(8/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^(1/
3) + (-x^3 + 1)^(1/3)) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

**mupad [B]** time = 4.86, size = 148, normalized size = 1.05

$$\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}}+\left(1-x^3\right)^{2/3}-\frac{2\left(1-x^3\right)^{5/3}}{5}+\frac{\left(1-x^3\right)^{8/3}}{8}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)^(8/3)/8 + (2^(2/3)*log((
1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 -
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*
1i + 1))/24
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

## 3.401 $\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=130

$$-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi** [A] time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 87, 43, 783, 78, 55, 617, 204, 31}

$$-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2
^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{x}{\sqrt[3]{1-x}}-\frac{x}{\sqrt[3]{1-x}\left(-1+x^2\right)}\right)dx, x, x^3\right)\\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}}\,dx, x, x^3\right)\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\left(-1+x^2\right)}\,dx, x, x^3\right)\\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}}-(1-x)^{2/3}\right)dx, x, x^3\right)\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(-1-x)(1-x)^{4/3}}\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt[3]{1-x}}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}}\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-
\end{aligned}$$

**Mathematica** [C] time = 0.02, size = 48, normalized size = 0.37

$$\frac{-5\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-2x^6-x^3+13}{10\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (13 - x^3 - 2*x^6 - 5*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(10*(1
 - x^3)^(1/3))
```

**IntegrateAlgebraic** [A] time = 0.16, size = 151, normalized size = 1.16

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}\left(2x^6+x^3-8\right)}{10\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-8 + x^3 + 2*x^6))/(10*(-1 + x^3)) - ArcTan[1/Sqrt[3] + (
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1
 - x^3)^(1/3)]/(6*2^(1/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 -
 x^3)^(2/3)]/(12*2^(1/3))
```

**fricas** [A] time = 0.45, size = 159, normalized size = 1.22

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}\left(-1\right)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\left(-1\right)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+5\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-10\cdot 2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^3-1\right)\log\left(-2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-12\left(2x^6+x^3-8\right)\left(-x^3+1\right)^{\frac{2}{3}}}{120\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/120*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(
6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 5*2^(2/3)*(-1)^(1/3)*(
x^3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x
^3 + 1)^(2/3)) - 10*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) +
(-x^3 + 1)^(1/3)) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

**giac** [A] time = 0.18, size = 120, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1
/3))) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

**maple** [C] time = 3.79, size = 497, normalized size = 3.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] -1/10*(2*x^6+x^3-8)/(-x^3+1)^(1/3)-1/12*RootOf(_Z^3-4)*ln((-6*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-45*RootOf(Root
Of(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+2*RootOf(_
Z^3-4)*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+63*R
ootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)+21*(-x^3+1)^(2/3)-14*RootOf(_Z^3-4)-105*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(_Z^3-4)*x^3+72*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+126*RootOf(_Z^3-4)*(-x^3+1)^(1/3)
*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2/3)-35*
RootOf(_Z^3-4)-168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x
+1)/(x^2-x+1))
```

**maxima [A]** time = 1.27, size = 119, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
 + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

**mupad [B]** time = 5.15, size = 139, normalized size = 1.07

$$\frac{1}{2\left(1-x^3\right)^{1/3}}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{\left(1-x^3\right)^{2/3}}{2}-\frac{\left(1-x^3\right)^{5/3}}{5}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2}{16}\right)\left(-1+\sqrt{3}\,1i\right)}{24}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1i\right)^2}{16}\right)\left(1+\sqrt{3}\,1i\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + (
1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^
(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 + (2^(2/3)*log((1 - x^3)
^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

## 3.402 $\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=115

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 87, 627, 51, 55, 617, 204, 31}

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(1-x)^{4/3}(1+x)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}}+\frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\right)dx,x,x^3\right)$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\,dx,x,x^3\right)$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx,x,x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx,x,\sqrt[3]{\cdots}\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x\cdots}\right.}{\cdots}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1\cdots}\right.}{4\sqrt[3]{2}}$$

**Mathematica [C]** time = 0.01, size = 41, normalized size = 0.36

$$\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-x^3+1}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3 + Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(2*(1 - x^3)^(1/3)
)
```

**IntegrateAlgebraic [A]** time = 0.15, size = 146, normalized size = 1.27

$$\frac{\left(1-x^3\right)^{2/3}\left(x^3-2\right)}{2\left(x^3-1\right)}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-2 + x^3))/(2*(-1 + x^3)) + ArcTan[1/Sqrt[3] + (2^(2/3)*(
1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(
1/3)]/(6*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/
3)]/(12*2^(1/3))
```

**fricas [A]** time = 0.47, size = 130, normalized size = 1.13

$$\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+2\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+12\left(x^3-2\right)\left(-x^3+1\right)^{\frac{2}{3}}}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) + 12*(x^3 - 2)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

**giac [A]** time = 0.18, size = 109, normalized size = 0.95

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2*(-x^3 + 1)^(
2/3) + 1/2/(-x^3 + 1)^(1/3)
```

**maple [C]** time = 3.78, size = 672, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] -1/2*(x^3-2)/(-x^3+1)^(1/3)+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*Roo
tOf(_Z^3-4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*R
ootOf(_Z^3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
x^3-5*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/
3)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4
))/(x+1)/(x^2-x+1))-1/12*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3
```

```
)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln((18*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21
*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima [A]** time = 1.27, size = 108, normalized size = 0.94

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/3)
+ 1/2/(-x^3 + 1)^(1/3)
```

**mupad [B]** time = 4.89, size = 128, normalized size = 1.11

$$\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}}+\frac{\left(1-x^3\right)^{2/3}}{2}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2}{16}\right)\left(-1+\sqrt{3}\,1i\right)}{24}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1i\right)^2}{16}\right)\left(1+\sqrt{3}\,1i\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3)/2 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i -
1)^2)/16)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*
(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.403 $\int \frac{x^5}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subs}}{} \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\right.}{2\sqrt[3]{2}} \\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 95, normalized size = 0.95

$$\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}} + 2^{2/3}\log\left(x^3+1\right) - 3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) - 2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
])/24
```

**IntegrateAlgebraic [A]** time = 0.14, size = 141, normalized size = 1.41

$$-\frac{\left(1-x^3\right)^{2/3}}{2\left(x^3-1\right)} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3)/(-1 + x^3) - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/
3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(
1/3)) + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1
/3))
```

**fricas [B]** time = 0.46, size = 148, normalized size = 1.48

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(-2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+12\left(-x^3+1\right)^{\frac{2}{3}}}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

**giac [A]** time = 0.18, size = 98, normalized size = 0.98

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^
(1/3)
```

**maple [C]** time = 3.76, size = 667, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-
4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-5*Roo
tOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+42*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/(x+1)/
(x^2-x+1))+1/12*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^
3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootO
f(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)+1/2*ln((18*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_
Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima [A]** time = 1.53, size = 97, normalized size = 0.97

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

**mupad [B]** time = 4.85, size = 117, normalized size = 1.17

$$\frac{1}{2\left(1-x^3\right)^{1/3}}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 - (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
i - 1))/24 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.404 $\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {444, 51, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subs}}{}\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\right.}{2\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 34, normalized size = 0.34

$$\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2]/(2*(1 - x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.12, size = 141, normalized size = 1.41

$$-\frac{\left(1-x^3\right)^{2/3}}{2\left(x^3-1\right)} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3)/(-1 + x^3) + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/
3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(
1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1
/3))
```

**fricas [A]** time = 0.47, size = 125, normalized size = 1.25

$$\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + 2\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - 12\left(-x^3+1\right)^{\frac{2}{3}}}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

**giac [A]** time = 0.19, size = 98, normalized size = 0.98

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^(
1/3)
```

**maple [C]** time = 3.87, size = 667, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-
4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-5*Roo
tOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+42*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/(x+1)/
(x^2-x+1))-1/12*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^
3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootO
f(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln((18*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_
Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima [A]** time = 1.34, size = 97, normalized size = 0.97

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

**mupad [B]** time = 4.90, size = 117, normalized size = 1.17

$$\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.405 $\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=154

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-$$

**Rubi [A]** time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 85, 156, 55, 618, 204, 31, 617}

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - A
rcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/2
 + Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (
1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{2+x}{\sqrt[3]{1-x}\,x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x,\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x}\right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right.}{4\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} +
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 54, normalized size = 0.35

$$\frac{2\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-x^3\right) - {}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2] + 2*Hypergeometric2F1[-1/3,
1, 2/3, 1 - x^3])/(2*(1 - x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.25, size = 219, normalized size = 1.42

$$-\frac{\left(1-x^3\right)^{2/3}}{2\left(x^3-1\right)} + \frac{1}{3}\log\left(\sqrt[3]{1-x^3}-1\right) - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}} - \frac{1}{6}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right) + \frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3)/(-1 + x^3) + ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sq
rt[3]]/Sqrt[3] - ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2
^(1/3)*Sqrt[3]) + Log[-1 + (1 - x^3)^(1/3)]/3 - Log[-2 + 2^(2/3)*(1 - x^3)^
(1/3)]/(6*2^(1/3)) - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6 + Log[2 +
 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))
```

**fricas [A]** time = 0.49, size = 226, normalized size = 1.47

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-8\sqrt{3}\left(x^3-1\right)\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+4\left(x^3-1\right)\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-8\left(x^3-1\right)\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)+12\left(-x^3+1\right)^{\frac{2}{3}}}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) - 8*sqrt(3)*(x^3 - 1)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/
3*sqrt(3)) + 4*(x^3 - 1)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*(
x^3 - 1)*log((-x^3 + 1)^(1/3) - 1) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

**giac [A]** time = 0.19, size = 160, normalized size = 1.04

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2/(-x^3 + 1)^(1/3) - 1/6*log(
(-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1
))
```

**maple [F]** time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(x^{3} + 1\right)} {\left(-x^{3} + 1\right)}^{\frac{4}{3}} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)
```

**mupad [B]** time = 5.40, size = 253, normalized size = 1.64

$$\frac{\ln\left(\frac{17}{4}-\frac{17\,{\left(1-x^3\right)}^{1/3}}{4}\right)}{3}+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\left(1458\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2-\frac{459\,{\left(1-x^3\right)}^{1/3}}{4}\right)-\frac{63}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\ln\left(\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\left(1458\,{\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2-\frac{459\,{\left(1-x^3\right)}^{1/3}}{4}\right)+\frac{63}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)-\frac{2^{2/3}\,\ln\left(\frac{2^{2/3}\,\left(\frac{81\,2^{1/3}}{4}-\frac{459\,{\left(1-x^3\right)}^{1/3}}{4}\right)}{12}+\frac{63}{4}\right)}{12}+\frac{1}{2\,{\left(1-x^3\right)}^{1/3}}+\frac{{\left(-1\right)}^{1/3}\,2^{2/3}\,\ln\left(\frac{{\left(-1\right)}^{1/3}\,2^{2/3}\,\left(\frac{81\,{\left(-1\right)}^{2/3}\,2^{1/3}}{4}-\frac{459\,{\left(1-x^3\right)}^{1/3}}{4}\right)}{12}-\frac{63}{4}\right)}{12}-\frac{{\left(-1\right)}^{1/3}\,2^{2/3}\,\ln\left(\frac{{\left(-1\right)}^{1/3}\,2^{2/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\left(\frac{459\,{\left(1-x^3\right)}^{1/3}}{4}-\frac{81\,{\left(-1\right)}^{2/3}\,2^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)}{24}-\frac{63}{4}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log(17/4 - (17*(1 - x^3)^(1/3))/4)/3 + log(((3^(1/2)*1i)/6 - 1/6)*(1458*((3
^(1/2)*1i)/6 - 1/6)^2 - (459*(1 - x^3)^(1/3))/4) - 63/4)*((3^(1/2)*1i)/6 -
1/6) - log(((3^(1/2)*1i)/6 + 1/6)*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - (459*(1
- x^3)^(1/3))/4) + 63/4)*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log((2^(2/3)*((8
1*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 + 63/4))/12 + 1/(2*(1 - x^3)^(1
/3)) + ((-1)^(1/3)*2^(2/3)*log(((-1)^(1/3)*2^(2/3)*((81*(-1)^(2/3)*2^(1/3))
/4 - (459*(1 - x^3)^(1/3))/4))/12 - 63/4))/12 - ((-1)^(1/3)*2^(2/3)*log(((-
1)^(1/3)*2^(2/3)*(3^(1/2)*1i + 1)*((459*(1 - x^3)^(1/3))/4 - (81*(-1)^(2/3)
*2^(1/3)*(3^(1/2)*1i + 1)^2)/16))/24 - 63/4)*(3^(1/2)*1i + 1))/24
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(- \left(x - 1\right) \left(x^{2} + x + 1\right)\right)^{\frac{4}{3}} \left(x + 1\right) \left(x^{2} - x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.406 $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=175

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{5}{6\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\right.}{2}$$

**Rubi [A]** time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 156, 51, 55, 618, 204, 31, 617}

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{5}{6\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 5/(6*(1 - x^3)^(1/3)) - 1/(3*x^3*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)
^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]
]/(2*2^(1/3)*Sqrt[3]) - Log[x]/6 - Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 -
 x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x^2(1+x)}\,dx,x,x^3\right)$$

$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{1}{3}\,\text{Subst}\left(\int \frac{-\frac{1}{3}-\frac{4x}{3}}{(1-x)^{4/3}x(1+x)}\,dx,x,x^3\right)$$

$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x}\,dx,x,x^3\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,x,x^3\right) + \frac{1}{6}\,\text{Subst}\left(\int$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{\right.$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log}{}$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6}$$

**Mathematica [C]** time = 0.03, size = 64, normalized size = 0.37

$$\frac{3x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)+2x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-x^3\right)-2}{6x^3\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-2 + 3*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2] + 2*x^3*Hypergeome
tric2F1[-1/3, 1, 2/3, 1 - x^3])/(6*x^3*(1 - x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.30, size = 232, normalized size = 1.33

$$\frac{\left(1-x^3\right)^{2/3}\left(2-5x^3\right)}{6x^3\left(x^3-1\right)}+\frac{1}{9}\log\left(\sqrt[3]{1-x^3}-1\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}-2\right)}{6\sqrt[3]{2}}-\frac{1}{18}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{1-x^3}+1\right)-\frac{\log\left(\sqrt[3]{2}\left(1-x^3\right)^{2/3}+2^{2/3}\sqrt[3]{1-x^3}+2\right)}{12\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((2 - 5*x^3)*(1 - x^3)^(2/3))/(6*x^3*(-1 + x^3)) + ArcTan[1/Sqrt[3] + (2*(1
 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x^3)
^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[-1 + (1 - x^3)^(1/3)]/9 + Log[-2
 + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (1 - x^3)^(1/3) + (1 - x^
3)^(2/3)]/18 - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)]/(
12*2^(1/3))
```

**fricas [A]** time = 0.46, size = 238, normalized size = 1.36

$$\frac{6\sqrt{6}\,2^{\frac{1}{6}}\left(x^6-x^3\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}\left(x^6-x^3\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+6\cdot 2^{\frac{2}{3}}\left(x^6-x^3\right)\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+8\sqrt{3}\left(x^6-x^3\right)\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-4\left(x^6-x^3\right)\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+8\left(x^6-x^3\right)\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)-12\left(5x^3-2\right)\left(-x^3+1\right)^{\frac{2}{3}}}{72\left(x^6-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(x^6 - x^3)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*(x^6 - x^3)*log(2^(2/3) + 2^(1/3)*(
-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*(x^6 - x^3)*log(-2^(1/3) +
(-x^3 + 1)^(1/3)) + 8*sqrt(3)*(x^6 - x^3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/
3) + 1/3*sqrt(3)) - 4*(x^6 - x^3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
 1) + 8*(x^6 - x^3)*log((-x^3 + 1)^(1/3) - 1) - 12*(5*x^3 - 2)*(-x^3 + 1)^(
2/3))/(x^6 - x^3)
```

**giac [A]** time = 0.21, size = 181, normalized size = 1.03

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{5x^3-2}{6\left(\left(-x^3+1\right)^{\frac{4}{3}}-\left(-x^3+1\right)^{\frac{1}{3}}\right)}-\frac{1}{18}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{9}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*(5*x^3 - 2)/((-x^3 + 1)^(4/3
) - (-x^3 + 1)^(1/3)) - 1/18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) +
 1/9*log(abs((-x^3 + 1)^(1/3) - 1))
```

**maple [F]** time = 2.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)
```

**mupad [B]** time = 5.02, size = 399, normalized size = 2.28

$$\frac{\ln\left(\frac{11\left(1-x^3\right)^{1/3}}{972}-\frac{11}{972}\right)}{9}+\frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(\frac{81\,2^{1/3}}{4}-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)}{12}-\frac{35}{12}\right)}{72}+\frac{\left(1-x^3\right)^{1/3}}{27}\right)}{12}+\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(1458\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)-\frac{35}{12}\right)+\frac{\left(1-x^3\right)^{1/3}}{27}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)-\ln\left(\frac{\left(1-x^3\right)^{1/3}}{27}-\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(1458\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)+\frac{35}{12}\right)\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)+\frac{\frac{5x^3}{6}-\frac{1}{3}}{\left(1-x^3\right)^{1/3}-\left(1-x^3\right)^{4/3}}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{27}+\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\left(\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{81\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)}{24}-\frac{35}{12}\right)}{288}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{27}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\left(\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{81\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)}{24}+\frac{35}{12}\right)}{288}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log((11*(1 - x^3)^(1/3))/972 - 11/972)/9 + (2^(2/3)*log((2^(1/3)*((2^(2/3)*
((81*2^(1/3))/4 - (75*(1 - x^3)^(1/3))/4))/12 - 35/12))/72 + (1 - x^3)^(1/3
)/27))/12 + log(((3^(1/2)*1i)/18 - 1/18)^2*(((3^(1/2)*1i)/18 - 1/18)*(1458*
((3^(1/2)*1i)/18 - 1/18)^2 - (75*(1 - x^3)^(1/3))/4) - 35/12) + (1 - x^3)^(
1/3)/27)*((3^(1/2)*1i)/18 - 1/18) - log((1 - x^3)^(1/3)/27 - ((3^(1/2)*1i)/
18 + 1/18)^2*(((3^(1/2)*1i)/18 + 1/18)*(1458*((3^(1/2)*1i)/18 + 1/18)^2 - (
75*(1 - x^3)^(1/3))/4) + 35/12))*((3^(1/2)*1i)/18 + 1/18) + ((5*x^3)/6 - 1/
3)/((1 - x^3)^(1/3) - (1 - x^3)^(4/3)) + (2^(2/3)*log((1 - x^3)^(1/3)/27 +
(2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2
)*1i - 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 - 35/12))/288)*(3^(1/2)*1i -
1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/27 - (2^(1/3)*(3^(1/2)*1i + 1)^2*((2^
(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2)*1i + 1)^2)/16 - (75*(1 - x^3)^
(1/3))/4))/24 + 35/12))/288)*(3^(1/2)*1i + 1))/24
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.407 $\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=174

$$\frac{5}{6}\left(1-x^3\right)^{2/3}x+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{1}{6}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [C]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.15, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{10}x^{10}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^10*AppellF1[10/3, 4/3, 1, 13/3, x^3, -x^3])/10
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{10}x^{10}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};x^3,-x^3\right)$$

**Mathematica [C]** time = 0.25, size = 152, normalized size = 0.87

$$\frac{1}{72}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)-\frac{12\left(2x^3-5\right)x}{\sqrt[3]{1-x^3}}-5\,2^{2/3}\left(2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*x*(-5 + 2*x^3))/(1 - x^3)^(1/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^
3, -x^3] - 5*2^(2/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3)
)/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3
)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/72
```

**IntegrateAlgebraic [A]** time = 0.57, size = 248, normalized size = 1.43

$$-\frac{1}{9}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}\left(2x^4-5x\right)}{6\left(x^3-1\right)}+\frac{1}{18}\log\left(-\sqrt[3]{1-x^3}x+\left(1-x^3\right)^{2/3}+x^2\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(-5*x + 2*x^4))/(6*(-1 + x^3)) - ArcTan[(Sqrt[3]*x)/(-x +
2*(1 - x^3)^(1/3))]/(3*Sqrt[3]) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3
)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[x + (1 - x^3)^(1/3)]/9 - Log[2*x + 2^(2
/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^
(2/3)]/18 + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3
)]/(12*2^(1/3))
```

**fricas [B]** time = 0.49, size = 271, normalized size = 1.56

$$\frac{6\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x+2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{6x}\right)+6\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(\frac{2^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^{2}+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x-\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)+8\sqrt{3}\left(x^{3}-1\right)\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3x}\right)-8\left(x^{3}-1\right)\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+4\left(x^{3}-1\right)\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)+12\left(2x^{4}-5x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}{72\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^
(1/3)*x + 2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3))/x) + 6*2^(2/3)*(-1)^(1/3)*
(x^3 - 1)*log((2^(1/3)*(-1)^(2/3)*x + (-x^3 + 1)^(1/3))/x) - 3*2^(2/3)*(-1)
^(1/3)*(x^3 - 1)*log(-(2^(2/3)*(-1)^(1/3)*x^2 + 2^(1/3)*(-1)^(2/3)*(-x^3 +
1)^(1/3)*x - (-x^3 + 1)^(2/3))/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt
(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1
/3))/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2
) + 12*(2*x^4 - 5*x)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{9}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**maple [F]** time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{x^{9}}{\left(-x^{3}+1\right)^{\frac{4}{3}}\left(x^{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{9}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**9/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.408 $\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)} dx$

**Optimal**. Leaf size=153

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [C]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.17, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^7*AppellF1[7/3, 4/3, 1, 10/3, x^3, -x^3])/7
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};x^3,-x^3\right)$$

**Mathematica [C]** time = 0.16, size = 142, normalized size = 0.93

$$\frac{1}{24}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)+\frac{12x}{\sqrt[3]{1-x^3}}+2^{2/3}\left(-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)+\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{(x^3-1)^{2/3}}+1\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^(
2/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Lo
g[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Lo
g[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/24
```

**IntegrateAlgebraic [A]** time = 0.46, size = 238, normalized size = 1.56

$$-\frac{(1-x^3)^{2/3}x}{2(x^3-1)}-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2\sqrt[3]{1-x^3}-x}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}x+(1-x^3)^{2/3}+x^2\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}(1-x^3)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(x*(1 - x^3)^(2/3))/(-1 + x^3) - ArcTan[(Sqrt[3]*x)/(-x + 2*(1 - x^3)^
(1/3))]/Sqrt[3] + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(
1/3)*Sqrt[3]) - Log[x + (1 - x^3)^(1/3)]/3 + Log[2*x + 2^(2/3)*(1 - x^3)^(1
/3)]/(6*2^(1/3)) + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6 - Log[-
2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))
```

**fricas [B]** time = 0.46, size = 239, normalized size = 1.56

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x-2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)-2\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)-8\sqrt{3}\left(x^3-1\right)\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3x}\right)+8\left(x^3-1\right)\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-4\left(x^3-1\right)\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)+12\left(-x^3+1\right)^{\frac{2}{3}}x}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(-1/6*2^(1/6)*(sqrt(6)*2^(1/3)*x -
 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^
3 + 1)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^
(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)
*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3)
)/x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) +
 12*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**maple [F]** time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.409 $\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [C]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 0.36, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{x^3+1}\right)}{4\left(x^3+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)])/(4*(1 + x^3)^(4/3
))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{1+x^3}\right)}{4\left(1+x^3\right)^{4/3}}$$

**Mathematica [C]** time = 0.01, size = 38, normalized size = 0.36

$$\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{x^3+1}\right)}{4\left(x^3+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)])/(4*(1 + x^3)^(4/3
))
```

**IntegrateAlgebraic [A]** time = 0.36, size = 152, normalized size = 1.43

$$-\frac{\left(1-x^3\right)^{2/3}x}{2\left(x^3-1\right)}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(x*(1 - x^3)^(2/3))/(-1 + x^3) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 -
 x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2
^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]
/(12*2^(1/3))
```

**fricas [B]** time = 2.58, size = 318, normalized size = 3.00

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^{9}-111x^{6}+33x^{3}-1\right)\right)}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}+1\right)+6\left(-x^{3}+1\right)^{\frac{2}{3}}x}{x^{3}+1}\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(-\frac{3\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(5x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(19x^{6}-16x^{3}+1\right)+12\left(2x^{5}-x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{6}+2x^{3}+1}\right)+36\left(-x^{3}+1\right)^{\frac{2}{3}}x}{72\left(x^{3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)
*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^
(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 -
111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1
/3)*(x^3 - 1)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)
^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*(x
^3 - 1)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(
-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 +
 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**maple [C]** time = 4.23, size = 627, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*ln(-(-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(
_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+30*(-x^3
+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-
4)*x^2+3*RootOf(_Z^3-4)*x^3+12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+
36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/12*RootOf(_Z^3-4)*ln((-3*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-2
7*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x
^3+6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Ro
otOf(_Z^3-4)^2*x+2*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-3*(-x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-3*Root
Of(_Z^3-4)*x^3-27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+
5*(-x^3+1)^(2/3)*x+RootOf(_Z^3-4)+9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.410 $\int \frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

**Rubi [A]** time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/
(1 - x^3)^(1/3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{6}\,\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\right.\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{\text{Su}}{}\\
&= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{2\sqrt[3]{2}}\\
&= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.05, size = 100, normalized size = 0.94

$$
\frac{-7\left(3x^3+4\right)\left(x^3-1\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2x^3}{x^3-1}\right)-12\left(x^9+x^6\right)\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+7\left(3x^3+4\right)\left(x^3-1\right)^2}{14x^2\left(1-x^3\right)^{7/3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (7*(-1 + x^3)^2*(4 + 3*x^3) - 7*(-1 + x^3)^2*(4 + 3*x^3)*Hypergeometric2F1[
1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] - 12*(x^6 + x^9)*Hypergeometric2F1[2, 7/3,
 10/3, (2*x^3)/(-1 + x^3)])/(14*x^2*(1 - x^3)^(7/3))
```

**IntegrateAlgebraic [A]** time = 0.34, size = 152, normalized size = 1.43

$$-\frac{\left(1-x^3\right)^{2/3} x}{2 \left(x^3-1\right)}+\frac{\log \left(2^{2/3} \sqrt[3]{1-x^3}+2 x\right)}{6 \sqrt[3]{2}}+\frac{\tan ^{-1}\left(\frac{\sqrt{3} x}{2^{2/3} \sqrt[3]{1-x^3}-x}\right)}{2 \sqrt[3]{2} \sqrt{3}}-\frac{\log \left(2^{2/3} \sqrt[3]{1-x^3} x-\sqrt[3]{2} \left(1-x^3\right)^{2/3}-2 x^2\right)}{12 \sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(x*(1 - x^3)^(2/3))/(-1 + x^3) + ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 -
 x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(6*2
^(1/3)) - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]
/(12*2^(1/3))
```

**fricas [B]** time = 2.60, size = 288, normalized size = 2.72

$$-\frac{2 \sqrt{6}\, 2^{\frac{1}{6}} \left(x^3-1\right) \arctan\left(\frac{2^{\frac{1}{6}} \left(6 \sqrt{6}\, 2^{\frac{2}{3}} \left(5 x^7+4 x^4-x\right) \left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}\, 2^{\frac{1}{3}} \left(71 x^9-111 x^6+33 x^3-1\right)+12 \sqrt{6} \left(19 x^8-16 x^5+x^2\right) \left(-x^3+1\right)^{\frac{1}{3}}\right)}{6 \left(109 x^9-105 x^6+3 x^3+1\right)}\right)-2 \cdot 2^{\frac{2}{3}} \left(x^3-1\right) \log\left(\frac{6 \cdot 2^{\frac{1}{3}} \left(-x^3+1\right)^{\frac{1}{3}} x^2+2^{\frac{2}{3}} \left(x^3+1\right)+6 \left(-x^3+1\right)^{\frac{2}{3}} x}{x^3+1}\right)+2^{\frac{2}{3}} \left(x^3-1\right) \log\left(\frac{3 \cdot 2^{\frac{2}{3}} \left(5 x^4-x\right) \left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(19 x^6-16 x^3+1\right)-12 \left(2 x^5-x^2\right) \left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2 x^3+1}\right)+36 \left(-x^3+1\right)^{\frac{2}{3}} x}{72 \left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 -
 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^(1/3
)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^3
 - 1)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^
3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(-x^3 +
 1)^(2/3)*x)/(x^3 - 1)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right) \left(-x^3+1\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**maple [C]** time = 3.01, size = 944, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2/(-x^3+1)^(1/3)*x+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z
^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)*RootOf(_Z^3-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1
)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)
*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3+10*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/12*ln(-(6*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2
```

```
/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x
+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x
+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(
x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Ro
otOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^
2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(
_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x+2*RootOf(_Z^3-4
)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

## 3.411 $\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx$

**Optimal**. Leaf size=124

$$\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{x^2}+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$

**Rubi [C]** time = 8.12, antiderivative size = 204, normalized size of antiderivative = 1.65, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{-18\left(x^3+1\right)^2x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-54x^{12}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-84x^9\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-30x^6\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-7\left(1-x^3\right)^2\left(9x^6+12x^3+2\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)+63x^{12}-42x^9-91x^6+56x^3+14}{14x^5\left(1-x^3\right)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(14 + 56*x^3 - 91*x^6 - 42*x^9 + 63*x^12 - 7*(1 - x^3)^2*(2 + 12*x^3 + 9*x
^6)*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 30*x^6*Hypergeomet
ric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 84*x^9*Hypergeometric2F1[2, 7/3,
 10/3, (-2*x^3)/(1 - x^3)] - 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^
3)/(1 - x^3)] - 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}
, (-2*x^3)/(1 - x^3)])/(14*x^5*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{14+56x^3-91x^6-42x^9+63x^{12}-7\left(1-x^3\right)^2\left(2+12x^3+9x^6\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\right.}{}$$

**Mathematica [C]** time = 3.11, size = 192, normalized size = 1.55

$$\frac{18\left(x^3+1\right)^2x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{2x^3}{x^3-1}\right)+54x^{12}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+84x^9\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+30x^6\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+7\left(x^3-1\right)^2\left(9x^6+12x^3+2\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2x^3}{x^3-1}\right)-63x^{12}+42x^9+91x^6-56x^3-14}{14x^5\left(1-x^3\right)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-14 - 56*x^3 + 91*x^6 + 42*x^9 - 63*x^12 + 7*(-1 + x^3)^2*(2 + 12*x^3 + 9*
x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] + 30*x^6*Hypergeome
tric2F1[2, 7/3, 10/3, (2*x^3)/(-1 + x^3)] + 84*x^9*Hypergeometric2F1[2, 7/3
, 10/3, (2*x^3)/(-1 + x^3)] + 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (2*x^
3)/(-1 + x^3)] + 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3
}, (2*x^3)/(-1 + x^3)])/(14*x^5*(1 - x^3)^(7/3))
```

**IntegrateAlgebraic [A]** time = 0.37, size = 161, normalized size = 1.30

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}\left(1-2x^3\right)}{2x^2\left(x^3-1\right)}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((1 - 2*x^3)*(1 - x^3)^(2/3))/(2*x^2*(-1 + x^3)) - ArcTan[(Sqrt[3]*x)/(-x +
 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[2*x + 2^(2/3)*(1 - x^3
)^(1/3)]/(6*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1
- x^3)^(2/3)]/(12*2^(1/3))
```

**fricas [B]** time = 2.60, size = 340, normalized size = 2.74

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^5-x^2\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-2\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^5-x^2\right)\log\left(\frac{6\,2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3+1\right)+6\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^5-x^2\right)\log\left(-\frac{3\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(5x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(19x^6-16x^3+1\right)+12\left(2x^5-x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+36\left(2x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}}{72\left(x^5-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^5 - x^2)*arctan(1/6*2^(1/6)*(6*sqrt(
6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1
)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9
- 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^
(1/3)*(x^5 - x^2)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*
(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3
)*(x^5 - x^2)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(
1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/
(x^6 + 2*x^3 + 1)) + 36*(2*x^3 - 1)*(-x^3 + 1)^(2/3))/(x^5 - x^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

**maple [C]** time = 2.92, size = 570, normalized size = 4.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2*(2*x^3-1)/x^2/(-x^3+1)^(1/3)-1/12*RootOf(_Z^3-4)*ln((-3*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-9*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-9*(-x^3+1)^(1
/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2
-3*RootOf(_Z^3-4)*x^3-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)*x^3+3*(-x^3+1)^(2/3)*x+RootOf(_Z^3-4)+3*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*ln(-(-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*RootOf(_Z^3-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
```

```
6*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(_Z^3-4)^2*(-x^3+1)^(
1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+12*RootOf(RootOf(_Z^3-4)^2+6*_
Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-12*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.412 $\int \frac{1}{x^6\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=144

$$-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{7\left(1-x^3\right)^{2/3}}{10x^5}+\frac{1}{2x^5\sqrt[3]{1-x^3}}-\frac{4\left(1-x^3\right)^{2/3}}{5x^2}$$

**Rubi** [C] time = 8.32, antiderivative size = 397, normalized size of antiderivative = 2.76, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{54\left(x^3+1\right)^2\left(6x^3+1\right)x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+54\left(x^3+1\right)^3x^6\,{}_4F_3\left(2,2,2,\frac{7}{3};1,1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+567x^{15}\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)+594x^{15}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-378x^{12}\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)+972x^{12}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-819x^9\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)+342x^9\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+476x^6\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)-36x^6\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+182x^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)-28\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)-567x^{15}+378x^{12}+819x^9-476x^6-182x^3+28}{70x^8\left(1-x^3\right)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(28 - 182*x^3 - 476*x^6 + 819*x^9 + 378*x^12 - 567*x^15 - 28*Hypergeometri
c2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 182*x^3*Hypergeometric2F1[1/3, 1, 4
/3, (-2*x^3)/(1 - x^3)] + 476*x^6*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(
1 - x^3)] - 819*x^9*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 37
8*x^12*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 567*x^15*Hyperg
eometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 36*x^6*Hypergeometric2F1[2,
7/3, 10/3, (-2*x^3)/(1 - x^3)] + 342*x^9*Hypergeometric2F1[2, 7/3, 10/3, (-
2*x^3)/(1 - x^3)] + 972*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 -
x^3)] + 594*x^15*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 54*x
^6*(1 + x^3)^2*(1 + 6*x^3)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (-2*x^
3)/(1 - x^3)] + 54*x^6*(1 + x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1,
 10/3}, (-2*x^3)/(1 - x^3)])/(70*x^8*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{28-182x^3-476x^6+819x^9+378x^{12}-567x^{15}-28\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)+182x^3 \cdots}{\cdots}$$

**Mathematica** [A] time = 5.16, size = 133, normalized size = 0.92

$$\frac{2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)}{12\sqrt[3]{2}}-\frac{-8x^6+x^3+2}{10x^5\sqrt[3]{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/10*(2 + x^3 - 8*x^6)/(x^5*(1 - x^3)^(1/3)) + (2*Sqrt[3]*ArcTan[(-1 + (2*
2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2
/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3
)])/(12*2^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.39, size = 164, normalized size = 1.14

$$\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}\,x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}+\frac{\left(1-x^3\right)^{2/3}\left(-8x^6+x^3+2\right)}{10x^5\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(2 + x^3 - 8*x^6))/(10*x^5*(-1 + x^3)) + ArcTan[(Sqrt[3]*x
)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(
1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1
/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))
```

**fricas [B]** time = 2.71, size = 316, normalized size = 2.19

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}\left(x^8-x^5\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)-10\cdot 2^{\frac{2}{3}}\left(x^8-x^5\right)\log\left(\frac{6\cdot 2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}\left(x^3+1\right)+6\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\right)+5\cdot 2^{\frac{2}{3}}\left(x^8-x^5\right)\log\left(\frac{3\cdot 2^{\frac{2}{3}}\left(5x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(19x^6-16x^3+1\right)-12\left(2x^5-x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}}{x^6+2x^3+1}\right)+36\left(8x^6-x^3-2\right)\left(-x^3+1\right)^{\frac{2}{3}}}{360\left(x^8-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(x^8 - x^5)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3
)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6
+ 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x
^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^3 +
1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(
2/3)*(x^8 - x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*
x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) +
 36*(8*x^6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

**maple [C]** time = 4.02, size = 851, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/10*(8*x^6-x^3-2)/x^5/(-x^3+1)^(1/3)+1/12*RootOf(_Z^3-4)*ln(-(3*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+54*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^
3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-6*(-x^3+1)^(1/3)*RootOf(RootO
```

```
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-RootOf(_Z^3-4)*
x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^
(2/3)*x+RootOf(_Z^3-4)+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2))/(x+1)/(x^2-x+1))-1/12*ln((3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+3*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+1
8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootO
f(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*x^3+6*(-x^3+1)^(2/3)*x-RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln(
(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^
3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^
2*x^3+3*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+18*(-x^3+1)^(1/3)*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)
*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+6*(-x^3+1)
^(2/3)*x-RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.413 $\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

**Optimal**. Leaf size=162

$$\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{5\left(1-x^3\right)^{2/3}}{8x^8}+\frac{1}{2x^8\sqrt[3]{1-x^3}}-\frac{13\left(1-x^3\right)^{2/3}}{20x^5}-\frac{49\left(1-x\right.}{40x^2}$$

**Rubi [C]** time = 10.59, antiderivative size = 643, normalized size of antiderivative = 3.97, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(70 - 308*x^3 + 1162*x^6 + 2856*x^9 - 4914*x^12 - 2268*x^15 + 3402*x^18 -
70*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 308*x^3*Hypergeomet
ric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 1162*x^6*Hypergeometric2F1[1/3, 1
, 4/3, (-2*x^3)/(1 - x^3)] - 2856*x^9*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^
3)/(1 - x^3)] + 4914*x^12*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)
] + 2268*x^15*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 3402*x^1
8*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 66*x^6*Hypergeometri
c2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 312*x^9*Hypergeometric2F1[2, 7/3,
10/3, (-2*x^3)/(1 - x^3)] - 2268*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x
^3)/(1 - x^3)] - 6696*x^15*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^
3)] - 4050*x^18*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 27*x^
6*(1 + x^3)^2*(7 - 18*x^3 - 105*x^6)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/
3}, (-2*x^3)/(1 - x^3)] + 54*x^6*(1 - 15*x^3)*(1 + x^3)^3*HypergeometricPFQ
[{2, 2, 2, 7/3}, {1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 81*x^6*HypergeometricP
FQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 324*x^9*Hyperg
eometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 486*x
^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3
)] - 324*x^15*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3
)/(1 - x^3)] - 81*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}
, (-2*x^3)/(1 - x^3)])/(280*x^11*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{70-308x^3+1162x^6+2856x^9-4914x^{12}-2268x^{15}+3402x^{18}-70\,{}_2F_1\left(\frac{1}{3},1\right.}{}$$

**Mathematica [A]** time = 5.21, size = 136, normalized size = 0.84

$$\frac{1}{120}\left(5\,2^{2/3}\left(-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)+\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)\right)-\frac{3\left(-49x^9+23x^6+x^3+5\right)}{x^8\sqrt[3]{1-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 5*2^(2/3)*(-2*Sqr
t[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Log[1 + (2^(2/
3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/
3)*x)/(-1 + x^3)^(1/3)]))/120
```

**IntegrateAlgebraic [A]** time = 0.39, size = 169, normalized size = 1.04

$$-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+2x\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{2^{2/3}\sqrt[3]{1-x^3}-x}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}x-\sqrt[3]{2}\left(1-x^3\right)^{2/3}-2x^2\right)}{12\sqrt[3]{2}}+\frac{\left(1-x^3\right)^{2/3}\left(-49x^9+23x^6+x^3+5\right)}{40x^8\left(x^3-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(2/3)*(5 + x^3 + 23*x^6 - 49*x^9))/(40*x^8*(-1 + x^3)) - ArcTan[
(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(2*2^(1/3)*Sqrt[3]) - Log[2*x +
 2^(2/3)*(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1
/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(12*2^(1/3))
```

**fricas [B]** time = 2.59, size = 351, normalized size = 2.17

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}\left(-1\right)^{\frac{1}{3}}\left(x^{11}-x^{8}\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(-1\right)^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-12\sqrt{6}\left(-1\right)^{\frac{1}{3}}\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^{9}-111x^{6}+33x^{3}-1\right)\right)}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}\right)-10\cdot2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{11}-x^{8}\right)\log\left(\frac{6\cdot2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x^{2}-2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{3}+1\right)+6\left(-x^{3}+1\right)^{\frac{2}{3}}x}{x^{3}+1}\right)+5\cdot2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{11}-x^{8}\right)\log\left(-\frac{3\cdot2^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(5x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(19x^{6}-16x^{3}+1\right)+12\left(2x^{5}-x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{x^{6}+2x^{3}+1}\right)+9\left(49x^{9}-23x^{6}-x^{3}-5\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}{360\left(x^{11}-x^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^11 - x^8)*arctan(1/6*2^(1/6)*(6*sq
rt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*
(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x
^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(
-1)^(1/3)*(x^11 - x^8)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(
2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(-
1)^(1/3)*(x^11 - x^8)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/
3) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)
^(1/3))/(x^6 + 2*x^3 + 1)) + 9*(49*x^9 - 23*x^6 - x^3 - 5)*(-x^3 + 1)^(2/3)
)/(x^11 - x^8)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

**maple [C]** time = 2.68, size = 649, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/40*(49*x^9-23*x^6-x^3-5)/x^8/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln(-(-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(_Z^3-4)^2*(-
x^3+1)^(1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+12*RootOf(RootOf(_Z^3-
4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-1
2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/
12*RootOf(_Z^3-4)*ln(-(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*RootOf(_Z^3-4)^3*x^3+27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)^2*RootOf(_Z^3-4)^2*x^3-6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-2*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)
*x^2+3*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+27*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*x^3-5*(-x^3+1)^(2/3)*x-RootOf(_Z^3-4)-9*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

## 3.414 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=264

$$\frac{\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{4b^3d^3}-\frac{\left(a+bx^3\right)^{7/3}(2ad+bc)}{7b^3d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^3\sqrt[3]{bc-ad}}{}$$

**Rubi** [A] time = 0.39, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{4b^3d^3}-\frac{\left(a+bx^3\right)^{7/3}(2ad+bc)}{7b^3d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}-\frac{c^3\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}-\frac{c^3\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c^3*(a + b*x^3)^(1/3))/d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)
^(4/3))/(4*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2) + (a +
b*x^3)^(10/3)/(10*b^3*d) - (c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^3*(b*
c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int\left(\frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx}}{b^2d^3}+\frac{(-bc-2ad)(a+bx)^{4/3}}{b^2d^2}+\frac{(a+bx)^{7/3}}{b^2d}-\frac{c^3}{d}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3d}-\frac{c^3\,\text{Subst}\left(\right.}{}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2}+\frac{\left(a+bx^3\right.}{10b^3d}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2}+\frac{\left(a+bx^3\right.}{10b^3d}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2}+\frac{\left(a+bx^3\right.}{10b^3d}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2}+\frac{\left(a+bx^3\right.}{10b^3d}$$

**Mathematica [A]** time = 0.68, size = 270, normalized size = 1.02

$$\frac{\frac{105d\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3}-\frac{60d^2\left(a+bx^3\right)^{7/3}(2ad+bc)}{b^3}+\frac{42d^3\left(a+bx^3\right)^{10/3}}{b^3}-\frac{70c^3\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)\right)}{\sqrt[3]{d}}-420c^3\sqrt[3]{a+bx^3}}{420d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-420*c^3*(a + b*x^3)^(1/3) + (105*d*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x
^3)^(4/3))/b^3 - (60*d^2*(b*c + 2*a*d)*(a + b*x^3)^(7/3))/b^3 + (42*d^3*(a
```

```
+ b*x^3)^(10/3))/b^3 - (70*c^3*(b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c - a*d)^
(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
 a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))/(420*
d^4)
```

**IntegrateAlgebraic [A]** time = 0.50, size = 340, normalized size = 1.29

$$\frac{\sqrt[3]{a+bx^3}\left(9a^3d^3+15a^2bcd^2-3a^2bd^3x^3+35ab^2c^2d-5ab^2cd^2x^3+2ab^2d^3x^6-140b^3c^3+35b^3c^2dx^3-20b^3cd^2x^6+14b^3d^3x^9\right)}{140b^3d^4}+\frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{13/3}}-\frac{c^3\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{13/3}}-\frac{c^3\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{13/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 9*a^3*
d^3 + 35*b^3*c^2*d*x^3 - 5*a*b^2*c*d^2*x^3 - 3*a^2*b*d^3*x^3 - 20*b^3*c*d^2
*x^6 + 2*a*b^2*d^3*x^6 + 14*b^3*d^3*x^9))/(140*b^3*d^4) - (c^3*(b*c - a*d)^
(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)
^(1/3))])/(Sqrt[3]*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(13/3)) - (c^3*(b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)])/(6*d^(13/3))
```

**fricas [A]** time = 0.45, size = 325, normalized size = 1.23

$$-\frac{140\sqrt{3}b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+70b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-140b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3\left(14b^3d^3x^9-2\left(10b^3cd^2-ab^2d^3\right)x^6-140b^3c^3+35ab^2c^2d+15a^2bcd^2+9a^3d^3+\left(35b^3c^2d-5ab^2cd^2-3a^2bd^3\right)x^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{420b^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b
*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/
3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c - a*d
)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*d^3*x
^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^3)*
(b*x^3 + a)^(1/3))/(b^3*d^4)
```

**giac [A]** time = 0.29, size = 379, normalized size = 1.44

$$-\frac{\left(b^{34}c^4d^6-ab^{33}c^3d^7\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{34}cd^{10}-ab^{33}d^{11}\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^5}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^5}-\frac{140\left(bx^3+a\right)^{\frac{1}{3}}b^{30}c^3d^6-35\left(bx^3+a\right)^{\frac{4}{3}}b^{29}c^2d^7+20\left(bx^3+a\right)^{\frac{7}{3}}b^{28}cd^8-35\left(bx^3+a\right)^{\frac{4}{3}}ab^{28}cd^8-14\left(bx^3+a\right)^{\frac{10}{3}}b^{27}d^9+40\left(bx^3+a\right)^{\frac{7}{3}}ab^{27}d^9-35\left(bx^3+a\right)^{\frac{4}{3}}a^2b^{27}d^9}{140b^{30}d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(-b*c*d^2 + a*d^
3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^30*c^3*d^
6 - 35*(b*x^3 + a)^(4/3)*b^29*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^28*c*d^8 - 3
5*(b*x^3 + a)^(4/3)*a*b^28*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^27*d^9 + 40*(b*x
^3 + a)^(7/3)*a*b^27*d^9 - 35*(b*x^3 + a)^(4/3)*a^2*b^27*d^9)/(b^30*d^10)
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{11}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.98, size = 442, normalized size = 1.67

$$\left(\frac{3a^2}{4b^3d}+\frac{\left(\frac{3a}{b^3d}+\frac{b^4c-ab^3d}{b^6d^2}\right)\left(b^4c-ab^3d\right)}{4b^3d}\right)\left(bx^3+a\right)^{4/3}-\left(\frac{3a}{7b^3d}+\frac{b^4c-ab^3d}{7b^6d^2}\right)\left(bx^3+a\right)^{7/3}-\left(bx^3+a\right)^{1/3}\left(\frac{a^3}{b^3d}+\frac{\left(\frac{3a^2}{b^3d}+\frac{\left(\frac{3a}{b^3d}+\frac{b^4c-ab^3d}{b^6d^2}\right)\left(b^4c-ab^3d\right)}{b^3d}\right)\left(b^4c-ab^3d\right)}{b^3d}\right)+\frac{\left(bx^3+a\right)^{10/3}}{10b^3d}-\frac{c^3\ln\left(\left(ad-bc\right)^{1/3}-d^{1/3}\left(bx^3+a\right)^{1/3}\right)\left(ad-bc\right)^{1/3}}{3d^{13/3}}-\frac{c^3\ln\left(\frac{3\left(bx^3+a\right)^{1/3}\left(bc^4-ac^3d\right)}{d^2}+\frac{3c^3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(ad-bc\right)^{4/3}}{d^{7/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(ad-bc\right)^{1/3}}{3d^{13/3}}+\frac{c^3\ln\left(\frac{3\left(bx^3+a\right)^{1/3}\left(bc^4-ac^3d\right)}{d^2}-\frac{9c^3\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(ad-bc\right)^{4/3}}{d^{7/3}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(ad-bc\right)^{1/3}}{d^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(4*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(4*b^3*d))*(a + b*x^3)^(4/3) - ((3*a)/(7*b^3*d) + (b^4*c - a*b^
3*d)/(7*b^6*d^2))*(a + b*x^3)^(7/3) - (a + b*x^3)^(1/3)*(a^3/(b^3*d) + (((3
*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b
^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(b^3*d)) + (a + b*x^3)^(10/3)/(10*b^3*d)
 - (c^3*log((a*d - b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3
))/(3*d^(13/3)) - (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d^2 + (3
*c^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(4/3))/d^(7/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(1/3))/(3*d^(13/3)) + (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4
- a*c^3*d))/d^2 - (9*c^3*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(4/3))/d^(7/3))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(1/3))/d^(13/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

## 3.415 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=220

$$-\frac{\left(a+bx^3\right)^{4/3}(ad+bc)}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\cdots$$

**Rubi [A]** time = 0.26, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{4/3}(ad+bc)}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{a+bx^3}}{d^3}+\frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c^2\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(1/3))/d^3 - ((b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*d^2) +
 (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c^2
*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2 \sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt[3]{a+bx}}{bd^2}+\frac{(a+bx)^{4/3}}{bd}+\frac{c^2\sqrt[3]{a+bx}}{d^2(c+dx)}\right)dx, x, x^3\right)\\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}\\
&= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}-\frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+d}\right.}{3d^3}\\
&= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{\left(c^2\sqrt[3]{\right.}{}\\
&= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{b}}{}\\
&= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}-
\end{aligned}$$

**Mathematica [A]** time = 0.41, size = 230, normalized size = 1.05

$$\frac{-\frac{21d\left(a+bx^3\right)^{4/3}(ad+bc)}{b^2}+\frac{12d^2\left(a+bx^3\right)^{7/3}}{b^2}+\frac{14c^2\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)\right)}{\sqrt[3]{d}}+84c^2\sqrt[3]{a+bx^3}}{84d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (84*c^2*(a + b*x^3)^(1/3) - (21*d*(b*c + a*d)*(a + b*x^3)^(4/3))/b^2 + (12*
d^2*(a + b*x^3)^(7/3))/b^2 + (14*c^2*(b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1
 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c -
```

```
a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*
(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))
/(84*d^3)
```

**IntegrateAlgebraic [A]** time = 0.37, size = 283, normalized size = 1.29

$$\frac{\sqrt[3]{a+bx^3}\left(-3a^2d^2-7abcd+abd^2x^3+28b^2c^2-7b^2cdx^3+4b^2d^2x^6\right)}{28b^2d^3}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{10/3}}+\frac{c^2\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{10/3}}+\frac{c^2\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - 7*b^2*c*d*x^3 + a*
b*d^2*x^3 + 4*b^2*d^2*x^6))/(28*b^2*d^3) + (c^2*(b*c - a*d)^(1/3)*ArcTan[1/
Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt
[3]*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
 b*x^3)^(1/3)])/(3*d^(10/3)) + (c^2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3)
 - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]
)/(6*d^(10/3))
```

**fricas [A]** time = 0.46, size = 282, normalized size = 1.28

$$-\frac{28\sqrt{3}b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+14b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-28b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3\left(4b^2d^2x^6+28b^2c^2-7abcd-3a^2d^2-\left(7b^2cd-abd^2\right)x^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{84b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1
/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*c -
a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b^2*d^
2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^3)*(b*
x^3 + a)^(1/3))/(b^2*d^3)
```

**giac [A]** time = 0.25, size = 320, normalized size = 1.45

$$\frac{\left(b^{17}c^3d^4-ab^{16}c^2d^5\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{17}cd^7-ab^{16}d^8\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^4}+\frac{28\left(bx^3+a\right)^{\frac{1}{3}}b^{14}c^2d^4-7\left(bx^3+a\right)^{\frac{4}{3}}b^{13}cd^5+4\left(bx^3+a\right)^{\frac{7}{3}}b^{12}d^6-7\left(bx^3+a\right)^{\frac{4}{3}}ab^{12}d^6}{28b^{14}d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - (-(b*c - a*d)/d)^(1/3))))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt(3
)*(-b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-
(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28*(28*(b*x^3 + a)^(1/3)*b^14*c^2*d^4 - 7
*(b*x^3 + a)^(4/3)*b^13*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^12*d^6 - 7*(b*x^3 + a
)^(4/3)*a*b^12*d^6)/(b^14*d^7)
```

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{8}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.93, size = 336, normalized size = 1.53

$$\left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}-\left(\frac{a}{2\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{4\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3}+\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^2\,d}+\frac{c^2\,\ln\left({\left(a\,d-b\,c\right)}^{1/3}-d^{1/3}\,{\left(b\,x^3+a\right)}^{1/3}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{10/3}}-\frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^3-a\,c^2\,d\right)}{d}-\frac{3\,c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{10/3}}+\frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^3-a\,c^2\,d\right)}{d}+\frac{9\,c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^
2*d))/(b^2*d))*(a + b*x^3)^(1/3) - (a/(2*b^2*d) + (b^3*c - a*b^2*d)/(4*b^4*
d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*log((a*d - b*c
)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3))/(3*d^(10/3)) - (c^2
*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d - (3*c^2*((3^(1/2)*1i)/2 + 1
/2)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(
3*d^(10/3)) + (c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d + (9*c^2*(
(3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/6 - 1/6)*(a
*d - b*c)^(1/3))/d^(10/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8} \sqrt[3]{a + b x^{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

## 3.416 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

**Optimal**. Leaf size=186

$$-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{c\sqrt[3]{a+}}{d^2}$$

**Rubi [A]** time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$-\frac{c\sqrt[3]{a+bx^3}}{d^2}-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}+\frac{\left(a+bx^3\right)^{4/3}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) - (c*(b*c - a*d)^(
1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(7/3)) + (c*
(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^
(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} + \frac{(c(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{\left(c\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+\ldots}\right.}{2d^{7/3}}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{\ldots}\right.}{2d^{7/3}}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\,\log\left(c+dx^3\ldots\right.}{6d^{7/3}}$$

**Mathematica [A]** time = 0.47, size = 204, normalized size = 1.10

$$\frac{c\sqrt[3]{bc-ad}\left(-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)\right)}{6d^{7/3}} - \frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) + (c*(b*c - a*d)^(
1/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)]))/(6*d^(7/3))
```

**IntegrateAlgebraic** [A] time = 0.22, size = 239, normalized size = 1.28

$$\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{7/3}}+\frac{\sqrt[3]{a+bx^3}\left(ad-4bc+bdx^3\right)}{4bd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-4*b*c + a*d + b*d*x^3))/(4*b*d^2) - (c*(b*c - a*d)^(1/
3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1
/3))])/(Sqrt[3]*d^(7/3)) + (c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(
1/3)*(a + b*x^3)^(1/3)])/(3*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)
^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^
(2/3)])/(6*d^(7/3))
```

**fricas** [A] time = 0.45, size = 222, normalized size = 1.19

$$-\frac{4\sqrt{3}bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-4bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-3\left(bdx^3-4bc+ad\right)\left(bx^3+a\right)^{\frac{1}{3}}}{12bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 +
a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*b*
c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a
*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log((b*
x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b*x^3
+ a)^(1/3))/(b*d^2)
```

**giac** [A] time = 0.25, size = 276, normalized size = 1.48

$$-\frac{\left(b^6c^2d^2-ab^5cd^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^6cd^4-ab^5d^5\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^3}-\frac{4\left(bx^3+a\right)^{\frac{1}{3}}b^4cd^2-\left(bx^3+a\right)^{\frac{4}{3}}b^3d^3}{4b^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^6*c^2*d^2 - a*b^5*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)
^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^6*c*d^4 - a*b^5*d^5) + 1/3*sqrt(3)*(-b
*c*d^2 + a*d^3)^(1/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*c
*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c
- a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b^4*c*d^2 - (b*x^3 + a)^(4/
3)*b^3*d^3)/(b^4*d^4)
```

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^5}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.62, size = 298, normalized size = 1.60

$$\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b\,d}-\left(b\,x^3+a\right)^{1/3}\,\left(\frac{a}{b\,d}+\frac{b^2\,c-a\,b\,d}{b^2\,d^2}\right)-\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,{\left(a\,d-b\,c\right)}^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{7/3}}-\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{7/3}}+\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,b\,c^2-3\,a\,c\,d\right)-\frac{c\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b*d) - (a + b*x^3)^(1/3)*(a/(b*d) + (b^2*c - a*b*d)/(b
^2*d^2)) - (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*(a*d - b*c)^(1
/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b*c)^(1/3))/(3*d^(7/3)) - (c
*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*((3^(1/2)*1i)/2 - 1/2)*(a*d
 - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 - 1/2)*(a
*d - b*c)^(1/3))/(3*d^(7/3)) + (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d)
 - (c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^
(7/3)))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(3*d^(7/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \sqrt[3]{a + b x^{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

## 3.417 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

**Optimal**. Leaf size=159

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}}+\frac{\sqrt[3]{a+bx^3}}{d}$$

**Rubi [A]** time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {444, 50, 58, 617, 204, 31}

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}}+\frac{\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)) + ((b*c - a*d)^(1/3)
*Log[c + d*x^3])/(6*d^(4/3)) - ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \cdots$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \cdots$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}} + \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\log\left(\cdots\right.}{\cdots}$$

**Mathematica [A]** time = 0.29, size = 205, normalized size = 1.29

$$\frac{\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}}{6d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3
)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)
*(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

**IntegrateAlgebraic [A]** time = 0.23, size = 215, normalized size = 1.35

$$-\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{4/3}}+\frac{\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{4/3}}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{4/3}}+\frac{\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(4/3)) - ((b*c - a*
```

```
d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(4/3)) +
((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b
*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

**fricas** [A] time = 0.44, size = 206, normalized size = 1.30

$$-\frac{2\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-6\left(bx^3+a\right)^{\frac{1}{3}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(
1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + (-(b*c
- a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^
(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^
(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3))/d
```

**giac** [A] time = 0.27, size = 223, normalized size = 1.40

$$\frac{\left(bc-ad\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd-ad^2\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^2}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{d}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c -
 a*d)/d)^(1/3)))/(b*c*d - a*d^2) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d
)/d)^(1/3))/d^2 + (b*x^3 + a)^(1/3)/d - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/
d)^(2/3))/d^2
```

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^2}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 4.62, size = 249, normalized size = 1.57

$$\frac{\left(b\,x^3+a\right)^{1/3}}{d}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-3\,b\,c\,d\right)-\frac{{\left(a\,d-b\,c\right)}^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{4/3}}-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-3\,b\,c\,d\right)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{4/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-3\,b\,c\,d\right)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(1/3)/d + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) - ((a*d -
b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*(a*d - b*c)^(1/3))/(3*d^(4/3
)) - (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(
a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*((3^(1/2)*1i)/2 + 1/2)
*(a*d - b*c)^(1/3))/(3*d^(4/3)) + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d
) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/d^(4/3
))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3))/d^(4/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt[3]{a + bx^3}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

## 3.418 $\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=246

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{d}}+\sqrt[3]{a}\log\Big(\cdots$$

**Rubi [A]** time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 83, 57, 617, 204, 31, 58}

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{d}}+\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,c}-\frac{\sqrt[3]{a}\log(x)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] -((a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(1/3)) - (a^(1/3)*Log[x])/(2*c) - ((b
*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c*d^(1/3)) + (a^(1/3)*Log[a^(1/3) - (a +
 b*x^3)^(1/3)])/(2*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)])/(2*c*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{2/3}\operatorname{Subs}}{}$$

$$= -\frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc}\right.}{2}$$

$$= -\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} - \frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c\right.}{6c\sqrt[3]{d}}$$

**Mathematica [A]** time = 0.47, size = 268, normalized size = 1.09

$$\frac{\frac{\sqrt[3]{bc-ad}\left(-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)\right)}{\sqrt[3]{d}}-\sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] (-(a^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] -
 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/
3) + (a + b*x^3)^(2/3)])) + ((b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))/(6*c)
```

**IntegrateAlgebraic** [A] time = 0.44, size = 334, normalized size = 1.36

$$-\frac{\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6c}-\frac{\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c\sqrt[3]{d}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c\sqrt[3]{d}}+\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3c}-\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] -((a^(1/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1
/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*c*d^(1/3)) + (a^(1/3)*Log[-a^(1
/3) + (a + b*x^3)^(1/3)])/(3*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3)
+ d^(1/3)*(a + b*x^3)^(1/3)])/(3*c*d^(1/3)) - (a^(1/3)*Log[a^(2/3) + a^(1/3
)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*c) - ((b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)])/(6*c*d^(1/3))
```

**fricas** [A] time = 0.45, size = 276, normalized size = 1.12

$$-\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-2a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-2\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1
/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(3)
*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a) +
a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + ((b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)
^(1/3) + ((b*c - a*d)/d)^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)
) - 2*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))
/c
```

**giac** [A] time = 0.80, size = 311, normalized size = 1.26

$$-\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3c}-\frac{a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6c}+\frac{a^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3c}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3cd}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b*c^2 - a*c*d) - 1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*
(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/c - 1/6*a^(1/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c + 1/3*a^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - a^(1/3)))/c + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*s
qrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3
))/(c*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d)
```

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

[Out] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)
```

**mupad [B]** time = 4.74, size = 1607, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x*(c + d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
 9*a^2*b^6*c^2*d^3) - (a/(27*c^3))^(1/3)*(((243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3) - (a + b*x^3)^(1/3)*(81*
a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4))*(a/(27*c^3))^(2/3) - 9*a*b^7*c^4*d^2 +
 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4))*(a/(27*c^3))^(1/3) + log((a + b*
x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*
c^2*d^3) - (((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5
)*(-(a*d - b*c)/(27*c^3*d))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 8
1*a^2*b^5*c^4*d^4))*(-(a*d - b*c)/(27*c^3*d))^(2/3) - 9*a*b^7*c^4*d^2 + 27*
a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(-(a
*d - b*c)/(27*c^3*d))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^
7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*
(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*((a + b*x^3)^(1/3
)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b
^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c
^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^
6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)/(27*c
^3*d))^(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*
a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(
27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5
*d^3 - 81*a^2*b^5*c^4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 72
9*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(
-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^6*c^3*d^3 + 18*
a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3) +
log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^(1/3)*(((3^(1/2)*
1i)/2 - 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) -
 ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*
b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d^2 - 27*
a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^
(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5
*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(a/(27*c^3))^(1/3)*(((
3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^
4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 +
486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d
^2 - 27*a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(a/(2
7*c^3))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)
```

## 3.419 $\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=340

$$\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{(bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}c^2}-\frac{\log(x)(bc-3ad)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+d\right.}{6c^2}$$

**Rubi [A]** time = 0.38, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{(bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}c^2}-\frac{\log(x)(bc-3ad)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}-\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}+\frac{d\sqrt[3]{a+bx^3}}{c^2}+\frac{\sqrt[3]{a+bx^3}(bc-3ad)}{3ac^2}-\frac{\left(a+bx^3\right)^{4/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(1/3))/c^2 + ((b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*a*c^2) - (
a + b*x^3)^(4/3)/(3*a*c*x^3) - ((b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^
3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d
)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]])/(Sqrt[3]*c^2) - ((b*c - 3*a*d)*Log[x])/(6*a^(2/3)*c^2) + (d^(2/3)*(b*c
- a*d)^(1/3)*Log[c + d*x^3])/(6*c^2) + ((b*c - 3*a*d)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(6*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{1}{3}(-bc+3ad)-\frac{bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x\right.}{9c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\,\text{lo}}{6c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\,\text{lo}}{6c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}c^2} + \frac{d^{2/3}}{}$$

**Mathematica [A]** time = 1.24, size = 366, normalized size = 1.08

$$\frac{\frac{(bc-3ad)\left(3\sqrt[3]{a+bx^3}-\frac{1}{2}\sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)}{3c}+\frac{ad^{2/3}\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c}-\frac{\left(a+bx^3\right)^{4/3}}{x^3}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-((a + b*x^3)^(4/3)/x^3) + ((b*c - 3*a*d)*(3*(a + b*x^3)^(1/3) - (a^(1/3)*
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)]))/2))/(3*c) + (a*d^(2/3)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt
[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d
)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(2*c))/(3*a*c)
```

**IntegrateAlgebraic [A]** time = 0.67, size = 383, normalized size = 1.13

$$\frac{(bc-3ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9a^{2/3}c^2}+\frac{(3ad-bc)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{18a^{2/3}c^2}+\frac{(3ad-bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}c^2}-\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,c^2}-\frac{\sqrt[3]{a+bx^3}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(1/3)/(c*x^3) + ((-(b*c) + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
 b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c
- a*d)^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c
 - a*d)^(1/3))])/(Sqrt[3]*c^2) + ((b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^
(1/3)])/(9*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3)
```

```
+ d^(1/3)*(a + b*x^3)^(1/3)])/(3*c^2) + ((-(b*c) + 3*a*d)*Log[a^(2/3) + a^(
1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(2/3)*c^2) + (d^(2/3)*(b
*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^
3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c^2)
```

**fricas [A]** time = 0.56, size = 429, normalized size = 1.26

$$-\frac{6\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\arctan\left(-\frac{2\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right)+3\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\right)-6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d-\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\right)+2\sqrt{3}\left(abc-3a^2d\right)x^3\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}(bc-3ad)x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}(bc-3ad)x^3\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)+6\left(bx^3+a\right)^{\frac{1}{3}}a^2c}{18a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*arctan(-1/3*(2*sqrt(3)*(-
b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
- a*d^2)) + 3*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(2/3)*d^2 +
(-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
6*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*
d^3)^(1/3)) + 2*sqrt(3)*(a*b*c - 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-1
/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt
(-(-a^2)^(1/3))/a^2) + (-a^2)^(2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 + a)^(2/3)
*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*(b*c
 - 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)
*a^2*c)/(a^2*c^2*x^3)
```

**giac [A]** time = 0.78, size = 351, normalized size = 1.03

$$\frac{\left(bcd-ad^2\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}-\frac{\sqrt{3}(bc-3ad)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{2}{3}}c^2}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3c^2}-\frac{(bc-3ad)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}c^2}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6c^2}+\frac{(bc-3ad)\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{2}{3}}c^2}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d - a*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b
*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c - 3*a*d)*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*c^2) - 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^2 - 1/18*(b*c - 3*a*d)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^2) - 1/6
*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
- a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^2 + 1/9*(b*c - 3*a*d)*log(abs((
b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)/(c*x^3)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(d\,x^{3}+c\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(dx^{3}+c\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 9.99, size = 1917, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a
^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6))^(2/3))/81 - (b^5*d^4*(27*a^3*d
^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(
a^2*c^6))^(1/3))/9 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 +
 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*(-(27*a^3*
d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) + log(
- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) -
81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^
(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 +
17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3 -
 (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2
- 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((a*d^3 - b*c*d^2)/(27*c^6))^(
1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3
*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*b^4*c^4*d^3*((3
^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^
6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3
 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/
3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d
^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*((a*
d^3 - b*c*d^2)/(27*c^6))^(1/3) - log((2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d
^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*
c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) + 81*a*b^4*c^4*d^3*((3^(1/
2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^(
1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 1
7*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3)*(
(3^(1/2)*1i)/2 + 1/2)*((a*d^3 - b*c*d^2)/(27*c^6))^(1/3) + log((((3^(1/2)*1
i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a
^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d
 - b*c)^3/(a^2*c^6))^(2/3))/81 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*
c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9 - (2*
b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32
*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d
^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) - log((
2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 -
32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(
1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 -
5*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 -
 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6)
)^(2/3))/81 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*
(-(27*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1
/3) - (a + b*x^3)^(1/3)/(3*c*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)
```

## 3.420 $\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=370

$$-\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}+\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}c^3}+\log(x)\left(-9a\right.$$

**Rubi [A]** time = 0.49, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 149, 156, 57, 617, 204, 31, 58}

$$-\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}+\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}c^3}+\frac{\log(x)\left(-9a^2d^2+3abcd+b^2c^2\right)}{18a^{5/3}c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c^3}+\frac{d^{5/3}\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^3}+\frac{\sqrt[3]{a+bx^3}\,(3ad+bc)}{9ac^2x^3}-\frac{\left(a+bx^3\right)^{4/3}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(9*a*c^2*x^3) - (a + b*x^3)^(4/3)/(6*a*c*
x^6) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/
(Sqrt[3]*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(5/3)*c^3)
 - (d^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 3*a*b*c
*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(5/3)*c^3) + (d^(5/
3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2
*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{2}{3}(bc+3ad)+\frac{2bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+3abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-6ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{6ac^2}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^3} - \frac{(b}{}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\,\log}{6c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\,\log}{6c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{}}{}$$

**Mathematica [A]** time = 1.84, size = 411, normalized size = 1.11

$$-\frac{\frac{2\left(-9a^2d^2+3abcd+b^2c^2\right)\left(3\sqrt[3]{a+bx^3}-\frac{1}{2}\sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)}{9ac^2}+\frac{ad^{5/3}\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{c^2}-\frac{2\left(a+bx^3\right)^{4/3}(3ad+bc)}{3acx^3}+\frac{\left(a+bx^3\right)^{4/3}}{x^6}}{6ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] -1/6*((a + b*x^3)^(4/3)/x^6 - (2*(b*c + 3*a*d)*(a + b*x^3)^(4/3))/(3*a*c*x^
3) + (2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(3*(a + b*x^3)^(1/3) - (a^(1/3)*(
2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/
3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)]))/2))/(9*a*c^2) + (a*d^(5/3)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*S
qrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
 - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/c^2)/(a*c)
```

**IntegrateAlgebraic [A]** time = 0.81, size = 448, normalized size = 1.21

$$\frac{\left(9a^2d^2-3abcd-b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{27a^{5/3}c^3}+\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54a^{5/3}c^3}-\frac{\left(9a^2d^2-3abcd-b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3}+\frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^3}-\frac{d^{5/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c^3}+\frac{\sqrt[3]{a+bx^3}\left(-3ac+6adx^3-bcx^3\right)}{18ac^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-3*a*c - b*c*x^3 + 6*a*d*x^3))/(18*a*c^2*x^6) - ((-(b^2
*c^2) - 3*a*b*c*d + 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt
[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1/3)*ArcTan[
1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sq
```

```
rt[3]*c^3) + ((-(b^2*c^2) - 3*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^
3)^(1/3)])/(27*a^(5/3)*c^3) + (d^(5/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d
^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a^(5/
3)*c^3) - (d^(5/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
 a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c^3)
```

**fricas** [A]  time = 2.00, size = 472, normalized size = 1.28

$$-\frac{18\,\sqrt{3}\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}a^{3}dx^{6}\arctan\left(-\frac{2\,\sqrt{3}\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^{2}\right)}{3\left(bcd-ad^{2}\right)}\right)+9\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}a^{3}dx^{6}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}d^{2}-\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}d+\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}\right)-18\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}a^{3}dx^{6}\log\left(\left(bx^{3}+a\right)^{\frac{1}{3}}d+\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}\right)-2\,\sqrt{3}\left(ab^{2}c^{2}+3\,a^{2}bcd-9\,a^{3}d^{2}\right)\left(a^{2}\right)^{\frac{1}{6}}x^{6}\arctan\left(\frac{\left(a^{2}\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}a+2\,\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)}{3\,a^{2}}\right)-\left(b^{2}c^{2}+3\,abcd-9\,a^{2}d^{2}\right)\left(a^{2}\right)^{\frac{2}{3}}x^{6}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)+2\left(b^{2}c^{2}+3\,abcd-9\,a^{2}d^{2}\right)\left(a^{2}\right)^{\frac{2}{3}}x^{6}\log\left(\left(bx^{3}+a\right)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+3\left(3\,a^{3}c^{2}+\left(a^{2}bc^{2}-6\,a^{3}cd\right)x^{3}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}}{54\,a^{3}c^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*arctan(-1/3*(2*sqrt(3)*
(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
 - a*d^2)) + 9*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(2/3)*d^2
- (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) -
18*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a
*d^3)^(1/3)) - 2*sqrt(3)*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2)^(1/6)*
x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(
1/3)*(a^2)^(2/3))/a^2) - (b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*
log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) +
2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(1/3)*a
 - (a^2)^(2/3)) + 3*(3*a^3*c^2 + (a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(
1/3))/(a^3*c^3*x^6)
```

**giac** [A]  time = 0.81, size = 465, normalized size = 1.26

$$-\frac{\left(bcd^{2}-ad^{3}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{4}-ac^{3}d\right)}+\frac{\sqrt{3}\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^{3}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}d\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^{3}}+\frac{\sqrt{3}\left(a^{\frac{1}{3}}b^{2}c^{2}+3\,a^{\frac{4}{3}}bcd-9\,a^{\frac{7}{3}}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{2}c^{3}}-\frac{\left(b^{2}c^{2}+3\,abcd-9\,a^{2}d^{2}\right)\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^{3}}+\frac{\left(a^{\frac{1}{3}}b^{2}c^{2}+3\,a^{\frac{4}{3}}bcd-9\,a^{\frac{7}{3}}d^{2}\right)\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{2}c^{3}}-\frac{\left(bx^{3}+a\right)^{\frac{4}{3}}b^{2}c+2\left(bx^{3}+a\right)^{\frac{1}{3}}ab^{2}c-6\left(bx^{3}+a\right)^{\frac{4}{3}}abd+6\left(bx^{3}+a\right)^{\frac{1}{3}}a^{2}bd}{18\,ab^{2}c^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (
-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^
(1/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(
-(b*c - a*d)/d)^(1/3))/c^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/c^3 + 1/27*sqrt(3)*(a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*c^3) - 1/27*(
b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(
5/3)*c^3) + 1/54*(a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^2*c^3) - 1/18*((b
*x^3 + a)^(4/3)*b^2*c + 2*(b*x^3 + a)^(1/3)*a*b^2*c - 6*(b*x^3 + a)^(4/3)*a
*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(a*b^2*c^2*x^6)
```

**maple** [F]  time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(d\,x^{3}+c\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)
```

**mupad [B]** time = 12.55, size = 2767, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*
c))/c^9)^(1/3) + (9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a
*b^2*c^2*d - 14*a^2*b*c*d^2))/a)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 - (b^5*d
^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a
^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a
*d - b*c))/c^9)^(1/3))/3 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c
^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564
*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*(-(a*d
^6 - b*c*d^5)/(27*c^9))^(1/3) + log((((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(1
2*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^4*c^4*d^3*(2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*
c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729 -
(b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 +
 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(
b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 - (b^4*d^6*(a + b*x
^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^
3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*
b*c*d^6))/(243*a^3*c^8))*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9
*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - (((a + b*x^3)^(1/3
)*(b^2*c + 3*a*b*d))/(9*c^2) - (b*(a + b*x^3)^(4/3)*(6*a*d - b*c))/(18*a*c^
2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log(- (((3^(1/2)*1i)/2 - 1/2)
*((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d -
14*a^2*b*c*d^2))/a + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2
*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d -
1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 - (b^4*d
^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3
*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d
- 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d^6 - b*c*d
^5)/(27*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((9*b^5*c^2*d^3*(a + b*
x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 81*a*
b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5
*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(-(d^5*(a*d - b*c))/c^9)^(
2/3))/9 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3
*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3
*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 + (b^4*d^6*(a + b*x^3)^(1/3)*(1458
*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^
3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*
a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log(
- (((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((9*b^5*c^2*d^3*(a + b*x
^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^
```

```
4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c
^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*
a*b*c*d)^3/(a^5*c^9))^(2/3))/729 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*
b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1
458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*
c^9))^(1/3))/27 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a
^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*
c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2
 - 1/2)*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 72
9*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((3^(
1/2)*1i)/2 - 1/2)*((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 +
 a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*
(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^
5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729
- (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3
 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(
-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 + (b^4*d^6*(a + b
*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*
d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^
6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(b^6*c^6 - 729*a^6*d^6
- 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(
1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^7 \left(c + dx^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)
```

## 3.421 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=336

$$-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3}-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{5/3}d^3}-\frac{c^{5/3}\sqrt[3]{bc-ad}\,\ldots}{6d\ldots}$$

**Rubi** [C] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica** [C] time = 0.37, size = 226, normalized size = 0.67

$$\frac{5cx^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(6bc-ad)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)+\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}\left(ad-6bc+3bdx^3\right)\right)-2x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}\left(a^2d^2+3abcd-9b^2c^2\right)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{90bcd^2\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-2*(-9*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^
3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^
2*((a + b*x^3)*(-6*b*c + a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b*c
- a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*
d)*x^3)/(a*(c + d*x^3))]))/(90*b*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2
/3))
```

**IntegrateAlgebraic** [C] time = 5.50, size = 613, normalized size = 1.82

$$\frac{\left(a^2d^2+3abcd-9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{27b^{5/3}d^3}-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{9\sqrt{3}\,b^{5/3}d^3}+\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54b^{5/3}d^3}+\frac{i\left(\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}+ic^{5/3}\sqrt[3]{bc-ad}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6d^3}-\frac{\sqrt{-1-i\sqrt{3}}\,c^{5/3}\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,d^3}+\frac{\left(c^{5/3}\sqrt[3]{bc-ad}-i\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^3}+\frac{\sqrt[3]{a+bx^3}\left(adx^2-6bcx^2+3bdx^5\right)}{18bd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-6*b*c*x^2 + a*d*x^2 + 3*b*d*x^5))/(18*b*d^2) - ((9*b^2
*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a +
b*x^3)^(1/3))])/(9*Sqrt[3]*b^(5/3)*d^3) - (Sqrt[-1 - I*Sqrt[3]]*c^(5/3)*(b*
c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x
- (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]])/(S
qrt[6]*d^3) + ((-9*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)])/(27*b^(5/3)*d^3) + ((I/6)*(I*c^(5/3)*(b*c - a*d)^(1/3) + Sqrt
[3]*c^(5/3)*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*
c^(1/3)*(a + b*x^3)^(1/3)])/d^3 + ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^
(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(5/3)*d
^3) + ((c^(5/3)*(b*c - a*d)^(1/3) - I*Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3))*Lo
g[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x
)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*d^3)
```

**fricas** [A] time = 1.95, size = 494, normalized size = 1.47

$$\frac{18\sqrt{3}\left(bc^3-ac^2d\right)^{\frac{1}{3}}b^3c\arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(bc^3-ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right)+18\left(bc^3-ac^2d\right)^{\frac{1}{3}}b^3c\log\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}c-\left(bc^3-ac^2d\right)^{\frac{1}{3}}x}{x}\right)-9\left(bc^3-ac^2d\right)^{\frac{1}{3}}b^3c\log\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}c^2+\left(bc^3-ac^2d\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}cx+\left(bc^3-ac^2d\right)^{\frac{2}{3}}x^2}{x^2}\right)+2\sqrt{3}\left(9b^3c^2-3ab^2cd-a^2bd^2\right)\left(b^2\right)^{\frac{1}{6}}\arctan\left(\frac{\left(\sqrt{3}\left(b^2\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{\frac{1}{6}}}{3b^2x}\right)-2\left(9b^2c^2-3abcd-a^2d^2\right)\left(b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(9b^2c^2-3abcd-a^2d^2\right)\left(b^2\right)^{\frac{2}{3}}\log\left(\frac{\left(b^2\right)^{\frac{1}{3}}bx^2+\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)+3\left(3b^3d^2x^5-\left(6b^3cd-ab^2d^2\right)x^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{54b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*arctan(-1/3*(sqrt(3)*(b*c^2
- a*c*d)*x + 2*sqrt(3)*(b*c^3 - a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 -
 a*c*d)*x)) + 18*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3 + a)^(1/3)*c - (
b*c^3 - a*c^2*d)^(1/3)*x)/x) - 9*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3
+ a)^(2/3)*c^2 + (b*c^3 - a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)*c*x + (b*c^3 - a
*c^2*d)^(2/3)*x^2)/x^2) + 2*sqrt(3)*(9*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*(
b^2)^(1/6)*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3
)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*(
b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 3*a
*b*c*d - a^2*d^2)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b
^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d - a
*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^3*d^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^7}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)

**maple** [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{7}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)

[Out] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}} x^{7}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7}\,\left(b\,x^{3}+a\right)^{1/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x)

[Out] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}\sqrt[3]{a+bx^{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)

## 3.422 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

**Optimal**. Leaf size=276

$$\frac{(3bc-ad)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^2}+\frac{c^{2/3}\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\,\log\left(\right.}{}$$

**Rubi [C]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.19, size = 185, normalized size = 0.67

$$\frac{x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(ad-3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}-a\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)\right)}{15cd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((-3*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1[5/
3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^2*((a + b*x^3)*(1 + (d*
x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((
-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(15*c*d*(a + b*x^3)^(2/3)*(1 + (d*x^3
)/c)^(2/3))
```

**IntegrateAlgebraic [C]** time = 4.32, size = 549, normalized size = 1.99

$$\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{2/3}d^2}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\,b^{2/3}d^2}+\frac{(ad-3bc)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18b^{2/3}d^2}+\frac{\left(c^{2/3}\sqrt[3]{bc-ad}-i\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6d^2}+\frac{\sqrt{-1-i\sqrt{3}}\,c^{2/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,d^2}+\frac{i\left(\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}+ic^{2/3}\sqrt[3]{bc-ad}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^2}+\frac{x^2\sqrt[3]{a+bx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*d) + ((3*b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(
b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^2) + (Sqrt[-1 - I*S
qrt[3]]*c^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a +
 b*x^3)^(1/3))])/(Sqrt[6]*d^2) + ((3*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(9*b^(2/3)*d^2) + ((c^(2/3)*(b*c - a*d)^(1/3) - I*Sqrt[3]*c^(2/
3)*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3)])/(6*d^2) + ((-3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
 + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(2/3)*d^2) + ((I/12)*(I*c^(2/3)
*(b*c - a*d)^(1/3) + Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a
*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/
3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/d^2
```

**fricas [B]** time = 0.55, size = 452, normalized size = 1.64

$$\frac{6\left(bx^3+a\right)^{\frac{1}{3}}b^2dx^2+6\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{1}{3}}b^2\arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right)+6\left(-bc^3+ac^2d\right)^{\frac{1}{3}}b^2\log\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}c+\left(-bc^3+ac^2d\right)^{\frac{1}{3}}x}{x}\right)-3\left(-bc^3+ac^2d\right)^{\frac{1}{3}}b^2\log\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}c^2-\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}cx+\left(-bc^3+ac^2d\right)^{\frac{2}{3}}x^2}{x^2}\right)-2\sqrt{3}\left(3b^2c-abd\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2x}\right)+2\left(-b^2\right)^{\frac{2}{3}}(3bc-ad)\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)-\left(-b^2\right)^{\frac{2}{3}}(3bc-ad)\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)}{18b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*sqrt(3)*(-b*c^3 + a*c^2*d)^(1/3)*b^
2*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 + a*c^2*d)^(2/
3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^(1/3)*b^2
*log(((b*x^3 + a)^(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) - 3*(-b*c^3 + a*
c^2*d)^(1/3)*b^2*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x
^3 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) - 2*sqrt(3)*(3*b^2*c
 - a*b*d)*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqr
t(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) + 2*(-b^2
)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - (-b^
2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^
(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{4}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}} x^{4}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{4}\left(b\,x^{3}+a\right)^{1/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\sqrt[3]{a+bx^{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

## 3.423 $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=234

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,d}+\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,d}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}\,d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}\right.}{2}$$

**Rubi [C]** time = 0.04, antiderivative size = 64, normalized size of antiderivative = 0.27, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{a+bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*((a + b*x^3)/a)^(1/3))
```

**IntegrateAlgebraic [C]** time = 3.79, size = 490, normalized size = 2.09

$$\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6d}+\frac{\left(\sqrt[3]{bc-ad}-i\sqrt{3}\sqrt[3]{bc-ad}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12\sqrt[3]{c}d}+\frac{i\left(\sqrt{3}\sqrt[3]{bc-ad}+i\sqrt[3]{bc-ad}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{c}d}-\frac{\sqrt{-1-i\sqrt{3}}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3d}-\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(
Sqrt[3]*d)) - (Sqrt[-1 - I*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)
^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) -
Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(Sqrt[6]*c^(1/3)*d) - (b^(1/3)*Log[-(b
^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*d) + ((I/6)*(I*(b*c - a*d)^(1/3) + Sqrt[
3]*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3)])/(c^(1/3)*d) + (b^(1/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*d) + (((b*c - a*d)^(1/3) - I*Sqrt[3]*
(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(
1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3
)^(2/3)])/(12*c^(1/3)*d)
```

**fricas [A]** time = 0.48, size = 330, normalized size = 1.41

$$\frac{2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3(bc-ad)x}\right)-2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3bx}\right)+2(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+2\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(-\frac{x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{x^2\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b*c - a*d)/c)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2
*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) - 2*sq
rt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)
^(2/3))/(b*x)) + 2*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2
*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 + a)^(1/3))/x
) - (-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^
3 + a)^(2/3))/x^2) - ((b*c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) +
 (b*x^3 + a)^(1/3)*x*((b*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}x}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(b\,x^3+a\right)^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

## 3.424 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx$

**Optimal**. Leaf size=168

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

**Rubi** [C] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 0.52, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(c*x*(1 + (b*x^3)/a)^(1/3))
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1+\frac{dx^3}{c}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.04, size = 81, normalized size = 0.48

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x*(1 + (b*x^3)/a)^(1/3)))
```

**IntegrateAlgebraic [C]** time = 2.27, size = 363, normalized size = 2.16

$$\frac{\left(\sqrt[3]{bc-ad}-i\sqrt{3}\sqrt[3]{bc-ad}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{4/3}}+\frac{\sqrt{-1-i\sqrt{3}}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}c^{4/3}}+\frac{i\left(\sqrt{3}\sqrt[3]{bc-ad}+i\sqrt[3]{bc-ad}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{4/3}}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(c*x)) + (Sqrt[-1 - I*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan
[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a +
b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(Sqrt[6]*c^(4/3)) + (((
b*c - a*d)^(1/3) - I*Sqrt[3]*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x +
 (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(4/3)) + ((I/12)*(I*(b*c
- a*d)^(1/3) + Sqrt[3]*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2
+ c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt
[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(4/3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)
```

## 3.425 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$

**Optimal**. Leaf size=204

$$-\frac{d\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}}-\frac{\sqrt[3]{a+bx^3}}{4a\ldots}$$

**Rubi [C]** time = 0.13, antiderivative size = 145, normalized size of antiderivative = 0.71, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{x^3\left(c-3dx^3\right)(-(bc-ad))\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+2c\left(a+bx^3\right)\left(c-3dx^3\right)}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(2*c*(a + b*x^3)*(c - 3*d*x^3) - (b*c - a*d)*x^3*(c - 3*d*x^3)*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*(b*c - a*d)*x^3*
(c + d*x^3)*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)])/(8*c^3*x^4*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$=-\frac{2c\left(a+bx^3\right)\left(c-3dx^3\right)-(bc-ad)x^3\left(c-3dx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)+3(bc-ad)x^3\left(c\ldots\right.}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

**Mathematica [C]** time = 0.78, size = 146, normalized size = 0.72

$$-\frac{x^3\left(3dx^3-c\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+2c\left(a+bx^3\right)\left(c-3dx^3\right)}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/8*(2*c*(a + b*x^3)*(c - 3*d*x^3) + (b*c - a*d)*x^3*(-c + 3*d*x^3)*Hyperg
eometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*(b*c - a*d)
*x^3*(c + d*x^3)*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))])/(c^3*x^4*(a + b*x^3)^(2/3))
```

**IntegrateAlgebraic [C]** time = 2.30, size = 394, normalized size = 1.93

$$\frac{i\left(\sqrt{3}d\sqrt[3]{bc-ad}+id\sqrt[3]{bc-ad}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{7/3}}-\frac{\sqrt{-1-i\sqrt{3}}\,d\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}c^{7/3}}+\frac{\left(d\sqrt[3]{bc-ad}-i\sqrt{3}d\sqrt[3]{bc-ad}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{7/3}}+\frac{\sqrt[3]{a+bx^3}\left(-ac+4adx^3-bcx^3\right)}{4ac^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-(a*c) - b*c*x^3 + 4*a*d*x^3))/(4*a*c^2*x^4) - (Sqrt[-1
 - I*Sqrt[3]]*d*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a +
 b*x^3)^(1/3))]])/(Sqrt[6]*c^(7/3)) + ((I/6)*(I*d*(b*c - a*d)^(1/3) + Sqrt[3
]*d*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3)])/c^(7/3) + ((d*(b*c - a*d)^(1/3) - I*Sqrt[3]*d*(b*c - a*
d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x
 - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/(12*c^(7/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)
```

## 3.426 $\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=258

$$\frac{\sqrt[3]{a+bx^3}\left(-28a^2d^2+7abcd+3b^2c^2\right)}{28a^2c^3x}+\frac{d^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{10/3}}-\frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}-$$

**Rubi [C]** time = 0.94, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-2x^3\left(2c^2-3cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-12bc^2dx^6\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+15bc^3x^3\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-15ac^2dx^3\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+27ad^3x^9\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-27bcd^2x^9\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+12acd^2x^6\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-12ac^2dx^3+8ac^3+36acd^2x^6-12bc^2dx^6+8bc^3x^3+36bcd^2x^9}{56c^4x^7\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2*x^6
+ 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*Hyperg
eometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3*x^3*
Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*a*c^
2*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
 12*b*c^2*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 12*a*c*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hypergeometr
icPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(56*c^4*x^
7*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{8ac^3+8bc^3x^3-12ac^2dx^3-12bc^2dx^6+36acd^2x^6+36bcd^2x^9-2(bc-ad)x^3\left(2c^2-3cd\right.}{}$$

**Mathematica [C]** time = 2.75, size = 451, normalized size = 1.75

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+15bc^3x^3\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-2x^3\left(2c^2-3cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-15ac^2dx^3\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-12bc^2dx^6\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+27ad^3x^9\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-27bcd^2x^9\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+12acd^2x^6\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+8ac^3-12ac^2dx^3+36acd^2x^6+8bc^3x^3-12bc^2dx^6+36bcd^2x^9}{56c^4x^7\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -1/56*(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2
*x^6 + 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*H
ypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3
*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15
*a*c^2*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] - 12*b*c^2*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 12*a*c*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hyperge
ometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*
x^7*(a + b*x^3)^(2/3))
```

**IntegrateAlgebraic [C]** time = 2.62, size = 444, normalized size = 1.72

$$\frac{\sqrt[3]{a+bx^3}\left(-4a^2c^2+7a^2cdx^3-28a^2d^2x^6-abc^2x^3+7abcdx^6+3b^2c^2x^6\right)}{28a^2c^3x^7}+\frac{\left(d^2\sqrt[3]{bc-ad}-i\sqrt{3}d^2\sqrt[3]{bc-ad}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{10/3}}+\frac{\sqrt{-1-i\sqrt{3}}\,d^2\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}c^{10/3}}+\frac{i\left(\sqrt{3}d^2\sqrt[3]{bc-ad}+id^2\sqrt[3]{bc-ad}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-4*a^2*c^2 - a*b*c^2*x^3 + 7*a^2*c*d*x^3 + 3*b^2*c^2*x^
6 + 7*a*b*c*d*x^6 - 28*a^2*d^2*x^6))/(28*a^2*c^3*x^7) + (Sqrt[-1 - I*Sqrt[3
]]*d^2*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d
)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(
1/3))])/(Sqrt[6]*c^(10/3)) + ((d^2*(b*c - a*d)^(1/3) - I*Sqrt[3]*d^2*(b*c -
 a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3)])/(6*c^(10/3)) + ((I/12)*(I*d^2*(b*c - a*d)^(1/3) + Sqrt[3]*d^2*(b*
c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3
)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(
2/3)])/c^(10/3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^8\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)
```

## 3.427 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=318

$$\frac{\sqrt[3]{a+bx^3}\left(-35a^2d^2+5abcd+3b^2c^2\right)}{140a^2c^3x^4}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3\right)}{140a^3c^4x}-\frac{d^3\sqrt[3]{bc-ad}\ \mathrm{lo}}{6c^{13/}}$$

**Rubi [C]** time = 2.64, antiderivative size = 905, normalized size of antiderivative = 2.85, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

---

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -(56*a*c^4 + 56*b*c^4*x^3 - 72*a*c^3*d*x^3 - 72*b*c^3*d*x^6 + 108*a*c^2*d^2
*x^6 + 108*b*c^2*d^2*x^9 - 324*a*c*d^3*x^9 - 324*b*c*d^3*x^12 - 28*b*c^4*x^
3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 28*a*
c^3*d*x^3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 36*b*c^3*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 36*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3
)/(c*(a + b*x^3))] - 54*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] + 54*a*c*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[2
/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 117*b*c^4*x^3*Hype
rgeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 117*a*c^3*d
*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99
*b*c^3*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] + 81*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] - 81*a*c*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[2/3, 2
, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F
1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c - a*d)*x^3*(2*c
 - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*Hypergeometri
cPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560*
c^5*x^10*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{56ac^4+56bc^4x^3-72ac^3dx^3-72bc^3dx^6+108ac^2d^2x^6+108bc^2d^2x^9-324acd^3x^9-324b}{}$$

**Mathematica [C]** time = 5.24, size = 214, normalized size = 0.67

$$\frac{\frac{\left(a+bx^3\right)\left(a^3\left(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9\right)+a^2bcx^3\left(-2c^2+5cdx^3-35d^2x^6\right)+3ab^2c^2x^6\left(c-5dx^3\right)-9b^3c^3x^9\right)}{70a^3c^4x^{10}}+\frac{d^3x^2\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^5\left(\frac{dx^3}{c}+1\right)^{2/3}}}{2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] (((a + b*x^3)*(-9*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(c - 5*d*x^3) + a^2*b*c*x^3
*(-2*c^2 + 5*c*d*x^3 - 35*d^2*x^6) + a^3*(-14*c^3 + 20*c^2*d*x^3 - 35*c*d^2
*x^6 + 140*d^3*x^9)))/(70*a^3*c^4*x^10) + (d^3*(-(b*c) + a*d)*x^2*(1 + (b*x
^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c +
d*x^3))])/(c^5*(1 + (d*x^3)/c)^(2/3)))/(2*(a + b*x^3)^(2/3))
```

**IntegrateAlgebraic [C]** time = 2.99, size = 503, normalized size = 1.58

$$\frac{\sqrt[3]{a+bx^3}\left(-14a^3c^3+20a^3c^2dx^3-35a^3cd^2x^6+140a^3d^3x^9-2a^2bc^3x^3+5a^2bc^2dx^6-35a^2bcd^2x^9+3ab^2c^3x^6-15ab^2c^2dx^9-9b^3c^3x^9\right)}{140a^3c^4x^{10}}+\frac{i\left(\sqrt{3}d^3\sqrt[3]{bc-ad}+id^3\sqrt[3]{bc-ad}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{13/3}}-\frac{\sqrt{-1-i\sqrt{3}}\,d^3\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}c^{13/3}}+\frac{\left(d^3\sqrt[3]{bc-ad}-i\sqrt{3}d^3\sqrt[3]{bc-ad}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{13/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-14*a^3*c^3 - 2*a^2*b*c^3*x^3 + 20*a^3*c^2*d*x^3 + 3*a*
b^2*c^3*x^6 + 5*a^2*b*c^2*d*x^6 - 35*a^3*c*d^2*x^6 - 9*b^3*c^3*x^9 - 15*a*b
^2*c^2*d*x^9 - 35*a^2*b*c*d^2*x^9 + 140*a^3*d^3*x^9))/(140*a^3*c^4*x^10) -
(Sqrt[-1 - I*Sqrt[3]]*d^3*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/
(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^
(1/3)*(a + b*x^3)^(1/3))])/(Sqrt[6]*c^(13/3)) + ((I/6)*(I*d^3*(b*c - a*d)^(
1/3) + Sqrt[3]*d^3*(b*c - a*d)^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sq
rt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(13/3) + ((d^3*(b*c - a*d)^(1/3) - I*S
qrt[3]*d^3*(b*c - a*d)^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b
*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)
*(a + b*x^3)^(2/3)])/(12*c^(13/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**11*(c + d*x**3)), x)
```

## 3.428 $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=266

$$\frac{\left(a+bx^3\right)^{5/3}\left(a^2d^2+abcd+b^2c^2\right)}{5b^3d^3}-\frac{\left(a+bx^3\right)^{8/3}(2ad+bc)}{8b^3d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^3d}+\frac{c^3(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{14/3}}-\frac{c^3(bc-a$$

**Rubi [A]** time = 0.32, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{5/3}\left(a^2d^2+abcd+b^2c^2\right)}{5b^3d^3}-\frac{\left(a+bx^3\right)^{8/3}(2ad+bc)}{8b^3d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^3d}-\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4}+\frac{c^3(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{14/3}}-\frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}}-\frac{c^3(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -(c^3*(a + b*x^3)^(2/3))/(2*d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^
3)^(5/3))/(5*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d^2) + (a
+ b*x^3)^(11/3)/(11*b^3*d) - (c^3*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(14/3)) + (c^3*(
b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[
(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{\left(b^2c^2+abcd+a^2d^2\right)(a+bx)^{2/3}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{5/3}}{b^2d^2} + \frac{(a+bx)^{8/3}}{b^2d}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} - \frac{c^3\,\text{Subs}}{}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+}{}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+}{}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+}{}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+}{}$$

**Mathematica [C]** time = 0.12, size = 148, normalized size = 0.56

$$\frac{\left(a+bx^3\right)^{2/3}\left(18a^3d^3+3a^2bd^2\left(11c-4dx^3\right)+220b^3c^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2ab^2d\left(44c^2-11cdx^3+5d^2x^6\right)+b^3\left(-220c^3+88c^2dx^3-55cd^2x^6+40d^3x^9\right)\right)}{440b^3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) + 2*a*b^2*d*(
44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*d^2*
```

```
x^6 + 40*d^3*x^9) + 220*b^3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^
3))/(-(b*c) + a*d)]))/(440*b^3*d^4)
```

**IntegrateAlgebraic [A]** time = 0.65, size = 340, normalized size = 1.28

$$\frac{\left(a+bx^3\right)^{2/3}\left(18a^3d^3+33a^2bcd^2-12a^2bd^3x^3+88ab^2c^2d-22ab^2cd^2x^3+10ab^2d^3x^6-220b^3c^3+88b^3c^2dx^3-55b^3cd^2x^6+40b^3d^3x^9\right)}{440b^3d^4}-\frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{14/3}}+\frac{c^3(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{14/3}}-\frac{c^3(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{14/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3
*d^3 + 88*b^3*c^2*d*x^3 - 22*a*b^2*c*d^2*x^3 - 12*a^2*b*d^3*x^3 - 55*b^3*c*
d^2*x^6 + 10*a*b^2*d^3*x^6 + 40*b^3*d^3*x^9))/(440*b^3*d^4) - (c^3*(b*c - a
*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c -
a*d)^(1/3))])/(Sqrt[3]*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(14/3)) + (c^3*(b*c - a*d)^(2/3)*Lo
g[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)
*(a + b*x^3)^(2/3)])/(6*d^(14/3))
```

**fricas [B]** time = 0.69, size = 455, normalized size = 1.71

$$-\frac{440\sqrt{3}b^3c^3\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+220b^3c^3\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{2}{3}}(bc-ad)+(bc-ad)\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)-440b^3c^3\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\right)-3\left(40b^3d^3x^9-5\left(11b^3cd^2-2ab^2d^3\right)x^6-220b^3c^3+88ab^2c^2d+33a^2bcd^2+18a^3d^3+2\left(44b^3c^2d-11ab^2cd^2-6a^2bd^3\right)x^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{1320b^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*a
rctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2 -
2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2
/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^3*c*d^2
- 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3
*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^3 + a)^(2/
3))/(b^3*d^4)
```

**giac [A]** time = 0.29, size = 409, normalized size = 1.54

$$-\frac{\left(b^{37}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{36}c^3d^8\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{37}cd^{11}-ab^{36}d^{12}\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^6}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^6}-\frac{220\left(bx^3+a\right)^{\frac{2}{3}}b^{33}c^3d^7-88\left(bx^3+a\right)^{\frac{5}{3}}b^{32}c^2d^8+55\left(bx^3+a\right)^{\frac{8}{3}}b^{31}cd^9-88\left(bx^3+a\right)^{\frac{5}{3}}ab^{31}cd^9-40\left(bx^3+a\right)^{\frac{11}{3}}b^{30}d^{10}+110\left(bx^3+a\right)^{\frac{8}{3}}ab^{30}d^{10}-88\left(bx^3+a\right)^{\frac{5}{3}}a^2b^{30}d^{10}}{440b^{33}d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d)
^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)
*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b
*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3 + a)^
(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/
d^6 - 1/440*(220*(b*x^3 + a)^(2/3)*b^33*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^32
*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^31*c*d^9 - 88*(b*x^3 + a)^(5/3)*a*b^31*c*
d^9 - 40*(b*x^3 + a)^(11/3)*b^30*d^10 + 110*(b*x^3 + a)^(8/3)*a*b^30*d^10 -
 88*(b*x^3 + a)^(5/3)*a^2*b^30*d^10)/(b^33*d^11)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} x^{11}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.13, size = 490, normalized size = 1.84

$$\left(\frac{3a^2}{5b^3d}+\frac{\left(\frac{3a}{b^3d}+\frac{b^4c-ab^3d}{b^6d^2}\right)\left(b^4c-ab^3d\right)}{5b^3d}\right)\left(bx^3+a\right)^{5/3}-\left(\frac{3a}{8b^3d}+\frac{b^4c-ab^3d}{8b^6d^2}\right)\left(bx^3+a\right)^{8/3}-\left(bx^3+a\right)^{2/3}\left(\frac{a^3}{2b^3d}+\frac{\left(\frac{3a^2}{b^3d}+\frac{\left(\frac{3a}{b^3d}+\frac{b^4c-ab^3d}{b^6d^2}\right)\left(b^4c-ab^3d\right)}{b^3d}\right)\left(b^4c-ab^3d\right)}{2b^3d}\right)+\frac{\left(bx^3+a\right)^{11/3}}{11b^3d}-\frac{c^3\ln\left(\frac{\left(bx^3+a\right)^{1/3}\left(a^2c^6d^2-2abc^7d+b^2c^8\right)}{d^7}-\frac{c^6\left(ad-bc\right)^{4/3}\left(9ad^3-9bcd^2\right)}{9d^{28/3}}\right)\left(ad-bc\right)^{2/3}}{3d^{14/3}}-\frac{c^3\ln\left(\frac{c^6\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(ad-bc\right)^{7/3}}{d^{22/3}}+\frac{c^6\left(bx^3+a\right)^{1/3}\left(ad-bc\right)^2}{d^7}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(ad-bc\right)^{2/3}}{3d^{14/3}}+\frac{c^3\ln\left(\frac{c^6\left(bx^3+a\right)^{1/3}\left(ad-bc\right)^2}{d^7}-\frac{c^6\left(1+\sqrt{3}\,1i\right)^2\left(ad-bc\right)^{7/3}}{4d^{22/3}}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(ad-bc\right)^{2/3}}{d^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(5*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(5*b^3*d))*(a + b*x^3)^(5/3) - ((3*a)/(8*b^3*d) + (b^4*c - a*b^
3*d)/(8*b^6*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*(a^3/(2*b^3*d) + ((
(3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a
*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(2*b^3*d)) + (a + b*x^3)^(11/3)/(11*b^
3*d) - (c^3*log(((a + b*x^3)^(1/3)*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))/d
^7 - (c^6*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(28/3)))*(a*d - b*c
)^(2/3))/(3*d^(14/3)) - (c^3*log((c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7
/3))/d^(22/3) + (c^6*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^7)*((3^(1/2)*1i)/2
- 1/2)*(a*d - b*c)^(2/3))/(3*d^(14/3)) + (c^3*log((c^6*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^7 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(7/3))/(4*d^(22/3)))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(2/3))/d^(14/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

## 3.429 $\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=223

$$-\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2}+\frac{(a+bx^3)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}$$

**Rubi [A]** time = 0.26, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$-\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2}+\frac{(a+bx^3)^{8/3}}{8b^2d}+\frac{c^2(a+bx^3)^{2/3}}{2d^3}-\frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(2/3))/(2*d^3) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^2*d^
2) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^
(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(11/3)) -
(c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(11/3)) + (c^2*(b*c - a*d)^(2/3
)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{2/3}}{bd^2}+\frac{(a+bx)^{5/3}}{bd}+\frac{c^2(a+bx)^{2/3}}{d^2(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}+\frac{c^2\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{\left(c^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\cdots}\right.}{3d^3}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}}+\cdots$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}}+\cdots$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}+\frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a}\cdots}{\sqrt[3]{bc-}\cdots}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}}\cdots$$

**Mathematica [C]** time = 0.10, size = 104, normalized size = 0.47

$$\frac{\left(a+bx^3\right)^{2/3}\left(-3a^2d^2-20b^2c^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2abd\left(dx^3-4c\right)+b^2\left(20c^2-8cdx^3+5d^2x^6\right)\right)}{40b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(20*c^2 - 8*c
*d*x^3 + 5*d^2*x^6) - 20*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x
^3))/(-(b*c) + a*d)]))/(40*b^2*d^3)
```

**IntegrateAlgebraic [A]** time = 0.40, size = 284, normalized size = 1.27

$$\frac{\left(a+bx^3\right)^{2/3}\left(-3a^2d^2-8abcd+2abd^2x^3+20b^2c^2-8b^2cdx^3+5b^2d^2x^6\right)}{40b^2d^3}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{11/3}}-\frac{c^2(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{11/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(20*b^2*c^2 - 8*a*b*c*d - 3*a^2*d^2 - 8*b^2*c*d*x^3 + 2*
a*b*d^2*x^3 + 5*b^2*d^2*x^6))/(40*b^2*d^3) + (c^2*(b*c - a*d)^(2/3)*ArcTan[
1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sq
rt[3]*d^(11/3)) + (c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
 + b*x^3)^(1/3)])/(3*d^(11/3)) - (c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/
3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3
)])/(6*d^(11/3))
```

**fricas [B]** time = 0.61, size = 398, normalized size = 1.78

$$\frac{40\sqrt{3}b^2c^2\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)-20b^2c^2\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{2}{3}}(bc-ad)-(bc-ad)\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)+40b^2c^2\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\right)+3\left(5b^2d^2x^6+20b^2c^2-8abcd-3a^2d^2-2\left(4b^2cd-abd^2\right)x^3\right)\left(bx^3+a\right)^{\frac{2}{3}}}{120b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arcta
n(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3
 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*d - 3*a^
2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3))/(b^2*d^3)
```

**giac [A]** time = 0.27, size = 350, normalized size = 1.57

$$\frac{\left(b^{19}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{18}c^2d^6\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{19}cd^8-ab^{18}d^9\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^5}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^5}+\frac{20\left(bx^3+a\right)^{\frac{2}{3}}b^{16}c^2d^5-8\left(bx^3+a\right)^{\frac{5}{3}}b^{15}cd^6+5\left(bx^3+a\right)^{\frac{8}{3}}b^{14}d^7-8\left(bx^3+a\right)^{\frac{5}{3}}ab^{14}d^7}{40b^{16}d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)^
(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c^
2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
- a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a)^(2/
3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5
 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^15*c*d^6
 + 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(b^16*d^8
)
```

**maple [F]** time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{8}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.11, size = 385, normalized size = 1.73

$$\left(\frac{a^2}{2\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{2\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\left(\frac{2\,a}{5\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{5\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^2\,d}+\frac{c^2\,\ln\left(\frac{{\left(b\,x^3+a\right)}^{1/3}\,\left(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6\right)}{d^5}-\frac{c^4\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{22/3}}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}}-\frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{16/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}}+\frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{7/3}}{4\,d^{16/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{d^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(2*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(2*b^2*d))*(a + b*x^3)^(2/3) - ((2*a)/(5*b^2*d) + (b^3*c - a*b^2*d)
/(5*b^4*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*log(((
a + b*x^3)^(1/3)*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d))/d^5 - (c^4*(a*d - b
*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(22/3)))*(a*d - b*c)^(2/3))/(3*d^(11/
3)) - (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*((3^(1/2)*1
i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(16/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(2/3))/(3*d^(11/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 -
 (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(16/3)))*((3^(1/2)*1i)/6 -
 1/6)*(a*d - b*c)^(2/3))/d^(11/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8} \left(a + b x^{3}\right)^{\frac{2}{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

## 3.430 $\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=188

$$\frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}-\frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}}-\frac{c(a}{}$$

**Rubi [A]** time = 0.20, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 56, 617, 204, 31}

$$-\frac{c\left(a+bx^3\right)^{2/3}}{2d^2}+\frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}-\frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}}+\frac{\left(a+bx^3\right)^{5/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -(c*(a + b*x^3)^(2/3))/(2*d^2) + (a + b*x^3)^(5/3)/(5*b*d) - (c*(b*c - a*d)
^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(8/3)) - (
c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5 \left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{5/3}}{5bd} - \frac{c\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{(c(bc-ad))\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{\left(c(bc-ad)^{2/3}\right)\operatorname{Subst}\left(\int\right.}{2}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-}\right.}{2d^{8/}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} - \frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}} + \frac{c(bc-ad)^{2/3}\log}{6d^{8/3}}$$

**Mathematica [C]** time = 0.05, size = 68, normalized size = 0.36

$$\frac{\left(a+bx^3\right)^{2/3}\left(5bc\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2ad-5bc+2bdx^3\right)}{10bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3 + 5*b*c*Hypergeometric2F1[2/
3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]))/(10*b*d^2)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 241, normalized size = 1.28

$$-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{8/3}}+\frac{c(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{8/3}}+\frac{\left(a+bx^3\right)^{2/3}\left(2ad-5bc+2bdx^3\right)}{10bd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3))/(10*b*d^2) - (c*(b*c - a*d
)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*
d)^(1/3))])/(Sqrt[3]*d^(8/3)) - (c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3)
+ d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[(b*c -
 a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*
x^3)^(2/3)])/(6*d^(8/3))
```

**fricas [B]** time = 0.41, size = 353, normalized size = 1.88

$$-\frac{10\sqrt{3}bc\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+5bc\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{2}{3}}(bc-ad)+(bc-ad)\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)-10bc\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(bc-ad)\right)-3(2bdx^3-5bc+2ad)(bx^3+a)^{\frac{2}{3}}}{30bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(
1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2
)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^
2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^
(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3))/(b*d^
2)
```

**giac [B]** time = 0.55, size = 306, normalized size = 1.63

$$-\frac{\left(b^7c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^6cd^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^7cd^5-ab^6d^6\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^4}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^4}-\frac{5(bx^3+a)^{\frac{2}{3}}b^5cd^3-2(bx^3+a)^{\frac{5}{3}}b^4d^4}{10b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^7*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b^6*c*d^4*(-(b*c - a*d)/d)^(1/
3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/
3)))/(b^7*c*d^5 - a*b^6*d^6) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c*arcta
n(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/
d)^(1/3))/d^4 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 - 1/10*(
5*(b*x^3 + a)^(2/3)*b^5*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^4*d^4)/(b^5*d^5)
```

**maple [F]** time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} x^5}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.06, size = 302, normalized size = 1.61

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b\,d}-\left(b\,x^3+a\right)^{2/3}\,\left(\frac{a}{2\,b\,d}+\frac{b^2\,c-a\,b\,d}{2\,b^2\,d^2}\right)-\frac{c\,\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}{d^3}-\frac{c^2\,\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{16/3}}\right)\,\left(a\,d-b\,c\right)^{2/3}}{3\,d^{8/3}}-\frac{c\,\ln\left(\frac{c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{7/3}}{d^{10/3}}+\frac{c^2\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{2/3}}{3\,d^{8/3}}+\frac{c\,\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)^2}{d^3}-\frac{c^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{7/3}}{d^{10/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{2/3}}{3\,d^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b*d) - (a + b*x^3)^(2/3)*(a/(2*b*d) + (b^2*c - a*b*d)/
(2*b^2*d^2)) - (c*log(((a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3
*d))/d^3 - (c^2*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(16/3)))*(a*d
 - b*c)^(2/3))/(3*d^(8/3)) - (c*log((c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(7/3))/d^(10/3) + (c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3)*((3^(1/2)*1i)
/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3)) + (c*log((c^2*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^3 - (c^2*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(10/3))
*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

**3.431** $\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=162

$$-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}}+\frac{(a+$$

**Rubi [A]** time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {444, 50, 56, 617, 204, 31}

$$-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}}+\frac{\left(a+bx^3\right)^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*
x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)) - ((b*c - a*d)^(
2/3)*Log[c + d*x^3])/(6*d^(5/3)) + ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+l\ldots}\right.}{2d^{5/3}}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\ldots\right.}{2d^{5/3}}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} + \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-a\ldots}{\ldots}$$

**Mathematica [C]** time = 0.03, size = 47, normalized size = 0.29

$$-\frac{\left(a+bx^3\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3)
)/(-(b*c) + a*d)]))/d
```

**IntegrateAlgebraic [A]** time = 0.25, size = 218, normalized size = 1.35

$$\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{3d^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{5/3}} + \frac{\left(a+bx^3\right)^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(5/3)) + ((b*c
- a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(5/3)
```

```
) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
 + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(5/3))
```

**fricas [B]** time = 0.44, size = 323, normalized size = 1.99

$$\frac{2\sqrt{3}\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}d\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}-\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)-\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\log\left(\left(bx^{3}+a\right)^{\frac{1}{3}}d\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{2}{3}}-\left(bx^{3}+a\right)^{\frac{2}{3}}\left(bc-ad\right)-\left(bc-ad\right)\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\right)+2\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{1}{3}}\log\left(-d\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{d^{2}}\right)^{\frac{2}{3}}-\left(bx^{3}+a\right)^{\frac{1}{3}}\left(bc-ad\right)\right)+3\left(bx^{3}+a\right)^{\frac{2}{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - sq
rt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/
3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b
*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(b
*x^3 + a)^(2/3))/d
```

**giac [B]** time = 0.27, size = 259, normalized size = 1.60

$$\frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^{2}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd^{2}-ad^{3}\right)}+\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{2\,d}+\frac{\sqrt{3}\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^{3}}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^2
 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (
b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{2}}{d\,x^{3}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 5.05, size = 238, normalized size = 1.47

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2\,d}+\frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{d}-\frac{(a\,d-b\,c)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{10/3}}\right)\,(a\,d-b\,c)^{2/3}}{3\,d^{5/3}}-\frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,(a\,d-b\,c)^2}{d}-\frac{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{7/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{2/3}}{3\,d^{5/3}}+\frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,(a\,d-b\,c)^2}{d}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{7/3}}{4\,d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{2/3}}{d^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + (log(((a + b*x^3)^(1/3)*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d))/d - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(10/3)))*(a*d -
b*c)^(2/3))/(3*d^(5/3)) - (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - (((3^(
1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^(2/3))/(3*d^(5/3)) + (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - ((3^(
1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(4/3)))*((3^(1/2)*1i)/6 - 1/6)*(a*d
- b*c)^(2/3))/d^(5/3)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b x^{3}\right)^{\frac{2}{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

## 3.432 $\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=245

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{\right.}}{}$$

**Rubi [A]** time = 0.23, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 83, 55, 617, 204, 31, 56}

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}-\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*c) - ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(2/3)) - (a^(2/3)*Log[x])/(2*c) + ((b*c
- a*d)^(2/3)*Log[c + d*x^3])/(6*c*d^(2/3)) + (a^(2/3)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*c*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a \operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} - \frac{a^{2/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} + \frac{a\operatorname{Sub}}{}$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\log}{}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} - \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{2/3}} - \frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log}{6cd^{2/3}}$$

**Mathematica [C]** time = 0.12, size = 115, normalized size = 0.47

$$\frac{a^{2/3}\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)+3\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (3*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c)
 + a*d)] + a^(2/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/(6*c)
```

**IntegrateAlgebraic [A]** time = 0.47, size = 333, normalized size = 1.36

$$\frac{a^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3c} - \frac{a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6c} + \frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,c} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3cd^{2/3}} + \frac{(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6cd^{2/3}} - \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,cd^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*c) - ((b*c - a*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3)
)/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*c*d^(2/3)) + (a^(2/3)*Log[-a^(1/3)
 + (a + b*x^3)^(1/3)])/(3*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(3*c*d^(2/3)) - (a^(2/3)*Log[a^(2/3) + a^(1/3)*(
a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*c) + ((b*c - a*d)^(2/3)*Log[(b*c
- a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b
*x^3)^(2/3)])/(6*c*d^(2/3))
```

**fricas [B]** time = 0.46, size = 425, normalized size = 1.73

$$-\frac{2\sqrt{3}\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3(bc-ad)}\right)-2\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{1}{3}}}{3a}\right)+\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{2}{3}}(bc-ad)+(bc-ad)\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\right)+\left(a^2\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-2\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(-d\left(-\frac{b^2c^2-2abcd+a^2d^2}{d^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\right)-2\left(a^2\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2
*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) +
 sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt
(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(1/3))/a) + (-(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2
- 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + (a^2)^(1/3)*log((b*x^3 + a)^(2/3)*a +
(a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(-(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) -
(b*x^3 + a)^(1/3)*(b*c - a*d)) - 2*(a^2)^(1/3)*log((b*x^3 + a)^(1/3)*a - (a
^2)^(2/3)))/c
```

**giac [A]** time = 0.80, size = 341, normalized size = 1.39

$$-\frac{\left(bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3c}-\frac{a^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6c}+\frac{a^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3c}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*(-(b*c - a*d)/d)^(1/3) - a*d*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*d
)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2 - a*
c*d) + 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3
))/a^(1/3))/c - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/
3) + a^(2/3))/c + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c - 1/3
*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) + 1/6*(-b*c*d^2 +
a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(d\,x^{3}+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)
```

**mupad [B]** time = 4.77, size = 1963, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x*(c + d*x^3)),x)
[Out] log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*
a^3*b^7*c^2*d^2) - (a^2/(27*c^3))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4
*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729
*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3)
)^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*
a*b^8*c^5*d))*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 -
 a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (-(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3
- 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729*a^2*
b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^
3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + 36*a
^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*
(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - log((a + b*x^3)^(1/
3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) +
((3^(1/2)*1i)/2 + 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a +
 b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^
5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 48
6*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) - 36*a^2*b^7*
c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/
2)*1i)/2 + 1/2)*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4
 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^(1/2)*1i)/2 -
 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54
*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i
)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5
)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*
b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(
a^2/(27*c^3))^(1/3) - log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d
- 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) + ((3^(1/2)*1i)/2 + 1/2)^2*(-(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a +
b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5
) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486
*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - 36*a^2*b^7*c^4*d^2 +
54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 +
 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + log((a + b*x^
3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d
^2) - ((3^(1/2)*1i)/2 - 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^
2))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 -
108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 - 1/2)^2*(243*a
*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*
```

```
c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27
*c^3*d^2))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2
*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)/(27*c^3*d^2))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)
```

## 3.433 $\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)} \, dx$

**Optimal**. Leaf size=347

$$\frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{\left(a+bx^3\right)^{2/3}(2bc-3ad)}{6ac^2}-\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2}+\cdots$$

**Rubi [A]** time = 0.39, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 55, 617, 204, 31, 56}

$$\frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{\left(a+bx^3\right)^{2/3}(2bc-3ad)}{6ac^2}-\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2}+\frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}+\frac{(2bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}c^2}+\frac{\sqrt[3]{d}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}-\frac{\log(x)(2bc-3ad)}{6\sqrt[3]{a}c^2}-\frac{\left(a+bx^3\right)^{5/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(2/3))/(2*c^2) + ((2*b*c - 3*a*d)*(a + b*x^3)^(2/3))/(6*a*c^
2) - (a + b*x^3)^(5/3)/(3*a*c*x^3) + ((2*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b
*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*c^2) - ((2*b*c - 3*a*d)*Log[x])/(6*a^(1/3)*c^2) - (d^(
1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^2) + ((2*b*c - 3*a*d)*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(c+dx)}\,dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3}-\frac{\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(-2bc+3ad)-\frac{2bdx}{3}\right)}{x(c+dx)}\,dx,x,x^3\right)}{3ac}\\
&= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)}{3c^2}+\frac{(2bc-3ad)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx,x,x^3\right)}{9ac^2}\\
&= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2}-\frac{\left(a+bx^3\right)^{5/3}}{3acx^3}+\frac{(2bc-3ad)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,c\right.}{9c^2}\\
&= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2}-\frac{\left(a+bx^3\right)^{5/3}}{3acx^3}-\frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}\,c^2}-\frac{\sqrt[3]{d}\,(bc-}{}\\
&= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2}-\frac{\left(a+bx^3\right)^{5/3}}{3acx^3}-\frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}\,c^2}-\frac{\sqrt[3]{d}\,(bc-}{}\\
&= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2}+\frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2}-\frac{\left(a+bx^3\right)^{5/3}}{3acx^3}+\frac{(2bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,c^2}
\end{aligned}
$$

**Mathematica [C]** time = 0.16, size = 202, normalized size = 0.58

$$
\frac{-9\sqrt[3]{a}\,dx^3\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2\sqrt{3}\,x^3(2bc-3ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-6\sqrt[3]{a}\,c\left(a+bx^3\right)^{2/3}+6bcx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-9adx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+9adx^3\log(x)-6bcx^3\log(x)}{18\sqrt[3]{a}\,c^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-6*a^(1/3)*c*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(2*b*c - 3*a*d)*x^3*ArcTan[(1 +
 (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 9*a^(1/3)*d*x^3*(a + b*x^3)^(2/3
)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)] - 6*b*c*x^
3*Log[x] + 9*a*d*x^3*Log[x] + 6*b*c*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)] -
9*a*d*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(1/3)*c^2*x^3)
```

**IntegrateAlgebraic [A]** time = 0.71, size = 384, normalized size = 1.11

$$
\frac{(3ad-2bc)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{18\sqrt[3]{a}\,c^2}-\frac{\sqrt[3]{d}\,(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9\sqrt[3]{a}\,c^2}+\frac{\sqrt[3]{d}\,(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^2}-\frac{(3ad-2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,c^2}+\frac{\sqrt[3]{d}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,c^2}-\frac{\left(a+bx^3\right)^{2/3}}{3cx^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(2/3)/(c*x^3) - ((-2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
 b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b*c
- a*d)^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c
 - a*d)^(1/3))])/(Sqrt[3]*c^2) + ((2*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3
)^(1/3)])/(9*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3
) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c^2) + ((-2*b*c + 3*a*d)*Log[a^(2/3) + a
^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(1/3)*c^2) - (d^(1/3)*
```

```
(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*
x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c^2)
```

**fricas [A]** time = 0.55, size = 1030, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3
 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)
^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) -
 6*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt
(3)*(b*c - a*d) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^
3 + a)^(1/3))/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)
^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(2*b*c - 3*a*d)*(-a
)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1
/3)*a*x^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(2/3)) + 6*(b*x^3 + a)^(2/3)*a*c)/(a*c^2*x^3), 1/18*(6*sqrt(1/3
)*(2*a*b*c - 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 +
a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) - (
2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1
/3) + (-a)^(1/3)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-
(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/
3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) - 6*(b*x^3
+ a)^(2/3)*a*c)/(a*c^2*x^3)]
```

**giac [A]** time = 0.81, size = 400, normalized size = 1.15

$$\frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)} - \frac{(2bc-3ad)\log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18a^{\frac{1}{3}}c^2} + \frac{\sqrt{3}\left(2a^{\frac{2}{3}}bc - 3a^{\frac{5}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9ac^2} + \frac{\left(2a^{\frac{1}{3}}bc - 3a^{\frac{4}{3}}d\right)\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{9a^{\frac{2}{3}}c^2} + \frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3c^2d} - \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6c^2d} - \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 -
 a*c^2*d) - 1/18*(2*b*c - 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
a^(1/3) + a^(2/3))/(a^(1/3)*c^2) + 1/9*sqrt(3)*(2*a^(2/3)*b*c - 3*a^(5/3)*d
)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a*c^2) + 1/9
*(2*a^(1/3)*b*c - 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c^2) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3
 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) - 1/6
*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
- a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) - 1/3*(b*x^3 + a)^(2/3)/(
c*x^3)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 10.57, size = 1908, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2
- 6*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d
- b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3
*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c
)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2
+ 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/
(27*c^6))^(1/3) + log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^
2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a
*b*c*d)*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1
/3))/9 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^
(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 + (a*b^5*d^4*(27
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 - (a*b^5*d^4*(27
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - (a + b*x^3)^(2/3)/(3*c*x^3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i
)/2 + 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^
2 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3
))/3 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^
2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4
*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i
)/2 + 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/(27*c^6))^(1/3) + log((((3^
```

```
(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a
*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2
)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2
/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3
 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/
9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a
*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c
*d^2)/(27*c^6))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^4\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)
```

## 3.434 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=370

$$-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}c^3}+\frac{\log(x)\left(-9a\right.}{}$$

**Rubi** [A] time = 0.50, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 149, 156, 55, 617, 204, 31, 56}

$$-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}c^3}+\frac{\log(x)\left(-9a^2d^2+6abcd+b^2c^2\right)}{18a^{4/3}c^3}+\frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^3}-\frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}-\frac{d^{4/3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^3}+\frac{\left(a+bx^3\right)^{2/3}(6ad+bc)}{18ac^2x^3}-\frac{\left(a+bx^3\right)^{5/3}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(18*a*c^2*x^3) - (a + b*x^3)^(5/3)/(6*a*c
*x^6) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^
(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(4/3)*c^3
) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 6*a*b*
c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(4/3)*c^3) - (d^(4
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(bc+6ad)+\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+6abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-3ad)x}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{6ac^2}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c^3} - \ldots$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\ldots}{6\ldots}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\ldots}{6\ldots}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{4/3}c^3} - \frac{d^{4/3}\ldots}{\ldots}$$

**Mathematica** [C]     time = 0.38, size = 240, normalized size = 0.65

$$\frac{27a^{4/3}d^2x^6\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+3x^6\log(x)\left(-9a^2d^2+6abcd+b^2c^2\right)-3x^6\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\,x^6\left(-9a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+3\sqrt[3]{a}\,c\left(a+bx^3\right)^{2/3}\left(-3ac+6adx^3-2bcx^3\right)}{54a^{4/3}c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (3*a^(1/3)*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3) - 2*Sqrt[3]
*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^
(1/3))/Sqrt[3]] + 27*a^(4/3)*d^2*x^6*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/
3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2
*d^2)*x^6*Log[x] - 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*Log[a^(1/3) - (a
 + b*x^3)^(1/3)])/(54*a^(4/3)*c^3*x^6)
```

**IntegrateAlgebraic** [A]     time = 0.88, size = 448, normalized size = 1.21

$$\frac{\left(9a^2d^2-6abcd-b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{27a^{4/3}c^3}+\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54a^{4/3}c^3}+\frac{\left(9a^2d^2-6abcd-b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}c^3}-\frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^3}+\frac{d^{4/3}(bc-ad)^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^3}-\frac{d^{4/3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,c^3}+\frac{\left(a+bx^3\right)^{2/3}\left(-3ac+6adx^3-2bcx^3\right)}{18ac^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3))/(18*a*c^2*x^6) + ((-(b
^2*c^2) - 6*a*b*c*d + 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sq
rt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(2/3)*ArcTa
n[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(
Sqrt[3]*c^3) + ((-(b^2*c^2) - 6*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*
x^3)^(1/3)])/(27*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^
```

```
(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2
*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a^(
4/3)*c^3) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
 - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c^3)
```

**fricas** [A]  time = 1.62, size = 1151, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arc
tan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d
^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a
^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*
d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 -
a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)
^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a
*b*c*d^2 - a^2*d^3)^(2/3)) + 3*sqrt(1/3)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d
^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^
(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/
3)*a^(2/3) + 3*a)/x^3) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a
*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(3*a^2
*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6), -1/54
*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arctan(-1
/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*
a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)
*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(2/3)) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a*b
*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*sqrt(1/3
)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d^2)*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)
^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) + 3*(3*a^2*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)
*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6)]
```

**giac** [A]  time = 0.78, size = 493, normalized size = 1.33

$$-\frac{\left(bcd^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)} - \frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3c^3} + \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6c^3} - \frac{\sqrt{3}\left(b^2c^2+6abcd-9a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{27a^{\frac{4}{3}}c^3} - \frac{\left(a^{\frac{1}{3}}b^2c^2+6a^{\frac{4}{3}}bcd-9a^{\frac{7}{3}}d^2\right)\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27a^{\frac{5}{3}}c^3} + \frac{\left(a^{\frac{2}{3}}b^2c^2+6a^{\frac{5}{3}}bcd-9a^{\frac{8}{3}}d^2\right)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54a^2c^3} - \frac{2\left(bx^3+a\right)^{\frac{5}{3}}b^2c+\left(bx^3+a\right)^{\frac{2}{3}}ab^2c-6\left(bx^3+a\right)^{\frac{5}{3}}abd+6\left(bx^3+a\right)^{\frac{2}{3}}a^2bd}{18ab^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2*(-(b*c - a*d)/d)^(1/3) - a*d^3*(-(b*c - a*d)/d)^(1/3))*(-(b*c
 - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^
4 - a*c^3*d) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 + 1/
6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(b^2*c^2 + 6*
a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(4/3)*c^3) - 1/27*(a^(1/3)*b^2*c^2 + 6*a^(4/3)*b*c*d - 9*a^(7/3)*d
^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^3) + 1/54*(a^(2/3)*b^2
*c^2 + 6*a^(5/3)*b*c*d - 9*a^(8/3)*d^2)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*a^(1/3) + a^(2/3))/(a^2*c^3) - 1/18*(2*(b*x^3 + a)^(5/3)*b^2*c + (b*
x^3 + a)^(2/3)*a*b^2*c - 6*(b*x^3 + a)^(5/3)*a*b*d + 6*(b*x^3 + a)^(2/3)*a^
2*b*d)/(a*b^2*c^2*x^6)
```

**maple [F]** time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(d\,x^{3}+c\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{\left(dx^{3}+c\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)
```

**mupad [B]** time = 15.19, size = 2788, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x)
[Out] log(((((27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*
c)^2)/c^9)^(2/3) - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 +
b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^
2))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 +
 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5
*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))
/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*
a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(a^2*d^6 + b^2*c^2*d^4 -
 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((((a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2
 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3 - (
b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2
*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2))*(-(b^2*c^2 - 9*a
^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^
6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*
b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b
*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c
)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))
*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*
a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
 - (((a + b*x^3)^(2/3)*(b^2*c + 6*a*b*d))/(18*c^2) - (b*(a + b*x^3)^(5/3)*(
3*a*d - b*c))/(9*a*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) - log((((3
^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a
*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d -
108*a^3*b*c*d^3))/(3*a^2*c^2) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*
d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^
4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*
a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^
7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d -
15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)*(-(a^2*d^6 + b^2*
```

```
c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((3^(1/2)*1i)/2 + 1/2)*((((3^
(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 +
 b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c
^2) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*
c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3
+ (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^
3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4)
)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d
 - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^
2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27
*c^9))^(1/3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d
^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*
d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) - (a*b^4*c^4*d^3*((3^(
1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2
+ 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(
a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2
- 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*
c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3
))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3
+ 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)
*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*
a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
 + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^
3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b
^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) + (a*b^4*c^4*d^3*((3^(1/2)*1i)/2 +
 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)
^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1
/3))/27 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^
3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*
a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*
b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^
2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(b^6*c^6
- 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*a^4*b^2*c^2*
d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3\right)^{\frac{2}{3}}}{x^7\left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)
```

## 3.435 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=334

$$-\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}d^3}+\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}d^3}-\frac{c^{4/3}(bc-ad)^{2/}}{6}$$

**Rubi [C]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica [C]** time = 0.76, size = 525, normalized size = 1.57

$$\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\sqrt[3]{bc-ad}\left(-a^2d^2-6abcd+9b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2c\left(-a^2\sqrt[3]{c}\,d\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6a^2dx\sqrt[3]{bc-ad}+9b^2dx^7\sqrt[3]{bc-ad}-18b^2cx^4\sqrt[3]{bc-ad}+3abc^{4/3}\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+15abdx^4\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\sqrt[3]{a+bx^3}(ad-3bc)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)-2\sqrt{3}\,a\sqrt[3]{c}\sqrt[3]{a+bx^3}(ad-3bc)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-18abcx\sqrt[3]{bc-ad}\right)}{108bcd^2\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (3*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*x^4*(1 + (b*x^3)/a)^
(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-18*a*b
*c*(b*c - a*d)^(1/3)*x + 6*a^2*d*(b*c - a*d)^(1/3)*x - 18*b^2*c*(b*c - a*d)
^(1/3)*x^4 + 15*a*b*d*(b*c - a*d)^(1/3)*x^4 + 9*b^2*d*(b*c - a*d)^(1/3)*x^7
 - 2*Sqrt[3]*a*c^(1/3)*(-3*b*c + a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c
 - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*(-3*b*
c + a*d)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^
(1/3)] + 3*a*b*c^(4/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x
^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] -
a^2*c^(1/3)*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b +
a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*b*c*
d^2*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [C]** time = 6.45, size = 608, normalized size = 1.82

$$\frac{\left(a^2d^2+6abcd-9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27b^{4/3}d^3}+\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}b^{4/3}d^3}+\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54b^{4/3}d^3}-\frac{i\left(\sqrt{3}c^{4/3}(bc-ad)^{2/3}-ic^{4/3}(bc-ad)^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6d^3}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}c^{4/3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{d^3}+\frac{\left(c^{4/3}(bc-ad)^{2/3}+i\sqrt{3}c^{4/3}(bc-ad)^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^3}+\frac{\left(a+bx^3\right)^{2/3}\left(2adx-6bcx+3bdx^4\right)}{18bd^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-6*b*c*x + 2*a*d*x + 3*b*d*x^4))/(18*b*d^2) + ((9*b^2*c
^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*
x^3)^(1/3))])/(9*Sqrt[3]*b^(4/3)*d^3) + (Sqrt[(-1 + I*Sqrt[3])/6]*c^(4/3)*(
b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*
x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/
d^3 + ((-9*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1
/3)])/(27*b^(4/3)*d^3) - ((I/6)*((-I)*c^(4/3)*(b*c - a*d)^(2/3) + Sqrt[3]*c
^(4/3)*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/
3)*(a + b*x^3)^(1/3)])/d^3 + ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[b^(2/3)
*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(4/3)*d^3) +
 ((c^(4/3)*(b*c - a*d)^(2/3) + I*Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3))*Log[(-2
*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a
+ b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*d^3)
```

**fricas [B]** time = 1.60, size = 1164, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*arctan(-
1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^
(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d + a^2
*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*x - (b*x
^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(
1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2
+ (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x + (b*x^3 +
a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2
*b*d^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*
sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2
/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d - a^2*d^
2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2) - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(b^2*d^3), -1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2
*c*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d +
```

```
a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/
3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c
- a*d)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x
+ (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*
d^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c
*d - a^2*d^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x
^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2*b*d^2)*arc
tan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3
*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))/(b^2*d^3)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^6}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(b\,x^3+a\right)^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

## 3.436 $\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=272

$$\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^2}-\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6\sqrt[3]{b}\,d^2}$$

**Rubi [C]** time = 0.06, antiderivative size = 64, normalized size of antiderivative = 0.24, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica [C]** time = 0.59, size = 286, normalized size = 1.05

$$\frac{\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,(2ad-3bc)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{a+bx^3}}+\frac{2\left(-a\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6x\left(a+bx^3\right)^{2/3}\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)-2\sqrt{3}\,a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}}{36d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*(-3*b*c + 2*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(c*(a + b*x^3)^(1/3)) + (2*(6*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)
*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*Log[c^(1/3) - ((b*c
 - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^
(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)]))/(b*c - a*d)^(1/3))/(36*d)
```

**IntegrateAlgebraic [C]** time = 4.96, size = 549, normalized size = 2.02

$$\frac{(2ad-3bc)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18\sqrt[3]{b}\,d^2}-\frac{i\left(\sqrt{3}\sqrt[3]{c}\,(bc-ad)^{2/3}-i\sqrt[3]{c}\,(bc-ad)^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^2}+\frac{(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9\sqrt[3]{b}\,d^2}+\frac{\left(\sqrt[3]{c}\,(bc-ad)^{2/3}+i\sqrt{3}\sqrt[3]{c}\,(bc-ad)^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6d^2}-\frac{(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\sqrt[3]{b}\,d^2}-\frac{\sqrt{-1+i\sqrt{3}}\sqrt[3]{c}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,d^2}+\frac{x\left(a+bx^3\right)^{2/3}}{3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*d) - ((3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(
b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(1/3)*d^2) - (Sqrt[-1 + I*S
qrt[3]]*c^(1/3)*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a +
 b*x^3)^(1/3))])/(Sqrt[6]*d^2) + ((3*b*c - 2*a*d)*Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)])/(9*b^(1/3)*d^2) + ((c^(1/3)*(b*c - a*d)^(2/3) + I*Sqrt[3]*c^(
1/3)*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)
*(a + b*x^3)^(1/3)])/(6*d^2) + ((-3*b*c + 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*
x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(1/3)*d^2) - ((I/12)*((-I)*
c^(1/3)*(b*c - a*d)^(2/3) + Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))*Log[(-2*I)*(
b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x
^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/d^2
```

**fricas [B]** time = 0.57, size = 1091, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - 2*a*b*d)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d
^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b
*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c -
2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b
^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*
d))/x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*
a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^
2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2
))/(b*d^2), 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(1/3)*(3*b^2*c - 2*a*b*
d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3
))*sqrt(-(-b)^(1/3)/b)/x) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^
(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2
*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a
)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c
^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/
```

```
x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*a*b*
c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^2*c*
d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2))/(
b*d^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^3}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

## 3.437 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=233

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\right.}{2c^{2/3}d}$$

**Rubi** [C] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica** [C] time = 0.06, size = 161, normalized size = 0.69

$$\frac{4acx\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*b*c*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

**IntegrateAlgebraic [C]** time = 0.00, size = 487, normalized size = 2.09

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,d}+\frac{b^{2/3}\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6d}-\frac{i\left(\sqrt{3}\,(bc-ad)^{2/3}-i(bc-ad)^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{2/3}d}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}d}+\frac{\left((bc-ad)^{2/3}+i\sqrt{3}\,(bc-ad)^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(Sq
rt[3]*d) + (Sqrt[(-1 + I*Sqrt[3])/6]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d
)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) -
 Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*
x) + (a + b*x^3)^(1/3)])/(3*d) - ((I/6)*((-I)*(b*c - a*d)^(2/3) + Sqrt[3]*(
b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)])/(c^(2/3)*d) + (b^(2/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)])/(6*d) + (((b*c - a*d)^(2/3) + I*Sqrt[3]*(b*c
 - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)
*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2
/3)])/(12*c^(2/3)*d)
```

**fricas [B]** time = 0.45, size = 469, normalized size = 2.01

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}}{3bx}\right)-2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)}{x}\right)-2\left(-b^2\right)^{\frac{1}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(-b^2\right)^{\frac{1}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)+\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{2/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

## 3.438 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(c+dx^3\right)} dx$

**Optimal**. Leaf size=169

$$\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}}-\frac{\left(a+bx^3\right)}{2cx^2}$$

**Rubi [C]** time = 0.06, antiderivative size = 89, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{2cx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3
, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(2*c*x^2*(1 + (b*x^3)/a)^(2/
3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(c+dx^3\right)}dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3\left(c+dx^3\right)}dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}\left(1+\frac{dx^3}{c}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{2cx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

**Mathematica [C]** time = 0.05, size = 83, normalized size = 0.49

$$-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{2cx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3,
 1/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x^2*(1 + (b*x^3)/a)^(2/3))
```

**IntegrateAlgebraic [C]** time = 2.38, size = 365, normalized size = 2.16

$$\frac{\left((bc-ad)^{2/3}+i\sqrt{3}(bc-ad)^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{5/3}}-\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}(bc-ad)^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{5/3}}-\frac{i\left(\sqrt{3}(bc-ad)^{2/3}-i(bc-ad)^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{5/3}}-\frac{\left(a+bx^3\right)^{2/3}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(c*x^2) - (Sqrt[(-1 + I*Sqrt[3])/6]*(b*c - a*d)^(2/3
)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/
3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]])/c^(5/3) + (((b*
c - a*d)^(2/3) + I*Sqrt[3]*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (
1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(6*c^(5/3)) - ((I/12)*((-I)*(b*c
 - a*d)^(2/3) + Sqrt[3]*(b*c - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2
 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqr
t[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/c^(5/3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^3\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)
```

## 3.439 $\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx$

**Optimal**. Leaf size=206

$$-\frac{d(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{8/3}}+\frac{d(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}}-\frac{d(bc-ad)^{2/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}}-\frac{(a+bx\cdots}{\cdots}$$

**Rubi [C]** time = 0.13, antiderivative size = 148, normalized size of antiderivative = 0.72, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-2x^3\left(2c-3dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+c\left(a+bx^3\right)\left(2c-3dx^3\right)}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -(c*(a + b*x^3)*(2*c - 3*d*x^3) - 2*(b*c - a*d)*x^3*(2*c - 3*d*x^3)*Hyperge
ometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c - a*d)*
x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(10*c^3*x^5*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx=\frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$=-\frac{c\left(a+bx^3\right)\left(2c-3dx^3\right)-2(bc-ad)x^3\left(2c-3dx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)+6(bc-ad)x^3\left(c\cdots\right.}{10c^3x^5\sqrt[3]{a+bx^3}}$$

**Mathematica [C]** time = 0.64, size = 148, normalized size = 0.72

$$-\frac{2x^3\left(3dx^3-2c\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+c\left(a+bx^3\right)\left(2c-3dx^3\right)}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/10*(c*(a + b*x^3)*(2*c - 3*d*x^3) + 2*(b*c - a*d)*x^3*(-2*c + 3*d*x^3)*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c -
 a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))])/(c^3*x^5*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [C]** time = 2.50, size = 392, normalized size = 1.90

$$-\frac{i\left(\sqrt{3}d(bc-ad)^{2/3}-id(bc-ad)^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{8/3}}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}d(bc-ad)^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{8/3}}+\frac{\left(d(bc-ad)^{2/3}+i\sqrt{3}d(bc-ad)^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{8/3}}+\frac{\left(a+bx^3\right)^{2/3}\left(-2ac+5adx^3-2bcx^3\right)}{10ac^2x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-2*a*c - 2*b*c*x^3 + 5*a*d*x^3))/(10*a*c^2*x^5) + (Sqrt
[(-1 + I*Sqrt[3])/6]*d*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sq
rt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/
3)*(a + b*x^3)^(1/3))]])/c^(8/3) - ((I/6)*((-I)*d*(b*c - a*d)^(2/3) + Sqrt[3
]*d*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3)])/c^(8/3) + ((d*(b*c - a*d)^(2/3) + I*Sqrt[3]*d*(b*c - a*
d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x
 - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/(12*c^(8/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3 + a\right)^{2/3}}{x^6\,\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3\right)^{\frac{2}{3}}}{x^6\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)
```

## 3.440 $\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx$

**Optimal**. Leaf size=257

$$\frac{(a+bx^3)^{2/3}\left(-20a^2d^2+8abcd+3b^2c^2\right)}{40a^2c^3x^2}+\frac{d^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{11/3}}-\frac{d^2(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right.}{2c^{11/3}}$$

**Rubi [C]** time = 0.97, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-2x^3\left(5c^2-6cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-6bc^2dx^6\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+21bc^3x^3\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-21ac^2dx^3\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+27ad^3x^9\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-27bcd^2x^9\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+6acd^2x^6\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-6ac^2dx^3+5ac^3+9acd^2x^6-6bc^2dx^6+5bc^3x^3+9bcd^2x^9}{40c^4x^8\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^6 + 9
*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3*Hype
rgeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c^2*d*
x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6*b
*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[
{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*x^8*(a +
 b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{5ac^3+5bc^3x^3-6ac^2dx^3-6bc^2dx^6+9acd^2x^6+9bcd^2x^9-2(bc-ad)x^3\left(5c^2-6cdx^3+9d\right.}{}$$

**Mathematica [C]** time = 2.20, size = 451, normalized size = 1.75

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+21bc^3x^3\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-2x^3\left(5c^2-6cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-21ac^2dx^3\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-6bc^2dx^6\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+27ad^3x^9\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-27bcd^2x^9\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6acd^2x^6\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+5ac^3-6ac^2dx^3+9acd^2x^6+5bc^3x^3-6bc^2dx^6+9bcd^2x^9}{40c^4x^8\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -1/40*(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^
6 + 9*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3
*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c
^2*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 6*b*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hypergeometri
cPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*x^8*(a
 + b*x^3)^(1/3))
```

**IntegrateAlgebraic [C]** time = 2.91, size = 444, normalized size = 1.73

$$\frac{\left(a+bx^3\right)^{2/3}\left(-5a^2c^2+8a^2cdx^3-20a^2d^2x^6-2abc^2x^3+8abcdx^6+3b^2c^2x^6\right)}{40a^2c^3x^8}+\frac{\left(d^2(bc-ad)^{2/3}+i\sqrt{3}\,d^2(bc-ad)^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{11/3}}-\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,d^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{c^{11/3}}-\frac{i\left(\sqrt{3}\,d^2(bc-ad)^{2/3}-id^2(bc-ad)^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{11/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*a^2*c^2 - 2*a*b*c^2*x^3 + 8*a^2*c*d*x^3 + 3*b^2*c^2*
x^6 + 8*a*b*c*d*x^6 - 20*a^2*d^2*x^6))/(40*a^2*c^3*x^8) - (Sqrt[(-1 + I*Sqr
t[3])/6]*d^2*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*
x^3)^(1/3))])/c^(11/3) + ((d^2*(b*c - a*d)^(2/3) + I*Sqrt[3]*d^2*(b*c - a*d
)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)])/(6*c^(11/3)) - ((I/12)*((-I)*d^2*(b*c - a*d)^(2/3) + Sqrt[3]*d^2*(b*c
 - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)
*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2
/3)])/c^(11/3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^9\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^9\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)
```

## 3.441 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx$

**Optimal**. Leaf size=320

$$\frac{(a+bx^3)^{2/3}(-44a^2d^2+11abcd+6b^2c^2)}{220a^2c^3x^5}-\frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3)}{440a^3c^4x^2}-\frac{d^3(bc-ad)}{\cdots}$$

**Rubi [C]** time = 2.63, antiderivative size = 819, normalized size of antiderivative = 2.56, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^2*d^2*
x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^12 - 80*b*c^4*x^3*Hy
pergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*a*c^3*
d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9
0*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1, 4/3,
((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[1/3
, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeometri
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3*Hyperg
eometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c^3*d*x
^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99*b
*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] - 27*(b*c - a*d)*x^3*(5*c - 6*d*x^3)*(c + d*x
^3)^2*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2},
 {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(440*c^5*x^11*(a + b*x^3)
^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40ac^4+40bc^4x^3-45ac^3dx^3-45bc^3dx^6+54ac^2d^2x^6+54bc^2d^2x^9-81acd^3x^9-81bcd}{}$$

**Mathematica [C]** time = 4.99, size = 819, normalized size = 2.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -1/440*(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^
2*d^2*x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^12 - 80*b*c^4*
x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*
a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] + 90*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2
F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hyperge
ometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3*
Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c
^3*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 99*b*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[1/3, 2, 4/3,
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^2*(-5*
c + 6*d*x^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2,
 2, 2}, {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*x^11*(a + b*x
^3)^(1/3))
```

**IntegrateAlgebraic [C]** time = 3.35, size = 501, normalized size = 1.57

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-40*a^3*c^3 - 10*a^2*b*c^3*x^3 + 55*a^3*c^2*d*x^3 + 12*
a*b^2*c^3*x^6 + 22*a^2*b*c^2*d*x^6 - 88*a^3*c*d^2*x^6 - 18*b^3*c^3*x^9 - 33
*a*b^2*c^2*d*x^9 - 88*a^2*b*c*d^2*x^9 + 220*a^3*d^3*x^9))/(440*a^3*c^4*x^11
) + (Sqrt[(-1 + I*Sqrt[3])/6]*d^3*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(
1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sq
rt[3]*c^(1/3)*(a + b*x^3)^(1/3))]])/c^(14/3) - ((I/6)*((-I)*d^3*(b*c - a*d)^
(2/3) + Sqrt[3]*d^3*(b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/c^(14/3) + ((d^3*(b*c - a*d)^(2/3) + I*
```

```
Sqrt[3]*d^3*(b*c - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(
b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3
)*(a + b*x^3)^(2/3)])/(12*c^(14/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(d\,x^{3}+c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^{12}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.442 $\int \frac{x^8\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=251

$$-\frac{\left(a+bx^3\right)^{7/3}(ad+bc)}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right.}{2d^{13/3}}$$

**Rubi** [A] time = 0.36, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{7/3}(ad+bc)}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{c^2\sqrt[3]{a+bx^3}(bc-ad)}{d^4}-\frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}-\frac{c^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -((c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4) + (c^2*(a + b*x^3)^(4/3))/(4*d^3
) - ((b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (a + b*x^3)^(10/3)/(10*b^
2*d) - (c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[c
 + d*x^3])/(6*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^
(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{4/3}}{bd^2}+\frac{(a+bx)^{7/3}}{bd}+\frac{c^2(a+bx)^{4/3}}{d^2(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{c^2\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+b}}{c+dx}\right.}{3d^3}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{(c}{}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2}{}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2}{}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2}{}$$

**Mathematica [A]** time = 0.54, size = 258, normalized size = 1.03

$$\frac{-\frac{60d\left(a+bx^3\right)^{7/3}(ad+bc)}{b^2}+\frac{42d^2\left(a+bx^3\right)^{10/3}}{b^2}-\frac{70c^2(bc-ad)\left(\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{d^{4/3}}+105c^2\left(a+bx^3\right)^{4/3}}{420d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (105*c^2*(a + b*x^3)^(4/3) - (60*d*(b*c + a*d)*(a + b*x^3)^(7/3))/b^2 + (42
*d^2*(a + b*x^3)^(10/3))/b^2 - (70*c^2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(
1/3) + (b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3
))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^
(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])))/d^(4/3))/(420*d^3)
```

**IntegrateAlgebraic [A]** time = 0.46, size = 340, normalized size = 1.35

$$\frac{\sqrt[3]{a+bx^3}\left(-6a^3d^3-20a^2bcd^2+2a^2bd^3x^3+175ab^2c^2d-40ab^2cd^2x^3+22ab^2d^3x^6-140b^3c^3+35b^3c^2dx^3-20b^3cd^2x^6+14b^3d^3x^9\right)}{140b^2d^4}+\frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{13/3}}-\frac{c^2(bc-ad)^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{13/3}}-\frac{c^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{13/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^3
*d^3 + 35*b^3*c^2*d*x^3 - 40*a*b^2*c*d^2*x^3 + 2*a^2*b*d^3*x^3 - 20*b^3*c*d
^2*x^6 + 22*a*b^2*d^3*x^6 + 14*b^3*d^3*x^9))/(140*b^2*d^4) - (c^2*(b*c - a*
d)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a
*d)^(1/3))])/(Sqrt[3]*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*
(a + b*x^3)^(2/3)])/(6*d^(13/3))
```

**fricas [A]** time = 0.56, size = 369, normalized size = 1.47

$$\frac{140\sqrt{3}\left(b^3c^3-ab^2c^2d\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+70\left(b^3c^3-ab^2c^2d\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-140\left(b^3c^3-ab^2c^2d\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)+3\left(14b^3d^3x^9-2\left(10b^3cd^2-11ab^2d^3\right)x^6-140b^3c^3+175ab^2c^2d-20a^2bcd^2-6a^3d^3+\left(35b^3c^2d-40ab^2cd^2+2a^2bd^3\right)x^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{420b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1
/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a
*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*c*d^2
- 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^
3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 + a)^(1/3
))/(b^2*d^4)
```

**giac [A]** time = 0.30, size = 394, normalized size = 1.57

$$-\frac{\left(b^{24}c^4d^6-2ab^{23}c^3d^7+a^2b^{22}c^2d^8\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{23}cd^{10}-ab^{22}d^{11}\right)}+\frac{\sqrt{3}\left(bc^3-ac^2d\right)\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^5}+\frac{\left(bc^3-ac^2d\right)\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^5}-\frac{140\left(bx^3+a\right)^{\frac{1}{3}}b^{21}c^3d^6-35\left(bx^3+a\right)^{\frac{4}{3}}b^{20}c^2d^7-140\left(bx^3+a\right)^{\frac{1}{3}}ab^{20}c^2d^7+20\left(bx^3+a\right)^{\frac{7}{3}}b^{19}cd^8-14\left(bx^3+a\right)^{\frac{10}{3}}b^{18}d^9+20\left(bx^3+a\right)^{\frac{7}{3}}ab^{18}d^9}{140b^{20}d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)^
(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 - a
*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b
^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b
^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*a*b^18*d^
9)/(b^20*d^10)
```

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} x^{8}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 5.12, size = 477, normalized size = 1.90

$$\left(\frac{a^2}{4\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{4\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3}-\left(\frac{2\,a}{7\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{7\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3}+\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^2\,d}+\frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}{d^2}-\frac{c^2\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{13/3}}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{13/3}}-\frac{\left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}-\frac{c^2\,\ln\left(\frac{3\,c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{7/3}}+\frac{3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{13/3}}+\frac{c^2\,\ln\left(\frac{3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^2}-\frac{9\,c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{7/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(4*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(4*b^2*d))*(a + b*x^3)^(4/3) - ((2*a)/(7*b^2*d) + (b^3*c - a*b^2*d)
/(7*b^4*d^2))*(a + b*x^3)^(7/3) + (a + b*x^3)^(10/3)/(10*b^2*d) + (c^2*log(
(3*(a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^2 - (c^2*(a*d
 - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(13/3)))*(a*d - b*c)^(4/3))/(3*d^
(13/3)) - ((a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b
^3*c - a*b^2*d))/(b^2*d))*(a + b*x^3)^(1/3)*(b^3*c - a*b^2*d))/(b^2*d) - (c
^2*log((3*c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(7/3) + (3*c^2*(a
 + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3
))/(3*d^(13/3)) + (c^2*log((3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2 - (9
*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(7/3))/d^(7/3))*((3^(1/2)*1i)/6 - 1
/6)*(a*d - b*c)^(4/3))/d^(13/3)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.443 $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=211

$$\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}+c\sqrt[3]{}$$

**Rubi [A]** time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$-\frac{c\left(a+bx^3\right)^{4/3}}{4d^2}+\frac{c\sqrt[3]{a+bx^3}\,(bc-ad)}{d^3}+\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}+\frac{\left(a+bx^3\right)^{7/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (c*(b*c - a*d)*(a + b*x^3)^(1/3))/d^3 - (c*(a + b*x^3)^(4/3))/(4*d^2) + (a
+ b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b
*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c*(b*c - a*
d)^(4/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{7/3}}{7bd} - \frac{c\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{(c(bc-ad))\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} - \frac{\left(c(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c \cdots}\right.}{3d^3}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}} - \cdots$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}} - \cdots$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}} \cdots\right.}{\sqrt{3}\,d^{10/3}}$$

**Mathematica [A]** time = 0.34, size = 255, normalized size = 1.21

$$\frac{c(bc-ad)\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{6d^{10/3}}-\frac{c\left(a+bx^3\right)^{4/3}}{4d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -1/4*(c*(a + b*x^3)^(4/3))/d^2 + (a + b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)
*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2
```

```
*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/
3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*L
og[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3
)*(a + b*x^3)^(2/3)]))/(6*d^(10/3))
```

**IntegrateAlgebraic [A]** time = 0.27, size = 278, normalized size = 1.32

$$\frac{\sqrt[3]{a+bx^3}\left(4a^2d^2-35abcd+8abd^2x^3+28b^2c^2-7b^2cdx^3+4b^2d^2x^6\right)}{28bd^3}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{10/3}}+\frac{c(bc-ad)^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{10/3}}+\frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - 7*b^2*c*d*x^3 + 8
*a*b*d^2*x^3 + 4*b^2*d^2*x^6))/(28*b*d^3) + (c*(b*c - a*d)^(4/3)*ArcTan[1/S
qrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[
3]*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*
x^3)^(1/3)])/(3*d^(10/3)) + (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^
(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*
d^(10/3))
```

**fricas [A]** time = 0.43, size = 298, normalized size = 1.41

$$\frac{28\sqrt{3}\left(b^2c^2-abcd\right)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+14\left(b^2c^2-abcd\right)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-28\left(b^2c^2-abcd\right)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)+3\left(4b^2d^2x^6+28b^2c^2-35abcd+4a^2d^2-\left(7b^2cd-8abd^2\right)x^3\right)\left(bx^3+a\right)^{\frac{1}{3}}}{84bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*
c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2
/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 28
*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c -
a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - (
7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3))/(b*d^3)
```

**giac [B]** time = 0.29, size = 348, normalized size = 1.65

$$\frac{\left(b^{10}c^3d^4-2ab^9c^2d^5+a^2b^8cd^6\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^9cd^7-ab^8d^8\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bc^2-acd\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bc^2-acd\right)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^4}+\frac{28\left(bx^3+a\right)^{\frac{1}{3}}b^8c^2d^4-7\left(bx^3+a\right)^{\frac{4}{3}}b^7cd^5-28\left(bx^3+a\right)^{\frac{1}{3}}ab^7cd^5+4\left(bx^3+a\right)^{\frac{7}{3}}b^6d^6}{28b^7d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3)
*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*d^
8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^
4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3) + (b
*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28
*(28*(b*x^3 + a)^(1/3)*b^8*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^7*c*d^5 - 28*(b*
x^3 + a)^(1/3)*a*b^7*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^6*d^6)/(b^7*d^7)
```

**maple [F]** time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} x^{5}}{d\,x^{3}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.06, size = 348, normalized size = 1.65

$$\frac{\left(b\,x^3+a\right)^{7/3}}{7\,b\,d}-\left(b\,x^3+a\right)^{4/3}\,\left(\frac{a}{4\,b\,d}+\frac{b^2\,c-a\,b\,d}{4\,b^2\,d^2}\right)-\frac{c\,\ln\left(\frac{3\,\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3\right)}{d}-\frac{c\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{10/3}}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{10/3}}-\frac{c\,\ln\left(\frac{3\,c\,\left(b\,x^3+a\right)^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d}-\frac{3\,c\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{10/3}}+\frac{c\,\ln\left(\frac{3\,c\,\left(b\,x^3+a\right)^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d}+\frac{3\,c\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{10/3}}+\frac{\left(b\,x^3+a\right)^{1/3}\,\left(b^2\,c-a\,b\,d\right)\,\left(\frac{a}{b\,d}+\frac{b^2\,c-a\,b\,d}{b^2\,d^2}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(7/3)/(7*b*d) - (a + b*x^3)^(4/3)*(a/(4*b*d) + (b^2*c - a*b*d)/
(4*b^2*d^2)) - (c*log((3*(a + b*x^3)^(1/3)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2
*d))/d - (c*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(10/3)))*(a*d - b
*c)^(4/3))/(3*d^(10/3)) - (c*log((3*c*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d -
(3*c*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + (c*log((3*c*(a + b*x^3)^(1/3)*(a*d -
b*c)^2)/d + (3*c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/
2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + ((a + b*x^3)^(1/3)*(b^2*c
 - a*b*d)*(a/(b*d) + (b^2*c - a*b*d)/(b^2*d^2)))/(b*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.444 $\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=187

$$-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{\sqrt[3]{a+}}{}$$

**Rubi [A]** time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {444, 50, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}(bc-ad)}{d^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}+\frac{\left(a+bx^3\right)^{4/3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*d) - ((b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]])/(Sqrt[3]*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(7/3)) +
 ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} + \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\operatorname{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}} - \frac{(bc-ad)^{4/3}\,l\cdots}{6d\cdots}$$

**Mathematica [A]** time = 0.32, size = 232, normalized size = 1.24

$$\frac{(ad-bc)\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{6d^{7/3}}+\frac{\left(a+bx^3\right)^{4/3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(4/3)/(4*d) + ((-(b*c) + a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*
Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
 a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(6*d^(7/3))
```

**IntegrateAlgebraic [A]** time = 0.24, size = 234, normalized size = 1.25

$$\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{7/3}}+\frac{\sqrt[3]{a+bx^3}\left(5ad-4bc+bdx^3\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-4*b*c + 5*a*d + b*d*x^3))/(4*d^2) - ((b*c - a*d)^(4/3)
*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3
))])/(Sqrt[3]*d^(7/3)) + ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(3*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3)
 - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]
)/(6*d^(7/3))
```

**fricas [A]** time = 0.44, size = 246, normalized size = 1.32

$$\frac{4\sqrt{3}(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-4(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)+3\left(bdx^3-4bc+5ad\right)(bx^3+a)^{\frac{1}{3}}}{12d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(
b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*d)*(
-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(
b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2
```

**giac [A]** time = 0.30, size = 297, normalized size = 1.59

$$-\frac{\left(b^2c^2d^2-2abcd^3+a^2d^4\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd^4-ad^5\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3d^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6d^3}-\frac{4(bx^3+a)^{\frac{1}{3}}bcd^2-(bx^3+a)^{\frac{4}{3}}d^3-4(bx^3+a)^{\frac{1}{3}}ad^3}{4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs((
b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(3)
*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 +
a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c -
 a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b*c
*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} x^{2}}{d\,x^{3}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.72, size = 304, normalized size = 1.63

$$\frac{\left(b\,x^3+a\right)^{4/3}}{4\,d}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)-\frac{\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(a\,d-b\,c\right)^{4/3}}{3\,d^{7/3}}+\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}{d^2}-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{4/3}}{3\,d^{7/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{d^{7/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^{4/3}}{d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*d) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6
*a*b*c*d) - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b
*c)^(4/3))/(3*d^(7/3)) + ((a + b*x^3)^(1/3)*(a*d - b*c))/d^2 - (log((a + b*
x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(a
*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^(4/3))/(3*d^(7/3)) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*
c^2 - 6*a*b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b
*c*d^2))/d^(7/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(7/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b x^{3}\right)^{\frac{4}{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

## 3.445 $\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)} \, dx$

**Optimal**. Leaf size=261

$$\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{\right.}$$

**Rubi [A]** time = 0.30, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {446, 84, 156, 57, 617, 204, 31, 58}

$$\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}}+\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}}+\frac{b\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(4/3)) - (a^(4
/3)*Log[x])/(2*c) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c*d^(4/3)) + (a^(
4/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(4/3)*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{\operatorname{Subst}\left(\int \frac{a^2d+b(-bc+2ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3 \right.}{3cd}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} - \frac{a^{4/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+b\right.}{2c}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc}{}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} + \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{4/3}} - \frac{a^{4/3}\log(x)}{2c} + \frac{(}{}$$

**Mathematica [A]** time = 0.67, size = 331, normalized size = 1.27

$$\frac{-\left(a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)+\frac{(bc-ad)\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{d^{4/3}}+6a\sqrt[3]{a+bx^3}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (6*a*(a + b*x^3)^(1/3) - a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a
^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]) + ((b*c - a*d)*(6*d^(1/3)*(a
 + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a +
b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c -
a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)
^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^
(2/3)]))/d^(4/3))/(6*c)
```

**IntegrateAlgebraic [A]** time = 0.65, size = 349, normalized size = 1.34

$$\frac{a^{4/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3c}-\frac{a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6c}-\frac{a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}c}-\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3cd^{4/3}}+\frac{(bc-ad)^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6cd^{4/3}}+\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}cd^{4/3}}+\frac{b\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))
/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[1/Sqrt[3] - (2
*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*c*d^(4/3
)) + (a^(4/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(3*c) - ((b*c - a*d)^(4/3)
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c*d^(4/3)) - (a^(4/
3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*c) + ((
b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x
^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c*d^(4/3))
```

**fricas [A]** time = 0.45, size = 320, normalized size = 1.23

$$-\frac{2\sqrt{3}a^{\frac{4}{3}}d\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{4}{3}}d\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{4}{3}}d\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-2\sqrt{3}(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)-6\left(bx^3+a\right)^{\frac{1}{3}}bc-(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)+2(bc-ad)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{6cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(b*
c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(
(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 6*(b*x^3 + a)^(1
/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3
+ a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c - a*d)*(
(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))/(c*d)
```

**giac [A]** time = 0.84, size = 357, normalized size = 1.37

$$-\frac{\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3c}-\frac{a^{\frac{4}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6c}+\frac{a^{\frac{4}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3c}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b}{d}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3cd^2}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}(bc-ad)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/c - 1/6*a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a
^(2/3))/c + 1/3*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c + 1/3*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3)
 - (-(b*c - a*d)/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (b*x^3 + a)^(1/3)*b/d - 1
/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^
3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) - 1/
6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^
(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

**maple** [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{4}{3}}}{\left(dx^{3}+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)
```

**mupad** [B] time = 6.08, size = 796, normalized size = 3.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x*(c + d*x^3)),x)
```

```
[Out] log(c*d*(-(a*d - b*c)^4/(c^3*d^4))^(1/3) + a*d*(a + b*x^3)^(1/3) - b*c*(a +
 b*x^3)^(1/3))*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) + log(c*(a^4/c^3)^(1/3) - a*(a + b*x^3)^(
1/3))*(a^4/(27*c^3))^(1/3) + (b*(a + b*x^3)^(1/3))/d - log(c*(a^4/c^3)^(1/3
) + 2*a*(a + b*x^3)^(1/3) + 3^(1/2)*c*(a^4/c^3)^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*(a^4/(27*c^3))^(1/3) + log(c*(a^4/c^3)^(1/3)*1i + a*(a + b*x^3)^(1/3)
*2i + 3^(1/2)*c*(a^4/c^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a^4/(27*c^3))^(1/3
) + log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6
*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + 3*a^2*b^4*c*((3^(1/2
)*1i)/2 - 1/2)*(-(a*d - b*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a
^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(
1/2)*1i)/2 - 1/2)*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) - log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*
d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3
*b*c*d^3))/d - 3*a^2*b^4*c*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)^4/(c^3*d^4)
)^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*
a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^4 + b^4*c^4 +
 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{4}{3}}}{x\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)
```

## 3.446 $\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)} \, dx$

**Optimal**. Leaf size=399

$$\frac{d(a+bx^3)^{4/3}}{4c^2}+\frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2}+\frac{\sqrt[3]{a+bx^3}\,(4bc-3ad)}{3c^2}-\frac{\sqrt[3]{a+bx^3}\,(bc-ad)}{c^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{d}}+\cdots$$

**Rubi** [A] time = 0.48, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{d(a+bx^3)^{4/3}}{4c^2}+\frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2}+\frac{\sqrt[3]{a+bx^3}\,(4bc-3ad)}{3c^2}-\frac{\sqrt[3]{a+bx^3}\,(bc-ad)}{c^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{d}}+\frac{\sqrt[3]{a}\,(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}}-\frac{\sqrt[3]{a}\,(4bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}c^2}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{d}}-\frac{\sqrt[3]{a}\log(x)(4bc-3ad)}{6c^2}-\frac{(a+bx^3)^{7/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((4*b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*c^2) - ((b*c - a*d)*(a + b*x^3)^(1/3
))/c^2 + (d*(a + b*x^3)^(4/3))/(4*c^2) + ((4*b*c - 3*a*d)*(a + b*x^3)^(4/3)
)/(12*a*c^2) - (a + b*x^3)^(7/3)/(3*a*c*x^3) - (a^(1/3)*(4*b*c - 3*a*d)*Arc
Tan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*c^2) - (
(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(Sqrt[3]*c^2*d^(1/3)) - (a^(1/3)*(4*b*c - 3*a*d)*Log[x])/(6*c
^2) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^2*d^(1/3)) + (a^(1/3)*(4*b*c
- 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c^2) + ((b*c - a*d)^(4/3)*Log
[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-4bc+3ad)-\frac{4bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x\right.}{9c^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \ldots$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \ldots$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \ldots$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \ldots$$

**Mathematica [A]** time = 1.55, size = 389, normalized size = 0.97

$$\frac{\frac{(4bc-3ad)\left(-\frac{1}{2}a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)+3a\sqrt[3]{a+bx^3}+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{3c}+\frac{a\left(3d^{4/3}\left(a+bx^3\right)^{4/3}-2(bc-ad)\left(\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)\right)}{4c\sqrt[3]{d}}-\frac{\left(a+bx^3\right)^{7/3}}{x^3}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-((a + b*x^3)^(7/3)/x^3) + ((4*b*c - 3*a*d)*(3*a*(a + b*x^3)^(1/3) + (3*(a
 + b*x^3)^(4/3))/4 - (a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/
a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/2))/(3*c) + (a*(3*d^(4/3)*(a +
 b*x^3)^(4/3) - 2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) + (b*c - a*d)^(1
/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)]))))/(4*c*d^(1/3)))/(3*a*c)
```

**IntegrateAlgebraic [A]** time = 0.77, size = 398, normalized size = 1.00

$$\frac{\left(4\sqrt[3]{a}\,bc-3a^{4/3}d\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9c^2}+\frac{\left(3a^{4/3}d-4\sqrt[3]{a}\,bc\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{18c^2}+\frac{\left(3a^{4/3}d-4\sqrt[3]{a}\,bc\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}c^2}-\frac{(bc-ad)^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^2\sqrt[3]{d}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^2\sqrt[3]{d}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c^2\sqrt[3]{d}}-\frac{a\sqrt[3]{a+bx^3}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] -1/3*(a*(a + b*x^3)^(1/3))/(c*x^3) + ((-4*a^(1/3)*b*c + 3*a^(4/3)*d)*ArcTan
[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*c^2) - ((b*
c - a*d)^(4/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b
*c - a*d)^(1/3))])/(Sqrt[3]*c^2*d^(1/3)) + ((4*a^(1/3)*b*c - 3*a^(4/3)*d)*L
og[-a^(1/3) + (a + b*x^3)^(1/3)])/(9*c^2) + ((b*c - a*d)^(4/3)*Log[(b*c - a
*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c^2*d^(1/3)) + ((-4*a^(1/3)*b*c
+ 3*a^(4/3)*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]
)/(18*c^2) - ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)
^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c^2*d^(1/3))
```

**fricas [A]** time = 0.70, size = 383, normalized size = 0.96

$$\frac{6\sqrt{3}(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2\sqrt{3}(4bc-3ad)(-a)^{\frac{1}{3}}x^3\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+(4bc-3ad)(-a)^{\frac{1}{3}}x^3\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+3(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)-2(4bc-3ad)(-a)^{\frac{1}{3}}x^3\log\left((bx^3+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)-6(bx^3+a)^{\frac{1}{3}}ac}{18c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(
3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c -
 a*d)) + 2*sqrt(3)*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*arctan(1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + (4*b*c - 3*a*d)*(-a)^(1/3)*x^3*
log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 3*(b*c
 - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(4*b*c - 3*a*d)*(-a)
^(1/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*c - a*d)*x^3*(-(b*c -
 a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 +
 a)^(1/3)*a*c)/(c^2*x^3)
```

**giac [A]** time = 0.86, size = 394, normalized size = 0.99

$$-\frac{(b^2c^2-2abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^3-ac^2d)}-\frac{\sqrt{3}\left(4a^{\frac{1}{3}}bc-3a^{\frac{4}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9c^2}-\frac{\left(4a^{\frac{1}{3}}bc-3a^{\frac{4}{3}}d\right)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18c^2}+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3c^2d}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6c^2d}+\frac{(4abc-3a^2d)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{2}{3}}c^2}-\frac{(bx^3+a)^{\frac{1}{3}}a}{3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(4*a^
(1/3)*b*c - 3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))
/a^(1/3))/c^2 - 1/18*(4*a^(1/3)*b*c - 3*a^(4/3)*d)*log((b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c^2 + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(
1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)
^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c
 - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
(-(b*c - a*d)/d)^(2/3))/(c^2*d) + 1/9*(4*a*b*c - 3*a^2*d)*log(abs((b*x^3 +
a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)*a/(c*x^3)
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 10.88, size = 2047, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(c^2*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3) + 3*a*d*(a + b*x^3)^(1/3) - 4*b*
c*(a + b*x^3)^(1/3))*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 108
*a^3*b*c*d^2)/(729*c^6))^(1/3) + log(((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*
c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3) - 108*a*b^5*c^3*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2)*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*
a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^
4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^
4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^
3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^
4))*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3
)/(27*c^6*d))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*
(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 81*a*b^4*c^4*d^3*((3^(
1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))
^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*
c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4
*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4*d^2*(a + b*x^3)
^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a
^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4))*((3^(1/2)*1i)
/2 - 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b
*c*d^3)/(27*c^6*d))^(1/3) - log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*
(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*
a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/
2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 + 81*a*b
^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d -
 b*c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 - (a*b^5*d^2*(27*a
^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4
*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3)*((3^(1/
2)*1i)/2 + 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b
*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3
+ 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 - 1/2)*(((
(3^(1/2)*1i)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 -
27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(
-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(2/3))/81 +
 (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^
2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*a*d - 4*b*c)^3)/
c^6)^(1/3))/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^
2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - log((a*b^4*d^2*(a + b*x^3
)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*
a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*
1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*
(a*d - b*c)^2 + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^
```

```
2 - 3*a*b*c*d)*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/
c^6)^(2/3))/81 - (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2
+ 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*
a*d - 4*b*c)^3)/c^6)^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^4*d^3 - 64*a*
b^3*c^3 + 144*a^2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - (a*(a + b
*x^3)^(1/3))/(3*c*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3\right)^{\frac{4}{3}}}{x^4 \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)
```

## 3.447 $\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx$

**Optimal**. Leaf size=440

$$\frac{\sqrt[3]{a+bx^3}\,(9a^2d^2-12abcd+2b^2c^2)}{9ac^3}+\frac{(9a^2d^2-12abcd+2b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3}-\frac{(9a^2d^2-12abcd+2b^2c^2)}{9\sqrt{3}\,a^{2}}$$

**Rubi** [A] time = 0.62, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {446, 103, 149, 156, 50, 57, 617, 204, 31, 58}

$$\frac{\sqrt[3]{a+bx^3}\,(9a^2d^2-12abcd+2b^2c^2)}{9ac^3}+\frac{(9a^2d^2-12abcd+2b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3}-\frac{(9a^2d^2-12abcd+2b^2c^2)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{2/3}c^3}-\frac{\log(x)(9a^2d^2-12abcd+2b^2c^2)}{18a^{2/3}c^3}+\frac{d^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6c^3}-\frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}+\frac{d^{2/3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3}-\frac{(a+bx^3)^{4/3}(bc-6ad)}{18ac^2x^3}+\frac{d\sqrt[3]{a+bx^3}(bc-ad)}{c^3}-\frac{(a+bx^3)^{7/3}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (d*(b*c - a*d)*(a + b*x^3)^(1/3))/c^3 + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^
2)*(a + b*x^3)^(1/3))/(9*a*c^3) - ((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(18*a*c
^2*x^3) - (a + b*x^3)^(7/3)/(6*a*c*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*
d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3
)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^3) - ((2*b^2*c^2 - 12*a*b*
c*d + 9*a^2*d^2)*Log[x])/(18*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[
c + d*x^3])/(6*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(18*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3(c+dx)}dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-bc+6ad)-\frac{bdx}{3}\right)}{x^2(c+dx)}dx, x, x^3\right)}{6ac}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{2}{9}\left(2b^2c^2-12abcd+9a^2d^2\right)-\frac{2}{9}bd(2bc-3ad)x\right)}{x(c+dx)}\right.}{6ac^2}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}dx, x, x^3\right)}{3c^3} + \frac{\left(2b^2c^2\right.}{}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

**Mathematica [A]** time = 1.25, size = 429, normalized size = 0.98

$$\frac{x^6\left(4\left(9a^2d^2-12abcd+2b^2c^2\right)\left(-\frac{1}{2}a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)+3a\sqrt[3]{a+bx^3}+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)-9a^2d^{2/3}\left(3d^{4/3}\left(a+bx^3\right)^{4/3}-2(bc-ad)\left(\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)\right)\right)-18ac^2\left(a+bx^3\right)^{7/3}-6cx^3\left(a+bx^3\right)^{7/3}(bc-6ad)}{108a^2c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (-18*a*c^2*(a + b*x^3)^(7/3) - 6*c*(b*c - 6*a*d)*x^3*(a + b*x^3)^(7/3) + x^
6*(4*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(3*a*(a + b*x^3)^(1/3) + (3*(a +
b*x^3)^(4/3))/4 - (a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)
*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/2) - 9*a^2*d^(2/3)*(3*d^(4/3)*(a
+ b*x^3)^(4/3) - 2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) + (b*c - a*d)^(
1/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
 + b*x^3)^(2/3)])))))/(108*a^2*c^3*x^6)
```

**IntegrateAlgebraic [A]** time = 1.07, size = 445, normalized size = 1.01

$$\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{27a^{2/3}c^3}+\frac{\left(-9a^2d^2+12abcd-2b^2c^2\right)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54a^{2/3}c^3}-\frac{\left(9a^2d^2-12abcd+2b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}c^3}-\frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^3}+\frac{d^{2/3}(bc-ad)^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^3}+\frac{d^{2/3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c^3}+\frac{\sqrt[3]{a+bx^3}\left(-3ac+6adx^3-7bcx^3\right)}{18c^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

[Out] ((a + b*x^3)^(1/3)*(-3*a*c - 7*b*c*x^3 + 6*a*d*x^3))/(18*c^2*x^6) - ((2*b^2
*c^2 - 12*a*b*c*d + 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt
[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[
1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sq
rt[3]*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^
3)^(1/3)])/(27*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c^3) + ((-2*b^2*c^2 + 12*a*b*c*d - 9*a
^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*a
^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b
*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c^3)

**fricas** [A] time = 2.02, size = 503, normalized size = 1.14

$$\frac{18\sqrt{3}\left(a^{2}bc-a^{3}d\right)\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}x^{6}\arctan\left(-\frac{2\sqrt{3}\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^{2}\right)}{3\left(bcd-ad^{2}\right)}\right)-2\sqrt{3}\left(2ab^{2}c^{2}-12a^{2}bcd+9a^{3}d^{2}\right)\left(a^{2}\right)^{\frac{1}{6}}x^{6}\arctan\left(\frac{\left(a^{2}\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)}{3a^{2}}\right)-\left(2b^{2}c^{2}-12abcd+9a^{2}d^{2}\right)\left(a^{2}\right)^{\frac{2}{3}}x^{6}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(a^{2}\right)^{\frac{2}{3}}\right)+2\left(2b^{2}c^{2}-12abcd+9a^{2}d^{2}\right)\left(a^{2}\right)^{\frac{2}{3}}x^{6}\log\left(\left(bx^{3}+a\right)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+9\left(a^{2}bc-a^{3}d\right)\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}x^{6}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}d^{2}-\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}d+\left(bcd^{2}-ad^{3}\right)^{\frac{2}{3}}\right)-18\left(a^{2}bc-a^{3}d\right)\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}x^{6}\log\left(\left(bx^{3}+a\right)^{\frac{1}{3}}d+\left(bcd^{2}-ad^{3}\right)^{\frac{1}{3}}\right)-3\left(3a^{3}c^{2}+\left(7a^{2}bc^{2}-6a^{3}cd\right)x^{3}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}}{54a^{2}c^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")

[Out] 1/54*(18*sqrt(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(1/3)*x^6*arctan(-1/3*
(2*sqrt(3)*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d
^2))/(b*c*d - a*d^2)) - 2*sqrt(3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*d^2)*
(a^2)^(1/6)*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(
a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(a^2)^(2/3)) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((
b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1
/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 18*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(3*a^3*c^2
+ (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(1/3))/(a^2*c^3*x^6)

**giac** [A] time = 0.86, size = 481, normalized size = 1.09

$$\frac{\left(b^{2}c^{2}d-2abcd^{2}+a^{2}d^{3}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{4}-ac^{3}d\right)}-\frac{\sqrt{3}\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3c^{3}}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}(bc-ad)\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6c^{3}}-\frac{\sqrt{3}\left(2b^{2}c^{2}-12abcd+9a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{27a^{\frac{2}{3}}c^{3}}-\frac{\left(2b^{2}c^{2}-12abcd+9a^{2}d^{2}\right)\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54a^{\frac{2}{3}}c^{3}}+\frac{\left(2b^{2}c^{2}-12abcd+9a^{2}d^{2}\right)\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27a^{\frac{2}{3}}c^{3}}-\frac{7\left(bx^{3}+a\right)^{\frac{4}{3}}b^{2}c-4\left(bx^{3}+a\right)^{\frac{1}{3}}ab^{2}c-6\left(bx^{3}+a\right)^{\frac{4}{3}}abd+6\left(bx^{3}+a\right)^{\frac{1}{3}}a^{2}bd}{18b^{2}c^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x
^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) - 1/3*sqrt(3)*(-
b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3)
+ (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 - 1/6*(-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(2*b^2*c^2 - 12*a*
b*c*d + 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/(a^(2/3)*c^3) - 1/54*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^3) + 1/27*(2*b^2
*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c^3) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2*c - 4*(b*x^3 + a)^(1/3)*a*b^2*c - 6
*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(b^2*c^2*x^6)

**maple** [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)

```
[Out] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)
```

**mupad [B]** time = 13.25, size = 2841, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*
b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*
a*b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c
^9))^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2
+ 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*
b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3
))/27 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^
6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 -
1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((729*a^6*d^6 + 8*b^6*c^6
+ 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d^4 - 144*a
*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3) + log(((((18*b^5*c^2*
d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*(2*a^2
*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d -
 b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c
^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 36
45*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 - (b^4*d^5*(
a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c
^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d -
 6804*a^5*b*c*d^5))/(243*c^8))*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 +
 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) + (((2*a*b^2*c - 3*a^2*
b*d)*(a + b*x^3)^(1/3))/(9*c^2) + (b*(a + b*x^3)^(4/3)*(6*a*d - 7*b*c))/(18
*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log((((3^(1/2)*1i)/2 - 1/2
)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*
(6*a*d - b*c) + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)
^(2/3))/9 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939
*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^
5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5*(a + b*x^3)^(1
/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 1242
0*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c
*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3
*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) - log((((3^(1
/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)
*(a*d - b*c)^2*(6*a*d - b*c) - 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a
^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d
 - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4
*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d +
3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5
*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4
*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d
```

```
 - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^6 + b^4*c^
4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3
) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a
 + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*b^4*c^4*d^3*((3^(1/2)*1i)
/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*
b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9
))^(2/3))/729 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 +
6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*
c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))
/27 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6
+ 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 17
64*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*((729
*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^
4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)
- log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a +
 b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*
c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))
^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 69
39*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*
d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))/2
7 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 +
6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764
*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*((729*a
^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*
b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.448 $\int \frac{x^4\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

**Optimal**. Leaf size=334

$$-\frac{\left(2a^2d^2-12abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3}-\frac{\left(2a^2d^2-12abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{2/3}d^3}-\frac{c^{2/3}(bc-ad)^{4/3}}{6d}$$

**Rubi [C]** time = 0.06, antiderivative size = 65, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{ax^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -4/3, 1, 8/3, -((b*x^3)/a), -((d*x^3
)/c)])/(5*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{ax^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.31, size = 225, normalized size = 0.67

$$\frac{2x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}\left(2a^2d^2-12abcd+9b^2c^2\right)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(6bc-7ad)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)+\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}\left(7ad-6bc+3bdx^3\right)\right)}{90cd^2\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x
^3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x
^2*((a + b*x^3)*(-6*b*c + 7*a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b
*c - 7*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c)
 + a*d)*x^3)/(a*(c + d*x^3))]))/(90*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)
^(2/3))
```

**IntegrateAlgebraic** [C] time = 9.96, size = 611, normalized size = 1.83

$$\frac{\left(-2a^2d^2+12abcd-9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27b^{2/3}d^3}-\frac{\left(2a^2d^2-12abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}b^{2/3}d^3}+\frac{\left(2a^2d^2-12abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54b^{2/3}d^3}+\frac{i\left(\sqrt{3}c^{2/3}(bc-ad)^{4/3}+ic^{2/3}(bc-ad)^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6d^3}-\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}c^{2/3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{d^3}+\frac{\left(c^{2/3}(bc-ad)^{4/3}-i\sqrt{3}c^{2/3}(bc-ad)^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^3}+\frac{\sqrt[3]{a+bx^3}\left(7adx^2-6bcx^2+3bdx^5\right)}{18d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-6*b*c*x^2 + 7*a*d*x^2 + 3*b*d*x^5))/(18*d^2) - ((9*b^2
*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a
 + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(2/3)*d^3) - (Sqrt[(-1 - I*Sqrt[3])/6]*c^(2
/3)*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(
1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3
))])/d^3 + ((-9*b^2*c^2 + 12*a*b*c*d - 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)])/(27*b^(2/3)*d^3) + ((I/6)*(I*c^(2/3)*(b*c - a*d)^(4/3) + Sqrt
[3]*c^(2/3)*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*
c^(1/3)*(a + b*x^3)^(1/3)])/d^3 + ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log
[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(2/3
)*d^3) + ((c^(2/3)*(b*c - a*d)^(4/3) - I*Sqrt[3]*c^(2/3)*(b*c - a*d)^(4/3))
*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3
]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*d^3)
```

**fricas** [A] time = 2.05, size = 550, normalized size = 1.65

$$\frac{2\sqrt{3}\left(9b^3c^2-12ab^2cd+2a^2bd^2\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2x}\right)-18\sqrt{3}\left(b^3c-ab^2d\right)\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right)-2\left(9b^2c^2-12abcd+2a^2d^2\right)\left(-b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(9b^2c^2-12abcd+2a^2d^2\right)\left(-b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)-18\left(b^3c-ab^2d\right)\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\log\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}c+\left(-bc^3+ac^2d\right)^{\frac{1}{3}}x}{x}\right)+9\left(b^3c-ab^2d\right)\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\log\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}c^2-\left(-bc^3+ac^2d\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}cx+\left(-bc^3+ac^2d\right)^{\frac{2}{3}}x^2}{x^2}\right)+3\left(3b^3d^2x^5-\left(6b^3cd-7ab^2d^2\right)x^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{54b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(9*b^3*c^2 - 12*a*b^2*c*d + 2*a^2*b*d^2)*sqrt(-(-b^2)^(1/3)
)*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2
)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 18*sqrt(3)*(b^3*c - a*b^2*d)*(-b*c^
3 + a*c^2*d)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c
^3 + a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) - 2*(9*b^2*c^2
- 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(
1/3)*b)/x) + (9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2)
 - 18*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1/3)*log(((b*x^3 + a)^(1/3)*c +
 (-b*c^3 + a*c^2*d)^(1/3)*x)/x) + 9*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1
/3)*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)
*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d -
 7*a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} x^{4}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{4}{3}} x^{4}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + bx^{3}\right)^{\frac{4}{3}}}{c + dx^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

## 3.449 $\int \frac{x(a+bx^3)^{4/3}}{c+dx^3} \, dx$

**Optimal**. Leaf size=277

$$\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,d^2}+\frac{\sqrt[3]{b}\,(3bc-4ad)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6d^2}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,d^2}+\cdots$$

**Rubi [C]** time = 0.04, antiderivative size = 65, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{ax^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -4/3, 1, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{ax^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica [C]** time = 0.32, size = 198, normalized size = 0.71

$$\frac{2bx^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(4ad-3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5x^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(3ad-2bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)+2bc\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}\right)}{30cd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (2*b*(-3*b*c + 4*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Appel
lF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*x^2*(2*b*c*(a + b*x^3
)*(1 + (d*x^3)/c)^(2/3) + a*(-2*b*c + 3*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(30*c*d*(a
 + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))
```

**IntegrateAlgebraic [C]** time = 7.07, size = 552, normalized size = 1.99

$$\frac{\left(3b^{4/3}c-4a\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9d^2}+\frac{\left(3b^{4/3}c-4a\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\,d^2}+\frac{\left(4a\sqrt[3]{b}\,d-3b^{4/3}c\right)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18d^2}+\frac{i\left(\sqrt{3}\,(bc-ad)^{4/3}+i(bc-ad)^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12\sqrt[3]{c}\,d^2}+\frac{\left((bc-ad)^{4/3}-i\sqrt{3}\,(bc-ad)^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{c}\,d^2}+\frac{\sqrt{-1-i\sqrt{3}}\,(bc-ad)^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,\sqrt[3]{c}\,d^2}+\frac{bx^2\sqrt[3]{a+bx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (b*x^2*(a + b*x^3)^(1/3))/(3*d) + ((3*b^(4/3)*c - 4*a*b^(1/3)*d)*ArcTan[(Sq
rt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*d^2) + (Sqr
t[-1 - I*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]
*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a
 + b*x^3)^(1/3))])/(Sqrt[6]*c^(1/3)*d^2) + ((3*b^(4/3)*c - 4*a*b^(1/3)*d)*L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*d^2) + (((b*c - a*d)^(4/3) - I*Sqr
t[3]*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)
*(a + b*x^3)^(1/3)])/(6*c^(1/3)*d^2) + ((-3*b^(4/3)*c + 4*a*b^(1/3)*d)*Log[
b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*d^2) +
((I/12)*(I*(b*c - a*d)^(4/3) + Sqrt[3]*(b*c - a*d)^(4/3))*Log[(-2*I)*(b*c -
 a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(
1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(1/3)*d^2)
```

**fricas [A]** time = 0.71, size = 396, normalized size = 1.43

$$\frac{6\left(bx^3+a\right)^{\frac{1}{3}}bdx^2-6\sqrt{3}\,(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\,(3\,bc-4\,ad)\,(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3\,bx}\right)-2\,(3\,bc-4\,ad)\,(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-6\,(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(-\frac{x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+(3\,bc-4\,ad)\,(-b)^{\frac{1}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+3\,(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\log\left(\frac{x^2\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}x\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{18\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b*d*x^2 - 6*sqrt(3)*(b*c - a*d)*((b*c - a*d)/c)^(
1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b
*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b*c - 4*a*d)*(-b)^(1/3)
*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^(2/3))/(b*x)) -
 2*(3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 6
*(b*c - a*d)*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 +
 a)^(1/3))/x) + (3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a
)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(b*c - a*d)*((b*c - a*d)
/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) + (b*x^3 + a)^(1/3)*x*((b*c - a*d)
/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} x}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{4}{3}} x}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(b\,x^{3}+a\right)^{4/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^{3}\right)^{\frac{4}{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

## 3.450 $\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$

**Optimal**. Leaf size=254

$$-\frac{b^{4/3}\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2d}-\frac{b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{4/3}d}+\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\cdots}\right.}{2c^{4/3}d}$$

**Rubi [C]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{1}{3};-\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3)*AppellF1[-1/3, -4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*x*(1 + (b*x^3)/a)^(1/3)))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{4/3}}{x^2\left(c+dx^3\right)}\,dx=\frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^2(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$=-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{1}{3};-\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}$$

**Mathematica [C]** time = 0.55, size = 161, normalized size = 0.63

$$\frac{2b^2cx^6\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-\frac{5ax^3\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-2bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{\left(\frac{dx^3}{c}+1\right)^{2/3}}-10ac\left(a+bx^3\right)}{10c^2x\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-10*a*c*(a + b*x^3) + 2*b^2*c*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3,
 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] - (5*a*(-2*b*c + a*d)*x^3*(1 + (b*x^3)
/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x
^3))])/(1 + (d*x^3)/c)^(2/3))/(10*c^2*x*(a + b*x^3)^(2/3))
```

**IntegrateAlgebraic [C]** time = 5.54, size = 509, normalized size = 2.00

$$-\frac{b^{4/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3d}-\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}d}+\frac{b^{4/3}\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6d}+\frac{i\left(\sqrt{3}(bc-ad)^{4/3}+i(bc-ad)^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{4/3}d}-\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}(bc-ad)^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{4/3}d}+\frac{\left((bc-ad)^{4/3}-i\sqrt{3}(bc-ad)^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{4/3}d}-\frac{a\sqrt[3]{a+bx^3}}{cx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(c*x)) - (b^(4/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/
3)*x + 2*(a + b*x^3)^(1/3))])/(Sqrt[3]*d) - (Sqrt[(-1 - I*Sqrt[3])/6]*(b*c
- a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x -
(3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(
4/3)*d) - (b^(4/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*d) + ((I/6)*(I
*(b*c - a*d)^(4/3) + Sqrt[3]*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x +
 (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(4/3)*d) + (b^(4/3)*Log[b^(
2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*d) + (((b*c
 - a*d)^(4/3) - I*Sqrt[3]*(b*c - a*d)^(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x
^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + S
qrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3)*d)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3 + a)^{4/3}}{x^2\,(d\,x^3 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{4}{3}}}{x^2\,(c + dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)
```

## 3.451 $\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$

**Optimal**. Leaf size=201

$$\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}}-\frac{\sqrt[3]{a+bx^3}}{4}$$

**Rubi [C]** time = 0.06, antiderivative size = 90, normalized size of antiderivative = 0.45, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{4cx^4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/3, -
1/3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(4*c*x^4*(1 + (b*x^3)/a)^
(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{4/3}}{x^5\left(c+dx^3\right)}\,dx&=\frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^5\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\&=-\frac{a\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4cx^4\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

**Mathematica** [C] time = 0.04, size = 84, normalized size = 0.42

$$-\frac{a\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{4cx^4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/
3, -1/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x^4*(1 + (b*x^3)/a)^(1/3
))
```

**IntegrateAlgebraic** [C] time = 2.90, size = 383, normalized size = 1.91

$$\frac{\left((bc-ad)^{4/3}-i\sqrt{3}(bc-ad)^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{7/3}}+\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}(bc-ad)^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{7/3}}+\frac{i\left(\sqrt{3}(bc-ad)^{4/3}+i(bc-ad)^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{7/3}}+\frac{\sqrt[3]{a+bx^3}\left(-ac+4adx^3-5bcx^3\right)}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-(a*c) - 5*b*c*x^3 + 4*a*d*x^3))/(4*c^2*x^4) + (Sqrt[(-
1 - I*Sqrt[3])/6]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]
*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a
 + b*x^3)^(1/3))])/c^(7/3) + (((b*c - a*d)^(4/3) - I*Sqrt[3]*(b*c - a*d)^(4
/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]
)/(6*c^(7/3)) + ((I/12)*(I*(b*c - a*d)^(4/3) + Sqrt[3]*(b*c - a*d)^(4/3))*L
og[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*
x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(7/3)
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

**maple** [F] time = 0.63, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^5\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)
```

## 3.452 $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx$

**Optimal**. Leaf size=250

$$-\frac{\sqrt[3]{a+bx^3}\left(28a^2d^2-35abcd+4b^2c^2\right)}{28ac^3x}-\frac{d(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{10/3}}+\frac{d(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}+\frac{d(\cdots}{}$$

**Rubi [C]** time = 0.51, antiderivative size = 169, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{12cx^3\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx^3\right)+5bcx^3\right)-2x^6(bc-ad)^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)\right)}{28c^4x^7\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] (12*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)*Hypergeometric2F1[-1/3, 2, 2/
3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (4*c - 3*d*x^3)*(c*(a + b*x^3)*(5*b
*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(28*c^4*x^7*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^8\left(c+dx^3\right)}\,dx=\frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^8\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$=\frac{12c(bc-ad)x^3\left(a+bx^3\right)\left(c+dx^3\right)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)-\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(5bcx^3\cdots\right.\right.}{28c^4x^7\left(a+bx^3\right)^{2/3}}$$

**Mathematica [C]** time = 0.60, size = 179, normalized size = 0.72

$$-\frac{a\left(\frac{bx^3}{a}+1\right)\left(12cx^3\left(a+bx^3\right)\left(c+dx^3\right)(ad-bc)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx^3\right)+5bcx^3\right)-2x^6(bc-ad)^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)\right)\right)}{28c^4x^7\left(a+bx^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]

[Out] -1/28*(a*(1 + (b*x^3)/a)*(12*c*(-(b*c) + a*d)*x^3*(a + b*x^3)*(c + d*x^3)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + (4*c - 3*d*x^3)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])))/(c^4*x^7*(a + b*x^3)^(5/3))

**IntegrateAlgebraic [C]** time = 3.20, size = 434, normalized size = 1.74

$$\frac{\sqrt[3]{a+bx^3}\left(-4a^2c^2+7a^2cdx^3-28a^2d^2x^6-8abc^2x^3+35abcdx^6-4b^2c^2x^6\right)}{28ac^3x^7}+\frac{i\left(\sqrt{3}d(bc-ad)^{4/3}+id(bc-ad)^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{10/3}}-\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}d(bc-ad)^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{10/3}}+\frac{\left(d(bc-ad)^{4/3}-i\sqrt{3}d(bc-ad)^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{10/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(1/3)*(-4*a^2*c^2 - 8*a*b*c^2*x^3 + 7*a^2*c*d*x^3 - 4*b^2*c^2*x^6 + 35*a*b*c*d*x^6 - 28*a^2*d^2*x^6))/(28*a*c^3*x^7) - (Sqrt[(-1 - I*Sqrt[3])/6]*d*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]])/c^(10/3) + ((I/6)*(I*d*(b*c - a*d)^(4/3) + Sqrt[3]*d*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(10/3) + ((d*(b*c - a*d)^(4/3) - I*Sqrt[3]*d*(b*c - a*d)^(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(10/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^8\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.453 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)} dx$

**Optimal**. Leaf size=318

$$-\frac{\sqrt[3]{a+bx^3}\left(35a^2d^2-40abcd+2b^2c^2\right)}{140ac^3x^4}+\frac{\sqrt[3]{a+bx^3}\left(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3\right)}{140a^2c^4x}+\frac{d^2(bc-ad)^{4/3}}{6c^{\cdots}}$$

**Rubi [C]** time = 1.80, antiderivative size = 260, normalized size of antiderivative = 0.82, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$\frac{-18cx^3\left(a+bx^3\right)\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(-\frac{1}{3},2,2;\frac{2}{3},1;\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+6cx^3\left(a+bx^3\right)\left(11c^2+2cdx^3-9d^2x^6\right)(bc-ad)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-\left(14c^2-12cdx^3+9d^2x^6\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx^3\right)+5bcx^3\right)-2x^6(bc-ad)^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)\right)}{140c^5x^{10}\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] (6*c*(b*c - a*d)*x^3*(a + b*x^3)*(11*c^2 + 2*c*d*x^3 - 9*d^2*x^6)*Hypergeom
etric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (14*c^2 - 12*c*
d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c -
a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
]) - 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{-1/3
, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(140*c^5*x^10*(a + b
*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)} dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{11}\left(c+dx^3\right)} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{6c(bc-ad)x^3\left(a+bx^3\right)\left(11c^2+2cdx^3-9d^2x^6\right)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)-\left(14c^2-12cdx^{3}\cdots\right.}{\cdots}$$

**Mathematica [C]** time = 2.70, size = 270, normalized size = 0.85

$$-\frac{a\left(\frac{bx^3}{a}+1\right)\left(18cx^3\left(a+bx^3\right)\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(-\frac{1}{3},2,2;\frac{2}{3},1;\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+6cx^3\left(a+bx^3\right)\left(-11c^2-2cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+\left(14c^2-12cdx^3+9d^2x^6\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx^3\right)+5bcx^3\right)-2x^6(bc-ad)^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)\right)\right)}{140c^5x^{10}\left(a+bx^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -1/140*(a*(1 + (b*x^3)/a)*(6*c*(b*c - a*d)*x^3*(a + b*x^3)*(-11*c^2 - 2*c*d
*x^3 + 9*d^2*x^6)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + (14*c^2 - 12*c*d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a
*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))]) + 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3
)^2*HypergeometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))]))/(c^5*x^10*(a + b*x^3)^(5/3))
```

**IntegrateAlgebraic [C]** time = 3.69, size = 501, normalized size = 1.58

$$\frac{\sqrt[3]{a+bx^3}\left(-14a^3c^3+20a^3c^2dx^3-35a^3cd^2x^6+140a^3d^3x^9-22a^2bc^3x^3+40a^2bc^2dx^6-175a^2bcd^2x^9-2ab^2c^3x^6+20ab^2c^2dx^9+6b^3c^3x^9\right)}{140a^2c^4x^{10}}+\frac{\left(d^2(bc-ad)^{4/3}-i\sqrt{3}d^2(bc-ad)^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{13/3}}+\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}d^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{13/3}}+\frac{i\left(\sqrt{3}d^2(bc-ad)^{4/3}+id^2(bc-ad)^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{13/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-14*a^3*c^3 - 22*a^2*b*c^3*x^3 + 20*a^3*c^2*d*x^3 - 2*a
*b^2*c^3*x^6 + 40*a^2*b*c^2*d*x^6 - 35*a^3*c*d^2*x^6 + 6*b^3*c^3*x^9 + 20*a
*b^2*c^2*d*x^9 - 175*a^2*b*c*d^2*x^9 + 140*a^3*d^3*x^9))/(140*a^2*c^4*x^10)
 + (Sqrt[(-1 - I*Sqrt[3])/6]*d^2*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1
/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqr
t[3]*c^(1/3)*(a + b*x^3)^(1/3))])/c^(13/3) + ((d^2*(b*c - a*d)^(4/3) - I*Sq
rt[3]*d^2*(b*c - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^
(1/3)*(a + b*x^3)^(1/3)])/(6*c^(13/3)) + ((I/12)*(I*d^2*(b*c - a*d)^(4/3) +
 Sqrt[3]*d^2*(b*c - a*d)^(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*
(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/
3)*(a + b*x^3)^(2/3)])/c^(13/3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^{11}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)
```

```
[Out] Timed out
```

## 3.454 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=392

$$-\frac{\sqrt[3]{a+bx^3}\left(130a^2d^2-143abcd+4b^2c^2\right)}{910ac^3x^7}+\frac{\sqrt[3]{a+bx^3}\left(455a^3d^3-520a^2bcd^2+26ab^2c^2d+12b^3c^3\right)}{1820a^2c^4x^4}-\frac{\sqrt[3]{a+bx^3}\left(18\right.}{}$$

**Rubi [C]** time = 4.39, antiderivative size = 1446, normalized size of antiderivative = 3.69, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] -(140*a^2*c^5 + 840*a*b*c^5*x^3 - 686*a^2*c^4*d*x^3 + 700*b^2*c^5*x^6 - 131
6*a*b*c^4*d*x^6 + 612*a^2*c^3*d^2*x^6 - 630*b^2*c^4*d*x^9 + 1152*a*b*c^3*d^
2*x^9 - 513*a^2*c^2*d^3*x^9 + 540*b^2*c^3*d^2*x^12 - 918*a*b*c^2*d^3*x^12 +
 324*a^2*c*d^4*x^12 - 405*b^2*c^2*d^3*x^15 + 324*a*b*c*d^4*x^15 - 828*a*b*c
^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/
3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*b^2*c^4*d*x^9*Hypergeome
tric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 234*a*b*c^3*d^2*
x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 32
4*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + 324*b^2*c^3*d^2*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] - 918*a*b*c^2*d^3*x^12*Hypergeometric2F1[-1/3, 2,
2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a^2*c*d^4*x^12*Hypergeometric
2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*b^2*c^2*d^3*x^15
*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a
*b*c*d^4*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] - 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*b^2*c^4*d*x^9*Hypergeometric2F
1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 504*a*b*c^3*d^2*x^9*Hyp
ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*a^2*c^
2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 216*b^2*c^3*d^2*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 432*a*b*c^2*d^3*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 216*a^2*c*d^4*x^12*Hypergeometric2F1[2/3, 1,
 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b^2*c^2*d^3*x^15*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*a*b*c*d^4*x^15
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*a^
2*d^5*x^15*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(7*c - 6*d*x^3)*(c + d*x^3)^2*Hypergeo
metricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*c
*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2, 2
}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(1820*c^6*x^13*(a + b*x
^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{140a^2c^5 + 840abc^5x^3 - 686a^2c^4dx^3 + 700b^2c^5x^6 - 1316abc^4dx^6 + 612a^2c^3d^2x^6 - 630b}{}$$

**Mathematica [C]** time = 5.44, size = 1446, normalized size = 3.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] (-140*a^2*c^5 - 840*a*b*c^5*x^3 + 686*a^2*c^4*d*x^3 - 700*b^2*c^5*x^6 + 131
6*a*b*c^4*d*x^6 - 612*a^2*c^3*d^2*x^6 + 630*b^2*c^4*d*x^9 - 1152*a*b*c^3*d^
2*x^9 + 513*a^2*c^2*d^3*x^9 - 540*b^2*c^3*d^2*x^12 + 918*a*b*c^2*d^3*x^12 -
 324*a^2*c*d^4*x^12 + 405*b^2*c^2*d^3*x^15 - 324*a*b*c*d^4*x^15 + 828*a*b*c
^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] + 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/
3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*b^2*c^4*d*x^9*Hypergeome
tric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 234*a*b*c^3*d^2*
x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 32
4*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] - 324*b^2*c^3*d^2*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] + 918*a*b*c^2*d^3*x^12*Hypergeometric2F1[-1/3, 2,
2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a^2*c*d^4*x^12*Hypergeometric
2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*b^2*c^2*d^3*x^15
*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a
*b*c*d^4*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] + 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*b^2*c^4*d*x^9*Hypergeometric2F
1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 504*a*b*c^3*d^2*x^9*Hyp
```

```
ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*a^2*c^
2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 216*b^2*c^3*d^2*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 432*a*b*c^2*d^3*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] + 216*a^2*c*d^4*x^12*Hypergeometric2F1[2/3, 1,
 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*b^2*c^2*d^3*x^15*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 324*a*b*c*d^4*x^15
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a^
2*d^5*x^15*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*(-7*c + 6*d*x^3)*Hyperge
ometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 54*
c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2,
2}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(1820*c^6*x^13*(a + b*
x^3)^(2/3))
```

**IntegrateAlgebraic [C]** time = 4.75, size = 577, normalized size = 1.47

$$\frac{\sqrt[3]{a+bx^3}\left(-140a^4c^4+182a^4c^3dx^3-260a^4c^2d^2x^6+455a^4cd^3x^9-1820a^4d^4x^{12}-196a^3bc^4x^3+286a^3bc^3dx^6-520a^3bc^2d^2x^9+2275a^3bcd^3x^{12}-8a^2b^2c^4x^6+26a^2b^2c^3dx^9-260a^2b^2c^2d^2x^{12}+12ab^3c^4x^9-78ab^3c^3dx^{12}-36b^4c^4x^{12}\right)}{1820a^3c^5x^{13}}+\frac{i\left(\sqrt{3}d^3(bc-ad)^{4/3}+id^3(bc-ad)^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{16/3}}-\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}d^3(bc-ad)^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{16/3}}+\frac{\left(d^3(bc-ad)^{4/3}-i\sqrt{3}d^3(bc-ad)^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{16/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-140*a^4*c^4 - 196*a^3*b*c^4*x^3 + 182*a^4*c^3*d*x^3 -
8*a^2*b^2*c^4*x^6 + 286*a^3*b*c^3*d*x^6 - 260*a^4*c^2*d^2*x^6 + 12*a*b^3*c^
4*x^9 + 26*a^2*b^2*c^3*d*x^9 - 520*a^3*b*c^2*d^2*x^9 + 455*a^4*c*d^3*x^9 -
36*b^4*c^4*x^12 - 78*a*b^3*c^3*d*x^12 - 260*a^2*b^2*c^2*d^2*x^12 + 2275*a^3
*b*c*d^3*x^12 - 1820*a^4*d^4*x^12))/(1820*a^3*c^5*x^13) - (Sqrt[(-1 - I*Sqr
t[3])/6]*d^3*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*
x^3)^(1/3))])/c^(16/3) + ((I/6)*(I*d^3*(b*c - a*d)^(4/3) + Sqrt[3]*d^3*(b*c
 - a*d)^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x
^3)^(1/3)])/c^(16/3) + ((d^3*(b*c - a*d)^(4/3) - I*Sqrt[3]*d^3*(b*c - a*d)^
(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x -
Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(1
2*c^(16/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}}}{\left(dx^3 + c\right)x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3 + a\right)^{4/3}}{x^{14}\,\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)
[Out] Timed out
```

## 3.455 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=290

$$-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)\left(a^2d^2+b^2c^2\right)}{2b^4d^4}+\frac{\left(a+bx^3\right)^{5/3}\left(3a^2d^2+2abcd+b^2c^2\right)}{5b^4d^3}-\frac{\left(a+bx^3\right)^{8/3}(3ad+bc)}{8b^4d^2}+\frac{\left(a+bx^3\right)^{11/}}{11b^4d}$$

**Rubi [A]** time = 0.32, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{5/3}\left(3a^2d^2+2abcd+b^2c^2\right)}{5b^4d^3}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)\left(a^2d^2+b^2c^2\right)}{2b^4d^4}-\frac{\left(a+bx^3\right)^{8/3}(3ad+bc)}{8b^4d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{c^4\log\left(c+dx^3\right)}{6d^{14/3}\sqrt[3]{bc-ad}}-\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}-\frac{c^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{14/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^4) + ((b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - ((b*c + 3*a*d)*
(a + b*x^3)^(8/3))/(8*b^4*d^2) + (a + b*x^3)^(11/3)/(11*b^4*d) - (c^4*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(6*d^(14/3)*(b*c - a*d)^
(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/
3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{a+bx}\,(c+dx)}dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(bc+ad)\left(-b^2c^2-a^2d^2\right)}{b^3d^4\sqrt[3]{a+bx}} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)(a+bx)^{2/3}}{b^3d^3} + \frac{(-\right.\right.$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(b}{}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(b}{}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(b}{}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(b}{}$$

**Mathematica [C]** time = 0.26, size = 157, normalized size = 0.54

$$\frac{\left(a+bx^3\right)^{2/3}\left(\frac{-81a^3d^3+9a^2bd^2\left(6dx^3-11c\right)-3ab^2d\left(44c^2-22cdx^3+15d^2x^6\right)+b^3\left(-220c^3+88c^2dx^3-55cd^2x^6+40d^3x^9\right)}{b^4}+\frac{220c^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad}\right)}{440d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*((-81*a^3*d^3 + 9*a^2*b*d^2*(-11*c + 6*d*x^3) - 3*a*b^2*
d*(44*c^2 - 22*c*d*x^3 + 15*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*
d^2*x^6 + 40*d^3*x^9))/b^4 + (220*c^4*Hypergeometric2F1[2/3, 1, 5/3, (d*(a
+ b*x^3))/(-(b*c) + a*d)])/(b*c - a*d)))/(440*d^4)
```

**IntegrateAlgebraic [A]** time = 0.63, size = 340, normalized size = 1.17

$$\frac{\left(a+bx^3\right)^{2/3}\left(-81a^3d^3-99a^2bcd^2+54a^2bd^3x^3-132ab^2c^2d+66ab^2cd^2x^3-45ab^2d^3x^6-220b^3c^3+88b^3c^2dx^3-55b^3cd^2x^6+40b^3d^3x^9\right)}{440b^4d^4} - \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{14/3}\sqrt[3]{bc-ad}} + \frac{c^4\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{14/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-220*b^3*c^3 - 132*a*b^2*c^2*d - 99*a^2*b*c*d^2 - 81*a^
3*d^3 + 88*b^3*c^2*d*x^3 + 66*a*b^2*c*d^2*x^3 + 54*a^2*b*d^3*x^3 - 55*b^3*c
*d^2*x^6 - 45*a*b^2*d^3*x^6 + 40*b^3*d^3*x^9))/(440*b^4*d^4) - (c^4*ArcTan[
1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sq
rt[3]*d^(14/3)*(b*c - a*d)^(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
 + b*x^3)^(1/3)])/(3*d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[(b*c - a*d)^(2/
3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3
)])/(6*d^(14/3)*(b*c - a*d)^(1/3))
```

**fricas [A]** time = 0.76, size = 1004, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 + (
-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 4
40*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a
*d^3)^(1/3)) + 660*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt((-b*c*d^2 + a
*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(
2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a
*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)
/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c))
- 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d
^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*
b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a
^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7
), 1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 +
 (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 +
 a*d^3)^(1/3)) + 1320*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt(-(-b*c*d^2
 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*
c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(2
20*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d^5 - 8
1*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*b^3*c*
d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a^2*b^2
*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7)]
```

**giac [A]** time = 0.33, size = 454, normalized size = 1.57

$$-\frac{b^{48}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{49}cd^{11}-ab^{48}d^{12}\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^6-\sqrt{3}\,ad^7}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^6-ad^7\right)}-\frac{220\left(bx^3+a\right)^{\frac{2}{3}}b^{43}c^3d^7-88\left(bx^3+a\right)^{\frac{5}{3}}b^{42}c^2d^8+220\left(bx^3+a\right)^{\frac{2}{3}}ab^{42}c^2d^8+55\left(bx^3+a\right)^{\frac{8}{3}}b^{41}cd^9-176\left(bx^3+a\right)^{\frac{5}{3}}ab^{41}cd^9+220\left(bx^3+a\right)^{\frac{2}{3}}a^2b^{41}cd^9-40\left(bx^3+a\right)^{\frac{11}{3}}b^{40}d^{10}+165\left(bx^3+a\right)^{\frac{8}{3}}ab^{40}d^{10}-264\left(bx^3+a\right)^{\frac{5}{3}}a^2b^{40}d^{10}+220\left(bx^3+a\right)^{\frac{2}{3}}a^3b^{40}d^{10}}{440\,b^{44}d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^48*c^4*d^7*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^49*c*d^11 - a*b^48*d^12) - (-b*c*d^2 + a*d^3)^(2/3)*c
^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^6 - sqrt(3)*a*d^7) + 1/6*(-b*c*d^2 + a*d^3
)^(2/3)*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3
) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^6 - a*d^7) - 1/440*(220*(b*x^3 + a)^(2/3
)*b^43*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^42*c^2*d^8 + 220*(b*x^3 + a)^(2/3)*
a*b^42*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^41*c*d^9 - 176*(b*x^3 + a)^(5/3)*a*
b^41*c*d^9 + 220*(b*x^3 + a)^(2/3)*a^2*b^41*c*d^9 - 40*(b*x^3 + a)^(11/3)*b
^40*d^10 + 165*(b*x^3 + a)^(8/3)*a*b^40*d^10 - 264*(b*x^3 + a)^(5/3)*a^2*b^
40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^40*d^10)/(b^44*d^11)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.11, size = 438, normalized size = 1.51

$$\left(\frac{6a^2}{5b^4 d}+\frac{\left(\frac{4a}{b^4 d}+\frac{b^5 c-a b^4 d}{b^8 d^2}\right)\left(b^5 c-a b^4 d\right)}{5b^4 d}\right)\left(b x^3+a\right)^{5/3}-\left(\frac{a}{2b^4 d}+\frac{b^5 c-a b^4 d}{8b^8 d^2}\right)\left(b x^3+a\right)^{8/3}-\left(b x^3+a\right)^{2/3}\left(\frac{2a^3}{b^4 d}+\frac{\left(\frac{6a^2}{b^4 d}+\frac{\left(\frac{4a}{b^4 d}+\frac{b^5 c-a b^4 d}{b^8 d^2}\right)\left(b^5 c-a b^4 d\right)}{b^4 d}\right)\left(b^5 c-a b^4 d\right)}{2b^4 d}\right)+\frac{\left(b x^3+a\right)^{11/3}}{11 b^4 d}+\frac{c^4 \ln\left(\frac{c^8\left(b x^3+a\right)^{1/3}}{d^7}-\frac{c^8\left(a d-b c\right)^{1/3}}{d^{22/3}}\right)}{3 d^{14/3}\left(a d-b c\right)^{1/3}}-\frac{\ln\left(\frac{c^8\left(b x^3+a\right)^{1/3}}{d^7}-\frac{c^8\left(1+\sqrt{3}\,1i\right)^2\left(a d-b c\right)^{1/3}}{4 d^{22/3}}\right)\left(c^4+\sqrt{3}\,c^4\,1i\right)}{6 d^{14/3}\left(a d-b c\right)^{1/3}}+\frac{c^4 \ln\left(\frac{c^8\left(b x^3+a\right)^{1/3}}{d^7}-\frac{c^8\left(-1+\sqrt{3}\,1i\right)^2\left(a d-b c\right)^{1/3}}{4 d^{22/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)}{d^{14/3}\left(a d-b c\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((6*a^2)/(5*b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c
- a*b^4*d))/(5*b^4*d))*(a + b*x^3)^(5/3) - (a/(2*b^4*d) + (b^5*c - a*b^4*d)
/(8*b^8*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*((2*a^3)/(b^4*d) + (((6
*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c - a*b
^4*d))/(b^4*d))*(b^5*c - a*b^4*d))/(2*b^4*d)) + (a + b*x^3)^(11/3)/(11*b^4*
d) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(a*d - b*c)^(1/3))/d^(22/3
)))/(3*d^(14/3)*(a*d - b*c)^(1/3)) - (log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^
8*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*(3^(1/2)*c^4*1i + c^4
))/(6*d^(14/3)*(a*d - b*c)^(1/3)) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 -
(c^8*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*((3^(1/2)*1i)/6 -
1/6))/(d^(14/3)*(a*d - b*c)^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^{3}}\,\left(c+dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.456 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=244

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}-\frac{\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{\cdots}\right.}{2d^{11/3}\sqrt[3]{bc-\cdots}}$$

**Rubi [A]** time = 0.24, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}-\frac{\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - ((b*c + 2*a
*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (a + b*x^3)^(8/3)/(8*b^3*d) + (c^3*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d
)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3\sqrt[3]{a+bx}} + \frac{(-bc-2ad)(a+bx)^{2/3}}{b^2d^2} + \frac{(a+bx)^{5/3}}{b^2d} - \frac{\ldots}{d^3\sqrt[3]{a}\ldots}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\,\text{S}\ldots}{\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\,\text{lo}\ldots}{6d^{11}\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\,\text{lo}\ldots}{6d^{11}\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} + \frac{c^3\,\text{ta}\ldots}{\sqrt{\ldots}}$$

**Mathematica [C]** time = 0.14, size = 145, normalized size = 0.59

$$-\frac{\left(a+bx^3\right)^{2/3}\left(9a^3d^3+3a^2bd^2\left(c-2dx^3\right)+20b^3c^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+ab^2d\left(8c^2-2cdx^3+5d^2x^6\right)+b^3c\left(-20c^2+8cdx^3-5d^2x^6\right)\right)}{40b^3d^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*((a + b*x^3)^(2/3)*(9*a^3*d^3 + 3*a^2*b*d^2*(c - 2*d*x^3) + b^3*c*(-2
0*c^2 + 8*c*d*x^3 - 5*d^2*x^6) + a*b^2*d*(8*c^2 - 2*c*d*x^3 + 5*d^2*x^6) +
20*b^3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]))
/(b^3*d^3*(b*c - a*d))
```

**IntegrateAlgebraic [A]** time = 0.42, size = 284, normalized size = 1.16

$$\frac{\left(a+bx^3\right)^{2/3}\left(9a^2d^2+12abcd-6abd^2x^3+20b^2c^2-8b^2cdx^3+5b^2d^2x^6\right)}{40b^3d^3}+\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{11/3}\sqrt[3]{bc-ad}}-\frac{c^3\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(20*b^2*c^2 + 12*a*b*c*d + 9*a^2*d^2 - 8*b^2*c*d*x^3 - 6
*a*b*d^2*x^3 + 5*b^2*d^2*x^6))/(40*b^3*d^3) + (c^3*ArcTan[1/Sqrt[3] - (2*d^
```

```
(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(11/3)*(b
*c - a*d)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(3*d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
 - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(11/3)*(
b*c - a*d)^(1/3))
```

**fricas** [A] time = 0.74, size = 873, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b*
c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*(b
*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(
1/3)) - 60*sqrt(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1
/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d
^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (
b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*
d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(20*b^3
*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*d^4 - a*b
^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*(b*x^3 + a
)^(2/3))/(b^4*c*d^5 - a*b^3*d^6), -1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^
3*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d +
 (b*c*d^2 - a*d^3)^(2/3)) - 40*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 +
 a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 120*sqrt(1/3)*(b^4*c^4*d - a*b^3*c
^3*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^
3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c
 - a*d))/d) - 3*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d
^5 + 5*(b^3*c*d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2
*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6)]
```

**giac** [A] time = 0.28, size = 371, normalized size = 1.52

$$\frac{b^{27}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{28}cd^8-ab^{27}d^9\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^5-\sqrt{3}\,ad^6}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^5-ad^6\right)}+\frac{20\left(bx^3+a\right)^{\frac{2}{3}}b^{23}c^2d^5-8\left(bx^3+a\right)^{\frac{5}{3}}b^{22}cd^6+20\left(bx^3+a\right)^{\frac{2}{3}}ab^{22}cd^6+5\left(bx^3+a\right)^{\frac{8}{3}}b^{21}d^7-16\left(bx^3+a\right)^{\frac{5}{3}}ab^{21}d^7+20\left(bx^3+a\right)^{\frac{2}{3}}a^2b^{21}d^7}{40\,b^{24}d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^3)^(
2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/3)*b^2
3*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*b^22*c*
d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^7 + 20*(
b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.09, size = 339, normalized size = 1.39

$$\left(\frac{3\,a^2}{2\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\left(\frac{3\,a}{5\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{5\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^3\,d}-\frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{b\,c^7-a\,c^6\,d}{d^{16/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}+\frac{\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}-\frac{c^6\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{16/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}-\frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{c^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{16/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(2*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(2*b^3*d))*(a + b*x^3)^(2/3) - ((3*a)/(5*b^3*d) + (b^4*c - a*b^
3*d)/(5*b^6*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^3*d) - (c^3*lo
g((c^6*(a + b*x^3)^(1/3))/d^5 + (b*c^7 - a*c^6*d)/(d^(16/3)*(a*d - b*c)^(2/
3))))/(3*d^(11/3)*(a*d - b*c)^(1/3)) + (log((c^6*(a + b*x^3)^(1/3))/d^5 - (
c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(16/3)))*(3^(1/2)*c^3*1i + c
^3))/(6*d^(11/3)*(a*d - b*c)^(1/3)) - (c^3*log((c^6*(a + b*x^3)^(1/3))/d^5
+ (c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/d^(16/3))*((3^(1/2)*1i)/2
- 1/2))/(3*d^(11/3)*(a*d - b*c)^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.457 $\int \frac{x^8}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=203

$$-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}$$

**Rubi [A]** time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 56, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^2*d^2) + (a + b*x^3)^(5/3)/(5*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}\,(c+dx)}dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{bd^2\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{bd} + \frac{c^2}{d^2\sqrt[3]{a+bx}\,(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\operatorname{Subst}\left(\int \frac{}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\cdots}\right.}{}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad}+\cdots\right.}{2d^{8/3}\sqrt[3]{bc-\cdots}}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\log\left(c+dx^{\cdots}\right.}{6d^{8/3}\sqrt[3]{bc-a\cdots}}$$

**Mathematica [C]** time = 0.09, size = 103, normalized size = 0.51

$$\frac{\left(a+bx^3\right)^{2/3}\left(3a^2d^2+5b^2c^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2abd\left(c-dx^3\right)+b^2c\left(2dx^3-5c\right)\right)}{10b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(3*a^2*d^2 + 2*a*b*d*(c - d*x^3) + b^2*c*(-5*c + 2*d*x^3
) + 5*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)
]))/(10*b^2*d^2*(b*c - a*d))
```

**IntegrateAlgebraic [A]** time = 0.40, size = 247, normalized size = 1.22

$$\frac{\left(a+bx^3\right)^{2/3}\left(-3ad-5bc+2bdx^3\right)}{10b^2d^2} - \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*b*c - 3*a*d + 2*b*d*x^3))/(10*b^2*d^2) - (c^2*ArcTan
[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(S
qrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
 + b*x^3)^(1/3)])/(3*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[(b*c - a*d)^(2/3
) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)
])/(6*d^(8/3)*(b*c - a*d)^(1/3))
```

**fricas [B]** time = 0.79, size = 768, normalized size = 3.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c
*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-
b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)
^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d^3)^
(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*
c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c
- a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(5
*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^
3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2
*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b
*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(b^3*c^3*d - a
*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^
2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5)]
```

**giac [A]** time = 0.26, size = 313, normalized size = 1.54

$$-\frac{b^{12}c^{2}d^{3}\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{13}cd^{5}-ab^{12}d^{6}\right)}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^{4}-\sqrt{3}\,ad^{5}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c^{2}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^{4}-ad^{5}\right)}-\frac{5\left(bx^{3}+a\right)^{\frac{2}{3}}b^{9}cd^{3}-2\left(bx^{3}+a\right)^{\frac{5}{3}}b^{8}d^{4}+5\left(bx^{3}+a\right)^{\frac{2}{3}}ab^{8}d^{4}}{10\,b^{10}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^12*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^13*c*d^5 - a*b^12*d^6) - (-b*c*d^2 + a*d^3)^(2/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) + 1/6*(-b*c*d^2 + a*d^3)^
(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) - 1/10*(5*(b*x^3 + a)^(2/3)*b^9
*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^10
*d^5)
```

**maple [F]** time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.11, size = 267, normalized size = 1.32

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b^2\,d}-\left(\frac{a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{2\,b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{2/3}+\frac{c^2\,\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}+\frac{b\,c^5-a\,c^4\,d}{d^{10/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}-\frac{\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}-\frac{c^4\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{10/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)}{6\,d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}+\frac{c^2\,\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}-\frac{c^4\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{10/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b^2*d) - (a/(b^2*d) + (b^3*c - a*b^2*d)/(2*b^4*d^2))*(
a + b*x^3)^(2/3) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 + (b*c^5 - a*c^4*d)
/(d^(10/3)*(a*d - b*c)^(2/3))))/(3*d^(8/3)*(a*d - b*c)^(1/3)) - (log((c^4*(
a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(10
/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(8/3)*(a*d - b*c)^(1/3)) + (c^2*log((c^4
*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(
10/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(8/3)*(a*d - b*c)^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.458 $\int \frac{x^5}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=168

$$-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}}+\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

**Rubi [A]** time = 0.16, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 56, 617, 204, 31}

$$-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}}+\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) - (c*Log[c +
 d*x^3])/(6*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} - \frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^2} +$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} - \frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\right.}{2d^{5/3}\sqrt[3]{bc-ad}}$$

**Mathematica [C]** time = 0.03, size = 69, normalized size = 0.41

$$-\frac{\left(a+bx^3\right)^{2/3}\left(bc\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+ad-bc\right)}{2bd(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*(-(b*c) + a*d + b*c*Hypergeometric2F1[2/3, 1, 5/3,
(d*(a + b*x^3))/(-(b*c) + a*d)]))/(b*d*(b*c - a*d))
```

**IntegrateAlgebraic [A]** time = 0.21, size = 224, normalized size = 1.33

$$\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{3d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}} + \frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1
/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) + (c
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(5/3)*(b*c - a*d)
```

```
^(1/3)) - (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^
(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(5/3)*(b*c - a*d)^(1/3))
```

**fricas [B]** time = 0.73, size = 667, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a
*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a
*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*lo
g((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1
/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 -
a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*(b*x^3 +
 a)^(2/3))/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d +
(b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((b*c*d^
2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*
c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(b*c
*d^2 - a*d^3)*(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4)]
```

**giac [A]** time = 0.28, size = 257, normalized size = 1.53

$$\frac{\frac{2\,bcd\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^2-ad^3}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^3-\sqrt{3}\,ad^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^3-ad^4}+\frac{3\left(bx^3+a\right)^{\frac{2}{3}}}{d}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*
d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3
*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1
/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^(2/3)/d)/b
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maximima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.10, size = 219, normalized size = 1.30

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2\,b\,d}+\frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{4/3}}\right)\,\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}}+\frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{4/3}}\right)\,\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{6\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}}-\frac{c\,\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}+\frac{b\,c^3-a\,c^2\,d}{d^{4/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{5/3}\,\left(a\,d-b\,c\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*
1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c - 3^(1/2)*c*1i))/(6*d^(5/3)*(a
*d - b*c)^(1/3)) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*1i + 1)^2
*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c + 3^(1/2)*c*1i))/(6*d^(5/3)*(a*d - b*c)
^(1/3)) - (c*log((c^2*(a + b*x^3)^(1/3))/d + (b*c^3 - a*c^2*d)/(d^(4/3)*(a*
d - b*c)^(2/3))))/(3*d^(5/3)*(a*d - b*c)^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.459 $\int \frac{x^2}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=145

$$\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}}$$

**Rubi [A]** time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 56, 617, 204, 31}

$$\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(6*d^(2/3)*(b*c - a*d)^(
1/3)) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c
- a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-a\ldots}}{\sqrt[3]{d}}\ldots}\right.}{2d^{2/3}\ldots}$$

$$= \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{\ldots}}{\ldots}\right.}{d^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

**Mathematica [C]** time = 0.02, size = 50, normalized size = 0.34

$$\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) +
 a*d)])/(2*b*c - 2*a*d)
```

**IntegrateAlgebraic [A]** time = 0.17, size = 201, normalized size = 1.39

$$-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{3d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/
3))]/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(1/3))) - Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)]/(3*d^(2/3)*(b*c - a*d)^(1/3)) + Log[(b*c - a*d)^(2/3) -
 d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(
6*d^(2/3)*(b*c - a*d)^(1/3))
```

**fricas [B]** time = 0.82, size = 592, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)
)*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2
/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c
*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^3)^(2
/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*
d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^
(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt(1/3)*(b
*c*d - a*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
 - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^
(1/3)))/(b*c*d^2 - a*d^3)]
```

**giac [A]** time = 0.29, size = 226, normalized size = 1.56

$$-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^2-\sqrt{3}\,ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2-ad^3\right)}-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
- a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3)
+ 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-
(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3) - 1/3*(-(b
*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c - a*d)
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.93, size = 208, normalized size = 1.43

$$\frac{\ln\left(d\left(b\,x^3+a\right)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{9\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{1/3}}+\frac{\ln\left(d\left(b\,x^3+a\right)^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}}-\frac{\ln\left(d\left(b\,x^3+a\right)^{1/3}-\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(d*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(9*d^(4/3)*(a*d - b*c)^(2/3
)))/(3*d^(2/3)*(a*d - b*c)^(1/3)) + (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*1i
 - 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i
- 1))/(6*d^(2/3)*(a*d - b*c)^(1/3)) - (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*
1i + 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1
i + 1))/(6*d^(2/3)*(a*d - b*c)^(1/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.460 $\int \frac{1}{x\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=244

$$-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,c}+\tan$$

**Rubi [A]** time = 0.21, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 86, 55, 617, 204, 31, 56}

$$-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,c}-\frac{\log(x)}{2\sqrt[3]{a}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
c) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) - Log[x]/(2*a^(1/3)*c) - (d^(1/3)*L
og[c + d*x^3])/(6*c*(b*c - a*d)^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(
2*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*c*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{\text{Subst}\,\Big(}{}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,c} + \frac{\sqrt[3]{d}\,\log\Big(\sqrt[3]{bc-ad}+\sqrt[3]{}}{2c\sqrt[3]{bc-ad}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,c} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\text{lo}}{}$$

**Mathematica [C]** time = 0.18, size = 140, normalized size = 0.57

$$\frac{3\sqrt[3]{a}\,d\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)-(bc-ad)\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)}{6\sqrt[3]{a}\,c(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (3*a^(1/3)*d*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3
))/(-(b*c) + a*d)] - (b*c - a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/(6*a
^(1/3)*c*(-(b*c) + a*d))
```

**IntegrateAlgebraic [A]** time = 0.49, size = 332, normalized size = 1.36

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6\sqrt[3]{a}\,c}-\frac{\sqrt[3]{d}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3\sqrt[3]{a}\,c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3
)*c) + (d^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(
b*c - a*d)^(1/3))])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) + Log[-a^(1/3) + (a + b*x
^3)^(1/3)]/(3*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*
x^3)^(1/3)])/(3*c*(b*c - a*d)^(1/3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1
/3) + (a + b*x^3)^(2/3)]/(6*a^(1/3)*c) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d
^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6
*c*(b*c - a*d)^(1/3))
```

**fricas [A]** time = 0.45, size = 628, normalized size = 2.57

$$\left[\frac{3\sqrt{\frac{1}{3}}a\sqrt{-\frac{1}{a^{2/3}}}\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{2/3}a^{2/3}-\left(bx^3+a\right)^{1/3}a-a^{4/3}\right)\sqrt{-\frac{1}{a^{2/3}}}-3\left(bx^3+a\right)^{1/3}a^{2/3}+3a}{x^3}\right)-2\sqrt{3}a\left(\frac{d}{bc-ad}\right)^{1/3}\arctan\left(\frac{2}{3}\sqrt{3}\left(bx^3+a\right)^{1/3}\left(\frac{d}{bc-ad}\right)^{1/3}-\frac{1}{3}\sqrt{3}\right)-a\left(\frac{d}{bc-ad}\right)^{1/3}\log\left(-\left(bx^3+a\right)^{1/3}(bc-ad)\left(\frac{d}{bc-ad}\right)^{2/3}+\left(bx^3+a\right)^{2/3}d+(bc-ad)\left(\frac{d}{bc-ad}\right)^{1/3}\right)+2a\left(\frac{d}{bc-ad}\right)^{1/3}\log\left((bc-ad)\left(\frac{d}{bc-ad}\right)^{2/3}+\left(bx^3+a\right)^{1/3}d\right)-a^{2/3}\log\left(\left(bx^3+a\right)^{2/3}+\left(bx^3+a\right)^{1/3}a^{1/3}+a^{2/3}\right)+2a^{2/3}\log\left(\left(bx^3+a\right)^{1/3}-a^{1/3}\right)}{6ac},\ -\frac{2\sqrt{3}a\left(\frac{d}{bc-ad}\right)^{1/3}\arctan\left(\frac{2}{3}\sqrt{3}\left(bx^3+a\right)^{1/3}\left(\frac{d}{bc-ad}\right)^{1/3}-\frac{1}{3}\sqrt{3}\right)+a\left(\frac{d}{bc-ad}\right)^{1/3}\log\left(-\left(bx^3+a\right)^{1/3}(bc-ad)\left(\frac{d}{bc-ad}\right)^{2/3}+\left(bx^3+a\right)^{2/3}d+(bc-ad)\left(\frac{d}{bc-ad}\right)^{1/3}\right)-2a\left(\frac{d}{bc-ad}\right)^{1/3}\log\left((bc-ad)\left(\frac{d}{bc-ad}\right)^{2/3}+\left(bx^3+a\right)^{1/3}d\right)-6\sqrt{\frac{1}{3}}a^{2/3}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{1/3}+a^{1/3}\right)}{a^{1/3}}\right)+a^{2/3}\log\left(\left(bx^3+a\right)^{2/3}+\left(bx^3+a\right)^{1/3}a^{1/3}+a^{2/3}\right)-2a^{2/3}\log\left(\left(bx^3+a\right)^{1/3}-a^{1/3}\right)}{6ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 +
 a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*arct
an(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - a*(
d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/
3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(d/(b*c
 - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d)
 - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2
*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*a*(d/(b*c
 - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) -
 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*
(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(
1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (
b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c)]
```

**giac [A]** time = 0.74, size = 326, normalized size = 1.34

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2d-acd^2\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}c}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}c}+\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3a^{\frac{1}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b*c^2 - a*c*d) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*
x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b
*c^2*d - sqrt(3)*a*c*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b
*c^2*d - a*c*d^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a
^(1/3))/a^(1/3))/(a^(1/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3))/(a^(1/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3))
)/(a^(1/3)*c)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right) x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}} \left(dx^{3}+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)
```

**mupad [B]** time = 6.44, size = 702, normalized size = 2.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2
*d^2 + b^2*c^2 - 2*a*b*c*d) - 243*a*b^4*c^4*d^3*(d/(27*b*c^4 - 27*a*c^3*d))
^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)))/(27*b*c^4 - 27*a*c^3*d))*(d/(27*
b*c^4 - 27*a*c^3*d))^(1/3) + log((a + b*x^3)^(1/3) - a*c^2*(1/(a*c^3))^(2/3
))*(1/(27*a*c^3))^(1/3) + (log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(3^(1/2)*1i -
 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) -
 (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^(2/3)*(2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*
1i - 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - (log(b^5*d^4*(a + b*x^3)^(1/
3) + (d*(3^(1/2)*1i + 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)^2*(d/(27*b*c^4 -
27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27
*a*c^3*d)))*(3^(1/2)*1i + 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - log((a
+ b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i + 3^(1/2)*a*c^2*(1/(a*c^3))^
(2/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*c^3))^(1/3) + log((a + b*x^3)^(1/3)*
2i + a*c^2*(1/(a*c^3))^(2/3)*1i - 3^(1/2)*a*c^2*(1/(a*c^3))^(2/3))*((3^(1/2
)*1i)/2 - 1/2)*(1/(27*a*c^3))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt[3]{a+bx^{3}} \left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.461 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=296

$$-\frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}c^2}+\frac{\log(x)(3ad+bc)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\,\mathrm{l}}{}$$

**Rubi [A]** time = 0.31, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 103, 156, 55, 617, 204, 31, 56}

$$-\frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}c^2}+\frac{\log(x)(3ad+bc)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(3*a*c*x^3) - ((b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*
x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*c^2) - (d^(4/3)*ArcTan[(
1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^2
*(b*c - a*d)^(1/3)) + ((b*c + 3*a*d)*Log[x])/(6*a^(4/3)*c^2) + (d^(4/3)*Log
[c + d*x^3])/(6*c^2*(b*c - a*d)^(1/3)) - ((b*c + 3*a*d)*Log[a^(1/3) - (a +
b*x^3)^(1/3)])/(6*a^(4/3)*c^2) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(bc+3ad)+\frac{bdx}{3}}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+b}}\right.}{9ac^2}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} + \frac{d\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-}\right.}{}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} - \frac{(bc+3ad)\log\left(\sqrt[3]{a}-\right.}{6a^{4/3}c^2}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} - \frac{(bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}c^2} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2\sqrt[3]{bc-ad}} + \frac{(bc}{}$$

**Mathematica [C]** time = 0.61, size = 156, normalized size = 0.53

$$-\frac{\frac{(3ad+bc)\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)}{a^{4/3}}-\frac{9d^2\left(a+bx^3\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad}+\frac{6c\left(a+bx^3\right)^{2/3}}{ax^3}}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/18*((6*c*(a + b*x^3)^(2/3))/(a*x^3) - (9*d^2*(a + b*x^3)^(2/3)*Hypergeom
etric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(b*c - a*d) + ((b*c
+ 3*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3
*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/a^(4/3))/c^2
```

**IntegrateAlgebraic [A]** time = 0.71, size = 386, normalized size = 1.30

$$\frac{(-3ad-bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9a^{4/3}c^2}+\frac{(3ad+bc)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{18a^{4/3}c^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^2\sqrt[3]{bc-ad}}+\frac{d^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(2/3)/(a*c*x^3) - ((b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*c^2) - (d^(4/3)*ArcTan
[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(S
qrt[3]*c^2*(b*c - a*d)^(1/3)) + ((-(b*c) - 3*a*d)*Log[-a^(1/3) + (a + b*x^3
)^(1/3)])/(9*a^(4/3)*c^2) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b
*x^3)^(1/3)])/(3*c^2*(b*c - a*d)^(1/3)) + ((b*c + 3*a*d)*Log[a^(2/3) + a^(1
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(4/3)*c^2) + (d^(4/3)*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*
(a + b*x^3)^(2/3)])/(6*c^2*(b*c - a*d)^(1/3))
```

**fricas [A]** time = 0.62, size = 837, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3
 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c -
a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*
x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b
*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)
*d) + 3*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3
*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/
3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + (b*
c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1
/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) +
(-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a*c)/(a^2*c^2*x^3), 1/18*(6*sqrt(3)*a^2*d
*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c -
 a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log(-(b*x^
3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*
c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log((
b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*(a*b
*c + 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)
 - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x
^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d
```

```
)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*
a*c)/(a^2*c^2*x^3)]
```

**giac [A]** time = 0.88, size = 383, normalized size = 1.29

$$-\frac{d^{2} \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3 \left(bc^{3}-ac^{2}d\right)} - \frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2 \left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\, bc^{3}-\sqrt{3}\, ac^{2}d} + \frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}} \log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6 \left(bc^{3}-ac^{2}d\right)} + \frac{(bc+3\,ad) \log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\, a^{\frac{4}{3}} c^{2}} - \frac{\sqrt{3}\left(a^{\frac{2}{3}} bc+3\, a^{\frac{5}{3}} d\right) \arctan\left(\frac{\sqrt{3}\left(2 \left(bx^{3}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\, a^{\frac{1}{3}}}\right)}{9\, a^{2} c^{2}} - \frac{\left(a^{\frac{1}{3}} bc+3\, a^{\frac{4}{3}} d\right) \log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\, a^{\frac{5}{3}} c^{2}} - \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{3\, acx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) - (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(
2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt
(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/(b*c^3 - a*c^2*d) + 1/18*(b*c + 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^2) - 1/9*sqrt(3)*(a^(2/3)*b*c + 3*a^(5
/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*c^2
) - 1/9*(a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(
a^(5/3)*c^2) - 1/3*(b*x^3 + a)^(2/3)/(a*c*x^3)
```

**maple [F]** time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right) x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}} \left(dx^{3}+c\right)x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 11.36, size = 1929, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(- (((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^
2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*(2*a^2*d^2 +
 b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*(-(3*a*d + b*c)^3
/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*
a^2*b*c*d^2))/(3*a^2*c))*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^
7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-(27*a^3*d^3 + b^3*c^3
+ 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log(- (-d^4/(27*b*
c^7 - 27*a*c^6*d))^(2/3)*((-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*
(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d
 - 12*a^3*b*c*d^3))/a^2 - 243*a*b^4*c^4*d^3*(-d^4/(27*b*c^7 - 27*a*c^6*d))^
```

```
(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)) + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 +
 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c)) - (4*b^5*d^7*(a + b*x^3)^(1/3)
*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3) - log(
(((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2
+ 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 -
1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*
((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 - (b^5*d^4*(b^
3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*
1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3
)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3
 + b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log((((
((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4
*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 + 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2
)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*((3
^(1/2)*1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c
^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*1i)
/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3)^(
1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 +
b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + (log(- ((3
^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((
3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a
*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(
-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/
2 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^
2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4*b^5*d^7*(a + b*x^3)^(1/3
)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^
6*d))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(-d^4/(27*b*c^
7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4
- 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4
*d^3*(3^(1/2)*1i + 1)^2*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d))/4))/2 - (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d
 + 18*a^2*b*c*d^2))/(3*a^2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4
*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i + 1)*
(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3))/2 - (a + b*x^3)^(2/3)/(3*a*c*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.462 $\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=273

$$\frac{(ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}d^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2}+\frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}}-\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\right.}{2d^2\sqrt[3]{bc-ad}}$$

**Rubi [A]** time = 0.51, antiderivative size = 394, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {494, 470, 522, 200, 31, 634, 617, 204, 628}

$$\frac{(ad+3bc)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2}-\frac{(ad+3bc)\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{4/3}d^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2}-\frac{c^{4/3}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}}+\frac{c^{4/3}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6d^2\sqrt[3]{bc-ad}}+\frac{c^{4/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^2\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d^2) + (c^(4/3)*ArcTan[(c^(1/
3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3
]*d^2*(b*c - a*d)^(1/3)) + ((3*b*c + a*d)*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(
1/3)])/(9*b^(4/3)*d^2) - ((3*b*c + a*d)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(4/3)*d^2) - (c^(4/3)*Log[c^(1
/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^(1/3)) +
 (c^(4/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3
)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = a^2\,\text{Subst}\left(\int \frac{x^6}{(1-bx^3)^2\,(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{c+(2bc+ad)x^3}{(1-bx^3)(c+(-bc+ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{1}{1-b\ldots}\right.}{3bd^2}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{2\ldots}{c^{2/3}+\sqrt[3]{c}\ldots}\right.}{\ldots}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{c^{4/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + \frac{c^{5/3}\,\text{Su}\ldots}{\ldots}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{(3bc+ad)\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\ldots}{\sqrt[3]{a\ldots}}\right.}{18b^{4/3}d^2}$$

$$= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{(3bc+ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2} + \frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2\sqrt[3]{bc-ad}} + \frac{(3bc+\ldots}{\ldots}$$

**Mathematica** [C]  time = 0.82, size = 288, normalized size = 1.05

$$\frac{\frac{2\left(-a\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}}+c^{2/3}\right)+6x(a+bx^3)^{2/3}\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)-2\sqrt{3}\,a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}-\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,(ad+3bc)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{a+bx^3}}}{36bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*(3*b*c + a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)])/(c*(a + b*x^3)^(1/3)) + (2*(6*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x
)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*Log[c^(1/3) - ((b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)]))/(b*c - a*d)^(1/3))/(36*b*d)
```

**IntegrateAlgebraic** [C]  time = 4.25, size = 525, normalized size = 1.92

$$\frac{(ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{4/3}d^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\,b^{4/3}d^2}+\frac{(-ad-3bc)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}+b^{2/3}x^2\right)}{18b^{4/3}d^2}+\frac{(c^{4/3}+i\sqrt{3}c^{4/3})\log\left(2x\sqrt[3]{bc-ad}+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6d^2\sqrt[3]{bc-ad}}-\frac{\sqrt{-1+i\sqrt{3}}\,c^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,d^2\sqrt[3]{bc-ad}}-\frac{i(\sqrt{3}c^{4/3}-ic^{4/3})\log\left((\sqrt{3}+i)c^{2/3}(a+bx^3)^{2/3}+\sqrt[3]{c}(-\sqrt{3}x+ix)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^2\sqrt[3]{bc-ad}}+\frac{x(a+bx^3)^{2/3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(
b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^2) - (Sqrt[-1 + I*S
qrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x
 - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(
Sqrt[6]*d^2*(b*c - a*d)^(1/3)) + ((3*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x
```

```
^3)^(1/3)])/(9*b^(4/3)*d^2) + ((c^(4/3) + I*Sqrt[3]*c^(4/3))*Log[2*(b*c - a
*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d
)^(1/3)) + ((-3*b*c - a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) +
(a + b*x^3)^(2/3)])/(18*b^(4/3)*d^2) - ((I/12)*((-I)*c^(4/3) + Sqrt[3]*c^(4
/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sq
rt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(d^2
*(b*c - a*d)^(1/3))
```

**fricas [A]** time = 0.63, size = 826, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*
d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)
/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))
^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 +
a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c + a*b*d
)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1
/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)
*sqrt(-1/b^(2/3)) + 2*a) - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3
)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d^2), -1/18*(6*sqrt(3)*b^2*c*(-
c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(
-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*c*(-c/(b*c - a
*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)
*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2) - 6*(b*x^
3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1
/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c + a*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*d^2)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3 + a\right)^{\frac{1}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\, x^3 + a\right)^{\frac{1}{3}} \left(d\, x^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3 + a\right)^{\frac{1}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.463 $\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

**Optimal**. Leaf size=233

$$-\frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}\,d}+\frac{\tan^{-1}\left(\cdots\right)}{\sqrt{3}\,\sqrt[3]{\cdots}}$$

**Rubi [A]** time = 0.29, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 481, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{b}\,d}-\frac{\sqrt[3]{c}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d\sqrt[3]{bc-ad}}-\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d) -
 (c^(1/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d*(b*c - a*d)^(1/3)) - Log[1 - (b^(1/3)*x)/(a + b*
x^3)^(1/3)]/(3*b^(1/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1
/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(1/3)*d) + (c^(1/3)*Log[c^(1/3) - ((b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d*(b*c - a*d)^(1/3)) - (c^(1/3)*Log[c^(2
/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3
)*x)/(a + b*x^3)^(1/3)])/(6*d*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx &= a\,\text{Subst}\left(\int \frac{x^3}{\left(1-bx^3\right)\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d} + \frac{\text{Subst}\left(\int \frac{2+\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d} - \frac{\sqrt[3]{c}\,\text{Sub}}{}\\
&= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+}}\right.}{2d}\\
&= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}d} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} - \\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\log\left(1+\frac{b^{2/3}x}{\left(a+bx^3\right.}\right.}{6\sqrt[3]{}}
\end{aligned}
$$

**Mathematica [C]** time = 0.04, size = 65, normalized size = 0.28

$$
\frac{x^4\sqrt[3]{\frac{a+bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{a+bx^3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*((a + b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [C]** time = 3.13, size = 474, normalized size = 2.03

$$\frac{\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6\sqrt[3]{b}\,d}+\frac{\left(\sqrt[3]{c}+i\sqrt{3}\sqrt[3]{c}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d\sqrt[3]{bc-ad}}-\frac{i\left(\sqrt{3}\sqrt[3]{c}-i\sqrt[3]{c}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt{-1+i\sqrt{3}}\sqrt[3]{c}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}d\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3\sqrt[3]{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(1
/3)*d) + (Sqrt[-1 + I*Sqrt[3]]*c^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt
[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)
*(a + b*x^3)^(1/3))])/(Sqrt[6]*d*(b*c - a*d)^(1/3)) - Log[-(b^(1/3)*x) + (a
 + b*x^3)^(1/3)]/(3*b^(1/3)*d) - ((I/6)*((-I)*c^(1/3) + Sqrt[3]*c^(1/3))*Lo
g[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(d*(b
*c - a*d)^(1/3)) + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x
^3)^(2/3)]/(6*b^(1/3)*d) + ((c^(1/3) + I*Sqrt[3]*c^(1/3))*Log[(-2*I)*(b*c -
 a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(
1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*d*(b*c - a*d)^(1/3))
```

**fricas [A]** time = 0.47, size = 761, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-
b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*b*(c/
(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(
b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b
*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*log(((
b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(
b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((-b)^(1/3
)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^
(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(6*sqrt(1/3)*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3
)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*b*(c/(b*c -
 a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*
c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3)
+ (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)
*c)/x^2) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (-b)^(2
/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3
))/x^2))/(b*d)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{{\left(bx^{3}+a\right)}^{\frac{1}{3}} {\left(dx^{3}+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt[3]{a + b x^{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.464 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=148

$$\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

**Rubi [A]** time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/
3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)
*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c^(2/3) + ((b*c
- a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*
x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}\,x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \cdots\right)}{3c^{2/3}}\\
&= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{c}} + \cdots\\
&= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx \cdots\right)}{c^{2/3}\cdots}\\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 168, normalized size = 1.14

$$\frac{\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))
/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[
c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^
(1/3)*x)/(a + b*x^3)^(1/3)]])/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

**IntegrateAlgebraic [C]** time = 0.00, size = 320, normalized size = 2.16

$$\frac{\left(1+i\sqrt{3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{i\left(\sqrt{3}-i\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((Sqrt[-1 + I*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)
^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1
/3))])/(Sqrt[6]*c^(2/3)*(b*c - a*d)^(1/3))) + ((1 + I*Sqrt[3])*Log[2*(b*c -
 a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(2/3)*(b*c
 - a*d)^(1/3)) - ((I/12)*(-I + Sqrt[3])*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 +
c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3
])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{1}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3 + a\right)^{\frac{1}{3}} \left(d\,x^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{1}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^3}\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.465 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx$

**Optimal**. Leaf size=176

$$-\frac{d \log \left(c+dx^3\right)}{6c^{5/3} \sqrt[3]{bc-ad}}+\frac{d \log \left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3} \sqrt[3]{bc-ad}}-\frac{d \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c} \sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\, c^{5/3} \sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

**Rubi [A]** time = 0.23, antiderivative size = 235, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$\frac{d \log \left(\sqrt[3]{c}-\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3} \sqrt[3]{bc-ad}}-\frac{d \log \left(\frac{x^2 (bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\, x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{5/3} \sqrt[3]{bc-ad}}-\frac{d \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3} \sqrt[3]{c}}\right)}{\sqrt{3}\, c^{5/3} \sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(2*a*c*x^2) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*
x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3
)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(5/3)*
(b*c - a*d)^(1/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c -
a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^3\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}} - \frac{d\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}\ldots}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc\ldots}}\right.}{\ldots}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\ldots\right.}{2c^{4/3}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{5/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log\left(c^{2/3}+\ldots\right.}{6c\ldots}$$

**Mathematica** [C] time = 0.07, size = 124, normalized size = 0.70

$$\frac{-3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-4c\left(a+bx^3\right)\left(c+3dx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{8c^3x^2\left(a+bx^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-4*c*(a + b*x^3)*(c + 3*d*x^3)*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[4/
3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(8*c^3*x^2*(a + b*x^3)^(4/3)
)
```

**IntegrateAlgebraic** [C] time = 1.96, size = 349, normalized size = 1.98

$$-\frac{i\left(\sqrt{3}d-id\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{5/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{-1+i\sqrt{3}}\,d\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}c^{5/3}\sqrt[3]{bc-ad}}+\frac{\left(d+i\sqrt{3}d\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{5/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*c*x^2) + (Sqrt[-1 + I*Sqrt[3]]*d*ArcTan[(3*(b*c -
 a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/
3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(Sqrt[6]*c^(5/3)*(b*c - a*d)^(1/3
)) - ((I/6)*((-I)*d + Sqrt[3]*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3]
)*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(5/3)*(b*c - a*d)^(1/3)) + ((d + I*Sqrt[3]
*d)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqr
t[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c
^(5/3)*(b*c - a*d)^(1/3))
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.466 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\left(c+dx^3\right)} \, dx$

**Optimal**. Leaf size=214

$$\frac{\left(a+bx^3\right)^{2/3}(5ad+3bc)}{10a^2c^2x^2}+\frac{d^2\log\left(c+dx^3\right)}{6c^{8/3}\sqrt[3]{bc-ad}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{5acx^5}$$

**Rubi [A]** time = 0.29, antiderivative size = 271, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2a^2c^2x^2}-\frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5}-\frac{d^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*a^2*c^2*x^2) - (a + b*x^3)^(5/3)/(5*a^2*
c*x^5) + (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/
(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) - (d^2*Log[c^(1/3)
- ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(8/3)*(b*c - a*d)^(1/3)) +
 (d^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(8/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
```

```
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^2}{x^6\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^6} + \frac{-bc-ad}{c^2x^3} + \frac{a^2d^2}{c^2\left(c-(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c^2}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\log\left(c^{2/3}+\cdots\right.}{6\cdots}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}} - \frac{d^2\log\left(\sqrt[3]{c}-\cdots\right.}{3c^{8/3}\sqrt[3]{bc-\cdots}}$$

**Mathematica [C]** time = 1.03, size = 207, normalized size = 0.97

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{4}{3},2,2;1,\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right) - 3x^3\left(-c^2+8cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right) + 8c\left(a+bx^3\right)\left(c^2-3cdx^3-9d^2x^6\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{40c^4x^5\left(a+bx^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*(8*c*(a + b*x^3)*(c^2 - 3*c*d*x^3 - 9*d^2*x^6)*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(-c^2 + 8*c
*d*x^3 + 9*d^2*x^6)*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 2},
 {1, 7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*x^5*(a + b*x^3)^(4/3))
```

**IntegrateAlgebraic [C]** time = 2.31, size = 379, normalized size = 1.77

$$\frac{\left(a+bx^3\right)^{2/3}\left(-2ac+5adx^3+3bcx^3\right)}{10a^2c^2x^5}+\frac{\left(d^2+i\sqrt{3}\,d^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{8/3}\sqrt[3]{bc-ad}}-\frac{\sqrt{-1+i\sqrt{3}}\,d^2\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,c^{8/3}\sqrt[3]{bc-ad}}-\frac{i\left(\sqrt{3}\,d^2-id^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{8/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-2*a*c + 3*b*c*x^3 + 5*a*d*x^3))/(10*a^2*c^2*x^5) - (Sq
rt[-1 + I*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^
(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/
3))])/(Sqrt[6]*c^(8/3)*(b*c - a*d)^(1/3)) + ((d^2 + I*Sqrt[3]*d^2)*Log[2*(b
*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(8/3)*
(b*c - a*d)^(1/3)) - ((I/12)*((-I)*d^2 + Sqrt[3]*d^2)*Log[(-2*I)*(b*c - a*d
)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3)
 + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(8/3)*(b*c - a*d)^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

**maple [F]** time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6 \left(b x^3 + a\right)^{1/3} \left(d x^3 + c\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \sqrt[3]{a + bx^3} \left(c + dx^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.467 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\left(c+dx^3\right)} dx$

**Optimal**. Leaf size=262

$$\frac{\left(a+bx^3\right)^{2/3}(4ad+3bc)}{20a^2c^2x^5}-\frac{\left(a+bx^3\right)^{2/3}\left(20a^2d^2+12abcd+9b^2c^2\right)}{40a^3c^3x^2}-\frac{d^3\log\left(c+dx^3\right)}{6c^{11/3}\sqrt[3]{bc-ad}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right.}{2c^{11/3}\sqrt[3]{bc-ad}}$$

**Rubi [A]** time = 0.32, antiderivative size = 317, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2a^3c^3x^2}+\frac{\left(a+bx^3\right)^{5/3}(ad+2bc)}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+\frac{d^3\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{11/3}\sqrt[3]{bc-ad}}-\frac{d^3\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{11/3}\sqrt[3]{bc-ad}}-\frac{d^3\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*a^3*c^3*x^2) + ((2*b*
c + a*d)*(a + b*x^3)^(5/3))/(5*a^3*c^2*x^5) - (a + b*x^3)^(8/3)/(8*a^3*c*x^
8) - (d^3*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c^(11/3)*(b*c - a*d)^(1/3)) + (d^3*Log[c^(1/3) - (
(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(11/3)*(b*c - a*d)^(1/3)) - (
d^3*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c
 - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(11/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
```

```
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^3}{x^9\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^9}+\frac{-2bc-ad}{c^2x^6}+\frac{b^2c^2+abcd+a^2d^2}{c^3x^3}+\frac{a^3d^3}{c^3\left(-c+(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^3}$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+$$

$$= -\frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2a^3c^3x^2}+\frac{(2bc+ad)\left(a+bx^3\right)^{5/3}}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}-$$

**Mathematica [C]** time = 2.55, size = 821, normalized size = 3.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/320*(40*a*c^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] + 40*b*c^4*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] - 72*a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] - 72*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] + 216*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 216*b*c^2*d^2*x^9*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*a*c*d^3*x^9*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*b*c*d^3*
x^12*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*
b*c^4*x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 9*a*c^3*d*x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] - 45*b*c^3*d*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 45*a*c^2*d^2*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 243*b*c^2*d^2*x^9*Hypergeometric2F1[4/3, 2, 7/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 243*a*c*d^3*x^9*Hypergeometric2F1[4/
3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeome
tric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hy
pergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c -
 a*d)*x^3*(c - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 2}, {1, 7/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*H
ypergeometricPFQ[{4/3, 2, 2, 2}, {1, 1, 7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))])/(c^5*x^8*(a + b*x^3)^(4/3))
```

**IntegrateAlgebraic [C]** time = 3.11, size = 419, normalized size = 1.60

$$\frac{\left(a+bx^3\right)^{2/3}\left(-5a^2c^2+8a^2cdx^3-20a^2d^2x^6+6abc^2x^3-12abcdx^6-9b^2c^2x^6\right)}{40a^3c^3x^8}-\frac{i\left(\sqrt{3}\,d^3-id^3\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{11/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{-1+i\sqrt{3}}\,d^3\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,c^{11/3}\sqrt[3]{bc-ad}}+\frac{\left(d^3+i\sqrt{3}\,d^3\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{11/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*a^2*c^2 + 6*a*b*c^2*x^3 + 8*a^2*c*d*x^3 - 9*b^2*c^2*
x^6 - 12*a*b*c*d*x^6 - 20*a^2*d^2*x^6))/(40*a^3*c^3*x^8) + (Sqrt[-1 + I*Sqr
t[3]]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(Sqrt[6
]*c^(11/3)*(b*c - a*d)^(1/3)) - ((I/6)*((-I)*d^3 + Sqrt[3]*d^3)*Log[2*(b*c
- a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(11/3)*(b*c
 - a*d)^(1/3)) + ((d^3 + I*Sqrt[3]*d^3)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 +
c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3
])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(11/3)*(b*c - a*d)^(1/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right) x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}} \left(dx^{3}+c\right) x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{9}\,{\left(b\,x^{3}+a\right)}^{1/3}\,\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9}\sqrt[3]{a+bx^{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

## 3.468 $\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=241

$$\frac{\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3d^3}-\frac{\left(a+bx^3\right)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{\ldots}\right.}{2d^{10/3}(bc-a\ldots}$$

**Rubi [A]** time = 0.26, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 58, 617, 204, 31}

$$\frac{\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3d^3}-\frac{\left(a+bx^3\right)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}+\frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - ((b*c + 2*a*d
)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (a + b*x^3)^(7/3)/(7*b^3*d) + (c^3*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(6*d^(10/3)*(b*c - a*d)^
(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/
3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3(a+bx)^{2/3}} + \frac{(-bc-2ad)\sqrt[3]{a+bx}}{b^2d^2} + \frac{(a+bx)^{4/3}}{b^2d} - \frac{\ldots}{d^3(a+\ldots}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} - \frac{c^3\,\text{S}\ldots}{\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\ldots}{6d^{10}\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\ldots}{6d^{10}\ldots}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\tan\ldots}{\sqrt{\ldots}}$$

**Mathematica** [A] time = 0.66, size = 251, normalized size = 1.04

$$\frac{\frac{84\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3} - \frac{21d\left(a+bx^3\right)^{4/3}(2ad+bc)}{b^3} + \frac{12d^2\left(a+bx^3\right)^{7/3}}{b^3} + \frac{14c^3\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}}{84d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((84*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/b^3 - (21*d*(b*c + 2*
a*d)*(a + b*x^3)^(4/3))/b^3 + (12*d^2*(a + b*x^3)^(7/3))/b^3 + (14*c^3*(2*S
qrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^
(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(
2/3)]))/(d^(1/3)*(b*c - a*d)^(2/3)))/(84*d^3)
```

**IntegrateAlgebraic** [A] time = 0.50, size = 284, normalized size = 1.18

$$\frac{\sqrt[3]{a+bx^3}\left(18a^2d^2+21abcd-6abd^2x^3+28b^2c^2-7b^2cdx^3+4b^2d^2x^6\right)}{28b^3d^3} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{10/3}(bc-ad)^{2/3}} + \frac{c^3\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{10/3}(bc-ad)^{2/3}} + \frac{c^3\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{10/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(28*b^2*c^2 + 21*a*b*c*d + 18*a^2*d^2 - 7*b^2*c*d*x^3 -
6*a*b*d^2*x^3 + 4*b^2*d^2*x^6))/(28*b^3*d^3) + (c^3*ArcTan[1/Sqrt[3] - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))]])/(Sqrt[3]*d^(10/3)*(
b*c - a*d)^(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
)/(3*d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(10/3)*
(b*c - a*d)^(2/3))
```

**fricas** [B] time = 0.83, size = 1322, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c
- a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28
*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*
(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 42*sqrt(1/3
)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/
3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*
sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2
- a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-
b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*
x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3
*b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 -
 (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*
x^3 + a)^(1/3))/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6), 1/84*(14*(-b^2
*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(2/3)*(b*c*d
 - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28*(-b^2*c^2*d +
2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
 - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 84*sqrt(1/3)*(b^4*c^4*d -
a*b^3*c^3*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*arctan(s
qrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(-b^2*c^2*d +
 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 3*(28*b
^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3*b*c*d^4 + 18*a^4*d
^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8
*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(1/3))/(b^
5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)]
```

**giac** [A] time = 0.26, size = 372, normalized size = 1.54

$$\frac{b^{24}c^{3}d^{4}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{25}cd^{7}-ab^{24}d^{8}\right)}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^{4}-\sqrt{3}ad^{5}}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{3}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^{4}-ad^{5}\right)}+\frac{28\left(bx^{3}+a\right)^{\frac{1}{3}}b^{20}c^{2}d^{4}-7\left(bx^{3}+a\right)^{\frac{4}{3}}b^{19}cd^{5}+28\left(bx^{3}+a\right)^{\frac{1}{3}}ab^{19}cd^{5}+4\left(bx^{3}+a\right)^{\frac{7}{3}}b^{18}d^{6}-14\left(bx^{3}+a\right)^{\frac{4}{3}}ab^{18}d^{6}+28\left(bx^{3}+a\right)^{\frac{1}{3}}a^{2}b^{18}d^{6}}{28b^{21}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^24*c^3*d^4*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^25*c*d^7 - a*b^24*d^8) - (-b*c*d^2 + a*d^3)^(1/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(
1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 1/28*(28*(b*x^3 + a)^(1/3)*b^2
0*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^19*c*d^5 + 28*(b*x^3 + a)^(1/3)*a*b^19*c*
```

```
d^5 + 4*(b*x^3 + a)^(7/3)*b^18*d^6 - 14*(b*x^3 + a)^(4/3)*a*b^18*d^6 + 28*(
b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.01, size = 331, normalized size = 1.37

$$\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}-\left(\frac{3\,a}{4\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{4\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3}+\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d}+\frac{\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}+\frac{3\,c^3\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{2\,d^{4/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{c^3\,\ln\left(\frac{3\,c^3\,{\left(b\,x^3+a\right)}^{1/3}}{d}-\frac{3\,c^3\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
a*b^3*d))/(b^3*d))*(a + b*x^3)^(1/3) - ((3*a)/(4*b^3*d) + (b^4*c - a*b^3*d)
/(4*b^6*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^3*d) + (log((3*c^3
*(a + b*x^3)^(1/3))/d + (3*c^3*(3^(1/2)*1i + 1)*(a*d - b*c)^(1/3))/(2*d^(4/
3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(10/3)*(a*d - b*c)^(2/3)) - (c^3*log((3*c
^3*(a + b*x^3)^(1/3))/d - (3*c^3*(a*d - b*c)^(1/3))/d^(4/3)))/(3*d^(10/3)*(
a*d - b*c)^(2/3)) - (c^3*log((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*((3^(1/2)
*1i)/2 - 1/2)*(a*d - b*c)^(1/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1/2))/(3*d^(10/
3)*(a*d - b*c)^(2/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^{3}\right)^{\frac{2}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.469 $\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

**Optimal**. Leaf size=201

$$-\frac{\sqrt[3]{a+bx^3}\,(ad+bc)}{b^2d^2}+\frac{(a+bx^3)^{4/3}}{4b^2d}-\frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}}$$

**Rubi [A]** time = 0.21, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}\,(ad+bc)}{b^2d^2}+\frac{(a+bx^3)^{4/3}}{4b^2d}-\frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(((b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2)) + (a + b*x^3)^(4/3)/(4*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(6*d^(7/3)*(b
*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(7/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{bd^2(a+bx)^{2/3}} + \frac{\sqrt[3]{a+bx}}{bd} + \frac{c^2}{d^2(a+bx)^{2/3}(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\right.}{2d^{7/3}(bc-}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\right.}{2d^{7/3}(bc-a}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}} - \frac{c^2\log\left(c+dx^3\right.}{6d^{7/3}(bc-ad)^2}$$

**Mathematica [A]** time = 0.43, size = 211, normalized size = 1.05

$$\frac{-\frac{12\sqrt[3]{a+bx^3}(ad+bc)}{b^2}+\frac{3d\left(a+bx^3\right)^{4/3}}{b^2}-\frac{2c^2\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}}{12d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-12*(b*c + a*d)*(a + b*x^3)^(1/3))/b^2 + (3*d*(a + b*x^3)^(4/3))/b^2 - (2
*c^2*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*
c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)]))/(d^(1/3)*(b*c - a*d)^(2/3)))/(12*d^2)
```

**IntegrateAlgebraic [A]** time = 0.42, size = 246, normalized size = 1.22

$$\frac{\sqrt[3]{a+bx^3}\left(-3ad-4bc+bdx^3\right)}{4b^2d^2}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(-4*b*c - 3*a*d + b*d*x^3))/(4*b^2*d^2) - (c^2*ArcTan[1/
Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt
[3]*d^(7/3)*(b*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
b*x^3)^(1/3)])/(3*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[(b*c - a*d)^(2/3) -
 d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/
(6*d^(7/3)*(b*c - a*d)^(2/3))
```

**fricas [B]** time = 0.47, size = 1156, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a
)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c -
a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*
d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*(b^3*
c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*l
og((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*sqrt(1/
3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c))
+ 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4 - (b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c^2*d^3 - 2*a*
b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log
(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3
+ a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*
a^3*d^4 - (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))
/(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)]
```

**giac [A]** time = 0.30, size = 312, normalized size = 1.55

$$-\frac{b^{10}c^{2}d^{2}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{11}cd^{4}-ab^{10}d^{5}\right)}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^{3}-\sqrt{3}\,ad^{4}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}c^{2}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^{3}-ad^{4}\right)}-\frac{4\left(bx^{3}+a\right)^{\frac{1}{3}}b^{7}cd^{2}-\left(bx^{3}+a\right)^{\frac{4}{3}}b^{6}d^{3}+4\left(bx^{3}+a\right)^{\frac{1}{3}}ab^{6}d^{3}}{4\,b^{8}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^10*c^2*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^11*c*d^4 - a*b^10*d^5) + (-b*c*d^2 + a*d^3)^(1/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) - 1/4*(4*(b*x^3 + a)^(1/3)*b^7*
c*d^2 - (b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4
)
```

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 4.69, size = 292, normalized size = 1.45

$$\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b^2\,d}-\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{1/3}-\frac{\ln\left(3\,c^2\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)}{6\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{c^2\,\ln\left(3\,c^2\,\left(b\,x^3+a\right)^{1/3}-\frac{c^2\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{c^2\,\ln\left(3\,c^2\,\left(b\,x^3+a\right)^{1/3}-\frac{c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b^2*d) - ((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))
*(a + b*x^3)^(1/3) - (log(3*c^2*(a + b*x^3)^(1/3) + ((3^(1/2)*c^2*1i + c^2)
*(9*a*d^3 - 9*b*c*d^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*c^2*1i + c^
2))/(6*d^(7/3)*(a*d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2
*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)*(a*d - b*c)^(2/3))))/(3*d^(7/3)*(a*d - b
*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2*((3^(1/2)*1i)/6 - 1/6)
*(9*a*d^3 - 9*b*c*d^2))/(d^(7/3)*(a*d - b*c)^(2/3)))*((3^(1/2)*1i)/6 - 1/6)
)/(d^(7/3)*(a*d - b*c)^(2/3))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.470 $\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=165

$$\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}$$

**Rubi [A]** time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 58, 617, 204, 31}

$$\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[c + d
*x^3])/(6*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*d^(4/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)\\
&= \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{c\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}\\
&= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subs}\cdots}{\cdots}\\
&= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subst}\left(\int\cdots\right.}{\cdots}\\
&= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\cdots\right.}{2d^{4/3}(bc-ad\cdots}
\end{aligned}
$$

**Mathematica [A]** time = 0.18, size = 202, normalized size = 1.22

$$
\frac{bc\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}(bc-ad)^{2/3}-2bc\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-2\sqrt{3}\,bc\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)}{6bd^{4/3}(bc-ad)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*b*c*ArcTan[(-1 +
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*b*c*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) - d^
(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*
b*d^(4/3)*(b*c - a*d)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.21, size = 221, normalized size = 1.34

$$
-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{4/3}(bc-ad)^{2/3}}+\frac{c\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{4/3}(bc-ad)^{2/3}}+\frac{c\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3
))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) - (c*L
og[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(4/3)*(b*c - a*d)^(
2/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1
/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(4/3)*(b*c - a*d)^(2/3))
```

**fricas [B]** time = 0.61, size = 1060, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3)
*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) +
 (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^
2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^
2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^2*d -
 a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b^2*c^2
 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(2*(b*x^
3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(
b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*
sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*d + 2*a*b*
c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c)) + 6*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2 - 2*a*b^2*c
*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(
-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(
1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(
1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(
1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 6*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c -
a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt
(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)]
```

**giac [A]** time = 0.23, size = 253, normalized size = 1.53

$$-\frac{\frac{6\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^{2}-\sqrt{3}\,ad^{3}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}bc\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^{2}-ad^{3}}-\frac{2\,bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd-ad^{2}}-\frac{6\left(bx^{3}+a\right)^{\frac{1}{3}}}{d}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt
(3)*a*d^3) + (-b*c*d^2 + a*d^3)^(1/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3)
 - 2*b*c*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c*d - a*d^2) - 6*(b*x^3 + a)^(1/3)/d)/b
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.68, size = 232, normalized size = 1.41

$$\frac{\left(b\,x^3+a\right)^{1/3}}{b\,d}-\frac{c\,\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}-\frac{c\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(9\,a\,d^3-9\,b\,c\,d^2\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(9\,a\,d^3-9\,b\,c\,d^2\right)\,\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) - (c*log(3*c*d*(a + b*x^3)^(1/3) - (c*(9*a*d^3 - 9*
b*c*d^2))/(3*d^(4/3)*(a*d - b*c)^(2/3))))/(3*d^(4/3)*(a*d - b*c)^(2/3)) + (
log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c - 3^(1/2)*c*1i))/(6
*d^(4/3)*(a*d - b*c)^(2/3)))*(c - 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b*c)^(2/
3)) + (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c + 3^(1/2)*c*
1i))/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c + 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b
*c)^(2/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.471 $\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

**Optimal**. Leaf size=145

$$-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}$$

**Rubi [A]** time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 58, 617, 204, 31}

$$-\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(6*d^(1/3)*(b*c - a*d)^(
2/3)) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \cdots} [illegible]\right)}{2d^{2/}\cdots}$$

$$= -\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 \cdots\right)}{\sqrt[3]{d}\,(bc-ad \cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}\,(bc-ad)^{2/3}} - \frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

**Mathematica [A]** time = 0.06, size = 164, normalized size = 1.13

$$\frac{-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c -
a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(6*d^(1/3)*(b*c - a*d)^(2/3))
```

**IntegrateAlgebraic [A]** time = 0.16, size = 201, normalized size = 1.39

$$-\frac{\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{d}\,(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/
3))]/(Sqrt[3]*d^(1/3)*(b*c - a*d)^(2/3))) + Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)]/(3*d^(1/3)*(b*c - a*d)^(2/3)) - Log[(b*c - a*d)^(2/3) -
 d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]/(
6*d^(1/3)*(b*c - a*d)^(2/3))
```

**fricas [B]** time = 0.56, size = 927, normalized size = 6.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3
)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3
 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(
d*x^3 + c)) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
 (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)
*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
+ (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) + 2*(b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)]
```

**giac** [A] time = 0.29, size = 221, normalized size = 1.52

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd-\sqrt{3}\,ad^2}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd-ad^2\right)}-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
 a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) + 1
/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*
c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d - a*d^2) - 1/3*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c - a
*d)
```

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.85, size = 213, normalized size = 1.47

$$\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}+\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(3*d^2*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(3*d^(1/3)*(a*d - b*c)^
(2/3)))/(3*d^(1/3)*(a*d - b*c)^(2/3)) + (log(3*d^2*(a + b*x^3)^(1/3) - ((3^
(1/2)*1i - 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2
)*1i - 1))/(6*d^(1/3)*(a*d - b*c)^(2/3)) - (log(3*d^2*(a + b*x^3)^(1/3) + (
(3^(1/2)*1i + 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(
1/2)*1i + 1))/(6*d^(1/3)*(a*d - b*c)^(2/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.472 $\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=245

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}+$$

**Rubi [A]** time = 0.21, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 86, 57, 617, 204, 31, 58}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}+\frac{d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*c)) + (d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(2/3)) - Log[x]/(2*a^(2/3)*c) + (d^(2/3
)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)
]/(2*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*c*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{\text{Subst}\left(\int \cdots\right.}{}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\cdots\right.}{2c(bc-a\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}c} + \frac{d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}$$

**Mathematica [A]** time = 0.45, size = 308, normalized size = 1.26

$$-\frac{-\frac{2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{a^{2/3}}+\frac{\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{2/3}}+\frac{2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{(bc-ad)^{2/3}}+\frac{2\sqrt{3}\,d^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)}{(bc-ad)^{2/3}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/6*((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/
3) + (2*Sqrt[3]*d^(2/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a
*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(2/3) - (2*Log[a^(1/3) - (a + b*x^3)^(1/3)
])/a^(2/3) + (2*d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(b*c - a*d)^(2/3) + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)]/a^(2/3) - (d^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)
*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/c
```

**IntegrateAlgebraic [A]** time = 0.59, size = 333, normalized size = 1.36

$$\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3a^{2/3}c}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6a^{2/3}c}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}c}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c(bc-ad)^{2/3}}+\frac{d^{2/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c(bc-ad)^{2/3}}+\frac{d^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2
/3)*c)) + (d^(2/3)*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3
]*(b*c - a*d)^(1/3))])/(Sqrt[3]*c*(b*c - a*d)^(2/3)) + Log[-a^(1/3) + (a +
b*x^3)^(1/3)]/(3*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
 b*x^3)^(1/3)])/(3*c*(b*c - a*d)^(2/3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)
^(1/3) + (a + b*x^3)^(2/3)]/(6*a^(2/3)*c) + (d^(2/3)*Log[(b*c - a*d)^(2/3)
- d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])
/(6*c*(b*c - a*d)^(2/3))
```

**fricas [B]** time = 0.50, size = 472, normalized size = 1.93

$$-\frac{2\sqrt{3}a^2\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)+a^2\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(bx^3+a\right)^{\frac{1}{3}}\left(bcd-ad^2\right)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}+\left(b^2c^2-2abcd+a^2d^2\right)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{2}{3}}\right)-2a^2\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d-(bc-ad)\left(-\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\right)+2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}a\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)}{6a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/
3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(2/3) - sqrt(3)*d)/d) + a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
1/3)*log((b*x^3 + a)^(2/3)*d^2 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)*(-d^2/(b
^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-d^
2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 2*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*
d + a^2*d^2))^(1/3)*log((b*x^3 + a)^(1/3)*d - (b*c - a*d)*(-d^2/(b^2*c^2 -
2*a*b*c*d + a^2*d^2))^(1/3)) + 2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(a^2)^(1/
6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) +
 (a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a
^2)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/(a^2*c)
```

**giac [A]** time = 0.76, size = 321, normalized size = 1.31

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2-acd\right)}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{2}{3}}c}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}c}+\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3a^{\frac{2}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b*c^2 - a*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*
x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b
*c^2 - sqrt(3)*a*c*d) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c^2
 - a*c*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/
a^(1/3))/(a^(2/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3))/(a^(2/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)
```

**mupad [B]** time = 4.94, size = 1413, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log((((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (243*a*b^6*c
^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))
*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*d^5
*(a + b*x^3)^(1/3))*(1/(27*a^2*c^3))^(1/3) + log(- (((81*b^6*c^5*d^3 - 162*
a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*
a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^
4*d^5))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3) - 9*b^5*c
^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) - 6*b^4*d
^5*(a + b*x^3)^(1/3))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(
1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((81*b^6*c^5*d^
3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^
6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/
3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*
d^5*(a + b*x^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*c^3))^(1/3) - log(
6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 + 1/2
)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) + ((3^(1/2)*1i)
/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(
1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c
^3))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*c^3))^(1/3) + (log(6*b^4*d^5*
(a + b*x^3)^(1/3) + ((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((81*b^6*c^5*d^3
 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)*(-d^2/(27*b^2*c
^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54
*a*b*c^4*d))^(2/3))/4 - 9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 -
 54*a*b*c^4*d))^(1/3))/2)*(3^(1/2)*1i - 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d
^2 - 54*a*b*c^4*d))^(1/3))/2 - (log(6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)
*1i + 1)*(((3^(1/2)*1i + 1)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*
x^3)^(1/3) + ((3^(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*
c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^
5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3))/4 - 9*b^5
*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2)*(3^
(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.473 $\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=299

$$-\frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{(3ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}c^2}+\frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}$$

**Rubi [A]** time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 103, 156, 57, 617, 204, 31, 58}

$$-\frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{(3ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}c^2}+\frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}+\frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}}-\frac{d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(1/3)/(3*a*c*x^3) + ((2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*c^2) - (d^(5/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c
^2*(b*c - a*d)^(2/3)) + ((2*b*c + 3*a*d)*Log[x])/(6*a^(5/3)*c^2) - (d^(5/3)
*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(2/3)) - ((2*b*c + 3*a*d)*Log[a^(1/3) -
 (a + b*x^3)^(1/3)])/(6*a^(5/3)*c^2) + (d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(
1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)$$

$$=-\frac{\sqrt[3]{a+bx^3}}{3acx^3}-\frac{\text{Subst}\left(\int \frac{\frac{1}{3}(2bc+3ad)+\frac{2bdx}{3}}{x(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3ac}$$

$$=-\frac{\sqrt[3]{a+bx^3}}{3acx^3}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3c^2}-\frac{(2bc+3ad)\,\text{Subst}\left(\int \frac{1}{x(a+bx}\right.}{9ac^2}$$

$$=-\frac{\sqrt[3]{a+bx^3}}{3acx^3}+\frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}+\frac{d^{5/3}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\right.}{2c^2(bc-a}$$

$$=-\frac{\sqrt[3]{a+bx^3}}{3acx^3}+\frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}-\frac{(2bc+3ad)\log\left(\sqrt[3]{a}\right.-}{6a^{5/3}c^2}$$

$$=-\frac{\sqrt[3]{a+bx^3}}{3acx^3}+\frac{(2bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}c^2}-\frac{d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2(bc-ad)^{2/3}}+(2bc$$

**Mathematica [A]** time = 0.59, size = 303, normalized size = 1.01

$$\frac{\frac{(3ad+2bc)\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{a^{2/3}c}+\frac{3ad^{5/3}\left(-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)\right)}{c(bc-ad)^{2/3}}-\frac{6\sqrt[3]{a+bx^3}}{x^3}}{18ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-6*(a + b*x^3)^(1/3))/x^3 + ((2*b*c + 3*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a
 + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + L
og[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/(a^(2/3)*c) +
 (3*a*d^(5/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
 - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^
(2/3)*(a + b*x^3)^(2/3)]))/(c*(b*c - a*d)^(2/3)))/(18*a*c)
```

**IntegrateAlgebraic** [A] time = 0.73, size = 388, normalized size = 1.30

$$\frac{(-3ad-2bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9a^{5/3}c^2}+\frac{(3ad+2bc)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{18a^{5/3}c^2}+\frac{(3ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2}+\frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^2(bc-ad)^{2/3}}-\frac{d^{5/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^2(bc-ad)^{2/3}}-\frac{d^{5/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(1/3)/(a*c*x^3) + ((2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*c^2) - (d^(5/3)*ArcT
an[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/
(Sqrt[3]*c^2*(b*c - a*d)^(2/3)) + ((-2*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x
^3)^(1/3)])/(9*a^(5/3)*c^2) + (d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a +
 b*x^3)^(1/3)])/(3*c^2*(b*c - a*d)^(2/3)) + ((2*b*c + 3*a*d)*Log[a^(2/3) +
a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(5/3)*c^2) - (d^(5/3)
*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2
/3)*(a + b*x^3)^(2/3)])/(6*c^2*(b*c - a*d)^(2/3))
```

**fricas** [B] time = 1.25, size = 562, normalized size = 1.88

$$-\frac{6\sqrt{3}a^3d\left(\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)+3a^3d\left(\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(bcd-ad^2\right)\left(\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}+\left(b^2c^2-2abcd+a^2d^2\right)\left(\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{2}{3}}\right)-6a^3d\left(\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d+(bc-ad)\left(\frac{d^2}{b^2c^2-2abcd+a^2d^2}\right)^{\frac{1}{3}}\right)-2\sqrt{3}\left(2abc+3a^2d\right)x^3\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)-\left(-a^2\right)^{\frac{2}{3}}(2bc+3ad)x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)+2\left(-a^2\right)^{\frac{2}{3}}(2bc+3ad)x^3\log\left(\left(bx^3+a\right)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)+6\left(bx^3+a\right)^{\frac{1}{3}}a^2c}{18a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*arct
an(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2))^(2/3) - sqrt(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(1/3)*x^3*log((b*x^3 + a)^(2/3)*d^2 - (b*x^3 + a)^(1/3)*(b*c*d - a*d
^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 6*a^3*d*(d^2/(b^2*c^2
 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(1/3)*d + (b*c - a*d)*(d
^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) - 2*sqrt(3)*(2*a*b*c + 3*a^2*d)*
x^3*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) - (-a^2)^(2/3)*(2*b*c
 + 3*a*d)*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(-a^2)^(2/3)) + 2*(-a^2)^(2/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a
- (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2*c)/(a^3*c^2*x^3)
```

**giac** [A] time = 0.76, size = 377, normalized size = 1.26

$$-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)}+\frac{\sqrt{3}(2bc+3ad)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{5}{3}}c^2}+\frac{(2bc+3ad)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{5}{3}}c^2}-\frac{(2bc+3ad)\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{5}{3}}c^2}-\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{3acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) + (-b*c*d^2 + a*d^3)^(1/3)*d*arctan(1/3*sqrt(3)
```

```
*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sq
rt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2
/3))/(b*c^3 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^2) + 1/18*(2*b*c + 3*a*d)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/3)*c^2)
- 1/9*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^2) -
 1/3*(b*x^3 + a)^(1/3)/(a*c*x^3)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(dx^{3}+c\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 11.14, size = 1959, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*
d))/a - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/9 + (b^5*d^4*(8*b^3*c^3 -
 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))/3 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 +
 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(d^5/(27*b^2*c^8 + 27*a^2*c
^6*d^2 - 54*a*b*c^7*d))^(1/3) + log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)
*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))*(-(3*a*d + 2*b*c)^3/(
a^5*c^6))^(2/3))/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 1
8*a^2*b*c*d^2))/(3*a^3*c))*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4
*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*
c*d^2))/(9*a^3*c^4))*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*a^2*b*
c*d^2)/(729*a^5*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)*((((27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - (81*a*b^4*c^4*d^3*(3^
(1/2)*1i - 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(
1/3))/2)*(3^(1/2)*1i - 1)^2*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/36 + (b^5*d^4*
(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5
/(c^6*(a*d - b*c)^2))^(1/3))/6 + (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 +
 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(3^(1/2)*1i - 1
)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*a*b*c^7*d))^(1/3))/2 - (log(((3^(1
/2)*1i + 1)*((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a
*b*c*d))/a + (81*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*
```

```
b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/2)*(3^(1/2)*1i + 1)^2*(d^5/(c^6*(a*
d - b*c)^2))^(2/3))/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d
+ 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/6 - (2*b^4*d
^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*
d^2))/(9*a^3*c^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*
a*b*c^7*d))^(1/3))/2 + log((2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3
*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4) - (((((27*b^5*c^3*d^3*
(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(
(3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3
/(a^5*c^6))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2
/3))/81 - (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^
2))/(3*a^3*c))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))
/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*
a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - log((((((27*b^5*c^3*d^3*(a + b*x^3)^(1/
3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
+ 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/
3))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2/3))/81 + (b^5*
d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*
((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4*d^6
*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^
2))/(9*a^3*c^4))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^
2*c^2*d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - (a + b*x^3)^(1/3)/(3*a*c*x
^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right)^{\frac{2}{3}} \left(c + dx^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.474 $\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=279

$$\frac{(2ad+3bc)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2}+\frac{(2ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2}+\frac{c^{5/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{2/3}}-\frac{c^{5/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\right.}{2d^2(bc-ad)^{2/3}}$$

**Rubi [A]** time = 0.50, antiderivative size = 400, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {494, 470, 584, 292, 31, 634, 617, 204, 628}

$$\frac{(2ad+3bc)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2}-\frac{(2ad+3bc)\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{5/3}d^2}+\frac{(2ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2}-\frac{c^{5/3}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d^2(bc-ad)^{2/3}}+\frac{c^{5/3}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6d^2(bc-ad)^{2/3}}-\frac{c^{5/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^2(bc-ad)^{2/3}}+\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d^2) - (c^(5/3)*ArcTan[(c
^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sq
rt[3]*d^2*(b*c - a*d)^(2/3)) + ((3*b*c + 2*a*d)*Log[1 - (b^(1/3)*x)/(a + b*
x^3)^(1/3)])/(9*b^(5/3)*d^2) - ((3*b*c + 2*a*d)*Log[1 + (b^(2/3)*x^2)/(a +
b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(5/3)*d^2) - (c^(5/3)*
Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^
(2/3)) + (c^(5/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) +
 (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = a^2\,\text{Subst}\left(\int \frac{x^7}{\left(1-bx^3\right)^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{x\left(2c+(bc+2ad)x^3\right)}{\left(1-bx^3\right)\left(c+(-bc+ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \left(\frac{(3bc+2ad)x}{ad\left(1-bx^3\right)} + \frac{3bc^2x}{ad\left(-c+(bc-ad)x^3\right)}\right) dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^2\,\text{Subst}\left(\int \frac{x}{-c+(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - \frac{(3bc+2ad)\,\text{Subst}\left(\int \frac{x}{1-bx^3} \cdots\right.}{3bd^2}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^{5/3}\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{c}+\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + \frac{c^{5/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{\cdots}}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{\cdots}} \cdots\right.}{3d \cdots}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{c^{5/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d^2(bc-ad)^{2/3}} + \frac{c^{5/3}\,\text{Sub}\cdots}{\cdots}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{(3bc+2ad)\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\cdots}{\sqrt[3]{\cdots}}\right.}{18b^{5/3}d^2} \cdots$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2} - \frac{c^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2(bc-ad)^{2/3}} + \frac{(3bc+2\cdots}{\cdots}$$

**Mathematica** [C] time = 0.29, size = 190, normalized size = 0.68

$$\frac{5cx^2\left(\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}-a\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)\right)-x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(2ad+3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{15bcd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-((3*b*c + 2*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1
[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]) + 5*c*x^2*((a + b*x^3)*(1 +
 (d*x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(15*b*c*d*(a + b*x^3)^(2/3)*(1 +
(d*x^3)/c)^(2/3))
```

**IntegrateAlgebraic** [C] time = 4.57, size = 527, normalized size = 1.89

$$\frac{(2ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{5/3}d^2}+\frac{(2ad+3bc)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\,b^{5/3}d^2}+\frac{(-2ad-3bc)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18b^{5/3}d^2}+\frac{\left(c^{5/3}-i\sqrt{3}\,c^{5/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6d^2(bc-ad)^{2/3}}+\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}\,c^{5/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{d^2(bc-ad)^{2/3}}+\frac{i\left(\sqrt{3}\,c^{5/3}+ic^{5/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^2(bc-ad)^{2/3}}+\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*
x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(5/3)*d^2) + (Sqrt[(-1
- I*Sqrt[3])/6]*c^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)
^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1
```

```
/3))])/(d^2*(b*c - a*d)^(2/3)) + ((3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)])/(9*b^(5/3)*d^2) + ((c^(5/3) - I*Sqrt[3]*c^(5/3))*Log[2*(b*c -
 a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a
*d)^(2/3)) + ((-3*b*c - 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3)])/(18*b^(5/3)*d^2) + ((I/12)*(I*c^(5/3) + Sqrt[3]*c^(
5/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - S
qrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(d^
2*(b*c - a*d)^(2/3))
```

**fricas [B]** time = 1.28, size = 558, normalized size = 2.00

$$\frac{6\sqrt{3}\,b^{3}c\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{2}{3}}+\sqrt{3}\,cx}{3\,cx}\right)+6\left(bx^{3}+a\right)^{\frac{1}{3}}b^{2}dx^{2}+6\,b^{3}c\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\log\left(\frac{(bc-ad)\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}x+\left(bx^{3}+a\right)^{\frac{1}{3}}c}{x}\right)-3\,b^{3}c\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}\log\left(\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{2}{3}}x^{2}+\left(bx^{3}+a\right)^{\frac{2}{3}}c^{2}-\left(bx^{3}+a\right)^{\frac{1}{3}}\left(bc^{2}-acd\right)\left(-\frac{c^{2}}{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}\right)^{\frac{1}{3}}x}{x^{2}}\right)-2\sqrt{3}\left(3\,b^{2}c+2\,abd\right)\left(b^{2}\right)^{\frac{1}{6}}\arctan\left(\frac{\left(\sqrt{3}\left(b^{2}\right)^{\frac{1}{3}}bx+2\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(b^{2}\right)^{\frac{2}{3}}\right)\left(b^{2}\right)^{\frac{1}{6}}}{3\,b^{2}x}\right)+2\left(b^{2}\right)^{\frac{2}{3}}(3\,bc+2\,ad)\log\left(-\frac{\left(b^{2}\right)^{\frac{2}{3}}x-\left(bx^{3}+a\right)^{\frac{1}{3}}b}{x}\right)-\left(b^{2}\right)^{\frac{2}{3}}(3\,bc+2\,ad)\log\left(\frac{\left(b^{2}\right)^{\frac{1}{3}}bx^{2}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(b^{2}\right)^{\frac{2}{3}}x+\left(bx^{3}+a\right)^{\frac{2}{3}}b}{x^{2}}\right)}{18\,b^{3}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*b^
3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b*c - a*d)*(-c^2/(b^
2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x + (b*x^3 + a)^(1/3)*c)/x) - 3*b^3*c*(
-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*
c^2 - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(1/3)*x)/x^2) - 2*sqrt(3)*(3*b^2*c + 2*a*b*d)*(b^2)^(1/6)*arctan(1/3*(s
qrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/
6)/(b^2*x)) + 2*(b^2)^(2/3)*(3*b*c + 2*a*d)*log(-((b^2)^(2/3)*x - (b*x^3 +
a)^(1/3)*b)/x) - (b^2)^(2/3)*(3*b*c + 2*a*d)*log(((b^2)^(1/3)*b*x^2 + (b*x^
3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^3*d^2)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.475 $\int \frac{x^4}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=234

$$-\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}}+\frac{c^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}+\frac{c^{2/3}\tan^{-1}\left(\ldots\right.}{\sqrt{3}\,d(bc\ldots}$$

**Rubi** [A] time = 0.31, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 481, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}+\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}}-\frac{c^{2/3}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6d(bc-ad)^{2/3}}+\frac{c^{2/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d)
) + (c^(2/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(
Sqrt[3]*c^(1/3))])/(Sqrt[3]*d*(b*c - a*d)^(2/3)) - Log[1 - (b^(1/3)*x)/(a +
 b*x^3)^(1/3)]/(3*b^(2/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b
^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(2/3)*d) + (c^(2/3)*Log[c^(1/3) - ((b*c -
 a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d*(b*c - a*d)^(2/3)) - (c^(2/3)*Log[c
^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(
1/3)*x)/(a + b*x^3)^(1/3)])/(6*d*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = a\,\text{Subst}\left(\int \frac{x^4}{\left(1-bx^3\right)\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d} - \frac{\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d} - \frac{c^{2/3}\,\text{Subs}}{}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+b}}\right.}{6b^{2/3}d}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} - \frac{c^{2}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d} + \frac{c^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d(bc-ad)^{2/3}} - \frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(1+\frac{b^{2/3}}{(a+bx}\right.}{6b}$$

**Mathematica [C]** time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^5\left(\frac{a+bx^3}{a}\right)^{2/3} F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*((a + b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(2/3))
```

**IntegrateAlgebraic [C]** time = 3.42, size = 475, normalized size = 2.03

$$-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6b^{2/3}d}+\frac{i\left(\sqrt{3}\,c^{2/3}+ic^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6d(bc-ad)^{2/3}}-\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}\,c^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{d(bc-ad)^{2/3}}+\frac{\left(c^{2/3}-i\sqrt{3}\,c^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^
(2/3)*d)) - (Sqrt[(-1 - I*Sqrt[3])/6]*c^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x
)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*
c^(1/3)*(a + b*x^3)^(1/3))]])/(d*(b*c - a*d)^(2/3)) - Log[-(b^(1/3)*x) + (a
 + b*x^3)^(1/3)]/(3*b^(2/3)*d) + ((I/6)*(I*c^(2/3) + Sqrt[3]*c^(2/3))*Log[2*
(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(d*(b*c -
 a*d)^(2/3)) + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)]/(6*b^(2/3)*d) + ((c^(2/3) - I*Sqrt[3]*c^(2/3))*Log[(-2*I)*(b*c - a*d
)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3)
 + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(12*d*(b*c - a*d)^(2/3))
```

**fricas [B]** time = 0.55, size = 530, normalized size = 2.26

$$\frac{2\sqrt{3}\,b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}\,cx}{3\,cx}\right)+2\,b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}c}{x}\right)-b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}x^2+\left(bx^3+a\right)^{\frac{2}{3}}c^2+\left(bx^3+a\right)^{\frac{1}{3}}\left(bc^2-acd\right)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x}{x^2}\right)+2\sqrt{3}\,b\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3\,b^2x}\right)-2\left(-b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(-b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)}{6\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3*
(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 2*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2
))^(1/3)*log(-((b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x -
(b*x^3 + a)^(1/3)*c)/x) - b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*l
og(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
2/3)*x^2 + (b*x^3 + a)^(2/3)*c^2 + (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(c^2/(
b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x)/x^2) + 2*sqrt(3)*b*sqrt(-(-b^2)^(1
/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-
b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*log(-((-b^2)^(2/3
)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*
x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.476 $\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

**Optimal**. Leaf size=149

$$\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

**Rubi** [A] time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(
1/3))]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) - Log[c^(1/3) - ((b*c - a*d)^(1
/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(1/3)*(b*c - a*d)^(2/3)) + Log[c^(2/3) + ((b
*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a +
 b*x^3)^(1/3)]/(6*c^(1/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \right.}{3\sqrt[3]{c}\sqrt[3]{bc-ad}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \text{Subs}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, \right.}{\sqrt[3]{c}(bc-}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right.}{6\sqrt[3]{c}(bc-ad)^{2/3}}$$

**Mathematica** [C] time = 0.04, size = 83, normalized size = 0.56

$$\frac{x^2\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{2c\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)
*x^3)/(a*(c + d*x^3))])/(2*c*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))
```

**IntegrateAlgebraic** [C] time = 1.96, size = 319, normalized size = 2.14

$$\frac{i\left(\sqrt{3}+i\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\left(1-i\sqrt{3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\sqrt{-1-i\sqrt{3}}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\sqrt[3]{c}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(
1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3
))])/(Sqrt[6]*c^(1/3)*(b*c - a*d)^(2/3)) + ((1 - I*Sqrt[3])*Log[2*(b*c - a*
d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(1/3)*(b*c -
a*d)^(2/3)) + ((I/12)*(I + Sqrt[3])*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1
/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c
^(2/3)*(a + b*x^3)^(2/3)])/(c^(1/3)*(b*c - a*d)^(2/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + bx^{3}\right)^{\frac{2}{3}} \left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.477 $\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=173

$$-\frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

**Rubi [A]** time = 0.22, antiderivative size = 232, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*c*x)) + (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)
/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3))
 + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(4/3)*(b
*c - a*d)^(2/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/
3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(4/3)*(b*c - a*
d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\,\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}\sqrt[3]{bc-ad}} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}-\sqrt[3]{\cdots}}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-a\cdots}}\right.}{3c^{4/3}\cdots}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx,\cdots\right.}{6c^{4/3}(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{4/3}(bc-ad)^{2/3}} - \cdots$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc\cdots}{(a\cdots}\right.}{6c^{4/3}\cdots}$$

**Mathematica [C]** time = 0.06, size = 128, normalized size = 0.74

$$-\frac{\frac{6x^3(c+dx^3)(bc-ad)\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{a+bx^3}+5c\left(2c+3dx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{10c^3x\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/10*(5*c*(2*c + 3*d*x^3)*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + (6*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[5/3, 2
, 8/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(a + b*x^3))/(c^3*x*(a + b*x^3)^
(2/3))
```

**IntegrateAlgebraic [C]** time = 2.02, size = 347, normalized size = 2.01

$$\frac{i\left(\sqrt{3}\,d+id\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}\,d\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{c^{4/3}(bc-ad)^{2/3}}+\frac{\left(d-i\sqrt{3}\,d\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*c*x)) - (Sqrt[(-1 - I*Sqrt[3])/6]*d*ArcTan[(3*(b*c -
 a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/
3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(4/3)*(b*c - a*d)^(2/3)) + ((I
/6)*(I*d + Sqrt[3]*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3)])/(c^(4/3)*(b*c - a*d)^(2/3)) + ((d - I*Sqrt[3]*d)*Log[(-2
*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a
+ b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3)*(b*c
 - a*d)^(2/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.478 $\int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx$

**Optimal**. Leaf size=215

$$\frac{\sqrt[3]{a+bx^3}\,(4ad+3bc)}{4a^2c^2x}+\frac{d^2\log\left(c+dx^3\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

**Rubi [A]** time = 0.30, antiderivative size = 269, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a+bx^3}\,(ad+bc)}{a^2c^2x}-\frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4}-\frac{d^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}}+\frac{d^2\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((b*c + a*d)*(a + b*x^3)^(1/3))/(a^2*c^2*x) - (a + b*x^3)^(4/3)/(4*a^2*c*x^
4) - (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d)^(2/3)) - (d^2*Log[c^(1/3) - ((
b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(7/3)*(b*c - a*d)^(2/3)) + (d^
2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(7/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
```

```
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^2}{x^5\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^5}+\frac{-bc-ad}{c^2x^2}+\frac{a^2d^2x}{c^2\left(c-(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} + \frac{d^2\,\text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c^2}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}\sqrt[3]{bc-ad}} - \cdots$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\,\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\log\left(c^{2/3}+\cdots\right)}{6c^{7/3}\cdots}$$

$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}} - \frac{d^2\log\left(\sqrt[3]{c}-\cdots\right)}{3c^{7/3}(bc-ad)\cdots}$$

**Mathematica [C]** time = 1.89, size = 267, normalized size = 1.24

$$-\frac{-81x^3\left(c+dx^3\right)^2(bc-ad)\,{}_4F_3\left(\frac{2}{3},2,2,2;1,1,\frac{8}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+216dx^6\left(c+dx^3\right)(ad-bc)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{8}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-5\left(2c\left(a+bx^3\right)\left(c^2+10cdx^3+9d^2x^6\right)+\left(a\left(-8c^3+17c^2dx^3+46cd^2x^6+9d^3x^9\right)+3bcx^3\left(-3c^2+2cdx^3+9d^2x^6\right)\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)\right)}{120c^4x^4\left(a+bx^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] -1/120*(-5*(2*c*(a + b*x^3)*(c^2 + 10*c*d*x^3 + 9*d^2*x^6) + (3*b*c*x^3*(-3
*c^2 + 2*c*d*x^3 + 9*d^2*x^6) + a*(-8*c^3 + 17*c^2*d*x^3 + 46*c*d^2*x^6 + 9
*d^3*x^9))*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
]) + 216*d*(-(b*c) + a*d)*x^6*(c + d*x^3)*HypergeometricPFQ[{2/3, 2, 2}, {1
, 8/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*(b*c - a*d)*x^3*(c + d*x^3)
^2*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))])/(c^4*x^4*(a + b*x^3)^(5/3))

**IntegrateAlgebraic [C]** time = 2.41, size = 377, normalized size = 1.75

$$\frac{\sqrt[3]{a+bx^3}\left(-ac+4adx^3+3bcx^3\right)}{4a^2c^2x^4}+\frac{\left(d^2-i\sqrt{3}d^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{7/3}(bc-ad)^{2/3}}+\frac{\sqrt{\frac{1}{6}\left(-1-i\sqrt{3}\right)}d^2\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{7/3}(bc-ad)^{2/3}}+\frac{i\left(\sqrt{3}d^2+id^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]

[Out] ((a + b*x^3)^(1/3)*(-(a*c) + 3*b*c*x^3 + 4*a*d*x^3))/(4*a^2*c^2*x^4) + (Sqr
t[(-1 - I*Sqrt[3])/6]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^
(1/3))]])/(c^(7/3)*(b*c - a*d)^(2/3)) + ((d^2 - I*Sqrt[3]*d^2)*Log[2*(b*c -
a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(6*c^(7/3)*(b*c
- a*d)^(2/3)) + ((I/12)*(I*d^2 + Sqrt[3]*d^2)*Log[(-2*I)*(b*c - a*d)^(2/3)*
x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I +
Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(7/3)*(b*c - a*d)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

## 3.479 $\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=347

$$-\frac{a^4}{b^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}+\frac{a\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^4d^2}-\frac{\left(a+bx^3\right.}{5}$$

**Rubi [A]** time = 0.44, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}-\frac{a^4}{b^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{a\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^4d^2}-\frac{\left(a+bx^3\right)^{5/3}(ad+bc)}{5b^4d^2}-\frac{2a\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{c^4\log\left(c+dx^3\right)}{6d^{11/3}(bc-ad)^{4/3}}+\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}+\frac{c^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{11/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^4/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a^2*(a + b*x^3)^(2/3))/(2*b^4
*d) + (a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + ((b^2*c^2 + a*b*c*d +
 a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (2*a*(a + b*x^3)^(5/3))/(5*b^4*d
) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (a + b*x^3)^(8/3)/(8*b^4*
d) + (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(Sqrt[3]*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[c + d*x^3])/(6*d^(11/
3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a^4}{b^3(bc-ad)(a+bx)^{4/3}} + \frac{b^2c^2+abcd+a^2d^2}{b^3d^3\sqrt[3]{a+bx}} - \frac{(bc+ad)x}{b^2d^2\sqrt[3]{a+bx}} + \frac{}{bd\sqrt[3]{}}\right.\right.$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} + \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx,\right.}{3bd}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} - \frac{c^4\log\left(c+dx^3\right)}{6d^{11/3}(bc-ad)^{4/3}} +$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{\left(b^2c^2+abcd\right.}{}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{\left(b^2c^2+abcd\right.}{}$$

**Mathematica [C]** time = 0.27, size = 157, normalized size = 0.45

$$\frac{\frac{81a^3d^3+9a^2bd^2\left(8c+3dx^3\right)+3ab^2d\left(20c^2+8cdx^3-3d^2x^6\right)+b^3\left(40c^3+20c^2dx^3-8cd^2x^6+5d^3x^9\right)}{b^4} - \frac{40c^4\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad}}{40d^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((81*a^3*d^3 + 9*a^2*b*d^2*(8*c + 3*d*x^3) + 3*a*b^2*d*(20*c^2 + 8*c*d*x^3
- 3*d^2*x^6) + b^3*(40*c^3 + 20*c^2*d*x^3 - 8*c*d^2*x^6 + 5*d^3*x^9))/b^4 -
```

```
(40*c^4*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(
b*c - a*d))/(40*d^4*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.71, size = 393, normalized size = 1.13

$$-\frac{81a^4d^3-9a^3bcd^2+27a^3bd^3x^3-12a^2b^2c^2d-3a^2b^2cd^2x^3-9a^2b^2d^3x^6-20ab^3c^3-4ab^3c^2dx^3+ab^3cd^2x^6+5ab^3d^3x^9-20b^4c^3x^3+8b^4c^2dx^6-5b^4cd^2x^9}{40b^4d^3\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{11/3}(bc-ad)^{4/3}}-\frac{c^4\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{11/3}(bc-ad)^{4/3}}+\frac{c^4\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{11/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*(-20*a*b^3*c^3 - 12*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 81*a^4*d^3 - 20*b
^4*c^3*x^3 - 4*a*b^3*c^2*d*x^3 - 3*a^2*b^2*c*d^2*x^3 + 27*a^3*b*d^3*x^3 + 8
*b^4*c^2*d*x^6 + a*b^3*c*d^2*x^6 - 9*a^2*b^2*d^3*x^6 - 5*b^4*c*d^2*x^9 + 5*
a*b^3*d^3*x^9)/(b^4*d^3*(b*c - a*d)*(a + b*x^3)^(1/3)) + (c^4*ArcTan[1/Sqrt
[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*
d^(11/3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)])/(3*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[(b*c - a*d)^(2/3) - d
^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6
*d^(11/3)*(b*c - a*d)^(4/3))
```

**fricas [B]** time = 0.84, size = 1300, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c
^4*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 -
b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)
)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(
b*x^3 + a)^(1/3))/(d*x^3 + c)) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a
)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*
d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*
(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5
+ 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c
^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*
c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*
d^6)*x^3)*(b*x^3 + a)^(2/3))/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7
 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*(120*sqrt(1/3)*
(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2)*x^3)*sqrt(-(-b
*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d
+ (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d)
+ 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/
3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^
(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 +
 a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c
^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 -
 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a
^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^
2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(a
*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6
 + a^2*b^5*d^7)*x^3)]
```

**giac [A]** time = 0.26, size = 431, normalized size = 1.24

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^5-2\sqrt{3}\,abcd^6+\sqrt{3}\,a^2d^7}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d^5-2\,abcd^6+a^2d^7\right)}+\frac{c^4\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)}-\frac{a^4}{\left(b^5c-ab^4d\right)\left(bx^3+a\right)^{\frac{1}{3}}}+\frac{20\left(bx^3+a\right)^{\frac{2}{3}}b^{30}c^2d^5-8\left(bx^3+a\right)^{\frac{5}{3}}b^{29}cd^6+40\left(bx^3+a\right)^{\frac{2}{3}}ab^{29}cd^6+5\left(bx^3+a\right)^{\frac{8}{3}}b^{28}d^7-24\left(bx^3+a\right)^{\frac{5}{3}}ab^{28}d^7+60\left(bx^3+a\right)^{\frac{2}{3}}a^2b^{28}d^7}{40\,b^{32}d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^5 - 2*sqrt(3
)*a*b*c*d^6 + sqrt(3)*a^2*d^7) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^4*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 1/3*c^4*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^3 - 2*a*
b*c*d^4 + a^2*d^5) - a^4/((b^5*c - a*b^4*d)*(b*x^3 + a)^(1/3)) + 1/40*(20*(
b*x^3 + a)^(2/3)*b^30*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^29*c*d^6 + 40*(b*x^3
+ a)^(2/3)*a*b^29*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^28*d^7 - 24*(b*x^3 + a)^(5/
3)*a*b^28*d^7 + 60*(b*x^3 + a)^(2/3)*a^2*b^28*d^7)/(b^32*d^8)
```

**maple [F]** time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.19, size = 564, normalized size = 1.63

$$\left(\frac{3\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\left(\frac{4\,a}{5\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{5\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^4\,d}+\frac{a^4}{b^4\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a\,d-b\,c\right)}+\frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{9\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}-\frac{\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{{\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{36\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}+\frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
a*b^4*d))/(2*b^4*d))*(a + b*x^3)^(2/3) - ((4*a)/(5*b^4*d) + (b^5*c - a*b^4*
d)/(5*b^8*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^4*d) + a^4/(b^4*
(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^4*log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*
c^9*d^4) - (c^8*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b
^2*c^2*d^13 - 36*a^3*b*c*d^14))/(9*d^(22/3)*(a*d - b*c)^(8/3))))/(3*d^(11/3
)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4) - ((3
^(1/2)*c^4*1i + c^4)^2*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 5
4*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(36*d^(22/3)*(a*d - b*c)^(8/3)))*(3^
(1/2)*c^4*1i + c^4))/(6*d^(11/3)*(a*d - b*c)^(4/3)) + (c^4*log((a + b*x^3)^
(1/3)*(a*c^8*d^5 - b*c^9*d^4) - (c^8*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^15 +
 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14
))/(d^(22/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(11/3)*(a*d - b
*c)^(4/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.480 $\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=253

$$\frac{a^3}{b^3\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^3d^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{\left(a+bx^3\right)^{5/3}}{5b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\cdots\right.}{2d^{8/3}(bc-\cdots}$$

**Rubi [A]** time = 0.34, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{a^3}{b^3\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^3d^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{\left(a+bx^3\right)^{5/3}}{5b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a^3/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (a*(a + b*x^3)^(2/3))/(2*b^3*d) -
 ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (a + b*x^3)^(5/3)/(5*b^3*d)
- (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(4/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int\left(-\frac{a^3}{b^2(bc-ad)(a+bx)^{4/3}} + \frac{-bc-ad}{b^2d^2\sqrt[3]{a+bx}} + \frac{x}{bd\sqrt[3]{a+bx}} - \frac{}{d^2(-bc}\right.\right.$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3bd} + \frac{c}{}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}} + \frac{\operatorname{Subst}\left(\int\left(\right.\right.}{}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} +$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} -$$

**Mathematica [C]** time = 0.11, size = 147, normalized size = 0.58

$$\frac{18a^3d^3 + 3a^2bd^2\left(2dx^3-c\right) + 10b^3c^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right) - ab^2d\left(5c^2+cdx^3+2d^2x^6\right) + b^3c\left(-10c^2-5cdx^3+2d^2x^6\right)}{10b^3d^3\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (18*a^3*d^3 + 3*a^2*b*d^2*(-c + 2*d*x^3) + b^3*c*(-10*c^2 - 5*c*d*x^3 + 2*d
^2*x^6) - a*b^2*d*(5*c^2 + c*d*x^3 + 2*d^2*x^6) + 10*b^3*c^3*Hypergeometric
2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(10*b^3*d^3*(b*c - a*d)*
(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.50, size = 322, normalized size = 1.27

$$-\frac{-18a^3d^2+3a^2bcd-6a^2bd^2x^3+5ab^2c^2+ab^2cdx^3+2ab^2d^2x^6+5b^3c^2x^3-2b^3cdx^6}{10b^3d^2\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{3d^{8/3}(bc-ad)^{4/3}}+\frac{c^3\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{8/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/10*(5*a*b^2*c^2 + 3*a^2*b*c*d - 18*a^3*d^2 + 5*b^3*c^2*x^3 + a*b^2*c*d*x
^3 - 6*a^2*b*d^2*x^3 - 2*b^3*c*d*x^6 + 2*a*b^2*d^2*x^6)/(b^3*d^2*(b*c - a*d
)*(a + b*x^3)^(1/3)) - (c^3*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3)
)/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) - (c^3*
Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(8/3)*(b*c - a*d)^
(4/3)) + (c^3*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)
^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(8/3)*(b*c - a*d)^(4/3))
```

**fricas [B]** time = 0.73, size = 1141, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^
3*d^2)*x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b
*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*s
qrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^
3 + a)^(1/3))/(d*x^3 + c)) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (b*c*d^2 - a*d^3)^(2/3)) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d
^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 3*(5*a*b^3*c
^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 -
 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^
2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b
^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3),
1/30*(30*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^3*
d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d))/d) + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log
((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(5*a*b^3*c^3*d^2 - 2*a^
2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 - 2*a*b^3*c*d^
4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^2*b^2*c*d^4 +
 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b^4*c*d^5 + a^
3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3)]
```

**giac [A]** time = 0.26, size = 372, normalized size = 1.47

$$-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^4-2\sqrt{3}\,abcd^5+\sqrt{3}\,a^2d^6}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d^4-2\,abcd^5+a^2d^6\right)}-\frac{c^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)}+\frac{a^3}{\left(b^4c-ab^3d\right)\left(bx^3+a\right)^{\frac{1}{3}}}-\frac{5\left(bx^3+a\right)^{\frac{2}{3}}b^{13}cd^3-2\left(bx^3+a\right)^{\frac{5}{3}}b^{12}d^4+10\left(bx^3+a\right)^{\frac{2}{3}}ab^{12}d^4}{10\,b^{15}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^4 - 2*sqrt(
3)*a*b*c*d^5 + sqrt(3)*a^2*d^6) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)
```

```
^(2/3))/(b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6) - 1/3*c^3*(-(b*c - a*d)/d)^(2
/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4) + a^3/((b^4*c - a*b^3*d)*(b*x^3 + a)^(1/3)) - 1/10*(5*(
b*x^3 + a)^(2/3)*b^13*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^12*d^4 + 10*(b*x^3 + a)
^(2/3)*a*b^12*d^4)/(b^15*d^5)
```

**maple** [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 5.16, size = 493, normalized size = 1.95

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b^3\,d}-\left(\frac{3\,a}{2\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{2\,b^6\,d^2}\right)\,\left(b\,x^3+a\right)^{2/3}-\frac{a^3}{b^3\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}-\frac{c^3\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^6\,d^4-b\,c^7\,d^3\right)-\frac{c^6\,\left(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8\right)}{9\,d^{16/3}\,\left(a\,d-b\,c\right)^{8/3}}\right)}{3\,d^{8/3}\,\left(a\,d-b\,c\right)^{4/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^6\,d^4-b\,c^7\,d^3\right)-\frac{{\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8\right)}{36\,d^{16/3}\,\left(a\,d-b\,c\right)^{8/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{8/3}\,\left(a\,d-b\,c\right)^{4/3}}-\frac{c^3\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^6\,d^4-b\,c^7\,d^3\right)-\frac{c^6\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\,\left(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8\right)}{9\,d^{16/3}\,\left(a\,d-b\,c\right)^{8/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{8/3}\,\left(a\,d-b\,c\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b^3*d) - ((3*a)/(2*b^3*d) + (b^4*c - a*b^3*d)/(2*b^6*d
^2))*(a + b*x^3)^(2/3) - a^3/(b^3*(a + b*x^3)^(1/3)*(a*d - b*c)) - (c^3*log
((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(9*a^4*d^12 + 9*b^4*c^4*d
^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)
*(a*d - b*c)^(8/3))))/(3*d^(8/3)*(a*d - b*c)^(4/3)) + (log((a + b*x^3)^(1/3
)*(a*c^6*d^4 - b*c^7*d^3) - ((3^(1/2)*c^3*1i + c^3)^2*(9*a^4*d^12 + 9*b^4*c
^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(36*d^(
16/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(8/3)*(a*d - b*c)^(4
/3)) - (c^3*log((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*((3^(1/2)*
1i)/2 - 1/2)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*
c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/
2 - 1/2))/(3*d^(8/3)*(a*d - b*c)^(4/3))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a + b x^{3}\right)^{\frac{4}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.481 $\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=203

$$-\frac{a^2}{b^2\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}$$

**Rubi [A]** time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 87, 56, 617, 204, 31}

$$-\frac{a^2}{b^2\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a + b*x^3)^(2/3)/(2*b^2*d) +
(c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(6*d^(5/3)*(b*c
 - a*d)^(4/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*d^(5/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}(c+dx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^{4/3}}+\frac{1}{bd\sqrt[3]{a+bx}}+\frac{c^2}{d(-bc+ad)\sqrt[3]{a+bx}\,(c+d\ldots}\right.\right.\\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx,x,x^3\right)}{3d(bc-ad)}\\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}\ldots}\right.}{2d^{5/3}(bc\ldots}\\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\ldots\right.}{2d^{5/3}(bc-\ldots}\\
&= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}-\frac{c^2\log\left(c+d\ldots\right.}{6d^{5/3}(bc-a\ldots}
\end{aligned}$$

**Mathematica [C]** time = 0.07, size = 101, normalized size = 0.50

$$\frac{-3a^2d^2-2b^2c^2\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+abd\left(c-dx^3\right)+b^2c\left(2c+dx^3\right)}{2b^2d^2\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (-3*a^2*d^2 + a*b*d*(c - d*x^3) + b^2*c*(2*c + d*x^3) - 2*b^2*c^2*Hypergeom
etric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(2*b^2*d^2*(b*c - a
*d)*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.46, size = 268, normalized size = 1.32

$$-\frac{3a^2d-abc+abdx^3-b^2cx^3}{2b^2d\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{5/3}(bc-ad)^{4/3}}-\frac{c^2\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*(-(a*b*c) + 3*a^2*d - b^2*c*x^3 + a*b*d*x^3)/(b^2*d*(b*c - a*d)*(a + b
*x^3)^(1/3)) + (c^2*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[
3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(4/3)) + (c^2*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(5/3)*(b*c - a*d)^(4/3)) -
 (c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) +
 d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(5/3)*(b*c - a*d)^(4/3))
```

**fricas [B]** time = 0.71, size = 1004, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*
d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c
*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*s
qrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x
^3 + a)^(1/3))/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*
c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*
d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5
 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*sqrt(1/3)*(a*b
^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*sqrt(-(-b*c*d
^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-
b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (b
^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
- 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3
)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*
d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*
b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4
+ a^2*b^3*d^5)*x^3)]
```

**giac [A]** time = 0.25, size = 325, normalized size = 1.60

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^3-2\sqrt{3}\,abcd^4+\sqrt{3}\,a^2d^5}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)}+\frac{c^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}-\frac{a^2}{\left(b^3c-ab^2d\right)\left(bx^3+a\right)^{\frac{1}{3}}}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^3 - 2*sqrt(3
)*a*b*c*d^4 + sqrt(3)*a^2*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 1/3*c^2*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) - a^2/((b^3*c - a*b^2*d)*(b*x^3 + a)^(1/3)) + 1/2*(b*x^3 +
 a)^(2/3)/(b^2*d)
```

**maple [F]** time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.99, size = 449, normalized size = 2.21

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2\,b^2\,d}+\frac{a^2}{b^2\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}+\frac{c^2\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^4\,d^3-b\,c^5\,d^2\right)-\frac{c^4\,\left(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5\right)}{9\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{5/3}\,{\left(a\,d-b\,c\right)}^{4/3}}-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^4\,d^3-b\,c^5\,d^2\right)-\frac{{\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5\right)}{36\,d^{10/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)}{6\,d^{5/3}\,{\left(a\,d-b\,c\right)}^{4/3}}+\frac{c^2\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^4\,d^3-b\,c^5\,d^2\right)-\frac{c^4\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2\,\left(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5\right)}{d^{10/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{5/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b^2*d) + a^2/(b^2*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^
2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*(9*a^4*d^9 + 9*b^4*c
^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(9*d^(10/
3)*(a*d - b*c)^(8/3))))/(3*d^(5/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1
/3)*(a*c^4*d^3 - b*c^5*d^2) - ((3^(1/2)*c^2*1i + c^2)^2*(9*a^4*d^9 + 9*b^4*
c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(36*d^(1
0/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(5/3)*(a*d - b*c)^(4/
3)) + (c^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*((3^(1/2)*1
i)/6 - 1/6)^2*(9*a^4*d^9 + 9*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^
2*d^7 - 36*a^3*b*c*d^8))/(d^(10/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/
6))/(d^(5/3)*(a*d - b*c)^(4/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(a + b x^{3}\right)^{\frac{4}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.482 $\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=174

$$\frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

**Rubi [A]** time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 56, 617, 204, 31}

$$\frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)) +
(c*Log[c + d*x^3])/(6*d^(2/3)*(b*c - a*d)^(4/3)) - (c*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right.}{2d^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} +$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-\right.}{2d^2}$$

**Mathematica [C]** time = 0.03, size = 77, normalized size = 0.44

$$\frac{bc\left(a+bx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2a(bc-ad)}{2b\sqrt[3]{a+bx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (2*a*(b*c - a*d) + b*c*(a + b*x^3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b
*x^3))/(-(b*c) + a*d)])/(2*b*(b*c - a*d)^2*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.26, size = 230, normalized size = 1.32

$$-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3d^{2/3}(bc-ad)^{4/3}}+\frac{c\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[1/Sqrt[3] - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(
4/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*d^(2/3)*(
b*c - a*d)^(4/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
 + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*d^(2/3)*(b*c - a*d)^(4/3))
```

**fricas [B]** time = 0.82, size = 872, normalized size = 5.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x
^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3
*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*
c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(
1/3))/(d*x^3 + c)) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3
 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 -
a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)
^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b
^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 +
 (b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c
*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)
^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2
/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*(a*b*c*d^2 - a^2
*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

**giac [B]** time = 0.25, size = 301, normalized size = 1.73

$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}} bc \arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^2-2\sqrt{3}\,abcd^3+\sqrt{3}\,a^2d^4} - \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}} bc \log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2abcd^3+a^2d^4} + \frac{2bc\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{b^2c^2-2abcd+a^2d^2} - \frac{6a}{\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2 -
2*sqrt(3)*a*b*c*d^3 + sqrt(3)*a^2*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 2*b*c*(-(b*c - a*d)/d)^(
2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2 - 2*a*b*
c*d + a^2*d^2) - 6*a/((b*x^3 + a)^(1/3)*(b*c - a*d)))/b
```

**maple [F]** time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [**B**] time = 4.98, size = 412, normalized size = 2.37

$$-\frac{a}{b\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}-\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^2\,d^2-b\,c^3\,d\right)-\frac{c^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{9\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{2/3}\,{\left(a\,d-b\,c\right)}^{4/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^2\,d^2-b\,c^3\,d\right)-\frac{{\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{36\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{2/3}\,{\left(a\,d-b\,c\right)}^{4/3}}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^2\,d^2-b\,c^3\,d\right)-\frac{{\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{36\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c+\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{2/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - ((c - 3^(1/2)*c*1i)^2*(9*a^4
*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d
^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c - 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d -
b*c)^(4/3)) - (c*log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - (c^2*(9*a^4*
d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^
5))/(9*d^(4/3)*(a*d - b*c)^(8/3))))/(3*d^(2/3)*(a*d - b*c)^(4/3)) - a/(b*(a
 + b*x^3)^(1/3)*(a*d - b*c)) + (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d)
 - ((c + 3^(1/2)*c*1i)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54
*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c + 3^
(1/2)*c*1i))/(6*d^(2/3)*(a*d - b*c)^(4/3))
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a + b x^{3}\right)^{\frac{4}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.483 $\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=167

$$-\frac{1}{\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}$$

**Rubi [A]** time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {444, 51, 56, 617, 204, 31}

$$-\frac{1}{\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) - (
d^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) + (d^(1/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+b\right.}{2(bc-ad)^{4/3}}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right.}{2(bc-ad)^{4/3}}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\log}{}$$

**Mathematica [C]** time = 0.02, size = 50, normalized size = 0.30

$$-\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)]/((b*c - a
*d)*(a + b*x^3)^(1/3)))
```

**IntegrateAlgebraic [A]** time = 0.24, size = 223, normalized size = 1.34

$$-\frac{\sqrt[3]{d}\,\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6(bc-ad)^{4/3}} - \frac{1}{\sqrt[3]{a+bx^3}\,(bc-ad)} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{3(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[1/Sqrt[3] - (2*d^(1/
3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*(b*c - a*d)^(4
```

```
/3)) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*(b*c
 - a*d)^(4/3)) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)
*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*(b*c - a*d)^(4/3))
```

**fricas [A]** time = 0.67, size = 262, normalized size = 1.57

$$-\frac{2\sqrt{3}\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}d-(bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\right)+2\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left((bc-ad)\left(-\frac{d}{bc-ad}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}d\right)+6\left(bx^3+a\right)^{\frac{2}{3}}}{6\left(\left(b^2c-abd\right)x^3+abc-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b*x^3 + a)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^
3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - (b*x^3 + a)*(-d/(b*c -
 a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b
*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 2*(b*x^3 + a)*(-d
/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1
/3)*d) + 6*(b*x^3 + a)^(2/3))/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)
```

**giac [B]** time = 0.26, size = 285, normalized size = 1.71

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d-2\sqrt{3}\,abcd^2+\sqrt{3}\,a^2d^3}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}-\frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/
3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(
1/3))/(sqrt(3)*b^2*c^2*d - 2*sqrt(3)*a*b*c*d^2 + sqrt(3)*a^2*d^3) - 1/6*(-b
*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) -
 1/((b*x^3 + a)^(1/3)*(b*c - a*d))
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.92, size = 389, normalized size = 2.33

$$\frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}+\frac{d^{1/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,d^4-b\,c\,d^3\right)-\frac{d^{2/3}\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{9\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,{\left(a\,d-b\,c\right)}^{4/3}}-\frac{d^{1/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,d^4-b\,c\,d^3\right)-\frac{d^{2/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{9\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,{\left(a\,d-b\,c\right)}^{4/3}}+\frac{d^{1/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,d^4-b\,c\,d^3\right)-\frac{d^{2/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{{\left(a\,d-b\,c\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] 1/((a + b*x^3)^(1/3)*(a*d - b*c)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 -
 b*c*d^3) - (d^(2/3)*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2
*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3))))/(3*(a*d - b*c)^(4/3
)) - (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d^(2/3)*((3^(1/2)*
1i)/2 + 1/2)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c
^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/2 + 1/2))/(3
*(a*d - b*c)^(4/3)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d
^(2/3)*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d
^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(a*d - b*c)^(8/3))*((3^(1/2)*1i)
/6 - 1/6))/(a*d - b*c)^(4/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b x^3\right)^{\frac{4}{3}} \left(c+d x^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.484 $\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=271

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}-\cdots$$

**Rubi [A]** time = 0.32, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 85, 156, 55, 617, 204, 31, 56}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}-\frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{4/3}}+\frac{b}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3)
)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(4/3
)) - Log[x]/(2*a^(4/3)*c) + (d^(4/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(4/3)
) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*
d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{-bc+ad-bdx}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3ac} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\right.}{3c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a}\right.}{2a^{4/3}c}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} -$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}c} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c}$$

**Mathematica [C]** time = 0.05, size = 86, normalized size = 0.32

$$\frac{ad\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+(bc-ad)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^3}{a}+1\right)}{ac\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*d*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + (b*c
 - a*d)*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^3)/a])/(a*c*(b*c - a*d)*(a
 + b*x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.85, size = 360, normalized size = 1.33

$$\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{3a^{4/3}c}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6a^{4/3}c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}c}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c(bc-ad)^{4/3}}+\frac{d^{4/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,c(bc-ad)^{4/3}}-\frac{b}{a\sqrt[3]{a+bx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(b/(a*(-(b*c) + a*d)*(a + b*x^3)^(1/3))) + ArcTan[1/Sqrt[3] + (2*(a + b*x^
3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[1/Sqrt[3
] - (2*d^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*c*
(b*c - a*d)^(4/3)) + Log[-a^(1/3) + (a + b*x^3)^(1/3)]/(3*a^(4/3)*c) - (d^(
4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*c*(b*c - a*d)^(
4/3)) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*a^(
4/3)*c) + (d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b
*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c*(b*c - a*d)^(4/3))
```

**fricas [B]** time = 0.80, size = 975, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c -
 a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(
2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 +
 a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a
*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3
*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(
2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a^3*d)
*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a^2*b*
d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^
3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*
c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3)
- 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a
^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*
d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a
^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(1/3)*d) + 6*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c - a^2
*b*d)*x^3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3
))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

**giac [A]** time = 0.79, size = 389, normalized size = 1.44

$$-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^3-2\sqrt{3}\,abc^2d+\sqrt{3}\,a^2cd^2}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}+\frac{b}{\left(bx^3+a\right)^{\frac{1}{3}}\left(abc-a^2d\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}c}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}c}+\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*ar
ctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*
d)/d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) +
1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b
*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2) + b/((b*x^3 + a)^(1/3)*(a*b*c - a^2*d)) + 1/3*sqrt(3)*arctan(1/3*sqrt
(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c) - 1/6*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c) + 1/3*log(abs(
(b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*c)
```

**maple** [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{4}{3}} \left(dx^{3}+c\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)
```

**mupad** [B] time = 5.34, size = 3804, normalized size = 14.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^
8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a
^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*
a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a
^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d
^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243
*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 -
51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^1
0*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7
*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4
*c^4*d^14))*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^
2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^
12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^
9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c
^3*d^12 - 27*a^16*b^5*c^2*d^13)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^
3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) + log(9*a^7*b^1
4*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d
^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*
d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*
```

d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d
^13 - 54*a^18*b^4*c^2*d^14) - (1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^
3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^
12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^1
6*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*
a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*(1/(27
*a^4*c^3))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*
b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b
^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c
^2*d^13)*(1/(27*a^4*c^3))^(1/3) + (log(((3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7
+ 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*
d))^(1/3)*((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4
+ 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7
 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^1
0 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13
 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i - 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c
^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*
(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^
5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10
*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18
*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21
*b^4*c^4*d^14))/4))/2 - 9*a^7*b^14*c^11*d^4 + 90*a^8*b^13*c^10*d^5 - 405*a^
9*b^12*c^9*d^6 + 1071*a^10*b^11*c^8*d^7 - 1827*a^11*b^10*c^7*d^8 + 2079*a^1
2*b^9*c^6*d^9 - 1575*a^13*b^8*c^5*d^10 + 765*a^14*b^7*c^4*d^11 - 216*a^15*b
^6*c^3*d^12 + 27*a^16*b^5*c^2*d^13)*(3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7 + 27
*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*
a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)*((a + b*x^3)^
(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^
5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*
d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d
^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14
) - ((3^(1/2)*1i + 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*
d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3
 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^1
2*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16
*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a
^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))/4))/2 +
 9*a^7*b^14*c^11*d^4 - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a
^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^
13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b
^5*c^2*d^13)*(3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*
b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3))/2 - b/((a + b*x^
3)^(1/3)*(a^2*d - a*b*c)) + log(((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 -
297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 +
8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 -
 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 +
 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i)/2 - 1/2)^2*(
1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 1
5795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*
d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8
*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b
^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^4*c^3
))^(1/3) - 9*a^7*b^14*c^11*d^4 + 90*a^8*b^13*c^10*d^5 - 405*a^9*b^12*c^9*d^
6 + 1071*a^10*b^11*c^8*d^7 - 1827*a^11*b^10*c^7*d^8 + 2079*a^12*b^9*c^6*d^9
 - 1575*a^13*b^8*c^5*d^10 + 765*a^14*b^7*c^4*d^11 - 216*a^15*b^6*c^3*d^12 +
 27*a^16*b^5*c^2*d^13)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^4*c^3))^(1/3) - log(
((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9
*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a

```
^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a
^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18
*b^4*c^2*d^14) - ((3^(1/2)*1i)/2 + 1/2)^2*(1/(27*a^4*c^3))^(2/3)*(243*a^10*
b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*
a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8
+ 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d
^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d
^14))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^4*c^3))^(1/3) + 9*a^7*b^14*c^11*d^4 -
 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 182
7*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*
a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*((3^(1/2)
*1i)/2 + 1/2)*(1/(27*a^4*c^3))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.485 $\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=357

$$-\frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}-\frac{(3ad+4bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}c^2}+\frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2}-\frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{d^{7/3}}{6c}$$

**Rubi [A]** time = 0.39, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 51, 55, 617, 204, 31, 56}

$$-\frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}-\frac{(3ad+4bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}c^2}+\frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2}-\frac{d^2}{c^2\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{d^{7/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{4/3}}+\frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}}+\frac{d^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2(bc-ad)^{4/3}}-\frac{1}{3acx^3\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(d^2/(c^2*(b*c - a*d)*(a + b*x^3)^(1/3))) - (4*b*c + 3*a*d)/(3*a^2*c^2*(a
+ b*x^3)^(1/3)) - 1/(3*a*c*x^3*(a + b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan
[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*c^2
) + (d^(7/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(Sqrt[3]*c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[x])/(6*a^(7
/3)*c^2) - (d^(7/3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(4/3)) - ((4*b*c + 3
*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(7/3)*c^2) + (d^(7/3)*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(4bc+3ad)+\frac{4bdx}{3}}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(4bc+3ad)\,\text{Subst}\left(\int \right.}{9ac}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{d^3\,\text{Subst}\left(\int \frac{}{\sqrt[3]{a+b}}\right.}{3c^2(bc}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3ad)\log(}{6a^{7/3}c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3ad)\log(}{6a^{7/3}c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{(4bc+3ad)\tan^{-1}}{3\sqrt{3}\,a^{7/}}$$

**Mathematica [C]** time = 0.05, size = 117, normalized size = 0.33

$$\frac{3a^2d^2x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right) + (bc-ad)\left(x^3(3ad+4bc)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^3}{a}+1\right)+ac\right)}{3a^2c^2x^3\sqrt[3]{a+bx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (3*a^2*d^2*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*
d)] + (b*c - a*d)*(a*c + (4*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/3, 1, 2/3
, 1 + (b*x^3)/a]))/(3*a^2*c^2*(-(b*c) + a*d)*x^3*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [A]** time = 0.99, size = 425, normalized size = 1.19

$$\frac{(-3ad-4bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{a}\right)}{9a^{7/3}c^2} + \frac{(3ad+4bc)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{18a^{7/3}c^2} - \frac{(3ad+4bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}c^2} + \frac{-a^2d+abc-abdx^3+4b^2cx^3}{3a^2cx^3\sqrt[3]{a+bx^3}\,(ad-bc)} + \frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3c^2(bc-ad)^{4/3}} - \frac{d^{7/3}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{6c^2(bc-ad)^{4/3}} + \frac{d^{7/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{bc-ad}}\right)}{\sqrt{3}\,c^2(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*b*c - a^2*d + 4*b^2*c*x^3 - a*b*d*x^3)/(3*a^2*c*(-(b*c) + a*d)*x^3*(a +
b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sq
rt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*c^2) + (d^(7/3)*ArcTan[1/Sqrt[3] - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3))])/(Sqrt[3]*c^2*(b*c -
 a*d)^(4/3)) + ((-4*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(9*a^(7
/3)*c^2) + (d^(7/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*
c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)
```

```
^(1/3) + (a + b*x^3)^(2/3)])/(18*a^(7/3)*c^2) - (d^(7/3)*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/
3)])/(6*c^2*(b*c - a*d)^(4/3))
```

**fricas [B]** time = 1.39, size = 1386, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*
b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*s
qrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)
*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6*sqr
t(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3
)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c^2 - a
*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-
a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3))
 - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d
- 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(a^3*b
*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c
- a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c
- a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log
((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*(a^2*b*c^2 -
 a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3
- a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6*sqrt(1/3)*((4*a
*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2 - a^3*b*c*d - 3*
a^4*d^2)*x^3)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (
-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(
-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))
^(1/3) + 1/3*sqrt(3)) - ((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b
^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*
x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2
*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*
x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a
*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*(a^3*b*d^2*x^6 +
a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d) + 6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)
*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^
5*c^2*d)*x^3)]
```

**giac [A]** time = 0.79, size = 486, normalized size = 1.36

$$\frac{d^{3}\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{2}c^{4}-2\sqrt{3}\,abc^{3}d+\sqrt{3}\,a^{2}c^{2}d^{2}}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}d\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)}-\frac{4\left(bx^{3}+a\right)b^{2}c-3\,ab^{2}c-\left(bx^{3}+a\right)abd}{3\left(a^{2}bc^{2}-a^{3}cd\right)\left(\left(bx^{3}+a\right)^{\frac{4}{3}}-\left(bx^{3}+a\right)^{\frac{1}{3}}a\right)}-\frac{\sqrt{3}\left(4\,bc+3\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}c^{2}}-\frac{\left(4\,a^{\frac{1}{3}}bc+3\,a^{\frac{4}{3}}d\right)\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{8}{3}}c^{2}}+\frac{\left(4\,a^{\frac{2}{3}}bc+3\,a^{\frac{5}{3}}d\right)\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*d
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^
2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d
+ a^2*c^2*d^2) - 1/3*(4*(b*x^3 + a)*b^2*c - 3*a*b^2*c - (b*x^3 + a)*a*b*d)/
((a^2*b*c^2 - a^3*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^(1/3)*a)) - 1/9*sqr
t(3)*(4*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(7/3)*c^2) - 1/9*(4*a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)
```

```
^(1/3) - a^(1/3)))/(a^(8/3)*c^2) + 1/18*(4*a^(2/3)*b*c + 3*a^(5/3)*d)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^3*c^2)
```

**maple [F]** time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{4}{3}}\left(dx^{3}+c\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)
```

**mupad [B]** time = 6.39, size = 5875, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] log((d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^
8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(
1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^1
6*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^
7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^
21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^
12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^
26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (d^7/(27*b^4*c^10 + 27*a^4*
c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)
*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21
*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*
d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 258280
3260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^
6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(
d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2
 - 108*a*b^3*c^9*d))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10
*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 78
01029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*
d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^1
3*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^
17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 10
7892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18
))*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8
*d^2 - 108*a*b^3*c^9*d))^(1/3) + (b^2/(a^2*d - a*b*c) + (b*(a + b*x^3)*(a*d
 - 4*b*c))/(3*a^2*c*(a*d - b*c)))/(a*(a + b*x^3)^(1/3) - (a + b*x^3)^(4/3))
 + log((-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729
*a^7*c^6))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(1/3)*(8975448*a
```

^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((3^(1/2)*1i - 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i - 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))/4))/2 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16))/4 - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(3^(1/2)*1i - 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(((3^(1/2)*1i + 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i + 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^

4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782
969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*
c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 -
3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a
^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15
*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))/4))/2 +
419904*a^13*b^17*c^20*d^4 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10
*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 78
01029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*
d^16))/4 - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*
c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356
*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 +
107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d
^18))*(3^(1/2)*1i + 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d
^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3))/2 - log(((3^(1/2)*1i)/2
+ 1/2)^2*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(
729*a^7*c^6))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a + b*x^3)^(1/3)*(8975448*a^1
5*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 +
83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19
*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10
+ 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^2
4*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15
+ 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i)/2 + 1/2)^2*(-(27*a^3*d^3 + 64*
b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*
a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22
*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214
155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*
b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^1
2 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d
^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) +
419904*a^13*b^17*c^20*d^4 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10
*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 78
01029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*
d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^1
3*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^
17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 10
7892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18
))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 10
8*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + log(((3^(1/2)*1i)/2 - 1/2)^2*(-(27*a^
3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3
)*(419904*a^13*b^17*c^20*d^4 - ((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(8
975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*
c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 565
09893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10
*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38
736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*
c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i)/2 - 1/2)^2*(-(27*a^3
*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)
*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21
*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*
d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 258280
3260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^
6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(
-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6
))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781

```
933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^
15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404
123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^1
0*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3
)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^1
5*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 -
 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*
d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*((3^(1/2)*1i)/2
 - 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(72
9*a^7*c^6))^(1/3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.486 $\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=322

$$\frac{(4ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{7/3}d^2}-\frac{(4ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}d^2}+\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3b^2d(bc-ad)}+\frac{c^{7/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{4/3}}-$$

**Rubi** [C] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 0.21, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^{10}\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^10*(1 + (b*x^3)/a)^(1/3)*AppellF1[10/3, 4/3, 1, 13/3, -((b*x^3)/a), -((d
*x^3)/c)])/(10*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^9}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^{10}\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac\sqrt[3]{a+bx^3}}\end{aligned}$$

**Mathematica** [C] time = 1.08, size = 504, normalized size = 1.57

$$-\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,\sqrt[3]{bc-ad}\left(-4a^2d^2+abcd+3b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2c\left(-4a^2\sqrt[3]{c}\,d\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+24a^2dx\sqrt[3]{bc-ad}-6b^2cx^4\sqrt[3]{bc-ad}+abc^{4/3}\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6abdx^4\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\,\sqrt[3]{a+bx^3}(4ad-bc)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)-2\sqrt{3}\,a\sqrt[3]{c}\,\sqrt[3]{a+bx^3}(4ad-bc)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-6abcx\sqrt[3]{bc-ad}\right)}{36b^2cd\sqrt[3]{a+bx^3}(bc-ad)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/36*(3*(b*c - a*d)^(1/3)*(3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*x^4*(1 + (b*x^
3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-
6*a*b*c*(b*c - a*d)^(1/3)*x + 24*a^2*d*(b*c - a*d)^(1/3)*x - 6*b^2*c*(b*c -
 a*d)^(1/3)*x^4 + 6*a*b*d*(b*c - a*d)^(1/3)*x^4 - 2*Sqrt[3]*a*c^(1/3)*(-(b*
c) + 4*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*(-(b*c) + 4*a*d)*(a + b*x^3)^(1/
3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + a*b*c^(4/3)*(a
+ b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c
^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - 4*a^2*c^(1/3)*d*(a + b*x^3
)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*
(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b^2*c*d*(b*c - a*d)^(4/3)*(a + b
*x^3)^(1/3))
```

**IntegrateAlgebraic [C]** time = 9.27, size = 565, normalized size = 1.75

$$-\frac{4a^2dx-abcx+abdx^4-b^2cx^4}{3b^2d\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{(4ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{7/3}d^2}-\frac{(4ad+3bc)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\,b^{7/3}d^2}+\frac{(-4ad-3bc)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18b^{7/3}d^2}+\frac{\left(c^{7/3}+i\sqrt{3}\,c^{7/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6d^2(bc-ad)^{4/3}}-\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,c^{7/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{d^2(bc-ad)^{4/3}}-\frac{i\left(\sqrt{3}\,c^{7/3}-ic^{7/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d^2(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(-(a*b*c*x) + 4*a^2*d*x - b^2*c*x^4 + a*b*d*x^4)/(b^2*d*(b*c - a*d)*(a
 + b*x^3)^(1/3)) - ((3*b*c + 4*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +
 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(7/3)*d^2) - (Sqrt[(-1 + I*Sqrt[3])/6]
*c^(7/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(d^2*(b*
c - a*d)^(4/3)) + ((3*b*c + 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(
9*b^(7/3)*d^2) + ((c^(7/3) + I*Sqrt[3]*c^(7/3))*Log[2*(b*c - a*d)^(1/3)*x +
 (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(4/3)) + ((
-3*b*c - 4*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)
^(2/3)])/(18*b^(7/3)*d^2) - ((I/12)*((-I)*c^(7/3) + Sqrt[3]*c^(7/3))*Log[(-
2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a
 + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(d^2*(b*c - a*d
)^(4/3))
```

**fricas [B]** time = 1.36, size = 1329, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 +
a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a
)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 -
2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4*c^2
*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c*d - 4
*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*
c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^
(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/
(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c -
 a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c -
 a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^2*d^2)*x^4 + (
a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*d^3 +
 (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^3 + a*b^3*c^2)*(
```

```
c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c
/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c
^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4
*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (
b*x^3 + a)^(2/3))/x^2) + 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*
log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - 3*(b^
4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c -
 a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*c)/x^2) - 6*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^
2 + (3*b^4*c^2 + a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*(b^(1/3)*
x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 6*((b^3*c*d - a*b^2*d^2)*x^
4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*
d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.487 $\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=260

$$-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}-\frac{c^{4/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{4/3}}+\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}}-\frac{c^{4/3}\tan^{-1}\left(\frac{2x\sqrt[3]{b\ldots}}{\sqrt[3]{c}\sqrt[3]{\ldots}}\ldots\right)}{\sqrt{3}\,d(bc-a\ldots}$$

**Rubi [C]** time = 0.06, antiderivative size = 67, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(1/3)*AppellF1[7/3, 4/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}\end{aligned}$$

**Mathematica [C]** time = 0.45, size = 309, normalized size = 1.19

$$\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,(bc-ad)^{4/3}F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2ac\left(\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\,\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\,\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\,\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-6x\sqrt[3]{bc-ad}\right)}{12bc\sqrt[3]{a+bx^3}\,(bc-ad)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (3*(b*c - a*d)^(4/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] - 2*a*c*(-6*(b*c - a*d)^(1/3)*x + 2*Sqrt[3]*c^(1
/3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)] + c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((
b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b
+ a*x^3)^(1/3)]))/(12*b*c*(b*c - a*d)^(4/3)*(a + b*x^3)^(1/3))
```

**IntegrateAlgebraic [C]** time = 5.72, size = 500, normalized size = 1.92

$$-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{4/3}d}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,b^{4/3}d}+\frac{\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6b^{4/3}d}-\frac{i\left(\sqrt{3}\,c^{4/3}-ic^{4/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6d(bc-ad)^{4/3}}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,c^{4/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{d(bc-ad)^{4/3}}+\frac{\left(c^{4/3}+i\sqrt{3}\,c^{4/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12d(bc-ad)^{4/3}}+\frac{ax}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*x)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/
3)*x + 2*(a + b*x^3)^(1/3))]/(Sqrt[3]*b^(4/3)*d) + (Sqrt[(-1 + I*Sqrt[3])/6
]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]])/(d*(b*c
 - a*d)^(4/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b^(4/3)*d) - ((I/
6)*((-I)*c^(4/3) + Sqrt[3]*c^(4/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(d*(b*c - a*d)^(4/3)) + Log[b^(2/3)*x^2 +
b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(6*b^(4/3)*d) + ((c^(4/3)
+ I*Sqrt[3]*c^(4/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)
^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x
^3)^(2/3)])/(12*d*(b*c - a*d)^(4/3))
```

**fricas [B]** time = 0.48, size = 1127, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*c
 - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^
(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*
x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3*c*x
^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*c
- a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c - a
*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/
3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(b*
c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1
/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*
x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d
))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d -
 a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1/3)*(a*b^2*c -
 a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-
b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*sqrt(3)*(b^3*c
*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c -
 a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(
1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(
b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
```

```
)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a
*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d
 - a*b^3*d^2)*x^3)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.488 $\int \frac{x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)} \, dx$

**Optimal**. Leaf size=172

$$-\frac{x}{\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}-\frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}$$

**Rubi [C]** time = 0.07, antiderivative size = 92, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{4ac\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((c*((b*x^3)/a
 - (d*x^3)/c))/(c + d*x^3))])/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3
))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)} \, dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)} \, dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4ac\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}}\end{aligned}$$

**Mathematica [C]** time = 0.06, size = 86, normalized size = 0.50

$$\frac{x^4 \sqrt[3]{\frac{bx^3}{a}+1}\ {}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{4ac\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, ((-(b*c) + a*d)
*x^3)/(a*(c + d*x^3))])/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3))
```

**IntegrateAlgebraic [C]** time = 2.56, size = 355, normalized size = 2.06

$$-\frac{i\left(\sqrt{3}\sqrt[3]{c}-i\sqrt[3]{c}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12(bc-ad)^{4/3}}-\frac{x}{\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{\left(\sqrt[3]{c}+i\sqrt{3}\sqrt[3]{c}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6(bc-ad)^{4/3}}-\frac{\sqrt{-1+i\sqrt{3}}\sqrt[3]{c}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(x/((b*c - a*d)*(a + b*x^3)^(1/3))) - (Sqrt[-1 + I*Sqrt[3]]*c^(1/3)*ArcTan
[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a +
b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(Sqrt[6]*(b*c - a*d)^(4
/3)) + ((c^(1/3) + I*Sqrt[3]*c^(1/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sq
rt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*(b*c - a*d)^(4/3)) - ((I/12)*((-I)*c^
(1/3) + Sqrt[3]*c^(1/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c -
a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a +
 b*x^3)^(2/3)])/(b*c - a*d)^(4/3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.489 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

**Optimal**. Leaf size=179

$$-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

**Rubi [A]** time = 0.20, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(4/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3
*c^(2/3)*(b*c - a*d)^(4/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a +
 b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3
)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)} - \frac{d\,\mathrm{Subst}\left(\int \frac{\cdots}{c^{2/3}+\cdots}\right.}{\cdots}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}+2(bc-ad)\cdots}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-a\cdots}\right.}{6c^{2/3}(bc-ad)\cdots}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{b\cdots}}{\sqrt[3]{a+\cdots}}\right.}{6c^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(\cdots\right.}{\cdots}$$

**Mathematica [C]** time = 0.12, size = 256, normalized size = 1.43

$$-\frac{28c^3\left(a+bx^3\right)^2{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-28c^3\left(a+bx^3\right)^2+21c^2dx^3\left(a+bx^3\right)^2{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-21c^2dx^3\left(a+bx^3\right)^2+3dx^9(bc-ad)^2{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+3cx^6(bc-ad)^2{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{7c^3x^2\left(a+bx^3\right)^{7/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/7*(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^
3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21
*c^2*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2
, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(-(b*c) + a*d)*x^2*(a
 + b*x^3)^(7/3))
```

**IntegrateAlgebraic [C]** time = 0.00, size = 352, normalized size = 1.97

$$-\frac{i\left(\sqrt{3}\,d-id\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,d\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}}+\frac{\left(d+i\sqrt{3}\,d\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}(bc-ad)^{4/3}}-\frac{bx}{a\sqrt[3]{a+bx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -((b*x)/(a*(-(b*c) + a*d)*(a + b*x^3)^(1/3))) + (Sqrt[(-1 + I*Sqrt[3])/6]*d
*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3
)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c -
a*d)^(4/3)) - ((I/6)*((-I)*d + Sqrt[3]*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 +
I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d +
I*Sqrt[3]*d)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(
I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3
)])/(12*c^(2/3)*(b*c - a*d)^(4/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.490 $\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=229

$$-\frac{\left(a+bx^3\right)^{2/3}(3bc-ad)}{2a^2cx^2(bc-ad)}+\frac{d^2\log\left(c+dx^3\right)}{6c^{5/3}(bc-ad)^{4/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}+\frac{b}{ax^2\sqrt[3]{a+bx^3}}$$

**Rubi [C]** time = 1.29, antiderivative size = 542, normalized size of antiderivative = 2.37, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{-9c^2x^6(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-9d^2x^{12}(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-18cdx^9(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-126c^2d^2x^6\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+126c^2d^2x^6\left(a+bx^3\right)^2-15c^2x^6(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-168c^3dx^3\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-28c^4\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+168c^3dx^3\left(a+bx^3\right)^2+28c^4\left(a+bx^3\right)^2-27d^2x^{12}(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-42cdx^9(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{14c^4x^5\left(a+bx^3\right)^{7/3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (28*c^4*(a + b*x^3)^2 + 168*c^3*d*x^3*(a + b*x^3)^2 + 126*c^2*d^2*x^6*(a +
b*x^3)^2 - 28*c^4*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 168*c^3*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 126*c^2*d^2*x^6*(a + b*x^3)^2
*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*c^2
*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 42*c*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] - 27*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1
[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^2*x^6
*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] - 18*c*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^2*x^12*HypergeometricPF
Q[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(14*c^4*(b*c
- a*d)*x^5*(a + b*x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{28c^4\left(a+bx^3\right)^2+168c^3dx^3\left(a+bx^3\right)^2+126c^2d^2x^6\left(a+bx^3\right)^2-28c^4\left(a+bx^3\right.}{}$$

**Mathematica** [C] time = 0.83, size = 542, normalized size = 2.37

$$\frac{9c^2x^6(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)+9d^2x^{12}(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)+18cdx^9(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)+28c^4\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)-28c^4\left(a+bx^3\right)^2+168c^3dx^3\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)-168c^3dx^3\left(a+bx^3\right)^2+126c^2d^2x^6\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)-126c^2d^2x^6\left(a+bx^3\right)^2+15c^2x^6(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)+27d^2x^{12}(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)+42cdx^9(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{x^3(bc-ad)}{c(a+bx^3)}\right)}{14c^4x^5\left(a+bx^3\right)^{7/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (-28*c^4*(a + b*x^3)^2 - 168*c^3*d*x^3*(a + b*x^3)^2 - 126*c^2*d^2*x^6*(a +
 b*x^3)^2 + 28*c^4*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] + 168*c^3*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3
, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 126*c^2*d^2*x^6*(a + b*x^3)^
2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*c^
2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 42*c*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 27*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F
1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*c^2*(b*c - a*d)^2*x^
6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 18*c*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*d^2*(b*c - a*d)^2*x^12*HypergeometricP
FQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(14*c^4*(-(b
*c) + a*d)*x^5*(a + b*x^3)^(7/3))
```

**IntegrateAlgebraic** [C] time = 3.19, size = 397, normalized size = 1.73

$$\frac{-a^2d+abc-abdx^3+3b^2cx^3}{2a^2cx^2\sqrt[3]{a+bx^3}\,(ad-bc)}+\frac{\left(d^2+i\sqrt{3}d^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}-\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,d^2\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{5/3}(bc-ad)^{4/3}}-\frac{i\left(\sqrt{3}d^2-id^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*b*c - a^2*d + 3*b^2*c*x^3 - a*b*d*x^3)/(2*a^2*c*(-(b*c) + a*d)*x^2*(a +
b*x^3)^(1/3)) - (Sqrt[(-1 + I*Sqrt[3])/6]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x
)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*
c^(1/3)*(a + b*x^3)^(1/3))])/(c^(5/3)*(b*c - a*d)^(4/3)) + ((d^2 + I*Sqrt[3
]*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3
)])/(6*c^(5/3)*(b*c - a*d)^(4/3)) - ((I/12)*((-I)*d^2 + Sqrt[3]*d^2)*Log[(-
2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a
 + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(5/3)*(b*c -
 a*d)^(4/3))
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

**maple [F]** time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.491 $\int \frac{1}{x^6\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=287

$$-\frac{\left(a+bx^3\right)^{2/3}(6bc-ad)}{5a^2cx^5(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-3abcd+18b^2c^2\right)}{10a^3c^2x^2(bc-ad)}-\frac{d^3\log\left(c+dx^3\right)}{6c^{8/3}(bc-ad)^{4/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\right.}{2c^{8/3}(bc-ad)^{4/3}}$$

**Rubi [C]** time = 3.86, antiderivative size = 950, normalized size of antiderivative = 3.31, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

---

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^6*(a +
 b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*x^3*(a
 + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeometric2
F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*d)^2*x
^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 171
*c^2*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 486*c*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 1
0/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*d^3*(b*c - a*d)^2*x^15*Hyperg
eometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c
- a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 216*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3},
 {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 351*c*d^2*(b*c - a*d)^2*x^
12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] + 162*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3},
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*Hypergeometr
icPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81
*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c*d^2*(b*c - a*d)^2*x^12*Hypergeometr
icPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27
*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))])/(70*c^5*(b*c - a*d)*x^8*(a + b*x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^6 \left(1+\frac{bx^3}{a}\right)^{4/3} \left(c+dx^3\right)} \, dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{56c^5 \left(a+bx^3\right)^2 - 252c^4dx^3 \left(a+bx^3\right)^2 - 1512c^3d^2x^6 \left(a+bx^3\right)^2 - 1134c^2d^3x^9 (a}{}$$

**Mathematica [C]** time = 2.31, size = 950, normalized size = 3.31

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/70*(56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^
6*(a + b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hyp
ergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*
x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*
d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 171*c^2*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] + 486*c*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2,
7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*d^3*(b*c - a*d)^2*x^15*
Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3
*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 216*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2,
 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 351*c*d^2*(b*c - a*d
)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] + 162*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1,
10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*Hyperg
eometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 81*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c*d^2*(b*c - a*d)^2*x^12*Hyperg
eometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 27*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*(-(b*c) + a*d)*x^8*(a + b*x^3)^
(7/3))
```

**IntegrateAlgebraic [C]** time = 4.82, size = 452, normalized size = 1.57

$$\frac{-2a^3cd+5a^3d^2x^3+2a^2bc^2+a^2bcdx^3+5a^2bd^2x^6-6ab^2c^2x^3+3ab^2cdx^6-18b^3c^2x^6}{10a^3c^2x^5\sqrt[3]{a+bx^3}\,(ad-bc)} - \frac{i\left(\sqrt{3}d^3-id^3\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{8/3}(bc-ad)^{4/3}} + \frac{\sqrt{\frac{1}{6}}\left(-1+i\sqrt{3}\right)d^3\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{8/3}(bc-ad)^{4/3}} + \frac{\left(d^3+i\sqrt{3}d^3\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{8/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (2*a^2*b*c^2 - 2*a^3*c*d - 6*a*b^2*c^2*x^3 + a^2*b*c*d*x^3 + 5*a^3*d^2*x^3
 - 18*b^3*c^2*x^6 + 3*a*b^2*c*d*x^6 + 5*a^2*b*d^2*x^6)/(10*a^3*c^2*(-(b*c) +
```

```
a*d)*x^5*(a + b*x^3)^(1/3)) + (Sqrt[(-1 + I*Sqrt[3])/6]*d^3*ArcTan[(3*(b*c
 - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(
1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(8/3)*(b*c - a*d)^(4/3)) - (
(I/6)*((-I)*d^3 + Sqrt[3]*d^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*
c^(1/3)*(a + b*x^3)^(1/3)])/(c^(8/3)*(b*c - a*d)^(4/3)) + ((d^3 + I*Sqrt[3]
*d^3)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - S
qrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12
*c^(8/3)*(b*c - a*d)^(4/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

**maple [F]** time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.492 $\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

**Optimal**. Leaf size=351

$$\frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{20a^3c^2x^5(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8a^2cx^8(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}\left(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b\right.}{40a^4c^3x^2(bc-ad)}$$

**Rubi [C]** time = 8.30, antiderivative size = 1486, normalized size of antiderivative = 4.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (280*c^6*(a + b*x^3)^2 - 672*c^5*d*x^3*(a + b*x^3)^2 + 3024*c^4*d^2*x^6*(a
+ b*x^3)^2 + 18144*c^3*d^3*x^9*(a + b*x^3)^2 + 13608*c^2*d^4*x^12*(a + b*x^
3)^2 - 280*c^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] + 672*c^5*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3024*c^4*d^2*x^6*(a + b*x^3)^2*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18144*c^
3*d^3*x^9*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 13608*c^2*d^4*x^12*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 66*c^4*(b*c - a*d)^2*x^6*Hyperge
ometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 312*c^3*d*(b*
c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 2268*c^2*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 10/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6696*c*d^3*(b*c - a*d)^2*x^15*Hypergeome
tric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4050*d^4*(b*c -
a*d)^2*x^18*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 189*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] - 108*c^3*d*(b*c - a*d)^2*x^9*Hypergeomet
ricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3618*c^
2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] - 6156*c*d^3*(b*c - a*d)^2*x^15*HypergeometricPF
Q[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2835*d^4*(b*
c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 54*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 7/
3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 648*c^3*d*(b*c - a*d
)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 2268*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2,
2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2376*c*d^3*(b*c
 - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 810*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2
, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*c^4*(b*c -
 a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 324*c^3*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{
2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 486
*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1,
10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*c*d^3*(b*c - a*d)^2*x^15*Hy
pergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] - 81*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3}
, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560*c^6*(b*c - a*d)
*x^11*(a + b*x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^9\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{280c^6\left(a+bx^3\right)^2-672c^5dx^3\left(a+bx^3\right)^2+3024c^4d^2x^6\left(a+bx^3\right)^2+18144c^3d^3x^9 \cdots}{\cdots}$$

**Mathematica** [A] time = 5.86, size = 278, normalized size = 0.79

$$\frac{\left(a+bx^3\right)^{2/3}\left(-\frac{x^6\left(20a^2d^2+32abcd+41b^2c^2\right)}{c^3}-\frac{5a^2}{c}+\frac{40b^4x^9}{\left(a+bx^3\right)(ad-bc)}+\frac{2ax^3(4ad+7bc)}{c^2}\right)}{40a^4x^8}+\frac{d^4\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{6c^{11/3}(bc-ad)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*((-5*a^2)/c + (2*a*(7*b*c + 4*a*d)*x^3)/c^2 - ((41*b^2*c
^2 + 32*a*b*c*d + 20*a^2*d^2)*x^6)/c^3 + (40*b^4*x^9)/((-(b*c) + a*d)*(a +
b*x^3))))/(40*a^4*x^8) + (d^4*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x
)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3
)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)
^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(6*c^(11/3)*(b*
c - a*d)^(4/3))
```

**IntegrateAlgebraic** [C] time = 11.24, size = 528, normalized size = 1.50

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*a^3*b*c^3 - 5*a^4*c^2*d - 9*a^2*b^2*c^3*x^3 + a^3*b*c^2*d*x^3 + 8*a^4*c*
d^2*x^3 + 27*a*b^3*c^3*x^6 - 3*a^2*b^2*c^2*d*x^6 - 4*a^3*b*c*d^2*x^6 - 20*a
^4*d^3*x^6 + 81*b^4*c^3*x^9 - 9*a*b^3*c^2*d*x^9 - 12*a^2*b^2*c*d^2*x^9 - 20
*a^3*b*d^3*x^9)/(40*a^4*c^3*(-(b*c) + a*d)*x^8*(a + b*x^3)^(1/3)) - (Sqrt[(
-1 + I*Sqrt[3])/6]*d^4*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^
```

```
(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/
3))])/(c^(11/3)*(b*c - a*d)^(4/3)) + ((d^4 + I*Sqrt[3]*d^4)*Log[2*(b*c - a*
d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(11/3)*(b*c -
 a*d)^(4/3)) - ((I/12)*((-I)*d^4 + Sqrt[3]*d^4)*Log[(-2*I)*(b*c - a*d)^(2/3
)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I
+ Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(11/3)*(b*c - a*d)^(4/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

**maple [F]** time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

## 3.493 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx$

**Optimal**. Leaf size=90

$$-\frac{a^3 \log \left(a+bx^4\right)}{4b^3(bc-ad)}-\frac{x^4(ad+bc)}{4b^2d^2}+\frac{c^3 \log \left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

**Rubi** [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{a^3 \log \left(a+bx^4\right)}{4b^3(bc-ad)}-\frac{x^4(ad+bc)}{4b^2d^2}+\frac{c^3 \log \left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((b*c + a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^3*
(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{15}}{\left(a+bx^4\right)\left(c+dx^4\right)} dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^3}{(a+bx)(c+dx)} dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d^2}+\frac{x}{bd}-\frac{a^3}{b^2(bc-ad)(a+bx)}-\frac{c^3}{d^2(-bc+ad)(c+dx)}\right) dx, x\\
&= -\frac{(bc+ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3 \log \left(a+bx^4\right)}{4b^3(bc-ad)}+\frac{c^3 \log \left(c+dx^4\right)}{4d^3(bc-ad)}
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 92, normalized size = 1.02

$$-\frac{a^3 \log \left(a+bx^4\right)}{4b^3(bc-ad)}+\frac{x^4(-ad-bc)}{4b^2d^2}+\frac{c^3 \log \left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(b*c) - a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^
3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^15/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 6.24, size = 100, normalized size = 1.11

$$\frac{\left(b^3cd^2-ab^2d^3\right)x^8-2\,a^3d^3\log\left(bx^4+a\right)+2\,b^3c^3\log\left(dx^4+c\right)-2\left(b^3c^2d-a^2bd^3\right)x^4}{8\left(b^4cd^3-ab^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*log
(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)
```

**giac [A]** time = 0.17, size = 88, normalized size = 0.98

$$-\frac{a^3\log\left(\left|bx^4+a\right|\right)}{4\left(b^4c-ab^3d\right)}+\frac{c^3\log\left(\left|dx^4+c\right|\right)}{4\left(bcd^3-ad^4\right)}+\frac{bdx^8-2\,bcx^4-2\,adx^4}{8\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*a^3*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*c^3*log(abs(d*x^4 + c)
)/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*b*c*x^4 - 2*a*d*x^4)/(b^2*d^2)
```

**maple [A]** time = 0.06, size = 89, normalized size = 0.99

$$\frac{x^8}{8bd}-\frac{a\,x^4}{4b^2d}-\frac{c\,x^4}{4b\,d^2}+\frac{a^3\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,b^3}-\frac{c^3\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*x^8/b/d-1/4/b^2/d*x^4*a-1/4/b/d^2*x^4*c-1/4*c^3/d^3/(a*d-b*c)*ln(d*x^4+
c)+1/4*a^3/b^3/(a*d-b*c)*ln(b*x^4+a)
```

**maxima [A]** time = 0.47, size = 84, normalized size = 0.93

$$-\frac{a^3\log\left(bx^4+a\right)}{4\left(b^4c-ab^3d\right)}+\frac{c^3\log\left(dx^4+c\right)}{4\left(bcd^3-ad^4\right)}+\frac{bdx^8-2\,(bc+ad)x^4}{8\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^3
 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)
```

**mupad [B]** time = 5.88, size = 88, normalized size = 0.98

$$\frac{x^8}{8\,b\,d}-\frac{c^3\,\ln\left(d\,x^4+c\right)}{4\,\left(a\,d^4-b\,c\,d^3\right)}-\frac{a^3\,\ln\left(b\,x^4+a\right)}{4\,\left(b^4\,c-a\,b^3\,d\right)}-\frac{x^4\,\left(a\,d+b\,c\right)}{4\,b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] x^8/(8*b*d) - (c^3*log(c + d*x^4))/(4*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*x
^4))/(4*(b^4*c - a*b^3*d)) - (x^4*(a*d + b*c))/(4*b^2*d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.494 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=70

$$\frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)}-\frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)}-\frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*(c + d*x^4)),x]

[Out] x^4/(4*b*d) + (a^2*Log[a + b*x^4])/(4*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4
])/(4*d^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx&=\frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^4\right)\\&=\frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx,x,x^4\right)\\&=\frac{x^4}{4bd}+\frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)}-\frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log\left(a+bx^4\right)-b\left(dx^4(ad-bc)+bc^2\log\left(c+dx^4\right)\right)}{4b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]

[Out] (a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))/
(4*b^2*d^2*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^11/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 1.68, size = 72, normalized size = 1.03

$$\frac{\left(b^2cd-abd^2\right)x^4+a^2d^2\log\left(bx^4+a\right)-b^2c^2\log\left(dx^4+c\right)}{4\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4 +
 c))/(b^3*c*d^2 - a*b^2*d^3)
```

**giac [A]** time = 0.20, size = 70, normalized size = 1.00

$$\frac{x^4}{4\,bd}+\frac{a^2\log\left(\left|bx^4+a\right|\right)}{4\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(\left|dx^4+c\right|\right)}{4\left(bcd^2-ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d) - 1/4*c^2*log
(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)
```

**maple [A]** time = 0.05, size = 65, normalized size = 0.93

$$\frac{x^4}{4bd}-\frac{a^2\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,b^2}+\frac{c^2\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4*x^4/b/d+1/4*c^2/d^2/(a*d-b*c)*ln(d*x^4+c)-1/4*a^2/b^2/(a*d-b*c)*ln(b*x^
4+a)
```

**maxima [A]** time = 0.57, size = 68, normalized size = 0.97

$$\frac{x^4}{4\,bd}+\frac{a^2\log\left(bx^4+a\right)}{4\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(dx^4+c\right)}{4\left(bcd^2-ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x^
4 + c)/(b*c*d^2 - a*d^3)
```

**mupad [B]** time = 5.63, size = 68, normalized size = 0.97

$$\frac{a^2\,\ln\left(b\,x^4+a\right)}{4\,b^3\,c-4\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^4+c\right)}{4\,a\,d^3-4\,b\,c\,d^2}+\frac{x^4}{4\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (a^2*log(a + b*x^4))/(4*b^3*c - 4*a*b^2*d) + (c^2*log(c + d*x^4))/(4*a*d^3
- 4*b*c*d^2) + x^4/(4*b*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.495 $\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=53

$$\frac{c\log\left(c+dx^4\right)}{4d(bc-ad)}-\frac{a\log\left(a+bx^4\right)}{4b(bc-ad)}$$

**Rubi** [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c\log\left(c+dx^4\right)}{4d(bc-ad)}-\frac{a\log\left(a+bx^4\right)}{4b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(a*Log[a + b*x^4])/(4*b*(b*c - a*d)) + (c*Log[c + d*x^4])/(4*d*(b*c - a*d)
)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx,x,x^4\right)\\
&= -\frac{a\log\left(a+bx^4\right)}{4b(bc-ad)}+\frac{c\log\left(c+dx^4\right)}{4d(bc-ad)}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 43, normalized size = 0.81

$$-\frac{ad\log\left(a+bx^4\right)-bc\log\left(c+dx^4\right)}{4b^2cd-4abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^7/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 0.75, size = 42, normalized size = 0.79

$$-\frac{ad\log\left(bx^4+a\right)-bc\log\left(dx^4+c\right)}{4\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)
```

**giac [A]** time = 0.19, size = 51, normalized size = 0.96

$$-\frac{a\log\left(\left|bx^4+a\right|\right)}{4\left(b^2c-abd\right)}+\frac{c\log\left(\left|dx^4+c\right|\right)}{4\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*a*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*c*log(abs(d*x^4 + c))/(b*c
*d - a*d^2)
```

**maple [A]** time = 0.05, size = 50, normalized size = 0.94

$$\frac{a\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,b}-\frac{c\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/4*c/(a*d-b*c)/d*ln(d*x^4+c)+1/4*a/(a*d-b*c)/b*ln(b*x^4+a)
```

**maxima [A]** time = 0.62, size = 49, normalized size = 0.92

$$-\frac{a\log\left(bx^4+a\right)}{4\left(b^2c-abd\right)}+\frac{c\log\left(dx^4+c\right)}{4\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^2
)
```

**mupad [B]** time = 5.10, size = 51, normalized size = 0.96

$$-\frac{a\,\ln\left(b\,x^4+a\right)}{4\,b^2\,c-4\,a\,b\,d}-\frac{c\,\ln\left(d\,x^4+c\right)}{4\,a\,d^2-4\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - (a*log(a + b*x^4))/(4*b^2*c - 4*a*b*d) - (c*log(c + d*x^4))/(4*a*d^2 - 4*
b*c*d)
```

**sympy [B]** time = 60.85, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^4+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{4b\,(ad-bc)}-\frac{c\log\left(x^4+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{4d\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] a*log(x**4 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(4*b*(a*d - b*c)) - c*log(x**4 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(4*d*(a*d - b*c))
```

## 3.496 $\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=45

$$\frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}$$

**Rubi [A]** time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[a + b*x^4]/(4*(b*c - a*d)) - Log[c + d*x^4]/(4*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^4\right)\\ &= \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^4\right)}{4(bc-ad)}-\frac{d\operatorname{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^4\right)}{4(bc-ad)}\\ &= \frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^4)-\log(c+dx^4)}{4bc-4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

[Out] (Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/((a + b*x^4)*(c + d*x^4)),x]

[Out] IntegrateAlgebraic[x^3/((a + b*x^4)*(c + d*x^4)), x]

**fricas [A]** time = 0.42, size = 31, normalized size = 0.69

$$\frac{\log\left(bx^4+a\right)-\log\left(dx^4+c\right)}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] 1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)

**giac [A]** time = 0.34, size = 51, normalized size = 1.13

$$\frac{b\log\left(\left|bx^4+a\right|\right)}{4\left(b^2c-abd\right)}-\frac{d\log\left(\left|dx^4+c\right|\right)}{4\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] 1/4*b*log(abs(b*x^4 + a))/(b^2*c - a*b*d) - 1/4*d*log(abs(d*x^4 + c))/(b*c*d - a*d^2)

**maple [A]** time = 0.05, size = 42, normalized size = 0.93

$$-\frac{\ln\left(b\,x^4+a\right)}{4\,(ad-bc)}+\frac{\ln\left(d\,x^4+c\right)}{4ad-4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^4+a)/(d*x^4+c),x)

[Out] 1/4/(a*d-b*c)*ln(d*x^4+c)-1/4/(a*d-b*c)*ln(b*x^4+a)

**maxima [A]** time = 0.52, size = 41, normalized size = 0.91

$$\frac{\log\left(bx^4+a\right)}{4\,(bc-ad)}-\frac{\log\left(dx^4+c\right)}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")

[Out] 1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)

**mupad [B]** time = 4.99, size = 1012, normalized size = 22.49

$$-\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}+8\,b^4\,d^4\,x^4\right)\,1{}\mathrm{i}}{4\,a\,d-4\,b\,c}-\frac{\left(\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)-\frac{x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)-\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}-8\,b^4\,d^4\,x^4\right)\,1{}\mathrm{i}}{4\,a\,d-4\,b\,c}}{\frac{\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}+8\,b^4\,d^4\,x^4}{4\,a\,d-4\,b\,c}+\frac{\frac{x^4\,\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)-\frac{x^4\,\left(384\,a^2\,b^4\,d^6+768\,a\,b^5\,c\,d^5+384\,b^6\,c^2\,d^4\right)-\frac{x^4\,\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+512\,a\,b^5\,c^2\,d^4+512\,a^2\,b^4\,c\,d^5}{4\,a\,d-4\,b\,c}+64\,a\,b^4\,c\,d^4}{4\,a\,d-4\,b\,c}-8\,b^4\,d^4\,x^4}{4\,a\,d-4\,b\,c}}\right)\,2{}\mathrm{i}}{4\,a\,d-4\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] -(atan(((((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512
*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^
4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a
^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a
^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4
*x^4)*1i)/(4*a*d - 4*b*c) - (((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(38
4*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7
+ 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c
^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a
*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d
- 4*b*c) - 8*b^4*d^4*x^4)*1i)/(4*a*d - 4*b*c))/(((x^4*(96*a*b^4*d^5 + 96*b^
5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 +
1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^
5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a
*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a
*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c) + ((x^4*(96*a*
b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b
^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 +
1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^
5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d
- 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)/(4*a*d - 4*b*c)
))*2i)/(4*a*d - 4*b*c)
```

**sympy [B]** time = 1.91, size = 138, normalized size = 3.07

$$\frac{\log\left(x^4+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)}-\frac{\log\left(x^4+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(4*(a*d - b*c))
```

## 3.497 $\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=62

$$-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}+\frac{\log(x)}{ac}$$

**Rubi [A]** time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^4])/(4*a*(b*c - a*d)) + (d*Log[c + d*x^4])/(4
*c*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx, x, x^4\right)\\ &= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^4\right)+ad\log\left(c+dx^4\right)-4ad\log(x)+4bc\log(x)}{4abc^2-4a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4*
a*b*c^2 - 4*a^2*c*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 2.00, size = 54, normalized size = 0.87

$$-\frac{bc\log\left(bx^4+a\right)-ad\log\left(dx^4+c\right)-4\,(bc-ad)\log(x)}{4\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

**giac [A]** time = 0.18, size = 73, normalized size = 1.18

$$-\frac{b^2\log\left(\left|bx^4+a\right|\right)}{4\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^4+c\right|\right)}{4\left(bc^2d-acd^2\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*b^2*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) + 1/4*d^2*log(abs(d*x^4 +
c))/(b*c^2*d - a*c*d^2) + 1/4*log(x^4)/(a*c)
```

**maple [A]** time = 0.06, size = 59, normalized size = 0.95

$$\frac{b\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,a}-\frac{d\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,c}+\frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/4*d/c/(a*d-b*c)*ln(d*x^4+c)+1/4*b/a/(a*d-b*c)*ln(b*x^4+a)+1/a/c*ln(x)
```

**maxima [A]** time = 0.71, size = 61, normalized size = 0.98

$$-\frac{b\log\left(bx^4+a\right)}{4\left(abc-a^2d\right)}+\frac{d\log\left(dx^4+c\right)}{4\left(bc^2-acd\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*d
) + 1/4*log(x^4)/(a*c)
```

**mupad [B]** time = 5.49, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^4+a\right)}{4\,a^2\,d-4\,a\,b\,c}+\frac{d\,\ln\left(d\,x^4+c\right)}{4\,b\,c^2-4\,a\,c\,d}+\frac{\ln(x)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (b*log(a + b*x^4))/(4*a^2*d - 4*a*b*c) + (d*log(c + d*x^4))/(4*b*c^2 - 4*a*
c*d) + log(x)/(a*c)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.498 $\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=87

$$\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

**Rubi [A]** time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(4*a*c*x^4) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^4])/(4*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx,\right.\\ &= -\frac{1}{4acx^4}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^4\right)}{4a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4])
/(4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^4)*(c + d*x^4)), x]
```

**fricas** [A] time = 8.27, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^4\log\left(bx^4+a\right)-a^2d^2x^4\log\left(dx^4+c\right)-4\left(b^2c^2-a^2d^2\right)x^4\log(x)-abc^2+a^2cd}{4\left(a^2bc^3-a^3c^2d\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2 -
 a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

**giac** [A] time = 0.19, size = 112, normalized size = 1.29

$$\frac{b^3\log\left(\left|bx^4+a\right|\right)}{4\left(a^2b^2c-a^3bd\right)}-\frac{d^3\log\left(\left|dx^4+c\right|\right)}{4\left(bc^3d-ac^2d^2\right)}-\frac{(bc+ad)\log\left(x^4\right)}{4\,a^2c^2}+\frac{bcx^4+adx^4-ac}{4\,a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*b^3*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) - 1/4*d^3*log(abs(d*x^4 +
 c))/(b*c^3*d - a*c^2*d^2) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) + 1/4*(b*c*
x^4 + a*d*x^4 - a*c)/(a^2*c^2*x^4)
```

**maple** [A] time = 0.06, size = 87, normalized size = 1.00

$$-\frac{b^2\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,a^2}+\frac{d^2\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,c^2}-\frac{d\ln(x)}{a\,c^2}-\frac{b\ln(x)}{a^2c}-\frac{1}{4ac\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4*d^2/c^2/(a*d-b*c)*ln(d*x^4+c)-1/4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)-1/4/a/c
/x^4-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b
```

**maxima** [A] time = 0.55, size = 87, normalized size = 1.00

$$\frac{b^2\log\left(bx^4+a\right)}{4\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^4+c\right)}{4\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^4\right)}{4\,a^2c^2}-\frac{1}{4\,acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 -
a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)
```

**mupad [B]** time = 6.21, size = 87, normalized size = 1.00

$$-\frac{b^2\ln\left(b\,x^4+a\right)}{4\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\ln\left(d\,x^4+c\right)}{4\,\left(b\,c^3-a\,c^2\,d\right)}-\frac{1}{4\,a\,c\,x^4}-\frac{\ln(x)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - (b^2*log(a + b*x^4))/(4*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^4))/(4*(b*c
^3 - a*c^2*d)) - 1/(4*a*c*x^4) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.499 $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=112

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}+\frac{x^6}{6bd}$$

**Rubi** [A] time = 0.27, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 479, 582, 522, 205}

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}+\frac{x^6}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((b*c + a*d)*x^2)/(2*b^2*d^2) + x^6/(6*b*d) - (a^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*b^(5/2)*(b*c - a*d)) + (c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*d^(5/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
```

```
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^6}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right) \\
&= \frac{x^6}{6bd} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3ac+3(bc+ad)x^2)}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6bd} \\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} + \frac{\operatorname{Subst}\left(\int \frac{3ac(bc+ad)+3(b^2c^2+ad(bc+ad))x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6b^2d^2} \\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b^2(bc-ad)} + \frac{c^3\operatorname{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d^2(bc-ad)} \\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} + \frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 104, normalized size = 0.93

$$\frac{1}{6}\left(\frac{3a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{5/2}(ad-bc)} + \frac{x^2\left(-3ad-3bc+bdx^4\right)}{b^2d^2} + \frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x^
2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqr
t[c]])/(d^(5/2)*(b*c - a*d)))/6
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^13/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 2.64, size = 576, normalized size = 5.14

$$\left[\frac{2\left(b^2cd-abd^2\right)x^6 - 3a^2d^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right) - 3b^2c^2\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right) - 6\left(b^2c^2-a^2d^2\right)x^2}{12\left(b^3cd^2-ab^2d^3\right)}, \frac{2\left(b^2cd-abd^2\right)x^6 - 6a^2d^2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right) - 3b^2c^2\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right) - 6\left(b^2c^2-a^2d^2\right)x^2}{12\left(b^3cd^2-ab^2d^3\right)}, \frac{2\left(b^2cd-abd^2\right)x^6 + 6b^2c^2\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right) - 3a^2d^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right) - 6\left(b^2c^2-a^2d^2\right)x^2}{12\left(b^3cd^2-ab^2d^3\right)}, \frac{\left(b^2cd-abd^2\right)x^6 - 3a^2d^2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right) + 3b^2c^2\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right) - 3\left(b^2c^2-a^2d^2\right)x^2}{6\left(b^3cd^2-ab^2d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^
2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*
sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^
```

```
2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*x^2*
sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d
*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(b
^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*a
^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 6*(b^
2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b*d^2)*x^6
 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt(c/d)*arct
an(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3)]
```

**giac** [A]  time = 0.19, size = 112, normalized size = 1.00

$$-\frac{a^{3} \arctan\left(\frac{bx^{2}}{\sqrt{ab}}\right)}{2 \left(b^{3} c - ab^{2} d\right) \sqrt{ab}} + \frac{c^{3} \arctan\left(\frac{dx^{2}}{\sqrt{cd}}\right)}{2 \left(bcd^{2} - ad^{3}\right) \sqrt{cd}} + \frac{b^{2} d^{2} x^{6} - 3 \, b^{2} c d x^{2} - 3 \, a b d^{2} x^{2}}{6 \, b^{3} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b^2*d^2*x^6 - 3*
b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)
```

**maple** [A]  time = 0.06, size = 105, normalized size = 0.94

$$\frac{x^{6}}{6bd} + \frac{a^{3} \arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2 \left(ad - bc\right) \sqrt{ab}\; b^{2}} - \frac{c^{3} \arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2 \left(ad - bc\right) \sqrt{cd}\; d^{2}} - \frac{a\,x^{2}}{2b^{2}d} - \frac{c\,x^{2}}{2b\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/6*x^6/b/d-1/2/b^2/d*x^2*a-1/2/b/d^2*x^2*c-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/
2)*arctan(d*x^2/(c*d)^(1/2))+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(x^2*b
/(a*b)^(1/2))
```

**maxima** [A]  time = 1.32, size = 100, normalized size = 0.89

$$-\frac{a^{3} \arctan\left(\frac{bx^{2}}{\sqrt{ab}}\right)}{2 \left(b^{3} c - ab^{2} d\right) \sqrt{ab}} + \frac{c^{3} \arctan\left(\frac{dx^{2}}{\sqrt{cd}}\right)}{2 \left(bcd^{2} - ad^{3}\right) \sqrt{cd}} + \frac{bdx^{6} - 3 \, (bc + ad)x^{2}}{6 \, b^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b*d*x^6 - 3*(b*c
 + a*d)*x^2)/(b^2*d^2)
```

**mupad** [B]  time = 5.70, size = 532, normalized size = 4.75

$$\frac{\ln\left(d^{10}\left(-a^5 b^5\right)^{5/2} + b^{20} c^{10} \sqrt{-a^5 b^5} - a^2 b^{23} c^{10} x^2 - a^{12} b^{13} d^{10} x^2 + 2\,b^{10} c^5 d^5 \left(-a^5 b^5\right)^{3/2} + 2\,a^7 b^{18} c^5 d^5 x^2\right) \sqrt{-a^5 b^5}}{4\,b^6 c - 4\,a\,b^5 d} - \frac{\ln\left(d^{10}\left(-a^5 b^5\right)^{5/2} + b^{20} c^{10} \sqrt{-a^5 b^5} + a^2 b^{23} c^{10} x^2 + a^{12} b^{13} d^{10} x^2 + 2\,b^{10} c^5 d^5 \left(-a^5 b^5\right)^{3/2} - 2\,a^7 b^{18} c^5 d^5 x^2\right) \sqrt{-a^5 b^5}}{4\left(b^6 c - a\,b^5 d\right)} - \frac{\ln\left(b^{10}\left(-c^5 d^5\right)^{5/2} + a^{10} d^{20} \sqrt{-c^5 d^5} + a^{10} c^2 d^{23} x^2 + b^{10} c^{12} d^{13} x^2 + 2\,a^5 b^5 d^{10} \left(-c^5 d^5\right)^{3/2} - 2\,a^5 b^5 c^7 d^{18} x^2\right) \sqrt{-c^5 d^5}}{4\left(a\,d^6 - b\,c\,d^5\right)} + \frac{\ln\left(b^{10}\left(-c^5 d^5\right)^{5/2} + a^{10} d^{20} \sqrt{-c^5 d^5} - a^{10} c^2 d^{23} x^2 - b^{10} c^{12} d^{13} x^2 + 2\,a^5 b^5 d^{10} \left(-c^5 d^5\right)^{3/2} + 2\,a^5 b^5 c^7 d^{18} x^2\right) \sqrt{-c^5 d^5}}{4\,a\,d^6 - 4\,b\,c\,d^5} + \frac{x^6}{6\,b\,d} - \frac{x^2\,\left(a\,d + b\,c\right)}{2\,b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^10*(-a^5*b^5)^(5/2) + b^20*c^10*(-a^5*b^5)^(1/2) - a^2*b^23*c^10*x^2
 - a^12*b^13*d^10*x^2 + 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^7*b^18*c^5*d^
5*x^2)*(-a^5*b^5)^(1/2))/(4*b^6*c - 4*a*b^5*d) - (log(d^10*(-a^5*b^5)^(5/2)
```

```
 + b^20*c^10*(-a^5*b^5)^(1/2) + a^2*b^23*c^10*x^2 + a^12*b^13*d^10*x^2 + 2*
b^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^7*b^18*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(b^6*c - a*b^5*d)) - (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1
/2) + a^10*c^2*d^23*x^2 + b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3
/2) - 2*a^5*b^5*c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*(a*d^6 - b*c*d^5)) + (lo
g(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1/2) - a^10*c^2*d^23*x^2 -
b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^7*d^18*x
^2)*(-c^5*d^5)^(1/2))/(4*a*d^6 - 4*b*c*d^5) + x^6/(6*b*d) - (x^2*(a*d + b*c
))/(2*b^2*d^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.500 $\int \frac{x^9}{(a+bx^4)(c+dx^4)} dx$

**Optimal**. Leaf size=92

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

**Rubi [A]** time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 479, 522, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^2/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d
)) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{x^2}{2bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)}{2bd}$$

$$= \frac{x^2}{2bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b(bc-ad)} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d(bc-ad)}$$

$$= \frac{x^2}{2bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}$$

**Mathematica [A]** time = 0.14, size = 82, normalized size = 0.89

$$\frac{\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{b^{3/2}} + x^2\left(\frac{c}{d}-\frac{a}{b}\right) - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{d^{3/2}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^9/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 0.79, size = 416, normalized size = 4.52

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right) + bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right) - 2\,(bc-ad)x^2}{4\left(b^2cd-abd^2\right)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right) - bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right) + 2\,(bc-ad)x^2}{4\left(b^2cd-abd^2\right)}, -\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right) + ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right) - 2\,(bc-ad)x^2}{4\left(b^2cd-abd^2\right)}, \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right) - bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right) + (bc-ad)x^2}{2\left(b^2cd-abd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + b
*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c -
a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/
a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) + 2*(
b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^2*sqr
t(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)
) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arctan(b*x^2
*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*d)*x^2)/
(b^2*c*d - a*b*d^2)]
```

**giac [A]** time = 0.18, size = 80, normalized size = 0.87

$$\frac{a^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^2c-abd\right)\sqrt{ab}} - \frac{c^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd-ad^2\right)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

**maple [A]** time = 0.06, size = 81, normalized size = 0.88

$$-\frac{a^2 \arctan\left(\frac{b x^2}{\sqrt{ab}}\right)}{2\left(ad-bc\right)\sqrt{ab}\, b}+\frac{c^2 \arctan\left(\frac{d x^2}{\sqrt{cd}}\right)}{2\left(ad-bc\right)\sqrt{cd}\, d}+\frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2/b/d*x^2+1/2*c^2/d/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2
*a^2/b/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

**maxima [A]** time = 1.19, size = 80, normalized size = 0.87

$$\frac{a^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^2c-abd\right)\sqrt{ab}}-\frac{c^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd-ad^2\right)\sqrt{cd}}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

**mupad [B]** time = 5.72, size = 518, normalized size = 5.63

$$\frac{\ln\left(b^9c^6\sqrt{-a^3b^3}-a^3d^6\left(-a^3b^3\right)^{3/2}+ab^{11}c^6x^2+a^7b^5d^6x^2+2b^3c^3d^3\left(-a^3b^3\right)^{3/2}-2a^4b^8c^3d^3x^2\right)\sqrt{-a^3b^3}}{4b^4c-4ab^3d}-\frac{\ln\left(a^3d^6\left(-a^3b^3\right)^{3/2}-b^9c^6\sqrt{-a^3b^3}+ab^{11}c^6x^2+a^7b^5d^6x^2-2b^3c^3d^3\left(-a^3b^3\right)^{3/2}-2a^4b^8c^3d^3x^2\right)\sqrt{-a^3b^3}}{4\left(b^4c-ab^3d\right)}-\frac{\ln\left(b^6c^3\left(-c^3d^3\right)^{3/2}-a^6d^9\sqrt{-c^3d^3}+a^6cd^{11}x^2+b^6c^7d^5x^2-2a^3b^3d^3\left(-c^3d^3\right)^{3/2}-2a^3b^3c^4d^8x^2\right)\sqrt{-c^3d^3}}{4\left(ad^4-bcd^3\right)}+\frac{\ln\left(a^6d^9\sqrt{-c^3d^3}-b^6c^3\left(-c^3d^3\right)^{3/2}+a^6cd^{11}x^2+b^6c^7d^5x^2+2a^3b^3d^3\left(-c^3d^3\right)^{3/2}-2a^3b^3c^4d^8x^2\right)\sqrt{-c^3d^3}}{4ad^4-4bcd^3}+\frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(b^9*c^6*(-a^3*b^3)^(1/2) - a^3*d^6*(-a^3*b^3)^(3/2) + a*b^11*c^6*x^2 +
 a^7*b^5*d^6*x^2 + 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*
(-a^3*b^3)^(1/2))/(4*b^4*c - 4*a*b^3*d) - (log(a^3*d^6*(-a^3*b^3)^(3/2) - b
^9*c^6*(-a^3*b^3)^(1/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 - 2*b^3*c^3*d^3*
(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*(b^4*c - a*b
^3*d)) - (log(b^6*c^3*(-c^3*d^3)^(3/2) - a^6*d^9*(-c^3*d^3)^(1/2) + a^6*c*d
^11*x^2 + b^6*c^7*d^5*x^2 - 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*
d^8*x^2)*(-c^3*d^3)^(1/2))/(4*(a*d^4 - b*c*d^3)) + (log(a^6*d^9*(-c^3*d^3)^
(1/2) - b^6*c^3*(-c^3*d^3)^(3/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 + 2*a^3
*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*a*d
^4 - 4*b*c*d^3) + x^2/(2*b*d)
```

**sympy [B]** time = 29.70, size = 932, normalized size = 10.13

$$-\frac{\sqrt{-\frac{a^3}{b^3}}\log\left(x^2+\frac{-\frac{a^4d^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}-\frac{a^3b^3d^6\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^4cd^5\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ab^5c^2d^4\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^6c^3d^3\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^4c^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{4\left(ad-bc\right)}+\frac{\sqrt{-\frac{a^3}{b^3}}\log\left(x^2+\frac{\frac{a^4d^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}+\frac{a^3b^3d^6\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^4cd^5\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ab^5c^2d^4\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^6c^3d^3\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^4c^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{4\left(ad-bc\right)}-\frac{\sqrt{-\frac{c^3}{d^3}}\log\left(x^2+\frac{-\frac{a^4d^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}-\frac{a^3b^3d^6\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^4cd^5\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ab^5c^2d^4\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^6c^3d^3\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^4c^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{4\left(ad-bc\right)}+\frac{\sqrt{-\frac{c^3}{d^3}}\log\left(x^2+\frac{\frac{a^4d^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}+\frac{a^3b^3d^6\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^4cd^5\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ab^5c^2d^4\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^6c^3d^3\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^4c^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{4\left(ad-bc\right)}+\frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -sqrt(-a**3/b**3)*log(x**2 + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**
3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b*
```

```
*3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c
)**3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-
a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(4*(a*
d - b*c)) + sqrt(-a**3/b**3)*log(x**2 + (a**4*d**4*sqrt(-a**3/b**3)/(a*d -
b*c) + a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5
*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/
(a*d - b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**4*c
**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**
3))/(4*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x**2 + (-a**4*d**4*sqrt(-c**3/d*
*3)/(a*d - b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2*
b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d*
*3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)*
*3 - b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d +
 a*b**2*c**3))/(4*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x**2 + (a**4*d**4*sqr
t(-c**3/d**3)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)*
*3 - a**2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4
*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a
*d - b*c)**3 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*
b*c**2*d + a*b**2*c**3))/(4*(a*d - b*c)) + x**2/(2*b*d)
```

## 3.501 $\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=79

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)}$$

**Rubi** [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 481, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c - a*d)) + (Sqrt[c]
*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)\\ &= -\frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2(bc-ad)}+\frac{c\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2(bc-ad)}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt
[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^5/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 0.49, size = 325, normalized size = 4.11

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4(bc-ad)},\ -\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4(bc-ad)},\ \frac{2\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)}{4(bc-ad)},\ -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)}{2(bc-ad)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + sqrt(
-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), -1/4*
(2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sq
rt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(2*sqrt(c/d)*arctan(d*x^2*sqrt
(c/d)/c) - sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)))/(b
*c - a*d), -1/2*(sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x
^2*sqrt(c/d)/c))/(b*c - a*d)]
```

**giac [A]** time = 0.17, size = 59, normalized size = 0.75

$$-\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)}+\frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

**maple [A]** time = 0.06, size = 60, normalized size = 0.76

$$\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}}-\frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/2*c/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)+1/2*a/(a*d-b*c)/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

**maxima [A]** time = 1.16, size = 59, normalized size = 0.75

$$-\frac{a \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)} + \frac{c \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

**mupad [B]** time = 5.34, size = 379, normalized size = 4.80

$$\frac{\ln\left(d^2\,(-a\,b)^{5/2}+b^4\,c^2\,\sqrt{-a\,b}-b^5\,c^2\,x^2+2\,b^2\,c\,d\,(-a\,b)^{3/2}-a^2\,b^3\,d^2\,x^2+2\,a\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,b^2\,c-4\,a\,b\,d}-\frac{\ln\left(d^2\,(-a\,b)^{5/2}+b^4\,c^2\,\sqrt{-a\,b}+b^5\,c^2\,x^2+2\,b^2\,c\,d\,(-a\,b)^{3/2}+a^2\,b^3\,d^2\,x^2-2\,a\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,\left(b^2\,c-a\,b\,d\right)}-\frac{\ln\left(b^2\,(-c\,d)^{5/2}+a^2\,d^4\,\sqrt{-c\,d}+a^2\,d^5\,x^2+2\,a\,b\,d^2\,(-c\,d)^{3/2}+b^2\,c^2\,d^3\,x^2-2\,a\,b\,c\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,\left(a\,d^2-b\,c\,d\right)}+\frac{\ln\left(b^2\,(-c\,d)^{5/2}+a^2\,d^4\,\sqrt{-c\,d}-a^2\,d^5\,x^2+2\,a\,b\,d^2\,(-c\,d)^{3/2}-b^2\,c^2\,d^3\,x^2+2\,a\,b\,c\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,a\,d^2-4\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) - b^5*c^2*x^2 + 2*b^2*c*d*(-a*
b)^(3/2) - a^2*b^3*d^2*x^2 + 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*b^2*c - 4*a*
b*d) - (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) + b^5*c^2*x^2 + 2*b^2*c
*d*(-a*b)^(3/2) + a^2*b^3*d^2*x^2 - 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*(b^2*
c - a*b*d)) - (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) + a^2*d^5*x^2 +
2*a*b*d^2*(-c*d)^(3/2) + b^2*c^2*d^3*x^2 - 2*a*b*c*d^4*x^2)*(-c*d)^(1/2))/(
4*(a*d^2 - b*c*d)) + (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) - a^2*d^5
*x^2 + 2*a*b*d^2*(-c*d)^(3/2) - b^2*c^2*d^3*x^2 + 2*a*b*c*d^4*x^2)*(-c*d)^(
1/2))/(4*a*d^2 - 4*b*c*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.502 $\int \frac{x}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

**Optimal**. Leaf size=79

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}$$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {465, 391, 205}

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)\\ &= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2(bc-ad)}-\frac{d\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2(bc-ad)}\\ &= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{\sqrt{c}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]
*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 0.50, size = 325, normalized size = 4.11

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\ \frac{2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)}{4\,(bc-ad)},\ -\frac{2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\ -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)}{2\,(bc-ad)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + sqrt(
-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(
2*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - sqrt(-b/a)*log((b*x^4 - 2*a*x^2*s
qrt(-b/a) - a)/(b*x^4 + a)))/(b*c - a*d), -1/4*(2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*x^2)) + sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c))
)/(b*c - a*d), -1/2*(sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - sqrt(d/c)*arct
an(c*sqrt(d/c)/(d*x^2)))/(b*c - a*d)]
```

**giac [A]** time = 0.18, size = 59, normalized size = 0.75

$$\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

**maple [A]** time = 0.06, size = 60, normalized size = 0.76

$$-\frac{b\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2\,(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2\,(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2*d/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2*b/(a*d-b*c)/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

**maxima** [A] time = 1.28, size = 59, normalized size = 0.75

$$\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

**mupad** [B] time = 5.29, size = 399, normalized size = 5.05

$$\frac{\ln\left(a^2\,d^2\,{\left(-a\,b\right)}^{5/2}+b^2\,c^2\,{\left(-a\,b\right)}^{5/2}+2\,c\,d\,{\left(-a\,b\right)}^{7/2}-a^2\,b^5\,c^2\,x^2-a^4\,b^3\,d^2\,x^2+2\,a^3\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,a^2\,d-4\,a\,b\,c}-\frac{\ln\left(a^2\,d^2\,{\left(-a\,b\right)}^{5/2}+b^2\,c^2\,{\left(-a\,b\right)}^{5/2}+2\,c\,d\,{\left(-a\,b\right)}^{7/2}+a^2\,b^5\,c^2\,x^2+a^4\,b^3\,d^2\,x^2-2\,a^3\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,\left(a^2\,d-a\,b\,c\right)}-\frac{\ln\left(a^2\,d^2\,{\left(-c\,d\right)}^{5/2}+b^2\,c^2\,{\left(-c\,d\right)}^{5/2}+2\,a\,b\,{\left(-c\,d\right)}^{7/2}+a^2\,c^2\,d^5\,x^2+b^2\,c^4\,d^3\,x^2-2\,a\,b\,c^3\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,\left(b\,c^2-a\,c\,d\right)}+\frac{\ln\left(a^2\,d^2\,{\left(-c\,d\right)}^{5/2}+b^2\,c^2\,{\left(-c\,d\right)}^{5/2}+2\,a\,b\,{\left(-c\,d\right)}^{7/2}-a^2\,c^2\,d^5\,x^2-b^2\,c^4\,d^3\,x^2+2\,a\,b\,c^3\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,b\,c^2-4\,a\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b)^(7/2) - a^2
*b^5*c^2*x^2 - a^4*b^3*d^2*x^2 + 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*a^2*d
- 4*a*b*c) - (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b
)^(7/2) + a^2*b^5*c^2*x^2 + a^4*b^3*d^2*x^2 - 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/
2))/(4*(a^2*d - a*b*c)) - (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*(-c*d)^(5/2)
+ 2*a*b*(-c*d)^(7/2) + a^2*c^2*d^5*x^2 + b^2*c^4*d^3*x^2 - 2*a*b*c^3*d^4*x^
2)*(-c*d)^(1/2))/(4*(b*c^2 - a*c*d)) + (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*
(-c*d)^(5/2) + 2*a*b*(-c*d)^(7/2) - a^2*c^2*d^5*x^2 - b^2*c^4*d^3*x^2 + 2*a
*b*c^3*d^4*x^2)*(-c*d)^(1/2))/(4*b*c^2 - 4*a*c*d)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.503 $\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=92

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}-\frac{1}{2acx^2}$$

**Rubi** [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 480, 522, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(2*a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(3/2)*(b*c -
a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right) \\
&= -\frac{1}{2acx^2}+\frac{\text{Subst}\left(\int \frac{-bc-ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)}{2ac} \\
&= -\frac{1}{2acx^2}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2a(bc-ad)}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2c(bc-ad)} \\
&= -\frac{1}{2acx^2}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 169, normalized size = 1.84

$$\frac{-\frac{b^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{3/2}}-\frac{b^{3/2}x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{a^{3/2}}+\frac{b}{a}+\frac{d^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{3/2}}+\frac{d^{3/2}x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{c^{3/2}}-\frac{d}{c}}{2x^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2)
- (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2)*
x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 0.88, size = 432, normalized size = 4.70

$$\left[-\frac{bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)+2bc-2ad}{4\left(abc^2-a^2cd\right)x^2}, -\frac{2adx^2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)+bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+2bc-2ad}{4\left(abc^2-a^2cd\right)x^2}, \frac{2bcx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-2bc+2ad}{4\left(abc^2-a^2cd\right)x^2}, \frac{bcx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-adx^2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-bc+ad}{2\left(abc^2-a^2cd\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a))
 + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) + 2
*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*a*d*x^2*sqrt(d/c)*arctan(c
*sqrt(d/c)/(d*x^2)) + b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) -
a)/(b*x^4 + a)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*b*c*x^2*
sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c
*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2
), 1/2*(b*c*x^2*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(d/c)*a
rctan(c*sqrt(d/c)/(d*x^2)) - b*c + a*d)/((a*b*c^2 - a^2*c*d)*x^2)]
```

**giac** [A] time = 0.21, size = 80, normalized size = 0.87

$$-\frac{b^{2} \arctan\left(\frac{b x^{2}}{\sqrt{a b}}\right)}{2 \left(a b c - a^{2} d\right) \sqrt{a b}} + \frac{d^{2} \arctan\left(\frac{d x^{2}}{\sqrt{c d}}\right)}{2 \left(b c^{2} - a c d\right) \sqrt{c d}} - \frac{1}{2 \, a c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

**maple** [A] time = 0.06, size = 81, normalized size = 0.88

$$\frac{b^{2} \arctan\left(\frac{b x^{2}}{\sqrt{a b}}\right)}{2 \left(a d - b c\right) \sqrt{a b}\, a} - \frac{d^{2} \arctan\left(\frac{d x^{2}}{\sqrt{c d}}\right)}{2 \left(a d - b c\right) \sqrt{c d}\, c} - \frac{1}{2 a c \, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/2*d^2/c/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2/a/c/x^2+1/
2*b^2/a/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

**maxima** [A] time = 1.20, size = 80, normalized size = 0.87

$$-\frac{b^{2} \arctan\left(\frac{b x^{2}}{\sqrt{a b}}\right)}{2 \left(a b c - a^{2} d\right) \sqrt{a b}} + \frac{d^{2} \arctan\left(\frac{d x^{2}}{\sqrt{c d}}\right)}{2 \left(b c^{2} - a c d\right) \sqrt{c d}} - \frac{1}{2 \, a c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

**mupad** [B] time = 5.35, size = 354, normalized size = 3.85

$$\frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}-a^8\,b\,d^3+a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,a^4\,d-4\,a^3\,b\,c}-\frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}+a^8\,b\,d^3-a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,\left(a^4\,d-a^3\,b\,c\right)}-\frac{1}{2\,a\,c\,x^2}-\frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^8\,d-a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,\left(b\,c^4-a\,c^3\,d\right)}+\frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}-b^3\,c^8\,d+a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,b\,c^4-4\,a\,c^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^3*x^2*(-a^3*b^3)^(3/2) - a^8*b*d^3 + a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3
*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(4*a^4*d - 4*a^3*b*c) - (log(c^3*x^2*(-a^3*b
^3)^(3/2) + a^8*b*d^3 - a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^(1/2))*(-a^3*b
^3)^(1/2))/(4*(a^4*d - a^3*b*c)) - 1/(2*a*c*x^2) - (log(a^3*x^2*(-c^3*d^3)^
(3/2) + b^3*c^8*d - a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^
(1/2))/(4*(b*c^4 - a*c^3*d)) + (log(a^3*x^2*(-c^3*d^3)^(3/2) - b^3*c^8*d +
a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(4*b*c^4 - 4*
a*c^3*d)
```

**sympy** [B] time = 84.89, size = 1103, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -sqrt(-b**3/a**3)*log(x**2 + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*
c)**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2
*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**3)/
(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*
b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)/(
a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d - b*
c)) + sqrt(-b**3/a**3)*log(x**2 + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d
- b*c)**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*
b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3/a*
*3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a
**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a**
3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d
- b*c)) - sqrt(-d**3/c**3)*log(x**2 + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/
(a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2*
a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-d*
*3/c**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**
3 - a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d**
3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*
(a*d - b*c)) + sqrt(-d**3/c**3)*log(x**2 + (a**7*c**3*d**4*(-d**3/c**3)**(3
/2)/(a*d - b*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3
+ 2*a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt
(-d**3/c**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(
-d**3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))
/(4*(a*d - b*c)) - 1/(2*a*c*x**2)
```

## 3.504 $\int \frac{1}{x^7(a+bx^4)(c+dx^4)} dx$

**Optimal**. Leaf size=112

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

**Rubi [A]** time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 480, 583, 522, 205}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^2) + (b^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*c^(5/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{6acx^6}+\frac{\operatorname{Subst}\left(\int \frac{-3(bc+ad)-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)}{6ac}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\operatorname{Subst}\left(\int \frac{-3\left(b^2c^2+abcd+a^2d^2\right)-3bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)}{6a^2c^2}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2a^2(bc-ad)}-\frac{d^3\operatorname{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2c^2(bc-ad)}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 193, normalized size = 1.72

$$\frac{\frac{3b^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}}+\frac{3b^{5/2}x^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{a^{5/2}}-\frac{3b^2x^4}{a^2}+\frac{b}{a}-\frac{3d^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{5/2}}-\frac{3d^{5/2}x^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{c^{5/2}}+\frac{3d^2x^4}{c^2}-\frac{d}{c}}{6x^6(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^4)*(c + d*x^4)), x]
```

**fricas [A]** time = 3.45, size = 592, normalized size = 5.29

$$\left[-\frac{3b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+3a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-6\left(b^2c^2-a^2d^2\right)x^4+2abc^2-2a^2cd}{12\left(a^2bc^3-a^3c^2d\right)x^6}, \frac{6a^2d^2x^6\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-3b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+6\left(b^2c^2-a^2d^2\right)x^4-2abc^2+2a^2cd}{12\left(a^2bc^3-a^3c^2d\right)x^6}, -\frac{6b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)+3a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-6\left(b^2c^2-a^2d^2\right)x^4+2abc^2-2a^2cd}{12\left(a^2bc^3-a^3c^2d\right)x^6}, -\frac{3b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-3a^2d^2x^6\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-3\left(b^2c^2-a^2d^2\right)x^4+abc^2-a^2cd}{6\left(a^2bc^3-a^3c^2d\right)x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^
4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*
```

```
x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3
- a^3*c^2*d)*x^6), 1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)
) - 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 +
a)) + 6*(b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3 - a^3*
c^2*d)*x^6), -1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) + 3
*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) -
 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d
)*x^6), -1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - 3*a^2*d
^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*(b^2*c^2 - a^2*d^2)*x^4 +
a*b*c^2 - a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

**giac** [A] time = 0.18, size = 103, normalized size = 0.92

$$\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(a^2bc - a^3d\right)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^3 - ac^2d\right)\sqrt{cd}} + \frac{3\,bcx^4 + 3\,adx^4 - ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*b*c*x^4 + 3*a*d
*x^4 - a*c)/(a^2*c^2*x^6)
```

**maple** [A] time = 0.06, size = 105, normalized size = 0.94

$$-\frac{b^3 \arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2\,(ad - bc)\,\sqrt{ab}\;a^2} + \frac{d^3 \arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2\,(ad - bc)\,\sqrt{cd}\;c^2} + \frac{d}{2a\,c^2x^2} + \frac{b}{2a^2c\,x^2} - \frac{1}{6ac\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2*d^3/c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2*b^3/a^2/(
a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)-1/6/a/c/x^6+1/2/a/c^2/x^2*
d+1/2/a^2/c/x^2*b
```

**maxima** [A] time = 1.51, size = 101, normalized size = 0.90

$$\frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(a^2bc - a^3d\right)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^3 - ac^2d\right)\sqrt{cd}} + \frac{3\,(bc + ad)x^4 - ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*(b*c + a*d)*x^4
 - a*c)/(a^2*c^2*x^6)
```

**mupad** [B] time = 5.53, size = 535, normalized size = 4.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^10*(-a^5*b^5)^(5/2) + a^20*d^10*(-a^5*b^5)^(1/2) - a^12*b^13*c^10*x^
2 - a^22*b^3*d^10*x^2 + 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^17*b^8*c^5*d^
```

```
5*x^2)*(-a^5*b^5)^(1/2))/(4*a^6*d - 4*a^5*b*c) - (log(c^10*(-a^5*b^5)^(5/2)
 + a^20*d^10*(-a^5*b^5)^(1/2) + a^12*b^13*c^10*x^2 + a^22*b^3*d^10*x^2 + 2*
a^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^17*b^8*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(a^6*d - a^5*b*c)) - (1/(6*a*c) - (x^4*(a*d + b*c))/(2*a^2*c^2))/x^6 - (l
og(a^10*(-c^5*d^5)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) + a^10*c^12*d^13*x^2
+ b^10*c^22*d^3*x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) - 2*a^5*b^5*c^17*d^8*
x^2)*(-c^5*d^5)^(1/2))/(4*(b*c^6 - a*c^5*d)) + (log(a^10*(-c^5*d^5)^(5/2) +
 b^10*c^20*(-c^5*d^5)^(1/2) - a^10*c^12*d^13*x^2 - b^10*c^22*d^3*x^2 + 2*a^
5*b^5*c^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^17*d^8*x^2)*(-c^5*d^5)^(1/2))/(4*
b*c^6 - 4*a*c^5*d)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.505 $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=457

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}}{2}$$

**Rubi [A]** time = 0.44, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{x}{bd}-\frac{\int \frac{ac+(bc+ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{bd}\\
&= \frac{x}{bd}+\frac{a^2\int \frac{1}{a+bx^4}\,dx}{b(bc-ad)}-\frac{c^2\int \frac{1}{c+dx^4}\,dx}{d(bc-ad)}\\
&= \frac{x}{bd}+\frac{a^{3/2}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2b(bc-ad)}+\frac{a^{3/2}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2b(bc-ad)}-\frac{c^{3/2}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2d(bc-ad)}-\frac{c^{3/2}\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2d(bc-ad)}\\
&= \frac{x}{bd}+\frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)}+\frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)}-\frac{a^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,b^{5/4}(bc-ad)}-\\
&= \frac{x}{bd}-\frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}+\\
&= \frac{x}{bd}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}-
\end{aligned}
$$

**Mathematica [A]** time = 0.25, size = 377, normalized size = 0.82

$$
\frac{-\frac{\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{5/4}}+\frac{\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{5/4}}-\frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}}+\frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{b^{5/4}}-\frac{8ax}{b}+\frac{\sqrt{2}c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{d^{5/4}}-\frac{\sqrt{2}c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{d^{5/4}}+\frac{2\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{d^{5/4}}-\frac{2\sqrt{2}c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{d^{5/4}}+\frac{8cx}{d}}{8bc-8ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/d^(5
/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^8/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 0.72, size = 1378, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*
d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a
^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*x - (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5
*c*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 -
4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt((a^2*x^2 + (b^4*c^2 - 2*a*b^3*c*
d + a^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4
*a^3*b^6*c*d^3 + a^4*b^5*d^4)))/a^2))/a^4) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*b^3
*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*arctan((
(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4)*x
 - (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4
*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4
)*sqrt((c^2*x^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(-c^5/(b^4*c^4*
d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)))/c^2)
)/c^4) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d
^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*x + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a
^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (
-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b
^5*d^4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2
*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (-c^5/(b^4*
c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(
1/4)*b*d*log(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7
 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - (-c^5/(b^4*c^4*d^5 -
4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*l
og(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - 4*x)/(b*d)
```

**giac [A]** time = 0.22, size = 469, normalized size = 1.03

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^{3}c-\sqrt{2}ab^{2}d\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^{3}c-\sqrt{2}ab^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^{2}-\sqrt{2}ad^{3}\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^{2}-\sqrt{2}ad^{3}\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^{3}c-\sqrt{2}ab^{2}d\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^{3}c-\sqrt{2}ab^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \log\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^{2}-\sqrt{2}ad^{3}\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} c \log\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^{2}-\sqrt{2}ad^{3}\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^
2*d) - 1/2*(c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(
c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - 1/2*(c*d^3)^(1/4)*c*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqr
t(2)*a*d^3) + 1/4*(a*b^3)^(1/4)*a*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(a*b^3)^(1/4)*a*log(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(c*d^
3)^(1/4)*c*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^2 -
sqrt(2)*a*d^3) + 1/4*(c*d^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt
(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + x/(b*d)
```

**maple [A]** time = 0.06, size = 328, normalized size = 0.72

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, a \ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)b}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, c \ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8\left(ad-bc\right)d}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/b/d*x+1/8/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)
+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+1/4/d*c/(a*d-b*c)*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4/d*c/(a*d-b*c)*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8/b*a/(a*d-b*c)*(a/b)^(1/4)*
2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)
*x+(a/b)^(1/2)))-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x+1)-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)
```

**maxima [A]** time = 1.22, size = 375, normalized size = 0.82

$$\frac{\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{8\left(b^{2}c-abd\right)}-\frac{\frac{2\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,c^{\frac{5}{4}}\log\left(\sqrt{d}x^{2}+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}\,c^{\frac{5}{4}}\log\left(\sqrt{d}x^{2}-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{8\left(bcd-ad^{2}\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d) - 1/8*(2*sqrt(2)*c^(
3/2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2
*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*s
qrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqr
t(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x
 + sqrt(c))/d^(1/4))/(b*c*d - a*d^2) + x/(b*d)
```

**mupad [B]** time = 5.63, size = 6361, normalized size = 13.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*
b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a
^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^
5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4
)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^
6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(2
56*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 -
1024*a*b^8*c^3*d))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i - (-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*
d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c
^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^
5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c
^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b
^8*c^3*d))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i)/((-a^5/(256
*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 10
24*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a
^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3
*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c
^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 -
768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256
*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d
))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)) + (-a^5/(256*b^9*c^4 +
256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^
3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^
5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3
+ 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c^8*d^4 - 76
8*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5
*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^
4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4) +
(4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)
*2i - 2*atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1
536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3
*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*
a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d)
)^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 +
512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))
*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c
^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b
*d)) - (-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)
*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6
*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/
(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2
- 1024*a*b^8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)))/(
(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^
2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5
*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
```

^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i + (-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4) + atan(((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i)/((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)) + (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*2i - 2*atan(((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)) - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 51

```
2*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4
*c^3*d^9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*
d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i - (4*x*(a^4*b^4*c^
8 + a^8*c^4*d^4))/(b*d)))/((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^
3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*
c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-c^5/(256*a
^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024
*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b
^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^
9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1
536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8
*c^4*d^4))/(b*d))*1i + (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^
3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9
+ a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-c^5/(256*a^4*d
^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3
*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c
^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4
i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*
a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4
*d^4))/(b*d))*1i))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^
6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) + x/(b*d)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.506 $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=449

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan}{2\sqrt{2}}$$

**Rubi [A]** time = 0.28, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{3/4}(bc-ad)}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,d^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/
4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{a\int \frac{x^2}{a+bx^4}\,dx}{bc-ad}+\frac{c\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{a\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{b}\,(bc-ad)}-\frac{a\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{b}\,(bc-ad)}-\frac{c\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{d}\,(bc-ad)}+\frac{c\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{d}\,(bc-ad)}$$

$$= -\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\int \cdots}{4\sqrt{\cdots}}$$

$$= -\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\cdots$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\cdots}{\cdots}$$

**Mathematica [A]** time = 0.12, size = 340, normalized size = 0.76

$$\frac{-a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-2a^{3/4}d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)+b^{3/4}c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-b^{3/4}c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-2b^{3/4}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+2b^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,b^{3/4}d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(3
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^6/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 0.54, size = 1358, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4
*b^3*d^4))^(1/4)*arctan(((b^2*c - a*b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6
*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*x - (b^2*c - a*b*d
)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^
4*b^3*d^4))^(1/4)*sqrt((a*x^2 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-a
^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3
*d^4)))/a))/a) + (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 -
 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(((b*c*d - a*d^2)*(-c^3/(b^4*c^4*d^3
 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*x
- (b*c*d - a*d^2)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5
- 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*sqrt((c*x^2 - (b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a
^3*b*c*d^6 + a^4*d^7)))/c))/c) - 1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2
*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x + (b^5*c^3 -
 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^
3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*(-a^
3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*
d^4))^(1/4)*log(a^2*x - (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^
2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
 + a^4*b^3*d^4))^(3/4)) + 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*
b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x + (b^3*c^3*d^2 - 3*
a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d
^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/4*(-c^3/(b^4*
c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(
1/4)*log(c^2*x - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*
(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 +
a^4*d^7))^(3/4))
```

**giac [A]** time = 0.24, size = 453, normalized size = 1.01

$$-\frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)}-\frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)}+\frac{\left(cd^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}+\frac{\left(cd^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}+\frac{\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)}-\frac{\left(ab^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)}-\frac{\left(cd^3\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}+\frac{\left(cd^3\right)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d
) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^
(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*
d^4) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt
```

```
(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(c*d^3)^(3/4)*lo
g(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4
) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c*d^3 - sqrt(2)*a*d^4)
```

**maple [A]** time = 0.05, size = 320, normalized size = 0.71

$$\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,a\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}d}-\frac{\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}d}-\frac{\sqrt{2}\,c\ln\left(\frac{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^
(1/2))/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4*c/(a*d-b*c)/d/(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4*c/(a*d-b*c)/d/(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)+1/8*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)
*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

**maxima [A]** time = 1.42, size = 363, normalized size = 0.81

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\left(bc-ad\right)}+\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) + 1/8*c*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(
c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(
c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x
+ sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

**mupad [B]** time = 5.72, size = 2553, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((4*b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c
*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 4*a^3*b*d^3*x*(-a^
3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^
2 - 1024*a*b^6*c^3*d))^(1/4) + 2048*a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4) + 2048*b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^
3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a*b^7*
c^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*
```

```
a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4) + 12288*a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4))/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256*a^4*b^
3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4
) - atan((b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d
^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^3*b*d^3*x*(-a^3
/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2
 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*a^4
*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(
5/4)*2048i + b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*
b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*2048i - a*b^7*c
^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a
^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*8192i - a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4)*8192i + a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4)*12288i)/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256
*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d
))^(1/4)*2i - 2*atan((4*a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 10
24*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4) + 2048*b
^7*c^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 15
36*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 4*b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4) + 2048*a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d
^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) -
 8192*a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3
*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) - 8192*a^3*b^4*c*d^7
*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 12288*a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4))/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a^4*d^7
+ 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*
c*d^6))^(1/4) - atan((a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024
*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4)*4i + b^7*c
^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a
^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*2048i + b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4)*4i + a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3
 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*204
8i - a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*
d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i - a^3*b^4*c*d^7
*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i + a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4)*12288i)/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a
^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024
*a^3*b*c*d^6))^(1/4)*2i
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.507 $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=449

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\right.}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

**Rubi** [A] time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/
4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= -\frac{a\int \frac{1}{a+bx^4}\,dx}{bc-ad}+\frac{c\int \frac{1}{c+dx^4}\,dx}{bc-ad}\\
&= -\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}-\frac{\sqrt{a}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{c}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}\\
&= -\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}(bc-ad)}-\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}(bc-ad)}+\frac{\sqrt[4]{a}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}+\frac{\sqrt[4]{a}\ \cdots}{4\cdots}\\
&= \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\ \mathrm{lo}\cdots}{\cdots}\\
&= \frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\ \mathrm{ta}\cdots}{2\sqrt{\cdots}}
\end{aligned}
$$

**Mathematica [A]** time = 0.11, size = 340, normalized size = 0.76

$$
\frac{\sqrt[4]{a}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)-\sqrt[4]{b}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 0.47, size = 1238, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4))^(1/4)*arctan(((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)
*x*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4
*b*d^4))^(3/4) - (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sq
rt(x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)))*(-a/(b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4))/a) - (-c/(
b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))
^(1/4)*arctan(((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(3/4) - (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(x^
2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)))*(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4))/c) - 1/4*(-a/(b
^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^
(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(b*c - a*d)*(
-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4))^(1/4) + x) + 1/4*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
- 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*
c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2
*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x)
```

**giac [A]** time = 0.24, size = 437, normalized size = 0.97

$$-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}-\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) +
1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1
/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^2*
c - sqrt(2)*a*b*d) + 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt
```

```
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1/4*(c*d^3
)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(
2)*a*d^2)
```

**maple** [A] time = 0.06, size = 296, normalized size = 0.66

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4ad-4bc}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4ad-4bc}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8ad-8bc}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2
))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4/(a*d-b*c)*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x-1)+1/8/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(
1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/(a
*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a*d-b*c)*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

**maxima** [A] time = 1.39, size = 361, normalized size = 0.80

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{8\left(bc-ad\right)}+\frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{d}x^{2}+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{d}x^{2}-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{8\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b*c - a*d) + 1/8*(2*sqrt(2)*sqrt(c
)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*s
qrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt
(d)) + sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c
))/d^(1/4) - sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/d^(1/4))/(b*c - a*d)
```

**mupad** [B] time = 5.86, size = 5889, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (a^6*b*d^6*x*256i)/(256*b^5*c^4 + 256
*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d)
- (a*b^6*c^5*d*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 +
1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^5*b^2*c*d^5*x*768i)/(256*b^5*
c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^
4*c^3*d) + (a^2*b^5*c^4*d^2*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3
*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^3*b^4*c^3*d^3*x*
512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*
d^2 - 1024*a*b^4*c^3*d) - (a^4*b^3*c^2*d^4*x*512i)/(256*b^5*c^4 + 256*a^4*b
*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))/((-a/
(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 -
1024*a*b^4*c^3*d))^(1/4)*((a*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*
```

c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^
4 + 15360*a^4*b^3*c^2*d^5))/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d
^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a
*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b
^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*2i - atan((a^2*d
^2*x*1i + b^2*c^2*x*1i - (b^6*c^6*d*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d -
1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^5*b*c*d^
6*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*
c^2*d^3 - 1024*a^3*b*c*d^4) + (a*b^5*c^5*d^2*x*768i)/(256*a^4*d^5 + 256*b^4
*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a
^2*b^4*c^4*d^3*x*512i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 +
1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^3*b^3*c^3*d^4*x*512i)/(256*a^
4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^
3*b*c*d^4) + (a^4*b^2*c^2*d^5*x*768i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a
*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))/((-c/(256*a^4*d^5
+ 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*
d^4))^(1/4)*((c*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*c^5*d^2 - 614
4*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^4 + 15360*a^4
*b^3*c^2*d^5))/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2
*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a*b^2*c^2*d +
4*a^2*b*c*d^2)))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 15
36*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*2i - 2*atan(((x*(4*a^2*b^5*c^
4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^
2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9
*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d
^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256*
a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^
(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^
6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 +
4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2
*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i + 16*a^2*b^6*c^
5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(
-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2
- 1024*a*b^4*c^3*d))^(1/4) + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) -
(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^
2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d
^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1
024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^
3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4
- 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 6
1440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(
-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2
- 1024*a*b^4*c^3*d))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 +
16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d
^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4))/
((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*
d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*
((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2
048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(2
56*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 10
24*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 614
40*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*
a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d
^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1
i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b
^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 153
6*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*1i - (x*(4*a^2*b^5*c^4*d^3 + 4
*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 +
1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4

```
- 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072
*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4
 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(40
96*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920
*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8
*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 +
1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 1
6*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b
^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a
*b^4*c^3*d))^(1/4)*1i))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d
^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4) - 2*atan(((x*(4*a^2*b^
5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*
b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2
*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c
^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-c/(256*a^4*d^5 +
256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^
4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^
6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d
^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b
^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i + 16*a^2*b^
6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1
i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2
*d^3 - 1024*a^3*b*c*d^4))^(1/4) + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5
) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^
2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c
^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8
 + 1024*a^7*b^4*c^2*d^9) + (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^
3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*
d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7
 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1
i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2
*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^
4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4
*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/
4))/((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^
4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1
/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6
 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-
c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3
 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 +
 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24
576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4
*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/
4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a
^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 +
 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*1i - (x*(4*a^2*b^5*c^4*d^3
 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d
^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*
d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 -
3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-c/(256*a^4*d^5 + 256*b^4*c
^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)
*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 8
1920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096
*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^
2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i - 16*a^2*b^6*c^5*d^3
 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-c/(2
56*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 10
24*a^3*b*c*d^4))^(1/4)*1i))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c
^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

### 3.508 $\int \frac{x^2}{(a+bx^4)(c+dx^4)} \, dx$

**Optimal**. Leaf size=449

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}\right.}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

**Rubi [A]** time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {482, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c
- a*d)) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(1/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c
- a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\int \frac{x^2}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{b}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{b}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{d}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2(bc-ad)} - \frac{\sqrt{d}\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,d\cdots}{2(bc-ad)}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,\cdots}{4(bc-ad)}$$

$$= \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc-ad)} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc-ad)} - \cdots$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}\,(bc-ad)} - \cdots$$

**Mathematica [A]** time = 0.14, size = 340, normalized size = 0.76

$$\frac{\sqrt[4]{b}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-\sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)-\sqrt[4]{a}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+\sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*c^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*d^(1/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] + a^(1/4)*d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 0.50, size = 1258, normalized size = 2.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*
d^4))^(1/4)*arctan((b*c - a*d)*x*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b
^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4) - (b*c - a*d)*sqrt((b*x^2 - (a
*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)))/b)*(-b/(a*b^4*c^4 - 4*a^2*b^3*
c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)) - (-d/(b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*
arctan((b*c - a*d)*x*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a
^3*b*c^2*d^3 + a^4*c*d^4))^(1/4) - (b*c - a*d)*sqrt((d*x^2 - (b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2)*sqrt(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2
- 4*a^3*b*c^2*d^3 + a^4*c*d^4)))/d)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^
2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)) + 1/4*(-b/(a*b^4*c^4 - 4*a
^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x
+ (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 -
4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/
4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^
5*d^4))^(1/4)*log(b*x - (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*
d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4))^(3/4)) - 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x + (b^3*c^4 - 3*a*b^2*c^3*d + 3*
a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2
 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d
+ 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x - (b^3*c^
4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4
*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4))
```

**giac [A]** time = 0.23, size = 477, normalized size = 1.06

$$\frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}-\frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{(ab^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(ab^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{(cd^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{(cd^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^
2*b^2*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))
/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2
 - sqrt(2)*a*c*d^3) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/4*(a*b^3)^(3/4)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d)
```

```
+ 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^
(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3)
```

**maple [A]** time = 0.06, size = 296, normalized size = 0.66

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(\frac{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)
)/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x-1)-1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1
/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/(a*
d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/(a*d-b*c)/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

**maxima [A]** time = 1.21, size = 363, normalized size = 0.81

$$\frac{b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\left(bc-ad\right)}-\frac{d\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\,\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) - 1/8*d*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c
)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

**mupad [B]** time = 5.50, size = 6633, normalized size = 14.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)
*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^
2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10
 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 716
8*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a^6*b^4
*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 -
768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i + (x*(4*a*b^6*c^2*d^5
 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
```

```
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 40
96*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5
*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*
d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^
3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*
d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8
192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*
b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6
 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)
 - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^
4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(2
56*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
+ 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 10
24*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*
b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*
d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768
*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d +
1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4) + 2*a*b^5*c*d^5))*(-b/(256*
a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*
a^4*b*c*d^3))^(1/4)*2i + 2*atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a
*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(
1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 71
68*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6
*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^
4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 2
56*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3
))^(1/4) + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*
a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^
(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b
^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c
*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7
+ 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256
*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^
3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^
2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4))/((x*(4*a*b^6
*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b
^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*
d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*
b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c
^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8
 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a
^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^
2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^
5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5
*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4
*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*
c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^
8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 51
```

2*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256
*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024
*a^4*b*c*d^3))^(1/4)*1i + 2*a*b^5*c*d^5))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4
- 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4) + at
an(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d/(256*b^4*c^5 + 256*a^4*c*d
^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(
x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*
d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 -
4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*
a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c
*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 76
8*a^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 +
1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*1i + (x*(4*a*b^6*c^2*d^5 +
4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 +
1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d
/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 -
1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096
*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b
^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^
5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-d/(
256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1
024*a*b^3*c^4*d))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d/(
256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1
024*a*b^3*c^4*d))^(3/4)*(x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^
2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^
4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 819
2*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^
9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 +
512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d
^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4) -
(x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4
- 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256
*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 +
1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024
*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^
7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^
9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a
^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 15
36*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4) + 2*a*b^5*c*d^5))*(-d/(256*b^
4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*
b^3*c^4*d))^(1/4)*2i + 2*atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d
/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 -
1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c^5 + 256*a^4
*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/
4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168
*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b
^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*
d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b^4*c^5 + 256
*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))
^(1/4) + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^
4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3
/4)*(x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2
*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d
^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 +
7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a
^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*
d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*
b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4))/((x*(4*a*b^6*c
^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c

```
^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(x*(-d/(256*b^4*c^
5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*
c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6
*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 -
 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2
*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*
d^8)*1i)*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b
^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*
c*d^6) + (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b
^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c
^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3
*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^
6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8
- 4096*a^6*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*
a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b
^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a
*b^3*c^4*d))^(1/4)*1i + 2*a*b^5*c*d^5))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 -
1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.509 $\int \frac{1}{(a+bx^4)(c+dx^4)} \, dx$

**Optimal**. Leaf size=449

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}}{2}$$

**Rubi [A]** time = 0.26, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}(bc-ad)} - \frac{b^{3/4}\int}{4\sqrt{}}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}}{}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)} - \frac{d^{3/4}}{2}$$

**Mathematica [A]** time = 0.20, size = 340, normalized size = 0.76

$$\frac{a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a^{3/4}d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)-b^{3/4}c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+b^{3/4}c^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2b^{3/4}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 0.63, size = 1356, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4))^(3/4)*sqrt((b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4)))/b^2))/b^2) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5 - 3*a*b^
2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3*a
*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt((d^2*x^2 +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))/d^2))/d^2) + 1/4*(-b^
3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*
d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d
+ 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^
2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3
*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
+ (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^
3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 -
 a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^
3 + a^4*c^3*d^4))^(1/4))
```

**giac [A]** time = 0.22, size = 437, normalized size = 0.97

$$\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1
/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/
4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c
 - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)
^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2
)*a*c*d)
```

**maple** [A]   time = 0.06, size = 320, normalized size = 0.71

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\,(ad-bc)\,a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\,(ad-bc)\,a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\,(ad-bc)\,a}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\,(ad-bc)\,c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\,(ad-bc)\,c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8\,(ad-bc)\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(
1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d/(a*d-b*c)*(c/d)^(1/4)/
c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*
ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)
^(1/2)))-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

**maxima** [A]   time = 1.38, size = 365, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{8\,(bc-ad)}-\frac{\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,d^{\frac{3}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\,d^{\frac{3}{4}}\log\left(\sqrt{d}x^2-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

**mupad** [B]   time = 5.85, size = 6153, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a
^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3
*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)
*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*
c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^
11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7
 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10)
+ x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6
*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^
3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7
*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^
4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^
4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*
a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*
d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 +
```

256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i)/(((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3

*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^
4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*
b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10
*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^
4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d
^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*
a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^
6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7
*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^
2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) + ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^
4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((
-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2
*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 102
4*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 +
256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*
d^3))^(1/4)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1
536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*2i - 2*atan((b^3*d^3*x - (12
8*b^10*c^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^
5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (128*a^7*b^3*d^7*x)/(256*a^7*d^4 + 256*
a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3)
- (768*a^2*b^8*c^5*d^2*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^
3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^3*b^7*c^4*d^3*x)/(2
56*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
1024*a^6*b*c*d^3) + (384*a^4*b^6*c^3*d^4*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4
- 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^5*
b^5*c^2*d^5*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a
^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a*b^9*c^6*d*x)/(256*a^7*d^4 + 256
*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3
) + (512*a^6*b^4*c*d^6*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3
*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))/((-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*((b^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*
b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*
d^4 + 4608*a^6*b^2*c^3*d^5))/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*
c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + 2*a^2*b^2*d^4 + 2*b^4*c^
2*d^2 - 4*a*b^3*c*d^3)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^
3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) - 2*atan((b^3*d^3
*x - (128*a^7*d^10*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^
7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^
3*c^6*d) - (768*a^2*b^5*c^5*d^5*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d
^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*
c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (
768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3
+ 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c
^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b
^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3
*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7
+ 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*
c^6*d))^(1/4)*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2
560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*

```
b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*
a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 +
2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.510 $\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

**Optimal**. Leaf size=460

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan}{2\sqrt{2}}$$

**Rubi [A]** time = 0.45, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{d^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{d^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)) +
(b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= -\frac{1}{acx}+\frac{\int \frac{x^2\left(-bc-ad-bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{ac}\\
&= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx^2}{(bc-ad)\left(a+bx^4\right)}-\frac{ad^2x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx}{ac}\\
&= -\frac{1}{acx}-\frac{b^2\int \frac{x^2}{a+bx^4}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x^2}{c+dx^4}\,dx}{c(bc-ad)}\\
&= -\frac{1}{acx}+\frac{b^{3/2}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a(bc-ad)}-\frac{b^{3/2}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a(bc-ad)}-\frac{d^{3/2}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2c(bc-ad)}+\frac{d^{3/2}\int \ldots}{2c(b\ldots}\\
&= -\frac{1}{acx}-\frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)}-\frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)}-\frac{b^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,a^{5/4}(bc-ad)}\ldots\\
&= -\frac{1}{acx}-\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\ldots\right.}{4\sqrt{2}\,a^{5/4}(bc-ad)}\ldots\\
&= -\frac{1}{acx}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\ldots\right.}{2\sqrt{2}\,c^{5/4}(bc-a\ldots}
\end{aligned}
$$

**Mathematica [A]** time = 0.22, size = 385, normalized size = 0.84

$$
\frac{\frac{\sqrt{2}b^{5/4}x\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{5/4}}-\frac{\sqrt{2}b^{5/4}x\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{5/4}}-\frac{2\sqrt{2}b^{5/4}x\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{2\sqrt{2}b^{5/4}x\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{5/4}}+\frac{8b}{a}-\frac{\sqrt{2}d^{5/4}x\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{5/4}}+\frac{\sqrt{2}d^{5/4}x\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{5/4}}+\frac{2\sqrt{2}d^{5/4}x\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/4}}-\frac{2\sqrt{2}d^{5/4}x\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{c^{5/4}}-\frac{8d}{c}}{8adx-8bcx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c
^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(-8*b*c*x + 8*a*d*x)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^4\right) \left(c + dx^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 0.89, size = 1395, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(1/4)*a*c*x*arctan(((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d
+ 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*(a*b*c - a^2*d)*x - (
-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a
^9*d^4))^(1/4)*(a*b*c - a^2*d)*sqrt((b^3*x^2 - (a^3*b^2*c^2 - 2*a^4*b*c*d +
 a^5*d^2)*sqrt(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*
a^8*b*c*d^3 + a^9*d^4)))/b^3))/b) - 4*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^
2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*arctan(((-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
))^(1/4)*(b*c^2 - a*c*d)*x - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7
*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*(b*c^2 - a*c*d)*sqrt((d^3*x^2
- (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d
+ 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4)))/d^3))/d) + (-b^5/(a^
5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))
^(1/4)*a*c*x*log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a
^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2
*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x - (a^4*b^3*c^3 -
 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*
c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c
^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1
/4)*a*c*x*log(d^4*x + (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*
d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4))^(3/4)) + (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2
- 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x - (b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6
*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + 4)/(a*c*x)
```

**giac [A]** time = 0.21, size = 488, normalized size = 1.06

$$-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}}\log\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}}\log\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}\right)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)
*a^3*b*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4)
)/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d
- sqrt(2)*a*c^2*d^2) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/4*(a*b^3)^(3/4)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d
) - 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) - 1/(a*c*x)
```

**maple [A]** time = 0.06, size = 331, normalized size = 0.72

$$\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,b\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,d\ln\left(\frac{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^
(1/2))/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4*d/c/(a*d-b*c)/(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4*d/c/(a*d-b*c)/(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)+1/8*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)
*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-
1/a/c/x
```

**maxima [A]** time = 1.27, size = 384, normalized size = 0.83

$$-\frac{b^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\left(abc-a^{2}d\right)}+\frac{d^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^{2}+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^{2}-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\left(bc^{2}-acd\right)}-\frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b*c - a^2*d) + 1/8*d
^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^
```

```
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^
(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^2 - a*c*d) - 1/(a*c*x)
```

**mupad [B]** time = 6.08, size = 5962, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] 2*atan(((-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^
2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b
^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 +
1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256
*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d
))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^1
0*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^
7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*
b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a
^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^1
7*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i) + (-
d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*
d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^
9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b
^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d
^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(
1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6
 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9
 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d
^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c
^17*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13
*d^10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i))/((-d^5/(256*b
^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024
*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5
/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2
 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*
a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*
b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^
15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^
18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i -
256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 +
256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 76
8*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i)*1i - (-d^5/(256*b^4*c^9
+ 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*
c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5/(256*b
^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024
*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c
^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*b^12*c^
20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*
c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*
c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^1
1*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^1
4*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*
b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i)*1i))*(-d^5/(256*b^4*c^9 + 256*a
^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))
^(1/4) + atan((a^14*c*d^8*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6
*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^
8*c^9*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^
7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^4*d^5*x*(-b^5/(256*a
^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024
```

*a^8*b*c*d^3))^(1/4)*4i + a^5*b^5*c*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*4i - a^7*b^7*c^8*d*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i - a^13*b*c^2*d^7*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^8*b^6*c^7*d^2*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*6144i - a^9*b^5*c^6*d^3*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^10*b^4*c^5*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*2048i - a^11*b^3*c^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^12*b^2*c^3*d^6*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*6144i)/(b^9*c^4 + a^4*b^5*d^4 + a^3*b^6*c*d^3 + a^2*b^7*c^2*d^2 + a*b^8*c^3*d))*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*2i + atan((b^5*c^6*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*4i + a*b^8*c^14*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*1024i + a^9*c^6*d^8*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*1024i + a*b^4*c^5*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*4i - a^2*b^7*c^13*d*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i - a^8*b*c^7*d^7*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^3*b^6*c^12*d^2*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*6144i - a^4*b^5*c^11*d^3*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^5*b^4*c^10*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*2048i - a^6*b^3*c^9*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^7*b^2*c^8*d^6*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*6144i)/(a^4*d^9 + b^4*c^4*d^5 + a*b^3*c^3*d^6 + a^2*b^2*c^2*d^7 + a^3*b*c*d^8))*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*2i + 2*atan(((-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i) + (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17

```
*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^
10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i))/((-b^5/(256*a^9*
d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^
8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(2
56*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 -
1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6
*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^1
2*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*
b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*
b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i - 256
*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 + 256
*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 768*a
^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i)*1i - (-b^5/(256*a^9*d^4 + 2
56*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d
^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*
d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^
8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^
3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*
d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^1
7*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^1
4*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b
^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^14*b
^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*b^5
*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i)*1i))*(-b^5/(256*a^9*d^4 + 256*a^5*
b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1
/4) - 1/(a*c*x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

## 3.511 $\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$

**Optimal**. Leaf size=462

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\text{ta}}{2\sqrt{}}$$

**Rubi [A]** time = 0.43, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/
c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d))
- (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{1}{3acx^3}+\frac{\int \frac{-3(bc+ad)-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{3ac}$$

$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{1}{a+bx^4}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^4}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}-\frac{b^2\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}+\frac{d^2\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2c^{3/2}(bc-ad)}+\frac{d^2\int \frac{\sqrt{c}+\sqrt{d}\,x\cdots}{c+dx^4}}{2c^{3/2}(bc-\cdots}$$

$$= -\frac{1}{3acx^3}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}+\frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^{\cdots}}}{4\sqrt{2}\,a^{7/4}(bc-ad\cdots}$$

$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{\cdots}\right.}{4\sqrt{2}\,a^{7/4}(bc-ad)}$$

$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right.}{2\sqrt{2}\,c^{7/4}(bc-a\cdots}$$

**Mathematica [A]** time = 0.26, size = 406, normalized size = 0.88

$$\frac{-\frac{6\sqrt{2}b^{7/4}x^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}b^{7/4}x^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}-\frac{3\sqrt{2}b^{7/4}x^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}b^{7/4}x^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{8b}{a}+\frac{6\sqrt{2}d^{7/4}x^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{6\sqrt{2}d^{7/4}x^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{c^{7/4}}+\frac{3\sqrt{2}d^{7/4}x^3\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}}-\frac{3\sqrt{2}d^{7/4}x^3\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}}-\frac{8d}{c}}{24x^3(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[2]*d^(7/
4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(24
*(-(b*c) + a*d)*x^3)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^4\right) \left(c + dx^4\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 5.40, size = 1415, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/12*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*
b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*arctan(((a^5*b^3*c^3 - 3*a^6*b^2*c^2*d +
 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*x - (a^5*b^3*c^3 - 3*a^6*b^2*c^
2*d + 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9
*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt((b^4*x^2 + (a^4*b^2*c
^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^
9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4)))/b^4))/b^5) - 12*(-d^7/(b^4*c^1
1 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1
/4)*a*c*x^3*arctan(((b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^
3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 +
 a^4*c^7*d^4))^(3/4)*x - (b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c
^5*d^3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(3/4)*sqrt((d^4*x^2 + (b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*
d^2)*sqrt(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8
*d^3 + a^4*c^7*d^4)))/d^4))/d^5) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d +
 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x +
(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 +
 a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3
*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*
x - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d
^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*log(
d^2*x + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^3*
c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*
log(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*
c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 4)/(a*c*x^3)
```

**giac [A]** time = 0.19, size = 472, normalized size = 1.02

$$-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2 x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} a^{2} b c-\sqrt{2} a^{3} d\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2 x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} a^{2} b c-\sqrt{2} a^{3} d\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2 x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} b c^{3}-\sqrt{2} a c^{2} d\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2 x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} b c^{3}-\sqrt{2} a c^{2} d\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(x^{2}+\sqrt{2} x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2} a^{2} b c-\sqrt{2} a^{3} d\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(x^{2}-\sqrt{2} x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2} a^{2} b c-\sqrt{2} a^{3} d\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \log\left(x^{2}+\sqrt{2} x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2} b c^{3}-\sqrt{2} a c^{2} d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \log\left(x^{2}-\sqrt{2} x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2} b c^{3}-\sqrt{2} a c^{2} d\right)}-\frac{1}{3 a c x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a
^3*d) + 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/2*(c*d^3)^(1/4)*d*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt
(2)*a*c^2*d) - 1/4*(a*b^3)^(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(c*d
^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3 - s
qrt(2)*a*c^2*d) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqr
t(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/3/(a*c*x^3)
```

**maple [A]** time = 0.06, size = 343, normalized size = 0.74

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(a d-b c\right) a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(a d-b c\right) a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{2} \ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8\left(a d-b c\right) a^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(a d-b c\right) c^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(a d-b c\right) c^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, d^{2} \ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{c}{d}}}\right)}{8\left(a d-b c\right) c^{2}}-\frac{1}{3 a c x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c
/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4/c^2*d^2/(a*d-b*c)*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4/c^2*d^2/(a*d-b*c)*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/3/a/c/x^3+1/8/a^2*b^2/(
a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

**maxima [A]** time = 1.36, size = 390, normalized size = 0.84

$$-\frac{\frac{2 \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{b} x+\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}+\frac{2 \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{b} x-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}+\frac{\sqrt{2}\, b^{\frac{7}{4}} \log\left(\sqrt{b} x^{2}+\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\, b^{\frac{7}{4}} \log\left(\sqrt{b} x^{2}-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{8\left(a b c-a^{2} d\right)}+\frac{\frac{2 \sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{d} x+\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{2 \sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{d} x-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{\sqrt{2}\, d^{\frac{7}{4}} \log\left(\sqrt{d} x^{2}+\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\, d^{\frac{7}{4}} \log\left(\sqrt{d} x^{2}-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8\left(b c^{2}-a c d\right)}-\frac{1}{3 a c x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(a*b*c - a^2*d) + 1/8*(
2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan
(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2
)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c^2 - a*c*d) - 1/3/(a*c*x^
3)
```

**mupad [B]** time = 6.19, size = 7459, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^2*b^16*c^11*x*256i)/(256*a
^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 102
4*a^10*b*c*d^3) - (a^4*b^14*c^9*d^2*x*1536i)/(256*a^11*d^4 + 256*a^7*b^4*c^
4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^5*b
^13*c^8*d^3*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d +
 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^6*b^12*c^7*d^4*x*256i)/(256
*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1
024*a^10*b*c*d^3) - (a^7*b^11*c^6*d^5*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c
^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^8*
b^10*c^5*d^6*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
 + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^9*b^9*c^4*d^7*x*1536i)/(25
6*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 -
1024*a^10*b*c*d^3) + (a^10*b^8*c^3*d^8*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4
*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^
11*b^7*c^2*d^9*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
 + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^3*b^15*c^10*d*x*1024i)/(2
56*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 -
 1024*a^10*b*c*d^3))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*
c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((b^7*(1024*a^4*b^
8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5120*a^11*b*c^5*d^7 + 10
240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10240*a^8*b^4*c^8*d^4 - 1126
4*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256*a^11*d^4 + 256*a^7*b^4*c^
4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + 4*a^5*
b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4*b^4*c*d^7)))*(-b^7/(256*a
^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 102
4*a^10*b*c*d^3))^(1/4)*2i - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^
11*c^2*d^16*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 +
1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^2*b^9*c^11*d^7*x*256i)/(256*
b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 10
24*a*b^3*c^10*d) + (a^3*b^8*c^10*d^8*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d
^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^4*
b^7*c^9*d^9*x*1536i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 +
 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^5*b^6*c^8*d^10*x*1024i)/(25
6*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 -
1024*a*b^3*c^10*d) - (a^6*b^5*c^7*d^11*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*
d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^7
*b^4*c^6*d^12*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3
+ 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^8*b^3*c^5*d^13*x*1024i)/(2
56*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 -
 1024*a*b^3*c^10*d) - (a^9*b^2*c^4*d^14*x*1536i)/(256*b^4*c^11 + 256*a^4*c^
7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a
^10*b*c^3*d^15*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^
3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))/((-d^7/(256*b^4*c^11 + 256*a
^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d)
)^(1/4)*((d^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d
- 5120*a^11*b*c^5*d^7 + 10240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10
240*a^8*b^4*c^8*d^4 - 11264*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256
*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1
024*a*b^3*c^10*d) + 4*a^5*b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4
*b^4*c*d^7)))*(-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 +
1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*2i - 2*atan(((-d^7/(256*b^
4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024
*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^
```

7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d
^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 102
4*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024
*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4
096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12
 + 1024*a^20*b^4*c^11*d^13) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*
a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13
*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 4505
6*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 4
0960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^1
2)*1i)*1i - 16*a^9*b^12*c^14*d^7 + 16*a^10*b^11*c^13*d^8 + 16*a^13*b^8*c^10
*d^11 - 16*a^14*b^7*c^9*d^12)*1i) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 -
 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(
4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*
c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(
1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2
*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096
*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1
024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 +
6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13
) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b
^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^1
4*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 4096
0*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 -
20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i + 16*a^9*b^12*c^
14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^
12)*1i))/((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536
*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^
11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d
^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11
+ 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*
c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144
*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 10
24*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 -
4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13) - (-d^7/(256*b^4*c^11 +
256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^
10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^
15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 4505
6*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 +
 4096*a^21*b^4*c^13*d^12)*1i)*1i - 16*a^9*b^12*c^14*d^7 + 16*a^10*b^11*c^13
*d^8 + 16*a^13*b^8*c^10*d^11 - 16*a^14*b^7*c^9*d^12)*1i)*1i - (-d^7/(256*b^
4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024
*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^
7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d
^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 102
4*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024
*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4
096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12
 + 1024*a^20*b^4*c^11*d^13) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*
a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13
*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 4505
6*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 4
0960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^1
2)*1i)*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10
*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i))*(-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^
4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4) -
 2*atan(-((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536

```
*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^
11*b^9*c^9*d^11) - (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3
*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4
+ 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b
*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
+ 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4
 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^1
8*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*
c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i + x*(102
4*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 -
4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^1
2 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13
*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i) + (-b^7/(256*a^11*
d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^
10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) + (-b^7/(
256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2
- 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a
^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256
*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1
024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^
5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^1
7*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5
*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i - x*(1024*a^11*b^13*c^20*d^4 - 409
6*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 +
1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 +
 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^1
3))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^
11 + 16*a^14*b^7*c^9*d^12)*1i))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 10
24*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a
^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-b^7/(256*a^11*d^4 + 256*a^7*b^4
*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4
)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^
2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^
4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(
4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d
^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^1
6*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4
*c^13*d^12)*1i + x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 614
4*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1
024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 -
 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*
d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)
*1i)*1i - (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536
*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^
11*b^9*c^9*d^11) + (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3
*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4
+ 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b
*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
+ 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4
 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^1
8*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*
c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i - x*(102
4*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 -
4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^1
2 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13
*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i))*(-b^7/(256*a^
11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024
```

```
*a^10*b*c*d^3))^(1/4) - 1/(3*a*c*x^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.512 $\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx$

**Optimal**. Leaf size=479

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\,\text{ta}}{2\sqrt{}}$$

**Rubi** [A] time = 0.60, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)}+\frac{d^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(a^2*c^2*x) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^(9
/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)
) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)) +
 (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(9/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{5ac}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{x^2\left(-5\left(b^2c^2+abcd+a^2d^2\right)-5bd(bc+ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{5a^2c^2}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{\int \left(-\frac{5b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)}-\frac{5a^2d^3x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx}{5a^2c^2}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{x^2}{a+bx^4}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{x^2}{c+dx^4}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{5/2}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^2(bc-ad)}+\frac{b^{5/2}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^2(bc-ad)}+\frac{d^{5/2}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}}{2c^2(bc-ad)}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^2(bc-ad)}+\frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^2(bc-ad)}+\frac{b^{9/4}\int}{4\sqrt{2}\,a}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}\,a^{9/4}}$$

$$= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{d^{9/4}\,\mathrm{t}}{2}$$

**Mathematica** [A] time = 0.38, size = 428, normalized size = 0.89

$$\frac{\frac{10\sqrt{2}b^{9/4}x^5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}b^{9/4}x^5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{9/4}}-\frac{5\sqrt{2}b^{9/4}x^5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{9/4}}+\frac{5\sqrt{2}b^{9/4}x^5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{9/4}}-\frac{40b^2x^4}{a^2}+\frac{8b}{a}-\frac{10\sqrt{2}d^{9/4}x^5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{9/4}}+\frac{10\sqrt{2}d^{9/4}x^5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{c^{9/4}}+\frac{5\sqrt{2}d^{9/4}x^5\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{9/4}}-\frac{5\sqrt{2}d^{9/4}x^5\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{9/4}}+\frac{40d^2x^4}{c^2}-\frac{8d}{c}}{40x^5(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*b^(9/
4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4) + (5
*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^4)*(c + d*x^4)), x]
```

**fricas [B]** time = 9.20, size = 1456, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*arctan(((-b^9/(a^9*b^4*c^4 - 4*a^1
0*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b
*c - a^3*d)*x - (-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2
- 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b*c - a^3*d)*sqrt((b^5*x^2 - (a^5*
b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d
+ 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)))/b^5))/b^2) - 20*(-d^9/(
b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9
*d^4))^(1/4)*a^2*c^2*x^5*arctan(((-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b
^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(b*c^3 - a*c^2*d)*x -
(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 +
a^4*c^9*d^4))^(1/4)*(b*c^3 - a*c^2*d)*sqrt((d^5*x^2 - (b^2*c^7 - 2*a*b*c^6*
d + a^2*c^5*d^2)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2
- 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)))/d^5))/d^2) + 5*(-b^9/(a^9*b^4*c^4 - 4*a
^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*
c^2*x^5*log(b^7*x + (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d
^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c
*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11
*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x - (a
^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4
 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4
)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10
*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x + (b^3*c^10 - 3*a*b^2*c^9*
d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2
*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/(b^4*c^13
 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(
1/4)*a^2*c^2*x^5*log(d^7*x - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 -
a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*
b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 20*(b*c + a*d)*x^4 - 4*a*c)/(a^2*c^2*x^
5)
```

**giac [A]** time = 0.24, size = 483, normalized size = 1.01

$$\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{3}bc-\sqrt{2}a^{4}d\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{3}bc-\sqrt{2}a^{4}d\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^{4}-\sqrt{2}ac^{3}d\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^{4}-\sqrt{2}ac^{3}d\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^{3}bc-\sqrt{2}a^{4}d\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^{3}bc-\sqrt{2}a^{4}d\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} \log\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^{4}-\sqrt{2}ac^{3}d\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} \log\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^{4}-\sqrt{2}ac^{3}d\right)}+\frac{5\,bcx^{4}+5\,adx^{4}-ac}{5\,a^{2}c^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d)
 - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3
*d) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(
2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(c*d^3)^(3/4)*log
(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d)
 - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^4 - sqrt(2)*a*c^3*d) + 1/5*(5*b*c*x^4 + 5*a*d*x^4 - a*c)/(a^2*c^2*x^5)
```

**maple [A]** time = 0.07, size = 365, normalized size = 0.76

$$-\frac{\sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}-\frac{\sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}-\frac{\sqrt{2}\, b^{2} \ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{\sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}+\frac{\sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}+\frac{\sqrt{2}\, d^{2} \ln\left(\frac{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{c}{d}}}{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{c}{d}}}\right)}{8\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}+\frac{d}{a\,c^{2}x}+\frac{b}{a^{2}cx}-\frac{1}{5ac\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2))/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d^2/c^2/(a*d-b*c)/(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d^2/c^2/(a*d-b*c)/(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8*b^2/a^2/(a*d-b*c)/(a/b
)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2)))-1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x+1)-1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)-1/5/a/c/x^5+1/a/c^2/x*d+1/a^2/c/x*b
```

**maxima [A]** time = 1.23, size = 405, normalized size = 0.85

$$\frac{b^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{8\left(a^{2}bc-a^{3}d\right)}-\frac{d^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^{2}+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^{2}-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{8\left(bc^{3}-ac^{2}d\right)}+\frac{5\left(bc+ad\right)x^{4}-ac}{5a^{2}c^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b*c - a^3*d) - 1/8*
d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d
^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 1/5*(5*(
b*c + a*d)*x^4 - a*c)/(a^2*c^2*x^5)
```

**mupad [B]** time = 6.01, size = 4547, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((1024*a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 102
4*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*a^
11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d +
1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 1024*a^21*c^3*d^10*x*(-
b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c
^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^12*b^9*c^12*d*x*(-b^9/(256*a^13
*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024
*a^12*b*c*d^3))^(5/4) - 4096*a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9
*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3)
)^(5/4) + 4*a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*
a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 6144*a
^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^
```

3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^14*b^7*c^1
0*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*
a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^15*b^6*c^9*d^4*x*(-b^
9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2
*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*
d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*
a^12*b*c*d^3))^(5/4) - 4096*a^18*b^3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^
9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3
))^(5/4) + 6144*a^19*b^2*c^5*d^8*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 -
1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4))/(b
^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3
 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*
(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2
*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) - atan((a^11*b^10*c^13*x*(-b^9/(256*a^
13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 10
24*a^12*b*c*d^3))^(5/4)*1024i + a^11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^
9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3
))^(1/4)*4i + a^21*c^3*d^10*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*
a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i -
a^12*b^9*c^12*d*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3
*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i - a^20*b*c^4*d
^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^1
1*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^8*b^9*c^3*d^6*x*(-b^9/(
256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^
2 - 1024*a^12*b*c*d^3))^(1/4)*4i + a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4
+ 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12
*b*c*d^3))^(5/4)*6144i - a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*
b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))
^(5/4)*4096i + a^15*b^6*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1
024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024
i + a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^
3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i - a^18*b^
3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1
536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^19*b^2*c^5*d^8*x
*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^
2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*6144i)/(b^16*c^8 + a^8*b^8*d^8 + a^7*
b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^
11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(256*a^13*d^4 + 256*a^
9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3
))^(1/4)*2i - 2*atan((4*b^9*c^11*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^
4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4)
 + 1024*a^3*b^10*c^21*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*
c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^13*c^
11*d^10*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 153
6*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^4*b^9*c^20*d*x*(-d^
9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11
*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^12*b*c^12*d^9*x*(-d^9/(256*b^4*c^
13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a
*b^3*c^12*d))^(5/4) + 4*a^3*b^6*c^8*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9
*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1
/4) + 6144*a^5*b^8*c^19*d^2*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*
a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a
^6*b^7*c^18*d^3*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d
^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^7*b^6*c^17*
d^4*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^
2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^9*b^4*c^15*d^6*x*(-d^9/
(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d
^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^10*b^3*c^14*d^7*x*(-d^9/(256*b^4*c^
13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a

```
*b^3*c^12*d))^(5/4) + 6144*a^11*b^2*c^13*d^8*x*(-d^9/(256*b^4*c^13 + 256*a^
4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d
))^(5/4))/(a^8*d^16 + b^8*c^8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*
b^5*c^5*d^11 + a^4*b^4*c^4*d^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7
*b*c*d^15))*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1
536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4) - atan((b^9*c^11*d^6*x*(-d
^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^1
1*d^2 - 1024*a*b^3*c^12*d))^(1/4)*4i + a^3*b^10*c^21*x*(-d^9/(256*b^4*c^13
+ 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^
3*c^12*d))^(5/4)*1024i + a^13*c^11*d^10*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9
*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5
/4)*1024i - a^4*b^9*c^20*d*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a
^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i - a
^12*b*c^12*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^
3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i + a^3*b^6*c^8*d
^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2
*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4)*4i + a^5*b^8*c^19*d^2*x*(-d^9/(25
6*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2
- 1024*a*b^3*c^12*d))^(5/4)*6144i - a^6*b^7*c^18*d^3*x*(-d^9/(256*b^4*c^13
+ 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^
3*c^12*d))^(5/4)*4096i + a^7*b^6*c^17*d^4*x*(-d^9/(256*b^4*c^13 + 256*a^4*c
^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^
(5/4)*1024i + a^9*b^4*c^15*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 10
24*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*1024i
 - a^10*b^3*c^14*d^7*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c
^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i + a^11*b^
2*c^13*d^8*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 +
1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*6144i)/(a^8*d^16 + b^8*c^
8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*b^5*c^5*d^11 + a^4*b^4*c^4*d
^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7*b*c*d^15))*(-d^9/(256*b^4*c
^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*
a*b^3*c^12*d))^(1/4)*2i - (1/(5*a*c) - (x^4*(a*d + b*c))/(a^2*c^2))/x^5
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

## 3.513 $\int \frac{x^7 \sqrt{c+dx^4}}{a+bx^4} \, dx$

**Optimal**. Leaf size=93

$$\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}} - \frac{a\sqrt{c+dx^4}}{2b^2} + \frac{\left(c+dx^4\right)^{3/2}}{6bd}$$

**Rubi [A]** time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}} + \frac{\left(c+dx^4\right)^{3/2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] -(a*Sqrt[c + d*x^4])/(2*b^2) + (c + d*x^4)^(3/2)/(6*b*d) + (a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx,x,x^4\right)\\
&= \frac{\left(c+dx^4\right)^{3/2}}{6bd}-\frac{a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^4\right)}{4b}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}-\frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b^2}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}-\frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2b^2d}\\
&= -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}+\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.11, size = 88, normalized size = 0.95

$$
\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}+\frac{\sqrt{c+dx^4}\left(b\left(c+dx^4\right)-3ad\right)}{6b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-3*a*d + b*(c + d*x^4)))/(6*b^2*d) + (a*Sqrt[b*c - a*d]*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

**IntegrateAlgebraic [A]** time = 0.11, size = 98, normalized size = 1.05

$$
\frac{\sqrt{c+dx^4}\left(-3ad+bc+bdx^4\right)}{6b^2d}-\frac{a\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]*(b*c - 3*a*d + b*d*x^4))/(6*b^2*d) - (a*Sqrt[-(b*c) + a*d]
*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(2*b^(5/
2))
```

**fricas [A]** time = 0.43, size = 195, normalized size = 2.10

$$
\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\left(bdx^4+bc-3\,ad\right)\sqrt{dx^4+c}}{12\,b^2d},\frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\left(bdx^4+bc-3\,ad\right)\sqrt{dx^4+c}}{6\,b^2d}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
+ c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(b*d*x^4 + b*c - 3*a*d)*sqrt(d
*x^4 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
```

```
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^4 + b*c - 3*a*d)*sqrt(d*x^4 +
 c))/(b^2*d)]
```

**giac [A]** time = 0.16, size = 96, normalized size = 1.03

$$-\frac{\left(abc - a^2d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b^2} + \frac{\left(dx^4+c\right)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^4+c}\,abd^3}{6\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*b*c - a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^4 + c)*a*
b*d^3)/(b^3*d^3)
```

**maple [B]** time = 0.27, size = 1015, normalized size = 10.91

$$-\frac{a^2d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^3}-\frac{a^2d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^3}+\frac{ac\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{ac\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{\sqrt{-ab}\,a\sqrt{d}\,\ln\left(\frac{\left(x^2+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{4b^3}-\frac{\sqrt{-ab}\,a\sqrt{d}\,\ln\left(\frac{\left(x^2-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{4b^3}-\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}{4b^2}-\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}{4b^2}+\frac{\left(dx^4+c\right)^{\frac{3}{2}}}{6bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d-1/4*a/b^2*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)/
b*d*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/4*a/b^3*(-a*b)^(1/2)*d^(1/2)*
ln((-(-a*b)^(1/2)/b*d+d*(x^2+(-a*b)^(1/2)/b))/d^(1/2)+((x^2+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)/b*d*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^
3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*(-a*b)^(1/2)/b*d*(x^2+(-a*b)^(1
/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)/b*d*
(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d+1/4*a/b^2/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*(-a*b)^(1/2)/b*d*(x^2+(-a*b)^(1/2
)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)/b*d*(x
^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c-1/4*a/b^2*((
x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b
)^(1/2)-1/4*a/b^3*(-a*b)^(1/2)*d^(1/2)*ln(((-a*b)^(1/2)/b*d+d*(x^2-(-a*b)^(
1/2)/b))/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(
1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/
2))/(x^2-(-a*b)^(1/2)/b))*d+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)
)/(x^2-(-a*b)^(1/2)/b))*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.69, size = 87, normalized size = 0.94

$$\frac{\left(d\,x^4+c\right)^{3/2}}{6\,b\,d} - \frac{a\,\sqrt{d\,x^4+c}}{2\,b^2} + \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d) - (a*(c + d*x^4)^(1/2))/(2*b^2) + (a*atan((a*b^(1
/2)*(c + d*x^4)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/2)
)/(2*b^(5/2))
```

**sympy [A]** time = 17.44, size = 90, normalized size = 0.97

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^4\right)^{\frac{3}{2}}}{12b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**4)/(4*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**4)/sqrt((a*d - b*c)/b))/(4*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**4)**(3
/2)/(12*b))/d**2
```

## 3.514 $\int \frac{x^5 \sqrt{c+dx^4}}{a+bx^4} \, dx$

**Optimal**. Leaf size=120

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}+\frac{x^2\sqrt{c+dx^4}}{4b}$$

**Rubi** [A] time = 0.15, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 478, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}+\frac{x^2\sqrt{c+dx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a
*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2) + ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
```

```
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\text{Subst}\left(\int \frac{ac+(-bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^2} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx}\cdots}\right.}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^2} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad\cdots}\right.}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\sqrt{a}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

**Mathematica [A]** time = 0.19, size = 114, normalized size = 0.95

$$\frac{\frac{(bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}} - 2\sqrt{a}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right) + bx^2\sqrt{c+dx^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*
x^2)/(Sqrt[a]*Sqrt[c + d*x^4])] + ((b*c - 2*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c
 + d*x^4]])/Sqrt[d])/(4*b^2)
```

**IntegrateAlgebraic [A]** time = 0.47, size = 173, normalized size = 1.44

$$\frac{(bc-2ad)\log\left(\sqrt{c+dx^4}+\sqrt{d}\,x^2\right)}{4b^2\sqrt{d}} - \frac{\sqrt{a}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{2b^2} + \frac{x^2\sqrt{c+dx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt
[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*S
qrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*b^2) + ((b*c - 2*a*d)*Log[Sq
rt[d]*x^2 + Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

**fricas [A]** time = 0.56, size = 714, normalized size = 5.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sq
rt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c
 - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2
*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - 2*(b*c - 2*a*d)
*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + sqrt(-a*b*c + a^2*d)*d*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a
^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*
d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 -
 2*sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))
 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))
/(b^2*d), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt
(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)
*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 +
(a*b*c^2 - a^2*c*d)*x^2)))/(b^2*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

**maple [B]** time = 0.28, size = 1066, normalized size = 8.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/4*x^2*(d*x^4+c)^(1/2)/b+1/4/b*c/d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))+1
/4*a/b/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-
(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4*a/b/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c-1/4*a/b/(-a*b)^(
1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*
d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
```

```
(x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4*a/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}\, x^5}{bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^5/(b*x^4 + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\, \sqrt{d\, x^4 + c}}{b\, x^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{c + dx^4}}{a + bx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```

## 3.515 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$

**Optimal**. Leaf size=70

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b) - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[b*c - a*d]])/(2*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)$$
$$= \frac{\sqrt{c+dx^4}}{2b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}$$
$$= \frac{\sqrt{c+dx^4}}{2b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}$$
$$= \frac{\sqrt{c+dx^4}}{2b} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

**Mathematica [A]** time = 0.05, size = 69, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{b} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqr
t[b*c - a*d]])/b^(3/2))/2
```

**IntegrateAlgebraic [A]** time = 0.09, size = 80, normalized size = 1.14

$$\frac{\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{3/2}} + \frac{\sqrt{c+dx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a
*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(2*b^(3/2))
```

**fricas [A]** time = 0.45, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}}{4\,b}, -\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^4+c}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^4 + c))/b]
```

**giac [A]** time = 0.18, size = 66, normalized size = 0.94

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b}+\frac{\sqrt{dx^4+c}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b
```

**maple [B]** time = 0.20, size = 988, normalized size = 14.11

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}-\frac{c\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{c\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x^2+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{4b^2}+\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x^2-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{4b^2}+\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4b}+\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/4/b*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)-1/4/b^2*(-a*b)^(1/2)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a
*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x^2+(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x^2+(-a*b)^(1/2)/b))*c+1/4/b*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*(-a*b)^(1/2)*d^(1/2)*ln((
(x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.66, size = 54, normalized size = 0.77

$$\frac{\sqrt{d\,x^4+c}}{2\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b) - (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/
2))*(a*d - b*c)^(1/2))/(2*b^(3/2))
```

**sympy** [**A**] time = 8.18, size = 65, normalized size = 0.93

$$\frac{2\left(\frac{d\sqrt{c+dx^4}}{4b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d
 - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d
```

## 3.516 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

**Optimal**. Leaf size=91

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

**Rubi [A]** time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {465, 402, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(
2*Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(2*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx, x, x^2\right) \\
&= \frac{d\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b} \\
&= \frac{d\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b} \\
&= \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b} + \frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}
\end{aligned}
$$

**Mathematica** [A] time = 0.07, size = 89, normalized size = 0.98

$$
\frac{\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{\sqrt{a}} + \sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{c+dx^4}+dx^2\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] ((Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/
Sqrt[a] + Sqrt[d]*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/(2*b)
```

**IntegrateAlgebraic** [A] time = 0.35, size = 144, normalized size = 1.58

$$
\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2\sqrt{a}\,b} + \frac{\sqrt{d}\,\log\left(\sqrt{c+dx^4}+\sqrt{d}\,x^2\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)
/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*
d])])/(2*Sqrt[a]*b) + (Sqrt[d]*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/(2*b)
```

**fricas** [A] time = 0.49, size = 612, normalized size = 6.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-(
b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, -1/8*(4*sqrt(-d
)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4
*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
```

```
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d
)*x^2 - c))/b, -1/4*(2*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt
((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((
b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)))/b]
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

**maple** [**B**] time = 0.20, size = 1000, normalized size = 10.99

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{c\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}+\frac{c\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}+\frac{\sqrt{d}\,\ln\left(\frac{\left(x^2+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{4b}+\frac{\sqrt{d}\,\ln\left(\frac{\left(x^2-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{4b}-\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\sqrt{-ab}}+\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] -1/4/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)
^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*a*d+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c+1/4/(-a*b)^(1/2)*((x^2-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4
/b*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+
1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/
b))*a*d-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^
(1/2)/b))*c
```

**maxima** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}\, x}{bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x/(b*x^4 + a), x)
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{d\,x^4+c}}{b\,x^4+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c + dx^4}}{a + bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

## 3.517 $\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx$

**Optimal**. Leaf size=85

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

**Rubi [A]** time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a) + (Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx = \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^4\right)$$
$$= \frac{c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}$$
$$= \frac{c\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad}$$
$$= -\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a} + \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}$$

**Mathematica** [A] time = 0.09, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} - \sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] (-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/Sqrt[b])/(2*a)
```

**IntegrateAlgebraic** [A] time = 0.12, size = 95, normalized size = 1.12

$$-\frac{\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2a\sqrt{b}} - \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] -1/2*(Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4]
)/(b*c - a*d)])/(a*Sqrt[b]) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2
*a)
```

**fricas** [A] time = 0.46, size = 383, normalized size = 4.51

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right)}{4\,a}, \frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right)}{4\,a}, \frac{2\,\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)+\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)}{4\,a}, \frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-c}}{c}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*
sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 +
c)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^4 + a)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
```

```
(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
4 + c)*sqrt(-c)/c))/a]
```

**giac** [A] time = 0.18, size = 79, normalized size = 0.93

$$-\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a}+\frac{c\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 1/2*c*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

**maple** [B] time = 0.26, size = 1037, normalized size = 12.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x/(b*x^4+a),x)
```

```
[Out] -1/4/a*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)+1/4/a/b*(-a*b)^(1/2)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-
a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x^2+(-a*b)^(1/2)/b))*d+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)
*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
x^2+(-a*b)^(1/2)/b))*c-1/4/a*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a/b*(-a*b)^(1/2)*d^(1/2)*ln(((x^
2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/b/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4/a/(-(a*d-b*c)/b)^(1/2
)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b
)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c+1/2/a*(d*x^4+c)^(1/2)-1/2/a*c^(1
/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)
```

**mupad** [B] time = 4.87, size = 199, normalized size = 2.34

$$\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c}\left(\sqrt{d\,x^4+c}\left(\frac{a^2\,b\,d^4}{2}-a\,b^2\,c\,d^3+b^3\,c^2\,d^2\right)+\frac{c\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^4+c}}{16\,a^2}\right)}{2\,a\left(\frac{b^2\,c^2\,d^3}{4}-\frac{a\,b\,c\,d^4}{4}\right)}\right)}{2\,a}+\frac{\mathrm{atanh}\left(\frac{a\,b^2\,c\,d^3\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\left(\frac{a\,b^3\,c^2\,d^3}{4}-\frac{a^2\,b^2\,c\,d^4}{4}\right)}\right)\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x*(a + b*x^4)),x)
```

```
[Out] (c^(1/2)*atanh((c^(1/2)*((c + d*x^4)^(1/2)*((a^2*b*d^4)/2 + b^3*c^2*d^2 - a
*b^2*c*d^3) + (c*(8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2))/(16*
a^2)))/(2*a*((b^2*c^2*d^3)/4 - (a*b*c*d^4)/4))))/(2*a) + (atanh((a*b^2*c*d^
3*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*((a*b^3*c^2*d^3)/4 - (a^2*b^2
*c*d^4)/4)))*(b^2*c - a*b*d)^(1/2))/(2*a*b)
```

**sympy [A]** time = 12.05, size = 82, normalized size = 0.96

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x/(b*x**4+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/b)))/d
```

## 3.518 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx$

**Optimal**. Leaf size=76

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 475, 12, 377, 205}

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(2*a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/
(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^4}}{2ax^2} + \frac{\operatorname{Subst}\left(\int \frac{-bc+ad}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{\sqrt{c+dx^4}}{2ax^2} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{\sqrt{c+dx^4}}{2ax^2} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}$$

$$= -\frac{\sqrt{c+dx^4}}{2ax^2} - \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{3/2}}$$

**Mathematica [C]** time = 0.06, size = 53, normalized size = 0.70

$$-\frac{\sqrt{c+dx^4}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{(ad-bc)x^4}{a\left(dx^4+c\right)}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c + d*x^4]*Hypergeometric2F1[-1/2, 1, 1/2, ((-(b*c) + a*d)*x^4)/
(a*(c + d*x^4))])/(a*x^2)
```

**IntegrateAlgebraic [A]** time = 0.39, size = 128, normalized size = 1.68

$$-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2a^{3/2}} - \frac{\sqrt{c+dx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -1/2*Sqrt[c + d*x^4]/(a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sq
rt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c +
 d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2))
```

**fricas [A]** time = 0.46, size = 281, normalized size = 3.70

$$\left[\frac{x^2\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)-4\sqrt{dx^4+c}}{8\,ax^2}, -\frac{x^2\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^4-ac\right)\sqrt{dx^4+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^6+\left(bc^2-acd\right)x^2\right)}\right)+2\sqrt{dx^4+c}}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(x^2*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c
*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)) -
4*sqrt(d*x^4 + c))/(a*x^2), -1/4*(x^2*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + 2*sqrt(d*x^4 + c))/(a*x^2)]
```

**giac [B]** time = 1.29, size = 121, normalized size = 1.59

$$\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,a}+\frac{c\sqrt{d}}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2-c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + c*
sqrt(d)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)*a)
```

**maple [B]** time = 0.26, size = 1075, normalized size = 14.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x)
```

```
[Out] -1/2/a/c/x^2*(d*x^4+c)^(3/2)+1/2/a/c*d*x^2*(d*x^4+c)^(1/2)+1/2/a*d^(1/2)*ln
(d^(1/2)*x^2+(d*x^4+c)^(1/2))+1/4/a*b/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a*d^(1/2)*
ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^
(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4/a*
b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
*c-1/4/a*b/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+
(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b
)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-
b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4+c}}{x^3\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)),x)
```

```
[Out] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^3 \left(a + bx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)
```

## 3.519 $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$

**Optimal**. Leaf size=115

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*a*x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]
])/(4*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^4])/Sqrt[b*c - a*d]])/(2*a^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4}+\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4}+\frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2}-\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,d\right.}{8a^2}\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4}+\frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d}-\frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+}\right.}{4a^2d}\\
&= -\frac{\sqrt{c+dx^4}}{4ax^4}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}
\end{aligned}
$$

**Mathematica [A]** time = 0.13, size = 107, normalized size = 0.93

$$
\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}-2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)-\frac{a\sqrt{c+dx^4}}{x^4}}{4a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^4])/x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/S
qrt[b*c - a*d]])/(4*a^2)
```

**IntegrateAlgebraic [A]** time = 0.27, size = 125, normalized size = 1.09

$$
\frac{\sqrt{b}\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{c+dx^4}}{4ax^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*x^4) + (Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(2*a^2) + ((2*b*c - a*d)*
ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a^2*Sqrt[c])
```

**fricas [A]** time = 0.47, size = 513, normalized size = 4.46

$$
\left[\frac{2\sqrt{b^2c-abd}\,cx^4\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-(2bc-ad)\sqrt{c}\,x^4\log\left(\frac{dx^4-2\sqrt{dx^4+c}\sqrt{c}+2c}{x^4}\right)-2\sqrt{dx^4+c}\,ac}{8a^2cx^4}, \frac{4\sqrt{-b^2c+abd}\,cx^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2bc-ad)\sqrt{c}\,x^4\log\left(\frac{dx^4-2\sqrt{dx^4+c}\sqrt{c}+2c}{x^4}\right)-2\sqrt{dx^4+c}\,ac}{8a^2cx^4}, -\frac{(2bc-ad)\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}\,cx^4\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+\sqrt{dx^4+c}\,ac}{4a^2cx^4}, \frac{2\sqrt{-b^2c+abd}\,cx^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2bc-ad)\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)-\sqrt{dx^4+c}\,ac}{4a^2cx^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4
 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*
x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c
*x^4), 1/8*(4*sqrt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*x^4 - 2*sqrt(
d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), -1/4*(
(2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c -
a*b*d))/(b*x^4 + a)) + sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), 1/4*(2*sqrt(-b^2*c
 + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(d*
x^4 + c)*a*c)/(a^2*c*x^4)]
```

**giac** [A] time = 0.17, size = 107, normalized size = 0.93

$$\frac{\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}-\frac{\sqrt{dx^4+c}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/4*(2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/4*sqrt(d*x^4 + c)/(a*x^4)
```

**maple** [B] time = 0.27, size = 1107, normalized size = 9.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x)
```

```
[Out] -1/4/a/c/x^4*(d*x^4+c)^(3/2)-1/4/a/c^(1/2)*d*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1
/2))/x^2)+1/4/a/c*d*(d*x^4+c)^(1/2)+1/4/a^2*b*((x^2+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a^2*(-a*b)^(1/2
)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1
/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4/a^2
*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^
2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c+1/4/a^2*b
*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)+1/4/a^2*(-a*b)^(1/2)*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(
1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*
(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
^2-(-a*b)^(1/2)/b))*d-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-
(-a*b)^(1/2)/b))*c-1/2/a^2*b*(d*x^4+c)^(1/2)+1/2/a^2*b*c^(1/2)*ln((2*c+2*(d
*x^4+c)^(1/2)*c^(1/2))/x^2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)
```

**mupad [B]** time = 5.39, size = 269, normalized size = 2.34

$$\frac{\mathrm{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{16\,\left(\frac{a\,b^3\,d^5}{16}-\frac{b^4\,c\,d^4}{16}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{2\,a^2}-\frac{\sqrt{d\,x^4+c}}{4\,a\,x^4}-\frac{\mathrm{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^4+c}}{16\,\left(\frac{b^4\,c\,d^4}{16}-\frac{3\,a\,b^3\,d^5}{32}+\frac{a^2\,b^2\,d^6}{32\,c}\right)}-\frac{3\,b^3\,d^5\,\sqrt{d\,x^4+c}}{32\,\sqrt{c}\,\left(\frac{a\,b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32}+\frac{b^4\,c\,d^4}{16\,a}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}}{32\,c^{3/2}\,\left(\frac{b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32\,a}+\frac{b^4\,c\,d^4}{16\,a^2}\right)}\right)\,\left(a\,d-2\,b\,c\right)}{4\,a^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^5*(a + b*x^4)),x)
```

```
[Out] (atanh((b^3*d^4*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(16*((a*b^3*d^5)/1
6 - (b^4*c*d^4)/16)))*(b^2*c - a*b*d)^(1/2))/(2*a^2) - (c + d*x^4)^(1/2)/(4
*a*x^4) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^4)^(1/2))/(16*((b^4*c*d^4)/16 -
(3*a*b^3*d^5)/32 + (a^2*b^2*d^6)/(32*c))) - (3*b^3*d^5*(c + d*x^4)^(1/2))/(
32*c^(1/2)*((a*b^2*d^6)/(32*c) - (3*b^3*d^5)/32 + (b^4*c*d^4)/(16*a))) + (b
^2*d^6*(c + d*x^4)^(1/2))/(32*c^(3/2)*((b^2*d^6)/(32*c) - (3*b^3*d^5)/(32*a
) + (b^4*c*d^4)/(16*a^2))))*(a*d - 2*b*c))/(4*a^2*c^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^5\left(a + bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)
```

## 3.520 $\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)} dx$

**Optimal**. Leaf size=110

$$\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^4}\,(3bc-ad)}{6a^2cx^2}-\frac{\sqrt{c+dx^4}}{6ax^6}$$

**Rubi [A]** time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 475, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}\,(3bc-ad)}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(6*a*x^6) + ((3*b*c - a*d)*Sqrt[c + d*x^4])/(6*a^2*c*x^2)
+ (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])
])/(2*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{\text{Subst}\left(\int \frac{-3bc+ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6a}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}-\frac{\text{Subst}\left(\int -\frac{3bc(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6a^2c}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 5.33, size = 130, normalized size = 1.18

$$\frac{3bc^2x^4\sqrt{\frac{dx^4}{c}+1}\,\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}\,\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)+\left(c+dx^4\right)\left(3bcx^4-a\left(c+dx^4\right)\right)}{6a^2cx^6\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] ((c + d*x^4)*(3*b*c*x^4 - a*(c + d*x^4)) + 3*b*c^2*x^4*Sqrt[(b/a - d/c)*x^4
]*Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(6
*a^2*c*x^6*Sqrt[c + d*x^4])
```

**IntegrateAlgebraic [A]** time = 0.53, size = 151, normalized size = 1.37

$$\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^4}\,\left(-ac-adx^4+3bcx^4\right)}{6a^2cx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 - a*d*x^4))/(6*a^2*c*x^6) + (b*Sqrt[b*
c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a
```

```
]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2
*a^(5/2))
```

**fricas [A]** time = 0.52, size = 329, normalized size = 2.99

$$\left[\frac{3\,bcx^{6}\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{8}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{4}+a^{2}c^{2}+4\left(\left(abc-2\,a^{2}d\right)x^{6}-a^{2}cx^{2}\right)\sqrt{dx^{4}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{8}+2\,abx^{4}+a^{2}}\right)+4\left(\left(3\,bc-ad\right)x^{4}-ac\right)\sqrt{dx^{4}+c}}{24\,a^{2}cx^{6}},\ \frac{3\,bcx^{6}\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{4}-ac\right)\sqrt{dx^{4}+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{6}+\left(bc^{2}-acd\right)x^{2}\right)}\right)+2\left(\left(3\,bc-ad\right)x^{4}-ac\right)\sqrt{dx^{4}+c}}{12\,a^{2}cx^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6
- a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a
^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(3*b*
c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
 c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*((
3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]
```

**giac [B]** time = 1.75, size = 225, normalized size = 2.05

$$-\frac{\left(b^{2}c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{2}b-bc+2\,ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{2\sqrt{abcd-a^{2}d^{2}}\,a^{2}}-\frac{3\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{4}bc\sqrt{d}-3\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{4}ad^{\frac{3}{2}}-6\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}bc^{2}\sqrt{d}+3\,bc^{3}\sqrt{d}-ac^{2}d^{\frac{3}{2}}}{3\left(\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}-c\right)^{3}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 +
c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^
2) - 1/3*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2*
sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2 - c)^3*a^2)
```

**maple [B]** time = 0.26, size = 1116, normalized size = 10.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x)
```

```
[Out] 1/2/a^2*b/c/x^2*(d*x^4+c)^(3/2)-1/2/a^2*b/c*d*x^2*(d*x^4+c)^(1/2)-1/2/a^2*b
*d^(1/2)*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))-1/4/a^2*b^2/(-a*b)^(1/2)*((x^2+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
+1/4/a^2*b*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x
^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))-1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-
a*b)^(1/2)/b))*d+1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x^2+(-a*b)^(1/2)/b))*c+1/4/a^2*b^2/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a^2*b*
d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(
1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4
/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)
```

```
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b
))*d-1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-
a*b)^(1/2)/b))*c-1/6/a/x^6*(d*x^4+c)^(3/2)/c
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}}{(bx^4 + a)x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4 + c}}{x^7\,\left(b\,x^4 + a\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^7\left(a + bx^4\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)
```

## 3.521 $\int \frac{x^{11}}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}\,(ad+bc)}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}$$

**Rubi [A]** time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}\,(ad+bc)}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^4])/(2*b^2*d^2) + (c + d*x^4)^(3/2)/(6*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*
c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^4\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^4}\left(-3ad-2bc+bdx^4\right)}{6b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

**IntegrateAlgebraic [A]** time = 0.18, size = 101, normalized size = 0.97

$$\frac{\sqrt{c+dx^4}\left(-3ad-2bc+bdx^4\right)}{6b^2d^2}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{5/2}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) - (a^2*ArcTan[(Sqr
t[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(2*b^(5/2)*Sqrt[-(b*
c) + a*d])
```

**fricas [A]** time = 0.72, size = 289, normalized size = 2.78

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-2\left(2b^3c^2+ab^2cd-3a^2bd^2-\left(b^3cd-ab^2d^2\right)x^4\right)\sqrt{dx^4+c}}{12\left(b^4cd^2-ab^3d^3\right)},\ \frac{3\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-\left(2b^3c^2+ab^2cd-3a^2bd^2-\left(b^3cd-ab^2d^2\right)x^4\right)\sqrt{dx^4+c}}{6\left(b^4cd^2-ab^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*
x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a
^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*
d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

**giac** [A] time = 0.16, size = 106, normalized size = 1.02

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b^2}+\frac{\left(dx^4+c\right)^{\frac{3}{2}}b^2d^4-3\sqrt{dx^4+c}\,b^2cd^4-3\sqrt{dx^4+c}\,abd^5}{6\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^4 + c)*b^2*c*d^4 - 3*s
qrt(d*x^4 + c)*a*b*d^5)/(b^3*d^6)
```

**maple** [B] time = 0.28, size = 378, normalized size = 3.63

$$\frac{\sqrt{d\,x^4+c}\;x^4}{6bd}-\frac{a^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\;b^3}-\frac{a^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\;b^3}-\frac{\sqrt{d\,x^4+c}\;a}{2b^2d}-\frac{\sqrt{d\,x^4+c}\;c}{3b\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/6/b*(d*x^4+c)^(1/2)/d*x^4-1/3/b*(d*x^4+c)^(1/2)/d^2*c-1/2/b^2*a/d*(d*x^4+
c)^(1/2)-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/
2)/b))-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/
b))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 4.82, size = 102, normalized size = 0.98

$$\frac{{\left(d\,x^4+c\right)}^{3/2}}{6\,b\,d^2}-\left(\frac{c}{b\,d^2}+\frac{2\,a\,d^3-2\,b\,c\,d^2}{4\,b^2\,d^4}\right)\,\sqrt{d\,x^4+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d^2) - (c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^4
))*(c + d*x^4)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1
/2)))/(2*b^(5/2)*(a*d - b*c)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.522 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{2bd}$$

**Rubi [A]** time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)$$
$$= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}$$
$$= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}$$
$$= \frac{\sqrt{c+dx^4}}{2bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.08, size = 72, normalized size = 0.97

$$\frac{1}{2}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^4}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/2
```

**IntegrateAlgebraic** [A] time = 0.10, size = 84, normalized size = 1.14

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{3/2}\sqrt{ad-bc}} + \frac{\sqrt{c+dx^4}}{2bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*
x^4])/(b*c - a*d)])/(2*b^(3/2)*Sqrt[-(b*c) + a*d])
```

**fricas** [A] time = 0.74, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(b^2c-abd\right)}{4\left(b^3cd-ab^2d^2\right)}, -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-\sqrt{dx^4+c}\left(b^2c-abd\right)}{2\left(b^3cd-ab^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c
)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

**giac [A]** time = 0.19, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^4+c}}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^4 + c)/b)/d
```

**maple [B]** time = 0.21, size = 335, normalized size = 4.53

$$\frac{a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{\sqrt{d\,x^4+c}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b/d+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*
(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
^2+(-a*b)^(1/2)/b))+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-
a*b)^(1/2)/b))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^4+c}}{2\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b*d) - (a*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)
^(1/2)))/(2*b^(3/2)*(a*d - b*c)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.523 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(2*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

**IntegrateAlgebraic** [A] time = 0.05, size = 61, normalized size = 1.20

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)]/(Sqrt
[b]*Sqrt[-(b*c) + a*d])
```

**fricas** [A] time = 0.69, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)}{4\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)}{2\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b
*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]
```

**giac** [A] time = 0.16, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

**maple** [B] time = 0.21, size = 316, normalized size = 6.20

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4/b/(
-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [**B**] time = 4.80, size = 40, normalized size = 0.78

$$\frac{\mathrm{atan}\left(\frac{b\,\sqrt{d\,x^4+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{2\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^4)^(1/2))/(a*b*d - b^2*c)^(1/2))/(2*(a*b*d - b^2*c)^(1/2))
```

**sympy** [**A**] time = 14.27, size = 37, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c)/b))
```

## 3.524 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}} \, dx$

**Optimal**. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

**Rubi** [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^4\right)}{4a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.09, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^4])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(2*a)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 95, normalized size = 1.12

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2a\sqrt{ad-bc}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/
(2*a*Sqrt[-(b*c) + a*d]) - ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a*Sqrt[c])
```

**fricas [A]** time = 0.73, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\sqrt{c}+2c}{x^4}\right)}{4ac}, \frac{2c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+\sqrt{c}\log\left(\frac{dx^4-2\sqrt{dx^4+c}\sqrt{c}+2c}{x^4}\right)}{4ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)}{4ac}, \frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)}{2ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt
(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt
(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-
c)/c))/(a*c), 1/2*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^4 + c)*
sqrt(-c)/c))/(a*c)]
```

**giac [A]** time = 0.17, size = 71, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

**maple [B]** time = 0.29, size = 347, normalized size = 4.08

$$\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{\ln\left(\frac{2c+2\sqrt{dx^4+c}\,\sqrt{c}}{x^2}\right)}{2a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4/a/(-
(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/2/a/c^(1/2)*ln(
(2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)
```

**mupad [B]** time = 5.03, size = 652, normalized size = 7.67

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{dx^4+c}}{\sqrt{c}}\right)}{2a\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2c-abd}\left(b^3d^2\sqrt{dx^4+c}-\frac{\sqrt{b^2c-abd}\left(2a^2b^2d^3-\frac{\left(8a^3b^2d^3-16a^2b^3cd^2\right)\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{4\left(a^2d-abc\right)}\right)}{4\left(a^2d-abc\right)}\right)1\mathrm{i}}{4\left(a^2d-abc\right)}+\frac{\sqrt{b^2c-abd}\left(b^3d^2\sqrt{dx^4+c}+\frac{\sqrt{b^2c-abd}\left(2a^2b^2d^3+\frac{\left(8a^3b^2d^3-16a^2b^3cd^2\right)\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{4\left(a^2d-abc\right)}\right)}{4\left(a^2d-abc\right)}\right)1\mathrm{i}}{4\left(a^2d-abc\right)}}{\frac{\sqrt{b^2c-abd}\left(b^3d^2\sqrt{dx^4+c}-\frac{\sqrt{b^2c-abd}\left(2a^2b^2d^3-\frac{\left(8a^3b^2d^3-16a^2b^3cd^2\right)\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{4\left(a^2d-abc\right)}\right)}{4\left(a^2d-abc\right)}\right)}{4\left(a^2d-abc\right)}-\frac{\sqrt{b^2c-abd}\left(b^3d^2\sqrt{dx^4+c}+\frac{\sqrt{b^2c-abd}\left(2a^2b^2d^3+\frac{\left(8a^3b^2d^3-16a^2b^3cd^2\right)\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{4\left(a^2d-abc\right)}\right)}{4\left(a^2d-abc\right)}\right)}{4\left(a^2d-abc\right)}}\right)\sqrt{b^2c-abd}\,1\mathrm{i}}{2\left(a^2d-abc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^4)^(1/2)/c^(1/2))/(2*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 -
((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2)
)/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a*b*c)) + ((b^
2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a
^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c -
 a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a
```

```
*b*c)))/(((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*
d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(
1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))/(4*
(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^
2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c
+ d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b
*c))))/(4*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(2*(a^2*d - a*b*c))
```

**sympy** [A] time = 20.21, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**4)/sqrt(-c))/(2*a*sqrt(-c))
```

## 3.525 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

**Rubi** [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*a*c*x^4) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[
c]])/(4*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c
- a*d]])/(2*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+bx^4\right) \sqrt{c+dx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{8a^2c}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{4a^2cd}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.17, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2(ad-bc)} + \frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4ac^{3/2}} - \frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*c*x^4) + (b*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a^
2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a*c^(3/2)) + (b^(3/2)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a^2*
(-(b*c) + a*d))
```

**IntegrateAlgebraic [A]** time = 0.25, size = 127, normalized size = 1.09

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2\sqrt{ad-bc}} + \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*c*x^4) - (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^4])/(b*c - a*d)])/(2*a^2*Sqrt[-(b*c) + a*d]) + ((2*b*c + a*d
)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a^2*c^(3/2))
```

**fricas [A]** time = 0.70, size = 565, normalized size = 4.83

$$\left[\frac{2bc^2x^4\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+(2bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\sqrt{dx^4+c}\sqrt{c}+2c}{x^4}\right)-2\sqrt{dx^4+c}ac}{8a^2c^2x^4}, -\frac{4bc^2x^4\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)-(2bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\sqrt{dx^4+c}\sqrt{c}+2c}{x^4}\right)+2\sqrt{dx^4+c}ac}{8a^2c^2x^4}, \frac{bc^2x^4\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)-(2bc+ad)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)-\sqrt{dx^4+c}ac}{4a^2c^2x^4}, -\frac{2bc^2x^4\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+(2bc+ad)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)+\sqrt{dx^4+c}ac}{4a^2c^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d
*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + (2*b*c + a*d)*sqr
t(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4
+ c)*a*c)/(a^2*c^2*x^4), -1/8*(4*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*sqrt
(d*x^4 + c)*a*c)/(a^2*c^2*x^4), 1/4*(b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d
*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^4 + a)) - (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) -
 sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4), -1/4*(2*b*c^2*x^4*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + sqrt(d*x^
4 + c)*a*c)/(a^2*c^2*x^4)]
```

**giac [A]** time = 0.17, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}\,c} - \frac{\sqrt{dx^4+c}}{4\,acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/4*(2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/4*sqrt(d*x^4 + c)/(a*c*x^4)
```

**maple [B]** time = 0.29, size = 402, normalized size = 3.44

$$-\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a^2} - \frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a^2} + \frac{d\ln\left(\frac{2c+2\sqrt{d\,x^4+c}\,\sqrt{c}}{x^2}\right)}{4a\,c^{\frac{3}{2}}} + \frac{b\ln\left(\frac{2c+2\sqrt{d\,x^4+c}\,\sqrt{c}}{x^2}\right)}{2a^2\sqrt{c}} - \frac{\sqrt{d\,x^4+c}}{4ac\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4*(d*x^4+c)^(1/2)/a/c/x^4+1/4/a*d/c^(3/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1
/2))/x^2)-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2
)/b))-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
+1/2/a^2*b/c^(1/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)
```

**mupad [B]** time = 5.35, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^4+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,a^3\,d-4\,a^2\,b\,c} - \frac{\ln\left(\sqrt{d\,x^4+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,\left(a^3\,d-a^2\,b\,c\right)} - \frac{\sqrt{d\,x^4+c}}{4\,a\,c\,x^4} - \frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^4+c}\,3{}\mathrm{i}}{16\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,c}+\frac{5\,a\,b^3\,d^5}{32\,c^2}+\frac{a^2\,b^2\,d^6}{32\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}\,1{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{32\,a}+\frac{b^2\,d^6}{32\,c}+\frac{3\,b^4\,c\,d^4}{16\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^4+c}\,5{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,a}+\frac{5\,b^3\,d^5}{32\,c}+\frac{a\,b^2\,d^6}{32\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{4\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (log((c + d*x^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(4*a^3*d - 4*a^2*b*c) - (log((c + d*x^4
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(4*(a^3*d - a^2*b*c)) - (c + d*x^4)^(1/2)/(4*a*c*x^4)
- (atan((b^4*d^4*(c + d*x^4)^(1/2)*3i)/(16*(c^3)^(1/2)*((3*b^4*d^4)/(16*c)
+ (5*a*b^3*d^5)/(32*c^2) + (a^2*b^2*d^6)/(32*c^3))) + (b^2*d^6*(c + d*x^4)^
(1/2)*1i)/(32*(c^3)^(1/2)*((5*b^3*d^5)/(32*a) + (b^2*d^6)/(32*c) + (3*b^4*c
*d^4)/(16*a^2))) + (b^3*d^5*(c + d*x^4)^(1/2)*5i)/(32*(c^3)^(1/2)*((3*b^4*d
^4)/(16*a) + (5*b^3*d^5)/(32*c) + (a*b^2*d^6)/(32*c^2))))*(a*d + 2*b*c)*1i)
/(4*a^2*(c^3)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.526 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

**Rubi [A]** time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt
[a]*Sqrt[c + d*x^4])])/(2*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4bd}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2} - \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,\right.}{4b^2d}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2} - \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx\right.}{4b^2d}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}$$

**Mathematica [A]** time = 0.21, size = 118, normalized size = 0.96

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\,\sqrt{c+dx^4}+dx^2\right)}{d^{3/2}} + \frac{bx^2\sqrt{c+dx^4}}{d}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*x^2*Sqrt[c + d*x^4])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[
a]*Sqrt[c + d*x^4])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^2 + Sqrt[d]*
Sqrt[c + d*x^4]])/d^(3/2))/(4*b^2)
```

**IntegrateAlgebraic [A]** time = 0.70, size = 177, normalized size = 1.44

$$\frac{a^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2b^2\sqrt{bc-ad}} + \frac{(-2ad-bc)\log\left(\sqrt{c+dx^4}+\sqrt{d}\,x^2\right)}{4b^2d^{3/2}} + \frac{x^2\sqrt{c+dx^4}}{4bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c
- a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4]
)/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*b^2*Sqrt[b*c - a*d]) + ((-(b*c) - 2*a*d)*L
og[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))
```

**fricas** [A]  time = 0.75, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^4+c}\,bdx^2 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4\left((b^2c^2-3abcd+2a^2d^2)x^6-(abc^2-a^2cd)x^2\right)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2abx^4+a^2}\right) + (bc+2ad)\sqrt{d}\log\left(-2dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{8b^2d^2}, \frac{2\sqrt{dx^4+c}\,bdx^2 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4\left((b^2c^2-3abcd+2a^2d^2)x^6-(abc^2-a^2cd)x^2\right)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2abx^4+a^2}\right) + 2(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)}{8b^2d^2}, \frac{2\sqrt{dx^4+c}\,bdx^2 - 2ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^6+acx^2)}\right) + (bc+2ad)\sqrt{d}\log\left(-2dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{8b^2d^2}, \frac{\sqrt{dx^4+c}\,bdx^2 - ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^6+acx^2)}\right) + (bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)}{4b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^
4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/8*(2
*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-
d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2), 1/8*(2*sqrt(d*x^4 + c)*
b*d*x^2 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/4*(s
qrt(d*x^4 + c)*b*d*x^2 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*
d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b
*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2)]
```

**giac** [F(-2)]  time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

**maple** [B]  time = 0.32, size = 408, normalized size = 3.32

$$-\frac{a^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{a^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{\sqrt{d\,x^4+c}\;x^2}{4bd}-\frac{a\ln\left(\sqrt{d}\;x^2+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}-\frac{c\ln\left(\sqrt{d}\;x^2+\sqrt{d\,x^4+c}\right)}{4b\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4*x^2*(d*x^4+c)^(1/2)/b/d-1/4/b*c/d^(3/2)*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))
-1/2/b^2*a*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)+1/4*a^2/b^2/(-a*b)^(1/2)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4*a^2/b^2/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

**maxima** [F]  time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.527 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

**Rubi [A]** time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 90, normalized size = 0.99

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{\sqrt{bc-ad}}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/Sqrt[b
*c - a*d]) + Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]]/Sqrt[d])/(2*b)
```

**IntegrateAlgebraic [A]** time = 0.46, size = 144, normalized size = 1.58

$$\frac{\log\left(\sqrt{c+dx^4}+\sqrt{d}\,x^2\right)}{2b\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\sqrt{bc-ad}} + \frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{2b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(S
qrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])
])/(b*Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]]/(2*b*Sqrt[d])
```

**fricas [A]** time = 0.67, size = 632, normalized size = 6.95

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^8-2\left(3abc^2-4a^2cd\right)x^4+a^2c^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^6-\left(abc^2-a^2cd\right)x^2\right)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2abx^4+a^2}\right)+2\sqrt{d}\log\left(-2dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{8bd}, \frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^8-2\left(3abc^2-4a^2cd\right)x^4+a^2c^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^6-\left(abc^2-a^2cd\right)x^2\right)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2abx^4+a^2}\right)-4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)}{8bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^4-ac\right)\sqrt{dx^4+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^6+acx^2\right)}\right)+\sqrt{d}\log\left(-2dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)}{4bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^4-ac\right)\sqrt{dx^4+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^6+acx^2\right)}\right)-2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)}{4bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^8 + 2*a*b*x^4 + a^2)) + 2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt
(d)*x^2 - c))/(b*d), 1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
```

```
+ 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 -
3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(
-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*
x^2/sqrt(d*x^4 + c)))/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c -
 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2))
 + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b*d), 1/4*(d
*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - 2*sqrt(-d)*arctan(sqrt(-d)*x^2/s
qrt(d*x^4 + c)))/(b*d)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

**maple** [B] time = 0.33, size = 356, normalized size = 3.91

$$\frac{a \ln \left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{a \ln \left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}+\frac{\ln \left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)}{2b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/b*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)-1/4*a/b/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4*a/b/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(bx^{4}+a\right)\sqrt{dx^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5}}{\left(b\,x^{4}+a\right)\,\sqrt{d\,x^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.528 $\int \frac{x}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

**Rubi [A]** time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 95, normalized size = 1.76

$$\frac{x^2\sqrt{\frac{dx^4}{c}+1}\tanh^{-1}\left(\frac{\sqrt{\frac{dx^4}{c}-\frac{bx^4}{a}}}{\sqrt{\frac{dx^4}{c}+1}}\right)}{2a\sqrt{c+dx^4}\sqrt{\frac{dx^4}{c}-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^4)/c]*ArcTanh[Sqrt[-((b*x^4)/a) + (d*x^4)/c]/Sqrt[1 + (d
*x^4)/c]])/(2*a*Sqrt[c + d*x^4]*Sqrt[-((b*x^4)/a) + (d*x^4)/c])
```

**IntegrateAlgebraic** [A] time = 0.32, size = 106, normalized size = 1.96

$$\frac{\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\sqrt{bc-ad}}+\frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*
c - a*d]) + (b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])]/(2*Sqrt[a]*S
qrt[b*c - a*d])
```

**fricas** [B] time = 0.51, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{8\left(abc-a^2d\right)},\ \frac{\arctan\left(\frac{\left((bc-2\,ad)x^4-ac\right)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^6+\left(abc^2-a^2cd\right)x^2\right)}\right)}{4\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt
(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))/(a*b*c - a^2
*d), 1/4*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c -
a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/sqrt(a*b*c - a^
2*d)]
```

**giac** [A] time = 0.18, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

**maple [B]** time = 0.32, size = 322, normalized size = 5.96

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b
))-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)
/b))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^{4}+a\right)\sqrt{dx^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^{4}+a\right)\,\sqrt{d\,x^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^{4}\right)\sqrt{c+dx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.529 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(2*a*c*x^2) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqr
t[c + d*x^4])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2ac}\\ &= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\ &= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\ &= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}\end{aligned}$$

**Mathematica [C]** time = 4.83, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^4}{c}+1\right)\left(\frac{4x^4\left(c+dx^4\right)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)}{3c^2\left(a+bx^4\right)}+\frac{\left(c+2dx^4\right)\sin^{-1}\left(\sqrt{\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}}\right)}{c\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}}}\right)}{2x^2\left(a+bx^4\right)\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*((1 + (d*x^4)/c)*(((c + 2*d*x^4)*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a +
 b*x^4))]])/(c*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)^2)]) +
 (4*(b*c - a*d)*x^4*(c + d*x^4)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x
^4)/(c*(a + b*x^4))])/(3*c^2*(a + b*x^4))))/(x^2*(a + b*x^4)*Sqrt[c + d*x^4
])
```

**IntegrateAlgebraic [A]** time = 0.48, size = 142, normalized size = 1.78

$$\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\sqrt{bc-ad}}+\frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{2a^{3/2}(ad-bc)}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^4]/(a*c*x^2) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d]
)/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt
[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])]])/(2*a^(3/2)*(-(b*c) + a*d))
```

**fricas [B]** time = 0.53, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^2\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\sqrt{dx^4+c}\left(abc-a^2d\right)}{8\left(a^2bc^2-a^3cd\right)x^2},\ -\frac{\sqrt{abc-a^2d}\,bcx^2\arctan\left(\frac{\left((bc-2\,ad)x^4-ac\right)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^6+\left(abc^2-a^2cd\right)x^2\right)}\right)+2\sqrt{dx^4+c}\left(abc-a^2d\right)}{4\left(a^2bc^2-a^3cd\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*b*c + a^2*d)*b*c*x^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*
x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4
*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -1/4*(sqrt(a
*b*c - a^2*d)*b*c*x^2*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) +
2*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]
```

**giac [A]** time = 0.22, size = 116, normalized size = 1.45

$$\frac{1}{2}\,d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ad}+\frac{2}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2 - c)*a*d))
```

**maple [B]** time = 0.30, size = 350, normalized size = 4.38

$$\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{\sqrt{d\,x^4+c}}{2ac\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/2*(d*x^4+c)^(1/2)/a/c/x^2-1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^4\right) \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.530 $\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}\,(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

**Rubi [A]** time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}\,(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(6*a*c*x^6) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(6*a^2*c^2
*x^2) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^
(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}-\frac{\text{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6a^2c^2}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 5.55, size = 137, normalized size = 1.19

$$\frac{\frac{b^2x^2\sqrt{\frac{dx^4}{c}+1}\,\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{a^3\sqrt{\frac{x^4(bc-ad)}{ac}}}+\frac{\left(c+dx^4\right)\left(-ac+2adx^4+3bcx^4\right)}{3a^2c^2x^6}}{2\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (((c + d*x^4)*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(3*a^2*c^2*x^6) + (b^2*x^2*
Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(a^3
*Sqrt[((b*c - a*d)*x^4)/(a*c)]))/(2*Sqrt[c + d*x^4])
```

**IntegrateAlgebraic [A]** time = 0.89, size = 163, normalized size = 1.42

$$\frac{\sqrt{c+dx^4}\,\left(-ac+2adx^4+3bcx^4\right)}{6a^2c^2x^6}-\frac{b^2\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2a^{5/2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(6*a^2*c^2*x^6) - (b^2*S
qrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(
Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d]
)]])/(2*a^(5/2)*(-(b*c) + a*d))
```

**fricas [A]** time = 0.54, size = 418, normalized size = 3.63

$$\left[-\frac{3\,\sqrt{-abc+a^2d}\,b^2c^2x^6\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left(\left(bc-2\,ad\right)x^6-acx^2\right)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\left(a^2bc^2-a^3cd-\left(3\,ab^2c^2-a^2bcd-2\,a^3d^2\right)x^4\right)\sqrt{dx^4+c}}{24\left(a^3bc^3-a^4c^2d\right)x^6},\ \frac{3\,\sqrt{abc-a^2d}\,b^2c^2x^6\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^4-ac\right)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^6+\left(abc^2-a^2cd\right)x^2\right)}\right)-2\left(a^2bc^2-a^3cd-\left(3\,ab^2c^2-a^2bcd-2\,a^3d^2\right)x^4\right)\sqrt{dx^4+c}}{12\left(a^3bc^3-a^4c^2d\right)x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6
 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^
2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*s
qrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^2*d)*
b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(a^2*b*c
^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^4 + c))/
((a^3*b*c^3 - a^4*c^2*d)*x^6)]
```

**giac [B]** time = 1.68, size = 205, normalized size = 1.78

$$-\frac{1}{6}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\left(3\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-6\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc-6\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad+3\,bc^2+2\,acd\right)}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2-c\right)^3a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c +
 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)
*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2*d^2))
```

**maple [B]** time = 0.27, size = 383, normalized size = 3.33

$$-\frac{b^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^2}+\frac{b^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^2}+\frac{\sqrt{d\,x^4+c}\;b}{2a^2c\,x^2}-\frac{\sqrt{d\,x^4+c}\,\left(-2d\,x^4+c\right)}{6a\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/a^2*b/x^2*(d*x^4+c)^(1/2)/c+1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/6/a*(d*x^4+c)^(1/2)*
(-2*d*x^4+c)/x^6/c^2
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7}\left(a+bx^{4}\right)\sqrt{c+dx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

## 3.531 $\int \frac{x^{15}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx$

**Optimal**. Leaf size=175

$$-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{12b^3d^2(bc-ad)}+\frac{ax^8\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-}$$

**Rubi [A]** time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 147, 63, 208}

$$-\frac{\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{12b^3d^2(bc-ad)}-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}+\frac{ax^8\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^8*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[c + d*x^4]*(4
*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(12*b^3*d^2*(
b*c - a*d)) - (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{x\left(2ac+\frac{1}{2}(-2bc+5ad)x\right)}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}$$

**Mathematica [A]** time = 0.31, size = 175, normalized size = 1.00

$$\frac{a^2(5ad-6bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}\left(-15a^3d^2+2a^2bd\left(4c-5dx^4\right)+2ab^2\left(2c^2+3cdx^4+d^2x^8\right)+2b^3cx^4\left(2c-dx^4\right)\right)}{12b^3d^2\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*x
^4*(2*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(b^3*d^2*(b*c -
a*d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])
/Sqrt[b*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

**IntegrateAlgebraic [A]** time = 0.49, size = 198, normalized size = 1.13

$$\frac{\left(6a^2bc-5a^3d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{7/2}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx^4}\left(-15a^3d^2+8a^2bcd-10a^2bd^2x^4+4ab^2c^2+6ab^2cdx^4+2ab^2d^2x^8+4b^3c^2x^4-2b^3cdx^8\right)}{12b^3d^2\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*(Sqrt[c + d*x^4]*(4*a*b^2*c^2 + 8*a^2*b*c*d - 15*a^3*d^2 + 4*b^3*c^2*
x^4 + 6*a*b^2*c*d*x^4 - 10*a^2*b*d^2*x^4 - 2*b^3*c*d*x^8 + 2*a*b^2*d^2*x^8)
```

```
)/(b^3*d^2*(b*c - a*d)*(a + b*x^4)) + ((6*a^2*b*c - 5*a^3*d)*ArcTan[(Sqrt[b
]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(4*b^(7/2)*(-(b*c) + a*
d)^(3/2))
```

**fricas [A]** time = 0.55, size = 622, normalized size = 3.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)*
sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2
*c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*
x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 - 2*(
2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqrt(d*x^4
+ c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b
^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b
^2*c*d^2 - 5*a^3*b*d^3)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b
^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d
^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqr
t(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2
 - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)]
```

**giac [A]** time = 0.17, size = 180, normalized size = 1.03

$$\frac{\sqrt{dx^4+c}\,a^3d}{4\left(b^4c-ab^3d\right)\left(\left(dx^4+c\right)b-bc+ad\right)} + \frac{\left(6\,a^2bc-5\,a^3d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^4c-ab^3d\right)\sqrt{-b^2c+abd}} + \frac{\left(dx^4+c\right)^{\frac{3}{2}}b^4d^4-3\sqrt{dx^4+c}\,b^4cd^4-6\sqrt{dx^4+c}\,ab^3d^5}{6\,b^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d)) +
 1/4*(6*a^2*b*c - 5*a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
(b^4*c - a*b^3*d)*sqrt(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^(3/2)*b^4*d^4 -
3*sqrt(d*x^4 + c)*b^4*c*d^4 - 6*sqrt(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6)
```

**maple [B]** time = 0.25, size = 923, normalized size = 5.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/6/b^2*(d*x^4+c)^(1/2)/d*x^4-1/3/b^2*(d*x^4+c)^(1/2)/d^2*c-1/b^3*a/d*(d*x^
4+c)^(1/2)-3/4*a^2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(
1/2)/b))-3/4*a^2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2
)/b))-1/8*a^2/b^4*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8
*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/
2)/b))+1/8*a^2/b^4*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)
```

```
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/
8*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/
2)/b))
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [**B**] time = 5.19, size = 186, normalized size = 1.06

$$\frac{\left(d\,x^4+c\right)^{3/2}}{6\,b^2\,d^2}-\left(\frac{3\,c}{2\,b^2\,d^2}+\frac{a\,d-b\,c}{b^3\,d^2}\right)\,\sqrt{d\,x^4+c}+\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\left(5\,a\,d-6\,b\,c\right)}{\sqrt{a\,d-b\,c}\,\left(5\,a^3\,d-6\,a^2\,b\,c\right)}\right)\,\left(5\,a\,d-6\,b\,c\right)}{4\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a^3\,d\,\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b^4\,\left(d\,x^4+c\right)-2\,b^4\,c+2\,a\,b^3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b^2*d^2) - ((3*c)/(2*b^2*d^2) + (a*d - b*c)/(b^3*d^2))
*(c + d*x^4)^(1/2) + (a^2*atan((a^2*b^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 6*b*
c))/((a*d - b*c)^(1/2)*(5*a^3*d - 6*a^2*b*c)))*(5*a*d - 6*b*c))/(4*b^(7/2)*
(a*d - b*c)^(3/2)) - (a^3*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^4*(c + d
*x^4) - 2*b^4*c + 2*a*b^3*d))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.532 $\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^4}}{4b^2\left(a+bx^4\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}}{2b^2d}$$

**Rubi [A]** time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^4}}{4b^2\left(a+bx^4\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b^2*d) - (a^2*Sqrt[c + d*x^4])/(4*b^2*(b*c - a*d)*(a + b
*x^4)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(4*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} - \frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, \ldots\right.}{8b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} - \frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c}\ldots\right.}{4b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.36, size = 107, normalized size = 0.87

$$\frac{1}{4}\left(\frac{\sqrt{c+dx^4}\left(\frac{a^2}{\left(a+bx^4\right)(ad-bc)}+\frac{2}{d}\right)}{b^2}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((Sqrt[c + d*x^4]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^4))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/4
```

**IntegrateAlgebraic [A]** time = 0.33, size = 143, normalized size = 1.16

$$\frac{\left(3a^2d-4abc\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{5/2}(ad-bc)^{3/2}}-\frac{\sqrt{c+dx^4}\left(3a^2d-2abc+2abdx^4-2b^2cx^4\right)}{4b^2d\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(Sqrt[c + d*x^4]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^4 + 2*a*b*d*x^4))/(b^
2*d*(b*c - a*d)*(a + b*x^4)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-
(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(4*b^(5/2)*(-(b*c) + a*d)^(3/2)
)
```

**fricas [B]** time = 0.62, size = 475, normalized size = 3.86

$$\left[\frac{\left(4\,a^{2}bcd-3\,a^{3}d^{2}+\left(4\,ab^{2}cd-3\,a^{2}bd^{2}\right)x^{4}\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{bdx^{4}+2\,bc-ad+2\,\sqrt{dx^{4}+c}\,\sqrt{b^{2}c-abd}}{bx^{4}+a}\right)+2\left(2\,ab^{3}c^{2}-5\,a^{2}b^{2}cd+3\,a^{3}bd^{2}+2\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{4}\right)\sqrt{dx^{4}+c}}{8\left(ab^{5}c^{2}d-2\,a^{2}b^{4}cd^{2}+a^{3}b^{3}d^{3}+\left(b^{6}c^{2}d-2\,ab^{5}cd^{2}+a^{2}b^{4}d^{3}\right)x^{4}\right)},-\frac{\left(4\,a^{2}bcd-3\,a^{3}d^{2}+\left(4\,ab^{2}cd-3\,a^{2}bd^{2}\right)x^{4}\right)\sqrt{-b^{2}c+abd}\,\arctan\left(\frac{\sqrt{dx^{4}+c}\,\sqrt{-b^{2}c+abd}}{bdx^{4}+bc}\right)-\left(2\,ab^{3}c^{2}-5\,a^{2}b^{2}cd+3\,a^{3}bd^{2}+2\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{4}\right)\sqrt{dx^{4}+c}}{4\left(ab^{5}c^{2}d-2\,a^{2}b^{4}cd^{2}+a^{3}b^{3}d^{3}+\left(b^{6}c^{2}d-2\,ab^{5}cd^{2}+a^{2}b^{4}d^{3}\right)x^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2*
c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*
d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4), -1
/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4)]
```

**giac [A]** time = 0.17, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^{4}+c}\,a^{2}d}{4\left(b^{3}c-ab^{2}d\right)\left(\left(dx^{4}+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^{2}d\right)\arctan\left(\frac{\sqrt{dx^{4}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{4\left(b^{3}c-ab^{2}d\right)\sqrt{-b^{2}c+abd}}+\frac{\sqrt{dx^{4}+c}}{2\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d*x^4 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^4 + c)*b - b*c + a*d))
- 1/4*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/2*sqrt(d*x^4 + c)/(b^2*d)
```

**maple [B]** time = 0.19, size = 876, normalized size = 7.12

$$-\frac{a^{2}d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^{3}}-\frac{a^{2}d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^{3}}+\frac{a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^{3}}+\frac{a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^{3}}+\frac{\sqrt{-ab}\,\sqrt{\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}{8\left(ad-bc\right)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)b^{3}}-\frac{\sqrt{-ab}\,\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}{8\left(ad-bc\right)\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)b^{3}}+\frac{\sqrt{dx^{4}+c}}{2b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b^2/d+1/2*a/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2
)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x^2+(-a*b)^(1/2)/b))+1/2*a/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-
(-a*b)^(1/2)/b))+1/8*a/b^3*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^
2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)-1/8*a^2/b^3*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(
-a*b)^(1/2)/b))-1/8*a/b^3*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2)-1/8*a^2/b^3*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a
*b)^(1/2)/b))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.12, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^4+c}}{2\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{4\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b^3\,\left(d\,x^4+c\right)-2\,b^3\,c+2\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^4)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(4*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^3*(c
+ d*x^4) - 2*b^3*c + 2*a*b^2*d))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.533 $\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=99

$$\frac{a\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

**Rubi [A]** time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$
$$= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8b(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4bd(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.11, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*b^(
3/2))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 109, normalized size = 1.10

$$\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{3/2}(ad-bc)^{3/2}} + \frac{a\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + ((2*b*c - a*d)*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(4*b^(3/2)*(-(b*c
) + a*d)^(3/2))
```

**fricas [A]** time = 0.71, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^4+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)}{8\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^4\right)}, \frac{\left(\left(2\,b^2c-abd\right)x^4+2\,abc-a^2d\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)}{4\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) +
 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*b*d)
*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b
```

```
^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^4)]
```

**giac** [A] time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^4+c}\,ad^2}{(b^2c-abd)\left((dx^4+c)b-bc+ad\right)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

**maple** [B] time = 0.20, size = 851, normalized size = 8.60

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^2}-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}-\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}-\frac{\sqrt{-ab}\,\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\,(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)b^2}+\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\,(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-
2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4/b
^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8/b^2*(-a*
b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a/b^2*d/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8/b^2*(-a*b)^(1
/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a/b^2*d/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 4.93, size = 95, normalized size = 0.96

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)(a\,d-2\,b\,c)}{4\,b^{3/2}\,(a\,d-b\,c)^{3/2}}-\frac{a\,d\,\sqrt{d\,x^4+c}}{2\,b\,(a\,d-b\,c)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(4*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^4)^(1/2))/(2*b*(a*d - b*c)*(2*b*(c +
 d*x^4) + 2*a*d - 2*b*c))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.534 $\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

**Rubi [A]** time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*(b*c - a*d)*(a + b*x^4)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)\\ &= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{8(bc-ad)}\\ &= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{4(bc-ad)}\\ &= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{4}\left(\frac{\sqrt{c+dx^4}}{\left(a+bx^4\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/((-(b*c) + a*d)*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```

**IntegrateAlgebraic [A]** time = 0.11, size = 97, normalized size = 1.11

$$-\frac{\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}-\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{4\sqrt{b}\,(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(
b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(4*Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

**fricas [B]** time = 0.72, size = 302, normalized size = 3.47

$$\left[-\frac{\left(bdx^4+ad\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(b^2c-abd\right)}{8\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4\right)},-\frac{\left(bdx^4+ad\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+\sqrt{dx^4+c}\left(b^2c-abd\right)}{4\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*s
qrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b^2*c
```

```
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^4)]
```

**giac [A]** time = 0.16, size = 93, normalized size = 1.07

$$-\frac{d \arctan\left(\frac{\sqrt{dx^4+c}\, b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,(bc-ad)} - \frac{\sqrt{dx^4+c}\, d}{4\left(\left(dx^4+c\right)b - bc + ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/4*sqrt(d*x^4 + c)*d/(((d*x^4 + c)*b - b*c + a*d)*(b*c - a
*d))
```

**maple [B]** time = 0.21, size = 541, normalized size = 6.22

$$-\frac{d \ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8(ad-bc)\sqrt{-\frac{ad-bc}{b}}\, b} - \frac{d \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8(ad-bc)\sqrt{-\frac{ad-bc}{b}}\, b} + \frac{\sqrt{-ab}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)ab} - \frac{\sqrt{-ab}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*(-a*b
)^(1/2)/a/b/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+
2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.85, size = 84, normalized size = 0.97

$$\frac{d\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,{(a\,d-b\,c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c)) + (
d*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(1/2)*(a*d - b*
c)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.535 $\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

**Rubi [A]** time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]]/(2*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^4\right)}{4a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \right.}{8a^2(bc-}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2a^2d} - \frac{(b(2bc-3ad))\,\text{Sub}}{}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right.}{4a^2(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.33, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - (2*ArcTanh[Sqrt[c + d*x^
4]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*a^2)
```

**IntegrateAlgebraic [A]** time = 0.32, size = 146, normalized size = 1.11

$$\frac{\left(3a\sqrt{b}\,d-2b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}\,\sqrt{ad-bc}}{bc-ad}\right)}{4a^2(ad-bc)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} - \frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(b*Sqrt[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((-2*b^(3/2)*c +
3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a
*d)])/(4*a^2*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a^
2*Sqrt[c])
```

**fricas [A]** time = 0.68, size = 862, normalized size = 6.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 -
3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^4 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4
 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqr
t(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^4 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a*b
*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c
 - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*
a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 +
 b*c)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4)]
```

**giac [A]** time = 0.17, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^4+c}\,bd}{4\left(abc-a^2d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*b*d/((a*b*c - a^2*d)*((d*x^4 + c)*b - b*c + a*d)) - 1/4
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

**maple [B]** time = 0.27, size = 880, normalized size = 6.67

$$\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a^2}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a^2}-\frac{\ln\left(\frac{2c+2\sqrt{d\,x^4+c}\,\sqrt{c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{-ab}\,\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\left(ad-bc\right)\left(x^2+\frac{\sqrt{-ab}}{b}\right)a^2}+\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\left(ad-bc\right)\left(x^2-\frac{\sqrt{-ab}}{b}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2
*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4/a^
2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8/a^2*(-a*b
)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/a*d/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8/a^2*(-a*b)^(1/2)/(
a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d
```

```
-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/2/a^2/c^(1/2)*ln((2*c+2*(d*x^4
+c)^(1/2)*c^(1/2))/x^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x), x)
```

**mupad [B]** time = 5.87, size = 3017, normalized size = 22.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^
3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*
x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a
^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*
c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)
^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) +
((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*
(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4
 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1
/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*
c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*
c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*
1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b
^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c +
 d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^
2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*
b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*x^4)^(1/2)*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 -
 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2
*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^
2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^
5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^4)^(1
/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^2 + a^2*b^
2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^
3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1/2)*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b
^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d
)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^
2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d -
b*c)^3)^(1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2)) - (atan((((((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^
5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 -
```

```
256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^
(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4
)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((2*a^6*b^2*d^5 - 3*a^5*b
^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c
 + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 +
 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(4*a^2*c^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1
/2)))/(((3*a*b^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
c*d) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 +
a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*
b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a
^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4)^(1/2)*
(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^
4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^4)^(1/
2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*
c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c
^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(2*a^2
*c^(1/2)) - (b*d*(c + d*x^4)^(1/2))/(2*(a^2*d - a*b*c)*(2*b*(c + d*x^4) + 2
*a*d - 2*b*c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

## 3.536 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b\sqrt{c+dx^4}\,(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

**Rubi [A]** time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b\sqrt{c+dx^4}\,(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(a + b*x^4)) - Sqrt
[c + d*x^4]/(4*a*c*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4
]/Sqrt[c]])/(4*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{4a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c}}\right.}{8a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{4a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b^{3/2}}{}$$

**Mathematica [A]** time = 0.70, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^4}\left(a^2d+ab\left(dx^4-c\right)-2b^2cx^4\right)}{x^4\left(a+bx^4\right)(bc-ad)} + \frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[c + d*x^4]*(a^2*d - 2*b^2*c*x^4 + a*b*(-c + d*x^4)))/((b*c - a*d)*
x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(4*a^3*c)
```

**IntegrateAlgebraic** [A] time = 0.66, size = 187, normalized size = 1.01

$$\frac{\left(4b^{5/2}c-5ab^{3/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}\sqrt{ad-bc}}{bc-ad}\right)}{4a^3(ad-bc)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}+\frac{\sqrt{c+dx^4}\left(-a^2d+abc-abdx^4+2b^2cx^4\right)}{4a^2cx^4\left(a+bx^4\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(a*b*c - a^2*d + 2*b^2*c*x^4 - a*b*d*x^4))/(4*a^2*c*(-(b*c
) + a*d)*x^4*(a + b*x^4)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^4])/(b*c - a*d)])/(4*a^3*(-(b*c) + a*d)^(3/
2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a^3*c^(3/2))
```

**fricas** [A] time = 0.91, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)
*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 +
2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a
^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*l
og((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arcta
n(sqrt(d*x^4 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d -
 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/4*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4)]
```

**giac** [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^4+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^4+c}\,b^2c^2d-\left(dx^4+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^4+c}\,abcd^2-\sqrt{dx^4+c}\,a^2d^3}{4\left(a^2bc^2-a^3cd\right)\left(\left(dx^4+c\right)^2b-2\left(dx^4+c\right)bc+bc^2+\left(dx^4+c\right)ad-acd\right)}-\frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/4*(2*(d*x^4 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^4 + c)*b^2*c^2*d - (d*x^4 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^4 + c
)*a*b*c*d^2 - sqrt(d*x^4 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^4 + c)^
```

```
2*b - 2*(d*x^4 + c)*b*c + b*c^2 + (d*x^4 + c)*a*d - a*c*d)) - 1/4*(4*b*c +
a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

**maple [B]** time = 0.28, size = 938, normalized size = 5.07

$$-\frac{bd\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,a^2}-\frac{bd\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,a^2}-\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a^3}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a^3}+\frac{d\ln\left(\frac{2c+2\sqrt{d x^4+c}\,\sqrt{c}}{x^2}\right)}{4a^2c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{d x^4+c}\,\sqrt{c}}{x^2}\right)}{a^3\sqrt{c}}+\frac{\sqrt{-ab}\,\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{8\,(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)a^3}-\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{8\,(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)a^3}-\frac{\sqrt{d x^4+c}}{4a^2c\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/a^2/c/x^4*(d*x^4+c)^(1/2)+1/4/a^2*d/c^(3/2)*ln((2*c+2*(d*x^4+c)^(1/2)*
c^(1/2))/x^2)-1/2/a^3*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^
(1/2)/b))-1/2/a^3*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)
/b))+1/8/a^3*b*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/a^
2*b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)
)-1/8/a^3*b*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/a^2*b
*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b
*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+b/
a^3/c^(1/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

**mupad [B]** time = 7.05, size = 3822, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (((c + d*x^4)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^4)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^4)*(2*a*d - 4*b*c) + 2*b*(c + d*x^4)^2 + 2*b*c^2 - 2*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c
)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*
c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((
-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^
5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(6
4*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^
4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5
```

```
*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2
)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a
^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(
a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4
*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^
3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a
^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2
*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c
^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*
d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)
/2 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((
-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6
+ 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)
)/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(
1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^
4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(
a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2
- 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*(a^4
*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*
c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a
^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c
)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*
c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((
-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^
5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(6
4*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^
4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(4*(a^6*d^3 - a^3*
b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^4)^(1/2)*(
a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*
b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*
c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c
^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a
^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2
*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a
*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2)) + (
(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^
3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*
c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^
3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^(1/2)
*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3
*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2
- 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8
*a^3*(c^3)^(1/2)))/((((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)/2
 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((((
c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b
^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3
*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c
^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^(1/2)*(a
*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^
4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2
*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(8*a^3*(
c^3)^(1/2)) + ((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6
*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2
```

```
*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c
^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c
 + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 +
256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4
 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d
+ 4*b*c))/(8*a^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)/(4*a^3*(c^3)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a + bx^4\right)^2 \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

## 3.537 $\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=191

$$\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}+\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}$$

**Rubi [A]** time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {465, 470, 582, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}+\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*c - 2*a*d)*x^2*Sqrt[c + d*x^4])/(4*b^2*d*(b*c - a*d)) + (a*x^6*Sqrt[c +
 d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^3*(b*c - a*d)^(3/2)) -
 ((b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^3*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3ac-2(bc-2ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{-2ac(bc-2ad)-2(bc-ad)(bc+4a\cdots}{\left(a+bx^2\right)\sqrt{c+dx^2}}\cdots\right.}{8b^2d(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{\cdots}}\cdots\right.}{4b^3(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{a-(-bc+a\cdots}\cdots\right.}{4b^3(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.57, size = 150, normalized size = 0.79

$$\frac{\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{(bc-ad)^{3/2}} + bx^2\sqrt{c+dx^4}\left(\frac{a^2}{\left(a+bx^4\right)(ad-bc)}+\frac{1}{d}\right) - \frac{(4ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{d^{3/2}}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4]*(d^(-1) + a^2/((-(b*c) + a*d)*(a + b*x^4))) + (a^(3/
2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])
/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/d
^(3/2))/(4*b^3)
```

**IntegrateAlgebraic** [A] time = 2.29, size = 208, normalized size = 1.09

$$\frac{\left(5a^{3/2}bc-4a^{5/2}d\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\sqrt{c+dx^4}+b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,\left(2a^2dx^2-abcx^2+abdx^6-b^2cx^6\right)}{4b^2d\left(a+bx^4\right)(bc-ad)}+\frac{(-4ad-bc)\log\left(\sqrt{c+dx^4}+\sqrt{d}\,x^2\right)}{4b^3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(Sqrt[c + d*x^4]*(-(a*b*c*x^2) + 2*a^2*d*x^2 - b^2*c*x^6 + a*b*d*x^6))
/(b^2*d*(b*c - a*d)*(a + b*x^4)) + ((5*a^(3/2)*b*c - 4*a^(5/2)*d)*ArcTan[(a
*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])
])/(4*b^3*(b*c - a*d)^(3/2)) + ((-(b*c) - 4*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c +
 d*x^4]])/(4*b^3*d^(3/2))
```

**fricas** [A] time = 1.94, size = 1386, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*
a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) +
 (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2
- a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqr
t(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), 1/
16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2
*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^
2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)
*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*((b
^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqrt(d*x^4 + c))/(
a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), -1/8*((5*a^2*b*c*
d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*ar
ctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*
d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b
^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)
*x^2 - c) - 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*s
qrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4),
1/8*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^
2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - (5*a^2*b*c*d^
2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*
x^6 + a*c*x^2)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*
x^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4)]
```

**giac [B]** time = 0.59, size = 337, normalized size = 1.76

$$-\frac{\left(5\,a^2bc\sqrt{d}-4\,a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\left(b^4c-ab^3d\right)\sqrt{abcd-a^2d^2}}+\frac{\sqrt{dx^4+c}\,x^2}{4\,b^2d}+\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2a^2bc\sqrt{d}-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2a^3d^{\frac{3}{2}}-a^2bc^2\sqrt{d}}{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)\left(b^4c-ab^3d\right)}+\frac{\left(bc\sqrt{d}+4\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2\right)}{8\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(5*a^2*b*c*sqrt(d) - 4*a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^4*c - a*b^3*d)*sq
rt(a*b*c*d - a^2*d^2)) + 1/4*sqrt(d*x^4 + c)*x^2/(b^2*d) + 1/2*((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2*a^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*a^3*d^(3/2) - a^2*b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b -
2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))
^2*a*d + b*c^2)*(b^4*c - a*b^3*d)) + 1/8*(b*c*sqrt(d) + 4*a*d^(3/2))*log((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^3*d^2)
```

**maple [B]** time = 0.27, size = 953, normalized size = 4.99

$$-\frac{5a^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^3}+\frac{5a^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^3}-\frac{\sqrt{-ab}\,a^2d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^4}+\frac{\sqrt{-ab}\,a^2d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^4}+\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^2}{8\,(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)b^3}+\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^2}{8\,(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)b^3}+\frac{\sqrt{d\,x^4+c}\,x^2}{4b^2d}-\frac{a\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{b^3\sqrt{d}}-\frac{c\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{4b^2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/b^2*x^2/d*(d*x^4+c)^(1/2)-1/4/b^2*c/d^(3/2)*ln(d^(1/2)*x^2+(d*x^4+c)^(1
/2))-1/b^3*a*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)+5/8*a^2/b^3/(-a*b)^(1/
2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^
2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-5/8*a^2/b^3
/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1
/8*a^2/b^3/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8*a^2/b^4*(-a*b)^(1/2
)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1
/8*a^2/b^3/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a^2/b^4*(-a*b)^(1/2
)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(a + bx^4\right)^2 \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**13/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

## 3.538 $\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{ax^2\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

**Rubi [A]** time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{ax^2\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2} - \frac{(a(3bc-2ad))\operatorname{Subst}\left(\int \frac{}{(a}\right.}{4b^2(bc-}$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2} - \frac{(a(3bc-2ad))\operatorname{Subst}\left(\int\right.}{4b^2(}$$

$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

**Mathematica [A]** time = 0.22, size = 135, normalized size = 0.96

$$\frac{\frac{abx^2\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)} + \frac{\sqrt{a}\,(2ad-3bc)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\,\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/Sqrt[d])/(4*b^2)
```

**IntegrateAlgebraic [A]** time = 1.59, size = 166, normalized size = 1.18

$$\frac{\left(2a^{3/2}d-3\sqrt{a}\,bc\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\sqrt{c+dx^4}+b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{ax^2\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)} + \frac{\log\left(\sqrt{c+dx^4}+\sqrt{d}\,x^2\right)}{2b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + ((-3*Sqrt[a]*b*c +
2*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sq
rt[a]*Sqrt[b*c - a*d])])/(4*b^2*(b*c - a*d)^(3/2)) + Log[Sqrt[d]*x^2 + Sqrt
[c + d*x^4]]/(2*b^2*Sqrt[d])
```

**fricas [A]** time = 0.84, size = 1077, normalized size = 7.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4
*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x
^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^
2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 -
 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(
d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a
/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c
^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x
^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*s
qrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2
*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
+ c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2*c - a*b*d)*x^4 + a*
b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(a*
b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*
a*b*d*x^2 - 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d
)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^
4*c*d - a*b^3*d^2)*x^4)]
```

**giac [B]** time = 0.54, size = 298, normalized size = 2.11

$$-\frac{\left(3\,abc\sqrt{d} - 2\,a^{2}d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{4\,\left(b^{3}c-ab^{2}d\right)\sqrt{abcd-a^{2}d^{2}}} - \frac{\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{2}abc\sqrt{d} - 2\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{2}a^{2}d^{\frac{3}{2}} - abc^{2}\sqrt{d}}{2\left(\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{4}b - 2\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{2}bc + 4\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{2}ad + bc^{2}\right)\left(b^{3}c-ab^{2}d\right)} - \frac{\log\left(\left(\sqrt{d}x^{2}-\sqrt{dx^{4}+c}\right)^{2}\right)}{4\,b^{2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x
^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c
 + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
4*log((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^2*sqrt(d))
```

**maple [B]** time = 0.24, size = 893, normalized size = 6.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/b^2*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)-3/8*a/b^2/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+3/8*a/b^2/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8*a/b^2/
(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8*a/b^2/(a*d
-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**9/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

# 3.539 $\int \frac{x^5}{(a+bx^4)^2 \sqrt{c+dx^4}} \, dx$

**Optimal**. Leaf size=93

$$\frac{c \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}} - \frac{x^2 \sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

**Rubi [A]** time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c \tan^{-1}\left(\frac{x^2 \sqrt{bc-ad}}{\sqrt{a} \sqrt{c+dx^4}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}} - \frac{x^2 \sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(x^2*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\text{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}$$

**Mathematica** [A] time = 0.58, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^4}\left(-\frac{x^4(bc-ad)}{a+bx^4}-\frac{c\sqrt{x^4\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^4\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^4}{c}+1}}\right)}{\sqrt{\frac{dx^4}{c}+1}}\right)}{4x^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-(((b*c - a*d)*x^4)/(a + b*x^4)) - (c*Sqrt[(-(b/a) + d/c)
*x^4]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^4]/Sqrt[1 + (d*x^4)/c]])/Sqrt[1 + (d*x^
4)/c]))/(4*(b*c - a*d)^2*x^2)
```

**IntegrateAlgebraic** [A] time = 1.04, size = 145, normalized size = 1.56

$$\frac{c\tan^{-1}\left(\frac{b\sqrt{d}\,x^4}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^2\sqrt{c+dx^4}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[a]*S
qrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^4)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^
2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*Sqrt[a]*(b*c - a*d)^(3/2)
)
```

**fricas** [B] time = 0.54, size = 426, normalized size = 4.58

$$\left[-\frac{4\sqrt{dx^4+c}\left(abc-a^2d\right)x^2-\left(bcx^4+ac\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{16\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^4\right)},\ -\frac{2\sqrt{dx^4+c}\left(abc-a^2d\right)x^2-\left(bcx^4+ac\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\left((bc-2\,ad)x^4-ac\right)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^6+\left(abc^2-a^2cd\right)x^2\right)}\right)}{8\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(
-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d +
 a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(d*x^4
 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*
((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a
^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d
^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]
```

**giac [B]** time = 1.63, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{4\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 bc\sqrt{d}-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 ad^{\frac{3}{2}}-bc^2\sqrt{d}}{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4 b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 ad+bc^2\right)\left(b^2c-abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/2*((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*a*d + b*c^2)*(b^2*c - a*b*d))
```

**maple [B]** time = 0.24, size = 861, normalized size = 9.26

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{\sqrt{-ab}\,d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{\sqrt{-ab}\,d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\,(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)b}+\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\,(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*
(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)
/b))-1/8/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(
1/2)/b))+1/8/b/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b^2*(-a*b)^(1/2
)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1
/8/b/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/b^2*(-a*b)^(1/2)*d/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

## 3.540 $\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

**Rubi** [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

**Mathematica [C]** time = 6.01, size = 407, normalized size = 3.91

$$
\frac{x^2\sqrt{c+dx^4}\left(-30dx^4\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}}-45c\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}}+16dx^4\left(\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^4\right)}{c\left(a+bx^4\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+16c\left(\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^4\right)}{c\left(a+bx^4\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+30dx^4\sin^{-1}\left(\sqrt{\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}}\right)+45c\sin^{-1}\left(\sqrt{\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}}\right)\right)}{60c^2\left(a+bx^4\right)^2\left(\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)^{3/2}\sqrt{\frac{a\left(c+dx^4\right)}{c\left(a+bx^4\right)}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4]*(-45*c*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a +
b*x^4)^2)] - 30*d*x^4*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 30*d*x^4*ArcS
in[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 16*c*(((b*c - a*d)*x^4)/(c*(a
 + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 16*d*x^4*(((b*c - a*d)*x^4)/
(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]))/(60*c^2*(((b*c - a*d)*x
^4)/(c*(a + b*x^4)))^(3/2)*(a + b*x^4)^2*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4
))])
```

**IntegrateAlgebraic [A]** time = 0.64, size = 124, normalized size = 1.19

$$
\frac{(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\sqrt{c+dx^4}+b\sqrt{d}\,x^4}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}(bc-ad)^{3/2}}-\frac{bx^2\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(b*x^2*Sqrt[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(4*a^(3/2)*(b*c - a*d)^(3/2))
```

**fricas [B]** time = 0.60, size = 467, normalized size = 4.49

$$
\left[\frac{4\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)x^2-\left(\left(b^2c-2\,abd\right)x^4+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{16\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^4\right)},\ \frac{2\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)x^2+\left(\left(b^2c-2\,abd\right)x^4+abc-2\,a^2d\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\left((bc-2\,ad)x^4-ac\right)\sqrt{dx^4+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^6+\left(abc^2-a^2cd\right)x^2\right)}\right)}{8\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a
*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))
/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c - 2*a
*b*d)*x^4 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*
x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (
a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]
```

**giac [B]** time = 0.38, size = 237, normalized size = 2.28

$$-\frac{1}{4}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 bc-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 ad-bc^2\right)}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4 b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 bc+4\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2 ad+bc^2\right)\left(abcd-a^2d^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

**maple [B]** time = 0.19, size = 867, normalized size = 8.34

$$\frac{\sqrt{-ab}\,d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,ab}-\frac{\sqrt{-ab}\,d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,ab}-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\,(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)a}-\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{8\,(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/a/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/b/a*(-a*b)^(1/2)*d/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a/(-a
*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d
-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8/
a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-
1/8/a/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b/a*(-a*b)^(1/2)*d/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

## 3.541 $\int \frac{1}{x^3\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=149

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,(3bc-2ad)}{4a^2cx^2(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^2\left(a+bx^4\right)(bc-ad)}$$

**Rubi [A]** time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^4}\,(3bc-2ad)}{4a^2cx^2(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^4}}{4ax^2\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x^2) + (b*Sqrt[c +
d*x^4])/(4*a*(b*c - a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,d\right.}{4a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int\right.}{4a^2(bc-}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int\right.}{4a^2(bc}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-}}{\sqrt{a}\sqrt{}}\right.}{4a^{5/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 5.63, size = 155, normalized size = 1.04

$$\frac{a^2\left(c+dx^4\right)\left(\frac{b^2x^4}{\left(a+bx^4\right)(ad-bc)}-\frac{2}{c}\right)-\frac{bx^8\sqrt{\frac{dx^4}{c}+1}\,(3bc-4ad)\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{c\left(\frac{x^4(bc-ad)}{ac}\right)^{3/2}}}{4a^4x^2\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a^2*(c + d*x^4)*(-2/c + (b^2*x^4)/((-(b*c) + a*d)*(a + b*x^4))) - (b*(3*b*
c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b
*x^4)/a]])/(c*(((b*c - a*d)*x^4)/(a*c))^(3/2)))/(4*a^4*x^2*Sqrt[c + d*x^4])
```

**IntegrateAlgebraic [A]** time = 1.37, size = 159, normalized size = 1.07

$$\frac{\left(4abd-3b^2c\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\sqrt{c+dx^4}+b\sqrt{d}\,x^4}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^4}\left(-2a^2d+2abc-2abdx^4+3b^2cx^4\right)}{4a^2cx^2\left(a+bx^4\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^4 - 2*a*b*d*x^4))/(4*a^2*c*
(-(b*c) + a*d)*x^2*(a + b*x^4)) + ((-3*b^2*c + 4*a*b*d)*ArcTan[(a*Sqrt[d] +
 b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(5
/2)*(b*c - a*d)^(3/2))
```

**fricas [B]** time = 0.62, size = 612, normalized size = 4.11

$$\left[-\frac{\left((3b^3c^2-4ab^2cd)x^6+(3ab^2c^2-4a^2bcd)x^2\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^8-2(3abc^2-4a^2cd)x^4+a^2c^2+4((bc-2ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2abx^4+a^2}\right)+4\left(2a^2b^2c^2-4a^3bcd+2a^4d^2+(3ab^3c^2-5a^2b^2cd+2a^3bd^2)x^4\right)\sqrt{dx^4+c}}{16\left((a^3b^3c^3-2a^4b^2c^2d+a^5bcd^2)x^6+(a^4b^2c^3-2a^5bc^2d+a^6cd^2)x^2\right)}, -\frac{\left((3b^3c^2-4ab^2cd)x^6+(3ab^2c^2-4a^2bcd)x^2\right)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)+2\left(2a^2b^2c^2-4a^3bcd+2a^4d^2+(3ab^3c^2-5a^2b^2cd+2a^3bd^2)x^4\right)\sqrt{dx^4+c}}{8\left((a^3b^3c^3-2a^4b^2c^2d+a^5bcd^2)x^6+(a^4b^2c^3-2a^5bc^2d+a^6cd^2)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*c^2
- 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4
)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^
4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a*b^2*c*
d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*((
b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2
*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*
a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(d*x^4 + c))
/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*c^3 - 2*a^5*
b*c^2*d + a^6*c*d^2)*x^2)]
```

**giac [B]** time = 1.68, size = 418, normalized size = 2.81

$$\frac{1}{4}d^{\frac{5}{2}}\left(\frac{\left(3b^2c-4abd\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}+\frac{2\left(3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b^2c-4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4abd-6\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b^2c^2+14\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2abcd-8\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^2d^2+3b^2c^3-2abc^2d\right)}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^6b-3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4ad+3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc^2-4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2acd-bc^3\right)\left(a^2bcd^2-a^3d^3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b^2*c - 4*(sq
rt(d)*x^2 - sqrt(d*x^4 + c))^4*a*b*d - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*d - 8*(sqrt(d)*x^2 - s
qrt(d*x^4 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/((((sqrt(d)*x^2 - sqrt(
d*x^4 + c))^6*b - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c + 4*(sqrt(d)*x^2
 - sqrt(d*x^4 + c))^4*a*d + 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2 - 4*(s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

**maple [B]** time = 0.27, size = 885, normalized size = 5.94

$$\frac{3b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}a^2}-\frac{3b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}a^2}-\frac{\sqrt{-ab}\,d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8(ad-bc)\sqrt{-\frac{ad-bc}{b}}a^2}+\frac{\sqrt{-ab}\,d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8(ad-bc)\sqrt{-\frac{ad-bc}{b}}a^2}+\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{8(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)a^2}+\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{8(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)a^2}-\frac{\sqrt{dx^4+c}}{2a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/2/a^2/x^2*(d*x^4+c)^(1/2)/c-3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
```

```
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a^2*b/(a*d-b*c)/(x^2-(-
a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)-1/8/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a^2*b/(a*d-b*c)/(x^2+(-a*b)^(
1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)+1/8/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^2 \sqrt{dx^4 + c}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(b\, x^4 + a\right)^2 \sqrt{d\, x^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^4\right)^2 \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

## 3.542 $\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

**Optimal**. Leaf size=208

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{\sqrt{c+dx^4}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{12a^3c^2x^2(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6\left(a+bx^4\right)(b}$$

**Rubi** [A] time = 0.33, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{12a^3c^2x^2(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^6) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^4])/(12*a^3*c^2*(b*c - a*d)*x^2) +
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a\cdots}{x^2\left(a+bx^2\cdots\right.}\cdots\right.}{12a^2c(b\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+\cdots}}{4a(bc-ad)x\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+\cdots}}{4a(bc-ad)x\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+\cdots}}{4a(bc-ad)x\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+\cdots}}{4a(bc-ad)x\cdots}$$

**Mathematica** [A] time = 5.87, size = 175, normalized size = 0.84

$$\frac{a^2\left(c+dx^4\right)\left(\frac{3b^3x^8}{\left(a+bx^4\right)(bc-ad)}+\frac{4x^4(ad+3bc)}{c^2}-\frac{2a}{c}\right)+\frac{3b^2x^{12}\sqrt{\frac{dx^4}{c}+1}\,(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{c\left(\frac{x^4(bc-ad)}{ac}\right)^{3/2}}}{12a^5x^6\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a^2*(c + d*x^4)*((-2*a)/c + (4*(3*b*c + a*d)*x^4)/c^2 + (3*b^3*x^8)/((b*c
- a*d)*(a + b*x^4))) + (3*b^2*(5*b*c - 6*a*d)*x^12*Sqrt[1 + (d*x^4)/c]*ArcS
in[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(c*(((b*c - a*d)*x^4)/(a*c))
^(3/2)))/(12*a^5*x^6*Sqrt[c + d*x^4])
```

**IntegrateAlgebraic** [A]  time = 2.28, size = 217, normalized size = 1.04

$$\frac{\left(5b^3c-6ab^2d\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^2\sqrt{c+dx^4}+b\sqrt{d}\,x^4}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{7/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}\left(-2a^3cd+4a^3d^2x^4+2a^2bc^2+6a^2bcdx^4+4a^2bd^2x^8-10ab^2c^2x^4+8ab^2cdx^8-15b^3c^2x^8\right)}{12a^3c^2x^6\left(a+bx^4\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(2*a^2*b*c^2 - 2*a^3*c*d - 10*a*b^2*c^2*x^4 + 6*a^2*b*c*d*
x^4 + 4*a^3*d^2*x^4 - 15*b^3*c^2*x^8 + 8*a*b^2*c*d*x^8 + 4*a^2*b*d^2*x^8))/
(12*a^3*c^2*(-(b*c) + a*d)*x^6*(a + b*x^4)) + ((5*b^3*c - 6*a*b^2*d)*ArcTan
[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*
d])])/(4*a^(7/2)*(b*c - a*d)^(3/2))
```

**fricas** [A]  time = 0.78, size = 760, normalized size = 3.65

$$\left[-\frac{3\left(\left(5b^4c^3-6ab^3c^2d\right)x^{10}+\left(5ab^3c^3-6a^2b^2c^2d\right)x^6\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^8-2\left(3abc^2-4a^2cd\right)x^4+a^2c^2-4\left(\left(bc-2ad\right)x^6-acx^2\right)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2abx^4+a^2}\right)-4\left(\left(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3\right)x^8-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2+2\left(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3\right)x^4\right)\sqrt{dx^4+c}}{48\left(\left(a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2\right)x^{10}+\left(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2\right)x^6\right)},\ \frac{3\left(\left(5b^4c^3-6ab^3c^2d\right)x^{10}+\left(5ab^3c^3-6a^2b^2c^2d\right)x^6\right)\sqrt{abc-a^2d}\arctan\left(\frac{\left(\left(bc-2ad\right)x^4-ac\right)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^6+\left(abc^2-a^2cd\right)x^2\right)}\right)+2\left(\left(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3\right)x^8-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2+2\left(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3\right)x^4\right)\sqrt{dx^4+c}}{24\left(\left(a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2\right)x^{10}+\left(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2\right)x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sq
rt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((15*a
*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a^3*b^
2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d +
a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^
3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^6
), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^
4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x
^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)
*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a
^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4
 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a
^7*c^2*d^2)*x^6)]
```

**giac** [B]  time = 2.04, size = 395, normalized size = 1.90

$$\frac{1}{12}d^{\frac{7}{2}}\left(\frac{3\left(5b^3c-6ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b^3c-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ab^2d-b^3c^2\right)}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)}-\frac{8\left(3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-6\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc-3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+3bc^2+acd\right)}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)^3a^3d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^
4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b^3*c - 2
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^2 - sq
rt(d*x^4 + c))^2 - c)^3*a^3*d^3))
```

**maple** [B]  time = 0.25, size = 923, normalized size = 4.44

$$-\frac{5b^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,a^3}+\frac{5b^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,a^3}+\frac{\sqrt{-ab}\,bd\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,a^3}-\frac{\sqrt{-ab}\,bd\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,a^3}-\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^2}{8(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)a^3}-\frac{\sqrt{\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^2}{8(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)a^3}+\frac{\sqrt{dx^4+c}\,b}{a^3cx^2}-\frac{\sqrt{dx^4+c}\left(-2dx^4+c\right)}{6a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] b/a^3/x^2*(d*x^4+c)^(1/2)/c+5/8/a^3*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-5/8/a^3*b^2/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/6/a^2*(d*x^4+c)^(1/2)*(-
2*d*x^4+c)/x^6/c^2-1/8/a^3*b^2/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8
/a^3*b*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-
(-a*b)^(1/2)/b))-1/8/a^3*b^2/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/a
^3*b*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(
-a*b)^(1/2)/b))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^2 \sqrt{dx^4 + c}\, x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^7 \left(b\, x^4 + a\right)^2 \sqrt{d\, x^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(a + bx^4\right)^2 \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

## 3.543 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}\,(ad+bc)}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}$$

**Rubi [A]** time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}\,(ad+bc)}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^6])/(3*b^2*d^2) + (c + d*x^6)^(3/2)/(9*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*
c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{17}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^6\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

**Mathematica** [A] time = 0.28, size = 91, normalized size = 0.88

$$
\frac{\sqrt{c+dx^6}\left(-3ad-2bc+bdx^6\right)}{9b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

**IntegrateAlgebraic** [A] time = 0.18, size = 101, normalized size = 0.97

$$
\frac{\sqrt{c+dx^6}\left(-3ad-2bc+bdx^6\right)}{9b^2d^2}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{5/2}\sqrt{ad-bc}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) - (a^2*ArcTan[(Sqr
t[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(3*b^(5/2)*Sqrt[-(b*
c) + a*d])
```

**fricas** [A] time = 0.43, size = 288, normalized size = 2.77

$$
\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\left(\left(b^3cd-ab^2d^2\right)x^6-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{dx^6+c}}{18\left(b^4cd^2-ab^3d^3\right)},\ \frac{3\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\left(\left(b^3cd-ab^2d^2\right)x^6-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{dx^6+c}}{9\left(b^4cd^2-ab^3d^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*
x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 - a*b^2
*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

**giac** [A] time = 0.17, size = 106, normalized size = 1.02

$$\frac{a^2 \arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^2} + \frac{\left(dx^6+c\right)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^6+c}\,b^2cd^4 - 3\sqrt{dx^6+c}\,abd^5}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*a^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/9*((d*x^6 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^6 + c)*b^2*c*d^4 - 3*s
qrt(d*x^6 + c)*a*b*d^5)/(b^3*d^6)
```

**maple** [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad** [B] time = 4.87, size = 103, normalized size = 0.99

$$\frac{\left(d\,x^6+c\right)^{3/2}}{9\,b\,d^2} - \left(\frac{2\,c}{3\,b\,d^2} + \frac{3\,a\,d^3 - 3\,b\,c\,d^2}{9\,b^2\,d^4}\right)\sqrt{d\,x^6+c} + \frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(3/2)/(9*b*d^2) - ((2*c)/(3*b*d^2) + (3*a*d^3 - 3*b*c*d^2)/(9*b
^2*d^4))*(c + d*x^6)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b
*c)^(1/2)))/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**17/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.544 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{3bd}$$

**Rubi** [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a
*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b}$$
$$= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3bd}$$
$$= \frac{\sqrt{c+dx^6}}{3bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.13, size = 72, normalized size = 0.97

$$\frac{1}{3}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

**IntegrateAlgebraic [A]** time = 0.10, size = 84, normalized size = 1.14

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{3b^{3/2}\sqrt{ad-bc}} + \frac{\sqrt{c+dx^6}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*
x^6])/(b*c - a*d)])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

**fricas [A]** time = 0.42, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\left(b^2c-abd\right)}{6\left(b^3cd-ab^2d^2\right)}, -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-\sqrt{dx^6+c}\left(b^2c-abd\right)}{3\left(b^3cd-ab^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c
)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

**giac [A]** time = 0.19, size = 64, normalized size = 0.86

$$-\frac{\frac{ad \arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b} - \frac{\sqrt{dx^6+c}}{b}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^6 + c)/b)/d
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.75, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^6+c}}{3\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b*d) - (a*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)
^(1/2)))/(3*b^(3/2)*(a*d - b*c)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.545 $\int \frac{x^5}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,\sqrt{bc-ad}}$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(3*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] -1/3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 61, normalized size = 1.20

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{3\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] -1/3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)]/(Sqrt
[b]*Sqrt[-(b*c) + a*d])
```

**fricas [A]** time = 0.41, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)}{6\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)}{3\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] [1/6*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b
*x^6 + a))/sqrt(b^2*c - a*b*d), 1/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c))/(b^2*c - a*b*d)]
```

**giac [A]** time = 0.17, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

**maple [F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.72, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\,\sqrt{d\,x^6+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^6)^(1/2))/(a*b*d - b^2*c)^(1/2))/(3*(a*b*d - b^2*c)^(1/2))
```

**sympy [A]** time = 23.76, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

## 3.546 $\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

**Rubi** [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\frac{1}{6}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)$$
$$=\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^6\right)}{6a}-\frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6a}$$
$$=\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3ad}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3ad}$$
$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.13, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^6])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(3*a)
```

**IntegrateAlgebraic [A]** time = 0.12, size = 95, normalized size = 1.12

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{3a\sqrt{ad-bc}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/
(3*a*Sqrt[-(b*c) + a*d]) - ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a*Sqrt[c])
```

**fricas [A]** time = 0.45, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad+2\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+\sqrt{c}\log\left(\frac{dx^6-2\sqrt{dx^6+c}\sqrt{c}+2c}{x^6}\right)}{6ac},\ \frac{2c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+\sqrt{c}\log\left(\frac{dx^6-2\sqrt{dx^6+c}\sqrt{c}+2c}{x^6}\right)}{6ac},\ \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad+2\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right)}{6ac},\ \frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right)}{3ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sqrt
(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt
(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-
c)/c))/(a*c), 1/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^6 + c)*
sqrt(-c)/c))/(a*c)]
```

**giac [A]** time = 0.18, size = 71, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{\sqrt{dx^6+c}\, b}{\sqrt{-b^2c+abd}}\right)}{3 \sqrt{-b^2c + abd}\, a} + \frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3 a \sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c))
```

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6 + a\right) \sqrt{d\,x^6 + c}\; x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6 + a\right)\sqrt{dx^6 + c}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)
```

**mupad [B]** time = 5.19, size = 652, normalized size = 7.67

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)}{3\,a\,\sqrt{c}} - \frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\left(a^2\,d-a\,b\,c\right)}\right)}{6\left(a^2\,d-a\,b\,c\right)}\right)1i}{a^2\,d-a\,b\,c}+\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}+\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\left(a^2\,d-a\,b\,c\right)}\right)}{6\left(a^2\,d-a\,b\,c\right)}\right)1i}{a^2\,d-a\,b\,c}}{\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\left(a^2\,d-a\,b\,c\right)}\right)}{6\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}+\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\left(a^2\,d-a\,b\,c\right)}\right)}{6\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}}\right)\sqrt{b^2\,c-a\,b\,d}\,1i}{3\left(a^2\,d-a\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^6)^(1/2)/c^(1/2))/(3*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^
2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a
*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c
) + ((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*
b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)
))*1i)/(a^2*d - a*b*c))/(((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/
2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a
^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))
))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c) - ((b^2*c - a*b*d)^(1/2)*((2*b^3*d
^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*
a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(3
6*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c)))*(b^2*c - a*b*d
)^(1/2)*1i)/(3*(a^2*d - a*b*c))
```

**sympy [A]** time = 26.11, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**6)/sqrt(-c))/(3*a*sqrt(-c))
```

## 3.547 $\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}} dx$

**Optimal**. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

**Rubi [A]** time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(6*a*c*x^6) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[
c]])/(6*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c
- a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{12a^2c}$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3a^2d} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{6a^2cd}$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.18, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2(ad-bc)} + \frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6ac^{3/2}} - \frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/(a*c*x^6) + (b*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(3*a^
2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(6*a*c^(3/2)) + (b^(3/2)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a^2*
(-(b*c) + a*d))
```

**IntegrateAlgebraic [A]** time = 0.26, size = 127, normalized size = 1.09

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{3a^2\sqrt{ad-bc}} + \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}} - \frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/(a*c*x^6) - (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^6])/(b*c - a*d)])/(3*a^2*Sqrt[-(b*c) + a*d]) + ((2*b*c + a*d
)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(6*a^2*c^(3/2))
```

**fricas [A]** time = 0.47, size = 565, normalized size = 4.83

$$\left[\frac{2bc^2x^6\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad-2\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+(2bc+ad)\sqrt{c}x^6\log\left(\frac{dx^6+2\sqrt{dx^6+c}\sqrt{c}+2c}{x^6}\right)-2\sqrt{dx^6+c}\,ac}{12a^2c^2x^6}, -\frac{4bc^2x^6\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)-(2bc+ad)\sqrt{c}x^6\log\left(\frac{dx^6+2\sqrt{dx^6+c}\sqrt{c}+2c}{x^6}\right)+2\sqrt{dx^6+c}\,ac}{12a^2c^2x^6}, \frac{bc^2x^6\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2bc-ad-2\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)-(2bc+ad)\sqrt{-c}x^6\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right)-\sqrt{dx^6+c}\,ac}{6a^2c^2x^6}, -\frac{2bc^2x^6\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right)+(2bc+ad)\sqrt{-c}x^6\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right)+\sqrt{dx^6+c}\,ac}{6a^2c^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(
d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*sq
rt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*x^6
 + c)*a*c)/(a^2*c^2*x^6), -1/12*(4*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*sq
rt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), 1/6*(b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^6 + a)) - (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c)
 - sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/6*(2*b*c^2*x^6*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + sqrt(d*
x^6 + c)*a*c)/(a^2*c^2*x^6)]
```

**giac [A]** time = 0.18, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^2\sqrt{-c}\,c} - \frac{\sqrt{dx^6+c}}{6\,acx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/6*(2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/6*sqrt(d*x^6 + c)/(a*c*x^6)
```

**maple [F]** time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}\;x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)
```

**mupad [B]** time = 5.62, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\sqrt{b^4\,c-a\,b^3\,d}}{6\,a^3\,d-6\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\sqrt{b^4\,c-a\,b^3\,d}}{6\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^6+c}}{6\,a\,c\,x^6}-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^6+c}\,1{}\mathrm{i}}{18\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,c}+\frac{5\,a\,b^3\,d^5}{108\,c^2}+\frac{a^2\,b^2\,d^6}{108\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^6+c}\,1{}\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{108\,a}+\frac{b^2\,d^6}{108\,c}+\frac{b^4\,c\,d^4}{18\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^6+c}\,5{}\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,a}+\frac{5\,b^3\,d^5}{108\,c}+\frac{a\,b^2\,d^6}{108\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (log((c + d*x^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(6*a^3*d - 6*a^2*b*c) - (log((c + d*x^6
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
```

```
*c - a*b^3*d)^(1/2))/(6*(a^3*d - a^2*b*c)) - (c + d*x^6)^(1/2)/(6*a*c*x^6)
- (atan((b^4*d^4*(c + d*x^6)^(1/2)*1i)/(18*(c^3)^(1/2)*((b^4*d^4)/(18*c) +
(5*a*b^3*d^5)/(108*c^2) + (a^2*b^2*d^6)/(108*c^3))) + (b^2*d^6*(c + d*x^6)^
(1/2)*1i)/(108*(c^3)^(1/2)*((5*b^3*d^5)/(108*a) + (b^2*d^6)/(108*c) + (b^4*
c*d^4)/(18*a^2))) + (b^3*d^5*(c + d*x^6)^(1/2)*5i)/(108*(c^3)^(1/2)*((b^4*d
^4)/(18*a) + (5*b^3*d^5)/(108*c) + (a*b^2*d^6)/(108*c^2))))*(a*d + 2*b*c)*1
i)/(6*a^2*(c^3)^(1/2))
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(a + bx^6\right) \sqrt{c + dx^6}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.548 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}+\frac{x^3\sqrt{c+dx^6}}{6bd}$$

**Rubi [A]** time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}+\frac{x^3\sqrt{c+dx^6}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt
[a]*Sqrt[c + d*x^6])])/(3*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^3)/Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} - \frac{\operatorname{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6bd}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,\right.}{6b^2d}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx\right.}{6b^2d}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}$$

**Mathematica [A]** time = 0.24, size = 118, normalized size = 0.96

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\,\sqrt{c+dx^6}+dx^3\right)}{d^{3/2}} + \frac{bx^3\sqrt{c+dx^6}}{d}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ((b*x^3*Sqrt[c + d*x^6])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[
a]*Sqrt[c + d*x^6])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^3 + Sqrt[d]*
Sqrt[c + d*x^6]])/d^(3/2))/(6*b^2)
```

**IntegrateAlgebraic [A]** time = 1.11, size = 177, normalized size = 1.44

$$\frac{a^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{3b^2\sqrt{bc-ad}} + \frac{(-2ad-bc)\log\left(\sqrt{c+dx^6}+\sqrt{d}\,x^3\right)}{6b^2d^{3/2}} + \frac{x^3\sqrt{c+dx^6}}{6bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c
- a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt[c + d*x^6]
```

```
)/(Sqrt[a]*Sqrt[b*c - a*d])])/(3*b^2*Sqrt[b*c - a*d]) + ((-(b*c) - 2*a*d)*L
og[Sqrt[d]*x^3 + Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))
```

**fricas [A]** time = 0.55, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^6+c}\,bdx^3+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{12}-2\left(3abc^2-4a^2cd\right)x^6+a^2c^2+4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^6+2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)}{12b^2d^2},\ \frac{2\sqrt{dx^6+c}\,bdx^3+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{12}-2\left(3abc^2-4a^2cd\right)x^6+a^2c^2+4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)+2(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)}{12b^2d^2},\ \frac{2\sqrt{dx^6+c}\,bdx^3-2ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(\frac{\left((bc-2ad)x^6-ac\right)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^9+acx^3\right)}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^6+2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)}{12b^2d^2},\ \frac{\sqrt{dx^6+c}\,bdx^3-ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(\frac{\left((bc-2ad)x^6-ac\right)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^9+acx^3\right)}\right)+(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)}{6b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2*a*d
)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/1
2*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6
+ c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c + 2*a*d)*
sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2*sqrt(d*x^
6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6
 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + (b*c + 2
*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2),
 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c
 - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3
)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2)
]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

**maple [F]** time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.549 $\int \frac{x^8}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

**Rubi** [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b}\\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}
\end{aligned}
$$

**Mathematica [A]**   time = 0.09, size = 90, normalized size = 0.99

$$
\frac{\frac{\log\left(\sqrt{d}\,\sqrt{c+dx^6}+dx^3\right)}{\sqrt{d}} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{\sqrt{bc-ad}}}{3b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/Sqrt[b
*c - a*d]) + Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]]/Sqrt[d])/(3*b)
```

**IntegrateAlgebraic [A]**   time = 0.62, size = 144, normalized size = 1.58

$$
\frac{\log\left(\sqrt{c+dx^6}+\sqrt{d}\,x^3\right)}{3b\sqrt{d}} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{3b\sqrt{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*(Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(S
qrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])
])/(b*Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

**fricas [A]**   time = 0.52, size = 632, normalized size = 6.95

$$
\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2\,abx^6+a^2}\right)+2\sqrt{d}\,\log\left(-2\,dx^6-2\sqrt{dx^6+c}\,\sqrt{d}\,x^3-c\right)}{12\,bd},\ \frac{d\sqrt{-\frac{a}{bc-ad}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2\,abx^6+a^2}\right)-4\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x^3}{\sqrt{dx^6+c}}\right)}{12\,bd},\ \frac{d\sqrt{\frac{a}{bc-ad}}\,\arctan\left(-\frac{\left((bc-2\,ad)x^6-ac\right)\sqrt{dx^6+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^9+acx^3\right)}\right)+\sqrt{d}\,\log\left(-2\,dx^6-2\sqrt{dx^6+c}\,\sqrt{d}\,x^3-c\right)}{6\,bd},\ \frac{d\sqrt{\frac{a}{bc-ad}}\,\arctan\left(-\frac{\left((bc-2\,ad)x^6-ac\right)\sqrt{dx^6+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^9+acx^3\right)}\right)-2\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x^3}{\sqrt{dx^6+c}}\right)}{6\,bd}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*s
qrt(d)*x^3 - c))/(b*d), 1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*
```

```
c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*sqrt(-d)*arctan(sqr
t(-d)*x^3/sqrt(d*x^6 + c)))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*(
(b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c
*x^3)) + sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b*d),
1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) - 2*sqrt(-d)*arctan(sqrt(-d)
*x^3/sqrt(d*x^6 + c)))/(b*d)]
```

**giac [B]** time = 0.23, size = 156, normalized size = 1.71

$$-\frac{\left(a\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{3\sqrt{abc-a^2d}\,b\sqrt{-d}}+\frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}\,b\,\mathrm{sgn}(x)}-\frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b\sqrt{-d}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*d
)*arctan(sqrt(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b*sqrt(-d)) + 1/3*a
*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b*sgn(x
)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b*sqrt(-d)*sgn(x))
```

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.550 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

**Rubi** [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]

[Out] ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 95, normalized size = 1.76

$$\frac{x^3\sqrt{\frac{dx^6}{c}+1}\tanh^{-1}\left(\frac{\sqrt{\frac{dx^6}{c}-\frac{bx^6}{a}}}{\sqrt{\frac{dx^6}{c}+1}}\right)}{3a\sqrt{c+dx^6}\sqrt{\frac{dx^6}{c}-\frac{bx^6}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^6)/c]*ArcTanh[Sqrt[-((b*x^6)/a) + (d*x^6)/c]/Sqrt[1 + (d
*x^6)/c]])/(3*a*Sqrt[c + d*x^6]*Sqrt[-((b*x^6)/a) + (d*x^6)/c])
```

**IntegrateAlgebraic [A]** time = 0.40, size = 106, normalized size = 1.96

$$\frac{\tan^{-1}\left(\frac{b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{3\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*
c - a*d]) + (b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])]/(3*Sqrt[a]*S
qrt[b*c - a*d])
```

**fricas [B]** time = 0.50, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2-4((bc-2ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2abx^6+a^2}\right)}{12(abc-a^2d)}, \frac{\arctan\left(\frac{((bc-2ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)}{6\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sq
rt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))/(a*b*c -
a^2*d), 1/6*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/sqrt(a*b*c -
 a^2*d)]
```

**giac [A]** time = 0.25, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{3\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.551 $\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(3*a*c*x^3) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqr
t[c + d*x^6])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{3ac}$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{3a}$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{3a}$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

**Mathematica [C]** time = 1.17, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^6}{c}+1\right)\left(\frac{4x^6\left(c+dx^6\right)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)}{3c^2\left(a+bx^6\right)}+\frac{\left(c+2dx^6\right)\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}}\right)}{c\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}}\right)}{3x^3\left(a+bx^6\right)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*((1 + (d*x^6)/c)*(((c + 2*d*x^6)*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a +
 b*x^6))]])/(c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2)]) +
 (4*(b*c - a*d)*x^6*(c + d*x^6)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x
^6)/(c*(a + b*x^6))])/(3*c^2*(a + b*x^6))))/(x^3*(a + b*x^6)*Sqrt[c + d*x^6
])
```

**IntegrateAlgebraic [A]** time = 0.66, size = 142, normalized size = 1.78

$$\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{3a^{3/2}(ad-bc)}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^6]/(a*c*x^3) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d]
)/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt
[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])]])/(3*a^(3/2)*(-(b*c) + a*d))
```

**fricas [B]** time = 0.51, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^3\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left((bc-2\,ad)x^9-acx^3\right)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+4\sqrt{dx^6+c}\left(abc-a^2d\right)}{12\left(a^2bc^2-a^3cd\right)x^3},-\frac{\sqrt{abc-a^2d}\,bcx^3\arctan\left(\frac{\left((bc-2\,ad)x^6-ac\right)\sqrt{dx^6+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^9+\left(abc^2-a^2cd\right)x^3\right)}\right)+2\sqrt{dx^6+c}\left(abc-a^2d\right)}{6\left(a^2bc^2-a^3cd\right)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(sqrt(-a*b*c + a^2*d)*b*c*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*
c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))
+ 4*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*(sqr
t(a*b*c - a^2*d)*b*c*x^3*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))
 + 2*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] Timed out
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}\;x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{6}+a\right)\sqrt{dx^{6}+c}\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{4}\,\left(b\,x^{6}+a\right)\,\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\left(a+bx^{6}\right)\sqrt{c+dx^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.552 $\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}\,(2ad+3bc)}{9a^2c^2x^3}-\frac{\sqrt{c+dx^6}}{9acx^9}$$

**Rubi** [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}\,(2ad+3bc)}{9a^2c^2x^3}-\frac{\sqrt{c+dx^6}}{9acx^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(9*a*c*x^9) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(9*a^2*c^2
*x^3) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*a^
(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{\operatorname{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{9ac}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}-\frac{\operatorname{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{9a^2c^2}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{3a^2}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{3a^2}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [C]** time = 4.65, size = 253, normalized size = 2.20

$$-\frac{\left(\frac{dx^6}{c}+1\right)\left(-\frac{8x^6\left(c+dx^6\right)^2(bc-ad)\,{}_3F_2\left(2,2,2;1,\frac{5}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)}{a+bx^6}+\frac{3c\left(c^2-4cdx^6-8d^2x^{12}\right)\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}}\right)}{\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}}+\frac{24dx^{12}\left(c+dx^6\right)(ad-bc)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)}{a+bx^6}\right)}{27c^3x^9\left(a+bx^6\right)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/27*((1 + (d*x^6)/c)*((3*c*(c^2 - 4*c*d*x^6 - 8*d^2*x^12)*ArcSin[Sqrt[((b
*c - a*d)*x^6)/(c*(a + b*x^6))]])/Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2
*(a + b*x^6)^2)] + (24*d*(-(b*c) + a*d)*x^12*(c + d*x^6)*Hypergeometric2F1[
2, 2, 5/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))])/(a + b*x^6) - (8*(b*c - a*d)
*x^6*(c + d*x^6)^2*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((b*c - a*d)*x^6)
/(c*(a + b*x^6))])/(a + b*x^6)))/(c^3*x^9*(a + b*x^6)*Sqrt[c + d*x^6])
```

**IntegrateAlgebraic [A]** time = 1.30, size = 163, normalized size = 1.42

$$\frac{\sqrt{c+dx^6}\left(-ac+2adx^6+3bcx^6\right)}{9a^2c^2x^9}-\frac{b^2\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^6}{\sqrt{a}\sqrt{bc-ad}}+\frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{3a^{5/2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-(a*c) + 3*b*c*x^6 + 2*a*d*x^6))/(9*a^2*c^2*x^9) - (b^2*S
qrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(
Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d]
)])/(3*a^(5/2)*(-(b*c) + a*d))
```

**fricas [A]** time = 0.53, size = 416, normalized size = 3.62

$$\left[-\frac{3\,\sqrt{-abc+a^{2}d}\,b^{2}c^{2}x^{9}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{12}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{6}+a^{2}c^{2}-4\left(\left(bc-2\,ad\right)x^{9}-acx^{3}\right)\sqrt{dx^{6}+c}\,\sqrt{-abc+a^{2}d}}{b^{2}x^{12}+2\,abx^{6}+a^{2}}\right)-4\left(\left(3\,ab^{2}c^{2}-a^{2}bcd-2\,a^{3}d^{2}\right)x^{6}-a^{2}bc^{2}+a^{3}cd\right)\sqrt{dx^{6}+c}}{36\left(a^{3}bc^{3}-a^{4}c^{2}d\right)x^{9}},\ \frac{3\,\sqrt{abc-a^{2}d}\,b^{2}c^{2}x^{9}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{6}-ac\right)\sqrt{dx^{6}+c}\,\sqrt{abc-a^{2}d}}{2\left(\left(abcd-a^{2}d^{2}\right)x^{9}+\left(abc^{2}-a^{2}cd\right)x^{3}\right)}\right)+2\left(\left(3\,ab^{2}c^{2}-a^{2}bcd-2\,a^{3}d^{2}\right)x^{6}-a^{2}bc^{2}+a^{3}cd\right)\sqrt{dx^{6}+c}}{18\left(a^{3}bc^{3}-a^{4}c^{2}d\right)x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^
9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 +
a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)
*sqrt(d*x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c - a^2*d
)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b
*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*
b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^6 + c)
)/((a^3*b*c^3 - a^4*c^2*d)*x^9)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}\;x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(bx^{6}+a\right)}\sqrt{dx^{6}+c}\,x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\,\left(b\,x^{6}+a\right)\,\sqrt{d\,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

## 3.553 $\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^6}}{6b^2\left(a+bx^6\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}}{3b^2d}$$

**Rubi [A]** time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^6}}{6b^2\left(a+bx^6\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b^2*d) - (a^2*Sqrt[c + d*x^6])/(6*b^2*(b*c - a*d)*(a + b
*x^6)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(6*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{12b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.36, size = 107, normalized size = 0.87

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^6}\left(\frac{a^2}{\left(a+bx^6\right)(ad-bc)}+\frac{2}{d}\right)}{b^2} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((Sqrt[c + d*x^6]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^6))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/6
```

**IntegrateAlgebraic [A]** time = 0.33, size = 143, normalized size = 1.16

$$\frac{\left(3a^2d-4abc\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6b^{5/2}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx^6}\left(3a^2d-2abc+2abdx^6-2b^2cx^6\right)}{6b^2d\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(Sqrt[c + d*x^6]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^6 + 2*a*b*d*x^6))/(b^
2*d*(b*c - a*d)*(a + b*x^6)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-
(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(6*b^(5/2)*(-(b*c) + a*d)^(3/2)
)
```

**fricas [B]** time = 0.65, size = 475, normalized size = 3.86

$$\left[\frac{\left(\left(4\,ab^2cd-3\,a^2bd^2\right)x^6+4\,a^2bcd-3\,a^3d^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\left(2\,\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^6+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2\right)\sqrt{dx^6+c}}{12\,\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)x^6\right)},\ -\frac{\left(\left(4\,ab^2cd-3\,a^2bd^2\right)x^6+4\,a^2bcd-3\,a^3d^2\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-\left(2\,\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^6+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2\right)\sqrt{dx^6+c}}{6\,\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b
*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6), -
1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6)]
```

**giac [A]** time = 0.44, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^6+c}\,a^2d}{6\,\left(b^3c-ab^2d\right)\left(\left(dx^6+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\,\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^6+c}}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(d*x^6 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*d))
- 1/6*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/3*sqrt(d*x^6 + c)/(b^2*d)
```

**maple [F]** time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.09, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^6+c}}{3\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^6+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^6+c}}{2\,\left(a\,d-b\,c\right)\,\left(3\,b^3\,\left(d\,x^6+c\right)-3\,b^3\,c+3\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^6)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(6*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b^3*(c
+ d*x^6) - 3*b^3*c + 3*a*b^2*d))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.554 $\int \frac{x^{11}}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=99

$$\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

**Rubi** [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{1}{6}\,\mathrm{Subst}\left(\int\frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)$$
$$=\frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}+\frac{(2bc-ad)\,\mathrm{Subst}\left(\int\frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12b(bc-ad)}$$
$$=\frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}+\frac{(2bc-ad)\,\mathrm{Subst}\left(\int\frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6bd(bc-ad)}$$
$$=\frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.10, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)}+\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{6b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*b^(
3/2))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 109, normalized size = 1.10

$$\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6b^{3/2}(ad-bc)^{3/2}}+\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) + ((2*b*c - a*d)*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(6*b^(3/2)*(-(b*c
) + a*d)^(3/2))
```

**fricas [A]** time = 0.69, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^6+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\left(ab^2c-a^2bd\right)}{12\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^6\right)},\ \frac{\left(\left(2\,b^2c-abd\right)x^6+2\,abc-a^2d\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\sqrt{dx^6+c}\left(ab^2c-a^2bd\right)}{6\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a))
+ 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b
^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a*b*d
)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-
```

```
b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*x^6)]
```

**giac [A]** time = 0.29, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^6+c}\,ad^2}{(b^2c-abd)\left((dx^6+c)b-bc+ad\right)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.99, size = 95, normalized size = 0.96

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^6+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(6*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^6)^(1/2))/(2*b*(a*d - b*c)*(3*b*(c +
 d*x^6) + 3*a*d - 3*b*c))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.555 $\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

**Rubi [A]** time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(6*(b*c - a*d)*(a + b*x^6)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[b*c - a*d]])/(6*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{12(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} - \frac{\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}$$

**Mathematica** [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^6}}{\left(a+bx^6\right)(ad-bc)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/((-(b*c) + a*d)*(a + b*x^6)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6
```

**IntegrateAlgebraic** [A] time = 0.11, size = 97, normalized size = 1.11

$$-\frac{\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)} - \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6\sqrt{b}\,(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(
b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(6*Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

**fricas** [B] time = 0.75, size = 302, normalized size = 3.47

$$\left[-\frac{\left(bdx^6+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\,\left(b^2c-abd\right)}{12\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}, -\frac{\left(bdx^6+ad\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\sqrt{dx^6+c}\,\left(b^2c-abd\right)}{6\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2*
sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(b^2*c
```

```
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

**giac [A]** time = 0.30, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^6+c}\,d}{6\left(\left(dx^6+c\right)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/6*sqrt(d*x^6 + c)*d/(((d*x^6 + c)*b - b*c + a*d)*(b*c - a
*d))
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.93, size = 84, normalized size = 0.97

$$\frac{d\,\sqrt{d\,x^6+c}}{2\,\left(a\,d-b\,c\right)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)) + (
d*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2)))/(6*b^(1/2)*(a*d - b*
c)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.556 $\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

**Rubi** [A] time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) - ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{}{(a+\ldots}\right.}{12a^2(bc-a\ldots}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3a^2d} - \frac{(b(2bc-3ad))\,\text{Subst}}{6a^2\ldots}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.35, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - (2*ArcTanh[Sqrt[c + d*x^
6]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*a^2)
```

**IntegrateAlgebraic [A]** time = 0.32, size = 146, normalized size = 1.11

$$\frac{\left(3a\sqrt{b}\,d-2b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6a^2(ad-bc)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(b*Sqrt[c + d*x^6])/(a*(-(b*c) + a*d)*(a + b*x^6)) + ((-2*b^(3/2)*c +
3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a
*d)])/(6*a^2*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a^
2*Sqrt[c])
```

**fricas [A]** time = 0.93, size = 862, normalized size = 6.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x^6
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
6 + b*c)) + ((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sq
rt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c
^2 - a^4*c*d), 1/12*(2*sqrt(d*x^6 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^6 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b
*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6
 + b*c)) + 2*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^6 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d)]
```

**giac [A]** time = 0.41, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^6+c}\,bd}{6\left(abc-a^2d\right)\left(\left(dx^6+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(d*x^6 + c)*b*d/((a*b*c - a^2*d)*((d*x^6 + c)*b - b*c + a*d)) - 1/6
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

**maple [F]** time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x), x)
```

**mupad [B]** time = 6.23, size = 3025, normalized size = 22.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b^2*d^5)/3 - 2*a
^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
 - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144*a^7*b^2
*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(216
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*
d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*
(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a
^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*
b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*a^6*b^2*d^5)
/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^
4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144
*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^
3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a
^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(
12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b
*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2
*c^2 - 2*a^4*b*c*d) - ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2
 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b
^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2
 - 2*a^4*b*c*d) - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/
2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4
*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*
d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2
*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2
*d^2 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*
a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^
2*c^2 - 2*a^4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3
)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^
5*b^4*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b
^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)
^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*
(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^
3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i
)/(6*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((
(((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2
 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a
^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - ((c + d*x^6)^(1/
2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b
^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - (((((4*a^6*b^2*d^5)/3 - 2*a^5*b
^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
 + ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^
3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^
2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(
a^2*c^(1/2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2*c^2 - 2
*a^4*b*c*d) + ((((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/
3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*
b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(
```

```
648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - (
(c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(
a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + ((((4*a^6*b^2*d^5)/3
 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a
^4*b*c*d)) + ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*
a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2
*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4
+ 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(a^2*c^(1/2))))*1i)/(3*a^2*c^(1/2)) - (b*d*(c + d*x^6)^(1/2))/(2*(a^2*
d - a*b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

## 3.557 $\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}\,(2bc-ad)}{6a^2c\left(a+bx^6\right)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}$$

**Rubi** [A] time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}\,(2bc-ad)}{6a^2c\left(a+bx^6\right)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*(a + b*x^6)) - Sqrt
[c + d*x^6]/(6*a*c*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6
]/Sqrt[c]])/(6*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)}{6ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc}{x(a+bx)\sqrt{c+dx}}\right.}{6a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{}{(a+bx)}\right.}{12a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+}\right.}{6a^3d(bc-a}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} -$$

**Mathematica [A]** time = 0.68, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^6}\left(a^2d+ab\left(dx^6-c\right)-2b^2cx^6\right)}{x^6\left(a+bx^6\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[c + d*x^6]*(a^2*d - 2*b^2*c*x^6 + a*b*(-c + d*x^6)))/((b*c - a*d)*
x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(6*a^3*c)
```

**IntegrateAlgebraic [A]** time = 0.66, size = 187, normalized size = 1.01

$$\frac{\left(4b^{5/2}c-5ab^{3/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}\sqrt{ad-bc}}{bc-ad}\right)}{6a^3(ad-bc)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}+\frac{\sqrt{c+dx^6}\left(-a^2d+abc-abdx^6+2b^2cx^6\right)}{6a^2cx^6\left(a+bx^6\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(a*b*c - a^2*d + 2*b^2*c*x^6 - a*b*d*x^6))/(6*a^2*c*(-(b*c
) + a*d)*x^6*(a + b*x^6)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^6])/(b*c - a*d)])/(6*a^3*(-(b*c) + a*d)^(3/
2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(6*a^3*c^(3/2))
```

**fricas [A]** time = 0.92, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6
 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12
+ (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sq
rt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt
(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*
b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/6*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^6 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6)]
```

**giac [A]** time = 0.41, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^6+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^6+c}\,b^2c^2d-\left(dx^6+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^6+c}\,abcd^2-\sqrt{dx^6+c}\,a^2d^3}{6\left(a^2bc^2-a^3cd\right)\left(\left(dx^6+c\right)^2b-2\left(dx^6+c\right)bc+bc^2+\left(dx^6+c\right)ad-acd\right)}-\frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/6*(2*(d*x^6 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^6 + c)*b^2*c^2*d - (d*x^6 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^6 + c
)*a*b*c*d^2 - sqrt(d*x^6 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^6 + c)^
```

```
2*b - 2*(d*x^6 + c)*b*c + b*c^2 + (d*x^6 + c)*a*d - a*c*d)) - 1/6*(4*b*c +
a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\;x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)
```

**mupad** [B] time = 7.29, size = 3860, normalized size = 20.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] (((c + d*x^6)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^6)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^6)*(3*a*d - 6*b*c) + 3*b*(c + d*x^6)^2 + 3*b*c^2 - 3*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576
*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2
*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b
*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*
a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3
 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(12*(a^6*d^3 - a^3*b^3*c^3 +
3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*
b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 +
 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*
a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c
*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(21
6*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2
)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*
d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c
^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1
i)/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/((5*a^3*
b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(108*(a^6*b^
2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d
- 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d
^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2
- 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b
```

```
^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)
/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^
(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*
c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a
^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3
*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32
*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(1
8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2
)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c
^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3
*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a
^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^
2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(6*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d
*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*
d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))
 + (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a
^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d
*x^6)^(1/2)*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*
a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 +
 a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d +
 4*b*c)*1i)/(12*a^3*(c^3)^(1/2)) + ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b
^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*
(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a
^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c
^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*
a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c
^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))
*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(12*a^3*(c^3)^(1/2)
))/((5*a^3*b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(
108*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((((c + d*x^6)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^
5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((144*a^9*b
^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)
/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d*x^6)^(1/2)*(a*
d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4
 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 -
2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(12*a^
3*(c^3)^(1/2)) + ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*
b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6
*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 5
76*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 -
 2*a^7*b*c^3*d)) + ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 -
720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*
(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(1
2*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)
/(6*a^3*(c^3)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.558 $\int \frac{x^{14}}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}} \, dx$

**Optimal**. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{ax^3\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

**Rubi [A]** time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{ax^3\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} - \frac{\operatorname{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6b(bc-ad)}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^3\right)}{3b^2} - \frac{(a(3bc-2ad))\operatorname{Subst}\left(\int \frac{}{(a}\right.}{6b^2(bc-}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2} - \frac{(a(3bc-2ad))\operatorname{Subst}\left(\int\right.}{6b^2(}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

**Mathematica [A]** time = 0.28, size = 135, normalized size = 0.96

$$\frac{\frac{abx^3\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)} + \frac{\sqrt{a}\,(2ad-3bc)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\,\sqrt{c+dx^6}+dx^3\right)}{\sqrt{d}}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]])/Sqrt[d])/(6*b^2)
```

**IntegrateAlgebraic [A]** time = 2.62, size = 166, normalized size = 1.18

$$\frac{\left(2a^{3/2}d-3\sqrt{a}\,bc\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\sqrt{c+dx^6}+b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{ax^3\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)} + \frac{\log\left(\sqrt{c+dx^6}+\sqrt{d}\,x^3\right)}{3b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) + ((-3*Sqrt[a]*b*c +
2*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sq
rt[a]*Sqrt[b*c - a*d])])/(6*b^2*(b*c - a*d)^(3/2)) + Log[Sqrt[d]*x^3 + Sqrt
[c + d*x^6]]/(3*b^2*Sqrt[d])
```

**fricas** [A]     time = 1.41, size = 1077, normalized size = 7.64

$$\left[\frac{4\sqrt{dx^6+c}\,abdx^3+4\left(\left(b^2c-abd\right)x^6+abc-a^2d\right)\sqrt{d}\log\left(-2dx^6-2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)+\left(\left(3b^2cd-2abd^2\right)x^6+3abcd-2a^2d^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{12}-2\left(3abc^2-4a^2cd\right)x^6+a^2c^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)}{24\left(\left(b^4cd-ab^3d^2\right)x^6+ab^3cd-a^2b^2d^2\right)},\ \frac{4\sqrt{dx^6+c}\,abdx^3-8\left(\left(b^2c-abd\right)x^6+abc-a^2d\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)+\left(\left(3b^2cd-2abd^2\right)x^6+3abcd-2a^2d^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{12}-2\left(3abc^2-4a^2cd\right)x^6+a^2c^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)}{24\left(\left(b^4cd-ab^3d^2\right)x^6+ab^3cd-a^2b^2d^2\right)},\ \frac{2\sqrt{dx^6+c}\,abdx^3+\left(\left(3b^2cd-2abd^2\right)x^6+3abcd-2a^2d^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2ad\right)x^6-ac\right)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^9+acx^3\right)}\right)+2\left(\left(b^2c-abd\right)x^6+abc-a^2d\right)\sqrt{d}\log\left(-2dx^6-2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)}{12\left(\left(b^4cd-ab^3d^2\right)x^6+ab^3cd-a^2b^2d^2\right)},\ \frac{2\sqrt{dx^6+c}\,abdx^3-4\left(\left(b^2c-abd\right)x^6+abc-a^2d\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^3}{\sqrt{dx^6+c}}\right)+\left(\left(3b^2cd-2abd^2\right)x^6+3abcd-2a^2d^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2ad\right)x^6-ac\right)\sqrt{dx^6+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^9+acx^3\right)}\right)}{12\left(\left(b^4cd-ab^3d^2\right)x^6+ab^3cd-a^2b^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a
*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3
 - 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqr
t(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(
-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*
b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a
*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12
*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d -
2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + 2*((b^2*c - a*b*d)*x^6
 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c)
)/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12*(2*sqrt(d*x^6
 + c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(s
qrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2
*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*
x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)))/((b^4*c*d - a*b^3*d^2)*x
^6 + a*b^3*c*d - a^2*b^2*d^2)]
```

**giac** [B]     time = 0.51, size = 343, normalized size = 2.43

$$-\frac{\left(3\,abc\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-2\,a^2\sqrt{-d}\,d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-2\sqrt{abc-a^2d}\,bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+2\sqrt{abc-a^2d}\,ad\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+\sqrt{abc-a^2d}\,a\sqrt{-d}\sqrt{d}\right)\mathrm{sgn}(x)}{6\left(\sqrt{abc-a^2d}\,b^3c\sqrt{-d}-\sqrt{abc-a^2d}\,ab^2\sqrt{-d}\,d\right)}+\frac{ac\sqrt{d+\frac{c}{x^6}}}{6\left(b^2c\,\mathrm{sgn}(x)-abd\,\mathrm{sgn}(x)\right)\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)}+\frac{\left(3\,abc-2\,a^2d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{6\left(b^3c\,\mathrm{sgn}(x)-ab^2d\,\mathrm{sgn}(x)\right)\sqrt{abc-a^2d}}-\frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b^2\sqrt{-d}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*c*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(-
d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*arct
an(sqrt(d)/sqrt(-d)) + 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)/sqrt(-d)) +
 sqrt(a*b*c - a^2*d)*a*sqrt(-d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^3*c*
sqrt(-d) - sqrt(a*b*c - a^2*d)*a*b^2*sqrt(-d)*d) + 1/6*a*c*sqrt(d + c/x^6)/
((b^2*c*sgn(x) - a*b*d*sgn(x))*(b*c + a*(d + c/x^6) - a*d)) + 1/6*(3*a*b*c
- 2*a^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/((b^3*c*sgn(x) - a
*b^2*d*sgn(x))*sqrt(a*b*c - a^2*d)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/
(b^2*sqrt(-d)*sgn(x))
```

**maple** [F]     time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.559 $\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

**Rubi [A]** time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(x^3*Sqrt[c + d*x^6])/(6*(b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{c\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}$$

**Mathematica** [A] time = 0.58, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^6}\left(-\frac{x^6(bc-ad)}{a+bx^6} - \frac{c\sqrt{x^6\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^6\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^6}{c}+1}}\right)}{\sqrt{\frac{dx^6}{c}+1}}\right)}{6x^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-(((b*c - a*d)*x^6)/(a + b*x^6)) - (c*Sqrt[(-(b/a) + d/c)
*x^6]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^6]/Sqrt[1 + (d*x^6)/c]])/Sqrt[1 + (d*x^
6)/c]))/(6*(b*c - a*d)^2*x^3)
```

**IntegrateAlgebraic** [A] time = 1.33, size = 145, normalized size = 1.56

$$\frac{c\tan^{-1}\left(\frac{b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{bx^3\sqrt{c+dx^6}}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}} - \frac{x^3\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[a]*S
qrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^6)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^
3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*Sqrt[a]*(b*c - a*d)^(3/2)
)
```

**fricas** [B] time = 0.79, size = 426, normalized size = 4.58

$$\left[-\frac{4\sqrt{dx^6+c}\left(abc-a^2d\right)x^3 - \left(bcx^6+ac\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left((bc-2\,ad)x^9-acx^3\right)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{24\left(\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}, -\frac{2\sqrt{dx^6+c}\left(abc-a^2d\right)x^3 - \left(bcx^6+ac\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\left((bc-2\,ad)x^6-ac\right)\sqrt{dx^6+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^9+\left(abc^2-a^2cd\right)x^3\right)}\right)}{12\left(\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt
(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c*
d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sqrt(d*
x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1
/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
- a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{{\left(bx^{6}+a\right)}^{2} \sqrt{dx^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{8}}{{\left(b\,x^{6}+a\right)}^{2}\,\sqrt{d\,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.560 $\int \frac{x^2}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}} \, dx$

**Optimal**. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

**Rubi** [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*x^3*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

**Mathematica [C]** time = 1.00, size = 407, normalized size = 3.91

$$\frac{x^3\sqrt{c+dx^6}\left(-30dx^6\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}-45c\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}+16dx^6\left(\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^6\right)}{c\left(a+bx^6\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)+16c\left(\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^6\right)}{c\left(a+bx^6\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)+30dx^6\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}}\right)+45c\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}}\right)\right)}{90c^2\left(a+bx^6\right)^2\left(\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}\right)^{3/2}\sqrt{\frac{a\left(c+dx^6\right)}{c\left(a+bx^6\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6]*(-45*c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a +
b*x^6)^2)] - 30*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 30*d*x^6*ArcS
in[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 16*c*(((b*c - a*d)*x^6)/(c*(a
 + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 16*d*x^6*(((b*c - a*d)*x^6)/
(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))]))/(90*c^2*(((b*c - a*d)*x
^6)/(c*(a + b*x^6)))^(3/2)*(a + b*x^6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6
))])
```

**IntegrateAlgebraic [A]** time = 0.81, size = 124, normalized size = 1.19

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\sqrt{c+dx^6}+b\sqrt{d}\,x^6}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{3/2}(bc-ad)^{3/2}}-\frac{bx^3\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(b*x^3*Sqrt[c + d*x^6])/(a*(-(b*c) + a*d)*(a + b*x^6)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(6*a^(3/2)*(b*c - a*d)^(3/2))
```

**fricas [B]** time = 0.59, size = 467, normalized size = 4.49

$$\left[\frac{4\sqrt{dx^6+c}\left(ab^2c-a^2bd\right)x^3-\left(\left(b^2c-2\,abd\right)x^6+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left((bc-2\,ad)x^9-acx^3\right)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{24\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^6\right)}, \frac{2\sqrt{dx^6+c}\left(ab^2c-a^2bd\right)x^3+\left(\left(b^2c-2\,abd\right)x^6+abc-2\,a^2d\right)\sqrt{abc-a^2d}\arctan\left(\frac{\left((bc-2\,ad)x^6-ac\right)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^9+\left(abc^2-a^2cd\right)x^3\right)}\right)}{12\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 -
a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)
))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^
4*b*d^2)*x^6), 1/12*(2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^2*c -
2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*
d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9
+ (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]
```

**giac [B]** time = 0.28, size = 237, normalized size = 2.28

$$-\frac{1}{6}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2 bc-2\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2 ad-bc^2\right)}{\left(\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^4 b-2\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2 bc+4\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2 ad+bc^2\right)\left(abcd-a^2d^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqrt(d*
x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

## 3.561 $\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=149

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(3bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^3\left(a+bx^6\right)(bc-ad)}$$

**Rubi [A]** time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^6}\,(3bc-2ad)}{6a^2cx^3(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^6}}{6ax^3\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c +
d*x^6])/(6*a*(b*c - a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x\ldots\right.}{6a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{\ldots}{(a+b\ldots}\right.}{6a^2(bc-a\ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{\ldots}{a-(-\ldots}\right.}{6a^2(bc-\ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x\ldots}{\sqrt{a}\sqrt{c+dx\ldots}}\right.}{6a^{5/2}(bc-ad)^{3/2}}$$

**Mathematica [C]** time = 5.40, size = 869, normalized size = 5.83

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/90*(Sqrt[c + d*x^6]*(-45*c^2*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(
a + b*x^6)^2)] - 180*c*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a +
 b*x^6)^2)] - 120*d^2*x^12*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b
*x^6)^2)] + 45*c^2*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 180*c*
d*x^6*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 120*d^2*x^12*ArcSin
[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 64*c^2*(((b*c - a*d)*x^6)/(c*(a
 + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 160*c*d*x^6*(((b*c - a*d)*x^
6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeomet
ric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 96*d^2*x^12*(((b*c -
 a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hyp
ergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 32*c^2*(((b*
c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*
HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] +
 64*c*d*x^6*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/
(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(
```

```
c*(a + b*x^6))] + 32*d^2*x^12*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqr
t[(a*(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, (
(b*c - a*d)*x^6)/(c*(a + b*x^6))]))/(c^3*x^3*(((b*c - a*d)*x^6)/(c*(a + b*x
^6)))^(3/2)*(a + b*x^6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))])
```

**IntegrateAlgebraic [A]** time = 1.63, size = 159, normalized size = 1.07

$$\frac{\left(4abd-3b^2c\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\sqrt{c+dx^6}+b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{6a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}\left(-2a^2d+2abc-2abdx^6+3b^2cx^6\right)}{6a^2cx^3\left(a+bx^6\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^6 - 2*a*b*d*x^6))/(6*a^2*c*
(-(b*c) + a*d)*x^3*(a + b*x^6)) + ((-3*b^2*c + 4*a*b*d)*ArcTan[(a*Sqrt[d] +
 b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(5
/2)*(b*c - a*d)^(3/2))
```

**fricas [B]** time = 0.61, size = 612, normalized size = 4.11

$$\left[-\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^9+\left(3\,ab^2c^2-4\,a^2bcd\right)x^3\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left(\left(bc-2\,ad\right)x^9-acx^3\right)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+4\left(\left(3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2\right)x^6+2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2\right)\sqrt{dx^6+c}}{24\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)x^9+\left(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2\right)x^3\right)},\ -\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^9+\left(3\,ab^2c^2-4\,a^2bcd\right)x^3\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^6-ac\right)\sqrt{dx^6+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^9+\left(abc^2-a^2cd\right)x^3\right)}\right)+2\left(\left(3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2\right)x^6+2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2\right)\sqrt{dx^6+c}}{12\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)x^9+\left(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6
 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3*c^2
 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d
^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (
a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 - 4*a*b^2
*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d -
a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2*c*d
+ 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^6 +
c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b^2*c^3 - 2*a
^5*b*c^2*d + a^6*c*d^2)*x^3)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}\;x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Timed out
```

## 3.562 $\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

**Optimal**. Leaf size=208

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(5bc-2ad)}{18a^2cx^9(bc-ad)}+\frac{\sqrt{c+dx^6}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{18a^3c^2x^3(bc-ad)}+\frac{b\sqrt{c+}}{6ax^9\left(a+bx^6\right.}$$

**Rubi [A]** time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^6}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{18a^3c^2x^3(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(5bc-2ad)}{18a^2cx^9(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^9\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^6])/(18*a^2*c*(b*c - a*d)*x^9) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^6])/(18*a^3*c^2*(b*c - a*d)*x^3) +
(b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2}{x^2\left(a+bx^2\right)\sqrt{\ldots}}\ldots\right)}{18a^2c(bc\ldots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d\ldots}}{6a(bc-ad)x^9\left(\ldots\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d\ldots}}{6a(bc-ad)x^9\left(\ldots\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d\ldots}}{6a(bc-ad)x^9\left(\ldots\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d\ldots}}{6a(bc-ad)x^9\left(\ldots\right.}$$

**Mathematica [A]** time = 5.86, size = 175, normalized size = 0.84

$$\frac{a^2\left(c+dx^6\right)\left(\frac{3b^3x^{12}}{\left(a+bx^6\right)(bc-ad)}+\frac{4x^6(ad+3bc)}{c^2}-\frac{2a}{c}\right)+\frac{3b^2x^{18}\sqrt{\frac{dx^6}{c}+1}\,(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^6\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^6}{a}+1}}\right)}{c\left(\frac{x^6(bc-ad)}{ac}\right)^{3/2}}}{18a^5x^9\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a^2*(c + d*x^6)*((-2*a)/c + (4*(3*b*c + a*d)*x^6)/c^2 + (3*b^3*x^12)/((b*c
 - a*d)*(a + b*x^6))) + (3*b^2*(5*b*c - 6*a*d)*x^18*Sqrt[1 + (d*x^6)/c]*Arc
Sin[Sqrt[(b/a - d/c)*x^6]/Sqrt[1 + (b*x^6)/a]])/(c*(((b*c - a*d)*x^6)/(a*c)
)^(3/2)))/(18*a^5*x^9*Sqrt[c + d*x^6])
```

**IntegrateAlgebraic [A]** time = 2.88, size = 217, normalized size = 1.04

$$\frac{\left(5b^3c-6ab^2d\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^3\sqrt{c+dx^6}+b\sqrt{d}\,x^6}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{6a^{7/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}\left(-2a^3cd+4a^3d^2x^6+2a^2bc^2+6a^2bcdx^6+4a^2bd^2x^{12}-10ab^2c^2x^6+8ab^2cdx^{12}-15b^3c^2x^{12}\right)}{18a^3c^2x^9\left(a+bx^6\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(2*a^2*b*c^2 - 2*a^3*c*d - 10*a*b^2*c^2*x^6 + 6*a^2*b*c*d*
x^6 + 4*a^3*d^2*x^6 - 15*b^3*c^2*x^12 + 8*a*b^2*c*d*x^12 + 4*a^2*b*d^2*x^12
))/(18*a^3*c^2*(-(b*c) + a*d)*x^9*(a + b*x^6)) + ((5*b^3*c - 6*a*b^2*d)*Arc
Tan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c -
 a*d])])/(6*a^(7/2)*(b*c - a*d)^(3/2))
```

**fricas [A]** time = 0.77, size = 760, normalized size = 3.65

$$\left[-\frac{3\left(\left(5b^4c^3-6ab^3c^2d\right)x^{15}+\left(5ab^3c^3-6a^2b^2c^2d\right)x^9\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{12}-2\left(3abc^2-4a^2cd\right)x^6+a^2c^2-4\left(\left(bc-2ad\right)x^9-acx^3\right)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2abx^6+a^2}\right)-4\left(\left(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3\right)x^{12}-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2+2\left(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3\right)x^6\right)\sqrt{dx^6+c}}{72\left(\left(a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2\right)x^{15}+\left(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2\right)x^9\right)},\ \frac{3\left(\left(5b^4c^3-6ab^3c^2d\right)x^{15}+\left(5ab^3c^3-6a^2b^2c^2d\right)x^9\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\left(\left(bc-2ad\right)x^6-ac\right)\sqrt{dx^6+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^9+\left(abc^2-a^2cd\right)x^3\right)}\right)+2\left(\left(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3\right)x^{12}-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2+2\left(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3\right)x^6\right)\sqrt{dx^6+c}}{36\left(\left(a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2\right)x^{15}+\left(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2\right)x^9\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^9)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*s
qrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*((15
*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^12 - 2*a^3
*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d
 + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3*c^4 - 2*a^5*b^2
*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*
x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*
c^2*d)*x^9)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d
)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d
^3)*x^12 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 -
 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3
*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d
 + a^7*c^2*d^2)*x^9)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [F]** time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}\;x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10}\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Timed out
```

## 3.563 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}\,(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

**Rubi [A]** time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}\,(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^8])/(4*b^2*d^2) + (c + d*x^8)^(3/2)/(12*b*d^2) -
 (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b
*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{23}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{1}{8}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}} + \frac{a^2}{b^2(a+bx)\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{bd}\right) dx, x, x^8\right)$$
$$= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{\left(c+dx^8\right)^{3/2}}{12bd^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b^2}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{\left(c+dx^8\right)^{3/2}}{12bd^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4b^2d}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{\left(c+dx^8\right)^{3/2}}{12bd^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.22, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^8}\left(-3ad-2bc+bdx^8\right)}{12b^2d^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) - (a^2*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b*c - a*d])
```

**IntegrateAlgebraic** [A] time = 0.16, size = 101, normalized size = 0.97

$$\frac{\sqrt{c+dx^8}\left(-3ad-2bc+bdx^8\right)}{12b^2d^2} - \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{5/2}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) - (a^2*ArcTan[(Sq
rt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(4*b^(5/2)*Sqrt[-(b
*c) + a*d])
```

**fricas** [A] time = 0.43, size = 288, normalized size = 2.77

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^8+2bc-ad-2\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\left(\left(b^3cd-ab^2d^2\right)x^8-2b^3c^2-ab^2cd+3a^2bd^2\right)\sqrt{dx^8+c}}{24\left(b^4cd^2-ab^3d^3\right)}, \frac{3\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\left(\left(b^3cd-ab^2d^2\right)x^8-2b^3c^2-ab^2cd+3a^2bd^2\right)\sqrt{dx^8+c}}{12\left(b^4cd^2-ab^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*
x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*
c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 - a*b^
2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

**giac [A]** time = 0.16, size = 106, normalized size = 1.02

$$\frac{a^{2} \arctan\left(\frac{\sqrt{dx^{8}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{4\sqrt{-b^{2}c+abd}\,b^{2}}+\frac{\left(dx^{8}+c\right)^{\frac{3}{2}}b^{2}d^{4}-3\sqrt{dx^{8}+c}\,b^{2}cd^{4}-3\sqrt{dx^{8}+c}\,abd^{5}}{12\,b^{3}d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/12*((d*x^8 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^8 + c)*b^2*c*d^4 - 3*
sqrt(d*x^8 + c)*a*b*d^5)/(b^3*d^6)
```

**maple [F]** time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.68, size = 103, normalized size = 0.99

$$\frac{\left(d\,x^{8}+c\right)^{3/2}}{12\,b\,d^{2}}-\left(\frac{c}{2\,b\,d^{2}}+\frac{4\,a\,d^{3}-4\,b\,c\,d^{2}}{16\,b^{2}\,d^{4}}\right)\sqrt{d\,x^{8}+c}+\frac{a^{2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^{8}+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(3/2)/(12*b*d^2) - (c/(2*b*d^2) + (4*a*d^3 - 4*b*c*d^2)/(16*b^2
*d^4))*(c + d*x^8)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c
)^(1/2)))/(4*b^(5/2)*(a*d - b*c)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.564 $\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}}{4bd}$$

**Rubi [A]** time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a
*d]])/(4*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b}$$
$$= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4bd}$$
$$= \frac{\sqrt{c+dx^8}}{4bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.08, size = 72, normalized size = 0.97

$$\frac{1}{4}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^8}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/4
```

**IntegrateAlgebraic** [A] time = 0.10, size = 84, normalized size = 1.14

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{3/2}\sqrt{ad-bc}} + \frac{\sqrt{c+dx^8}}{4bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*
x^8])/(b*c - a*d)])/(4*b^(3/2)*Sqrt[-(b*c) + a*d])
```

**fricas** [A] time = 0.46, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\,(b^2c-abd)}{8\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-\sqrt{dx^8+c}\,(b^2c-abd)}{4\,(b^3cd-ab^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c
)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

**giac [A]** time = 0.16, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^8+c}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^8 + c)/b)/d
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^8+c}}{4\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b*d) - (a*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)
^(1/2)))/(4*b^(3/2)*(a*d - b*c)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

## 3.565 $\int \frac{x^7}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(4*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

**IntegrateAlgebraic [A]** time = 0.05, size = 61, normalized size = 1.20

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{4\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)]/(Sqrt
[b]*Sqrt[-(b*c) + a*d])
```

**fricas [A]** time = 0.42, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)}{8\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)}{4\,\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b
*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]
```

**giac [A]** time = 0.17, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.59, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\,\sqrt{d\,x^8+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{4\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^8)^(1/2))/(a*b*d - b^2*c)^(1/2))/(4*(a*b*d - b^2*c)^(1/2))
```

**sympy [A]** time = 44.57, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c)/b))
```

## 3.566 $\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

**Rubi [A]** time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}$$

**Mathematica** [A] time = 0.10, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^8])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(4*a)
```

**IntegrateAlgebraic** [A] time = 0.11, size = 95, normalized size = 1.12

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{4a\sqrt{ad-bc}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/
(4*a*Sqrt[-(b*c) + a*d]) - ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a*Sqrt[c])
```

**fricas** [A] time = 0.48, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad+2\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+\sqrt{c}\log\left(\frac{dx^8-2\sqrt{dx^8+c}\sqrt{c}+2c}{x^8}\right)}{8ac}, \frac{2c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+\sqrt{c}\log\left(\frac{dx^8-2\sqrt{dx^8+c}\sqrt{c}+2c}{x^8}\right)}{8ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad+2\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)}{8ac}, \frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+\sqrt{-c}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)}{4ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sqrt
(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt
(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-
c)/c))/(a*c), 1/4*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^8 + c)*
sqrt(-c)/c))/(a*c)]
```

**giac [A]** time = 0.16, size = 71, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c))
```

**maple [F]** time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)
```

**mupad [B]** time = 4.81, size = 652, normalized size = 7.67

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^8+c}}{\sqrt{c}}\right)}{4\,a\,\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{8\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{8\,\left(a^2\,d-a\,b\,c\right)}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}}\right)\,\sqrt{b^2\,c-a\,b\,d}\,1{}\mathrm{i}}{4\,\left(a^2\,d-a\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^8)^(1/2)/c^(1/2))/(4*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3
- ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/
2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d - a*b*c)) + ((
b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2
)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2
*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d
 - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*
c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c))
))/(8*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2)
)/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*
d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2
```

```
*d - a*b*c))))/(8*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(4*(a^2*d -
a*b*c))
```

**sympy** [A] time = 37.86, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**8)/sqrt(-c))/(4*a*sqrt(-c))
```

## 3.567 $\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

**Rubi [A]** time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(8*a*c*x^8) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[
c]])/(8*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c
- a*d]])/(4*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8ac}$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a^2} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.}{16a^2c}$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4a^2d} - \frac{(2bc+ad)\operatorname{Subst}\left(\int \frac{}{-\frac{c}{d}}\right.}{8a^2c}$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}$$

**Mathematica [A]** time = 0.14, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2(ad-bc)} + \frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8ac^{3/2}} - \frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/(a*c*x^8) + (b*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(4*a^
2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a*c^(3/2)) + (b^(3/2)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a^2*
(-(b*c) + a*d))
```

**IntegrateAlgebraic [A]** time = 0.26, size = 127, normalized size = 1.09

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{4a^2\sqrt{ad-bc}} + \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/(a*c*x^8) - (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^8])/(b*c - a*d)])/(4*a^2*Sqrt[-(b*c) + a*d]) + ((2*b*c + a*d
)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a^2*c^(3/2))
```

**fricas [A]** time = 0.47, size = 565, normalized size = 4.83

$$\left[\frac{2bc^2x^8\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad-2\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+(2bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\sqrt{dx^8+c}\sqrt{c}+2c}{x^8}\right)-2\sqrt{dx^8+c}\,ac}{16a^2c^2x^8}, -\frac{4bc^2x^8\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)-(2bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\sqrt{dx^8+c}\sqrt{c}+2c}{x^8}\right)+2\sqrt{dx^8+c}\,ac}{16a^2c^2x^8}, \frac{bc^2x^8\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad-2\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)-(2bc+ad)\sqrt{-c}x^8\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)-\sqrt{dx^8+c}\,ac}{8a^2c^2x^8}, -\frac{2bc^2x^8\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+(2bc+ad)\sqrt{-c}x^8\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)+\sqrt{dx^8+c}\,ac}{8a^2c^2x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(
d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + (2*b*c + a*d)*sq
rt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*x^8
 + c)*a*c)/(a^2*c^2*x^8), -1/16*(4*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*sq
rt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), 1/8*(b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^8 + a)) - (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c)
 - sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), -1/8*(2*b*c^2*x^8*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + sqrt(d*
x^8 + c)*a*c)/(a^2*c^2*x^8)]
```

**giac [A]** time = 0.18, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^2\sqrt{-c}\,c} - \frac{\sqrt{dx^8+c}}{8\,acx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/8*(2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/8*sqrt(d*x^8 + c)/(a*c*x^8)
```

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)
```

**mupad [B]** time = 5.51, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^8+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,a^3\,d-8\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^8+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^8+c}}{8\,a\,c\,x^8}-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^8+c}\,3{}\mathrm{i}}{128\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,c}+\frac{5\,a\,b^3\,d^5}{256\,c^2}+\frac{a^2\,b^2\,d^6}{256\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^8+c}\,1{}\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{256\,a}+\frac{b^2\,d^6}{256\,c}+\frac{3\,b^4\,c\,d^4}{128\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^8+c}\,5{}\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,a}+\frac{5\,b^3\,d^5}{256\,c}+\frac{a\,b^2\,d^6}{256\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{8\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (log((c + d*x^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(8*a^3*d - 8*a^2*b*c) - (log((c + d*x^8
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
```

```
*c - a*b^3*d)^(1/2))/(8*(a^3*d - a^2*b*c)) - (c + d*x^8)^(1/2)/(8*a*c*x^8)
- (atan((b^4*d^4*(c + d*x^8)^(1/2)*3i)/(128*(c^3)^(1/2)*((3*b^4*d^4)/(128*c
) + (5*a*b^3*d^5)/(256*c^2) + (a^2*b^2*d^6)/(256*c^3))) + (b^2*d^6*(c + d*x
^8)^(1/2)*1i)/(256*(c^3)^(1/2)*((5*b^3*d^5)/(256*a) + (b^2*d^6)/(256*c) + (
3*b^4*c*d^4)/(128*a^2))) + (b^3*d^5*(c + d*x^8)^(1/2)*5i)/(256*(c^3)^(1/2)*
((3*b^4*d^4)/(128*a) + (5*b^3*d^5)/(256*c) + (a*b^2*d^6)/(256*c^2))))*(a*d
+ 2*b*c)*1i)/(8*a^2*(c^3)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9 \left(a + bx^8\right) \sqrt{c + dx^8}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

## 3.568 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}+\frac{x^4\sqrt{c+dx^8}}{8bd}$$

**Rubi [A]** time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}+\frac{x^4\sqrt{c+dx^8}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt
[a]*Sqrt[c + d*x^8])])/(4*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^4)/Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{19}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8bd}\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\right.}{8b^2d}\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\right.}{8b^2d}\\
&= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.24, size = 118, normalized size = 0.96

$$
\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\,\sqrt{c+dx^8}+dx^4\right)}{d^{3/2}} + \frac{bx^4\sqrt{c+dx^8}}{d}}{8b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ((b*x^4*Sqrt[c + d*x^8])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[
a]*Sqrt[c + d*x^8])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^4 + Sqrt[d]*
Sqrt[c + d*x^8]])/d^(3/2))/(8*b^2)
```

**IntegrateAlgebraic [A]** time = 1.90, size = 177, normalized size = 1.44

$$
\frac{a^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^8}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\,\sqrt{bc-ad}} + \frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{4b^2\sqrt{bc-ad}} + \frac{(-2ad-bc)\log\left(\sqrt{c+dx^8}+\sqrt{d}\,x^4\right)}{8b^2d^{3/2}} + \frac{x^4\sqrt{c+dx^8}}{8bd}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c
- a*d] + (b*Sqrt[d]*x^8)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt[c + d*x^8]
```

```
)/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*b^2*Sqrt[b*c - a*d]) + ((-(b*c) - 2*a*d)*L
og[Sqrt[d]*x^4 + Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))
```

**fricas [A]** time = 0.56, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^8+c}\,bdx^4+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{16}-2(3abc^2-4a^2cd)x^8+a^2c^2+4\left((b^2c^2-3abcd+2a^2d^2)x^{12}-(abc^2-a^2cd)x^4\right)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2abx^8+a^2}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^8+2\sqrt{dx^8+c}\sqrt{d}x^4-c\right)}{16b^2d^2}, \frac{2\sqrt{dx^8+c}\,bdx^4+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{16}-2(3abc^2-4a^2cd)x^8+a^2c^2+4\left((b^2c^2-3abcd+2a^2d^2)x^{12}-(abc^2-a^2cd)x^4\right)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2abx^8+a^2}\right)+2(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^4}{\sqrt{dx^8+c}}\right)}{16b^2d^2}, \frac{2\sqrt{dx^8+c}\,bdx^4-2ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^8-ac)\sqrt{dx^8+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^{12}+acx^4)}\right)+(bc+2ad)\sqrt{d}\log\left(-2dx^8+2\sqrt{dx^8+c}\sqrt{d}x^4-c\right)}{16b^2d^2}, \frac{\sqrt{dx^8+c}\,bdx^4-ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^8-ac)\sqrt{dx^8+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^{12}+acx^4)}\right)+(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^4}{\sqrt{dx^8+c}}\right)}{8b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c + 2*a*
d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^
8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*c + 2*a*d
)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x
^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + (b*c
+ 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^
2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((
b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c
*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*
d^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(bx^{8}+a\right)\sqrt{dx^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**19/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

# 3.569 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

**Rubi [A]** time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4b}-\frac{a\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4b}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 90, normalized size = 0.99

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^8}+dx^4\right)}{\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{\sqrt{bc-ad}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/Sqrt[b
*c - a*d]) + Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]]/Sqrt[d])/(4*b)
```

**IntegrateAlgebraic [A]** time = 0.94, size = 144, normalized size = 1.58

$$\frac{\log\left(\sqrt{c+dx^8}+\sqrt{d}\,x^4\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{b\sqrt{d}\,x^8}{\sqrt{a}\sqrt{bc-ad}}+\frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{4b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*(Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)/(S
qrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])
])/(b*Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^4 + Sqrt[c + d*x^8]]/(4*b*Sqrt[d])
```

**fricas [A]** time = 0.52, size = 632, normalized size = 6.95

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{16}-2\left(3abc^2-4a^2cd\right)x^8+a^2c^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^{12}-\left(abc^2-a^2cd\right)x^4\right)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2abx^8+a^2}\right)+2\sqrt{d}\log\left(-2dx^8-2\sqrt{dx^8+c}\sqrt{d}x^4-c\right)}{16bd}, \frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{16}-2\left(3abc^2-4a^2cd\right)x^8+a^2c^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^{12}-\left(abc^2-a^2cd\right)x^4\right)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2abx^8+a^2}\right)-4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^4}{\sqrt{dx^8+c}}\right)}{16bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^8-ac\right)\sqrt{dx^8+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^{12}+acx^4\right)}\right)+\sqrt{d}\log\left(-2dx^8-2\sqrt{dx^8+c}\sqrt{d}x^4-c\right)}{8bd}, \frac{d\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2ad)x^8-ac\right)\sqrt{dx^8+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^{12}+acx^4\right)}\right)-2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^4}{\sqrt{dx^8+c}}\right)}{8bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*
sqrt(d)*x^4 - c))/(b*d), 1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
```

```
*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c
)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*sqrt(-d)*arctan(s
qrt(-d)*x^4/sqrt(d*x^8 + c)))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2
*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 +
a*c*x^4)) + sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b*d
), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*
x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) - 2*sqrt(-d)*arctan(sqrt
(-d)*x^4/sqrt(d*x^8 + c)))/(b*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(bx^{8}+a\right)\sqrt{dx^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\left(b\,x^{8}+a\right)\,\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

# 3.570 $\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

**Rubi** [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 95, normalized size = 1.76

$$\frac{x^4\sqrt{\frac{dx^8}{c}+1}\,\tanh^{-1}\left(\frac{\sqrt{\frac{dx^8}{c}-\frac{bx^8}{a}}}{\sqrt{\frac{dx^8}{c}+1}}\right)}{4a\sqrt{c+dx^8}\sqrt{\frac{dx^8}{c}-\frac{bx^8}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^8)/c]*ArcTanh[Sqrt[-((b*x^8)/a) + (d*x^8)/c]/Sqrt[1 + (d
*x^8)/c]])/(4*a*Sqrt[c + d*x^8]*Sqrt[-((b*x^8)/a) + (d*x^8)/c])
```

**IntegrateAlgebraic [A]** time = 0.49, size = 106, normalized size = 1.96

$$\frac{\tan^{-1}\left(\frac{b\sqrt{d}\,x^8}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{4\sqrt{a}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)/(Sqrt[a]*Sqrt[b*
c - a*d]) + (b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])]/(4*Sqrt[a]*S
qrt[b*c - a*d])
```

**fricas [B]** time = 0.49, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4\left((bc-2\,ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{16\left(abc-a^2d\right)},\ \frac{\arctan\left(\frac{\left((bc-2\,ad)x^8-ac\right)\sqrt{dx^8+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^{12}+\left(abc^2-a^2cd\right)x^4\right)}\right)}{8\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*s
qrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*c -
 a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*
c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(a*b*c
 - a^2*d)]
```

**giac [A]** time = 0.20, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{4\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

## 3.571 $\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

**Rubi [A]** time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(4*a*c*x^4) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqr
t[c + d*x^8])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+bx^8\right) \sqrt{c+dx^8}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2 \left(a+bx^2\right) \sqrt{c+dx^2}}\,dx, x, x^4\right)$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4ac}$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4a}$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4a}$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

**Mathematica** [**C**] time = 0.83, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^8}{c}+1\right)\left(\frac{4x^8\left(c+dx^8\right)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)}{3c^2\left(a+bx^8\right)}+\frac{\left(c+2dx^8\right)\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}}\right)}{c\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}}\right)}{4x^4\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*((1 + (d*x^8)/c)*(((c + 2*d*x^8)*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a +
 b*x^8))]])/(c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)^2)]) +
 (4*(b*c - a*d)*x^8*(c + d*x^8)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x
^8)/(c*(a + b*x^8))])/(3*c^2*(a + b*x^8))))/(x^4*(a + b*x^8)*Sqrt[c + d*x^8
])
```

**IntegrateAlgebraic** [**A**] time = 0.80, size = 142, normalized size = 1.78

$$\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^8}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{4a^{3/2}(ad-bc)}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^8]/(a*c*x^4) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d]
)/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt
[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])]])/(4*a^(3/2)*(-(b*c) + a*d))
```

**fricas** [**B**] time = 0.50, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^4\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4\left((bc-2\,ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)+4\sqrt{dx^8+c}\left(abc-a^2d\right)}{16\left(a^2bc^2-a^3cd\right)x^4}, -\frac{\sqrt{abc-a^2d}\,bcx^4\arctan\left(\frac{\left((bc-2\,ad)x^8-ac\right)\sqrt{dx^8+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^{12}+\left(abc^2-a^2cd\right)x^4\right)}\right)+2\sqrt{dx^8+c}\left(abc-a^2d\right)}{8\left(a^2bc^2-a^3cd\right)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a*b*c + a^2*d)*b*c*x^4*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a
*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))
 + 4*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8*(sq
rt(a*b*c - a^2*d)*b*c*x^4*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 +
 c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4
)) + 2*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]
```

**giac [A]** time = 0.23, size = 116, normalized size = 1.45

$$\frac{1}{4}\,d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,ad}+\frac{2}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
4 - sqrt(d*x^8 + c))^2 - c)*a*d))
```

**maple [F]** time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\,\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

## 3.572 $\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}\,(2ad+3bc)}{12a^2c^2x^4}-\frac{\sqrt{c+dx^8}}{12acx^{12}}$$

**Rubi** [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}\,(2ad+3bc)}{12a^2c^2x^4}-\frac{\sqrt{c+dx^8}}{12acx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(12*a*c*x^12) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(12*a^2*
c^2*x^4) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4
*a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{12ac}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}-\frac{\text{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{12a^2c^2}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4a^2}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4a^2}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [C]** time = 2.27, size = 253, normalized size = 2.20

$$-\frac{\left(\frac{dx^8}{c}+1\right)\left(-\frac{8x^8\left(c+dx^8\right)^2(bc-ad)\,{}_3F_2\left(2,2,2;1,\frac{5}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)}{a+bx^8}+\frac{3c\left(c^2-4cdx^8-8d^2x^{16}\right)\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}}\right)}{\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}}+\frac{24dx^{16}\left(c+dx^8\right)(ad-bc)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)}{a+bx^8}\right)}{36c^3x^{12}\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/36*((1 + (d*x^8)/c)*((3*c*(c^2 - 4*c*d*x^8 - 8*d^2*x^16)*ArcSin[Sqrt[((b
*c - a*d)*x^8)/(c*(a + b*x^8))]])/Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2
*(a + b*x^8)^2)] + (24*d*(-(b*c) + a*d)*x^16*(c + d*x^8)*Hypergeometric2F1[
2, 2, 5/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))])/(a + b*x^8) - (8*(b*c - a*d)
*x^8*(c + d*x^8)^2*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((b*c - a*d)*x^8)
/(c*(a + b*x^8))])/(a + b*x^8)))/(c^3*x^12*(a + b*x^8)*Sqrt[c + d*x^8])
```

**IntegrateAlgebraic [A]** time = 2.18, size = 163, normalized size = 1.42

$$\frac{\sqrt{c+dx^8}\left(-ac+2adx^8+3bcx^8\right)}{12a^2c^2x^{12}}-\frac{b^2\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^8}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{4a^{5/2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-(a*c) + 3*b*c*x^8 + 2*a*d*x^8))/(12*a^2*c^2*x^12) - (b^2
*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)
/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*
d])])/(4*a^(5/2)*(-(b*c) + a*d))
```

**fricas [A]** time = 0.52, size = 416, normalized size = 3.62

$$\left[-\frac{3\,\sqrt{-abc+a^2d}\,b^2c^2x^{12}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\,\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4\,\left(\left(bc-2\,ad\right)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)-4\,\left(\left(3\,ab^2c^2-a^2bcd-2\,a^3d^2\right)x^8-a^2bc^2+a^3cd\right)\sqrt{dx^8+c}}{48\,\left(a^3bc^3-a^4c^2d\right)x^{12}},\ \frac{3\,\sqrt{abc-a^2d}\,b^2c^2x^{12}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^8-ac\right)\sqrt{dx^8+c}\,\sqrt{abc-a^2d}}{2\,\left(\left(abcd-a^2d^2\right)x^{12}+\left(abc^2-a^2cd\right)x^4\right)}\right)+2\,\left(\left(3\,ab^2c^2-a^2bcd-2\,a^3d^2\right)x^8-a^2bc^2+a^3cd\right)\sqrt{dx^8+c}}{24\,\left(a^3bc^3-a^4c^2d\right)x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x
^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8
+ a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*
d)*sqrt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c - a^
2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt
(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*(
(3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^8
 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12)]
```

**giac [B]** time = 1.69, size = 205, normalized size = 1.78

$$-\frac{1}{12}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\,\left(3\,\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-6\,\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc-6\,\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+3\,bc^2+2\,acd\right)}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2-c\right)^3a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
*c - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d
)*x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^2*d^2))
```

**maple [F]** time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^13), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13}\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**13*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

## 3.573 $\int \frac{x^{23}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^8}}{8b^2\left(a+bx^8\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

**Rubi [A]** time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^8}}{8b^2\left(a+bx^8\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b^2*d) - (a^2*Sqrt[c + d*x^8])/(8*b^2*(b*c - a*d)*(a + b
*x^8)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{23}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= -\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)} - \frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, \right.}{16b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)} - \frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c}\right.}{8b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.26, size = 107, normalized size = 0.87

$$\frac{1}{8}\left(\frac{\sqrt{c+dx^8}\left(\frac{a^2}{\left(a+bx^8\right)(ad-bc)}+\frac{2}{d}\right)}{b^2} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((Sqrt[c + d*x^8]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^8))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/8
```

**IntegrateAlgebraic [A]** time = 0.33, size = 143, normalized size = 1.16

$$\frac{\left(3a^2d-4abc\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{8b^{5/2}(ad-bc)^{3/2}} - \frac{\sqrt{c+dx^8}\left(3a^2d-2abc+2abdx^8-2b^2cx^8\right)}{8b^2d\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(Sqrt[c + d*x^8]*(-2*a*b*c + 3*a^2*d - 2*b^2*c*x^8 + 2*a*b*d*x^8))/(b^
2*d*(b*c - a*d)*(a + b*x^8)) + ((-4*a*b*c + 3*a^2*d)*ArcTan[(Sqrt[b]*Sqrt[-
(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(8*b^(5/2)*(-(b*c) + a*d)^(3/2)
)
```

**fricas [B]** time = 0.47, size = 475, normalized size = 3.86

$$\left[\frac{\left(\left(4\,ab^2cd-3\,a^2bd^2\right)x^8+4\,a^2bcd-3\,a^3d^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\left(2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^8+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2\right)\sqrt{dx^8+c}}{16\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)x^8\right)},-\frac{\left(\left(4\,ab^2cd-3\,a^2bd^2\right)x^8+4\,a^2bcd-3\,a^3d^2\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-\left(2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^8+2\,ab^3c^2-5\,a^2b^2cd+3\,a^3bd^2\right)\sqrt{dx^8+c}}{8\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)x^8\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b
*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8), -
1/8*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8)]
```

**giac [A]** time = 0.18, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^8+c}\,a^2d}{8\left(b^3c-ab^2d\right)\left(\left(dx^8+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^8+c}}{4\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(d*x^8 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^8 + c)*b - b*c + a*d))
 - 1/8*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/4*sqrt(d*x^8 + c)/(b^2*d)
```

**maple [F]** time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 5.02, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^8+c}}{4\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^8+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{8\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^8+c}}{2\,\left(a\,d-b\,c\right)\,\left(4\,b^3\,\left(d\,x^8+c\right)-4\,b^3\,c+4\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^8)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(8*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b^3*(c
+ d*x^8) - 4*b^3*c + 4*a*b^2*d))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.574 $\int \frac{x^{15}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}} \, dx$

**Optimal**. Leaf size=99

$$\frac{a\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

**Rubi [A]** time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16b(bc-ad)}$$

$$= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8bd(bc-ad)}$$

$$= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.09, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*b^(
3/2))
```

**IntegrateAlgebraic [A]** time = 0.17, size = 109, normalized size = 1.10

$$\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{8b^{3/2}(ad-bc)^{3/2}} + \frac{a\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) + ((2*b*c - a*d)*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(8*b^(3/2)*(-(b*c
) + a*d)^(3/2))
```

**fricas [A]** time = 0.43, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2b^2c-abd\right)x^8+2abc-a^2d\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2bc-ad-2\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\sqrt{dx^8+c}\left(ab^2c-a^2bd\right)}{16\left(\left(b^5c^2-2ab^4cd+a^2b^3d^2\right)x^8+ab^4c^2-2a^2b^3cd+a^3b^2d^2\right)}, \frac{\left(\left(2b^2c-abd\right)x^8+2abc-a^2d\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\sqrt{dx^8+c}\left(ab^2c-a^2bd\right)}{8\left(\left(b^5c^2-2ab^4cd+a^2b^3d^2\right)x^8+ab^4c^2-2a^2b^3cd+a^3b^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a))
+ 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a*b*d
)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-
```

```
b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2)]
```

**giac [A]** time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^8+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^8+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

**maple [F]** time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.84, size = 95, normalized size = 0.96

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{8\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^8+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(4\,b\,\left(d\,x^8+c\right)+4\,a\,d-4\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(8*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^8)^(1/2))/(2*b*(a*d - b*c)*(4*b*(c +
 d*x^8) + 4*a*d - 4*b*c))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.575 $\int \frac{x^7}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

**Rubi** [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(8*(b*c - a*d)*(a + b*x^8)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16(bc-ad)}\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8(bc-ad)}\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}
$$

**Mathematica** [A] time = 0.10, size = 85, normalized size = 0.98

$$
\frac{1}{8}\left(\frac{\sqrt{c+dx^8}}{\left(a+bx^8\right)(ad-bc)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/((-(b*c) + a*d)*(a + b*x^8)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8
```

**IntegrateAlgebraic** [A] time = 0.11, size = 97, normalized size = 1.11

$$
-\frac{\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)} - \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}\,\sqrt{ad-bc}}{bc-ad}\right)}{8\sqrt{b}\,(ad-bc)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(
b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(8*Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

**fricas** [B] time = 0.44, size = 302, normalized size = 3.47

$$
\left[-\frac{\left(bdx^8+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\left(b^2c-abd\right)}{16\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}, -\frac{\left(bdx^8+ad\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\sqrt{dx^8+c}\left(b^2c-abd\right)}{8\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2*
sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(b^2*c
```

```
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

**giac [A]** time = 0.16, size = 93, normalized size = 1.07

$$-\frac{d \arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\sqrt{-b^2c+abd}\,(bc-ad)} - \frac{\sqrt{dx^8+c}\,d}{8\left(\left(dx^8+c\right)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/8*sqrt(d*x^8 + c)*d/(((d*x^8 + c)*b - b*c + a*d)*(b*c - a
*d))
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

**mupad [B]** time = 4.80, size = 84, normalized size = 0.97

$$\frac{d\sqrt{d\,x^8+c}}{2\,\left(a\,d-b\,c\right)\,\left(4\,b\,\left(d\,x^8+c\right)+4\,a\,d-4\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c)) + (
d*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.576 $\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

**Rubi [A]** time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]]/(4*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^8\right)}{8a^2} - \frac{(b(2bc-3ad))\operatorname{Subst}\left(\int\right.}{16a^2(bc-}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{4a^2d} - \frac{(b(2bc-3ad))\operatorname{Sub}}{}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right.}{8a^2(bc-ad)^{3/2}}$$

**Mathematica [A]** time = 0.36, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - (2*ArcTanh[Sqrt[c + d*x^
8]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*a^2)
```

**IntegrateAlgebraic [A]** time = 0.30, size = 146, normalized size = 1.11

$$\frac{\left(3a\sqrt{b}\,d-2b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{8a^2(ad-bc)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b*Sqrt[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((-2*b^(3/2)*c +
3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a
*d)])/(8*a^2*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a^
2*Sqrt[c])
```

**fricas [A]** time = 0.49, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^8+c}\,abc+\left((2b^2c^2-3abcd)x^8+2abc^2-3a^2cd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad+2\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+2\left((b^2c-abd)x^8+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^8-2\sqrt{dx^8+c}\sqrt{c}+2c}{x^8}\right)}{16\left((a^2b^2c^2-a^3bcd)x^8+a^3bc^2-a^4cd\right)}, \frac{\sqrt{dx^8+c}\,abc+\left((2b^2c^2-3abcd)x^8+2abc^2-3a^2cd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+\left((b^2c-abd)x^8+abc-a^2d\right)\sqrt{c}\log\left(\frac{dx^8-2\sqrt{dx^8+c}\sqrt{c}+2c}{x^8}\right)}{8\left((a^2b^2c^2-a^3bcd)x^8+a^3bc^2-a^4cd\right)}, \frac{2\sqrt{dx^8+c}\,abc+4\left((b^2c-abd)x^8+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)+\left((2b^2c^2-3abcd)x^8+2abc^2-3a^2cd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2bc-ad+2\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)}{16\left((a^2b^2c^2-a^3bcd)x^8+a^3bc^2-a^4cd\right)}, \frac{\sqrt{dx^8+c}\,abc+\left((2b^2c^2-3abcd)x^8+2abc^2-3a^2cd\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+2\left((b^2c-abd)x^8+abc-a^2d\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)}{8\left((a^2b^2c^2-a^3bcd)x^8+a^3bc^2-a^4cd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x^8
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
8 + b*c)) + ((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sq
rt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c
^2 - a^4*c*d), 1/16*(2*sqrt(d*x^8 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^8 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b
*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8
 + b*c)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^8 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)]
```

**giac [A]** time = 0.21, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^8+c}\,bd}{8\left(abc-a^2d\right)\left(\left(dx^8+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^8 + c)*b*d/((a*b*c - a^2*d)*((d*x^8 + c)*b - b*c + a*d)) - 1/8
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

**maple [F]** time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x), x)
```

**mupad [B]** time = 5.82, size = 3017, normalized size = 22.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d^5 - (3*a^5*b^
3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - (
(c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2*d^5
 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(512*(
a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^
3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a
*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4
*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^
4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^2*d^5 - (3*
a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b
^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/
(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b
^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(
a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*
(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
 3*a^4*b*c*d^2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*
c^2 - 2*a^4*b*c*d) - ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d
^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*
a^4*b*c*d) - ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(2
56*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^
2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 +
 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2
))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d -
 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^
2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^
2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^
2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 -
 2*a^4*b*c*d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)
*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4
*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*
d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2
*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(8*
(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((((a^6
*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2
*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c
*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*
d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c + d*x^8)^(1/2)*(13
*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2
 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2
 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d
*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1
280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(
1/2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4
*b*c*d) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a
^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5
- 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a
```

```
^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c +
d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d
^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((a^6*b^2*d^5 - (3*a^5*b
^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
) + ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*
c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 -
 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^
5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(
a^2*c^(1/2))))*1i)/(4*a^2*c^(1/2)) - (b*d*(c + d*x^8)^(1/2))/(2*(a^2*d - a*
b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.577 $\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b\sqrt{c+dx^8}\,(2bc-ad)}{8a^2c\left(a+bx^8\right)(bc-ad)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}$$

**Rubi [A]** time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b\sqrt{c+dx^8}\,(2bc-ad)}{8a^2c\left(a+bx^8\right)(bc-ad)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(a + b*x^8)) - Sqrt
[c + d*x^8]/(8*a*c*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8
]/Sqrt[c]])/(8*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)$$

$$=-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)}{8ac}$$

$$=-\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{8a^2c(bc-ad)}$$

$$=-\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}+\frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c}}\right.}{16a^3(bc-ad)}$$

$$=-\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}+\frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{8a^3d(bc-ad)}$$

$$=-\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}+\frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b^{3/2}}{}$$

**Mathematica [A]** time = 0.60, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^8}\left(a^2d+ab\left(dx^8-c\right)-2b^2cx^8\right)}{x^8\left(a+bx^8\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[c + d*x^8]*(a^2*d - 2*b^2*c*x^8 + a*b*(-c + d*x^8)))/((b*c - a*d)*
x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(8*a^3*c)
```

**IntegrateAlgebraic** [A] time = 0.65, size = 187, normalized size = 1.01

$$\frac{\left(4b^{5/2}c-5ab^{3/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}\sqrt{ad-bc}}{bc-ad}\right)}{8a^3(ad-bc)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}+\frac{\sqrt{c+dx^8}\left(-a^2d+abc-abdx^8+2b^2cx^8\right)}{8a^2cx^8\left(a+bx^8\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(a*b*c - a^2*d + 2*b^2*c*x^8 - a*b*d*x^8))/(8*a^2*c*(-(b*c
) + a*d)*x^8*(a + b*x^8)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^8])/(b*c - a*d)])/(8*a^3*(-(b*c) + a*d)^(3/
2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a^3*c^(3/2))
```

**fricas** [A] time = 0.56, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8
 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16
+ (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sq
rt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt
(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*
b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/8*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^8 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8)]
```

**giac** [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^8+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^8+c}\,b^2c^2d-\left(dx^8+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^8+c}\,abcd^2-\sqrt{dx^8+c}\,a^2d^3}{8\left(a^2bc^2-a^3cd\right)\left(\left(dx^8+c\right)^2b-2\left(dx^8+c\right)bc+bc^2+\left(dx^8+c\right)ad-acd\right)}-\frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/8*(2*(d*x^8 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^8 + c)*b^2*c^2*d - (d*x^8 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^8 + c
)*a*b*c*d^2 - sqrt(d*x^8 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^8 + c)^
```

```
2*b - 2*(d*x^8 + c)*b*c + b*c^2 + (d*x^8 + c)*a*d - a*c*d)) - 1/8*(4*b*c +
a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

**maple [F]** time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}\;x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{8}+a\right)^{2}\sqrt{dx^{8}+c}\,x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^9), x)
```

**mupad [B]** time = 7.85, size = 3832, normalized size = 20.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] (((c + d*x^8)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^8)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^8)*(4*a*d - 8*b*c) + 4*b*(c + d*x^8)^2 + 4*b*c^2 - 4*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b
^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*
d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6
*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^
2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*
c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c
+ d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4
*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*
d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6
*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*
c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5
*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^
3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3
*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d
^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^
3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/256 + (b^7
*c^3*d^3)/8 - (3*a*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 +
a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*
(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^
3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b
*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2
```

```
+ a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 +
 a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/
2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b
^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*
b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(
a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 -
a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/
2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a
*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^
6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(
((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5
)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1
/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c
^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a
^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(-b^3*
(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32
*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(3
2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 + a^6
*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*
c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^
5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))
/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d +
4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(16*a^3*(c^3)^(1/2)) + ((((
c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b
^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^
3*d)) - ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b
^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^(
1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b
^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*
c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*
c)*1i)/(16*a^3*(c^3)^(1/2)))/(((5*a^3*b^4*d^6)/256 + (b^7*c^3*d^3)/8 - (3*a
*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^
7*b*c^3*d) - ((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*
c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2
*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4
*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)
 - ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4
*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a
^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1
/2)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)) + ((((c + d*x^8)^(1/2)*(a^4*b^3*d
^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d
^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((((a^9*b^2*c*d^6)/
2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4
 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6
*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^
2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*
(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2))))*
(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.578 $\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=141

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

**Rubi [A]** time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8b(bc-ad)}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{}{(a}\right.}{8b^2(bc-}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int\right.}{8b^2(}$$

$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

**Mathematica [A]** time = 0.26, size = 135, normalized size = 0.96

$$\frac{\frac{abx^4\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)} + \frac{\sqrt{a}\,(2ad-3bc)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\,\sqrt{c+dx^8}+dx^4\right)}{\sqrt{d}}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]])/Sqrt[d])/(8*b^2)
```

**IntegrateAlgebraic [A]** time = 3.67, size = 166, normalized size = 1.18

$$\frac{\left(2a^{3/2}d-3\sqrt{a}\,bc\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\sqrt{c+dx^8}+b\sqrt{d}\,x^8}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)} + \frac{\log\left(\sqrt{c+dx^8}+\sqrt{d}\,x^4\right)}{4b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) + ((-3*Sqrt[a]*b*c +
2*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sq
rt[a]*Sqrt[b*c - a*d])])/(8*b^2*(b*c - a*d)^(3/2)) + Log[Sqrt[d]*x^4 + Sqrt
[c + d*x^8]]/(4*b^2*Sqrt[d])
```

**fricas [A]** time = 0.82, size = 1077, normalized size = 7.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d -
a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^
4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sq
rt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt
(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (
a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2
*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + 2*((b^2*c - a*b*d)*
x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 -
 c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*a*b*d*x^4 - 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)))/((b^4*c*d - a*b^3*d^
2)*x^8 + a*b^3*c*d - a^2*b^2*d^2)]
```

**giac [B]** time = 0.65, size = 298, normalized size = 2.11

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^{2}d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^{4}-\sqrt{dx^{8}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{8\,\left(b^{3}c-ab^{2}d\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}x^{4}-\sqrt{dx^{8}+c}\right)^{2}abc\sqrt{d}-2\left(\sqrt{d}x^{4}-\sqrt{dx^{8}+c}\right)^{2}a^{2}d^{\frac{3}{2}}-abc^{2}\sqrt{d}}{4\left(\left(\sqrt{d}x^{4}-\sqrt{dx^{8}+c}\right)^{4}b-2\left(\sqrt{d}x^{4}-\sqrt{dx^{8}+c}\right)^{2}bc+4\left(\sqrt{d}x^{4}-\sqrt{dx^{8}+c}\right)^{2}ad+bc^{2}\right)\left(b^{3}c-ab^{2}d\right)}-\frac{\log\left(\left(\sqrt{d}x^{4}-\sqrt{dx^{8}+c}\right)^{2}\right)}{8\,b^{2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x
^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/4*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c
 + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
8*log((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2)/(b^2*sqrt(d))
```

**maple [F]** time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^19/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

## 3.579 $\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

**Rubi [A]** time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(x^4*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$
$$= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\operatorname{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8(bc-ad)}$$
$$= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{c\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8(bc-ad)}$$
$$= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{c\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8(bc-ad)}$$
$$= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}\,(bc-ad)^{3/2}}$$

**Mathematica** [A] time = 0.55, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^8}\left(-\frac{x^8(bc-ad)}{a+bx^8}-\frac{c\sqrt{x^8\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^8\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^8}{c}+1}}\right)}{\sqrt{\frac{dx^8}{c}+1}}\right)}{8x^4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-(((b*c - a*d)*x^8)/(a + b*x^8)) - (c*Sqrt[(-(b/a) + d/c)
*x^8]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^8]/Sqrt[1 + (d*x^8)/c]])/Sqrt[1 + (d*x^
8)/c]))/(8*(b*c - a*d)^2*x^4)
```

**IntegrateAlgebraic** [A] time = 1.82, size = 145, normalized size = 1.56

$$\frac{c\tan^{-1}\left(\frac{b\sqrt{d}\,x^8}{\sqrt{a}\sqrt{bc-ad}}+\frac{bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{8\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[a]*S
qrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^8)/(Sqrt[a]*Sqrt[b*c - a*d]) + (b*x^
4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])]])/(8*Sqrt[a]*(b*c - a*d)^(3/2)
)
```

**fricas** [B] time = 0.54, size = 426, normalized size = 4.58

$$\left[-\frac{4\sqrt{dx^8+c}\left(abc-a^2d\right)x^4-\left(bcx^8+ac\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{16}-2\left(3abc^2-4a^2cd\right)x^8+a^2c^2+4\left((bc-2ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2abx^8+a^2}\right)}{32\left(\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^8+a^2b^2c^2-2a^3bcd+a^4d^2\right)}, -\frac{2\sqrt{dx^8+c}\left(abc-a^2d\right)x^4-\left(bcx^8+ac\right)\sqrt{abc-a^2d}\arctan\left(\frac{\left((bc-2ad)x^8-ac\right)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^{12}+\left(abc^2-a^2cd\right)x^4\right)}\right)}{16\left(\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^8+a^2b^2c^2-2a^3bcd+a^4d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqr
t(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c
*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*sqrt(d
*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*arctan(
1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
 - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

**giac** [B]  time = 1.59, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{8\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 bc\sqrt{d}-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 ad^{\frac{3}{2}}-bc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4 b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 ad+bc^2\right)\left(b^2c-abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/4*((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 +
 c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*a*d + b*c^2)*(b^2*c - a*b*d))
```

**maple** [F]  time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima** [F]  time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

**mupad** [F]  time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.580 $\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

**Rubi [A]** time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^4*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}$$

$$= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8a(bc-ad)}$$

$$= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

**Mathematica** [C] time = 0.94, size = 407, normalized size = 3.91

$$\frac{x^4\sqrt{c+dx^8}\left(-30dx^8\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}-45c\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}+16dx^8\left(\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^8\right)}{c\left(a+bx^8\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)+16c\left(\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^8\right)}{c\left(a+bx^8\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)+30dx^8\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}}\right)+45c\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}}\right)\right)}{120c^2\left(a+bx^8\right)^2\left(\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}\right)^{3/2}\sqrt{\frac{a\left(c+dx^8\right)}{c\left(a+bx^8\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8]*(-45*c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a +
b*x^8)^2)] - 30*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 30*d*x^8*ArcS
in[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 16*c*(((b*c - a*d)*x^8)/(c*(a
 + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 16*d*x^8*(((b*c - a*d)*x^8)/
(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))]))/(120*c^2*(((b*c - a*d)*
x^8)/(c*(a + b*x^8)))^(3/2)*(a + b*x^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^
8))])
```

**IntegrateAlgebraic** [A] time = 0.99, size = 124, normalized size = 1.19

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\sqrt{c+dx^8}+b\sqrt{d}\,x^8}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{3/2}(bc-ad)^{3/2}}-\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b*x^4*Sqrt[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(8*a^(3/2)*(b*c - a*d)^(3/2))
```

**fricas** [B] time = 0.58, size = 467, normalized size = 4.49

$$\left[\frac{4\sqrt{dx^8+c}\left(ab^2c-a^2bd\right)x^4-\left(\left(b^2c-2\,abd\right)x^8+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4\left((bc-2\,ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{32\left(\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^8+a^3b^2c^2-2\,a^4bcd+a^5d^2\right)}, \frac{2\sqrt{dx^8+c}\left(ab^2c-a^2bd\right)x^4+\left(\left(b^2c-2\,abd\right)x^8+abc-2\,a^2d\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\left((bc-2\,ad)x^8-ac\right)\sqrt{dx^8+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^{12}+\left(abc^2-a^2cd\right)x^4\right)}\right)}{16\left(\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^8+a^3b^2c^2-2\,a^4bcd+a^5d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 -
 a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2
)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*
c*d + a^5*d^2), 1/16*(2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b^2*c -
 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a
*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^1
2 + (a*b*c^2 - a^2*c*d)*x^4)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x
^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]
```

**giac [B]** time = 0.39, size = 237, normalized size = 2.28

$$-\frac{1}{8}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2 bc-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2 ad-bc^2\right)}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4 b-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2 bc+4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2 ad+bc^2\right)\left(abcd-a^2d^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*
x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

## 3.581 $\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=149

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}\,(3bc-2ad)}{8a^2cx^4(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^4\left(a+bx^8\right)(bc-ad)}$$

**Rubi [A]** time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^8}\,(3bc-2ad)}{8a^2cx^4(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^8}}{8ax^4\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^4) + (b*Sqrt[c +
d*x^8])/(8*a*(b*c - a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,d\ldots\right.}{8a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \ldots\right.}{8a^2(bc-\ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \ldots\right.}{8a^2(bc\ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-\ldots}}{\sqrt{a}\sqrt{\ldots}}\right.}{8a^{5/2}(bc-ad)^{3/2}}$$

**Mathematica [C]** time = 2.03, size = 869, normalized size = 5.83

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/120*(Sqrt[c + d*x^8]*(-45*c^2*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*
(a + b*x^8)^2)] - 180*c*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a
+ b*x^8)^2)] - 120*d^2*x^16*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a +
b*x^8)^2)] + 45*c^2*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 180*c
*d*x^8*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 120*d^2*x^16*ArcSi
n[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 64*c^2*(((b*c - a*d)*x^8)/(c*(
a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[
2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 160*c*d*x^8*(((b*c - a*d)*x
^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeome
tric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 96*d^2*x^16*(((b*c
- a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hy
pergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 32*c^2*(((b
*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]
*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8))]
+ 64*c*d*x^8*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))
/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/
```

```
(c*(a + b*x^8))] + 32*d^2*x^16*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sq
rt[(a*(c + d*x^8))/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2},
((b*c - a*d)*x^8)/(c*(a + b*x^8))]))/(c^3*x^4*(((b*c - a*d)*x^8)/(c*(a + b*
x^8)))^(3/2)*(a + b*x^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))])
```

**IntegrateAlgebraic [A]** time = 1.97, size = 159, normalized size = 1.07

$$\frac{\left(4abd-3b^2c\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\sqrt{c+dx^8}+b\sqrt{d}x^8}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}\left(-2a^2d+2abc-2abdx^8+3b^2cx^8\right)}{8a^2cx^4\left(a+bx^8\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^8 - 2*a*b*d*x^8))/(8*a^2*c*
(-(b*c) + a*d)*x^4*(a + b*x^8)) + ((-3*b^2*c + 4*a*b*d)*ArcTan[(a*Sqrt[d] +
 b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5
/2)*(b*c - a*d)^(3/2))
```

**fricas [B]** time = 0.61, size = 612, normalized size = 4.11

$$\left[-\frac{\left(\left(3b^3c^2-4ab^2cd\right)x^{12}+\left(3ab^2c^2-4a^2bcd\right)x^4\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{16}-2\left(3abc^2-4a^2cd\right)x^8+a^2c^2+4\left(\left(bc-2ad\right)x^{12}-acx^4\right)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2abx^8+a^2}\right)+4\left(\left(3ab^3c^2-5a^2b^2cd+2a^3bd^2\right)x^8+2a^2b^2c^2-4a^3bcd+2a^4d^2\right)\sqrt{dx^8+c}}{32\left(\left(a^3b^3c^3-2a^4b^2c^2d+a^5bcd^2\right)x^{12}+\left(a^4b^2c^3-2a^5bc^2d+a^6cd^2\right)x^4\right)},-\frac{\left(\left(3b^3c^2-4ab^2cd\right)x^{12}+\left(3ab^2c^2-4a^2bcd\right)x^4\right)\sqrt{abc-a^2d}\arctan\left(\frac{\left(\left(bc-2ad\right)x^8-ac\right)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^{12}+\left(abc^2-a^2cd\right)x^4\right)}\right)+2\left(\left(3ab^3c^2-5a^2b^2cd+2a^3bd^2\right)x^8+2a^2b^2c^2-4a^3bcd+2a^4d^2\right)\sqrt{dx^8+c}}{16\left(\left(a^3b^3c^3-2a^4b^2c^2d+a^5bcd^2\right)x^{12}+\left(a^4b^2c^3-2a^5bc^2d+a^6cd^2\right)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x
^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b^3*c
^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4
*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12
+ (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2 - 4*a*
b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d)*arctan
(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*
d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2
*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x
^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 + (a^4*b^2*c^3
 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]
```

**giac [B]** time = 1.76, size = 418, normalized size = 2.81

$$\frac{1}{8}d^{\frac{5}{2}}\left(\frac{\left(3b^2c-4abd\right)\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}+\frac{2\left(3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b^2c-4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4abd-6\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b^2c^2+14\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2abcd-8\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2a^2d^2+3b^2c^3-2abc^2d\right)}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^6b-3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4ad+3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc^2-4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2acd-bc^3\right)\left(a^2bcd^2-a^3d^3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b^2*c - 4*(sq
rt(d)*x^4 - sqrt(d*x^8 + c))^4*a*b*d - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*d - 8*(sqrt(d)*x^4 - s
qrt(d*x^8 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^4 - sqrt(
d*x^8 + c))^6*b - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b*c + 4*(sqrt(d)*x^4
 - sqrt(d*x^8 + c))^4*a*d + 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c^2 - 4*(s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}\;x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{8}+a\right)^{2} \sqrt{dx^{8}+c}\,x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5}\,\left(b\,x^{8}+a\right)^{2}\,\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

## 3.582 $\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

**Optimal**. Leaf size=208

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}\,(5bc-2ad)}{24a^2cx^{12}(bc-ad)}+\frac{\sqrt{c+dx^8}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{24a^3c^2x^4(bc-ad)}+\frac{b\sqrt{c+dx\cdots}}{8ax^{12}\left(a+bx^8\right)\cdots}$$

**Rubi** [A] time = 0.30, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^8}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{24a^3c^2x^4(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}\,(5bc-2ad)}{24a^2cx^{12}(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^{12}\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^12) + ((15*b^2*c
^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^8])/(24*a^3*c^2*(b*c - a*d)*x^4) +
 (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+\cdots}{x^2\left(a+\cdots\right.}\cdots\right.}{24a^2\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c\cdots}}{8a(bc-ad)\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c\cdots}}{8a(bc-ad)\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c\cdots}}{8a(bc-ad)\cdots}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c\cdots}}{8a(bc-ad)\cdots}$$

**Mathematica [A]** time = 5.85, size = 175, normalized size = 0.84

$$\frac{a^2\left(c+dx^8\right)\left(\frac{3b^3x^{16}}{\left(a+bx^8\right)(bc-ad)}+\frac{4x^8(ad+3bc)}{c^2}-\frac{2a}{c}\right)+\frac{3b^2x^{24}\sqrt{\frac{dx^8}{c}+1}\,(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^8\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^8}{a}+1}}\right)}{c\left(\frac{x^8(bc-ad)}{ac}\right)^{3/2}}}{24a^5x^{12}\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a^2*(c + d*x^8)*((-2*a)/c + (4*(3*b*c + a*d)*x^8)/c^2 + (3*b^3*x^16)/((b*c
 - a*d)*(a + b*x^8))) + (3*b^2*(5*b*c - 6*a*d)*x^24*Sqrt[1 + (d*x^8)/c]*Arc
Sin[Sqrt[(b/a - d/c)*x^8]/Sqrt[1 + (b*x^8)/a]])/(c*(((b*c - a*d)*x^8)/(a*c)
)^(3/2)))/(24*a^5*x^12*Sqrt[c + d*x^8])
```

**IntegrateAlgebraic [A]** time = 4.94, size = 217, normalized size = 1.04

$$\frac{\left(5b^3c-6ab^2d\right)\tan^{-1}\left(\frac{a\sqrt{d}+bx^4\sqrt{c+dx^8}+b\sqrt{d}\,x^8}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{8a^{7/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}\,\left(-2a^3cd+4a^3d^2x^8+2a^2bc^2+6a^2bcdx^8+4a^2bd^2x^{16}-10ab^2c^2x^8+8ab^2cdx^{16}-15b^3c^2x^{16}\right)}{24a^3c^2x^{12}\left(a+bx^8\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(2*a^2*b*c^2 - 2*a^3*c*d - 10*a*b^2*c^2*x^8 + 6*a^2*b*c*d*
x^8 + 4*a^3*d^2*x^8 - 15*b^3*c^2*x^16 + 8*a*b^2*c*d*x^16 + 4*a^2*b*d^2*x^16
))/(24*a^3*c^2*(-(b*c) + a*d)*x^12*(a + b*x^8)) + ((5*b^3*c - 6*a*b^2*d)*Ar
cTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c
- a*d]])]/(8*a^(7/2)*(b*c - a*d)^(3/2))
```

**fricas [A]** time = 0.76, size = 760, normalized size = 3.65

$$\left[-\frac{3\left(\left(5b^4c^3-6ab^3c^2d\right)x^{20}+\left(5ab^3c^3-6a^2b^2c^2d\right)x^{12}\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^{16}-2\left(3abc^2-4a^2cd\right)x^8+a^2c^2-4\left(\left(bc-2ad\right)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2abx^8+a^2}\right)-4\left(\left(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3\right)x^{16}+2\left(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3\right)x^8-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2\right)\sqrt{dx^8+c}}{96\left(\left(a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2\right)x^{20}+\left(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2\right)x^{12}\right)},\ \frac{3\left(\left(5b^4c^3-6ab^3c^2d\right)x^{20}+\left(5ab^3c^3-6a^2b^2c^2d\right)x^{12}\right)\sqrt{abc-a^2d}\arctan\left(\frac{\left(\left(bc-2ad\right)x^8-ac\right)\sqrt{dx^8+c}\,\sqrt{abc-a^2d}}{2\left(\left(abcd-a^2d^2\right)x^{12}+\left(abc^2-a^2cd\right)x^4\right)}\right)+2\left(\left(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3\right)x^{16}+2\left(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3\right)x^8-2a^3b^2c^3+4a^4bc^2d-2a^5cd^2\right)\sqrt{dx^8+c}}{48\left(\left(a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2\right)x^{20}+\left(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2\right)x^{12}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^12)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)
*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*((
15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^16 + 2*(
5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*
c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b
^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2
)*x^12), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b
^2*c^2*d)*x^12)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sq
rt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^
2*c*d)*x^4)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^16 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^
3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^
4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*
c^3*d + a^7*c^2*d^2)*x^12)]
```

**giac [B]** time = 2.28, size = 395, normalized size = 1.90

$$\frac{1}{24}\,d^{\frac{7}{2}}\left(\frac{3\left(5b^3c-6ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b^3c-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ab^2d-b^3c^2\right)}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)}-\frac{8\left(3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc-3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad+3bc^2+acd\right)}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2-c\right)^3a^3d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^
8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b^3*c - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 3*(
sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^4 - sq
rt(d*x^8 + c))^2 - c)^3*a^3*d^3))
```

**maple [F]** time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\;x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + c}\, x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{13} \left(b\, x^8 + a\right)^2 \sqrt{d\, x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

## 3.583 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\, x^5\, dx$

**Optimal**. Leaf size=123

$$-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

**Rubi [A]** time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c^2) + ((2*b*c - a*d)*Sqrt[c + d/
x^2]*x^4)/(8*c) + (a*(c + d/x^2)^(3/2)*x^6)/(6*c) - (d^2*(2*b*c - a*d)*ArcT
anh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^5\,dx=-\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^4}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c}-\frac{\left(3bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3}\,dx,x,\frac{1}{x^2}\right)}{6c}$$

$$=\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c}-\frac{(d(2bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx,x,\cdots\right.}{16c}$$

$$=\frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c}+\frac{\left(d^2(2bc-a\cdots\right.}{\cdots}$$

$$=\frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c}+\frac{(d(2bc-ad\cdots}{\cdots}$$

$$=\frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c}-\frac{d^2(2bc-ad\cdots}{\cdots}$$

**Mathematica [A]** time = 0.26, size = 121, normalized size = 0.98

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{d}+1}\left(a\left(8c^2x^4+2cdx^2-3d^2\right)+6bc\left(2cx^2+d\right)\right)+3d^{3/2}(ad-2bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\right)}{48c^{5/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(d + 2*c*x^2) + a*
(-3*d^2 + 2*c*d*x^2 + 8*c^2*x^4)) + 3*d^(3/2)*(-2*b*c + a*d)*ArcSinh[(Sqrt[
c]*x)/Sqrt[d]]))/(48*c^(5/2)*Sqrt[1 + (c*x^2)/d])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 111, normalized size = 0.90

$$\frac{\left(ad^3-2bcd^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(8ac^2x^6+2acdx^4-3ad^2x^2+12bc^2x^4+6bcdx^2\right)}{48c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(6*b*c*d*x^2 - 3*a*d^2*x^2 + 12*b*c^2*x^4 + 2*a*c*d*
x^4 + 8*a*c^2*x^6))/(48*c^2) + ((-2*b*c*d^2 + a*d^3)*ArcTanh[Sqrt[(d + c*x^
2)/x^2]/Sqrt[c]])/(16*c^(5/2))
```

**fricas [A]** time = 0.44, size = 242, normalized size = 1.97

$$\left[-\frac{3\left(2\,bcd^2-ad^3\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(8\,ac^3x^6+2\left(6\,bc^3+ac^2d\right)x^4+3\left(2\,bc^2d-acd^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96\,c^3},\ \frac{3\left(2\,bcd^2-ad^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(8\,ac^3x^6+2\left(6\,bc^3+ac^2d\right)x^4+3\left(2\,bc^2d-acd^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{48\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^
2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d^3)
*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c^3
*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2
 + d)/x^2))/c^3]
```

**giac [A]** time = 0.19, size = 143, normalized size = 1.16

$$\frac{1}{48}\left(2\left(4\,ax^2\mathrm{sgn}(x)+\frac{6\,bc^4\mathrm{sgn}(x)+ac^3d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\left(2\,bc^3d\mathrm{sgn}(x)-ac^2d^2\mathrm{sgn}(x)\right)}{c^4}\right)\sqrt{cx^2+d}\,x+\frac{\left(2\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{5}{2}}}-\frac{\left(2\,bcd^2\log\left(|d|\right)-ad^3\log\left(|d|\right)\right)\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(2
*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*d^
2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2) - 1
/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)
```

**maple [A]** time = 0.06, size = 162, normalized size = 1.32

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(8\left(c\,x^2+d\right)^{\frac{3}{2}}a\,c^{\frac{3}{2}}x^3+3a\,d^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-6bc\,d^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)+3\sqrt{c\,x^2+d}\,a\sqrt{c}\,d^2x-6\sqrt{c\,x^2+d}\,b\,c^{\frac{3}{2}}dx-6\left(c\,x^2+d\right)^{\frac{3}{2}}a\sqrt{c}\,dx+12\left(c\,x^2+d\right)^{\frac{3}{2}}b\,c^{\frac{3}{2}}x\right)x}{48\sqrt{c\,x^2+d}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)*x*(8*c^(3/2)*(c*x^2+d)^(3/2)*x^3*a-6*c^(1/2)*(c*
x^2+d)^(3/2)*x*a*d+12*c^(3/2)*(c*x^2+d)^(3/2)*x*b+3*c^(1/2)*(c*x^2+d)^(1/2)
*x*a*d^2-6*c^(3/2)*(c*x^2+d)^(1/2)*x*b*d+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*
d^3-6*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d^2)/(c*x^2+d)^(1/2)/c^(5/2)
```

**maxima [B]** time = 1.36, size = 243, normalized size = 1.98

$$-\frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}\,c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c^2-3\left(c+\frac{d}{x^2}\right)^2c^3+3\left(c+\frac{d}{x^2}\right)c^4-c^5}\right)a+\frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c
^(5/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 - 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c
+ d/x^2)*c^2*d^3)/((c + d/x^2)^3*c^2 - 3*(c + d/x^2)^2*c^3 + 3*(c + d/x^2)*
c^4 - c^5))*a + 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2)
+ sqrt(c)))/c^(3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c
 + d/x^2)^2*c - 2*(c + d/x^2)*c^2 + c^3))*b
```

**mupad [B]** time = 5.71, size = 134, normalized size = 1.09

$$\frac{a\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}+\frac{b\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{6\,c}-\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{5/2}}{16\,c^2}+\frac{b\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}-\frac{b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}-\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{16\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(1/2))/16 + (b*x^4*(c + d/x^2)^(1/2))/8 + (a*x^6*(c + d/
x^2)^(3/2))/(6*c) - (a*x^6*(c + d/x^2)^(5/2))/(16*c^2) + (b*x^4*(c + d/x^2)
^(3/2))/(8*c) - (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c^(5/2)
) - (b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

**sympy [B]** time = 72.29, size = 226, normalized size = 1.84

$$\frac{acx^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{5a\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}}+\frac{bcx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3b\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**5*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2/
d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c**2
*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b*c*
x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d + 1
)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sqrt(d)
)/(8*c**(3/2))
```

## 3.584 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\, x^3\, dx$

**Optimal**. Leaf size=90

$$\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

**Rubi [A]** time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] ((4*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(8*c) + (a*(c + d/x^2)^(3/2)*x^4)/(4*c)
 + (d*(4*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^3\,dx=-\left(\frac{1}{2}\text{Subst}\left(\int\frac{(a+bx)\sqrt{c+dx}}{x^3}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$=\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}-\frac{\left(2bc-\frac{ad}{2}\right)\text{Subst}\left(\int\frac{\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x^2}\right)}{4c}$$
$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}-\frac{(d(4bc-ad))\,\text{Subst}\left(\int\frac{1}{x\sqrt{c+dx}}\,dx,x,\right.}{16c}$$
$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}-\frac{(4bc-ad)\,\text{Subst}\left(\int\frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+}\right.}{8c}$$
$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}+\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

**Mathematica [A]** time = 0.14, size = 100, normalized size = 1.11

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{d}+1}\,\left(a\left(2cx^2+d\right)+4bc\right)+\sqrt{d}\,(4bc-ad)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\right)}{8c^{3/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(4*b*c + a*(d + 2*c*x^2))
 + Sqrt[d]*(4*b*c - a*d)*ArcSinh[(Sqrt[c]*x)/Sqrt[d]]))/(8*c^(3/2)*Sqrt[1 +
 (c*x^2)/d])
```

**IntegrateAlgebraic [A]** time = 0.15, size = 87, normalized size = 0.97

$$\frac{\left(4bcd-ad^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{\sqrt{\frac{cx^2+d}{x^2}}\,\left(2acx^4+adx^2+4bcx^2\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(4*b*c*x^2 + a*d*x^2 + 2*a*c*x^4))/(8*c) + ((4*b*c*d
 - a*d^2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*c^(3/2))
```

**fricas [A]** time = 0.44, size = 191, normalized size = 2.12

$$\left[-\frac{\left(4\,bcd-ad^2\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^2x^4+\left(4\,bc^2+acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^2},\,-\frac{\left(4\,bcd-ad^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-\left(2\,ac^2x^4+\left(4\,bc^2+acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2
+ d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2
+ d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2
+ d)/x^2))/c^2]
```

**giac [A]** time = 0.26, size = 105, normalized size = 1.17

$$\frac{1}{8}\left(2\,ax^2\mathrm{sgn}(x)+\frac{4\,bc^2\mathrm{sgn}(x)+acd\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}\,x-\frac{\left(4\,bcd\mathrm{sgn}(x)-ad^2\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{3}{2}}}+\frac{\left(4\,bcd\log\left(|d|\right)-ad^2\log\left(|d|\right)\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*
x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d
)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3/2)
```

**maple [A]** time = 0.06, size = 122, normalized size = 1.36

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(-a\,d^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)+4bcd\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-\sqrt{c\,x^2+d}\,a\sqrt{c}\,dx+4\sqrt{c\,x^2+d}\,bc^{\frac{3}{2}}x+2\left(c\,x^2+d\right)^{\frac{3}{2}}a\sqrt{c}\,x\right)x}{8\sqrt{c\,x^2+d}\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(1/2)*x*(2*(c*x^2+d)^(3/2)*c^(1/2)*x*a-(c*x^2+d)^(1/2)*
c^(1/2)*x*a*d+4*(c*x^2+d)^(1/2)*c^(3/2)*x*b-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a
*d^2+4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d)/(c*x^2+d)^(1/2)/c^(3/2)
```

**maxima [B]** time = 1.24, size = 159, normalized size = 1.77

$$\frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)a+\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3
/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c -
2*(c + d/x^2)*c^2 + c^3))*a + 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c +
d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*b
```

**mupad [B]** time = 5.31, size = 93, normalized size = 1.03

$$\frac{a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}+\frac{a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}+\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}-\frac{a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^4*(c + d/x^2)^(1/2))/8 + (b*x^2*(c + d/x^2)^(1/2))/2 + (a*x^4*(c + d/x
^2)^(3/2))/(8*c) + (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2)) - (a*
d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

**sympy [A]** time = 59.05, size = 144, normalized size = 1.60

$$\frac{acx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3a\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}+\frac{b\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*sqrt(d)*x**3/(8*sqrt(c*x**2/d
 + 1)) + a*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - a*d**2*asinh(sqrt(c)*x/sqr
t(d))/(8*c**(3/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + b*d*asinh(sqrt(c)*x
/sqrt(d))/(2*sqrt(c))
```

## 3.585 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x \, dx$

**Optimal**. Leaf size=84

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad+2bc)}{2c}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

**Rubi [A]** time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad+2bc)}{2c}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] -((2*b*c + a*d)*Sqrt[c + d/x^2])/(2*c) + (a*(c + d/x^2)^(3/2)*x^2)/(2*c) +
((2*b*c + a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*Sqrt[c])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{1}{4}(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x,\right.$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\right.}{2d}$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

**Mathematica [A]** time = 0.19, size = 71, normalized size = 0.85

$$\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\left(\frac{x(ad+2bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}} + ax^2 - 2b\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-2*b + a*x^2 + ((2*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[
d]])/(Sqrt[c]*Sqrt[d]*Sqrt[1 + (c*x^2)/d])))/2
```

**IntegrateAlgebraic [A]** time = 0.11, size = 68, normalized size = 0.81

$$\frac{1}{2}\left(ax^2-2b\right)\sqrt{\frac{cx^2+d}{x^2}} + \frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] ((-2*b + a*x^2)*Sqrt[(d + c*x^2)/x^2])/2 + ((2*b*c + a*d)*ArcTanh[Sqrt[(d +
 c*x^2)/x^2]/Sqrt[c]])/(2*Sqrt[c])
```

**fricas [A]** time = 0.43, size = 155, normalized size = 1.85

$$\left[\frac{(2\,bc+ad)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\left(acx^2-2\,bc\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\,c}, -\frac{(2\,bc+ad)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-\left(acx^2-2\,bc\right)\sqrt{\frac{cx^2+d}{x^2}}}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x
^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a*d
)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c*x^
2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]
```

**giac [A]** time = 0.29, size = 92, normalized size = 1.10

$$\frac{1}{2}\sqrt{cx^2+d}\,ax\mathrm{sgn}(x)+\frac{2\,b\sqrt{c}\,d\mathrm{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d}-\frac{\left(2\,bc^{\frac{3}{2}}\mathrm{sgn}(x)+a\sqrt{c}\,d\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d) - 1/4*(2*b*c^(3/2)*sgn(x) + a*sqrt(c)*d*sgn(x))*log((sqrt(
c)*x - sqrt(c*x^2 + d))^2)/c
```

**maple [A]** time = 0.06, size = 129, normalized size = 1.54

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(-a\,d^2x\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-2bcdx\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-\sqrt{c\,x^2+d}\,a\sqrt{c}\,d\,x^2-2\sqrt{c\,x^2+d}\,b\,c^{\frac{3}{2}}x^2+2\left(c\,x^2+d\right)^{\frac{3}{2}}b\sqrt{c}\right)}{2\sqrt{c\,x^2+d}\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*((c*x^2+d)/x^2)^(1/2)*(-(c*x^2+d)^(1/2)*c^(1/2)*x^2*a*d-2*(c*x^2+d)^(1
/2)*c^(3/2)*x^2*b+2*(c*x^2+d)^(3/2)*c^(1/2)*b-ln(c^(1/2)*x+(c*x^2+d)^(1/2))
*x*a*d^2-2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*b*c*d)/(c*x^2+d)^(1/2)/d/c^(1/2)
```

**maxima [A]** time = 1.21, size = 108, normalized size = 1.29

$$\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)a-\frac{1}{2}\left(\sqrt{c}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/
x^2) + sqrt(c)))/sqrt(c))*a - 1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) + 2*sqrt(c + d/x^2))*b
```

**mupad [B]** time = 5.10, size = 68, normalized size = 0.81

$$\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}-b\,\sqrt{c+\frac{d}{x^2}}+b\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^2*(c + d/x^2)^(1/2))/2 - b*(c + d/x^2)^(1/2) + b*c^(1/2)*atanh((c + d/
x^2)^(1/2)/c^(1/2)) + (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2))
```

**sympy [A]** time = 60.43, size = 107, normalized size = 1.27

$$\frac{a\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2\sqrt{c}}+b\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)-\frac{bcx}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + a*d*asinh(sqrt(c)*x/sqrt(d))/(2*sqrt(c))
 + b*sqrt(c)*asinh(sqrt(c)*x/sqrt(d)) - b*c*x/(sqrt(d)*sqrt(c*x**2/d + 1))
- b*sqrt(d)/(x*sqrt(c*x**2/d + 1))
```

## 3.586 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

**Optimal**. Leaf size=59

$$-a\sqrt{c+\frac{d}{x^2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

**Rubi [A]** time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-a\sqrt{c+\frac{d}{x^2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] -(a*Sqrt[c + d/x^2]) - (b*(c + d/x^2)^(3/2))/(3*d) + a*Sqrt[c]*ArcTanh[Sqrt
[c + d/x^2]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}(ac)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{(ac)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.15, size = 82, normalized size = 1.39

$$
\frac{\sqrt{c+\frac{d}{x^2}}\left(\frac{3a\sqrt{c}\sqrt{d}x^3\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{\frac{cx^2}{d}+1}} - 3adx^2 - b\left(cx^2+d\right)\right)}{3dx^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-3*a*d*x^2 - b*(d + c*x^2) + (3*a*Sqrt[c]*Sqrt[d]*x^3*Arc
Sinh[(Sqrt[c]*x)/Sqrt[d]])/Sqrt[1 + (c*x^2)/d]))/(3*d*x^2)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 74, normalized size = 1.25

$$
\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-3adx^2-bcx^2-bd\right)}{3dx^2} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d) - b*c*x^2 - 3*a*d*x^2))/(3*d*x^2) + a*Sqrt[c
]*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]]
```

**fricas [A]** time = 0.44, size = 166, normalized size = 2.81

$$
\left[\frac{3a\sqrt{c}dx^2\log\left(-2cx^2-2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left((bc+3ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{6dx^2}, -\frac{3a\sqrt{-c}dx^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left((bc+3ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{3dx^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2)
- d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(3*
a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (
(b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]
```

**giac [B]** time = 0.49, size = 163, normalized size = 2.76

$$-\frac{1}{2}a\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\mathrm{sgn}(x)+\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}\mathrm{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d\mathrm{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}d^2\mathrm{sgn}(x)+bc^{\frac{3}{2}}d^2\mathrm{sgn}(x)+3a\sqrt{c}d^3\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(3/2)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))
^4*a*sqrt(c)*d*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^2*sgn
(x) + b*c^(3/2)*d^2*sgn(x) + 3*a*sqrt(c)*d^3*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + d))^2 - d)^3
```

**maple [B]** time = 0.06, size = 109, normalized size = 1.85

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(-3acd\,x^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-3\sqrt{c\,x^2+d}\,a\,c^{\frac{3}{2}}x^4+3\left(c\,x^2+d\right)^{\frac{3}{2}}a\sqrt{c}\,x^2+\left(c\,x^2+d\right)^{\frac{3}{2}}b\sqrt{c}\right)}{3\sqrt{c\,x^2+d}\,\sqrt{c}\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x)
```

```
[Out] -1/3*((c*x^2+d)/x^2)^(1/2)/x^2*(-3*(c*x^2+d)^(1/2)*c^(3/2)*x^4*a+3*(c*x^2+d
)^(3/2)*c^(1/2)*x^2*a-3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x^3*a*c*d+(c*x^2+d)^(
3/2)*b*c^(1/2))/(c*x^2+d)^(1/2)/d/c^(1/2)
```

**maxima [A]** time = 1.23, size = 67, normalized size = 1.14

$$-\frac{1}{2}\left(\sqrt{c}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)a-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))
+ 2*sqrt(c + d/x^2))*a - 1/3*b*(c + d/x^2)^(3/2)/d
```

**mupad [B]** time = 5.21, size = 57, normalized size = 0.97

$$a\sqrt{c}\;\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-a\sqrt{c+\frac{d}{x^2}}-\frac{b\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x,x)
```

```
[Out] a*c^(1/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - a*(c + d/x^2)^(1/2) - (b*(c +
d/x^2)^(1/2)*(d + c*x^2))/(3*d*x^2)
```

**sympy [A]** time = 23.44, size = 75, normalized size = 1.27

$$\frac{a\left(-\frac{2c\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c+\frac{d}{x^2}}\right)}{2}+\frac{b\left(\begin{cases}-\frac{\sqrt{c}}{x^2} & \text{for } d=0\\ -\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)
```

```
[Out] a*(-2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*sqrt(c + d/x**2))/2 +
b*Piecewise((-sqrt(c)/x**2, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True)
)/2
```

## 3.587 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$

**Optimal**. Leaf size=46

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (b*(c + d/x^2)^(5/2))/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int (a+bx)\sqrt{c+dx}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 47, normalized size = 1.02

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(5adx^2-2bcx^2+3bd\right)}{15d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] -1/15*(Sqrt[c + d/x^2]*(d + c*x^2)*(3*b*d - 2*b*c*x^2 + 5*a*d*x^2))/(d^2*x^
4)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 66, normalized size = 1.43

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-5acdx^4-5ad^2x^2+2bc^2x^4-bcdx^2-3bd^2\right)}{15d^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-3*b*d^2 - b*c*d*x^2 - 5*a*d^2*x^2 + 2*b*c^2*x^4 -
5*a*c*d*x^4))/(15*d^2*x^4)
```

**fricas [A]** time = 0.43, size = 60, normalized size = 1.30

$$\frac{\left(\left(2\,bc^2-5\,acd\right)x^4-3\,bd^2-\left(bcd+5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2)/(d^2*x^4)
```

**giac [B]** time = 0.92, size = 250, normalized size = 5.43

$$\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{5}{2}}\mathrm{sgn}(x)-30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d\,\mathrm{sgn}(x)+10\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}d\,\mathrm{sgn}(x)+20\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d^2\mathrm{sgn}(x)+10\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{5}{2}}d^2\mathrm{sgn}(x)-10\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2ac^{\frac{3}{2}}d^3\mathrm{sgn}(x)-2\,bc^{\frac{5}{2}}d^3\mathrm{sgn}(x)+5\,ac^{\frac{3}{2}}d^4\mathrm{sgn}(x)\right)}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*sgn(x) + 30*(sqrt(c)*x -
 sqrt(c*x^2 + d))^6*b*c^(5/2)*sgn(x) - 30*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a
*c^(3/2)*d*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d*sgn(x) +
 20*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^2*sgn(x) + 10*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*b*c^(5/2)*d^2*sgn(x) - 10*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(3/2)*d^3*sgn(x) - 2*b*c^(5/2)*d^3*sgn(x) + 5*a*c^(3/2)*d^4*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

**maple [A]** time = 0.05, size = 48, normalized size = 1.04

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(5ad\,x^2-2bc\,x^2+3bd\right)\left(c\,x^2+d\right)}{15d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4
```

**maxima [A]** time = 0.72, size = 49, normalized size = 1.07

$$-\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c +
d/x^2)^(3/2)/d
```

**mupad [B]** time = 4.82, size = 91, normalized size = 1.98

$$\frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,c^2+a\,d\,c\right)}{5\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(5\,a\,d^2+b\,c\,d\right)}{15\,d^2\,x^2}-\frac{c\,\sqrt{c+\frac{d}{x^2}}\,\left(8\,a\,d+b\,c\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^3,x)
```

```
[Out] ((c + d/x^2)^(1/2)*(b*c^2 + a*c*d))/(5*d^2) - (b*(c + d/x^2)^(1/2))/(5*x^4)
 - ((c + d/x^2)^(1/2)*(5*a*d^2 + b*c*d))/(15*d^2*x^2) - (c*(c + d/x^2)^(1/2
)*(8*a*d + b*c))/(15*d^2)
```

**sympy [A]** time = 4.02, size = 58, normalized size = 1.26

$$-\frac{a\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}-\frac{b\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3,x)
```

```
[Out] -a*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True))
/2 - b*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2
```

## 3.588 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$

**Optimal**. Leaf size=74

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(5/
2))/(5*d^3) - (b*(c + d/x^2)^(7/2))/(7*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int x(a+bx)\sqrt{c+dx}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int\left(\frac{c(bc-ad)\sqrt{c+dx}}{d^2}+\frac{(-2bc+ad)(c+dx)^{3/2}}{d^2}+\frac{b(c+dx)^{5/2}}{d^2}\right)dx,x,\right.\right.\\
&= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 69, normalized size = 0.93

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(7adx^2\left(2cx^2-3d\right)+b\left(-8c^2x^4+12cdx^2-15d^2\right)\right)}{105d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(7*a*d*x^2*(-3*d + 2*c*x^2) + b*(-15*d^2 + 12*
c*d*x^2 - 8*c^2*x^4)))/(105*d^3*x^6)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 90, normalized size = 1.22

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(14ac^2dx^6-7acd^2x^4-21ad^3x^2-8bc^3x^6+4bc^2dx^4-3bcd^2x^2-15bd^3\right)}{105d^3x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-15*b*d^3 - 3*b*c*d^2*x^2 - 21*a*d^3*x^2 + 4*b*c^2*
d*x^4 - 7*a*c*d^2*x^4 - 8*b*c^3*x^6 + 14*a*c^2*d*x^6))/(105*d^3*x^6)
```

**fricas [A]** time = 0.43, size = 85, normalized size = 1.15

$$-\frac{\left(2\left(4\,bc^3-7\,ac^2d\right)x^6-\left(4\,bc^2d-7\,acd^2\right)x^4+15\,bd^3+3\left(bcd^2+7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d^
3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^6)
```

**giac [B]** time = 1.38, size = 310, normalized size = 4.19

$$\frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}\mathrm{sgn}(x)+280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d\,\mathrm{sgn}(x)+140\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{7}{2}}d\,\mathrm{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{5}{2}}d^2\mathrm{sgn}(x)+84\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{7}{2}}d^2\mathrm{sgn}(x)-42\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{5}{2}}d^3\mathrm{sgn}(x)-28\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{7}{2}}d^3\mathrm{sgn}(x)+49\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{5}{2}}d^4\mathrm{sgn}(x)+4\,bc^{\frac{7}{2}}d^4\mathrm{sgn}(x)-7\,ac^{\frac{5}{2}}d^5\mathrm{sgn}(x)\right)}{105\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(5/2)*sgn(x) + 280*(sqrt(c)
*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*sgn(x) - 175*(sqrt(c)*x - sqrt(c*x^2 + d)
)^8*a*c^(5/2)*d*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d^2*sgn(x) + 84*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*x^2
+ d))^4*a*c^(5/2)*d^3*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)
*d^3*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^4*sgn(x) + 4*b
*c^(7/2)*d^4*sgn(x) - 7*a*c^(5/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d)
)^2 - d)^7
```

**maple [A]** time = 0.05, size = 70, normalized size = 0.95

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(14acd\,x^4-8b\,c^2x^4-21a\,d^2x^2+12bcd\,x^2-15b\,d^2\right)\left(c\,x^2+d\right)}{105d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*d
*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6
```

**maxima** [A] time = 0.60, size = 84, normalized size = 1.14

$$-\frac{1}{105}\,b\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)-\frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/105*b*(15*(c + d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d
/x^2)^(3/2)*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2
)*c/d^2)
```

**mupad** [B] time = 4.92, size = 126, normalized size = 1.70

$$\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{15\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{15\,d\,x^2}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}+\frac{4\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^5,x)
```

```
[Out] (2*a*c^2*(c + d/x^2)^(1/2))/(15*d^2) - (b*(c + d/x^2)^(1/2))/(7*x^6) - (a*(
c + d/x^2)^(1/2))/(5*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*c*(c
 + d/x^2)^(1/2))/(15*d*x^2) - (b*c*(c + d/x^2)^(1/2))/(35*d*x^4) + (4*b*c^2
*(c + d/x^2)^(1/2))/(105*d^2*x^2)
```

**sympy** [A] time = 4.56, size = 78, normalized size = 1.05

$$-\frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5,x)
```

```
[Out] -a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d
/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3
```

## 3.589 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$

**Optimal**. Leaf size=104

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

**Rubi [A]** time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(5/2))/(5*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (b*(c + d/x^
2)^(9/2))/(9*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)\sqrt{c+dx}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{c^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{3/2}}{d^3}+\frac{(-3bc+ad)(c+dx}{d^3}\right.\right.\right.\\ &= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)}{9d^4}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 79, normalized size = 0.76

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{cx^2}{d}+1\right)\left(8c^2x^4-12cdx^2+15d^2\right)(6bc-9ad)-105bd^2\left(cx^2+d\right)\right)}{945d^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-105*b*d^2*(d + c*x^2) + (6*b*c - 9*a*d)*x^2*(1 + (c*x^2)
/d)*(15*d^2 - 12*c*d*x^2 + 8*c^2*x^4)))/(945*d^3*x^8)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 114, normalized size = 1.10

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-24ac^3dx^8+12ac^2d^2x^6-9acd^3x^4-45ad^4x^2+16bc^4x^8-8bc^3dx^6+6bc^2d^2x^4-5bcd^3x^2-35bd^4\right)}{315d^4x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-35*b*d^4 - 5*b*c*d^3*x^2 - 45*a*d^4*x^2 + 6*b*c^2*
d^2*x^4 - 9*a*c*d^3*x^4 - 8*b*c^3*d*x^6 + 12*a*c^2*d^2*x^6 + 16*b*c^4*x^8 -
 24*a*c^3*d*x^8))/(315*d^4*x^8)
```

**fricas [A]** time = 0.48, size = 109, normalized size = 1.05

$$\frac{\left(8\left(2\,bc^4-3\,ac^3d\right)x^8-4\left(2\,bc^3d-3\,ac^2d^2\right)x^6-35\,bd^4+3\left(2\,bc^2d^2-3\,acd^3\right)x^4-5\left(bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*b
*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*sqrt((c
*x^2 + d)/x^2)/(d^4*x^8)
```

**giac [B]** time = 2.06, size = 370, normalized size = 3.56

$$\frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{7}{2}}d\,\mathrm{sgn}(x)+378\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{9}{2}}d\,\mathrm{sgn}(x)+63\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{7}{2}}d^2\mathrm{sgn}(x)+168\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{9}{2}}d^2\mathrm{sgn}(x)-42\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{7}{2}}d^3\mathrm{sgn}(x)-72\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{9}{2}}d^3\mathrm{sgn}(x)+108\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{7}{2}}d^4\mathrm{sgn}(x)+18\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{9}{2}}d^4\mathrm{sgn}(x)-27\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{7}{2}}d^5\mathrm{sgn}(x)-2\,bc^{\frac{9}{2}}d^5\mathrm{sgn}(x)+3\,ac^{\frac{7}{2}}d^6\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(7/2)*sgn(x) + 630*(sqrt(c
)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 +
d))^10*a*c^(7/2)*d*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d
*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^2*sgn(x) + 168*(sq
rt(c)*x - sqrt(c*x^2 + d))^6*b*c^(9/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*
x^2 + d))^6*a*c^(7/2)*d^3*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(
9/2)*d^3*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^4*sgn(x)
+ 18*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(9/2)*d^4*sgn(x) - 27*(sqrt(c)*x -
 sqrt(c*x^2 + d))^2*a*c^(7/2)*d^5*sgn(x) - 2*b*c^(9/2)*d^5*sgn(x) + 3*a*c^(
7/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

**maple [A]** time = 0.05, size = 94, normalized size = 0.90

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(24a\,c^2d\,x^6-16b\,c^3x^6-36ac\,d^2x^4+24b\,c^2d\,x^4+45a\,d^3x^2-30bc\,d^2x^2+35b\,d^3\right)\left(c\,x^2+d\right)}{315d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x)
```

```
[Out] -1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+24
*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8
```

**maxima [A]** time = 0.57, size = 118, normalized size = 1.13

$$-\frac{1}{315}\,b\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)-\frac{1}{105}\,a\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c +
 d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c + d
/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d/x^2)^(3/2)*c^2/d^3
)
```

**mupad [B]** time = 5.22, size = 168, normalized size = 1.62

$$\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}+\frac{4\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}+\frac{2\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^7,x)
```

```
[Out] (16*b*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(9*x^8) - (8
*a*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*(c + d/x^2)^(1/2))/(7*x^6) - (a*c*
(c + d/x^2)^(1/2))/(35*d*x^4) - (b*c*(c + d/x^2)^(1/2))/(63*d*x^6) + (4*a*c
^2*(c + d/x^2)^(1/2))/(105*d^2*x^2) + (2*b*c^2*(c + d/x^2)^(1/2))/(105*d^2*
x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(315*d^3*x^2)
```

**sympy [A]** time = 5.21, size = 112, normalized size = 1.08

$$-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7,x)
```

```
[Out] -a*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**
(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)
/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4
```

## 3.590 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9} \, dx$

**Optimal**. Leaf size=134

$$-\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

**Rubi [A]** time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (3*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5) + ((4*b
*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (b*(c + d/x^2)^(11/2))/(11*d^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9} \, dx &= -\left(\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)\sqrt{c+dx} \, dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)\sqrt{c+dx}}{d^4}-\frac{c^2(4bc-3ad)(c+dx)^{3/2}}{d^4}+\frac{3c(2bc-ad)(c+\right.\right.\right.\\ &= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}+\ldots\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 90, normalized size = 0.67

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{cx^2}{d}+1\right)\left(-16c^3x^6+24c^2dx^4-30cd^2x^2+35d^3\right)(8bc-11ad)-315bd^3\left(cx^2+d\right)\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-315*b*d^3*(d + c*x^2) + (8*b*c - 11*a*d)*x^2*(1 + (c*x^2
)/d)*(35*d^3 - 30*c*d^2*x^2 + 24*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x^10)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 138, normalized size = 1.03

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(176ac^4dx^{10}-88ac^3d^2x^8+66ac^2d^3x^6-55acd^4x^4-385ad^5x^2-128bc^5x^{10}+64bc^4dx^8-48bc^3d^2x^6+40bc^2d^3x^4-35bcd^4x^2-315bd^5\right)}{3465d^5x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-315*b*d^5 - 35*b*c*d^4*x^2 - 385*a*d^5*x^2 + 40*b*
c^2*d^3*x^4 - 55*a*c*d^4*x^4 - 48*b*c^3*d^2*x^6 + 66*a*c^2*d^3*x^6 + 64*b*c
^4*d*x^8 - 88*a*c^3*d^2*x^8 - 128*b*c^5*x^10 + 176*a*c^4*d*x^10))/(3465*d^5
*x^10)
```

**fricas [A]** time = 0.53, size = 133, normalized size = 0.99

$$-\frac{\left(16\left(8\,bc^5-11\,ac^4d\right)x^{10}-8\left(8\,bc^4d-11\,ac^3d^2\right)x^8+6\left(8\,bc^3d^2-11\,ac^2d^3\right)x^6+315\,bd^5-5\left(8\,bc^2d^3-11\,acd^4\right)x^4+35\left(bcd^4+11\,ad^5\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3465\,d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^10 - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8
+ 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*c*
d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^10)
```

**giac [B]** time = 2.98, size = 430, normalized size = 3.21

$$\frac{32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\mathrm{sgn}(x)+11088\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}\mathrm{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{9}{2}}d\,\mathrm{sgn}(x)+7392\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{11}{2}}d\,\mathrm{sgn}(x)+231\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{9}{2}}d^2\mathrm{sgn}(x)+2640\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{11}{2}}d^2\mathrm{sgn}(x)-165\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{9}{2}}d^3\mathrm{sgn}(x)-1320\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{11}{2}}d^3\mathrm{sgn}(x)+1815\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{9}{2}}d^4\mathrm{sgn}(x)+440\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{11}{2}}d^4\mathrm{sgn}(x)-605\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{9}{2}}d^5\mathrm{sgn}(x)-88\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{11}{2}}d^5\mathrm{sgn}(x)+121\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{9}{2}}d^6\mathrm{sgn}(x)+8\,bc^{\frac{11}{2}}d^6\mathrm{sgn}(x)-11\,ac^{\frac{9}{2}}d^7\mathrm{sgn}(x)\right)}{3465\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*sgn(x) + 11088*(sq
rt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*sgn(x) - 4851*(sqrt(c)*x - sqrt(c*
x^2 + d))^12*a*c^(9/2)*d*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^2*sgn(x
) + 2640*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^2*sgn(x) - 165*(sqrt(
c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^3*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x
^2 + d))^6*b*c^(11/2)*d^3*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c
^(9/2)*d^4*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^4*sgn(
x) - 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^5*sgn(x) - 88*(sqrt(c)
*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^5*sgn(x) + 121*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*a*c^(9/2)*d^6*sgn(x) + 8*b*c^(11/2)*d^6*sgn(x) - 11*a*c^(9/2)*d^7*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

**maple [A]** time = 0.05, size = 118, normalized size = 0.88

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(176a\,c^3d\,x^8-128b\,c^4x^8-264a\,c^2d^2x^6+192b\,c^3d\,x^6+330ac\,d^3x^4-240b\,c^2d^2x^4-385a\,d^4x^2+280bc\,d^3x^2-315b\,d^4\right)\left(c\,x^2+d\right)}{3465d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x)
```

[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*x^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*c*d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^10

**maxima [A]** time = 0.47, size = 152, normalized size = 1.13

$$-\frac{1}{3465}\,b\left(\frac{315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^5}-\frac{1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^5}+\frac{2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^5}-\frac{2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^5}+\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4}{d^5}\right)-\frac{1}{315}\,a\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")

[Out] -1/3465*b*(315*(c + d/x^2)^(11/2)/d^5 - 1540*(c + d/x^2)^(9/2)*c/d^5 + 2970*(c + d/x^2)^(7/2)*c^2/d^5 - 2772*(c + d/x^2)^(5/2)*c^3/d^5 + 1155*(c + d/x^2)^(3/2)*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4)

**mupad [B]** time = 5.61, size = 210, normalized size = 1.57

$$\frac{16\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{128\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^5}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{99\,d\,x^8}+\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}+\frac{8\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{693\,d^2\,x^6}-\frac{16\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^4}+\frac{64\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^9,x)

[Out] (16*a*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(11*x^10) - (a*(c + d/x^2)^(1/2))/(9*x^8) - (128*b*c^5*(c + d/x^2)^(1/2))/(3465*d^5) - (a*c*(c + d/x^2)^(1/2))/(63*d*x^6) - (b*c*(c + d/x^2)^(1/2))/(99*d*x^8) + (2*a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^4) - (8*a*c^3*(c + d/x^2)^(1/2))/(315*d^3*x^2) + (8*b*c^2*(c + d/x^2)^(1/2))/(693*d^2*x^6) - (16*b*c^3*(c + d/x^2)^(1/2))/(1155*d^3*x^4) + (64*b*c^4*(c + d/x^2)^(1/2))/(3465*d^4*x^2)

**sympy [A]** time = 5.86, size = 146, normalized size = 1.09

$$-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{b\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9,x)

[Out] -a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5

## 3.591 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^{10}\,dx$

**Optimal**. Leaf size=150

$$-\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5}+\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4}-\frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{231c^3}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(11}{99c^2}$$

**Rubi [A]** time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4}-\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{99c^2}-\frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{231c^3}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (-16*d^3*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^3)/(3465*c^5) + (8*d^2*(11*b*
c - 8*a*d)*(c + d/x^2)^(3/2)*x^5)/(1155*c^4) - (2*d*(11*b*c - 8*a*d)*(c + d
/x^2)^(3/2)*x^7)/(231*c^3) + ((11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^9)/(99*c
^2) + (a*(c + d/x^2)^(3/2)*x^11)/(11*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^{10}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c} + \frac{(11bc-8ad)\int \sqrt{c+\frac{d}{x^2}}\,x^8\,dx}{11c}$$

$$= \frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c} - \frac{(2d(11bc-8ad))\int \sqrt{c+\frac{d}{x^2}}\,x^6 \ldots}{33c^2}$$

$$= -\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3} + \frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c} \ldots$$

$$= \frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4} - \frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3} + \frac{(11bc-8ad)\left(\ldots\right.}{99c\ldots}$$

$$= -\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3465c^5} + \frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4} - \frac{2d(11bc-\ldots}{\ldots}$$

**Mathematica [A]** time = 0.08, size = 108, normalized size = 0.72

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(315c^4x^8-280c^3dx^6+240c^2d^2x^4-192cd^3x^2+128d^4\right)+11bc\left(35c^3x^6-30c^2dx^4+24cd^2x^2-16d^3\right)\right)}{3465c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x
^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3*d
*x^6 + 315*c^4*x^8)))/(3465*c^5)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 112, normalized size = 0.75

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(315ac^4x^8-280ac^3dx^6+240ac^2d^2x^4-192acd^3x^2+128ad^4+385bc^4x^6-330bc^3dx^4+264bc^2d^2x^2-176bcd^3\right)}{3465c^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(-176*b*c*d^3 + 128*a*d^4 + 264*b*c^2*d^2*x^
2 - 192*a*c*d^3*x^2 - 330*b*c^3*d*x^4 + 240*a*c^2*d^2*x^4 + 385*b*c^4*x^6 -
 280*a*c^3*d*x^6 + 315*a*c^4*x^8))/(3465*c^5)
```

**fricas [A]** time = 0.43, size = 131, normalized size = 0.87

$$\frac{\left(315\,ac^5x^{11}+35\left(11\,bc^5+ac^4d\right)x^9+5\left(11\,bc^4d-8\,ac^3d^2\right)x^7-6\left(11\,bc^3d^2-8\,ac^2d^3\right)x^5+8\left(11\,bc^2d^3-8\,acd^4\right)x^3-16\left(11\,bcd^4-8\,ad^5\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a*
c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a*c
*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

**giac [A]** time = 0.22, size = 175, normalized size = 1.17

$$\frac{16\left(11\,bcd^{\frac{9}{2}}-8\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^5}+\frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1540\left(cx^2+d\right)^{\frac{9}{2}}ad\mathrm{sgn}(x)-1485\left(cx^2+d\right)^{\frac{7}{2}}bcd\mathrm{sgn}(x)+2970\left(cx^2+d\right)^{\frac{7}{2}}ad^2\mathrm{sgn}(x)+2079\left(cx^2+d\right)^{\frac{5}{2}}bcd^2\mathrm{sgn}(x)-2772\left(cx^2+d\right)^{\frac{5}{2}}ad^3\mathrm{sgn}(x)-1155\left(cx^2+d\right)^{\frac{3}{2}}bcd^3\mathrm{sgn}(x)+1155\left(cx^2+d\right)^{\frac{3}{2}}ad^4\mathrm{sgn}(x)}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] 16/3465*(11*b*c*d^(9/2) - 8*a*d^(11/2))*sgn(x)/c^5 + 1/3465*(315*(c*x^2 + d)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1540*(c*x^2 + d)^(9/2)*a*d*sgn(x) - 1485*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 2970*(c*x^2 + d)^(7/2)*a*d^2*sgn(x) + 2079*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 2772*(c*x^2 + d)^(5/2)*a*d^3*sgn(x) - 1155*(c*x^2 + d)^(3/2)*b*c*d^3*sgn(x) + 1155*(c*x^2 + d)^(3/2)*a*d^4*sgn(x))/c^5

**maple** [A] time = 0.05, size = 113, normalized size = 0.75

$$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(315 a\,x^{8} c^{4}-280 a\,c^{3} d\,x^{6}+385 b\,c^{4} x^{6}+240 a\,c^{2} d^{2} x^{4}-330 b\,c^{3} d\,x^{4}-192 a c\,d^{3} x^{2}+264 b\,c^{2} d^{2} x^{2}+128 a\,d^{4}-176 b c\,d^{3}\right) \left(c\,x^{2}+d\right) x}{3465 c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x)

[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^6+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*a*d^4-176*b*c*d^3)*(c*x^2+d)/c^5

**maxima** [A] time = 0.64, size = 158, normalized size = 1.05

$$\frac{\left(35 \left(c+\frac{d}{x^{2}}\right)^{\frac{9}{2}} x^{9}-135 \left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}} d x^{7}+189 \left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}} d^{2} x^{5}-105 \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}} d^{3} x^{3}\right) b}{315\,c^{4}}+\frac{\left(315 \left(c+\frac{d}{x^{2}}\right)^{\frac{11}{2}} x^{11}-1540 \left(c+\frac{d}{x^{2}}\right)^{\frac{9}{2}} d x^{9}+2970 \left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}} d^{2} x^{7}-2772 \left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}} d^{3} x^{5}+1155 \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}} d^{4} x^{3}\right) a}{3465\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*b/c^4 + 1/3465*(315*(c + d/x^2)^(11/2)*x^11 - 1540*(c + d/x^2)^(9/2)*d*x^9 + 2970*(c + d/x^2)^(7/2)*d^2*x^7 - 2772*(c + d/x^2)^(5/2)*d^3*x^5 + 1155*(c + d/x^2)^(3/2)*d^4*x^3)*a/c^5

**mupad** [B] time = 4.59, size = 117, normalized size = 0.78

$$\sqrt{c+\frac{d}{x^{2}}}\,\left(\frac{a\,x^{11}}{11}+\frac{x\,\left(128\,a\,d^{5}-176\,b\,c\,d^{4}\right)}{3465\,c^{5}}+\frac{x^{9}\,\left(385\,b\,c^{5}+35\,a\,d\,c^{4}\right)}{3465\,c^{5}}-\frac{d\,x^{7}\,\left(8\,a\,d-11\,b\,c\right)}{693\,c^{2}}+\frac{2\,d^{2}\,x^{5}\,\left(8\,a\,d-11\,b\,c\right)}{1155\,c^{3}}-\frac{8\,d^{3}\,x^{3}\,\left(8\,a\,d-11\,b\,c\right)}{3465\,c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(1/2),x)

[Out] (c + d/x^2)^(1/2)*((a*x^11)/11 + (x*(128*a*d^5 - 176*b*c*d^4))/(3465*c^5) + (x^9*(385*b*c^5 + 35*a*c^4*d))/(3465*c^5) - (d*x^7*(8*a*d - 11*b*c))/(693*c^2) + (2*d^2*x^5*(8*a*d - 11*b*c))/(1155*c^3) - (8*d^3*x^3*(8*a*d - 11*b*c))/(3465*c^4))

**sympy** [B] time = 6.44, size = 1386, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)*x**10*(c+d/x**2)**(1/2),x)

[Out] 315*a*c**9*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1295*a*c**8*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**

```
2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(3465
*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c*
*6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2)*x**10*s
qrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**
7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**4*d**(43/
2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 280*a*c*
*3*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**
17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20)
+ 560*a*c**2*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 448*a*c*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 128*a*d**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 35*b*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 11
0*b*c**6*d**(21/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*
d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 114*b*c**5*d**(23/2)*x
**10*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**
5*d**11*x**2 + 315*c**4*d**12) + 40*b*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) - 5*b*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 +
 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*b*c**2*d*
*(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
 945*c**5*d**11*x**2 + 315*c**4*d**12) - 40*b*c*d**(31/2)*x**2*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) - 16*b*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945
*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
```

## 3.592 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^8\,dx$

**Optimal**. Leaf size=117

$$\frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4}-\frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

**Rubi [A]** time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2}-\frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (8*d^2*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(315*c^4) - (4*d*(3*b*c - 2*a
*d)*(c + d/x^2)^(3/2)*x^5)/(105*c^3) + ((3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x
^7)/(21*c^2) + (a*(c + d/x^2)^(3/2)*x^9)/(9*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^8\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}+\frac{(9bc-6ad)\int\sqrt{c+\frac{d}{x^2}}\,x^6\,dx}{9c}$$

$$= \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}-\frac{(4d(3bc-2ad))\int\sqrt{c+\frac{d}{x^2}}\,x^4\,dx}{21c^2}$$

$$= -\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}+ -$$

$$= \frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{315c^4}-\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\right.}{21c^2}$$

**Mathematica** [A] time = 0.07, size = 86, normalized size = 0.74

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(35c^3x^6-30c^2dx^4+24cd^2x^2-16d^3\right)+3bc\left(15c^2x^4-12cdx^2+8d^2\right)\right)}{315c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(3*b*c*(8*d^2 - 12*c*d*x^2 + 15*c^2*x^4) + a
*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x^4 + 35*c^3*x^6)))/(315*c^4)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 88, normalized size = 0.75

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(35ac^3x^6-30ac^2dx^4+24acd^2x^2-16ad^3+45bc^3x^4-36bc^2dx^2+24bcd^2\right)}{315c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(24*b*c*d^2 - 16*a*d^3 - 36*b*c^2*d*x^2 + 24
*a*c*d^2*x^2 + 45*b*c^3*x^4 - 30*a*c^2*d*x^4 + 35*a*c^3*x^6))/(315*c^4)
```

**fricas** [A] time = 0.42, size = 107, normalized size = 0.91

$$\frac{\left(35\,ac^4x^9+5\left(9\,bc^4+ac^3d\right)x^7+3\left(3\,bc^3d-2\,ac^2d^2\right)x^5-4\left(3\,bc^2d^2-2\,acd^3\right)x^3+8\left(3\,bcd^3-2\,ad^4\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d^
2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^4
```

**giac** [A] time = 0.17, size = 140, normalized size = 1.20

$$-\frac{8\left(3\,bcd^{\frac{7}{2}}-2\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^4}+\frac{35\left(cx^2+d\right)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\left(cx^2+d\right)^{\frac{7}{2}}bc\mathrm{sgn}(x)-135\left(cx^2+d\right)^{\frac{7}{2}}ad\mathrm{sgn}(x)-126\left(cx^2+d\right)^{\frac{5}{2}}bcd\mathrm{sgn}(x)+189\left(cx^2+d\right)^{\frac{5}{2}}ad^2\mathrm{sgn}(x)+105\left(cx^2+d\right)^{\frac{3}{2}}bcd^2\mathrm{sgn}(x)-105\left(cx^2+d\right)^{\frac{3}{2}}ad^3\mathrm{sgn}(x)}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8/315*(3*b*c*d^(7/2) - 2*a*d^(9/2))*sgn(x)/c^4 + 1/315*(35*(c*x^2 + d)^(9/
2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 135*(c*x^2 + d)^(7/2)*a*d*s
gn(x) - 126*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 189*(c*x^2 + d)^(5/2)*a*d^2*sg
n(x) + 105*(c*x^2 + d)^(3/2)*b*c*d^2*sgn(x) - 105*(c*x^2 + d)^(3/2)*a*d^3*s
gn(x))/c^4
```

**maple [A]** time = 0.05, size = 89, normalized size = 0.76

$$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(35a\,x^{6}c^{3}-30a\,c^{2}d\,x^{4}+45b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-36b\,c^{2}d\,x^{2}-16a\,d^{3}+24bc\,d^{2}\right)\left(c\,x^{2}+d\right)x}{315c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24*
a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4
```

**maxima [A]** time = 0.56, size = 124, normalized size = 1.06

$$\frac{\left(15\left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}}x^{7}-42\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}dx^{5}+35\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}d^{2}x^{3}\right)b}{105\,c^{3}}+\frac{\left(35\left(c+\frac{d}{x^{2}}\right)^{\frac{9}{2}}x^{9}-135\left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}}dx^{7}+189\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}d^{2}x^{5}-105\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}d^{3}x^{3}\right)a}{315\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^
2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)
^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^
3)*a/c^4
```

**mupad [B]** time = 4.52, size = 97, normalized size = 0.83

$$\sqrt{c+\frac{d}{x^{2}}}\,\left(\frac{a\,x^{9}}{9}-\frac{x\,\left(16\,a\,d^{4}-24\,b\,c\,d^{3}\right)}{315\,c^{4}}+\frac{x^{7}\,\left(45\,b\,c^{4}+5\,a\,d\,c^{3}\right)}{315\,c^{4}}-\frac{d\,x^{5}\,\left(2\,a\,d-3\,b\,c\right)}{105\,c^{2}}+\frac{4\,d^{2}\,x^{3}\,\left(2\,a\,d-3\,b\,c\right)}{315\,c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^9)/9 - (x*(16*a*d^4 - 24*b*c*d^3))/(315*c^4) + (x^7
*(45*b*c^4 + 5*a*c^3*d))/(315*c^4) - (d*x^5*(2*a*d - 3*b*c))/(105*c^2) + (4
*d^2*x^3*(2*a*d - 3*b*c))/(315*c^3))
```

**sympy [B]** time = 4.92, size = 910, normalized size = 7.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**8*(c+d/x**2)**(1/2),x)
```

```
[Out] 35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5*a*c**3
*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x**4*sqrt(c
```

```
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d*
*9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*
a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x**10*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*b*c
**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x
**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**
5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**2*d**(15/2)*x**4
*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**
6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) + 8*b*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

## 3.593 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^6 \, dx$

**Optimal**. Leaf size=84

$$-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{105c^3}+\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

**Rubi [A]** time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{35c^2}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{105c^3}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (-2*d*(7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^3)/(105*c^3) + ((7*b*c - 4*a*d)*(
c + d/x^2)^(3/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(3/2)*x^7)/(7*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\, x^6\, dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}+\frac{(7bc-4ad)\int \sqrt{c+\frac{d}{x^2}}\, x^4\, dx}{7c}\\
&=\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}-\frac{(2d(7bc-4ad))\int \sqrt{c+\frac{d}{x^2}}\, x^2\, dx}{35c^2}\\
&=-\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{105c^3}+\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 64, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(15c^2x^4-12cdx^2+8d^2\right)+7bc\left(3cx^2-2d\right)\right)}{105c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(7*b*c*(-2*d + 3*c*x^2) + a*(8*d^2 - 12*c*d*
x^2 + 15*c^2*x^4)))/(105*c^3)
```

**IntegrateAlgebraic** [A] time = 0.09, size = 64, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(15ac^2x^4-12acdx^2+8ad^2+21bc^2x^2-14bcd\right)}{105c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(-14*b*c*d + 8*a*d^2 + 21*b*c^2*x^2 - 12*a*c
*d*x^2 + 15*a*c^2*x^4))/(105*c^3)
```

**fricas** [A] time = 0.42, size = 82, normalized size = 0.98

$$\frac{\left(15\,ac^3x^7+3\left(7\,bc^3+ac^2d\right)x^5+\left(7\,bc^2d-4\,acd^2\right)x^3-2\left(7\,bcd^2-4\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*x
^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3
```

**giac** [A] time = 0.17, size = 105, normalized size = 1.25

$$\frac{2\left(7\,bcd^{\frac{5}{2}}-4\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{105\,c^3}+\frac{15\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+21\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-42\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)-35\left(cx^2+d\right)^{\frac{3}{2}}bcd\mathrm{sgn}(x)+35\left(cx^2+d\right)^{\frac{3}{2}}ad^2\mathrm{sgn}(x)}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3 + 1/105*(15*(c*x^2 + d)^(7/2
)*a*sgn(x) + 21*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 42*(c*x^2 + d)^(5/2)*a*d*sgn
(x) - 35*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) + 35*(c*x^2 + d)^(3/2)*a*d^2*sgn(x)
)/c^3
```

**maple** [A] time = 0.06, size = 65, normalized size = 0.77

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(15a\,x^4c^2-12acd\,x^2+21b\,c^2x^2+8a\,d^2-14bcd\right)\left(c\,x^2+d\right)x}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*d
^2-14*b*c*d)*(c*x^2+d)/c^3
```

**maxima [A]** time = 0.53, size = 90, normalized size = 1.07

$$\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15\,c^2}+\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(1
5*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/2)
*d^2*x^3)*a/c^3
```

**mupad [B]** time = 4.49, size = 77, normalized size = 0.92

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{a\,x^7}{7}+\frac{x\,\left(8\,a\,d^3-14\,b\,c\,d^2\right)}{105\,c^3}+\frac{x^5\,\left(21\,b\,c^3+3\,a\,d\,c^2\right)}{105\,c^3}-\frac{d\,x^3\,\left(4\,a\,d-7\,b\,c\right)}{105\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^7)/7 + (x*(8*a*d^3 - 14*b*c*d^2))/(105*c^3) + (x^5*
(21*b*c^3 + 3*a*c^2*d))/(105*c^3) - (d*x^3*(4*a*d - 7*b*c))/(105*c^2))
```

**sympy [B]** time = 3.99, size = 422, normalized size = 5.02

$$\frac{15ac^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{33ac^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{3ac^2d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{12acd^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{8ad^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{b\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**6*(c+d/x**2)**(1/2),x)
```

```
[Out] 15*a*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*sqrt(c*x**2/d
 + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(1
9/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + b*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(3/2)*x**2*sqrt(c*x**2/
d + 1)/(15*c) - 2*b*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

## 3.594 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\, x^4 \, dx$

**Optimal**. Leaf size=53

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

**Rubi [A]** time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] ((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^5
)/(5*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\, x^4 \, dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}+\frac{(5bc-2ad)\int \sqrt{c+\frac{d}{x^2}}\, x^2 \, dx}{5c}\\ &= \frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 42, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(3acx^2-2ad+5bc\right)}{15c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 42, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(3acx^2-2ad+5bc\right)}{15c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

**fricas** [A] time = 0.42, size = 57, normalized size = 1.08

$$\frac{\left(3\,ac^2x^5+\left(5\,bc^2+acd\right)x^3+\left(5\,bcd-2\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c*
x^2 + d)/x^2)/c^2
```

**giac** [A] time = 0.21, size = 72, normalized size = 1.36

$$-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^2}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}a\mathrm{sgn}(x)+5\left(cx^2+d\right)^{\frac{3}{2}}bc\mathrm{sgn}(x)-5\left(cx^2+d\right)^{\frac{3}{2}}ad\mathrm{sgn}(x)}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2 + 1/15*(3*(c*x^2 + d)^(5/2)*
a*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c*sgn(x) - 5*(c*x^2 + d)^(3/2)*a*d*sgn(x))
/c^2
```

**maple** [A] time = 0.05, size = 43, normalized size = 0.81

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(3a\,x^2c-2ad+5bc\right)\left(c\,x^2+d\right)x}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2
```

**maxima** [A] time = 0.66, size = 55, normalized size = 1.04

$$\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)a}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^
2)^(3/2)*d*x^3)*a/c^2
```

**mupad [B]** time = 4.44, size = 54, normalized size = 1.02

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{a\,x^5}{5}-\frac{x\,\left(2\,a\,d^2-5\,b\,c\,d\right)}{15\,c^2}+\frac{x^3\,\left(5\,b\,c^2+a\,d\,c\right)}{15\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^5)/5 - (x*(2*a*d^2 - 5*b*c*d))/(15*c^2) + (x^3*(5*b
*c^2 + a*c*d))/(15*c^2))
```

**sympy [B]** time = 3.04, size = 119, normalized size = 2.25

$$\frac{a\sqrt{d}\,x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{b\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/(1
5*c) - 2*a*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*sqrt(d)*x**2*sqrt(c*x*
*2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
```

## 3.595 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx$

**Optimal**. Leaf size=66

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}+bx\sqrt{c+\frac{d}{x^2}}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)$$

**Rubi [A]** time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {451, 242, 277, 217, 206}

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}+bx\sqrt{c+\frac{d}{x^2}}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] b*Sqrt[c + d/x^2]*x + (a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*Sqrt[d]*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^2\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}+b\int \sqrt{c+\frac{d}{x^2}}\,dx \\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right) \\
&= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right) \\
&= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right) \\
&= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.06, size = 84, normalized size = 1.27

$$
\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{cx^2+d}\left(a\left(cx^2+d\right)+3bc\right)-3bc\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{3c\sqrt{cx^2+d}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[d + c*x^2]*(3*b*c + a*(d + c*x^2)) - 3*b*c*Sqrt[d]
*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(3*c*Sqrt[d + c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.09, size = 83, normalized size = 1.26

$$
\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\sqrt{cx^2+d}\left(acx^2+ad+3bc\right)}{3c}-b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{\sqrt{cx^2+d}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(3*b*c + a*d + a*c*x^2))/(3*c) - b*Sqr
t[d]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/Sqrt[d + c*x^2]
```

**fricas [A]** time = 0.44, size = 156, normalized size = 2.36

$$
\left[\frac{3\,bc\sqrt{d}\,\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(acx^3+(3\,bc+ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,c},\ \frac{3\,bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(acx^3+(3\,bc+ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/
x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c*s
```

```
qrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (a*c*x^3 + (
3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

**giac [B]** time = 0.19, size = 116, normalized size = 1.76

$$\frac{bd\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\mathrm{sgn}(x)}{\sqrt{-d}}-\frac{\left(3\,bcd\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+3\,bc\sqrt{-d}\,\sqrt{d}\,+a\sqrt{-d}\,d^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,c\sqrt{-d}}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}ac^2\mathrm{sgn}(x)+3\,\sqrt{cx^2+d}\,bc^3\mathrm{sgn}(x)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan(
sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(c*
sqrt(-d)) + 1/3*((c*x^2 + d)^(3/2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^3*s
gn(x))/c^3
```

**maple [A]** time = 0.06, size = 83, normalized size = 1.26

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(3bc\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-3\sqrt{c\,x^2+d}\,bc-\left(c\,x^2+d\right)^{\frac{3}{2}}a\right)x}{3\sqrt{c\,x^2+d}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/3*((c*x^2+d)/x^2)^(1/2)*x*(3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*b*c-a*(c*x^2+d)^(3/2)-3*(c*x^2+d)^(1/2)*b*c)/(c*x^2+d)^(1/2)/c
```

**maxima [A]** time = 1.38, size = 75, normalized size = 1.14

$$\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{1}{2}\left(2\,\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(c + d/x^2)^(3/2)*x^3/c + 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

**mupad [B]** time = 4.72, size = 80, normalized size = 1.21

$$b\,x\,\sqrt{c+\frac{d}{x^2}}+\frac{a\,x\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,c}+\frac{b\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] b*x*(c + d/x^2)^(1/2) + (a*x*(c + d/x^2)^(1/2)*(d + c*x^2))/(3*c) + (b*d^(1
/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2
) + 1)^(1/2))
```

**sympy [A]** time = 3.27, size = 107, normalized size = 1.62

$$\frac{a\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}+\frac{b\sqrt{c}\,x}{\sqrt{1+\frac{d}{cx^2}}}-b\sqrt{d}\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{bd}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c) +
 b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) +
b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))
```

## 3.596 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$

**Optimal**. Leaf size=85

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(2ad+bc)}{2cx}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

**Rubi [A]** time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 453, 195, 217, 206}

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(2ad+bc)}{2cx}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] -((b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + (a*(c + d/x^2)^(3/2)*x)/c - ((b*
c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = -\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{(-bc-2ad)\operatorname{Subst}\left(\int \sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{1}{2}(-bc-2ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x,$$

$$= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{1}{2}(-bc-2ad)\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \text{[illegible]}\right.$$

$$= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}$$

**Mathematica [A]** time = 0.09, size = 75, normalized size = 0.88

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-\frac{x^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{cx^2+d}}+2ax^2-b\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]
/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(2*x)
```

**IntegrateAlgebraic [A]** time = 0.15, size = 89, normalized size = 1.05

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\left(2ax^2-b\right)\sqrt{cx^2+d}}{2x^2}+\frac{(-2ad-bc)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{2\sqrt{d}}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(((-b + 2*a*x^2)*Sqrt[d + c*x^2])/(2*x^2) + ((-(b*c) - 2
*a*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*Sqrt[d])))/Sqrt[d + c*x^2]
```

**fricas [A]** time = 0.45, size = 164, normalized size = 1.93

$$\left[\frac{(bc+2ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)+2\left(2adx^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{4dx}, \frac{(bc+2ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(2adx^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{2dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^
2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((b*
c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d))
+ (2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]
```

**giac** [A] time = 0.25, size = 76, normalized size = 0.89

$$\frac{2\,\sqrt{cx^2+d}\,ac\mathrm{sgn}(x)+\frac{\left(bc^2\mathrm{sgn}(x)+2\,acd\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{\sqrt{cx^2+d}\,bc\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + d)*a*c*sgn(x) + (b*c^2*sgn(x) + 2*a*c*d*sgn(x))*arctan(
sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - sqrt(c*x^2 + d)*b*c*sgn(x)/x^2)/c
```

**maple** [A] time = 0.05, size = 135, normalized size = 1.59

$$-\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(2a\,d^{\frac{3}{2}}x^2\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)+bc\sqrt{d}\,x^2\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-2\sqrt{c\,x^2+d}\,ad\,x^2-\sqrt{c\,x^2+d}\,bc\,x^2+\left(c\,x^2+d\right)^{\frac{3}{2}}b\right)}{2\sqrt{c\,x^2+d}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*((c*x^2+d)/x^2)^(1/2)/x*(2*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)
*x^2*a+d^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*b*c-2*(c*x^2+d)^(1/2
)*x^2*a*d-(c*x^2+d)^(1/2)*x^2*b*c+(c*x^2+d)^(3/2)*b)/(c*x^2+d)^(1/2)/d
```

**maxima** [A] time = 1.22, size = 133, normalized size = 1.56

$$\frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(
c + d/x^2)*x + sqrt(d))))*a - 1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 -
 d) - c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sq
rt(d))*b
```

**mupad** [B] time = 5.11, size = 97, normalized size = 1.14

$$a\,x\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,x}-\frac{b\,c\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{2\,\sqrt{d}}+\frac{a\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] a*x*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2))/(2*x) - (b*c*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/(2*d^(1/2)) + (a*d^(1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))
*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2) + 1)^(1/2))
```

**sympy [A]** time = 4.48, size = 107, normalized size = 1.26

$$\frac{a\sqrt{c}\,x}{\sqrt{1+\frac{d}{cx^2}}}-a\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{ad}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\,\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) + a
*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x)
- b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```

## 3.597 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

**Optimal**. Leaf size=91

$$\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

**Rubi [A]** time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 195, 217, 206}

$$\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] ((b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d*x) - (b*(c + d/x^2)^(3/2))/(4*d*x) + (
c*(b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx &= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(-bc+4ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx}{4d}\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}-\frac{(-bc+4ad)\,\text{Subst}\left(\int \sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{4d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.19, size = 100, normalized size = 1.10

$$
-\frac{\sqrt{c+\frac{d}{x^2}}\left(\left(cx^2+d\right)\left(4adx^2+bcx^2+2bd\right)+cx^4\sqrt{\frac{cx^2}{d}+1}\,(4ad-bc)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{8dx^3\left(cx^2+d\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] -1/8*(Sqrt[c + d/x^2]*((d + c*x^2)*(2*b*d + b*c*x^2 + 4*a*d*x^2) + c*(-(b*c
) + 4*a*d)*x^4*Sqrt[1 + (c*x^2)/d]*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d*x^3*(d
 + c*x^2))
```

**IntegrateAlgebraic [A]** time = 0.18, size = 103, normalized size = 1.13

$$
\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\left(bc^2-4acd\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{8d^{3/2}}+\frac{\sqrt{cx^2+d}\left(-4adx^2-bcx^2-2bd\right)}{8dx^4}\right)}{\sqrt{cx^2+d}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-2*b*d - b*c*x^2 - 4*a*d*x^2))/(8*d*x
^4) + ((b*c^2 - 4*a*c*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(3/2))))/Sq
rt[d + c*x^2]
```

**fricas [A]** time = 0.44, size = 194, normalized size = 2.13

$$
\left[-\frac{\left(bc^2-4\,acd\right)\sqrt{d}\,x^3\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,bd^2+\left(bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^2x^3},\ -\frac{\left(bc^2-4\,acd\right)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(2\,bd^2+\left(bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\,d^2x^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2
 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*
sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/(d^2*x^3)]
```

**giac [A]** time = 0.24, size = 130, normalized size = 1.43

$$-\frac{\frac{\left(bc^3\mathrm{sgn}(x)-4\,ac^2d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\mathrm{sgn}(x)+\sqrt{cx^2+d}\,bc^3d\mathrm{sgn}(x)-4\sqrt{cx^2+d}\,ac^2d^2\mathrm{sgn}(x)}{c^2dx^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/8*((b*c^3*sgn(x) - 4*a*c^2*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(s
qrt(-d)*d) + ((c*x^2 + d)^(3/2)*b*c^3*sgn(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*
sgn(x) + sqrt(c*x^2 + d)*b*c^3*d*sgn(x) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x
))/(c^2*d*x^4))/c
```

**maple [B]** time = 0.06, size = 175, normalized size = 1.92

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(4ac\,d^{\frac{3}{2}}x^4\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-b\,c^2\sqrt{d}\,x^4\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-4\sqrt{c\,x^2+d}\,acd\,x^4+\sqrt{c\,x^2+d}\,b\,c^2x^4+4\left(c\,x^2+d\right)^{\frac{3}{2}}ad\,x^2-\left(c\,x^2+d\right)^{\frac{3}{2}}bc\,x^2+2\left(c\,x^2+d\right)^{\frac{3}{2}}bd\right)}{8\sqrt{c\,x^2+d}\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/8*((c*x^2+d)/x^2)^(1/2)/x^3*(4*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/
x)*x^4*a*c-d^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2-4*(c*x^2+d
)^(1/2)*x^4*a*c*d+(c*x^2+d)^(1/2)*x^4*b*c^2+4*(c*x^2+d)^(3/2)*x^2*a*d-(c*x^
2+d)^(3/2)*x^2*b*c+2*(c*x^2+d)^(3/2)*b*d)/(c*x^2+d)^(1/2)/d^2
```

**maxima [B]** time = 1.35, size = 193, normalized size = 2.12

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}\right)a-\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 - d) - c*log((sqrt(c + d/x^2)*
x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d))*a - 1/16*(c^2*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*((c +
d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*d*x^4 - 2*(c
 + d/x^2)*d^2*x^2 + d^3))*b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)
```

**sympy [A]** time = 6.97, size = 144, normalized size = 1.58

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2\sqrt{d}}-\frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{3}{2}}}-\frac{bd}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2,x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*s
qrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*sqr
t(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*d/(
4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

## 3.598 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

**Optimal**. Leaf size=123

$$-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

**Rubi [A]** time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 279, 321, 217, 206}

$$-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] ((b*c - 2*a*d)*Sqrt[c + d/x^2])/(8*d*x^3) - (b*(c + d/x^2)^(3/2))/(6*d*x^3)
 + (c*(b*c - 2*a*d)*Sqrt[c + d/x^2])/(16*d^2*x) - (c^2*(b*c - 2*a*d)*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx &= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{(-3bc+6ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx}{6d}\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}-\frac{(-3bc+6ad)\,\text{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{6d}\\
&= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{(c(bc-2ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d}\\
&= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{\left(c^2(bc-2ad)\right)\text{Subst}\left(\int \cdots\right.}{16d^2}\\
&= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{\left(c^2(bc-2ad)\right)\text{Subst}\left(\cdots\right.}{16\cdots}\\
&= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{c^2(bc-2ad)\tanh^{-1}\left(\cdots\right.}{16d^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.03, size = 68, normalized size = 0.55

$$
\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(c^2x^6(bc-2ad)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{d}+1\right)-bd^3\right)}{6d^4x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-(b*d^3) + c^2*(b*c - 2*a*d)*x^6*Hypergeometr
ic2F1[3/2, 3, 5/2, 1 + (c*x^2)/d]))/(6*d^4*x^5)
```

**IntegrateAlgebraic [A]** time = 0.24, size = 128, normalized size = 1.04

$$
\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\sqrt{cx^2+d}\left(-6acdx^4-12ad^2x^2+3bc^2x^4-2bcdx^2-8bd^2\right)}{48d^2x^6}+\frac{\left(2ac^2d-bc^3\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{16d^{5/2}}\right)}{\sqrt{cx^2+d}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-8*b*d^2 - 2*b*c*d*x^2 - 12*a*d^2*x^2
 + 3*b*c^2*x^4 - 6*a*c*d*x^4))/(48*d^2*x^6) + ((-(b*c^3) + 2*a*c^2*d)*ArcTa
nh[Sqrt[d + c*x^2]/Sqrt[d]])/(16*d^(5/2))))/Sqrt[d + c*x^2]
```

**fricas [A]** time = 0.46, size = 244, normalized size = 1.98

$$\left[-\frac{3\left(bc^3-2\,ac^2d\right)\sqrt{d}\,x^5\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(bc^2d-2\,acd^2\right)x^4-8\,bd^3-2\left(bcd^2+6\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96\,d^3x^5},\ \frac{3\left(bc^3-2\,ac^2d\right)\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(3\left(bc^2d-2\,acd^2\right)x^4-8\,bd^3-2\left(bcd^2+6\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{48\,d^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(
b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3 -
2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)
) + (3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqr
t((c*x^2 + d)/x^2))/(d^3*x^5)]
```

**giac [A]** time = 0.30, size = 153, normalized size = 1.24

$$\frac{\frac{3\left(bc^4\mathrm{sgn}(x)-2\,ac^3d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d^2}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\mathrm{sgn}(x)-6\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\mathrm{sgn}(x)-8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\mathrm{sgn}(x)-3\sqrt{cx^2+d}\,bc^4d^2\mathrm{sgn}(x)+6\sqrt{cx^2+d}\,ac^3d^3\mathrm{sgn}(x)}{c^3d^2x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b*c^4*sgn(x) - 2*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/
(sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) - 6*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4
*d^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c^3*d^3*sgn(x))/(c^3*d^2*x^6))/c
```

**maple [B]** time = 0.06, size = 220, normalized size = 1.79

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(6a\,c^2d^{\frac{3}{2}}x^6\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-3b\,c^3\sqrt{d}\,x^6\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-6\sqrt{c\,x^2+d}\,a\,c^2d\,x^6+3\sqrt{c\,x^2+d}\,b\,c^3x^6+6\left(c\,x^2+d\right)^{\frac{3}{2}}acd\,x^4-3\left(c\,x^2+d\right)^{\frac{3}{2}}b\,c^2x^4-12\left(c\,x^2+d\right)^{\frac{3}{2}}a\,d^2x^2+6\left(c\,x^2+d\right)^{\frac{3}{2}}bcd\,x^2-8\left(c\,x^2+d\right)^{\frac{3}{2}}b\,d^2\right)}{48\sqrt{c\,x^2+d}\,d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)/x^5*(6*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/
x)*x^6*a*c^2-3*d^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^6*b*c^3-6*(c*x
^2+d)^(1/2)*x^6*a*c^2*d+3*(c*x^2+d)^(1/2)*x^6*b*c^3+6*(c*x^2+d)^(3/2)*x^4*a
*c*d-3*(c*x^2+d)^(3/2)*x^4*b*c^2-12*(c*x^2+d)^(3/2)*x^2*a*d^2+6*(c*x^2+d)^(
3/2)*x^2*b*c*d-8*(c*x^2+d)^(3/2)*b*d^2)/(c*x^2+d)^(1/2)/d^3
```

**maxima [B]** time = 1.27, size = 277, normalized size = 2.25

$$-\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)a+\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}\,c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3d^2x^6-3\left(c+\frac{d}{x^2}\right)^2d^3x^4+3\left(c+\frac{d}{x^2}\right)d^4x^2-d^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*(c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))
/d^(3/2) + 2*((c + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/
```

```
x^2)^2*d*x^4 - 2*(c + d/x^2)*d^2*x^2 + d^3))*a + 1/96*(3*c^3*log((sqrt(c +
d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^
2)^(5/2)*c^3*x^5 - 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d^
2*x)/((c + d/x^2)^3*d^2*x^6 - 3*(c + d/x^2)^2*d^3*x^4 + 3*(c + d/x^2)*d^4*x
^2 - d^5))*b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4, x)
```

**sympy [B]** time = 11.42, size = 226, normalized size = 1.84

$$-\frac{ac^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{3}{2}}}-\frac{ad}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{5}{2}}}{16d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}}{48dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{5b\sqrt{c}}{24x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{16d^{\frac{5}{2}}}-\frac{bd}{6\sqrt{c}\,x^7\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)
```

```
[Out] -a*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*a*sqrt(c)/(8*x**3*sqrt(1 + d/(
c*x**2))) + a*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - a*d/(4*sqrt(c)
*x**5*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))) +
 b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqrt(1
+ d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d/(6*s
qrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

## 3.599 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx$

**Optimal**. Leaf size=123

$$\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(6bc-ad)}{16c}+\frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

**Rubi [A]** time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(6bc-ad)}{16c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (d*(6*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c) + ((6*b*c - a*d)*(c + d/x^2)^(
3/2)*x^4)/(24*c) + (a*(c + d/x^2)^(5/2)*x^6)/(6*c) + (d^2*(6*b*c - a*d)*Arc
Tanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{\left(3bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d(6bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2}\,dx, \ldots\right.}{16c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{\left(d^2(6bc \ldots\right.}{\ldots}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} - \frac{(d(6bc \ldots}{\ldots}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} + \frac{d^2(6bc \ldots}{\ldots}$$

**Mathematica [A]** time = 0.14, size = 123, normalized size = 1.00

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{d}+1}\left(a\left(8c^2x^4+14cdx^2+3d^2\right)+6bc\left(2cx^2+5d\right)\right)-3d^{3/2}(ad-6bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\right)}{48c^{3/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(5*d + 2*c*x^2) +
a*(3*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) - 3*d^(3/2)*(-6*b*c + a*d)*ArcSinh[(Sqr
t[c]*x)/Sqrt[d]]))/(48*c^(3/2)*Sqrt[1 + (c*x^2)/d])
```

**IntegrateAlgebraic [A]** time = 0.19, size = 112, normalized size = 0.91

$$\frac{\left(6bcd^2-ad^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(8ac^2x^6+14acdx^4+3ad^2x^2+12bc^2x^4+30bcdx^2\right)}{48c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(30*b*c*d*x^2 + 3*a*d^2*x^2 + 12*b*c^2*x^4 + 14*a*c*
d*x^4 + 8*a*c^2*x^6))/(48*c) + ((6*b*c*d^2 - a*d^3)*ArcTanh[Sqrt[(d + c*x^2
)/x^2]/Sqrt[c]])/(16*c^(3/2))
```

**fricas [A]** time = 0.45, size = 243, normalized size = 1.98

$$\left[-\frac{3\left(6\,bcd^2-ad^3\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(8\,ac^3x^6+2\left(6\,bc^3+7\,ac^2d\right)x^4+3\left(10\,bc^2d+acd^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96\,c^2},\ -\frac{3\left(6\,bcd^2-ad^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-\left(8\,ac^3x^6+2\left(6\,bc^3+7\,ac^2d\right)x^4+3\left(10\,bc^2d+acd^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{48\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b
*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 - a*
d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (8*a
*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/c^2]
```

**giac [A]** time = 0.23, size = 144, normalized size = 1.17

$$\frac{1}{48}\left(2\left(4\,acx^2\mathrm{sgn}\left(x\right)+\frac{6\,bc^5\mathrm{sgn}\left(x\right)+7\,ac^4d\mathrm{sgn}\left(x\right)}{c^4}\right)x^2+\frac{3\left(10\,bc^4d\mathrm{sgn}\left(x\right)+ac^3d^2\mathrm{sgn}\left(x\right)\right)}{c^4}\right)\sqrt{cx^2+d}\,x-\frac{\left(6\,bcd^2\mathrm{sgn}\left(x\right)-ad^3\mathrm{sgn}\left(x\right)\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{3}{2}}}+\frac{\left(6\,bcd^2\log\left(|d|\right)-ad^3\log\left(|d|\right)\right)\mathrm{sgn}\left(x\right)}{32\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*c*x^2*sgn(x) + (6*b*c^5*sgn(x) + 7*a*c^4*d*sgn(x))/c^4)*x^2 +
3*(10*b*c^4*d*sgn(x) + a*c^3*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x - 1/16*(6*b
*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(3/2
) + 1/32*(6*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(3/2)
```

**maple [A]** time = 0.06, size = 162, normalized size = 1.32

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(-3a\,d^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)+18bc\,d^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-3\sqrt{c\,x^2+d}\,a\sqrt{c}\,d^2x+18\sqrt{c\,x^2+d}\,b\,c^{\frac{3}{2}}dx-2\left(c\,x^2+d\right)^{\frac{3}{2}}a\sqrt{c}\,dx+12\left(c\,x^2+d\right)^{\frac{3}{2}}b\,c^{\frac{3}{2}}x+8\left(c\,x^2+d\right)^{\frac{5}{2}}a\sqrt{c}\,x\right)x^3}{48\left(c\,x^2+d\right)^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(3/2)*x^3*(8*c^(1/2)*(c*x^2+d)^(5/2)*x*a-2*(c*x^2+d)^(
3/2)*a*c^(1/2)*d*x+12*(c*x^2+d)^(3/2)*b*c^(3/2)*x-3*(c*x^2+d)^(1/2)*a*c^(1/
2)*d^2*x+18*(c*x^2+d)^(1/2)*b*c^(3/2)*d*x-3*a*d^3*ln(c^(1/2)*x+(c*x^2+d)^(1
/2))+18*b*c*d^2*ln(c^(1/2)*x+(c*x^2+d)^(1/2)))/(c*x^2+d)^(3/2)/c^(3/2)
```

**maxima [B]** time = 1.30, size = 240, normalized size = 1.95

$$\frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}\,c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c-3\left(c+\frac{d}{x^2}\right)^2c^2+3\left(c+\frac{d}{x^2}\right)c^3-c^4}\right)a-\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] 1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(3/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 + 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c +
 d/x^2)*c^2*d^3)/((c + d/x^2)^3*c - 3*(c + d/x^2)^2*c^2 + 3*(c + d/x^2)*c^3
 - c^4))*a - 1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) +
 sqrt(c)))/sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/
((c + d/x^2)^2 - 2*(c + d/x^2)*c + c^2))*b
```

**mupad [B]** time = 5.78, size = 130, normalized size = 1.06

$$\frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{3/2}}{6}+\frac{5\,b\,x^4\,\left(c+\frac{d}{x^2}\right)^{3/2}}{8}+\frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{5/2}}{16\,c}+\frac{3\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{a\,c\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}-\frac{3\,b\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(3/2))/6 + (5*b*x^4*(c + d/x^2)^(3/2))/8 + (a*x^6*(c + d
/x^2)^(5/2))/(16*c) + (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c
^(3/2)) + (3*b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(1/2)) - (a*c*x^6
*(c + d/x^2)^(1/2))/16 - (3*b*c*x^4*(c + d/x^2)^(1/2))/8
```

**sympy [B]** time = 100.22, size = 253, normalized size = 2.06

$$\frac{ac^2x^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{11ac\sqrt{d}\,x^5}{24\sqrt{\frac{cx^2}{d}+1}}+\frac{17ad^{\frac{3}{2}}x^3}{48\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{5}{2}}x}{16c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^3\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{16c^{\frac{3}{2}}}+\frac{bc^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3bc\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3bd^2\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)
```

```
[Out] a*c**2*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 11*a*c*sqrt(d)*x**5/(24*sqrt(c
*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*sqrt(c*x**2/d + 1)) + a*d**(5/2)*x/(
16*c*sqrt(c*x**2/d + 1)) - a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(3/2)) +
b*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + b*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c))
```

## 3.600 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^3\,dx$

**Optimal**. Leaf size=115

$$\frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c}-\frac{3d\sqrt{c+\frac{d}{x^2}}\,(ad+4bc)}{8c}+\frac{3d(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

**Rubi [A]** time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$\frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c}-\frac{3d\sqrt{c+\frac{d}{x^2}}\,(ad+4bc)}{8c}+\frac{3d(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (-3*d*(4*b*c + a*d)*Sqrt[c + d/x^2])/(8*c) + ((4*b*c + a*d)*(c + d/x^2)^(3/
2)*x^2)/(8*c) + (a*(c + d/x^2)^(5/2)*x^4)/(4*c) + (3*d*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx=-\left(\frac{1}{2}\operatorname{Subst}\left(\int\frac{(a+bx)(c+dx)^{3/2}}{x^3}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}-\frac{(4bc+ad)\operatorname{Subst}\left(\int\frac{(c+dx)^{3/2}}{x^2}\,dx,x,\frac{1}{x^2}\right)}{8c}$$

$$=\frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}-\frac{(3d(4bc+ad))\operatorname{Subst}\left(\int\frac{\sqrt{c+dx}}{x}\,d\right.}{16c}$$

$$=-\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c}+\frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}-\frac{1}{16}(3d$$

$$=-\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c}+\frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}-\frac{1}{8}(3(4b$$

$$=-\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c}+\frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c}+\frac{3d(4b}{}$$

**Mathematica [A]** time = 0.25, size = 89, normalized size = 0.77

$$\frac{1}{8}\sqrt{c+\frac{d}{x^2}}\left(\frac{3\sqrt{d}\,x(ad+4bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{d}+1}}+2acx^4+5adx^2+4bcx^2-8bd\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 + (3*Sqrt[d]*(
4*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/d]))
)/8
```

**IntegrateAlgebraic [A]** time = 0.14, size = 88, normalized size = 0.77

$$\frac{3\left(ad^2+4bcd\right)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{1}{8}\sqrt{\frac{cx^2+d}{x^2}}\left(2acx^4+5adx^2+4bcx^2-8bd\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4))/8 + (3
*(4*b*c*d + a*d^2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*Sqrt[c])
```

**fricas [A]** time = 0.45, size = 203, normalized size = 1.77

$$\left[\frac{3\left(4bcd+ad^{2}\right)\sqrt{c}\log\left(-2cx^{2}-2\sqrt{c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)+2\left(2ac^{2}x^{4}-8bcd+\left(4bc^{2}+5acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{16c},-\frac{3\left(4bcd+ad^{2}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)-\left(2ac^{2}x^{4}-8bcd+\left(4bc^{2}+5acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{8c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2
 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5
*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]
```

**giac [A]** time = 0.29, size = 126, normalized size = 1.10

$$\frac{2b\sqrt{c}d^{2}\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^{2}+d}\right)^{2}-d}+\frac{1}{8}\left(2acx^{2}\mathrm{sgn}(x)+\frac{4bc^{3}\mathrm{sgn}(x)+5ac^{2}d\mathrm{sgn}(x)}{c^{2}}\right)\sqrt{cx^{2}+d}\,x-\frac{3\left(4bc^{\frac{3}{2}}d\mathrm{sgn}(x)+a\sqrt{c}d^{2}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^{2}+d}\right)^{2}\right)}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*x
^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x - 3/
16*(4*b*c^(3/2)*d*sgn(x) + a*sqrt(c)*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^
2 + d))^2)/c
```

**maple [A]** time = 0.06, size = 174, normalized size = 1.51

$$\frac{\left(\frac{cx^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(3ad^{3}x\ln\left(\sqrt{c}\,x+\sqrt{cx^{2}+d}\right)+12bcd^{2}x\ln\left(\sqrt{c}\,x+\sqrt{cx^{2}+d}\right)+3\sqrt{cx^{2}+d}\,a\sqrt{c}\,d^{2}x^{2}+12\sqrt{cx^{2}+d}\,bc^{\frac{3}{2}}dx^{2}+2\left(cx^{2}+d\right)^{\frac{3}{2}}a\sqrt{c}\,dx^{2}+8\left(cx^{2}+d\right)^{\frac{3}{2}}bc^{\frac{3}{2}}x^{2}-8\left(cx^{2}+d\right)^{\frac{5}{2}}b\sqrt{c}\right)x^{2}}{8\left(cx^{2}+d\right)^{\frac{3}{2}}\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(3/2)*x^2*(8*c^(3/2)*(c*x^2+d)^(3/2)*x^2*b+12*c^(3/2)*(
c*x^2+d)^(1/2)*x^2*b*d+2*c^(1/2)*(c*x^2+d)^(3/2)*x^2*a*d-8*c^(1/2)*(c*x^2+d
)^(5/2)*b+3*c^(1/2)*(c*x^2+d)^(1/2)*x^2*a*d^2+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2
))*x*a*d^3+12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*b*c*d^2)/(c*x^2+d)^(3/2)/d/c^
(1/2)
```

**maxima [A]** time = 1.33, size = 171, normalized size = 1.49

$$-\frac{1}{16}\left(\frac{3d^{2}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}d^{2}-3\sqrt{c+\frac{d}{x^{2}}}\,cd^{2}\right)}{\left(c+\frac{d}{x^{2}}\right)^{2}-2\left(c+\frac{d}{x^{2}}\right)c+c^{2}}\right)a+\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^{2}}}\,cx^{2}-3\sqrt{c}\,d\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^{2}}}\,d\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/s
qrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)
^2 - 2*(c + d/x^2)*c + c^2))*a + 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d
*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c +
d/x^2)*d)*b
```

**mupad [B]** time = 5.70, size = 105, normalized size = 0.91

$$\frac{5\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8}-b\,d\,\sqrt{c+\frac{d}{x^2}}+\frac{3\,b\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{3\,a\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(3/2))/8 - b*d*(c + d/x^2)^(1/2) + (3*b*c^(1/2)*d*atan
h((c + d/x^2)^(1/2)/c^(1/2)))/2 + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2))
)/(8*c^(1/2)) - (3*a*c*x^4*(c + d/x^2)^(1/2))/8 + (b*c*x^2*(c + d/x^2)^(1/2
))/2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)
```

```
[Out] Timed out
```

## 3.601 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx$

**Optimal**. Leaf size=110

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c}-\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\,(3ad+2bc)+\frac{1}{2}\sqrt{c}\,(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

**Rubi [A]** time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c}-\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\,(3ad+2bc)+\frac{1}{2}\sqrt{c}\,(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] -((2*b*c + 3*a*d)*Sqrt[c + d/x^2])/2 - ((2*b*c + 3*a*d)*(c + d/x^2)^(3/2))/
(6*c) + (a*(c + d/x^2)^(5/2)*x^2)/(2*c) + (Sqrt[c]*(2*b*c + 3*a*d)*ArcTanh[
Sqrt[c + d/x^2]/Sqrt[c]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{\left(bc+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= -\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{1}{4}(2bc+3ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\right.$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{1}{4}(c(2bc$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{(c(2bc+}{}$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} + \frac{1}{2}\sqrt{c}\,(2b$$

**Mathematica [C]** time = 0.10, size = 78, normalized size = 0.71

$$\frac{1}{3}\sqrt{c+\frac{d}{x^2}}\left(-\frac{(3ad+2bc)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{d}\right)}{\sqrt{\frac{cx^2}{d}+1}} - \frac{b\left(cx^2+d\right)^2}{dx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-((b*(d + c*x^2)^2)/(d*x^2)) - ((2*b*c + 3*a*d)*Hypergeom
etric2F1[-3/2, -1/2, 1/2, -((c*x^2)/d)])/Sqrt[1 + (c*x^2)/d]))/3
```

**IntegrateAlgebraic [A]** time = 0.17, size = 93, normalized size = 0.85

$$\frac{1}{2}\left(3a\sqrt{c}\,d+2bc^{3/2}\right)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right) + \frac{\sqrt{\frac{cx^2+d}{x^2}}\left(3acx^4-6adx^2-8bcx^2-2bd\right)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-2*b*d - 8*b*c*x^2 - 6*a*d*x^2 + 3*a*c*x^4))/(6*x^2
) + ((2*b*c^(3/2) + 3*a*Sqrt[c]*d)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/
2
```

**fricas [A]** time = 0.44, size = 195, normalized size = 1.77

$$\left[\frac{3\,(2\,bc+3\,ad)\sqrt{c}\,x^2\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\left(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x^2}, -\frac{3\,(2\,bc+3\,ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-\left(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x
^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c*x
^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 -
2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]
```

**giac [B]** time = 0.54, size = 225, normalized size = 2.05

$$\frac{1}{2}\sqrt{cx^2+d}\,acx\mathrm{sgn}(x)-\frac{1}{4}\left(2bc^{\frac{3}{2}}\mathrm{sgn}(x)+3a\sqrt{c}\,d\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)+\frac{2\left(6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}d\mathrm{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}\,d^2\mathrm{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}d^2\mathrm{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}\,d^3\mathrm{sgn}(x)+4bc^{\frac{3}{2}}d^3\mathrm{sgn}(x)+3a\sqrt{c}\,d^4\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*c*x*sgn(x) - 1/4*(2*b*c^(3/2)*sgn(x) + 3*a*sqrt(c)*d*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2) + 2/3*(6*(sqrt(c)*x - sqrt(c*x
^2 + d))^4*b*c^(3/2)*d*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)
*d^2*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d^2*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^3*sgn(x) + 4*b*c^(3/2)*d^3*sgn(x)
 + 3*a*sqrt(c)*d^4*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3
```

**maple [B]** time = 0.06, size = 216, normalized size = 1.96

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(9acd^3x^3\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)+6bc^2d^2x^3\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)+9\sqrt{cx^2+d}\,ac^{\frac{3}{2}}d^2x^4+6\sqrt{cx^2+d}\,bc^{\frac{5}{2}}dx^4+6\left(cx^2+d\right)^{\frac{3}{2}}ac^{\frac{3}{2}}dx^4+4\left(cx^2+d\right)^{\frac{3}{2}}bc^{\frac{5}{2}}x^4-6\left(cx^2+d\right)^{\frac{5}{2}}a\sqrt{c}\,dx^2-4\left(cx^2+d\right)^{\frac{5}{2}}bc^{\frac{3}{2}}x^2-2\left(cx^2+d\right)^{\frac{5}{2}}b\sqrt{c}\,d\right)}{6\left(cx^2+d\right)^{\frac{3}{2}}\sqrt{c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x)
```

```
[Out] 1/6*((c*x^2+d)/x^2)^(3/2)*(4*c^(5/2)*(c*x^2+d)^(3/2)*x^4*b+6*c^(5/2)*(c*x^2
+d)^(1/2)*x^4*b*d+6*c^(3/2)*(c*x^2+d)^(3/2)*x^4*a*d-4*c^(3/2)*(c*x^2+d)^(5/
2)*x^2*b+9*c^(3/2)*(c*x^2+d)^(1/2)*x^4*a*d^2-6*c^(1/2)*(c*x^2+d)^(5/2)*x^2*
a*d+9*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x^3*a*c*d^3+6*ln(c^(1/2)*x+(c*x^2+d)^(1
/2))*x^3*b*c^2*d^2-2*c^(1/2)*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2/c^(1/
2)
```

**maxima [A]** time = 1.23, size = 134, normalized size = 1.22

$$\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,cx^2-3\sqrt{c}\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}\,d\right)a-\frac{1}{6}\left(3c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}\,c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c + d/x^2)*d)*a - 1/6*(3*c^(3/2)*log(
(sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3
/2) + 6*sqrt(c + d/x^2)*c)*b
```

**mupad [B]** time = 5.65, size = 95, normalized size = 0.86

$$b\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{3}-a\,d\,\sqrt{c+\frac{d}{x^2}}-b\,c\,\sqrt{c+\frac{d}{x^2}}+\frac{3\,a\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{a\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] b*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (b*(c + d/x^2)^(3/2))/3 - a*d*
(c + d/x^2)^(1/2) - b*c*(c + d/x^2)^(1/2) + (3*a*c^(1/2)*d*atanh((c + d/x^2
)^(1/2)/c^(1/2)))/2 + (a*c*x^2*(c + d/x^2)^(1/2))/2
```

**sympy [A]** time = 56.16, size = 187, normalized size = 1.70

$$\frac{3a\sqrt{c}\,d\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2}+\frac{ac\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{ac\sqrt{d}\,x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}+bc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)-\frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}+bd\left(\begin{cases}-\frac{\sqrt{c}}{2x^2} & \text{for } d=0\\ -\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)
```

```
[Out] 3*a*sqrt(c)*d*asinh(sqrt(c)*x/sqrt(d))/2 + a*c*sqrt(d)*x*sqrt(c*x**2/d + 1)
/2 - a*c*sqrt(d)*x/sqrt(c*x**2/d + 1) - a*d**(3/2)/(x*sqrt(c*x**2/d + 1)) +
 b*c**(3/2)*asinh(sqrt(c)*x/sqrt(d)) - b*c**2*x/(sqrt(d)*sqrt(c*x**2/d + 1)
) - b*c*sqrt(d)/(x*sqrt(c*x**2/d + 1)) + b*d*Piecewise((-sqrt(c)/(2*x**2),
Eq(d, 0)), (-(c + d/x**2)**(3/2)/(3*d), True))
```

## 3.602 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x} \, dx$

**Optimal**. Leaf size=76

$$ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-ac\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

**Rubi [A]** time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-ac\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] -(a*c*Sqrt[c + d/x^2]) - (a*(c + d/x^2)^(3/2))/3 - (b*(c + d/x^2)^(5/2))/(5
*d) + a*c^(3/2)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}(ac)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}\left(ac^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x,\right.\\
&= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{\left(ac^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\right.}{d}\\
&= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)
\end{aligned}
$$

**Mathematica [C]** time = 0.07, size = 90, normalized size = 1.18

$$
-\frac{\sqrt{c+\frac{d}{x^2}}\left(5ad^2x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{d}\right)+3b\left(cx^2+d\right)^2\sqrt{\frac{cx^2}{d}+1}\right)}{15dx^4\sqrt{\frac{cx^2}{d}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] -1/15*(Sqrt[c + d/x^2]*(3*b*(d + c*x^2)^2*Sqrt[1 + (c*x^2)/d] + 5*a*d^2*x^2
*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)/d)]))/(d*x^4*Sqrt[1 + (c*x^2
)/d])
```

**IntegrateAlgebraic [A]** time = 0.11, size = 96, normalized size = 1.26

$$
\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-20acdx^4-5ad^2x^2-3bc^2x^4-6bcdx^2-3bd^2\right)}{15dx^4}+ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-3*b*d^2 - 6*b*c*d*x^2 - 5*a*d^2*x^2 - 3*b*c^2*x^4
- 20*a*c*d*x^4))/(15*d*x^4) + a*c^(3/2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[
c]]
```

**fricas [A]** time = 0.43, size = 213, normalized size = 2.80

$$
\left[\frac{15\,ac^{\frac{3}{2}}dx^4\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(\left(3\,bc^2+20\,acd\right)x^4+3\,bd^2+\left(6\,bcd+5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,dx^4}, -\frac{15\,a\sqrt{-c}\,cdx^4\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(\left(3\,bc^2+20\,acd\right)x^4+3\,bd^2+\left(6\,bcd+5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,dx^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2
) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*s
qrt((c*x^2 + d)/x^2))/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(-c)
*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d
^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^4)]
```

**giac** [B] time = 1.07, size = 254, normalized size = 3.34

$$-\frac{1}{2}\,ac^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\mathrm{sgn}(x)+\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d\,\mathrm{sgn}(x)-90\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d^2\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}d^2\mathrm{sgn}(x)+110\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d^3\mathrm{sgn}(x)-70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2ac^{\frac{3}{2}}d^4\mathrm{sgn}(x)+3\,bc^{\frac{5}{2}}d^4\mathrm{sgn}(x)+20\,ac^{\frac{3}{2}}d^5\mathrm{sgn}(x)\right)}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/15*(15*(sqrt
(c)*x - sqrt(c*x^2 + d))^8*b*c^(5/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 +
d))^8*a*c^(3/2)*d*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2)*d^2
*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d^2*sgn(x) + 110*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + d))^2*a*c^(3/2)*d^4*sgn(x) + 3*b*c^(5/2)*d^4*sgn(x) + 20*a*c^(3/2)*d^
5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

**maple** [B] time = 0.07, size = 153, normalized size = 2.01

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(15ac^2d^2x^5\ln\left(\sqrt{c}\,x+\sqrt{cx^2+d}\right)+15\sqrt{cx^2+d}\,ac^{\frac{5}{2}}dx^6+10\left(cx^2+d\right)^{\frac{3}{2}}ac^{\frac{5}{2}}x^6-10\left(cx^2+d\right)^{\frac{5}{2}}ac^{\frac{3}{2}}x^4-5\left(cx^2+d\right)^{\frac{5}{2}}a\sqrt{c}\,dx^2-3\left(cx^2+d\right)^{\frac{5}{2}}b\sqrt{c}\,d\right)}{15\left(cx^2+d\right)^{\frac{3}{2}}\sqrt{c}\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(3/2)*(10*c^(5/2)*(c*x^2+d)^(3/2)*x^6*a+15*c^(5/2)*(c*
x^2+d)^(1/2)*x^6*a*d-10*c^(3/2)*(c*x^2+d)^(5/2)*x^4*a+15*ln(c^(1/2)*x+(c*x^
2+d)^(1/2))*x^5*a*c^2*d^2-5*(c*x^2+d)^(5/2)*a*c^(1/2)*d*x^2-3*(c*x^2+d)^(5/
2)*b*c^(1/2)*d)/x^2/(c*x^2+d)^(3/2)/d^2/c^(1/2)
```

**maxima** [A] time = 1.31, size = 80, normalized size = 1.05

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}\,c\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/5*b*(c + d/x^2)^(5/2)/d - 1/6*(3*c^(3/2)*log((sqrt(c + d/x^2) - sqrt(c))
/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3/2) + 6*sqrt(c + d/x^2)*c)*
a
```

**mupad** [B] time = 5.83, size = 72, normalized size = 0.95

$$a\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{a\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{3}-a\,c\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}\,{\left(c\,x^2+d\right)}^2}{5\,d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x,x)
```

```
[Out] a*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (a*(c + d/x^2)^(3/2))/3 - a*c*
(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2)*(d + c*x^2)^2)/(5*d*x^4)
```

**sympy** [**A**] time = 54.14, size = 73, normalized size = 0.96

$$-\frac{ac^2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-ac\sqrt{c+\frac{d}{x^2}}-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*c**2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - a*c*sqrt(c + d/x**2) - a
*(c + d/x**2)**(3/2)/3 - b*(c + d/x**2)**(5/2)/(5*d)
```

## 3.603 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=46

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

**Rubi [A]** time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (b*(c + d/x^2)^(7/2))/(7*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int (a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 1.07

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(7adx^2-2bcx^2+5bd\right)}{35d^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

[Out] -1/35*(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(d^2*
x^6)

**IntegrateAlgebraic [A]** time = 0.06, size = 90, normalized size = 1.96

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-7ac^2dx^6-14acd^2x^4-7ad^3x^2+2bc^3x^6-bc^2dx^4-8bcd^2x^2-5bd^3\right)}{35d^2x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]

[Out] (Sqrt[(d + c*x^2)/x^2]*(-5*b*d^3 - 8*b*c*d^2*x^2 - 7*a*d^3*x^2 - b*c^2*d*x^
4 - 14*a*c*d^2*x^4 + 2*b*c^3*x^6 - 7*a*c^2*d*x^6))/(35*d^2*x^6)

**fricas [B]** time = 0.42, size = 84, normalized size = 1.83

$$\frac{\left(\left(2\,bc^3-7\,ac^2d\right)x^6-\left(bc^2d+14\,acd^2\right)x^4-5\,bd^3-\left(8\,bcd^2+7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] 1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (8
*b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^2*x^6)

**giac [B]** time = 1.87, size = 370, normalized size = 8.04

$$\frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}\mathrm{sgn}(x)-70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}d\,\mathrm{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}d\,\mathrm{sgn}(x)+105\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d^2\mathrm{sgn}(x)+140\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{7}{2}}d^2\mathrm{sgn}(x)-140\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{5}{2}}d^3\mathrm{sgn}(x)+28\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{7}{2}}d^3\mathrm{sgn}(x)+77\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{5}{2}}d^4\mathrm{sgn}(x)+14\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{7}{2}}d^4\mathrm{sgn}(x)-14\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{5}{2}}d^5\mathrm{sgn}(x)-2\,bc^{\frac{7}{2}}d^5\mathrm{sgn}(x)+7\,ac^{\frac{5}{2}}d^6\mathrm{sgn}(x)\right)}{35\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")

[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)*sgn(x) + 70*(sqrt(c)*x
- sqrt(c*x^2 + d))^10*b*c^(7/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*x^2 + d))^1
0*a*c^(5/2)*d*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*d*sgn(x
) + 105*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^2*sgn(x) + 140*(sqrt(c)
*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^2*sgn(x) - 140*(sqrt(c)*x - sqrt(c*x^2
+ d))^6*a*c^(5/2)*d^3*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)
*d^3*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^4*sgn(x) + 14*
(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*d^4*sgn(x) - 14*(sqrt(c)*x - sqrt
(c*x^2 + d))^2*a*c^(5/2)*d^5*sgn(x) - 2*b*c^(7/2)*d^5*sgn(x) + 7*a*c^(5/2)*
d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7

**maple [A]** time = 0.05, size = 48, normalized size = 1.04

$$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(7ad\,x^2-2bc\,x^2+5bd\right)\left(c\,x^2+d\right)}{35d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x)

[Out] -1/35*((c*x^2+d)/x^2)^(3/2)*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4

**maxima [A]** time = 0.59, size = 49, normalized size = 1.07

$$-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/5*a*(c + d/x^2)^(5/2)/d - 1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^
(5/2)*c/d^2)*b
```

**mupad [B]** time = 5.33, size = 122, normalized size = 2.65

$$\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{5\,d}-\frac{2\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{5\,x^2}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x)
```

```
[Out] (2*b*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (a*c^2*(c + d/x^2)^(1/2))/(5*d) - (2
*a*c*(c + d/x^2)^(1/2))/(5*x^2) - (a*d*(c + d/x^2)^(1/2))/(5*x^4) - (8*b*c*
(c + d/x^2)^(1/2))/(35*x^4) - (b*d*(c + d/x^2)^(1/2))/(7*x^6) - (b*c^2*(c +
 d/x^2)^(1/2))/(35*d*x^2)
```

**sympy [A]** time = 14.16, size = 138, normalized size = 3.00

$$-\frac{ac\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for } d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}-\frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d}-\frac{bc\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] -a*c*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True
))/2 - a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d - b*c*(-c*(c
+ d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d/x**2)**(3
/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
```

## 3.604 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx$

**Optimal**. Leaf size=74

$$\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

**Rubi [A]** time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(7/
2))/(7*d^3) - (b*(c + d/x^2)^(9/2))/(9*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx&=-\left(\frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)\\&=-\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{c(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{(-2bc+ad)(c+dx)^{5/2}}{d^2}+\frac{b(c+dx)^{7/2}}{d^2}\right)d\right.\right.\\&=-\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 71, normalized size = 0.96

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(9adx^2\left(2cx^2-5d\right)+b\left(-8c^2x^4+20cdx^2-35d^2\right)\right)}{315d^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 + 2
0*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 114, normalized size = 1.54

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(18ac^3dx^8-9ac^2d^2x^6-72acd^3x^4-45ad^4x^2-8bc^4x^8+4bc^3dx^6-3bc^2d^2x^4-50bcd^3x^2-35bd^4\right)}{315d^3x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-35*b*d^4 - 50*b*c*d^3*x^2 - 45*a*d^4*x^2 - 3*b*c^2
*d^2*x^4 - 72*a*c*d^3*x^4 + 4*b*c^3*d*x^6 - 9*a*c^2*d^2*x^6 - 8*b*c^4*x^8 +
 18*a*c^3*d*x^8))/(315*d^3*x^8)
```

**fricas [A]** time = 0.48, size = 109, normalized size = 1.47

$$-\frac{\left(2\left(4\,bc^4-9\,ac^3d\right)x^8-\left(4\,bc^3d-9\,ac^2d^2\right)x^6+35\,bd^4+3\left(bc^2d^2+24\,acd^3\right)x^4+5\left(10\,bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b*
d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2)/(d^3*x^8)
```

**giac [B]** time = 2.69, size = 430, normalized size = 5.81

$$\frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{7}{2}}\mathrm{sgn}(x)+840\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}d\,\mathrm{sgn}(x)+1260\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}d\,\mathrm{sgn}(x)+315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{7}{2}}d^2\mathrm{sgn}(x)+1764\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{9}{2}}d^2\mathrm{sgn}(x)-819\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{7}{2}}d^3\mathrm{sgn}(x)+504\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{9}{2}}d^3\mathrm{sgn}(x)+441\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{7}{2}}d^4\mathrm{sgn}(x)+144\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{9}{2}}d^4\mathrm{sgn}(x)-9\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{7}{2}}d^5\mathrm{sgn}(x)-36\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{9}{2}}d^5\mathrm{sgn}(x)+81\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{7}{2}}d^6\mathrm{sgn}(x)+4\,bc^{\frac{9}{2}}d^6\mathrm{sgn}(x)-9\,ac^{\frac{7}{2}}d^7\mathrm{sgn}(x)\right)}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(7/2)*sgn(x) + 840*(sqrt(c)
*x - sqrt(c*x^2 + d))^12*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 + d
))^12*a*c^(7/2)*d*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*
d*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(7/2)*d^2*sgn(x) + 1764
*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d^2*sgn(x) - 819*(sqrt(c)*x - sq
rt(c*x^2 + d))^8*a*c^(7/2)*d^3*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^6
*b*c^(9/2)*d^3*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(7/2)*d^4*s
gn(x) + 144*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(9/2)*d^4*sgn(x) - 9*(sqrt(
c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^5*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*b*c^(9/2)*d^5*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(7/2
)*d^6*sgn(x) + 4*b*c^(9/2)*d^6*sgn(x) - 9*a*c^(7/2)*d^7*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + d))^2 - d)^9
```

**maple [A]** time = 0.05, size = 70, normalized size = 0.95

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(18acd\,x^4-8b\,c^2x^4-45a\,d^2x^2+20bcd\,x^2-35b\,d^2\right)\left(c\,x^2+d\right)}{315d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*d
*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6
```

**maxima [A]** time = 0.64, size = 84, normalized size = 1.14

$$-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)a-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^(5/2)*c/d^2)*a - 1/315*(35*(
c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x^2)^(5/2)*c^
2/d^3)*b
```

**mupad [B]** time = 5.76, size = 164, normalized size = 2.22

$$\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{8\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{10\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4}+\frac{4\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x)
```

```
[Out] (2*a*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (8*b*c^4*(c + d/x^2)^(1/2))/(315*d^3
) - (8*a*c*(c + d/x^2)^(1/2))/(35*x^4) - (a*d*(c + d/x^2)^(1/2))/(7*x^6) -
(10*b*c*(c + d/x^2)^(1/2))/(63*x^6) - (b*d*(c + d/x^2)^(1/2))/(9*x^8) - (a*
c^2*(c + d/x^2)^(1/2))/(35*d*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) +
 (4*b*c^3*(c + d/x^2)^(1/2))/(315*d^2*x^2)
```

**sympy [B]** time = 15.61, size = 194, normalized size = 2.62

$$-\frac{ac\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2}-\frac{bc\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -a*c*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - a*(c**2*(c +
 d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
 - b*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**
2)**(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(
5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3
```

## 3.605 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx$

**Optimal**. Leaf size=104

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

**Rubi [A]** time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(7/2))/(7*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (b*(c + d/x^
2)^(11/2))/(11*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{c^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{5/2}}{d^3}+\frac{(-3bc+ad)(c-\right.\right.\right.\\ &= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}-\frac{b\left(c+\frac{d}{x^2}\right.}{11d}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 94, normalized size = 0.90

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(-11adx^2\left(8c^2x^4-20cdx^2+35d^2\right)-3b\left(-16c^3x^6+40c^2dx^4-70cd^2x^2+105d^3\right)\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x^
4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x
^10)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 138, normalized size = 1.33

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-88ac^4dx^{10}+44ac^3d^2x^8-33ac^2d^3x^6-550acd^4x^4-385ad^5x^2+48bc^5x^{10}-24bc^4dx^8+18bc^3d^2x^6-15bc^2d^3x^4-420bcd^4x^2-315bd^5\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-315*b*d^5 - 420*b*c*d^4*x^2 - 385*a*d^5*x^2 - 15*b
*c^2*d^3*x^4 - 550*a*c*d^4*x^4 + 18*b*c^3*d^2*x^6 - 33*a*c^2*d^3*x^6 - 24*b
*c^4*d*x^8 + 44*a*c^3*d^2*x^8 + 48*b*c^5*x^10 - 88*a*c^4*d*x^10))/(3465*d^4
*x^10)
```

**fricas [A]** time = 0.54, size = 134, normalized size = 1.29

$$\frac{\left(8\left(6bc^5-11ac^4d\right)x^{10}-4\left(6bc^4d-11ac^3d^2\right)x^8+3\left(6bc^3d^2-11ac^2d^3\right)x^6-315bd^5-5\left(3bc^2d^3+110acd^4\right)x^4-35\left(12bcd^4+11ad^5\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3465d^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 +
3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c*d
^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^10)
```

**giac [B]** time = 3.28, size = 490, normalized size = 4.71

$$\frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{9}{2}}\mathrm{sgn}(x)+6930\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{11}{2}}\mathrm{sgn}(x)-1155\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}d\,\mathrm{sgn}(x)+12474\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}d\,\mathrm{sgn}(x)+231\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{9}{2}}d^2\mathrm{sgn}(x)+15246\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{11}{2}}d^2\mathrm{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{9}{2}}d^3\mathrm{sgn}(x)+4950\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{11}{2}}d^3\mathrm{sgn}(x)+2475\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{9}{2}}d^4\mathrm{sgn}(x)+990\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{11}{2}}d^4\mathrm{sgn}(x)+495\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{9}{2}}d^5\mathrm{sgn}(x)-330\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{11}{2}}d^5\mathrm{sgn}(x)+605\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{9}{2}}d^6\mathrm{sgn}(x)+66\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{11}{2}}d^6\mathrm{sgn}(x)-121\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{9}{2}}d^7\mathrm{sgn}(x)-6bc^{\frac{11}{2}}d^7\mathrm{sgn}(x)+11ac^{\frac{9}{2}}d^8\mathrm{sgn}(x)\right)}{3465\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sqr
t(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c*x
^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*sgn(x
) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 4851*(sq
rt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x - sqrt
(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11/2)*d^4*
sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x) - 330*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c
^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9/2)*d^7*sgn(
x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + d))^2 - d)^11
```

**maple [A]** time = 0.05, size = 94, normalized size = 0.90

$$-\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(88ac^2dx^6-48bc^3x^6-220acd^2x^4+120bc^2dx^4+385ad^3x^2-210bcd^2x^2+315bd^3\right)\left(cx^2+d\right)}{3465d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x)
```

```
[Out] -1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4+
120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^8
```

**maxima [A]** time = 0.58, size = 118, normalized size = 1.13

$$-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)a-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*(35*(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x
^2)^(5/2)*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)
^(9/2)*c/d^4 + 495*(c + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^
4)*b
```

**mupad [B]** time = 6.31, size = 206, normalized size = 1.98

$$\frac{16\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{10\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{4\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4}+\frac{4\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x)
```

```
[Out] (16*b*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(315*
d^3) - (10*a*c*(c + d/x^2)^(1/2))/(63*x^6) - (a*d*(c + d/x^2)^(1/2))/(9*x^8
) - (4*b*c*(c + d/x^2)^(1/2))/(33*x^8) - (b*d*(c + d/x^2)^(1/2))/(11*x^10)
- (a*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) + (4*a*c^3*(c + d/x^2)^(1/2))/(315*
d^2*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*b*c^3*(c + d/x^2)^(1/
2))/(385*d^2*x^4) - (8*b*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2)
```

**sympy [B]** time = 17.96, size = 262, normalized size = 2.52

$$-\frac{ac\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3}-\frac{bc\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{b\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -a*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)
**(7/2)/7)/d**3 - a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/
2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3 - b*c*(-c**3
*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(
7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)/3 - 4*c*
*3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)*
*(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4
```

## 3.606 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

**Optimal**. Leaf size=134

$$-\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right.}{13d}$$

**Rubi [A]** time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(7/2))/(7*d^5) - (c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5) + ((4*b*c
 - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (b*(c + d/x^2)^(13/2))/(13*d^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=-\left(\frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$=-\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{c^3(bc-ad)(c+dx)^{3/2}}{d^4}-\frac{c^2(4bc-3ad)(c+dx)^{5/2}}{d^4}+\frac{3c(2bc-ad)}{d^4}\right.\right.\right.$$

$$=-\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}-\frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5}+$$

**Mathematica [A]** time = 0.06, size = 115, normalized size = 0.86

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(13adx^2\left(16c^3x^6-40c^2dx^4+70cd^2x^2-105d^3\right)+b\left(-128c^4x^8+320c^3dx^6-560c^2d^2x^4+840cd^3x^2-1155d^4\right)\right)}{15015d^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c^
2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 + 32
0*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 162, normalized size = 1.21

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(208ac^5dx^{12}-104ac^4d^2x^{10}+78ac^3d^3x^8-65ac^2d^4x^6-1820acd^5x^4-1365ad^6x^2-128bc^6x^{12}+64bc^5dx^{10}-48bc^4d^2x^8+40bc^3d^3x^6-35bc^2d^4x^4-1470bcd^5x^2-1155bd^6\right)}{15015d^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-1155*b*d^6 - 1470*b*c*d^5*x^2 - 1365*a*d^6*x^2 - 3
5*b*c^2*d^4*x^4 - 1820*a*c*d^5*x^4 + 40*b*c^3*d^3*x^6 - 65*a*c^2*d^4*x^6 -
48*b*c^4*d^2*x^8 + 78*a*c^3*d^3*x^8 + 64*b*c^5*d*x^10 - 104*a*c^4*d^2*x^10
- 128*b*c^6*x^12 + 208*a*c^5*d*x^12))/(15015*d^5*x^12)
```

**fricas [A]** time = 0.61, size = 157, normalized size = 1.17

$$-\frac{\left(16\left(8bc^6-13ac^5d\right)x^{12}-8\left(8bc^5d-13ac^4d^2\right)x^{10}+6\left(8bc^4d^2-13ac^3d^3\right)x^8+1155bd^6-5\left(8bc^3d^3-13ac^2d^4\right)x^6+35\left(bc^2d^4+52acd^5\right)x^4+105\left(14bcd^5+13ad^6\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15015d^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^12 - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^1
0 + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13*a
*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a*d^
6)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^12)
```

**giac [B]** time = 4.63, size = 550, normalized size = 4.10

$$\frac{32\left(15015\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{18}ac^{\frac{11}{2}}\mathrm{sgn}(x)+48048\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}bc^{\frac{13}{2}}\mathrm{sgn}(x)-3003\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{11}{2}}d\,\mathrm{sgn}(x)+96096\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{13}{2}}d\,\mathrm{sgn}(x)-6006\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{11}{2}}d^2\mathrm{sgn}(x)+109824\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{13}{2}}d^2\mathrm{sgn}(x)-28314\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{11}{2}}d^3\mathrm{sgn}(x)+37752\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{13}{2}}d^3\mathrm{sgn}(x)+13728\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{11}{2}}d^4\mathrm{sgn}(x)+5720\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{13}{2}}d^4\mathrm{sgn}(x)+5720\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{11}{2}}d^5\mathrm{sgn}(x)-2288\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}bc^{\frac{13}{2}}d^5\mathrm{sgn}(x)+3718\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{6}ac^{\frac{11}{2}}d^6\mathrm{sgn}(x)+624\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}bc^{\frac{13}{2}}d^6\mathrm{sgn}(x)-1014\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{4}ac^{\frac{11}{2}}d^7\mathrm{sgn}(x)-104\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}bc^{\frac{13}{2}}d^7\mathrm{sgn}(x)+169\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{2}ac^{\frac{11}{2}}d^8\mathrm{sgn}(x)+8bc^{\frac{13}{2}}d^8\mathrm{sgn}(x)-13ac^{\frac{11}{2}}d^9\mathrm{sgn}(x)\right)}{15015\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + d))^18*a*c^(11/2)*sgn(x) + 48048*
(sqrt(c)*x - sqrt(c*x^2 + d))^16*b*c^(13/2)*sgn(x) - 3003*(sqrt(c)*x - sqrt
(c*x^2 + d))^16*a*c^(11/2)*d*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + d))^1
4*b*c^(13/2)*d*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(11/2)*d^
2*sgn(x) + 109824*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(13/2)*d^2*sgn(x) -
28314*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(11/2)*d^3*sgn(x) + 37752*(sqrt(
c)*x - sqrt(c*x^2 + d))^10*b*c^(13/2)*d^3*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + d))^10*a*c^(11/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*b*c^(13/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(11/2)*d^
5*sgn(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(13/2)*d^5*sgn(x) + 371
8*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(11/2)*d^6*sgn(x) + 624*(sqrt(c)*x -
sqrt(c*x^2 + d))^4*b*c^(13/2)*d^6*sgn(x) - 1014*(sqrt(c)*x - sqrt(c*x^2 + d
))^4*a*c^(11/2)*d^7*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(13/2)
*d^7*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(11/2)*d^8*sgn(x) + 8
*b*c^(13/2)*d^8*sgn(x) - 13*a*c^(11/2)*d^9*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + d))^2 - d)^13
```

**maple [A]** time = 0.05, size = 118, normalized size = 0.88

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(208ac^3dx^8-128bc^4x^8-520ac^2d^2x^6+320bc^3dx^6+910acd^3x^4-560bc^2d^2x^4-1365ad^4x^2+840bcd^3x^2-1155bd^4\right)\left(cx^2+d\right)}{15015d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2*
x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*b*
c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^10
```

**maxima [A]** time = 0.63, size = 152, normalized size = 1.13

$$-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a-\frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-\frac{5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}c}{d^5}+\frac{10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c^2}{d^5}-\frac{8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^3}{d^5}+\frac{3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4}{d^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)^(9/2)*c/d^4 + 495*(c
+ d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^4)*a - 1/15015*(1155*(
c + d/x^2)^(13/2)/d^5 - 5460*(c + d/x^2)^(11/2)*c/d^5 + 10010*(c + d/x^2)^(
9/2)*c^2/d^5 - 8580*(c + d/x^2)^(7/2)*c^3/d^5 + 3003*(c + d/x^2)^(5/2)*c^4/
d^5)*b
```

**mupad [B]** time = 6.81, size = 248, normalized size = 1.85

$$\frac{16\,a\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{128\,b\,c^6\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^5}-\frac{4\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{14\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{143\,x^{10}}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{13\,x^{12}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{429\,d\,x^8}+\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{3003\,d^2\,x^6}-\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{5005\,d^3\,x^4}+\frac{64\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x)
```

```
[Out] (16*a*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (128*b*c^6*(c + d/x^2)^(1/2))/(15
015*d^5) - (4*a*c*(c + d/x^2)^(1/2))/(33*x^8) - (a*d*(c + d/x^2)^(1/2))/(11
*x^10) - (14*b*c*(c + d/x^2)^(1/2))/(143*x^10) - (b*d*(c + d/x^2)^(1/2))/(1
3*x^12) - (a*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*a*c^3*(c + d/x^2)^(1/2
))/(385*d^2*x^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2) - (b*c^2*(c +
 d/x^2)^(1/2))/(429*d*x^8) + (8*b*c^3*(c + d/x^2)^(1/2))/(3003*d^2*x^6) - (
16*b*c^4*(c + d/x^2)^(1/2))/(5005*d^3*x^4) + (64*b*c^5*(c + d/x^2)^(1/2))/(
15015*d^4*x^2)
```

**sympy [B]** time = 19.45, size = 326, normalized size = 2.43

$$-\frac{ac\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{a\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^4}-\frac{bc\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5}-\frac{b\left(-\frac{c^5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+c^4\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}-\frac{10c^3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{10c^2\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}-\frac{5c\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{13}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -a*c*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c +
 d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - a*(c**4*(c + d/x**2)**(3/
2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c
 + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4 - b*c*(c**4*(c + d/x**2
)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 -
4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5 - b*(-c**5*(c + d
/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(5/2) - 10*c**3*(c + d/x**2)**(7/2)/7
+ 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c*(c + d/x**2)**(11/2)/11 + (c + d/x**2
)**(13/2)/13)/d**5
```

## 3.607 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx$

**Optimal**. Leaf size=150

$$-\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5}+\frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4}-\frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{429c^3}+\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}(\cdots}{143c\cdots}$$

**Rubi [A]** time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4}-\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5}+\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{143c^2}-\frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{429c^3}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (-16*d^3*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^5)/(15015*c^5) + (8*d^2*(13*b
*c - 8*a*d)*(c + d/x^2)^(5/2)*x^7)/(3003*c^4) - (2*d*(13*b*c - 8*a*d)*(c +
d/x^2)^(5/2)*x^9)/(429*c^3) + ((13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^11)/(14
3*c^2) + (a*(c + d/x^2)^(5/2)*x^13)/(13*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} + \frac{(13bc-8ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx}{13c}$$

$$= \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} - \frac{(6d(13bc-8ad))\int \left(c+\frac{d}{x^2}\right) \cdots}{143c^2}$$

$$= -\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{9}}{429c^3} + \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} \cdots}{13c}$$

$$= \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{7}}{3003c^4} - \frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{9}}{429c^3} + \cdots$$

$$= -\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{5}}{15015c^5} + \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{7}}{3003c^4} - \cdots$$

**Mathematica [A]** time = 0.11, size = 110, normalized size = 0.73

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(a\left(1155c^4x^8-840c^3dx^6+560c^2d^2x^4-320cd^3x^2+128d^4\right)+13bc\left(105c^3x^6-70c^2dx^4+40cd^2x^2-16d^3\right)\right)}{15015c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d
*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*c^
3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)
```

**IntegrateAlgebraic [A]** time = 0.13, size = 114, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(1155ac^4x^8-840ac^3dx^6+560ac^2d^2x^4-320acd^3x^2+128ad^4+1365bc^4x^6-910bc^3dx^4+520bc^2d^2x^2-208bcd^3\right)}{15015c^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(-208*b*c*d^3 + 128*a*d^4 + 520*b*c^2*d^2*
x^2 - 320*a*c*d^3*x^2 - 910*b*c^3*d*x^4 + 560*a*c^2*d^2*x^4 + 1365*b*c^4*x^
6 - 840*a*c^3*d*x^6 + 1155*a*c^4*x^8))/(15015*c^5)
```

**fricas [A]** time = 0.42, size = 155, normalized size = 1.03

$$\frac{\left(1155\,ac^6x^{13}+105\left(13\,bc^6+14\,ac^5d\right)x^{11}+35\left(52\,bc^5d+ac^4d^2\right)x^9+5\left(13\,bc^4d^2-8\,ac^3d^3\right)x^7-6\left(13\,bc^3d^3-8\,ac^2d^4\right)x^5+8\left(13\,bc^2d^4-8\,acd^5\right)x^3-16\left(13\,bcd^5-8\,ad^6\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*a*c^6*x^13 + 105*(13*b*c^6 + 14*a*c^5*d)*x^11 + 35*(52*b*c^5*
d + a*c^4*d^2)*x^9 + 5*(13*b*c^4*d^2 - 8*a*c^3*d^3)*x^7 - 6*(13*b*c^3*d^3 -
 8*a*c^2*d^4)*x^5 + 8*(13*b*c^2*d^4 - 8*a*c*d^5)*x^3 - 16*(13*b*c*d^5 - 8*a
*d^6)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

**giac [A]** time = 0.18, size = 175, normalized size = 1.17

$$\frac{16\left(13\,bcd^{\frac{11}{2}}-8\,ad^{\frac{13}{2}}\right)\mathrm{sgn}(x)}{15015\,c^5}+\frac{1155\left(cx^2+d\right)^{\frac{13}{2}}a\,\mathrm{sgn}(x)+1365\left(cx^2+d\right)^{\frac{11}{2}}bc\,\mathrm{sgn}(x)-5460\left(cx^2+d\right)^{\frac{11}{2}}ad\,\mathrm{sgn}(x)-5005\left(cx^2+d\right)^{\frac{9}{2}}bcd\,\mathrm{sgn}(x)+10010\left(cx^2+d\right)^{\frac{9}{2}}ad^2\mathrm{sgn}(x)+6435\left(cx^2+d\right)^{\frac{7}{2}}bcd^2\mathrm{sgn}(x)-8580\left(cx^2+d\right)^{\frac{7}{2}}ad^3\mathrm{sgn}(x)-3003\left(cx^2+d\right)^{\frac{5}{2}}bcd^3\mathrm{sgn}(x)+3003\left(cx^2+d\right)^{\frac{5}{2}}ad^4\mathrm{sgn}(x)}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")
```

```
[Out] 16/15015*(13*b*c*d^(11/2) - 8*a*d^(13/2))*sgn(x)/c^5 + 1/15015*(1155*(c*x^2
 + d)^(13/2)*a*sgn(x) + 1365*(c*x^2 + d)^(11/2)*b*c*sgn(x) - 5460*(c*x^2 +
d)^(11/2)*a*d*sgn(x) - 5005*(c*x^2 + d)^(9/2)*b*c*d*sgn(x) + 10010*(c*x^2 +
 d)^(9/2)*a*d^2*sgn(x) + 6435*(c*x^2 + d)^(7/2)*b*c*d^2*sgn(x) - 8580*(c*x^
2 + d)^(7/2)*a*d^3*sgn(x) - 3003*(c*x^2 + d)^(5/2)*b*c*d^3*sgn(x) + 3003*(c
*x^2 + d)^(5/2)*a*d^4*sgn(x))/c^5
```

**maple** [A] time = 0.05, size = 115, normalized size = 0.77

$$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(1155a\,x^{8}c^{4}-840a\,c^{3}d\,x^{6}+1365b\,c^{4}x^{6}+560a\,c^{2}d^{2}x^{4}-910b\,c^{3}d\,x^{4}-320ac\,d^{3}x^{2}+520b\,c^{2}d^{2}x^{2}+128a\,d^{4}-208bc\,d^{3}\right)\left(c\,x^{2}+d\right)x^{3}}{15015c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*x^3*(1155*a*c^4*x^8-840*a*c^3*d*x^6+1365*b*c^
4*x^6+560*a*c^2*d^2*x^4-910*b*c^3*d*x^4-320*a*c*d^3*x^2+520*b*c^2*d^2*x^2+1
28*a*d^4-208*b*c*d^3)*(c*x^2+d)/c^5
```

**maxima** [A] time = 0.63, size = 158, normalized size = 1.05

$$\frac{\left(105\left(c+\frac{d}{x^{2}}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^{2}}\right)^{\frac{9}{2}}dx^{9}+495\left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}}d^{2}x^{7}-231\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}d^{3}x^{5}\right)b}{1155\,c^{4}}+\frac{\left(1155\left(c+\frac{d}{x^{2}}\right)^{\frac{13}{2}}x^{13}-5460\left(c+\frac{d}{x^{2}}\right)^{\frac{11}{2}}dx^{11}+10010\left(c+\frac{d}{x^{2}}\right)^{\frac{9}{2}}d^{2}x^{9}-8580\left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}}d^{3}x^{7}+3003\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}d^{4}x^{5}\right)a}{15015\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")
```

```
[Out] 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c
+ d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(11
55*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c + d/
x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^(5/2
)*d^4*x^5)*a/c^5
```

**mupad** [B] time = 4.66, size = 137, normalized size = 0.91

$$\sqrt{c+\frac{d}{x^{2}}}\left(\frac{x\left(128\,a\,d^{6}-208\,b\,c\,d^{5}\right)}{15015\,c^{5}}+\frac{x^{11}\left(1365\,b\,c^{6}+1470\,a\,d\,c^{5}\right)}{15015\,c^{5}}+\frac{a\,c\,x^{13}}{13}+\frac{d\,x^{9}\,\left(a\,d+52\,b\,c\right)}{429\,c}-\frac{d^{2}\,x^{7}\,\left(8\,a\,d-13\,b\,c\right)}{3003\,c^{2}}+\frac{2\,d^{3}\,x^{5}\,\left(8\,a\,d-13\,b\,c\right)}{5005\,c^{3}}-\frac{8\,d^{4}\,x^{3}\,\left(8\,a\,d-13\,b\,c\right)}{15015\,c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(128*a*d^6 - 208*b*c*d^5))/(15015*c^5) + (x^11*(1365*
b*c^6 + 1470*a*c^5*d))/(15015*c^5) + (a*c*x^13)/13 + (d*x^9*(a*d + 52*b*c))
/(429*c) - (d^2*x^7*(8*a*d - 13*b*c))/(3003*c^2) + (2*d^3*x^5*(8*a*d - 13*b
*c))/(5005*c^3) - (8*d^4*x^3*(8*a*d - 13*b*c))/(15015*c^4))
```

**sympy** [B] time = 12.60, size = 3351, normalized size = 22.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)
```

```
[Out] 693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45
045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 4504
5*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt(c*
x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d
```

```
**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30
) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10
 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 +
 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x**16*sqr
t(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c*
*9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d
**30) + 315*a*c**9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d + 1)/(9009*c**
11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**
8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1295*a*c**8*d**(3
7/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 756*a
*c**7*d**(61/2)*x**12*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c*
*10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7
*d**29*x**2 + 9009*c**6*d**30) + 1990*a*c**7*d**(39/2)*x**14*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 63*a*c**6*d**(63/2)*x**10*sqrt
(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**
9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d*
*30) + 1358*a*c**6*d**(41/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) - 630*a*c**5*d**(65/2)*x**8*sqrt(c*x**2/d + 1)/(9009*c**11
*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*
d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 343*a*c**5*d**(43/2
)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 1680*a*c
**4*d**(67/2)*x**6*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10
*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d*
*29*x**2 + 9009*c**6*d**30) + 35*a*c**4*d**(45/2)*x**8*sqrt(c*x**2/d + 1)/(
3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 1386
0*c**6*d**19*x**2 + 3465*c**5*d**20) - 2016*a*c**3*d**(69/2)*x**4*sqrt(c*x*
*2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**
27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30)
+ 280*a*c**3*d**(47/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) - 1152*a*c**2*d**(71/2)*x**2*sqrt(c*x**2/d + 1)/(9009*c**11*d**25
*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*
x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 560*a*c**2*d**(49/2)*x**4
*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c
**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 256*a*c*d**(73/
2)*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90
090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*
c**6*d**30) + 448*a*c*d**(51/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 128*a*d**(53/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 315*b*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c
**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6
*d**19*x**2 + 3465*c**5*d**20) + 1295*b*c**9*d**(35/2)*x**16*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1990*b*c**8*d**(37/2)*x**14*sq
rt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7
*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*b*c**7*d**(39
/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
+ 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*b*c
**7*d**(21/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10
*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 343*b*c**6*d**(41/2)*x**10*
sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c*
```

```
*7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*b*c**6*d**(2
3/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 9
45*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**5*d**(43/2)*x**8*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 114*b*c**5*d**(25/2)*x**10*
sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**
11*x**2 + 315*c**4*d**12) + 280*b*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 40*b*c**4*d**(27/2)*x**8*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) + 560*b*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**
16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*
x**2 + 3465*c**5*d**20) - 5*b*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 448*b*c**2*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 138
60*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c
**5*d**20) - 30*b*c**2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**
6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*b*c*d
**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*b*c*
d**(33/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4
 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*b*d**(35/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12)
```

## 3.608 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx$

**Optimal**. Leaf size=117

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

**Rubi** [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (8*d^2*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^5)/(3465*c^4) - (4*d*(11*b*c -
6*a*d)*(c + d/x^2)^(5/2)*x^7)/(693*c^3) + ((11*b*c - 6*a*d)*(c + d/x^2)^(5/
2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(5/2)*x^11)/(11*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c} + \frac{(11bc-6ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx}{11c}$$

$$= \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c} - \frac{(4d(11bc-6ad))\int \left(c+\frac{d}{x^2}\right)^{3/2} x}{99c^2}$$

$$= -\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c}$$

$$= \frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{3465c^4} - \frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)(c}{99c^2}$$

**Mathematica** [A] time = 0.09, size = 89, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(3a\left(105c^3x^6-70c^2dx^4+40cd^2x^2-16d^3\right)+11bc\left(35c^2x^4-20cdx^2+8d^2\right)\right)}{3465c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)
+ 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)
```

**IntegrateAlgebraic** [A] time = 0.10, size = 90, normalized size = 0.77

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(315ac^3x^6-210ac^2dx^4+120acd^2x^2-48ad^3+385bc^3x^4-220bc^2dx^2+88bcd^2\right)}{3465c^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(88*b*c*d^2 - 48*a*d^3 - 220*b*c^2*d*x^2 +
 120*a*c*d^2*x^2 + 385*b*c^3*x^4 - 210*a*c^2*d*x^4 + 315*a*c^3*x^6))/(3465*
c^4)
```

**fricas** [A] time = 0.44, size = 132, normalized size = 1.13

$$\frac{\left(315\,ac^5x^{11}+35\left(11\,bc^5+12\,ac^4d\right)x^9+5\left(110\,bc^4d+3\,ac^3d^2\right)x^7+3\left(11\,bc^3d^2-6\,ac^2d^3\right)x^5-4\left(11\,bc^2d^3-6\,acd^4\right)x^3+8\left(11\,bcd^4-6\,ad^5\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3465\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d +
3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 - 6
*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4
```

**giac** [A] time = 0.18, size = 140, normalized size = 1.20

$$-\frac{8\left(11\,bcd^{\frac{9}{2}}-6\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^4}+\frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\,\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\,\mathrm{sgn}(x)-1155\left(cx^2+d\right)^{\frac{9}{2}}ad\,\mathrm{sgn}(x)-990\left(cx^2+d\right)^{\frac{7}{2}}bcd\,\mathrm{sgn}(x)+1485\left(cx^2+d\right)^{\frac{7}{2}}ad^2\,\mathrm{sgn}(x)+693\left(cx^2+d\right)^{\frac{5}{2}}bcd^2\,\mathrm{sgn}(x)-693\left(cx^2+d\right)^{\frac{5}{2}}ad^3\,\mathrm{sgn}(x)}{3465\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")
```

```
[Out] -8/3465*(11*b*c*d^(9/2) - 6*a*d^(11/2))*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + d
)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1155*(c*x^2 + d)^(9/
2)*a*d*sgn(x) - 990*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 1485*(c*x^2 + d)^(7/2)
*a*d^2*sgn(x) + 693*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 693*(c*x^2 + d)^(5/2
)*a*d^3*sgn(x))/c^4
```

**maple** [A] time = 0.05, size = 91, normalized size = 0.78

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(315a\,x^6c^3-210a\,c^2d\,x^4+385b\,c^3x^4+120ac\,d^2x^2-220b\,c^2d\,x^2-48a\,d^3+88bc\,d^2\right)\left(c\,x^2+d\right)x^3}{3465c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*x
^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4
```

**maxima** [A] time = 0.49, size = 124, normalized size = 1.06

$$\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315\,c^3}+\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)a}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^
2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/
x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^
3*x^5)*a/c^4
```

**mupad** [B] time = 4.57, size = 118, normalized size = 1.01

$$\sqrt{c+\frac{d}{x^2}}\left(\frac{x^9\left(385\,b\,c^5+420\,a\,d\,c^4\right)}{3465\,c^4}-\frac{x\left(48\,a\,d^5-88\,b\,c\,d^4\right)}{3465\,c^4}+\frac{a\,c\,x^{11}}{11}+\frac{d\,x^7\,\left(3\,a\,d+110\,b\,c\right)}{693\,c}-\frac{d^2\,x^5\,\left(6\,a\,d-11\,b\,c\right)}{1155\,c^2}+\frac{4\,d^3\,x^3\,\left(6\,a\,d-11\,b\,c\right)}{3465\,c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x^9*(385*b*c^5 + 420*a*c^4*d))/(3465*c^4) - (x*(48*a*d^
5 - 88*b*c*d^4))/(3465*c^4) + (a*c*x^11)/11 + (d*x^7*(3*a*d + 110*b*c))/(69
3*c) - (d^2*x^5*(6*a*d - 11*b*c))/(1155*c^2) + (4*d^3*x^3*(6*a*d - 11*b*c))
/(3465*c^3))
```

**sympy** [B] time = 9.65, size = 2304, normalized size = 19.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)
```

```
[Out] 315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16
*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x*
*2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(346
5*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c
**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2)*x**14*s
```

```
qrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**1
1*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**12*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d*
*16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19
*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt(c*x**2/d + 1)/(3
15*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d*
*12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 +
 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 34
65*c**5*d**20) + 40*a*c**4*d**(27/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9
*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 560*a
*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*
d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**2
0) - 5*a*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 448*a*c**2*d**(49/
2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 30*a*c**
2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x*
*4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*a*c*d**(51/2)*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*a*c*d**(33/2)*x**2*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 16*a*d**(35/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**8*
d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*b*c**7*d**(21/2)*x**12*sqrt
(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x
**2 + 315*c**4*d**12) + 114*b*c**6*d**(23/2)*x**10*sqrt(c*x**2/d + 1)/(315*
c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12
) + 40*b*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(11/2
)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) - 5*b*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
+ 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 33*b*c**4*d
**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 +
 105*c**3*d**6) - 30*b*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**
9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 17*b
*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5
*x**2 + 105*c**3*d**6) - 40*b*c**2*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 3*b*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) - 16*b*c*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**
7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) +
 12*b*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d*
*5*x**2 + 105*c**3*d**6) + 8*b*d**(21/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

## 3.609 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx$

**Optimal**. Leaf size=84

$$-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{315c^3}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{63c^2}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

**Rubi [A]** time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{63c^2}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{315c^3}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (-2*d*(9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^5)/(315*c^3) + ((9*b*c - 4*a*d)*(
c + d/x^2)^(5/2)*x^7)/(63*c^2) + (a*(c + d/x^2)^(5/2)*x^9)/(9*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}+\frac{(9bc-4ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx}{9c}\\
&= \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{63c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}-\frac{(2d(9bc-4ad))\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4}{63c^2}\\
&= -\frac{2d(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{315c^3}+\frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{63c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 66, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(a\left(35c^2x^4-20cdx^2+8d^2\right)+9bc\left(5cx^2-2d\right)\right)}{315c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c*
d*x^2 + 35*c^2*x^4)))/(315*c^3)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 66, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(35ac^2x^4-20acdx^2+8ad^2+45bc^2x^2-18bcd\right)}{315c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(-18*b*c*d + 8*a*d^2 + 45*b*c^2*x^2 - 20*a
*c*d*x^2 + 35*a*c^2*x^4))/(315*c^3)
```

**fricas [A]** time = 0.43, size = 106, normalized size = 1.26

$$\frac{\left(35\,ac^4x^9+5\left(9\,bc^4+10\,ac^3d\right)x^7+3\left(24\,bc^3d+ac^2d^2\right)x^5+\left(9\,bc^2d^2-4\,acd^3\right)x^3-2\left(9\,bcd^3-4\,ad^4\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2*
d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^3
```

**giac [A]** time = 0.17, size = 105, normalized size = 1.25

$$\frac{2\left(9\,bcd^{\frac{7}{2}}-4\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^3}+\frac{35\left(cx^2+d\right)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\left(cx^2+d\right)^{\frac{7}{2}}bc\mathrm{sgn}(x)-90\left(cx^2+d\right)^{\frac{7}{2}}ad\mathrm{sgn}(x)-63\left(cx^2+d\right)^{\frac{5}{2}}bcd\mathrm{sgn}(x)+63\left(cx^2+d\right)^{\frac{5}{2}}ad^2\mathrm{sgn}(x)}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] 2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3 + 1/315*(35*(c*x^2 + d)^(9/2
)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 90*(c*x^2 + d)^(7/2)*a*d*sgn
(x) - 63*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 63*(c*x^2 + d)^(5/2)*a*d^2*sgn(x)
)/c^3
```

**maple [A]** time = 0.06, size = 67, normalized size = 0.80

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(35a\,x^4c^2-20acd\,x^2+45b\,c^2x^2+8a\,d^2-18bcd\right)\left(c\,x^2+d\right)x^3}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*a
*d^2-18*b*c*d)*(c*x^2+d)/c^3
```

**maxima** [A] time = 0.76, size = 90, normalized size = 1.07

$$\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35\,c^2}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(3
5*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/2)
*d^2*x^5)*a/c^3
```

**mupad** [B] time = 4.55, size = 97, normalized size = 1.15

$$\sqrt{c+\frac{d}{x^2}}\left(\frac{x\left(8\,a\,d^4-18\,b\,c\,d^3\right)}{315\,c^3}+\frac{x^7\left(45\,b\,c^4+50\,a\,d\,c^3\right)}{315\,c^3}+\frac{a\,c\,x^9}{9}+\frac{d\,x^5\,\left(a\,d+24\,b\,c\right)}{105\,c}-\frac{d^2\,x^3\,\left(4\,a\,d-9\,b\,c\right)}{315\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(8*a*d^4 - 18*b*c*d^3))/(315*c^3) + (x^7*(45*b*c^4 +
50*a*c^3*d))/(315*c^3) + (a*c*x^9)/9 + (d*x^5*(a*d + 24*b*c))/(105*c) - (d^
2*x^3*(4*a*d - 9*b*c))/(315*c^2))
```

**sympy** [B] time = 7.64, size = 1340, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**8,x)
```

```
[Out] 35*a*c**8*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*a*c**
5*d**(11/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x*
*2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*
d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 3
3*a*c**4*d**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d
**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(31
5*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**
12) + 17*a*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210
*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*sqrt(c*x**2/d +
 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c
**4*d**12) + 3*a*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) + 12*a*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 2
10*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(21/2)*sqrt(c*x**2/d + 1)/(105*
c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 15*b*c**6*d**(9/2)*x
**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + 33*b*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**4*d**(13/2)*x**6*sqrt(c*x**2/
d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**3
*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2
```

```
 + 105*c**3*d**6) + 12*b*c**2*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*b*c*d**(19/2)*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + b*d**(
3/2)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(5/2)*x**2*sqrt(c*x**2/d + 1)/(15*c)
- 2*b*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

## 3.610 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx$

**Optimal**. Leaf size=53

$$\frac{x^5 \left(c+\frac{d}{x^2}\right)^{5/2} (7bc-2ad)}{35c^2}+\frac{ax^7 \left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{x^5 \left(c+\frac{d}{x^2}\right)^{5/2} (7bc-2ad)}{35c^2}+\frac{ax^7 \left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] ((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^7
)/(7*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx &= \frac{a \left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}+\frac{(7bc-2ad) \int \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx}{7c} \\
&= \frac{(7bc-2ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^5}{35c^2}+\frac{a \left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 44, normalized size = 0.83

$$\frac{x\sqrt{c+\frac{d}{x^2}} \left(cx^2+d\right)^2 \left(5acx^2-2ad+7bc\right)}{35c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

**IntegrateAlgebraic [A]** time = 0.08, size = 44, normalized size = 0.83

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(5acx^2-2ad+7bc\right)}{35c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

**fricas [A]** time = 0.41, size = 80, normalized size = 1.51

$$\frac{\left(5\,ac^3x^7+\left(7\,bc^3+8\,ac^2d\right)x^5+\left(14\,bc^2d+acd^2\right)x^3+\left(7\,bcd^2-2\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3
+ (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

**giac [A]** time = 0.18, size = 72, normalized size = 1.36

$$-\frac{\left(7\,bcd^{\frac{5}{2}}-2\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{35\,c^2}+\frac{5\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+7\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-7\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] -1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2 + 1/35*(5*(c*x^2 + d)^(7/2)*
a*sgn(x) + 7*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 7*(c*x^2 + d)^(5/2)*a*d*sgn(x))
/c^2
```

**maple [A]** time = 0.04, size = 45, normalized size = 0.85

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(5a\,x^2c-2ad+7bc\right)\left(c\,x^2+d\right)x^3}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x)
```

```
[Out] 1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2
```

**maxima [A]** time = 0.65, size = 55, normalized size = 1.04

$$\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)a}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^
2)^(5/2)*d*x^5)*a/c^2
```

**mupad [B]** time = 4.63, size = 77, normalized size = 1.45

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x^5\,\left(7\,b\,c^3+8\,a\,d\,c^2\right)}{35\,c^2}-\frac{x\,\left(2\,a\,d^3-7\,b\,c\,d^2\right)}{35\,c^2}+\frac{a\,c\,x^7}{7}+\frac{d\,x^3\,\left(a\,d+14\,b\,c\right)}{35\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x^5*(7*b*c^3 + 8*a*c^2*d))/(35*c^2) - (x*(2*a*d^3 - 7*b
*c*d^2))/(35*c^2) + (a*c*x^7)/7 + (d*x^3*(a*d + 14*b*c))/(35*c))
```

**sympy [B]** time = 5.95, size = 498, normalized size = 9.40

$$\frac{15ac^6d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{33ac^5d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^4d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{3ac^3d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{12ac^2d^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{8acd^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{ad^{\frac{3}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2ad^{\frac{7}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{bc\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)
```

```
[Out] 15*a*c**6*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**3*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*c*
d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105
*c**3*d**6) + a*d**(3/2)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*x**2*sqrt(c
*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*c*sqrt(
d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/5 + b*d
**(5/2)*sqrt(c*x**2/d + 1)/(5*c)
```

## 3.611 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx$

**Optimal**. Leaf size=86

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+bdx\sqrt{c+\frac{d}{x^2}}+\frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}$$

**Rubi [A]** time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {451, 335, 277, 217, 206}

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}+bdx\sqrt{c+\frac{d}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] b*d*Sqrt[c + d/x^2]*x + (b*(c + d/x^2)^(3/2)*x^3)/3 + (a*(c + d/x^2)^(5/2)*
x^5)/(5*c) - b*d^(3/2)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} + b\int \left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - b\,\text{Subst}\left(\int \frac{\left(c+dx^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - (bd)\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+d\!:}}\right.$$

$$= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\right.$$

$$= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right.$$

**Mathematica** [A] time = 0.15, size = 81, normalized size = 0.94

$$\frac{1}{15}x\sqrt{c+\frac{d}{x^2}}\left(\frac{3a\left(cx^2+d\right)^2}{c} - \frac{15bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{cx^2+d}} + 5b\left(cx^2+4d\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3/
2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/15
```

**IntegrateAlgebraic** [A] time = 0.12, size = 107, normalized size = 1.24

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\sqrt{cx^2+d}\left(3ac^2x^4+6acdx^2+3ad^2+5bc^2x^2+20bcd\right)}{15c} - bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(20*b*c*d + 3*a*d^2 + 5*b*c^2*x^2 + 6*
a*c*d*x^2 + 3*a*c^2*x^4))/(15*c) - b*d^(3/2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d
]]))/Sqrt[d + c*x^2]
```

**fricas** [A] time = 0.45, size = 203, normalized size = 2.36

$$\left[\frac{15\,bcd^{\frac{3}{2}}\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(3\,ac^2x^5+\left(5\,bc^2+6\,acd\right)x^3+\left(20\,bcd+3\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,c}, \frac{15\,bc\sqrt{-d}\,d\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(3\,ac^2x^5+\left(5\,bc^2+6\,acd\right)x^3+\left(20\,bcd+3\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d
)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*x)
*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqrt((
c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20
*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

**giac [A]** time = 0.20, size = 140, normalized size = 1.63

$$\frac{bd^2 \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\mathrm{sgn}(x)}{\sqrt{-d}} - \frac{\left(15\,bcd^2 \arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + 20\,bc\sqrt{-d}\,d^{\frac{3}{2}} + 3\,a\sqrt{-d}\,d^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c\sqrt{-d}} + \frac{3\left(cx^2+d\right)^{\frac{5}{2}}ac^4\mathrm{sgn}(x) + 5\left(cx^2+d\right)^{\frac{3}{2}}bc^5\mathrm{sgn}(x) + 15\sqrt{cx^2+d}\,bc^5 d\mathrm{sgn}(x)}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*a
rctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))*s
gn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 + d)
^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5
```

**maple [A]** time = 0.06, size = 99, normalized size = 1.15

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-15bc\,d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)+15\sqrt{c\,x^2+d}\;bcd+5\left(c\,x^2+d\right)^{\frac{3}{2}}bc+3\left(c\,x^2+d\right)^{\frac{5}{2}}a\right)x^3}{15\left(c\,x^2+d\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(3*a*(c*x^2+d)^(5/2)+5*(c*x^2+d)^(3/2)*b*c-1
5*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*b*c+15*(c*x^2+d)^(1/2)*b*c*d)
/(c*x^2+d)^(3/2)/c
```

**maxima [A]** time = 1.35, size = 91, normalized size = 1.06

$$\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}\,dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*a*(c + d/x^2)^(5/2)*x^5/c + 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d
/x^2)*d*x + 3*d^(3/2)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x
+ sqrt(d))))*b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+\frac{b}{x^2}\right)\,{\left(c+\frac{d}{x^2}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

**sympy [B]** time = 5.25, size = 184, normalized size = 2.14

$$\frac{ac\sqrt{d}\,x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}+\frac{b\sqrt{c}\,dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{bc\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-bd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{bd^2}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)
```

```
[Out] a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d + 1
)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x**
2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)
/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 + d/(
c*x**2)))
```

## 3.612 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx$

**Optimal**. Leaf size=121

$$\frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}\,(2ad+3bc)}{2cx}-\frac{1}{2}\sqrt{d}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

**Rubi [A]** time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 242, 277, 195, 217, 206}

$$\frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}\,(2ad+3bc)}{2cx}-\frac{1}{2}\sqrt{d}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] -(d*(3*b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + ((3*b*c + 2*a*d)*(c + d/x^2)
^(3/2)*x)/(3*c) + (a*(c + d/x^2)^(5/2)*x^3)/(3*c) - (Sqrt[d]*(3*b*c + 2*a*d
)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}+\frac{(3bc+2ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}dx}{3c}$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{(3bc+2ad)\,\text{Subst}\left(\int\frac{\left(c+dx^2\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)}{3c}$$

$$=\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{(d(3bc+2ad))\,\text{Subst}\left(\int\sqrt{c+d\right.}$$

$$=-\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{1}{2}(d(3$$

$$=-\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{1}{2}(d(3$$

$$=-\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{1}{2}\sqrt{d}\,($$

**Mathematica [A]** time = 0.09, size = 105, normalized size = 0.87

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{cx^2+d}\left(2acx^4+8adx^2+6bcx^2-3bd\right)-3\sqrt{d}\,x^2(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{6x\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*x
^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6*x
*Sqrt[d + c*x^2])
```

**IntegrateAlgebraic [A]** time = 0.18, size = 109, normalized size = 0.90

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{1}{2}\left(-2ad^{3/2}-3bc\sqrt{d}\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)+\frac{\sqrt{cx^2+d}\left(2acx^4+8adx^2+6bcx^2-3bd\right)}{6x^2}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*
c*x^4))/(6*x^2) + ((-3*b*c*Sqrt[d] - 2*a*d^(3/2))*ArcTanh[Sqrt[d + c*x^2]/S
qrt[d]])/2))/Sqrt[d + c*x^2]
```

**fricas [A]** time = 0.44, size = 190, normalized size = 1.57

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x},\ \frac{3\,(3\,bc+2\,ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 +
d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt((c
*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt
((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d
)*sqrt((c*x^2 + d)/x^2))/x]
```

**giac [A]** time = 0.27, size = 115, normalized size = 0.95

$$\frac{2\left(cx^2+d\right)^{\frac{3}{2}}ac\mathrm{sgn}(x)+6\,\sqrt{cx^2+d}\,bc^2\mathrm{sgn}(x)+6\,\sqrt{cx^2+d}\,acd\mathrm{sgn}(x)-\frac{3\,\sqrt{cx^2+d}\,bcd\mathrm{sgn}(x)}{x^2}+\frac{3\left(3\,bc^2d\mathrm{sgn}(x)+2\,acd^2\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + d)^(3/2)*a*c*sgn(x) + 6*sqrt(c*x^2 + d)*b*c^2*sgn(x) + 6*sq
rt(c*x^2 + d)*a*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d*sgn(x)/x^2 + 3*(3*b*c^
2*d*sgn(x) + 2*a*c*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d))/c
```

**maple [A]** time = 0.06, size = 170, normalized size = 1.40

$$-\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(6ad^{\frac{5}{2}}x^2\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)+9bcd^{\frac{3}{2}}x^2\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-6\sqrt{cx^2+d}\,ad^2x^2-9\sqrt{cx^2+d}\,bcdx^2-2\left(cx^2+d\right)^{\frac{3}{2}}adx^2-3\left(cx^2+d\right)^{\frac{3}{2}}bcx^2+3\left(cx^2+d\right)^{\frac{5}{2}}b\right)x}{6\left(cx^2+d\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x)
```

```
[Out] -1/6*((c*x^2+d)/x^2)^(3/2)*x*(-2*(c*x^2+d)^(3/2)*a*d*x^2-3*(c*x^2+d)^(3/2)*
b*c*x^2+6*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*a+9*d^(3/2)*ln(2*
(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*b*c+3*(c*x^2+d)^(5/2)*b-6*(c*x^2+d)^(1/2
)*x^2*a*d^2-9*(c*x^2+d)^(1/2)*x^2*b*c*d)/(c*x^2+d)^(3/2)/d
```

**maxima [A]** time = 1.36, size = 163, normalized size = 1.35

$$\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}\,dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a+\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}\,cx-\frac{2\sqrt{c+\frac{d}{x^2}}\,cdx}{\left(c+\frac{d}{x^2}\right)x^2-d}+3\,c\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d/x^2)*d*x + 3*d^(3/2)*log((sqrt(
c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*a + 1/4*(4*sqrt(c +
 d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d) + 3*c*sqrt(d)*l
og((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+\frac{b}{x^2}\right)\,{\left(c+\frac{d}{x^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

**sympy [A]** time = 7.65, size = 202, normalized size = 1.67

$$\frac{a\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{ac\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{ad^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{b\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)
```

```
[Out] a*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + a*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3
+ a*d**(3/2)*sqrt(c*x**2/d + 1)/3 - a*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) +
 a*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) + b*c**(3/2)*x/sqrt(1 + d/(c*x**2)
) - b*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x) + b*sqrt(c)*d/(x*sqrt(1 + d/(c*x
**2))) - 3*b*c*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))/2
```

## 3.613 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx$

**Optimal**. Leaf size=112

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}\,(4ad+bc)}{8x}-\frac{3c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

**Rubi** [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 453, 195, 217, 206}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}\,(4ad+bc)}{8x}-\frac{3c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (-3*(b*c + 4*a*d)*Sqrt[c + d/x^2])/(8*x) - ((b*c + 4*a*d)*(c + d/x^2)^(3/2)
)/(4*c*x) + (a*(c + d/x^2)^(5/2)*x)/c - (3*c*(b*c + 4*a*d)*ArcTanh[Sqrt[d]/
(Sqrt[c + d/x^2]*x)])/(8*Sqrt[d])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}+\frac{(-bc-4ad)\,\text{Subst}\left(\int \left(c+dx^2\right)^{3/2}\,dx,x,\frac{1}{x}\right)}{c}$$
$$=-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{1}{4}(3(bc+4ad))\,\text{Subst}\left(\int \sqrt{c+dx^2}\ d\right.$$
$$=-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{1}{8}(3c(bc+4a$$
$$=-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{1}{8}(3c(bc+4a$$
$$=-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{3c(bc+4ad)\,t}{8}$$

**Mathematica [C]** time = 0.03, size = 68, normalized size = 0.61

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(cx^4(4ad+bc)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{d}+1\right)-5bd^2\right)}{20d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^2 + c*(b*c + 4*a*d)*x^4*Hypergeometr
ic2F1[2, 5/2, 7/2, 1 + (c*x^2)/d]))/(20*d^3*x^3)
```

**IntegrateAlgebraic [A]** time = 0.22, size = 107, normalized size = 0.96

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\sqrt{cx^2+d}\left(8acx^4-4adx^2-5bcx^2-2bd\right)}{8x^4}-\frac{3\left(4acd+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{8\sqrt{d}}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-2*b*d - 5*b*c*x^2 - 4*a*d*x^2 + 8*a*
c*x^4))/(8*x^4) - (3*(b*c^2 + 4*a*c*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8
*Sqrt[d])))/Sqrt[d + c*x^2]
```

**fricas [A]** time = 0.44, size = 216, normalized size = 1.93

$$\left[\frac{3\left(bc^2+4\,acd\right)\sqrt{d}\,x^3\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(8\,acdx^4-2\,bd^2-\left(5\,bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,dx^3},\ \frac{3\left(bc^2+4\,acd\right)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(8\,acdx^4-2\,bd^2-\left(5\,bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\,dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^
2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^
2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3*ar
ctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c*d*x^4 - 2*b*d^2
 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]
```

**giac** [A]  time = 0.26, size = 145, normalized size = 1.29

$$\frac{8\sqrt{cx^2+d}\,ac^2\mathrm{sgn}(x)+\frac{3\left(bc^3\mathrm{sgn}(x)+4\,ac^2d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{5\left(cx^2+d\right)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\mathrm{sgn}(x)-3\sqrt{cx^2+d}\,bc^3d\mathrm{sgn}(x)-4\sqrt{cx^2+d}\,ac^2d^2\mathrm{sgn}(x)}{c^2x^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + d)*a*c^2*sgn(x) + 3*(b*c^3*sgn(x) + 4*a*c^2*d*sgn(x))*a
rctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - (5*(c*x^2 + d)^(3/2)*b*c^3*sgn(x
) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^3*d*sgn(x) -
 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x))/(c^2*x^4))/c
```

**maple** [B]  time = 0.06, size = 213, normalized size = 1.90

$$-\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(12ac\,d^{\frac{5}{2}}x^4\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)+3bc^2d^{\frac{3}{2}}x^4\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-12\sqrt{cx^2+d}\,acd^2x^4-3\sqrt{cx^2+d}\,bc^2dx^4-4\left(cx^2+d\right)^{\frac{3}{2}}acdx^4-\left(cx^2+d\right)^{\frac{3}{2}}bc^2x^4+4\left(cx^2+d\right)^{\frac{5}{2}}adx^2+\left(cx^2+d\right)^{\frac{5}{2}}bcx^2+2\left(cx^2+d\right)^{\frac{5}{2}}bd\right)}{8\left(cx^2+d\right)^{\frac{3}{2}}d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2),x)
```

```
[Out] -1/8*((c*x^2+d)/x^2)^(3/2)/x*(12*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x
)*x^4*a*c+3*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2-4*(c*x^2+
d)^(3/2)*a*c*d*x^4-(c*x^2+d)^(3/2)*b*c^2*x^4+4*(c*x^2+d)^(5/2)*x^2*a*d+(c*x
^2+d)^(5/2)*x^2*b*c-12*(c*x^2+d)^(1/2)*x^4*a*c*d^2-3*(c*x^2+d)^(1/2)*x^4*b*
c^2*d+2*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2
```

**maxima** [B]  time = 1.42, size = 207, normalized size = 1.85

$$\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}\,cx-\frac{2\sqrt{c+\frac{d}{x^2}}\,cdx}{\left(c+\frac{d}{x^2}\right)x^2-d}+3\,c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a+\frac{1}{16}\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-3\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4-2\left(c+\frac{d}{x^2}\right)dx^2+d^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*sqrt(c + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d)
+ 3*c*sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d
))))*a + 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x +
 sqrt(d)))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2
*d*x)/((c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*b
```

**mupad** [B]  time = 5.86, size = 78, normalized size = 0.70

$$\frac{a\,x\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{d}{c\,x^2}\right)}{{\left(\frac{d}{c}+x^2\right)}^{3/2}}-\frac{b\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{d}{c\,x^2}\right)}{x\,{\left(\frac{d}{c}+x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x*(d + c*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -d/(c*x^2)))/(d/c + x^2
)^(3/2) - (b*(d + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -d/(c*x^2)))/(x*
(d/c + x^2)^(3/2))
```

**sympy [B]** time = 11.78, size = 216, normalized size = 1.93

$$\frac{ac^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{a\sqrt{c}\,d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{a\sqrt{c}\,d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2}-\frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}\,d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8\sqrt{d}}-\frac{bd^2}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)
```

```
[Out] a*c**(3/2)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x)
+ a*sqrt(c)*d/(x*sqrt(1 + d/(c*x**2))) - 3*a*c*sqrt(d)*asinh(sqrt(d)/(sqrt(
c)*x))/2 - b*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - b*c**(3/2)/(8*x*sqrt(1 +
 d/(c*x**2))) - 3*b*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asin
h(sqrt(d)/(sqrt(c)*x))/(8*sqrt(d)) - b*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x
**2)))
```

## 3.614 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=123

$$\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-6ad)}{16dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

**Rubi [A]** time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 195, 217, 206}

$$\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-6ad)}{16dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] (c*(b*c - 6*a*d)*Sqrt[c + d/x^2])/(16*d*x) + ((b*c - 6*a*d)*(c + d/x^2)^(3/
2))/(24*d*x) - (b*(c + d/x^2)^(5/2))/(6*d*x) + (c^2*(b*c - 6*a*d)*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx &= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(-bc+6ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx}{6d} \\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}-\frac{(-bc+6ad)\,\mathrm{Subst}\left(\int \left(c+dx^2\right)^{3/2}\,dx,x,\frac{1}{x}\right)}{6d} \\
&= \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(c(bc-6ad))\,\mathrm{Subst}\left(\int \sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{8d} \\
&= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{\left(c^2(bc-6ad)\right)\mathrm{Sub}}{} \\
&= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{\left(c^2(bc-6ad)\right)\mathrm{Sub}}{} \\
&= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{c^2(bc-6ad)\tanh^{-}}{16d^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 126, normalized size = 1.02

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(\left(cx^2+d\right)\left(6adx^2\left(5cx^2+2d\right)+b\left(3c^2x^4+14cdx^2+8d^2\right)\right)+3c^2x^6\sqrt{\frac{cx^2}{d}+1}\,(6ad-bc)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{48dx^5\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] -1/48*(Sqrt[c + d/x^2]*((d + c*x^2)*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 +
 14*c*d*x^2 + 3*c^2*x^4)) + 3*c^2*(-(b*c) + 6*a*d)*x^6*Sqrt[1 + (c*x^2)/d]*
ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d*x^5*(d + c*x^2))
```

**IntegrateAlgebraic [A]** time = 0.25, size = 127, normalized size = 1.03

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\sqrt{cx^2+d}\left(-30acdx^4-12ad^2x^2-3bc^2x^4-14bcdx^2-8bd^2\right)}{48dx^6}+\frac{\left(bc^3-6ac^2d\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{16d^{3/2}}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-8*b*d^2 - 14*b*c*d*x^2 - 12*a*d^2*x^
2 - 3*b*c^2*x^4 - 30*a*c*d*x^4))/(48*d*x^6) + ((b*c^3 - 6*a*c^2*d)*ArcTanh[
Sqrt[d + c*x^2]/Sqrt[d]])/(16*d^(3/2))))/Sqrt[d + c*x^2]
```

**fricas [A]** time = 0.46, size = 246, normalized size = 2.00

$$\left[-\frac{3\left(bc^3-6ac^2d\right)\sqrt{d}\,x^5\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)+2\left(3\left(bc^2d+10acd^2\right)x^4+8bd^3+2\left(7bcd^2+6ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96d^2x^5},\ -\frac{3\left(bc^3-6ac^2d\right)\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(3\left(bc^2d+10acd^2\right)x^4+8bd^3+2\left(7bcd^2+6ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{48d^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*
(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b*c^
3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2
+ d)) + (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x
^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5)]
```

**giac** [A] time = 0.30, size = 173, normalized size = 1.41

$$-\frac{\frac{3\left(bc^4\mathrm{sgn}(x)-6\,ac^3d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\mathrm{sgn}(x)+30\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\mathrm{sgn}(x)+8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\mathrm{sgn}(x)-48\left(cx^2+d\right)^{\frac{3}{2}}ac^3d^2\mathrm{sgn}(x)-3\sqrt{cx^2+d}\,bc^4d^2\mathrm{sgn}(x)+18\sqrt{cx^2+d}\,ac^3d^3\mathrm{sgn}(x)}{c^3dx^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/48*(3*(b*c^4*sgn(x) - 6*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))
/(sqrt(-d)*d) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) + 30*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) + 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 48*(c*x^2 + d)^(3/2)*a*
c^3*d^2*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4*d^2*sgn(x) + 18*sqrt(c*x^2 + d)*a*
c^3*d^3*sgn(x))/(c^3*d*x^6))/c
```

**maple** [B] time = 0.07, size = 259, normalized size = 2.11

$$-\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(18ac^2d^{\frac{5}{2}}x^6\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-3bc^3d^{\frac{3}{2}}x^6\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-18\sqrt{cx^2+d}\,ac^2d^2x^6+3\sqrt{cx^2+d}\,bc^3dx^6-6\left(cx^2+d\right)^{\frac{3}{2}}ac^2dx^6+\left(cx^2+d\right)^{\frac{3}{2}}bc^3x^6+6\left(cx^2+d\right)^{\frac{5}{2}}acdx^4-\left(cx^2+d\right)^{\frac{5}{2}}bc^2x^4+12\left(cx^2+d\right)^{\frac{5}{2}}ad^2x^2-2\left(cx^2+d\right)^{\frac{5}{2}}bcdx^2+8\left(cx^2+d\right)^{\frac{5}{2}}bd^2\right)}{48\left(cx^2+d\right)^{\frac{3}{2}}d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x)
```

```
[Out] -1/48*((c*x^2+d)/x^2)^(3/2)/x^3*(18*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2)
)/x)*x^6*a*c^2-3*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^6*b*c^3-6*(c
*x^2+d)^(3/2)*x^6*a*c^2*d+(c*x^2+d)^(3/2)*x^6*b*c^3+6*(c*x^2+d)^(5/2)*x^4*a
*c*d-(c*x^2+d)^(5/2)*x^4*b*c^2-18*(c*x^2+d)^(1/2)*x^6*a*c^2*d^2+3*(c*x^2+d)
^(1/2)*x^6*b*c^3*d+12*(c*x^2+d)^(5/2)*x^2*a*d^2-2*(c*x^2+d)^(5/2)*x^2*b*c*d
+8*(c*x^2+d)^(5/2)*b*d^2)/(c*x^2+d)^(3/2)/d^3
```

**maxima** [B] time = 1.38, size = 275, normalized size = 2.24

$$\frac{1}{16}\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-3\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4-2\left(c+\frac{d}{x^2}\right)dx^2+d^2}\right)a-\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}\,c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))
)/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2*d*x)/((c
 + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*a - 1/96*(3*c^3*log((sqrt(c +
 d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*(3*(c + d/x
^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d
^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*x^4 + 3*(c + d/x^2)*d^3*x^
2 - d^4))*b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)
```

**sympy [B]** time = 18.95, size = 253, normalized size = 2.06

$$-\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}\,d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8\sqrt{d}}-\frac{ad^2}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}}-\frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{11b\sqrt{c}\,d}{24x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{16d^{\frac{3}{2}}}-\frac{bd^2}{6\sqrt{c}\,x^7\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - a*c**(3/2)/(8*x*sqrt(1 + d/(c*x**2
))) - 3*a*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*a*c**2*asinh(sqrt(d)/
(sqrt(c)*x))/(8*sqrt(d)) - a*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2))) - b
*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*b*c**(3/2)/(48*x**3*sqrt(1 + d
/(c*x**2))) - 11*b*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + b*c**3*asinh(
sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - b*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c*x
**2)))
```

# 3.615 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

**Optimal**. Leaf size=159

$$-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{128d^2x}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c+\right.}{8d}$$

**Rubi** [A] time = 0.09, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 279, 321, 217, 206}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{128d^2x}-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (c*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(64*d*x^3) + ((3*b*c - 8*a*d)*(c + d/x^
2)^(3/2))/(48*d*x^3) - (b*(c + d/x^2)^(5/2))/(8*d*x^3) + (c^2*(3*b*c - 8*a*
d)*Sqrt[c + d/x^2])/(128*d^2*x) - (c^3*(3*b*c - 8*a*d)*ArcTanh[Sqrt[d]/(Sqr
t[c + d/x^2]*x)])/(128*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{(-3bc+8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx}{8d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}-\frac{(-3bc+8ad)\,\mathrm{Subst}\left(\int x^2\left(c+dx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right)}{8d}$$

$$= \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{(c(3bc-8ad))\,\mathrm{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx,\right.}{16d}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{\left(c^2(3bc-8ad)\right)}{}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{}}{128d^2x}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{}}{128d^2x}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{}}{128d^2x}$$

**Mathematica [C]** time = 0.05, size = 71, normalized size = 0.45

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(c^3x^8(8ad-3bc)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{d}+1\right)-5bd^4\right)}{40d^5x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^4 + c^3*(-3*b*c + 8*a*d)*x^8*Hyperge
ometric2F1[5/2, 4, 7/2, 1 + (c*x^2)/d]))/(40*d^5*x^7)
```

**IntegrateAlgebraic [A]** time = 0.31, size = 152, normalized size = 0.96

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\left(8ac^3d-3bc^4\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{128d^{5/2}}+\frac{\sqrt{cx^2+d}\left(-24ac^2dx^6-112acd^2x^4-64ad^3x^2+9bc^3x^6-6bc^2dx^4-72bcd^2x^2-48bd^3\right)}{384d^2x^8}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-48*b*d^3 - 72*b*c*d^2*x^2 - 64*a*d^3
*x^2 - 6*b*c^2*d*x^4 - 112*a*c*d^2*x^4 + 9*b*c^3*x^6 - 24*a*c^2*d*x^6))/(38
4*d^2*x^8) + ((-3*b*c^4 + 8*a*c^3*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(128
*d^(5/2))))/Sqrt[d + c*x^2]
```

**fricas [A]** time = 0.47, size = 298, normalized size = 1.87

$$\left[-\frac{3\left(3\,bc^4-8\,ac^3d\right)\sqrt{d}\,x^7\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(3\,bc^3d-8\,ac^2d^2\right)x^6-48\,bd^4-2\left(3\,bc^2d^2+56\,acd^3\right)x^4-8\left(9\,bcd^3+8\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{768\,d^3x^7},\ \frac{3\left(3\,bc^4-8\,ac^3d\right)\sqrt{-d}\,x^7\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(3\left(3\,bc^3d-8\,ac^2d^2\right)x^6-48\,bd^4-2\left(3\,bc^2d^2+56\,acd^3\right)x^4-8\left(9\,bcd^3+8\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{384\,d^3x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqrt
((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d
^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7*arc
tan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(3*b*c^3*d - 8*a*c^2
*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*
a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]
```

**giac [A]** time = 0.37, size = 214, normalized size = 1.35

$$\frac{\frac{3\left(3\,bc^5\mathrm{sgn}(x)-8\,ac^4d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d^2}+\frac{9\left(cx^2+d\right)^{\frac{7}{2}}bc^5\mathrm{sgn}(x)-24\left(cx^2+d\right)^{\frac{7}{2}}ac^4d\mathrm{sgn}(x)-33\left(cx^2+d\right)^{\frac{5}{2}}bc^5d\mathrm{sgn}(x)-40\left(cx^2+d\right)^{\frac{5}{2}}ac^4d^2\mathrm{sgn}(x)-33\left(cx^2+d\right)^{\frac{3}{2}}bc^5d^2\mathrm{sgn}(x)+88\left(cx^2+d\right)^{\frac{3}{2}}ac^4d^3\mathrm{sgn}(x)+9\sqrt{cx^2+d}\,bc^5d^3\mathrm{sgn}(x)-24\sqrt{cx^2+d}\,ac^4d^4\mathrm{sgn}(x)}{c^4d^2x^8}}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d
))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7/2
)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^(5/
2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2 + d
)^(3/2)*a*c^4*d^3*sgn(x) + 9*sqrt(c*x^2 + d)*b*c^5*d^3*sgn(x) - 24*sqrt(c*x
^2 + d)*a*c^4*d^4*sgn(x))/(c^4*d^2*x^8))/c
```

**maple [B]** time = 0.07, size = 302, normalized size = 1.90

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(24a\,c^3d^{\frac{5}{2}}x^8\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-9b\,c^4d^{\frac{3}{2}}x^8\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-24\sqrt{cx^2+d}\,a\,c^3d^2x^8+9\sqrt{cx^2+d}\,b\,c^4d\,x^8-8\left(cx^2+d\right)^{\frac{3}{2}}a\,c^3d\,x^8+3\left(cx^2+d\right)^{\frac{3}{2}}b\,c^4x^8+8\left(cx^2+d\right)^{\frac{5}{2}}a\,c^2d\,x^6-3\left(cx^2+d\right)^{\frac{5}{2}}b\,c^3x^6+16\left(cx^2+d\right)^{\frac{5}{2}}ac\,d^2x^4-6\left(cx^2+d\right)^{\frac{5}{2}}b\,c^2d\,x^4-64\left(cx^2+d\right)^{\frac{5}{2}}a\,d^3x^2+24\left(cx^2+d\right)^{\frac{5}{2}}bc\,d^2x^2-48\left(cx^2+d\right)^{\frac{5}{2}}b\,d^3\right)}{384\left(cx^2+d\right)^{\frac{3}{2}}d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x)
```

```
[Out] 1/384*((c*x^2+d)/x^2)^(3/2)/x^5*(24*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2)
)/x)*x^8*a*c^3-9*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^8*b*c^4-8*(c
*x^2+d)^(3/2)*x^8*a*c^3*d+3*(c*x^2+d)^(3/2)*x^8*b*c^4+8*(c*x^2+d)^(5/2)*x^6
*a*c^2*d-3*(c*x^2+d)^(5/2)*x^6*b*c^3-24*(c*x^2+d)^(1/2)*x^8*a*c^3*d^2+9*(c*
x^2+d)^(1/2)*x^8*b*c^4*d+16*(c*x^2+d)^(5/2)*x^4*a*c*d^2-6*(c*x^2+d)^(5/2)*x
^4*b*c^2*d-64*(c*x^2+d)^(5/2)*x^2*a*d^3+24*(c*x^2+d)^(5/2)*x^2*b*c*d^2-48*(
c*x^2+d)^(5/2)*b*d^3)/(c*x^2+d)^(3/2)/d^4
```

**maxima [B]** time = 1.57, size = 354, normalized size = 2.23

$$-\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}\,c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4}\right)a+\frac{1}{256}\left(\frac{3\,c^4\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^4x^7-11\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4dx^5-11\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4d^2x^3+3\sqrt{c+\frac{d}{x^2}}\,c^4d^3x\right)}{\left(c+\frac{d}{x^2}\right)^4d^2x^8-4\left(c+\frac{d}{x^2}\right)^3d^3x^6+6\left(c+\frac{d}{x^2}\right)^2d^4x^4-4\left(c+\frac{d}{x^2}\right)d^5x^2+d^6}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(3*c^3*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)
))/d^(3/2) + 2*(3*(c + d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3
 - 3*sqrt(c + d/x^2)*c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*
x^4 + 3*(c + d/x^2)*d^3*x^2 - d^4))*a + 1/256*(3*c^4*log((sqrt(c + d/x^2)*x
 - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^2)^(7/2)
*c^4*x^7 - 11*(c + d/x^2)^(5/2)*c^4*d*x^5 - 11*(c + d/x^2)^(3/2)*c^4*d^2*x^
3 + 3*sqrt(c + d/x^2)*c^4*d^3*x)/((c + d/x^2)^4*d^2*x^8 - 4*(c + d/x^2)^3*d
^3*x^6 + 6*(c + d/x^2)^2*d^4*x^4 - 4*(c + d/x^2)*d^5*x^2 + d^6))*b
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4, x)
```

**sympy [B]** time = 29.16, size = 287, normalized size = 1.81

$$-\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}}-\frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{11a\sqrt{c}\,d}{24x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{16d^{\frac{3}{2}}}-\frac{ad^2}{6\sqrt{c}\,x^7\sqrt{1+\frac{d}{cx^2}}}+\frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{5}{2}}}{128dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{13bc^{\frac{3}{2}}}{64x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{5b\sqrt{c}\,d}{16x^7\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^4\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{128d^{\frac{5}{2}}}-\frac{bd^2}{8\sqrt{c}\,x^9\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] -a*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*a*c**(3/2)/(48*x**3*sqrt(1 +
 d/(c*x**2))) - 11*a*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + a*c**3*asin
h(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - a*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c
*x**2))) + 3*b*c**(7/2)/(128*d**2*x*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(128
*d*x**3*sqrt(1 + d/(c*x**2))) - 13*b*c**(3/2)/(64*x**5*sqrt(1 + d/(c*x**2))
) - 5*b*sqrt(c)*d/(16*x**7*sqrt(1 + d/(c*x**2))) - 3*b*c**4*asinh(sqrt(d)/(
sqrt(c)*x))/(128*d**(5/2)) - b*d**2/(8*sqrt(c)*x**9*sqrt(1 + d/(c*x**2)))
```

**3.616** $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=90

$$-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{8c^2}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{8c^2}-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] ((4*b*c - 3*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^2) + (a*Sqrt[c + d/x^2]*x^4)/(4*
c) - (d*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x^3\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} - \frac{\left(2bc-\frac{3ad}{2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c}\\
&= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} + \frac{(d(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c^2}\\
&= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} + \frac{(4bc-3ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{8c^2}\\
&= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} - \frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}$$

**Mathematica [A]**   time = 0.11, size = 95, normalized size = 1.06

$$\frac{\sqrt{c}\,x\left(cx^2+d\right)\left(2acx^2-3ad+4bc\right)+d\sqrt{cx^2+d}\,(3ad-4bc)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d}}\right)}{8c^{5/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c]*x*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + d*(-4*b*c + 3*a*d)*Sqr
t[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[d + c*x^2]])/(8*c^(5/2)*Sqrt[c + d/x^
2]*x)
```

**IntegrateAlgebraic [A]**   time = 0.14, size = 88, normalized size = 0.98

$$\frac{\left(3ad^2-4bcd\right)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(2acx^4-3adx^2+4bcx^2\right)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(4*b*c*x^2 - 3*a*d*x^2 + 2*a*c*x^4))/(8*c^2) + ((-4*
b*c*d + 3*a*d^2)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*c^(5/2))
```

**fricas [A]**   time = 0.44, size = 192, normalized size = 2.13

$$\left[-\frac{\left(4\,bcd-3\,ad^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^2x^4+\left(4\,bc^2-3\,acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^3},\ \frac{\left(4\,bcd-3\,ad^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(2\,ac^2x^4+\left(4\,bc^2-3\,acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^
2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 +
d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c
*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c^3]
```

**giac [A]** time = 0.26, size = 113, normalized size = 1.26

$$\frac{1}{8}\sqrt{cx^4+dx^2}\left(\frac{2\,ax^2}{c}+\frac{4\,bc-3\,ad}{c^2}\right)+\frac{\left(4\,bcd-3\,ad^2\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)\sqrt{c}-d\right|\right)}{16\,c^{\frac{5}{2}}}-\frac{4\,bcd\log\left(|d|\right)-3\,ad^2\log\left(|d|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + d*x^2)*(2*a*x^2/c + (4*b*c - 3*a*d)/c^2) + 1/16*(4*b*c*d -
 3*a*d^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))*sqrt(c) - d))/c^(5
/2) - 1/16*(4*b*c*d*log(abs(d)) - 3*a*d^2*log(abs(d)))/c^(5/2)
```

**maple [A]** time = 0.06, size = 129, normalized size = 1.43

$$\frac{\sqrt{c\,x^2+d}\,\left(2\sqrt{c\,x^2+d}\;a\,c^{\frac{5}{2}}x^3+3ac\,d^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-4b\,c^2d\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-3\sqrt{c\,x^2+d}\;a\,c^{\frac{3}{2}}dx+4\sqrt{c\,x^2+d}\;b\,c^{\frac{5}{2}}x\right)}{8\sqrt{\frac{c\,x^2+d}{x^2}}\;c^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/8*(c*x^2+d)^(1/2)*(2*(c*x^2+d)^(1/2)*c^(5/2)*x^3*a-3*(c*x^2+d)^(1/2)*c^(3
/2)*x*a*d+4*(c*x^2+d)^(1/2)*c^(5/2)*x*b+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c
*d^2-4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c^2*d)/((c*x^2+d)/x^2)^(1/2)/x/c^(7/
2)
```

**maxima [B]** time = 1.17, size = 178, normalized size = 1.98

$$\frac{1}{4}\,b\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{1}{16}\,a\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-5\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c^2-2\left(c+\frac{d}{x^2}\right)c^3+c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/16*a*(3*d^2*log((sqrt(c
 + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/2) + 2*(3*(c + d/x^2
)^(3/2)*d^2 - 5*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c^2 - 2*(c + d/x^2)*c
^3 + c^4))
```

**mupad [B]** time = 5.35, size = 99, normalized size = 1.10

$$\frac{5\,a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8\,c}-\frac{3\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c^2}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c}-\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}+\frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(1/2))/(8*c) - (3*a*x^4*(c + d/x^2)^(3/2))/(8*c^2) + (
b*x^2*(c + d/x^2)^(1/2))/(2*c) - (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*
c^(3/2)) + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(5/2))
```

**sympy [A]** time = 71.06, size = 150, normalized size = 1.67

$$\frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{a\sqrt{d}\,x^3}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}}+\frac{b\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) - a*sqrt(d)*x**3/(8*c*sqrt(c*x**2/d +
 1)) - 3*a*d**(3/2)*x/(8*c**2*sqrt(c*x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*
x/sqrt(d))/(8*c**(5/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - b*d*asinh(
sqrt(c)*x/sqrt(d))/(2*c**(3/2))
```

## 3.617 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$

**Optimal**. Leaf size=59

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

**Rubi [A]** time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 78, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x^2)/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/(2*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{a+bx}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}$$

**Mathematica** [A] time = 0.06, size = 79, normalized size = 1.34

$$\frac{\sqrt{cx^2+d}\,(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d}}\right)+a\sqrt{c}\,x\left(cx^2+d\right)}{2c^{3/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c]*x*(d + c*x^2) + (2*b*c - a*d)*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x
)/Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic** [A] time = 0.10, size = 67, normalized size = 1.14

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{\frac{cx^2+d}{x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*x^2*Sqrt[(d + c*x^2)/x^2])/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[(d + c*x^
2)/x^2]/Sqrt[c]])/(2*c^(3/2))
```

**fricas** [A] time = 0.44, size = 146, normalized size = 2.47

$$\left[\frac{2\,acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{4\,c^2}, \frac{acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2 +
```

```
 d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)
/(c*x^2 + d)))/c^2]
```

**giac [A]** time = 0.26, size = 88, normalized size = 1.49

$$\frac{\sqrt{cx^4+dx^2}\,a}{2\,c}-\frac{(2\,bc-ad)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)\sqrt{c}-d\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{2\,bc\log\left(|d|\right)-ad\log\left(|d|\right)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + d*x^2)*a/c - 1/4*(2*b*c - a*d)*log(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + d*x^2))*sqrt(c) - d))/c^(3/2) + 1/4*(2*b*c*log(abs(d)) - a*d*lo
g(abs(d)))/c^(3/2)
```

**maple [A]** time = 0.05, size = 90, normalized size = 1.53

$$\frac{\sqrt{c\,x^2+d}\,\left(-acd\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)+2b\,c^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)+\sqrt{c\,x^2+d}\,a\,c^{\frac{3}{2}}x\right)}{2\sqrt{\frac{c\,x^2+d}{x^2}}\,c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/2*(c*x^2+d)^(1/2)*(c^(3/2)*(c*x^2+d)^(1/2)*x*a+2*b*ln(c^(1/2)*x+(c*x^2+d)
^(1/2))*c^2-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d)/((c*x^2+d)/x^2)^(1/2)/x/c^
(5/2)
```

**maxima [B]** time = 1.20, size = 109, normalized size = 1.85

$$\frac{1}{4}\,a\left(\frac{2\,\sqrt{c+\frac{d}{x^2}}\,d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/2*b*log((sqrt(c + d/x^2
) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
```

**mupad [B]** time = 5.08, size = 59, normalized size = 1.00

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c}-\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) + (a*x^2*(c + d/x^2)^(1/2))/(2
*c) - (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(3/2))
```

**sympy [A]** time = 84.41, size = 66, normalized size = 1.12

$$\frac{a\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(
3/2)) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)
```

## 3.618 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx$

**Optimal**. Leaf size=43

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

**Rubi [A]** time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] -((b*Sqrt[c + d/x^2])/d) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}$$

**Mathematica [A]** time = 0.05, size = 73, normalized size = 1.70

$$\frac{adx\sqrt{cx^2+d}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d}}\right) - b\sqrt{c}\left(cx^2+d\right)}{\sqrt{c}\,dx^2\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] (-(b*Sqrt[c]*(d + c*x^2)) + a*d*x*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[
d + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 51, normalized size = 1.19

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\sqrt{\frac{cx^2+d}{x^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] -((b*Sqrt[(d + c*x^2)/x^2])/d) + (a*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])
/Sqrt[c]
```

**fricas [A]** time = 0.44, size = 130, normalized size = 3.02

$$\left[\frac{a\sqrt{c}\,d\log\left(-2\,cx^2 - 2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2\,bc\sqrt{\frac{cx^2+d}{x^2}}}{2\,cd}, -\frac{a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + bc\sqrt{\frac{cx^2+d}{x^2}}}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) -
 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*sqr
t((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}
+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%
}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters val
ues [86,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0
,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parameters values [-64,61.7937478349,70]S
ign error (%%%{d,0%%%}+%%%{-2*sqrt(c)*sqrt(d),1/2%%%}+%%%{2*c,1%%%}+%%%{-c*
sqrt(c)*sqrt(d)/d,3/2%%%}+%%%{c^2*sqrt(c)*sqrt(d)/(4*d^2),5/2%%%}+%%%{undef
,7/2%%%})Limit: Max order reached or unable to make series expansion Error:
 Bad Argument Value
```

**maple [A]** time = 0.05, size = 70, normalized size = 1.63

$$-\frac{\sqrt{c\,x^{2}+d}\,\left(-adx\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)+\sqrt{c\,x^{2}+d}\,b\sqrt{c}\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\sqrt{c}\,d\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x/(c+d/x^2)^(1/2),x)
```

```
[Out] -(c*x^2+d)^(1/2)*(-a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*d*x+b*(c*x^2+d)^(1/2)*c^
(1/2))/((c*x^2+d)/x^2)^(1/2)/x^2/c^(1/2)/d
```

**maxima [A]** time = 1.10, size = 54, normalized size = 1.26

$$-\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{2\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
 - b*sqrt(c + d/x^2)/d
```

**mupad [B]** time = 4.85, size = 35, normalized size = 0.81

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\,\sqrt{c+\frac{d}{x^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(1/2)),x)
```

```
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) - (b*(c + d/x^2)^(1/2))/d
```

**sympy** [A] time = 22.34, size = 63, normalized size = 1.47

$$-\frac{a\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\sqrt{c+\frac{d}{x^2}}}\right)}{c\sqrt{-\frac{1}{c}}}+\frac{b\left(\begin{cases}-\frac{1}{\sqrt{c}\,x^2} & \text{for } d=0\\ -\frac{2\sqrt{c+\frac{d}{x^2}}}{d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*atan(1/(sqrt(-1/c)*sqrt(c + d/x**2)))/(c*sqrt(-1/c)) + b*Piecewise((-1/(
sqrt(c)*x**2), Eq(d, 0)), (-2*sqrt(c + d/x**2)/d, True))/2
```

## 3.619 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^3}\,dx$

**Optimal**. Leaf size=43

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

**Rubi** [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (b*(c + d/x^2)^(3/2))/(3*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^3}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)\\ &= \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 39, normalized size = 0.91

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(3adx^2+b\left(d-2cx^2\right)\right)}{3d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

```
[Out] -1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(d^2*x^2)
```

**IntegrateAlgebraic [A]** time = 0.05, size = 44, normalized size = 1.02

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-3adx^2+2bcx^2-bd\right)}{3d^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d) + 2*b*c*x^2 - 3*a*d*x^2))/(3*d^2*x^2)
```

**fricas [A]** time = 0.41, size = 39, normalized size = 0.91

$$\frac{\left((2\,bc-3\,ad)x^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)
```

**giac [B]** time = 0.41, size = 88, normalized size = 2.05

$$\frac{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^2a+3\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)b\sqrt{c}+bd}{3\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^2*a + 3*(sqrt(c)*x^2 - sqrt(c*x^
4 + d*x^2))*b*sqrt(c) + b*d)/(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^3
```

**maple [A]** time = 0.05, size = 47, normalized size = 1.09

$$-\frac{\left(3ad\,x^2-2bc\,x^2+bd\right)\left(c\,x^2+d\right)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\;d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x)
[Out] -1/3*(3*a*d*x^2-2*b*c*x^2+b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^2/x^4
```

**maxima [A]** time = 0.53, size = 48, normalized size = 1.12

$$-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}\,c}{d^2}\right)-\frac{a\sqrt{c+\frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^2
)/d
```

**mupad [B]** time = 4.56, size = 35, normalized size = 0.81

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,d+3\,a\,d\,x^2-2\,b\,c\,x^2\right)}{3\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(1/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d + 3*a*d*x^2 - 2*b*c*x^2))/(3*d^2*x^2)
```

**sympy [A]** time = 7.05, size = 138, normalized size = 3.21

$$\frac{\begin{cases} \frac{-\frac{a}{x^2}-\frac{b}{2x^4}}{\sqrt{c}} & \text{for } d=0 \\ \frac{\frac{2ac}{\sqrt{c+\frac{d}{x^2}}}+2a\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)+\frac{2bc\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2b\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-a/x**2 - b/(2*x**4))/sqrt(c), Eq(d, 0)), ((2*a*c/sqrt(c + d/x*
*2) + 2*a*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2)) + 2*b*c*(-c/sqrt(c + d/x
**2) - sqrt(c + d/x**2))/d + 2*b*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x*
*2) - (c + d/x**2)**(3/2)/3)/d)/d, True))/2
```

## 3.620 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^5} \, dx$

**Optimal**. Leaf size=72

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] -((c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3) + ((2*b*c - a*d)*(c + d/x^2)^(3/2))/
(3*d^3) - (b*(c + d/x^2)^(5/2))/(5*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^5} \, dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)}{\sqrt{c+dx}} \, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2\sqrt{c+dx}}+\frac{(-2bc+ad)\sqrt{c+dx}}{d^2}+\frac{b(c+dx)^{3/2}}{d^2}\right) dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.83

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(b\left(-8c^2x^4+4cdx^2-3d^2\right)-5adx^2\left(d-2cx^2\right)\right)}{15d^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] (Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2*
x^4)))/(15*d^3*x^4)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 66, normalized size = 0.92

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(10acdx^4-5ad^2x^2-8bc^2x^4+4bcdx^2-3bd^2\right)}{15d^3x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-3*b*d^2 + 4*b*c*d*x^2 - 5*a*d^2*x^2 - 8*b*c^2*x^4
+ 10*a*c*d*x^4))/(15*d^3*x^4)
```

**fricas [A]** time = 0.43, size = 62, normalized size = 0.86

$$-\frac{\left(2\left(4\,bc^2-5\,acd\right)x^4+3\,bd^2-\left(4\,bcd-5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2)/(d^3*x^4)
```

**giac [B]** time = 0.50, size = 153, normalized size = 2.12

$$\frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^3a\sqrt{c}+20\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^2bc+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^2ad+15\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)b\sqrt{c}\,d+3\,bd^2}{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^3*a*sqrt(c) + 20*(sqrt(c)*x^2
- sqrt(c*x^4 + d*x^2))^2*b*c + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^2*a*d
+ 15*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))*b*sqrt(c)*d + 3*b*d^2)/(sqrt(c)*x^
2 - sqrt(c*x^4 + d*x^2))^5
```

**maple [A]** time = 0.05, size = 70, normalized size = 0.97

$$\frac{\left(10acd\,x^4-8b\,c^2x^4-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\;d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)*(c*x^2+d)/(
(c*x^2+d)/x^2)^(1/2)/d^3/x^6
```

**maxima [A]** time = 0.49, size = 83, normalized size = 1.15

$$-\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^3}\right)-\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}\,c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c +
 d/x^2)*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2)
```

**mupad** [B] time = 4.68, size = 58, normalized size = 0.81

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(8\,b\,c^2\,x^4-10\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+5\,a\,d^2\,x^2+3\,b\,d^2\right)}{15\,d^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(1/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(3*b*d^2 + 5*a*d^2*x^2 + 8*b*c^2*x^4 - 10*a*c*d*x^4 - 4
*b*c*d*x^2))/(15*d^3*x^4)
```

**sympy** [A] time = 13.17, size = 204, normalized size = 2.83

$$\frac{\begin{cases} \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{\sqrt{c}} & \text{for } d=0 \\ \frac{\frac{2ac\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2a\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}+\frac{2bc\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2b\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**5/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-a/(2*x**4) - b/(3*x**6))/sqrt(c), Eq(d, 0)), ((2*a*c*(-c/sqrt(
c + d/x**2) - sqrt(c + d/x**2))/d + 2*a*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c
 + d/x**2) - (c + d/x**2)**(3/2)/3)/d + 2*b*c*(c**2/sqrt(c + d/x**2) + 2*c*
sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*b*(-c**3/sqrt(c + d/x**2
) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5
)/d**2)/d, True))/2
```

# 3.621 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^7}\,dx$

**Optimal**. Leaf size=101

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

**Rubi** [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] (c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2
))/(3*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (b*(c + d/x^2)^(7/
2))/(7*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^7}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(a+bx)}{\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{c(3bc-2ad)\sqrt{c+dx}}{d^3}+\frac{(-3bc+ad)(c+dx)^{3/2}}{d^3}+\frac{b(c+dx)^{5/}}{d^3}\right.\right.\right.\\
&= \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 91, normalized size = 0.90

$$\frac{\left(\frac{cx^2}{d}+1\right)\left(8c^2x^4-4cdx^2+3d^2\right)(6bc-7ad)}{105d^3x^6\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(cx^2+d\right)}{7dx^8\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] -1/7*(b*(d + c*x^2))/(d*Sqrt[c + d/x^2]*x^8) + ((6*b*c - 7*a*d)*(1 + (c*x^2
)/d)*(3*d^2 - 4*c*d*x^2 + 8*c^2*x^4))/(105*d^3*Sqrt[c + d/x^2]*x^6)
```

**IntegrateAlgebraic [A]** time = 0.07, size = 90, normalized size = 0.89

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-56ac^2dx^6+28acd^2x^4-21ad^3x^2+48bc^3x^6-24bc^2dx^4+18bcd^2x^2-15bd^3\right)}{105d^4x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-15*b*d^3 + 18*b*c*d^2*x^2 - 21*a*d^3*x^2 - 24*b*c^
2*d*x^4 + 28*a*c*d^2*x^4 + 48*b*c^3*x^6 - 56*a*c^2*d*x^6))/(105*d^4*x^6)
```

**fricas [A]** time = 0.43, size = 86, normalized size = 0.85

$$\frac{\left(8\left(6\,bc^3-7\,ac^2d\right)x^6-4\left(6\,bc^2d-7\,acd^2\right)x^4-15\,bd^3+3\left(6\,bcd^2-7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*b*c^3 - 7*a*c^2*d)*x^6 - 4*(6*b*c^2*d - 7*a*c*d^2)*x^4 - 15*b*d
^3 + 3*(6*b*c*d^2 - 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^6)
```

**giac [B]** time = 0.62, size = 219, normalized size = 2.17

$$\frac{140\left(\sqrt{c}x^2-\sqrt{cx^4+dx^2}\right)^4ac+210\left(\sqrt{c}x^2-\sqrt{cx^4+dx^2}\right)^3bc^{\frac{3}{2}}+105\left(\sqrt{c}x^2-\sqrt{cx^4+dx^2}\right)^3a\sqrt{c}d+252\left(\sqrt{c}x^2-\sqrt{cx^4+dx^2}\right)^2bcd+21\left(\sqrt{c}x^2-\sqrt{cx^4+dx^2}\right)^2ad^2+105\left(\sqrt{c}x^2-\sqrt{cx^4+dx^2}\right)b\sqrt{c}d^2+15\,bd^3}{105\left(\sqrt{c}x^2-\sqrt{cx^4+dx^2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(140*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^4*a*c + 210*(sqrt(c)*x^2 - s
qrt(c*x^4 + d*x^2))^3*b*c^(3/2) + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^3
*a*sqrt(c)*d + 252*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^2*b*c*d + 21*(sqrt(c
)*x^2 - sqrt(c*x^4 + d*x^2))^2*a*d^2 + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^
2))*b*sqrt(c)*d^2 + 15*b*d^3)/(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^7
```

**maple [A]** time = 0.05, size = 94, normalized size = 0.93

$$-\frac{\left(56a\,c^2d\,x^6-48b\,c^3x^6-28ac\,d^2x^4+24b\,c^2d\,x^4+21a\,d^3x^2-18bc\,d^2x^2+15b\,d^3\right)\left(c\,x^2+d\right)}{105\sqrt{\frac{c\,x^2+d}{x^2}}\;d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/105*(56*a*c^2*d*x^6-48*b*c^3*x^6-28*a*c*d^2*x^4+24*b*c^2*d*x^4+21*a*d^3*
x^2-18*b*c*d^2*x^2+15*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^4/x^8
```

**maxima [A]** time = 0.58, size = 118, normalized size = 1.17

$$-\frac{1}{35}b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^4}-\frac{21\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^4}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^4}-\frac{35\sqrt{c+\frac{d}{x^2}}\,c^3}{d^4}\right)-\frac{1}{15}a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^4 - 21*(c + d/x^2)^(5/2)*c/d^4 + 35*(c + d/x
^2)^(3/2)*c^2/d^4 - 35*sqrt(c + d/x^2)*c^3/d^4) - 1/15*a*(3*(c + d/x^2)^(5/
2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3)
```

**mupad [B]** time = 4.72, size = 102, normalized size = 1.01

$$\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3-56\,a\,c^2\,d\right)}{105\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,d\,x^6}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(24\,b\,c^2-28\,a\,c\,d\right)}{105\,d^3\,x^2}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(7\,a\,d-6\,b\,c\right)}{35\,d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(1/2)),x)
```

```
[Out] ((c + d/x^2)^(1/2)*(48*b*c^3 - 56*a*c^2*d))/(105*d^4) - (b*(c + d/x^2)^(1/2
))/(7*d*x^6) - ((c + d/x^2)^(1/2)*(24*b*c^2 - 28*a*c*d))/(105*d^3*x^2) - ((
c + d/x^2)^(1/2)*(7*a*d - 6*b*c))/(35*d^2*x^4)
```

**sympy [A]** time = 18.00, size = 269, normalized size = 2.66

$$\frac{\begin{cases}\frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{\sqrt{c}} & \text{for}\ d=0\\ \frac{\frac{2ac\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2a\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{2bc\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^3}+\frac{2b\left(\frac{c^4}{\sqrt{c+\frac{d}{x^2}}}+4c^3\sqrt{c+\frac{d}{x^2}}-2c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{otherwise}\end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**7/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-a/(3*x**6) - b/(4*x**8))/sqrt(c), Eq(d, 0)), ((2*a*c*(c**2/sqr
t(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*a*(-
c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (
c + d/x**2)**(5/2)/5)/d**2 + 2*b*c*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c
+ d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**3 + 2*b*(c**4
/sqrt(c + d/x**2) + 4*c**3*sqrt(c + d/x**2) - 2*c**2*(c + d/x**2)**(3/2) +
4*c*(c + d/x**2)**(5/2)/5 - (c + d/x**2)**(7/2)/7)/d**3)/d, True))/2
```

## 3.622 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$

**Optimal**. Leaf size=82

$$-\frac{2dx\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^3}+\frac{x^3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^2}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

**Rubi** [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$\frac{x^3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^2}-\frac{2dx\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^3}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*(5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(15*c^3) + ((5*b*c - 4*a*d)*Sqrt[c
 + d/x^2]*x^3)/(15*c^2) + (a*Sqrt[c + d/x^2]*x^5)/(5*c)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx &= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}+\frac{(5bc-4ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx}{5c}\\
&= \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}-\frac{(2d(5bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^2}\\
&= -\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^3}+\frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 56, normalized size = 0.68

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(a\left(3c^2x^4-4cdx^2+8d^2\right)+5bc\left(cx^2-2d\right)\right)}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^4
)))/(15*c^3)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 57, normalized size = 0.70

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(3ac^2x^4-4acdx^2+8ad^2+5bc^2x^2-10bcd\right)}{15c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(-10*b*c*d + 8*a*d^2 + 5*b*c^2*x^2 - 4*a*c*d*x^2 + 3*a*c
^2*x^4))/(15*c^3)
```

**fricas** [A] time = 0.41, size = 59, normalized size = 0.72

$$\frac{\left(3\,ac^2x^5+\left(5\,bc^2-4\,acd\right)x^3-2\left(5\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqrt
((c*x^2 + d)/x^2)/c^3
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple** [A] time = 0.05, size = 67, normalized size = 0.82

$$\frac{\left(3a\,x^4c^2-4acd\,x^2+5b\,c^2x^2+8a\,d^2-10bcd\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\;c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15/x*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*(c*x^2+d)/((c
*x^2+d)/x^2)^(1/2)/c^3
```

**maxima [A]** time = 0.49, size = 85, normalized size = 1.04

$$\frac{\left(\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^3 - 3\sqrt{c + \frac{d}{x^2}}\, dx\right) b}{3\,c^2} + \frac{\left(3\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}} x^5 - 10\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} dx^3 + 15\sqrt{c + \frac{d}{x^2}}\, d^2 x\right) a}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)*b/c^2 + 1/15*(3*(c + d/
x^2)^(5/2)*x^5 - 10*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)*a/c
^3
```

**mupad [B]** time = 5.31, size = 53, normalized size = 0.65

$$\frac{x\,\sqrt{c + \frac{d}{x^2}}\,\left(3\,a\,c^2\,x^4 + 5\,b\,c^2\,x^2 - 4\,a\,c\,d\,x^2 - 10\,b\,c\,d + 8\,a\,d^2\right)}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (x*(c + d/x^2)^(1/2)*(8*a*d^2 + 3*a*c^2*x^4 + 5*b*c^2*x^2 - 10*b*c*d - 4*a*
c*d*x^2))/(15*c^3)
```

**sympy [B]** time = 3.55, size = 338, normalized size = 4.12

$$\frac{3ac^4d^{\frac{9}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{2ac^3d^{\frac{11}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{3ac^2d^{\frac{13}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{12acd^{\frac{15}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{8ad^{\frac{17}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{b\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] 3*a*c**4*d**(9/2)*x**8*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5
*x**2 + 15*c**3*d**6) + 2*a*c**3*d**(11/2)*x**6*sqrt(c*x**2/d + 1)/(15*c**5
*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 3*a*c**2*d**(13/2)*x**4*sq
rt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 1
2*a*c*d**(15/2)*x**2*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x
**2 + 15*c**3*d**6) + 8*a*d**(17/2)*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 +
 30*c**4*d**5*x**2 + 15*c**3*d**6) + b*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c
) - 2*b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c**2)
```

## 3.623 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

**Optimal**. Leaf size=51

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

**Rubi [A]** time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 191}

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] ((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx &= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}+\frac{(3bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c}\\ &= \frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 34, normalized size = 0.67

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(acx^2-2ad+3bc\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

**IntegrateAlgebraic** [A] time = 0.06, size = 34, normalized size = 0.67

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,\left(acx^2-2ad+3bc\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

**fricas** [A] time = 0.42, size = 36, normalized size = 0.71

$$\frac{\left(acx^3+(3\,bc-2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple** [A] time = 0.04, size = 44, normalized size = 0.86

$$\frac{\left(a\,x^2c-2ad+3bc\right)\left(c\,x^2+d\right)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\;c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x)
[Out] 1/3/x*(a*c*x^2-2*a*d+3*b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/c^2
```

**maxima** [A] time = 0.59, size = 49, normalized size = 0.96

$$\frac{b\sqrt{c+\frac{d}{x^2}}\,x}{c}+\frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\,\sqrt{c+\frac{d}{x^2}}\,dx\right)a}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)
*a/c^2
```

**mupad [B]** time = 4.92, size = 67, normalized size = 1.31

$$\frac{a\,x^3\,\sqrt{c+\frac{d}{x^2}}\,\left(c-\frac{2\,d}{x^2}\right)}{3\,c^2}+\frac{b\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^3*(c + d/x^2)^(1/2)*(c - (2*d)/x^2))/(3*c^2) + (b*x*((c*x^2)/d + 1)^(1
/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
```

**sympy [A]** time = 2.76, size = 70, normalized size = 1.37

$$\frac{a\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3c}-\frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}+\frac{b\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c) - 2*a*d**(3/2)*sqrt(c*x**2/d + 1)/(
3*c**2) + b*sqrt(d)*sqrt(c*x**2/d + 1)/c
```

## 3.624 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}} dx$

**Optimal**. Leaf size=47

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

**Rubi** [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {375, 451, 217, 206}

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\operatorname{Subst}\left(\int \frac{a+bx^2}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{\sqrt{d}}$$

**Mathematica** [A] time = 0.04, size = 71, normalized size = 1.51

$$\frac{a\sqrt{d}\left(cx^2+d\right) - bc\sqrt{cx^2+d}\,\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{c\sqrt{d}\,x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d
]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 67, normalized size = 1.43

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{a\sqrt{cx^2+d}}{c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{d}}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((a*Sqrt[d + c*x^2])/c - (b*ArcTanh[Sqrt[d + c*x^2]/Sqrt
[d]])/Sqrt[d]))/Sqrt[d + c*x^2]
```

**fricas** [A] time = 0.44, size = 131, normalized size = 2.79

$$\left[\frac{2\,adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{d}\,\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,cd}, \frac{adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x
*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) + b
*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple [A]** time = 0.05, size = 73, normalized size = 1.55

$$\frac{\sqrt{c\,x^{2}+d}\,\left(-b c \ln\left(\frac{2 d+2 \sqrt{c\,x^{2}+d}\,\sqrt{d}}{x}\right)+\sqrt{c\,x^{2}+d}\,a \sqrt{d}\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,c \sqrt{d}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(1/2),x)
```

```
[Out] (c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2
))/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)
```

**maxima [A]** time = 1.28, size = 58, normalized size = 1.23

$$\frac{a \sqrt{c+\frac{d}{x^{2}}}\,x}{c}+\frac{b \log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{2 \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a*sqrt(c + d/x^2)*x/c + 1/2*b*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d
/x^2)*x + sqrt(d)))/sqrt(d)
```

**mupad [B]** time = 5.00, size = 65, normalized size = 1.38

$$\frac{a\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x*((c*x^2)/d + 1)^(1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
 - (b*log((c + d/x^2)^(1/2) + d^(1/2)/x))/d^(1/2)
```

**sympy [A]** time = 2.86, size = 39, normalized size = 0.83

$$\frac{a\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)
```

## 3.625 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx$

**Optimal**. Leaf size=61

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

**Rubi [A]** time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 335, 217, 206}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] -(b*Sqrt[c + d/x^2])/(2*d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x
^2]*x)])/(2*d^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} + \frac{(-bc+2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{2d}$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} - \frac{(-bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{2d}$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} - \frac{(-bc+2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d}$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{3/2}}$$

**Mathematica** [A] time = 0.06, size = 80, normalized size = 1.31

$$\frac{x^2\sqrt{cx^2+d}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right) - b\sqrt{d}\,\left(cx^2+d\right)}{2d^{3/2}x^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] (-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt[
d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)
```

**IntegrateAlgebraic** [A] time = 0.12, size = 82, normalized size = 1.34

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{2d^{3/2}} - \frac{b\sqrt{cx^2+d}}{2dx^2}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(-1/2*(b*Sqrt[d + c*x^2])/(d*x^2) + ((b*c - 2*a*d)*ArcTa
nh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(3/2))))/Sqrt[d + c*x^2]
```

**fricas** [A] time = 0.42, size = 144, normalized size = 2.36

$$\left[-\frac{(bc-2\,ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,bd\sqrt{\frac{cx^2+d}{x^2}}}{4\,d^2x}, -\frac{(bc-2\,ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bd\sqrt{\frac{cx^2+d}{x^2}}}{2\,d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x
^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*d)
*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*d*sqrt
((c*x^2 + d)/x^2))/(d^2*x)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple [B]** time = 0.06, size = 105, normalized size = 1.72

$$-\frac{\sqrt{c\,x^{2}+d}\,\left(2 a\,d^{2} x^{2} \ln \left(\frac{2 d+2 \sqrt{c\,x^{2}+d}\,\sqrt{d}}{x}\right)-b c d\,x^{2} \ln \left(\frac{2 d+2 \sqrt{c\,x^{2}+d}\,\sqrt{d}}{x}\right)+\sqrt{c\,x^{2}+d}\,b\,d^{\frac{3}{2}}\right)}{2 \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{\frac{5}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*(c*x^2+d)^(1/2)*(2*a*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*d^2-ln(2*
(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*b*c*d+(c*x^2+d)^(1/2)*d^(3/2)*b)/((c*x^2
+d)/x^2)^(1/2)/x^3/d^(5/2)
```

**maxima [B]** time = 1.18, size = 121, normalized size = 1.98

$$-\frac{1}{4}\left(\frac{2 \sqrt{c+\frac{d}{x^{2}}}\,c x}{\left(c+\frac{d}{x^{2}}\right) d x^{2}-d^{2}}+\frac{c \log \left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right) b+\frac{a \log \left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{2 \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x
^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*b + 1/2*a*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d)
```

**mupad [B]** time = 5.53, size = 94, normalized size = 1.54

$$\begin{cases} -\frac{3\,a\,x^2+b}{3\,\sqrt{c}\,x^3} & \text{if } d=0\\ \frac{b\,c\,\ln\left(2\,\sqrt{c+\frac{d}{x^2}}+\frac{2\,\sqrt{d}}{x}\right)}{2\,d^{3/2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,d\,x}-\frac{a\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}} & \text{if } d\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(1/2)),x)
```

```
[Out] piecewise(d == 0, -(b + 3*a*x^2)/(3*c^(1/2)*x^3), d ~= 0, - (a*log((c + d/x
^2)^(1/2) + d^(1/2)/x))/d^(1/2) - (b*(c + d/x^2)^(1/2))/(2*d*x) + (b*c*log(
2*(c + d/x^2)^(1/2) + (2*d^(1/2))/x))/(2*d^(3/2)))
```

**sympy** [**A**] time = 4.92, size = 66, normalized size = 1.08

$$-\frac{a\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{\sqrt{d}}-\frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d
*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))
```

## 3.626 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx$

**Optimal**. Leaf size=93

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{8d^2x}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

**Rubi** [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 321, 217, 206}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -(b*Sqrt[c + d/x^2])/(4*d*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^2*x
) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(-3bc+4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx}{4d}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}-\frac{(-3bc+4ad)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{4d}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{8d^2}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^2}$$

$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{5/2}}$$

**Mathematica** [A] time = 0.18, size = 107, normalized size = 1.15

$$-\frac{\left(cx^2+d\right)\left(d\sqrt{\frac{cx^2}{d}+1}\,\left(4adx^2-3bcx^2+2bd\right)+cx^4(3bc-4ad)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{8d^3x^5\sqrt{c+\frac{d}{x^2}}\,\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -1/8*((d + c*x^2)*(d*Sqrt[1 + (c*x^2)/d]*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2) +
c*(3*b*c - 4*a*d)*x^4*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d^3*Sqrt[c + d/x^2]*x
^5*Sqrt[1 + (c*x^2)/d])
```

**IntegrateAlgebraic** [A] time = 0.20, size = 104, normalized size = 1.12

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{\left(4acd-3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{8d^{5/2}}+\frac{\sqrt{cx^2+d}\left(-4adx^2+3bcx^2-2bd\right)}{8d^2x^4}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((Sqrt[d + c*x^2]*(-2*b*d + 3*b*c*x^2 - 4*a*d*x^2))/(8*d
^2*x^4) + ((-3*b*c^2 + 4*a*c*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(5/2
))))/Sqrt[d + c*x^2]
```

**fricas** [A] time = 0.45, size = 201, normalized size = 2.16

$$\left[-\frac{\left(3\,bc^2-4\,acd\right)\sqrt{d}\,x^3\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,bd^2-\left(3\,bcd-4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^3x^3},\ \frac{\left(3\,bc^2-4\,acd\right)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-\left(2\,bd^2-\left(3\,bcd-4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\,d^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x
^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-
d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^
2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple [A]** time = 0.06, size = 146, normalized size = 1.57

$$-\frac{\sqrt{c x^2+d}\,\left(-4 a c d^2 x^4 \ln\left(\frac{2d+2\sqrt{c x^2+d}\,\sqrt{d}}{x}\right)+3 b c^2 d x^4 \ln\left(\frac{2d+2\sqrt{c x^2+d}\,\sqrt{d}}{x}\right)+4\sqrt{c x^2+d}\, a d^{\frac{5}{2}} x^2-3\sqrt{c x^2+d}\, b c d^{\frac{3}{2}} x^2+2\sqrt{c x^2+d}\, b d^{\frac{5}{2}}\right)}{8\sqrt{\frac{c x^2+d}{x^2}}\, d^{\frac{7}{2}} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+d)^(1/2)*(-4*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*a*c*d^2+3*
ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2*d+4*(c*x^2+d)^(1/2)*d^(5/2)*x
^2*a-3*(c*x^2+d)^(1/2)*d^(3/2)*x^2*b*c+2*(c*x^2+d)^(1/2)*d^(5/2)*b)/((c*x^2
+d)/x^2)^(1/2)/x^5/d^(7/2)
```

**maxima [B]** time = 1.23, size = 200, normalized size = 2.15

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,c x}{\left(c+\frac{d}{x^2}\right)d x^2-d^2}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)a+\frac{1}{16}\,b\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-5\sqrt{c+\frac{d}{x^2}}\,c^2 d x\right)}{\left(c+\frac{d}{x^2}\right)^2 d^2x^4-2\left(c+\frac{d}{x^2}\right)d^3x^2+d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x
^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*a + 1/16*b*(3*c^2*
log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) +
2*(3*(c + d/x^2)^(3/2)*c^2*x^3 - 5*sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*
d^2*x^4 - 2*(c + d/x^2)*d^3*x^2 + d^4))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^4\,\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)),x)
```

```
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)
```

**sympy [A]** time = 8.46, size = 150, normalized size = 1.61

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}+\frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{5}{2}}}-\frac{b}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2
*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*d*
x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(5/2
)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

## 3.627 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=118

$$-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^2(4bc-5ad)}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

**Rubi** [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{3x^2\sqrt{c+\frac{d}{x^2}}(4bc-5ad)}{8c^3}-\frac{x^2(4bc-5ad)}{4c^2\sqrt{c+\frac{d}{x^2}}}-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((4*b*c - 5*a*d)*x^2)/(4*c^2*Sqrt[c + d/x^2]) + (3*(4*b*c - 5*a*d)*Sqrt[c
+ d/x^2]*x^2)/(8*c^3) + (a*x^4)/(4*c*Sqrt[c + d/x^2]) - (3*d*(4*b*c - 5*a*d
)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x^3(c+dx)^{3/2}}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$= \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(2bc-\frac{5ad}{2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2(c+dx)^{3/2}}\,dx,x,\frac{1}{x^2}\right)}{4c}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{(3(4bc-5ad))\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{8c^2}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3d(4bc-5ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.}{16c^3}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3(4bc-5ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx\right.}{8c^3}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

**Mathematica [A]** time = 0.19, size = 111, normalized size = 0.94

$$\frac{\sqrt{c}\,x\left(a\left(2c^2x^4-5cdx^2-15d^2\right)+4bc\left(cx^2+3d\right)\right)+3d^{3/2}\sqrt{\frac{cx^2}{d}+1}\,(5ad-4bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{7/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 3*
d^(3/2)*(-4*b*c + 5*a*d)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/
(8*c^(7/2)*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic [A]** time = 0.18, size = 119, normalized size = 1.01

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(2ac^2x^6-5acdx^4-15ad^2x^2+4bc^2x^4+12bcdx^2\right)}{8c^3\left(cx^2+d\right)} - \frac{3\left(4bcd-5ad^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(12*b*c*d*x^2 - 15*a*d^2*x^2 + 4*b*c^2*x^4 - 5*a*c*d
*x^4 + 2*a*c^2*x^6))/(8*c^3*(d + c*x^2)) - (3*(4*b*c*d - 5*a*d^2)*ArcTanh[S
qrt[(d + c*x^2)/x^2]/Sqrt[c]])/(8*c^(7/2))
```

**fricas [A]** time = 0.46, size = 304, normalized size = 2.58

$$\left[-\frac{3\left(4\,bcd^{2}-5\,ad^{3}+\left(4\,bc^{2}d-5\,acd^{2}\right)x^{2}\right)\sqrt{c}\,\log\left(-2\,cx^{2}-2\,\sqrt{c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\left(2\,ac^{3}x^{6}+\left(4\,bc^{3}-5\,ac^{2}d\right)x^{4}+3\left(4\,bc^{2}d-5\,acd^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{16\left(c^{5}x^{2}+c^{4}d\right)},\ \frac{3\left(4\,bcd^{2}-5\,ad^{3}+\left(4\,bc^{2}d-5\,acd^{2}\right)x^{2}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)+\left(2\,ac^{3}x^{6}+\left(4\,bc^{3}-5\,ac^{2}d\right)x^{4}+3\left(4\,bc^{2}d-5\,acd^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{8\left(c^{5}x^{2}+c^{4}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*b*
c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)
)/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*
x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a
*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

**maple [A]** time = 0.06, size = 140, normalized size = 1.19

$$\frac{\left(c\,x^{2}+d\right)\left(2 a\,c^{\frac{7}{2}} x^{5}-5 a\,c^{\frac{5}{2}} d\,x^{3}+4 b\,c^{\frac{7}{2}} x^{3}-15 a\,c^{\frac{3}{2}} d^{2} x+12 b\,c^{\frac{5}{2}} d x+15 \sqrt{c\,x^{2}+d}\;a c\,d^{2} \ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)-12 \sqrt{c\,x^{2}+d}\;b\,c^{2} d \ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\right)}{8 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{\frac{9}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/8*(c*x^2+d)*(2*c^(7/2)*x^5*a-5*c^(5/2)*x^3*a*d+4*c^(7/2)*x^3*b-15*c^(3/2)
*x*a*d^2+12*c^(5/2)*x*b*d+15*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*
a*c*d^2-12*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c^2*d)/((c*x^2+d
)/x^2)^(3/2)/x^3/c^(9/2)
```

**maxima [B]** time = 1.38, size = 215, normalized size = 1.82

$$-\frac{1}{16}\,a\left(\frac{2\left(15\left(c+\frac{d}{x^{2}}\right)^{2} d^{2}-25\left(c+\frac{d}{x^{2}}\right)c d^{2}+8\,c^{2} d^{2}\right)}{\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}} c^{3}-2\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}} c^{4}+\sqrt{c+\frac{d}{x^{2}}}\,c^{5}}+\frac{15\,d^{2}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)+\frac{1}{4}\,b\left(\frac{2\left(3\left(c+\frac{d}{x^{2}}\right)d-2\,c d\right)}{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}} c^{2}-\sqrt{c+\frac{d}{x^{2}}}\,c^{3}}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*a*(2*(15*(c + d/x^2)^2*d^2 - 25*(c + d/x^2)*c*d^2 + 8*c^2*d^2)/((c +
d/x^2)^(5/2)*c^3 - 2*(c + d/x^2)^(3/2)*c^4 + sqrt(c + d/x^2)*c^5) + 15*d^2*
log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(7/2)) + 1/4
*b*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)*c^
3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/
2))
```

**mupad [B]** time = 6.34, size = 134, normalized size = 1.14

$$\frac{a\,x^4}{4\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{15\,a\,d^2}{8\,c^3\,\sqrt{c+\frac{d}{x^2}}}+\frac{b\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{3\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{15\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{7/2}}+\frac{3\,b\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}-\frac{5\,a\,d\,x^2}{8\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^4)/(4*c*(c + d/x^2)^(1/2)) - (15*a*d^2)/(8*c^3*(c + d/x^2)^(1/2)) + (b
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (15*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(7/2)) + (3*b*
d)/(2*c^2*(c + d/x^2)^(1/2)) - (5*a*d*x^2)/(8*c^2*(c + d/x^2)^(1/2))
```

**sympy [A]** time = 103.90, size = 177, normalized size = 1.50

$$a\left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{5\sqrt{d}\,x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}}+\frac{15d^2\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}}\right)+b\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}\,x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**5/(4*c*sqrt(d)*sqrt(c*x**2/d + 1)) - 5*sqrt(d)*x**3/(8*c**2*sqrt(c*x*
*2/d + 1)) - 15*d**(3/2)*x/(8*c**3*sqrt(c*x**2/d + 1)) + 15*d**2*asinh(sqrt
(c)*x/sqrt(d))/(8*c**(7/2))) + b*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3
*sqrt(d)*x/(2*c**2*sqrt(c*x**2/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c*
*(5/2)))
```

## 3.628 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=86

$$\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

**Rubi [A]** time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 51, 63, 208}

$$-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] -(2*b*c - 3*a*d)/(2*c^2*Sqrt[c + d/x^2]) + (a*x^2)/(2*c*Sqrt[c + d/x^2]) +
((2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(bc-\frac{3ad}{2}\right)\operatorname{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c^2}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{2c^2 d}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} + \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

**Mathematica** [A] time = 0.22, size = 89, normalized size = 1.03

$$\frac{\sqrt{c}\,x\left(acx^2+3ad-2bc\right)-\sqrt{d}\sqrt{\frac{cx^2}{d}+1}\,(3ad-2bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{5/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(-2*b*c + 3*a*d + a*c*x^2) - Sqrt[d]*(-2*b*c + 3*a*d)*Sqrt[1 + (
c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(2*c^(5/2)*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic** [A] time = 0.18, size = 93, normalized size = 1.08

$$\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}} + \frac{\sqrt{\frac{cx^2+d}{x^2}}\left(acx^4+3adx^2-2bcx^2\right)}{2c^2\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-2*b*c*x^2 + 3*a*d*x^2 + a*c*x^4))/(2*c^2*(d + c*x^
2)) + ((2*b*c - 3*a*d)*ArcTanh[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/(2*c^(5/2))
```

**fricas [A]** time = 0.44, size = 249, normalized size = 2.90

$$\left[-\frac{\left(2\,bcd-3\,ad^{2}+\left(2\,bc^{2}-3\,acd\right)x^{2}\right)\sqrt{c}\log\left(-2\,cx^{2}+2\,\sqrt{c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\,\left(ac^{2}x^{4}-\left(2\,bc^{2}-3\,acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{4\,\left(c^{4}x^{2}+c^{3}d\right)},-\frac{\left(2\,bcd-3\,ad^{2}+\left(2\,bc^{2}-3\,acd\right)x^{2}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)-\left(ac^{2}x^{4}-\left(2\,bc^{2}-3\,acd\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{2\,\left(c^{4}x^{2}+c^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a*c
*d)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*d^2
 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x
^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/(c^4*x^2 + c^3*d)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

**maple [A]** time = 0.06, size = 115, normalized size = 1.34

$$-\frac{\left(c\,x^{2}+d\right)\left(-a\,c^{\frac{5}{2}}x^{3}-3a\,c^{\frac{3}{2}}dx+2b\,c^{\frac{5}{2}}x+3\sqrt{c\,x^{2}+d}\,acd\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)-2\sqrt{c\,x^{2}+d}\,b\,c^{2}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\right)}{2\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}c^{\frac{7}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(3/2),x)
```

```
[Out] -1/2*(c*x^2+d)*(-c^(5/2)*x^3*a-3*c^(3/2)*x*a*d+2*c^(5/2)*x*b+3*(c*x^2+d)^(1
/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d-2*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x
^2+d)^(1/2))*b*c^2)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(7/2)
```

**maxima [B]** time = 1.20, size = 144, normalized size = 1.67

$$\frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^{2}}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}c^{2}-\sqrt{c+\frac{d}{x^{2}}}\,c^{3}}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)-\frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^{2}}}\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)
*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(5/2)) - 1/2*b*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))
)/c^(3/2) + 2/(sqrt(c + d/x^2)*c))
```

**mupad** [B] time = 5.61, size = 90, normalized size = 1.05

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{b}{c\,\sqrt{c+\frac{d}{x^2}}}+\frac{a\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{3\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{3\,a\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - b/(c*(c + d/x^2)^(1/2)) + (a
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (3*a*d)/(2*c^2*(c + d/x^2)^(1/2))
```

**sympy** [B] time = 52.42, size = 264, normalized size = 3.07

$$a\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}\,x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)+b\left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^3x^2\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^3x^2\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^2d\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^2d\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2/
d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(5/2))) + b*(-2*c**3*x**2*sqrt
(1 + d/(c*x**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**3*x**2*log(d/(c*x**2
))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) + 2*c**3*x**2*log(sqrt(1 + d/(c*x**2))
+ 1)/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**2*d*log(d/(c*x**2))/(2*c**(9/2)*
x**2 + 2*c**(7/2)*d) + 2*c**2*d*log(sqrt(1 + d/(c*x**2)) + 1)/(2*c**(9/2)*x
**2 + 2*c**(7/2)*d))
```

## 3.629 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx$

**Optimal**. Leaf size=52

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^
(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$
$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}$$
$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

**Mathematica [A]** time = 0.09, size = 73, normalized size = 1.40

$$\frac{\sqrt{c}\,x(bc-ad)+ad^{3/2}\sqrt{\frac{cx^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}dx\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (Sqrt[c]*(b*c - a*d)*x + a*d^(3/2)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/
Sqrt[d]])/(c^(3/2)*d*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 73, normalized size = 1.40

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{\frac{cx^2+d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{x^2\sqrt{\frac{cx^2+d}{x^2}}\,(ad-bc)}{cd\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] -(((-(b*c) + a*d)*x^2*Sqrt[(d + c*x^2)/x^2])/(c*d*(d + c*x^2))) + (a*ArcTan
h[Sqrt[(d + c*x^2)/x^2]/Sqrt[c]])/c^(3/2)
```

**fricas [B]** time = 0.46, size = 200, normalized size = 3.85

$$\left[\frac{2\left(bc^2-acd\right)x^2\sqrt{\frac{cx^2+d}{x^2}}+\left(acdx^2+ad^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{2\left(c^3dx^2+c^2d^2\right)}, \frac{\left(bc^2-acd\right)x^2\sqrt{\frac{cx^2+d}{x^2}}-\left(acdx^2+ad^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{c^3dx^2+c^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) + (a*c*d*x^2 + a*d^2)*sqr
t(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/(c^3*d*x^2 +
c^2*d^2), ((b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) - (a*c*d*x^2 + a*d^2)*
sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^3*d*x^2
 + c^2*d^2)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

**maple [A]** time = 0.06, size = 75, normalized size = 1.44

$$\frac{\left(c\,x^{2}+d\right)\left(-a\,c^{\frac{3}{2}}dx+b\,c^{\frac{5}{2}}x+\sqrt{c\,x^{2}+d}\;acd\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+d}\right)\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}c^{\frac{5}{2}}d\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x)
```

```
[Out] (c*x^2+d)*(-a*c^(3/2)*d*x+b*c^(5/2)*x+(c*x^2+d)^(1/2)*a*c*d*ln(c^(1/2)*x+(c
*x^2+d)^(1/2)))/((c*x^2+d)/x^2)^(3/2)/x^3/c^(5/2)/d
```

**maxima [A]** time = 1.26, size = 69, normalized size = 1.33

$$-\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^{2}}}\,c}\right)+\frac{b}{\sqrt{c+\frac{d}{x^{2}}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2
) + 2/(sqrt(c + d/x^2)*c)) + b/(sqrt(c + d/x^2)*d)
```

**mupad [B]** time = 5.06, size = 54, normalized size = 1.04

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a}{c\,\sqrt{c+\frac{d}{x^{2}}}}+\frac{b\,\sqrt{x^{2}}}{d\,\sqrt{c\,x^{2}+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(3/2)),x)
```

```
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - a/(c*(c + d/x^2)^(1/2)) + (b
*(x^2)^(1/2))/(d*(d + c*x^2)^(1/2))
```

**sympy [A]** time = 20.33, size = 49, normalized size = 0.94

$$-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{c\sqrt{-c}}-\frac{ad-bc}{cd\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*atan(sqrt(c + d/x**2)/sqrt(-c))/(c*sqrt(-c)) - (a*d - b*c)/(c*d*sqrt(c +
 d/x**2))
```

## 3.630 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx$

**Optimal**. Leaf size=42

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

**Rubi** [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

```
[Out] -((b*c - a*d)/(d^2*Sqrt[c + d/x^2])) - (b*Sqrt[c + d/x^2])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{a+bx}{(c+dx)^{3/2}} \, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right) dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 36, normalized size = 0.86

$$\frac{adx^2-b\left(2cx^2+d\right)}{d^2x^2\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
[Out] (a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)
```

**IntegrateAlgebraic [A]** time = 0.06, size = 46, normalized size = 1.10

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(adx^2-2bcx^2-bd\right)}{d^2\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d) - 2*b*c*x^2 + a*d*x^2))/(d^2*(d + c*x^2))
```

**fricas [A]** time = 0.42, size = 46, normalized size = 1.10

$$-\frac{\left((2\,bc-ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2+d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] -((2*b*c - a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2)/(c*d^2*x^2 + d^3)
```

**giac [A]** time = 0.26, size = 37, normalized size = 0.88

$$-\frac{\frac{(2\,bc-ad)x^2}{d^2}+\frac{b}{d}}{\sqrt{cx^4+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
[Out] -((2*b*c - a*d)*x^2/d^2 + b/d)/sqrt(c*x^4 + d*x^2)
```

**maple [A]** time = 0.05, size = 46, normalized size = 1.10

$$\frac{\left(ad\,x^2-2bc\,x^2-bd\right)\left(c\,x^2+d\right)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x)
[Out] (a*d*x^2-2*b*c*x^2-b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^2/x^4
```

**maxima [A]** time = 0.46, size = 46, normalized size = 1.10

$$-b\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)+\frac{a}{\sqrt{c+\frac{d}{x^2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] -b*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2)) + a/(sqrt(c + d/x^2)*d)
```

**mupad [B]** time = 4.52, size = 46, normalized size = 1.10

$$\frac{x\,\sqrt{c+\frac{d}{x^2}}\,\left(x^2\,\left(\frac{a}{d}-\frac{2\,b\,c}{d^2}\right)-\frac{b}{d}\right)}{c\,x^3+d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(3/2)),x)
```

```
[Out] (x*(c + d/x^2)^(1/2)*(x^2*(a/d - (2*b*c)/d^2) - b/d))/(d*x + c*x^3)
```

**sympy [A]** time = 3.42, size = 68, normalized size = 1.62

$$\begin{cases} \frac{a}{d\sqrt{c+\frac{d}{x^2}}} - \frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}} - \frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for}\ d \neq 0 \\ \frac{-\frac{a}{2x^2}-\frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x*
*2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), True
))
```

## 3.631 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx$

**Optimal**. Leaf size=68

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + ((2*b*c - a*d)*Sqrt[c + d/x^2])/d^3
 - (b*(c + d/x^2)^(3/2))/(3*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)}{(c+dx)^{3/2}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{-2bc+ad}{d^2\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d^2}\right)dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 60, normalized size = 0.88

$$\frac{b\left(8c^2x^4+4cdx^2-d^2\right)-3adx^2\left(2cx^2+d\right)}{3d^3x^4\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[c
 + d/x^2]*x^4)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 75, normalized size = 1.10

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-6acdx^4-3ad^2x^2+8bc^2x^4+4bcdx^2-bd^2\right)}{3d^3x^2\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-(b*d^2) + 4*b*c*d*x^2 - 3*a*d^2*x^2 + 8*b*c^2*x^4
- 6*a*c*d*x^4))/(3*d^3*x^2*(d + c*x^2))
```

**fricas** [A] time = 0.42, size = 73, normalized size = 1.07

$$\frac{\left(2\left(4\,bc^2-3\,acd\right)x^4-bd^2+\left(4\,bcd-3\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(cd^3x^4+d^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(4*b*c^2 - 3*a*c*d)*x^4 - b*d^2 + (4*b*c*d - 3*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2)/(c*d^3*x^4 + d^4*x^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^5), x)
```

**maple** [A] time = 0.05, size = 69, normalized size = 1.01

$$-\frac{\left(6acd\,x^4-8b\,c^2x^4+3a\,d^2x^2-4bcd\,x^2+b\,d^2\right)\left(c\,x^2+d\right)}{3\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x)
```

```
[Out] -1/3*(6*a*c*d*x^4-8*b*c^2*x^4+3*a*d^2*x^2-4*b*c*d*x^2+b*d^2)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^3/x^6
```

**maxima [A]** time = 0.61, size = 81, normalized size = 1.19

$$-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\sqrt{c+\frac{d}{x^2}}\,c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right)-a\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^3 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d
/x^2)*d^3)) - a*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2))
```

**mupad [B]** time = 4.64, size = 66, normalized size = 0.97

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(-8\,b\,c^2\,x^4+6\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+3\,a\,d^2\,x^2+b\,d^2\right)}{3\,d^3\,x^2\,\left(c\,x^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d^2 + 3*a*d^2*x^2 - 8*b*c^2*x^4 + 6*a*c*d*x^4 - 4*b*
c*d*x^2))/(3*d^3*x^2*(d + c*x^2))
```

**sympy [A]** time = 11.07, size = 61, normalized size = 0.90

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d^3}-\frac{c\,(ad-bc)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad-2bc)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -b*(c + d/x**2)**(3/2)/(3*d**3) - c*(a*d - b*c)/(d**3*sqrt(c + d/x**2)) - s
qrt(c + d/x**2)*(a*d - 2*b*c)/d**3
```

## 3.632 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx$

**Optimal**. Leaf size=100

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

**Rubi** [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] -((c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2])) - (c*(3*b*c - 2*a*d)*Sqrt[c + d/
x^2])/d^4 + ((3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (b*(c + d/x^2)^(5/2
))/(5*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(a+bx)}{(c+dx)^{3/2}} \, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3(c+dx)^{3/2}}+\frac{c(3bc-2ad)}{d^3\sqrt{c+dx}}+\frac{(-3bc+ad)\sqrt{c+dx}}{d^3}+\frac{b(c+dx)^{3/2}}{d^3}\right)\right.\right.\\
&= -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 81, normalized size = 0.81

$$\frac{-5adx^2\left(-8c^2x^4-4cdx^2+d^2\right)-3b\left(16c^3x^6+8c^2dx^4-2cd^2x^2+d^3\right)}{15d^4x^6\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] (-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2*
d*x^4 + 16*c^3*x^6))/(15*d^4*Sqrt[c + d/x^2]*x^6)
```

**IntegrateAlgebraic** [A] time = 0.10, size = 99, normalized size = 0.99

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(40ac^2dx^6+20acd^2x^4-5ad^3x^2-48bc^3x^6-24bc^2dx^4+6bcd^2x^2-3bd^3\right)}{15d^4x^4\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-3*b*d^3 + 6*b*c*d^2*x^2 - 5*a*d^3*x^2 - 24*b*c^2*d
*x^4 + 20*a*c*d^2*x^4 - 48*b*c^3*x^6 + 40*a*c^2*d*x^6))/(15*d^4*x^4*(d + c*
x^2))
```

**fricas** [A] time = 0.43, size = 98, normalized size = 0.98

$$-\frac{\left(8\left(6\,bc^3-5\,ac^2d\right)x^6+4\left(6\,bc^2d-5\,acd^2\right)x^4+3\,bd^3-\left(6\,bcd^2-5\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(cd^4x^6+d^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d^
3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^4*x^6 + d^5*x^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^7), x)
```

**maple** [A] time = 0.05, size = 94, normalized size = 0.94

$$\frac{\left(40a\,c^2d\,x^6-48b\,c^3x^6+20ac\,d^2x^4-24b\,c^2d\,x^4-5a\,d^3x^2+6bc\,d^2x^2-3b\,d^3\right)\left(c\,x^2+d\right)}{15\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x)
```

```
[Out] 1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^2
+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^4/x^8
```

**maxima [A]** time = 0.62, size = 116, normalized size = 1.16

$$-\frac{1}{5}b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}\,d^4}\right)-\frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\sqrt{c+\frac{d}{x^2}}\,c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/5*b*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c + d/x
^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4)) - 1/3*a*((c + d/x^2)^(3/2)/d^3 -
 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d/x^2)*d^3))
```

**mupad [B]** time = 4.84, size = 91, normalized size = 0.91

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3\,x^6-40\,a\,c^2\,d\,x^6+24\,b\,c^2\,d\,x^4-20\,a\,c\,d^2\,x^4-6\,b\,c\,d^2\,x^2+5\,a\,d^3\,x^2+3\,b\,d^3\right)}{15\,d^4\,x^4\,\left(c\,x^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(3*b*d^3 + 5*a*d^3*x^2 + 48*b*c^3*x^6 - 20*a*c*d^2*x^4
- 40*a*c^2*d*x^6 - 6*b*c*d^2*x^2 + 24*b*c^2*d*x^4))/(15*d^4*x^4*(d + c*x^2)
)
```

**sympy [A]** time = 13.28, size = 90, normalized size = 0.90

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d^4}+\frac{c^2\,(ad-bc)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}(ad-3bc)}{3d^4}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(-2acd+3bc^2\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -b*(c + d/x**2)**(5/2)/(5*d**4) + c**2*(a*d - b*c)/(d**4*sqrt(c + d/x**2))
- (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(3*d**4) - sqrt(c + d/x**2)*(-2*a*c*d +
 3*b*c**2)/d**4
```

## 3.633 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx$

**Optimal**. Leaf size=126

$$\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

**Rubi [A]** time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{d^5}+\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (c^2*(4*b*c - 3*a*d)*Sqrt[c + d/x
^2])/d^5 - (c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + ((4*b*c - a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (b*(c + d/x^2)^(7/2))/(7*d^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(a+bx)}{(c+dx)^{3/2}} \, dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c^3(bc-ad)}{d^4(c+dx)^{3/2}}-\frac{c^2(4bc-3ad)}{d^4\sqrt{c+dx}}+\frac{3c(2bc-ad)\sqrt{c+dx}}{d^4}+\frac{(-4bc+ad)(c+}{d^4}\right.\right.\right.$$

$$= \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5}-\frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-$$

**Mathematica [A]** time = 0.04, size = 104, normalized size = 0.83

$$\frac{b\left(128c^4x^8+64c^3dx^6-16c^2d^2x^4+8cd^3x^2-5d^4\right)-7adx^2\left(16c^3x^6+8c^2dx^4-2cd^2x^2+d^3\right)}{35d^5x^8\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8*
c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*Sqrt[c +
d/x^2]*x^8)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 123, normalized size = 0.98

$$\frac{\sqrt{\frac{cx^2+d}{x^2}}\left(-112ac^3dx^8-56ac^2d^2x^6+14acd^3x^4-7ad^4x^2+128bc^4x^8+64bc^3dx^6-16bc^2d^2x^4+8bcd^3x^2-5bd^4\right)}{35d^5x^6\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (Sqrt[(d + c*x^2)/x^2]*(-5*b*d^4 + 8*b*c*d^3*x^2 - 7*a*d^4*x^2 - 16*b*c^2*d
^2*x^4 + 14*a*c*d^3*x^4 + 64*b*c^3*d*x^6 - 56*a*c^2*d^2*x^6 + 128*b*c^4*x^8
 - 112*a*c^3*d*x^8))/(35*d^5*x^6*(d + c*x^2))
```

**fricas [A]** time = 0.45, size = 121, normalized size = 0.96

$$\frac{\left(16\left(8\,bc^4-7\,ac^3d\right)x^8+8\left(8\,bc^3d-7\,ac^2d^2\right)x^6-5\,bd^4-2\left(8\,bc^2d^2-7\,acd^3\right)x^4+\left(8\,bcd^3-7\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\left(cd^5x^8+d^6x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b*
d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((c*
x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^9), x)
```

**maple [A]** time = 0.05, size = 118, normalized size = 0.94

$$-\frac{\left(112a\,c^3d\,x^8-128b\,c^4x^8+56a\,c^2d^2x^6-64b\,c^3d\,x^6-14ac\,d^3x^4+16b\,c^2d^2x^4+7a\,d^4x^2-8bc\,d^3x^2+5b\,d^4\right)\left(c\,x^2+d\right)}{35\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x)
```

```
[Out] -1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*c
*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^5/x^10
```

**maxima** [A] time = 0.55, size = 151, normalized size = 1.20

$$-\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\sqrt{c+\frac{d}{x^2}}\,c^3}{d^5}-\frac{35\,c^4}{\sqrt{c+\frac{d}{x^2}}\,d^5}\right)-\frac{1}{5}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/x
^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)*d
^5)) - 1/5*a*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c
 + d/x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4))
```

**mupad** [B] time = 4.92, size = 154, normalized size = 1.22

$$\frac{c\sqrt{c+\frac{d}{x^2}}\,(21\,a\,d-29\,b\,c)}{35\,d^4\,x^2}-\frac{b\sqrt{c+\frac{d}{x^2}}}{7\,d^2\,x^6}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(7\,a\,d^2-13\,b\,c\,d\right)}{35\,d^4\,x^4}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(x^2\,\left(\frac{58\,b\,c^4-42\,a\,c^3\,d}{35\,d^5}+\frac{2\,c^3\,(77\,a\,d-93\,b\,c)}{35\,d^5}\right)+\frac{c^2\,(77\,a\,d-93\,b\,c)}{35\,d^4}\right)}{c\,x^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^9*(c + d/x^2)^(3/2)),x)
```

```
[Out] (c*(c + d/x^2)^(1/2)*(21*a*d - 29*b*c))/(35*d^4*x^2) - (b*(c + d/x^2)^(1/2)
)/(7*d^2*x^6) - ((c + d/x^2)^(1/2)*(7*a*d^2 - 13*b*c*d))/(35*d^4*x^4) - ((c
 + d/x^2)^(1/2)*(x^2*((58*b*c^4 - 42*a*c^3*d)/(35*d^5) + (2*c^3*(77*a*d - 9
3*b*c))/(35*d^5)) + (c^2*(77*a*d - 93*b*c))/(35*d^4)))/(d + c*x^2)
```

**sympy** [A] time = 16.14, size = 122, normalized size = 0.97

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7d^5}-\frac{c^3\,(ad-bc)}{d^5\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{5d^5}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\left(-3acd+6bc^2\right)}{3d^5}-\frac{\sqrt{c+\frac{d}{x^2}}\left(3ac^2d-4bc^3\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -b*(c + d/x**2)**(7/2)/(7*d**5) - c**3*(a*d - b*c)/(d**5*sqrt(c + d/x**2))
- (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(5*d**5) - (c + d/x**2)**(3/2)*(-3*a*c*
d + 6*b*c**2)/(3*d**5) - sqrt(c + d/x**2)*(3*a*c**2*d - 4*b*c**3)/d**5
```

## 3.634 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=111

$$-\frac{8dx\sqrt{c+\frac{d}{x^2}}\,(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

**Rubi** [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 271, 192, 191}

$$\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}-\frac{8dx\sqrt{c+\frac{d}{x^2}}\,(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (4*d*(5*b*c - 6*a*d)*x)/(15*c^3*Sqrt[c + d/x^2]) - (8*d*(5*b*c - 6*a*d)*Sqr
t[c + d/x^2]*x)/(15*c^4) + ((5*b*c - 6*a*d)*x^3)/(15*c^2*Sqrt[c + d/x^2]) +
 (a*x^5)/(5*c*Sqrt[c + d/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}} + \frac{(5bc-6ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{5c}$$

$$= \frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}} - \frac{(4d(5bc-6ad))\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{15c^2}$$

$$= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}} + \frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}} - \frac{(8d(5bc-6ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^3}$$

$$= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}} - \frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4} + \frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

**Mathematica [A]** time = 0.08, size = 80, normalized size = 0.72

$$\frac{3a\left(c^3x^6-2c^2dx^4+8cd^2x^2+16d^3\right)+5bc\left(c^2x^4-4cdx^2-8d^2\right)}{15c^4x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*d
*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 90, normalized size = 0.81

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(3ac^3x^6-6ac^2dx^4+24acd^2x^2+48ad^3+5bc^3x^4-20bc^2dx^2-40bcd^2\right)}{15c^4\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(-40*b*c*d^2 + 48*a*d^3 - 20*b*c^2*d*x^2 + 24*a*c*d^2*x^
2 + 5*b*c^3*x^4 - 6*a*c^2*d*x^4 + 3*a*c^3*x^6))/(15*c^4*(d + c*x^2))
```

**fricas [A]** time = 0.43, size = 95, normalized size = 0.86

$$\frac{\left(3\,ac^3x^7+\left(5\,bc^3-6\,ac^2d\right)x^5-4\left(5\,bc^2d-6\,acd^2\right)x^3-8\left(5\,bcd^2-6\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(c^5x^2+c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*x
^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple** [A] time = 0.06, size = 91, normalized size = 0.82

$$\frac{\left(3a\,x^6c^3-6a\,c^2d\,x^4+5b\,c^3x^4+24ac\,d^2x^2-20b\,c^2d\,x^2+48a\,d^3-40bc\,d^2\right)\left(c\,x^2+d\right)}{15\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/15*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+4
8*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4
```

**maxima** [A] time = 0.56, size = 128, normalized size = 1.15

$$\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\sqrt{c+\frac{d}{x^2}}\,dx}{c^3}-\frac{3\,d^2}{\sqrt{c+\frac{d}{x^2}}\,c^3x}\right)+\frac{1}{5}\,a\left(\frac{5\,d^3}{\sqrt{c+\frac{d}{x^2}}\,c^4x}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\sqrt{c+\frac{d}{x^2}}\,d^2x}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c
+ d/x^2)*c^3*x)) + 1/5*a*(5*d^3/(sqrt(c + d/x^2)*c^4*x) + ((c + d/x^2)^(5/2
)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)/c^4)
```

**mupad** [B] time = 5.77, size = 79, normalized size = 0.71

$$\frac{3\,a\,c^3\,x^6+5\,b\,c^3\,x^4-6\,a\,c^2\,d\,x^4-20\,b\,c^2\,d\,x^2+24\,a\,c\,d^2\,x^2-40\,b\,c\,d^2+48\,a\,d^3}{15\,c^4\,x\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (48*a*d^3 + 3*a*c^3*x^6 + 5*b*c^3*x^4 - 40*b*c*d^2 + 24*a*c*d^2*x^2 - 6*a*c
^2*d*x^4 - 20*b*c^2*d*x^2)/(15*c^4*x*(c + d/x^2)^(1/2))
```

**sympy** [B] time = 7.83, size = 561, normalized size = 5.05

$$a\left(\frac{c^5d^{\frac{19}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{5c^3d^{\frac{23}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{30c^2d^{\frac{25}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{40cd^{\frac{27}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}+\frac{16d^{\frac{29}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}\right)+b\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**5*d**(19/2)*x**10*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**1
0*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*sqrt(c*
```

```
x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5
*c**4*d**12) + 30*c**2*d**(25/2)*x**4*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6
+ 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(27/2)*
x**2*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d*
*11*x**2 + 5*c**4*d**12) + 16*d**(29/2)*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**
6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12)) + b*(c**3*d**(
9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*
d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4
*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d + 1)/(3*c**5*
d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(c*x**2/d + 1
)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6))
```

## 3.635 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

**Optimal**. Leaf size=79

$$\frac{2x\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

**Rubi [A]** time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 192, 191}

$$\frac{2x\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((3*b*c - 4*a*d)*x)/(3*c^2*Sqrt[c + d/x^2]) + (2*(3*b*c - 4*a*d)*Sqrt[c +
d/x^2]*x)/(3*c^3) + (a*x^3)/(3*c*Sqrt[c + d/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}} + \frac{(3bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{3c}$$

$$= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}} + \frac{(2(3bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c^2}$$

$$= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}} + \frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3} + \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

**Mathematica [A]** time = 0.05, size = 57, normalized size = 0.72

$$\frac{a\left(c^2x^4-4cdx^2-8d^2\right)+3bc\left(cx^2+2d\right)}{3c^3x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] (3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d/
x^2]*x)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 65, normalized size = 0.82

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(ac^2x^4-4acdx^2-8ad^2+3bc^2x^2+6bcd\right)}{3c^3\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(6*b*c*d - 8*a*d^2 + 3*b*c^2*x^2 - 4*a*c*d*x^2 + a*c^2*x
^4))/(3*c^3*(d + c*x^2))
```

**fricas [A]** time = 0.43, size = 70, normalized size = 0.89

$$\frac{\left(ac^2x^5+\left(3\,bc^2-4\,acd\right)x^3+2\left(3\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(c^4x^2+c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((c
*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple [A]** time = 0.06, size = 66, normalized size = 0.84

$$\frac{\left(a\,x^{4} c^{2}-4 a c d\,x^{2}+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 b c d\right) \left(c\,x^{2}+d\right)}{3 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{3} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+d
)/x^2)^(3/2)/c^3/x^3
```

**maxima [A]** time = 0.53, size = 90, normalized size = 1.14

$$b\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x}{c^{2}}+\frac{d}{\sqrt{c+\frac{d}{x^{2}}}\,c^{2} x}\right)+\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}} x^{3}-6\,\sqrt{c+\frac{d}{x^{2}}}\,dx}{c^{3}}-\frac{3\,d^{2}}{\sqrt{c+\frac{d}{x^{2}}}\,c^{3} x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2)
^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))
```

**mupad [B]** time = 5.20, size = 81, normalized size = 1.03

$$\frac{b\,c^{2}\,x^{4}+3\,b\,c\,d\,x^{2}+2\,b\,d^{2}}{c^{2}\,x^{3}\,{\left(c+\frac{d}{x^{2}}\right)}^{3/2}}-\frac{-a\,c^{2}\,x^{4}+4\,a\,c\,d\,x^{2}+8\,a\,d^{2}}{3\,c^{3}\,x\,\sqrt{c+\frac{d}{x^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (2*b*d^2 + b*c^2*x^4 + 3*b*c*d*x^2)/(c^2*x^3*(c + d/x^2)^(3/2)) - (8*a*d^2
- a*c^2*x^4 + 4*a*c*d*x^2)/(3*c^3*x*(c + d/x^2)^(1/2))
```

**sympy [B]** time = 7.31, size = 267, normalized size = 3.38

$$a\left(\frac{c^{3} d^{\frac{9}{2}} x^{6} \sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}-\frac{3c^{2}d^{\frac{11}{2}} x^{4} \sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}-\frac{12cd^{\frac{13}{2}} x^{2} \sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}-\frac{8d^{\frac{15}{2}} \sqrt{\frac{cx^{2}}{d}+1}}{3c^{5}d^{4}x^{4}+6c^{4}d^{5}x^{2}+3c^{3}d^{6}}\right)+b\left(\frac{x^{2}}{c\sqrt{d}\sqrt{\frac{cx^{2}}{d}+1}}+\frac{2\sqrt{d}}{c^{2}\sqrt{\frac{cx^{2}}{d}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x*
*2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x
**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d +
 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(
c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b*(x**
2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)))
```

## 3.636 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx$

**Optimal**. Leaf size=45

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

**Rubi [A]** time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 453, 191}

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} \, dx &= -\text{Subst}\left(\int \frac{a+bx^2}{x^2\left(c+dx^2\right)^{3/2}} \, dx,x,\frac{1}{x}\right)\\
&= \frac{ax}{c\sqrt{c+\frac{d}{x^2}}}+\frac{(-bc+2ad)\,\text{Subst}\left(\int \frac{1}{\left(c+dx^2\right)^{3/2}} \, dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 33, normalized size = 0.73

$$\frac{acx^2+2ad-bc}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2)^(3/2),x]
[Out] (-(b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic** [A] time = 0.08, size = 40, normalized size = 0.89

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(acx^2+2ad-bc\right)}{c^2\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(c + d/x^2)^(3/2),x]
[Out] (Sqrt[c + d/x^2]*x*(-(b*c) + 2*a*d + a*c*x^2))/(c^2*(d + c*x^2))
```

**fricas** [A] time = 0.48, size = 47, normalized size = 1.04

$$\frac{\left(acx^3-(bc-2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
[Out] (a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple** [A] time = 0.04, size = 43, normalized size = 0.96

$$\frac{\left(a\,x^2c+2ad-bc\right)\left(c\,x^2+d\right)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2),x)
[Out] (a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^2
```

**maxima** [A] time = 0.68, size = 53, normalized size = 1.18

$$a\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}\,c^2x}\right)-\frac{b}{\sqrt{c+\frac{d}{x^2}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)*
c*x)
```

**mupad** [B] time = 4.90, size = 38, normalized size = 0.84

$$\frac{\left(c\,x^2+d\right)\,\left(a\,c\,x^2+2\,a\,d-b\,c\right)}{c^2\,x^3\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2)^(3/2),x)
```

```
[Out] ((d + c*x^2)*(2*a*d - b*c + a*c*x^2))/(c^2*x^3*(c + d/x^2)^(3/2))
```

**sympy** [A] time = 7.51, size = 65, normalized size = 1.44

$$a\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)-\frac{b}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)
)) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))
```

**3.637** $\displaystyle\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx$

**Optimal**. Leaf size=59

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {452, 335, 217, 206}

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x
)])/d^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} + \frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{d}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d^{3/2}}$$

**Mathematica** [A] time = 0.04, size = 71, normalized size = 1.20

$$\frac{\sqrt{d}\,(bc-ad) - bc\sqrt{cx^2+d}\,\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{cd^{3/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]
)/(c*d^(3/2)*Sqrt[c + d/x^2]*x)
```

**IntegrateAlgebraic** [A] time = 0.12, size = 77, normalized size = 1.31

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{bc-ad}{cd\sqrt{cx^2+d}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{d^{3/2}}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((b*c - a*d)/(c*d*Sqrt[d + c*x^2]) - (b*ArcTanh[Sqrt[d +
 c*x^2]/Sqrt[d]])/d^(3/2)))/Sqrt[d + c*x^2]
```

**fricas** [A] time = 0.46, size = 195, normalized size = 3.31

$$\left[\frac{2\left(bcd-ad^2\right)x\sqrt{\frac{cx^2+d}{x^2}} + \left(bc^2x^2+bcd\right)\sqrt{d}\,\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\left(c^2d^2x^2+cd^3\right)}, \frac{\left(bcd-ad^2\right)x\sqrt{\frac{cx^2+d}{x^2}} + \left(bc^2x^2+bcd\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{c^2d^2x^2+cd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt(
d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*x^
2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*
sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^2*d^2*x^2
 + c*d^3)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple [A]** time = 0.06, size = 79, normalized size = 1.34

$$-\frac{\left(c\,x^{2}+d\right)\left(\sqrt{c\,x^{2}+d}\;b c d\ln\left(\frac{2 d+2\sqrt{c\,x^{2}+d}\;\sqrt{d}}{x}\right)+a\,d^{\frac{5}{2}}-b c\,d^{\frac{3}{2}}\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c\,d^{\frac{5}{2}} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x)
```

```
[Out] -(c*x^2+d)*((c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*b*c*d+a*d^(
5/2)-d^(3/2)*b*c)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)
```

**maxima [A]** time = 1.21, size = 80, normalized size = 1.36

$$\frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^{2}}}\,dx}\right)-\frac{a}{\sqrt{c+\frac{d}{x^{2}}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(
3/2) + 2/(sqrt(c + d/x^2)*d*x)) - a/(sqrt(c + d/x^2)*c*x)
```

**mupad [B]** time = 5.11, size = 60, normalized size = 1.02

$$\frac{b}{d\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{a}{c\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(3/2)),x)
```

```
[Out] b/(d*x*(c + d/x^2)^(1/2)) - a/(c*x*(c + d/x^2)^(1/2)) - (b*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/d^(3/2)
```

**sympy [B]** time = 11.71, size = 206, normalized size = 3.49

$$-\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+b\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7
/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**
(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2
*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*l
og(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))
```

## 3.638 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx$

**Optimal**. Leaf size=92

$$\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

**Rubi [A]** time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 288, 217, 206}

$$-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}+\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] -b/(2*d*Sqrt[c + d/x^2]*x^3) - (3*b*c - 2*a*d)/(2*d^2*Sqrt[c + d/x^2]*x) +
((3*b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{(-3bc+2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx}{2d}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{(-3bc+2ad)\,\text{Subst}\left(\int \frac{x^2}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2d}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{2d^2}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^2}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{5/2}}$$

**Mathematica [C]** time = 0.03, size = 57, normalized size = 0.62

$$\frac{x^2(2ad-3bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{d}+1\right)-bd}{2d^2x^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] (-(b*d) + (-3*b*c + 2*a*d)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/
d])/(2*d^2*Sqrt[c + d/x^2]*x^3)
```

**IntegrateAlgebraic [A]** time = 0.24, size = 101, normalized size = 1.10

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{2adx^2-3bcx^2-bd}{2d^2x^2\sqrt{cx^2+d}}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((-(b*d) - 3*b*c*x^2 + 2*a*d*x^2)/(2*d^2*x^2*Sqrt[d + c*
x^2]) + ((3*b*c - 2*a*d)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(5/2))))/Sq
rt[d + c*x^2]
```

**fricas [A]** time = 0.44, size = 248, normalized size = 2.70

$$\left[-\frac{\left(\left(3\,bc^{2}-2\,acd\right)x^{3}+\left(3\,bcd-2\,ad^{2}\right)x\right)\sqrt{d}\log\left(-\frac{cx^{2}-2\,\sqrt{d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}+2\,d}{x^{2}}\right)+2\,\left(bd^{2}+\left(3\,bcd-2\,ad^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{4\,\left(cd^{3}x^{3}+d^{4}x\right)},-\frac{\left(\left(3\,bc^{2}-2\,acd\right)x^{3}+\left(3\,bcd-2\,ad^{2}\right)x\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^{2}+d}{x^{2}}}}{cx^{2}+d}\right)+\left(bd^{2}+\left(3\,bcd-2\,ad^{2}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{2\,\left(cd^{3}x^{3}+d^{4}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x^
2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d - 2
*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2 -
2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^
2 + d)/x^2)/(c*x^2 + d)) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(c*d^3*x^3 + d^4*x)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple [A]** time = 0.06, size = 132, normalized size = 1.43

$$\frac{\left(c\,x^{2}+d\right)\left(-2\sqrt{c\,x^{2}+d}\;a\,d^{2}x^{2}\ln\left(\frac{2d+2\sqrt{c\,x^{2}+d}\;\sqrt{d}}{x}\right)+3\sqrt{c\,x^{2}+d}\;bcd\,x^{2}\ln\left(\frac{2d+2\sqrt{c\,x^{2}+d}\;\sqrt{d}}{x}\right)+2a\,d^{\frac{5}{2}}x^{2}-3bc\,d^{\frac{3}{2}}x^{2}-b\,d^{\frac{5}{2}}\right)}{2\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{\frac{7}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x)
```

```
[Out] 1/2*(c*x^2+d)*(2*d^(5/2)*x^2*a-3*d^(3/2)*x^2*b*c-2*ln(2*(d+(c*x^2+d)^(1/2)*
d^(1/2))/x)*(c*x^2+d)^(1/2)*x^2*a*d^2+3*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)
*(c*x^2+d)^(1/2)*x^2*b*c*d-d^(5/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^5/d^(7/2)
```

**maxima [B]** time = 1.22, size = 162, normalized size = 1.76

$$-\frac{1}{4}\,b\left(\frac{2\left(3\left(c+\frac{d}{x^{2}}\right)cx^{2}-2\,cd\right)}{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}d^{2}x^{3}-\sqrt{c+\frac{d}{x^{2}}}\,d^{3}x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)+\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^{2}}}\,dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)*d^2*x^3 - sqrt(c
 + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x
```

```
 + sqrt(d)))/d^(5/2)) + 1/2*a*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c +
d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^4\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)),x)
```

```
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)), x)
```

**sympy [B]** time = 18.44, size = 262, normalized size = 2.85

$$a\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)+b\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x*
*2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sq
rt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c
*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(
7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) +
3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 +
d/(c*x**2))))
```

### 3.639 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} \, dx$

**Optimal**. Leaf size=123

$$-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}+\frac{3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

**Rubi [A]** time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {459, 335, 288, 321, 217, 206}

$$\frac{3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] -b/(4*d*Sqrt[c + d/x^2]*x^5) - (5*b*c - 4*a*d)/(4*d^2*Sqrt[c + d/x^2]*x^3)
+ (3*(5*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^3*x) - (3*c*(5*b*c - 4*a*d)*ArcT
anh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6}\,dx = -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} + \frac{(-5bc+4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6}\,dx}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{(-5bc+4ad)\,\mathrm{Subst}\left(\int \frac{x^4}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{(3(5bc-4ad))\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{4d^2}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+d}}\right.}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\right.}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right.}{8d^{7/2}}$$

**Mathematica [C]** time = 0.03, size = 60, normalized size = 0.49

$$\frac{cx^4(5bc-4ad)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{d}+1\right)-bd^2}{4d^3x^5\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] (-(b*d^2) + c*(5*b*c - 4*a*d)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x^
2)/d])/(4*d^3*Sqrt[c + d/x^2]*x^5)
```

**IntegrateAlgebraic** [A] time = 0.25, size = 126, normalized size = 1.02

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\frac{-12acdx^4-4ad^2x^2+15bc^2x^4+5bcdx^2-2bd^2}{8d^3x^4\sqrt{cx^2+d}}-\frac{3\left(5bc^2-4acd\right)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{8d^{7/2}}\right)}{\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((-2*b*d^2 + 5*b*c*d*x^2 - 4*a*d^2*x^2 + 15*b*c^2*x^4 -
12*a*c*d*x^4)/(8*d^3*x^4*Sqrt[d + c*x^2]) - (3*(5*b*c^2 - 4*a*c*d)*ArcTanh[
Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(7/2))))/Sqrt[d + c*x^2]
```

**fricas** [A] time = 0.45, size = 314, normalized size = 2.55

$$\left[-\frac{3\left(\left(5\,bc^3-4\,ac^2d\right)x^5+\left(5\,bc^2d-4\,acd^2\right)x^3\right)\sqrt{d}\,\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(5\,bc^2d-4\,acd^2\right)x^4-2\,bd^3+\left(5\,bcd^2-4\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\left(cd^4x^5+d^5x^3\right)},\ \frac{3\left(\left(5\,bc^3-4\,ac^2d\right)x^5+\left(5\,bc^2d-4\,acd^2\right)x^3\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(3\left(5\,bc^2d-4\,acd^2\right)x^4-2\,bd^3+\left(5\,bcd^2-4\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\left(cd^4x^5+d^5x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d)
*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*c^
2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d
)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2
*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^
2 + d)) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*
x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

**maple** [A] time = 0.06, size = 157, normalized size = 1.28

$$\frac{\left(cx^2+d\right)\left(12\sqrt{cx^2+d}\,acd^2x^4\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-15\sqrt{cx^2+d}\,bc^2dx^4\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-12acd^{\frac{5}{2}}x^4+15bc^2d^{\frac{3}{2}}x^4-4ad^{\frac{7}{2}}x^2+5bcd^{\frac{5}{2}}x^2-2bd^{\frac{7}{2}}\right)}{8\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}d^{\frac{9}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x)
```

```
[Out] 1/8*(c*x^2+d)*(12*(c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*a
*c*d^2-15*(c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2*d-1
2*d^(5/2)*x^4*a*c+15*d^(3/2)*x^4*b*c^2-4*d^(7/2)*x^2*a+5*d^(5/2)*x^2*b*c-2*
d^(7/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^7/d^(9/2)
```

**maxima [B]** time = 1.28, size = 243, normalized size = 1.98

$$\frac{1}{16}\,b\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2c^2x^4-25\left(c+\frac{d}{x^2}\right)c^2dx^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3+\sqrt{c+\frac{d}{x^2}}\,d^5x}+\frac{15\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right)-\frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}\,d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/16*b*(2*(15*(c + d/x^2)^2*c^2*x^4 - 25*(c + d/x^2)*c^2*d*x^2 + 8*c^2*d^2)
/((c + d/x^2)^(5/2)*d^3*x^5 - 2*(c + d/x^2)^(3/2)*d^4*x^3 + sqrt(c + d/x^2)
*d^5*x) + 15*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqr
t(d)))/d^(7/2)) - 1/4*a*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)
*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(
sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^6\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)),x)
```

```
[Out] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)), x)
```

**sympy [A]** time = 29.81, size = 180, normalized size = 1.46

$$a\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}\,dx^3\sqrt{1+\frac{d}{cx^2}}}\right)+b\left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}}+\frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{15c^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{7}{2}}}-\frac{1}{4\sqrt{c}\,dx^5\sqrt{1+\frac{d}{cx^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**6,x)
```

```
[Out] a*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)*
x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2)))) + b*(15*c**(3
/2)/(8*d**3*x*sqrt(1 + d/(c*x**2))) + 5*sqrt(c)/(8*d**2*x**3*sqrt(1 + d/(c*
x**2))) - 15*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(7/2)) - 1/(4*sqrt(c)*d*
x**5*sqrt(1 + d/(c*x**2))))
```

## 3.640 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}\,dx$

**Optimal**. Leaf size=135

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}$$

**Rubi** [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {280, 323, 330, 52}

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{5}{64}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/64 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/96 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/24 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 - (5*ArcCosh[Sq
rt[x]])/64
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}\,dx = \frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}-\frac{1}{8}\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{24}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}-\frac{5}{48}\int \frac{x}{\sqrt{-1+\sqrt{x}}}$$

$$= -\frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{1}{4}\sqrt{-1+\sqrt{x}}$$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}$$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}$$

$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}$$

**Mathematica [A]** time = 0.07, size = 111, normalized size = 0.82

$$\frac{\sqrt{\sqrt{x}+1}\,\sqrt{x}\left(48x^{7/2}-8x^{5/2}-10x^{3/2}-48x^3+8x^2+10x-15\sqrt{x}+15\right)+30\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{192\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 - 15*Sqrt[x] + 10*x - 10*x^(3/2) + 8*x^2 - 8
*x^(5/2) - 48*x^3 + 48*x^(7/2)) + 30*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt
[x]]/Sqrt[2]])/(192*Sqrt[-1 + Sqrt[x]])
```

**IntegrateAlgebraic [A]** time = 2.44, size = 216, normalized size = 1.60

$$\frac{\left(\frac{15\left(\sqrt{x}-1\right)^7}{\left(\sqrt{x}+1\right)^7}+\frac{397\left(\sqrt{x}-1\right)^6}{\left(\sqrt{x}+1\right)^6}+\frac{895\left(\sqrt{x}-1\right)^5}{\left(\sqrt{x}+1\right)^5}+\frac{1765\left(\sqrt{x}-1\right)^4}{\left(\sqrt{x}+1\right)^4}+\frac{1765\left(\sqrt{x}-1\right)^3}{\left(\sqrt{x}+1\right)^3}+\frac{895\left(\sqrt{x}-1\right)^2}{\left(\sqrt{x}+1\right)^2}+\frac{397\left(\sqrt{x}-1\right)}{\sqrt{x}+1}+15\right)\sqrt{\sqrt{x}-1}}{96\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}-1\right)^8\sqrt{\sqrt{x}+1}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] ((15 + (15*(-1 + Sqrt[x])^7)/(1 + Sqrt[x])^7 + (397*(-1 + Sqrt[x])^6)/(1 +
Sqrt[x])^6 + (895*(-1 + Sqrt[x])^5)/(1 + Sqrt[x])^5 + (1765*(-1 + Sqrt[x])^
4)/(1 + Sqrt[x])^4 + (1765*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (895*(-1 + S
qrt[x])^2)/(1 + Sqrt[x])^2 + (397*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 +
Sqrt[x]])/(96*(-1 + (-1 + Sqrt[x])/(1 + Sqrt[x]))^8*Sqrt[1 + Sqrt[x]]) - (5
*ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]])/32
```

**fricas [A]** time = 0.39, size = 62, normalized size = 0.46

$$\frac{1}{192}\left(48\,x^3-8\,x^2-10\,x-15\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{5}{128}\,\log\left(2\,\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac [A]** time = 0.29, size = 162, normalized size = 1.20

$$\frac{1}{6720}\left(\left(2\left(\left(4\left(5\left(6\left(7\sqrt{x}-50\right)\left(\sqrt{x}+1\right)+1219\right)\left(\sqrt{x}+1\right)-12463\right)\left(\sqrt{x}+1\right)+64233\right)\left(\sqrt{x}+1\right)-53963\right)\left(\sqrt{x}+1\right)+59465\right)\left(\sqrt{x}+1\right)-23205\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{840}\left(\left(2\left(\left(4\left(5\left(6\sqrt{x}-37\right)\left(\sqrt{x}+1\right)+661\right)\left(\sqrt{x}+1\right)-4551\right)\left(\sqrt{x}+1\right)+4781\right)\left(\sqrt{x}+1\right)-6335\right)\left(\sqrt{x}+1\right)+2835\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{5}{32}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6720*((2*((4*(5*(6*(7*sqrt(x) - 50)*(sqrt(x) + 1) + 1219)*(sqrt(x) + 1) -
 12463)*(sqrt(x) + 1) + 64233)*(sqrt(x) + 1) - 53963)*(sqrt(x) + 1) + 59465
)*(sqrt(x) + 1) - 23205)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/840*((2*((
4*(5*(6*sqrt(x) - 37)*(sqrt(x) + 1) + 661)*(sqrt(x) + 1) - 4551)*(sqrt(x) +
 1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) + 1)*s
qrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

**maple [A]** time = 0.05, size = 75, normalized size = 0.56

$$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{x-1}\,x^{\frac{7}{2}}+8\sqrt{x-1}\,x^{\frac{5}{2}}+10\sqrt{x-1}\,x^{\frac{3}{2}}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)+15\sqrt{x-1}\,\sqrt{x}\right)}{192\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/192*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-48*x^(7/2)*(x-1)^(1/2)+8*x^(5/
2)*(x-1)^(1/2)+10*x^(3/2)*(x-1)^(1/2)+15*(x-1)^(1/2)*x^(1/2)+15*ln(x^(1/2)+
(x-1)^(1/2)))/(x-1)^(1/2)
```

**maxima [A]** time = 0.55, size = 57, normalized size = 0.42

$$\frac{1}{4}(x-1)^{\frac{3}{2}}x^{\frac{5}{2}}+\frac{5}{24}(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{5}{32}(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{5}{64}\sqrt{x-1}\,\sqrt{x}-\frac{5}{64}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2)
*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))
```

**mupad [B]** time = 52.03, size = 831, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] ((1723*((x^(1/2) - 1)^(1/2) - 1i)^5)/(48*((x^(1/2) + 1)^(1/2) - 1)^5) - (23
5*((x^(1/2) - 1)^(1/2) - 1i)^3)/(48*((x^(1/2) + 1)^(1/2) - 1)^3) + (72283*(
(x^(1/2) - 1)^(1/2) - 1i)^7)/(16*((x^(1/2) + 1)^(1/2) - 1)^7) + (848801*((x
^(1/2) - 1)^(1/2) - 1i)^9)/(16*((x^(1/2) + 1)^(1/2) - 1)^9) + (4181067*((x^
(1/2) - 1)^(1/2) - 1i)^11)/(16*((x^(1/2) + 1)^(1/2) - 1)^11) + (10994181*((
x^(1/2) - 1)^(1/2) - 1i)^13)/(16*((x^(1/2) + 1)^(1/2) - 1)^13) + (17457599*
((x^(1/2) - 1)^(1/2) - 1i)^15)/(16*((x^(1/2) + 1)^(1/2) - 1)^15) + (1745759
9*((x^(1/2) - 1)^(1/2) - 1i)^17)/(16*((x^(1/2) + 1)^(1/2) - 1)^17) + (10994
181*((x^(1/2) - 1)^(1/2) - 1i)^19)/(16*((x^(1/2) + 1)^(1/2) - 1)^19) + (418
1067*((x^(1/2) - 1)^(1/2) - 1i)^21)/(16*((x^(1/2) + 1)^(1/2) - 1)^21) + (84
8801*((x^(1/2) - 1)^(1/2) - 1i)^23)/(16*((x^(1/2) + 1)^(1/2) - 1)^23) + (72
```

```
283*((x^(1/2) - 1)^(1/2) - 1i)^25)/(16*((x^(1/2) + 1)^(1/2) - 1)^25) + (172
3*((x^(1/2) - 1)^(1/2) - 1i)^27)/(48*((x^(1/2) + 1)^(1/2) - 1)^27) - (235*(
(x^(1/2) - 1)^(1/2) - 1i)^29)/(48*((x^(1/2) + 1)^(1/2) - 1)^29) + (5*((x^(1
/2) - 1)^(1/2) - 1i)^31)/(16*((x^(1/2) + 1)^(1/2) - 1)^31) + (5*((x^(1/2) -
 1)^(1/2) - 1i))/(16*((x^(1/2) + 1)^(1/2) - 1)))/((120*((x^(1/2) - 1)^(1/2)
 - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (16*((x^(1/2) - 1)^(1/2) - 1i)^2)/(
(x^(1/2) + 1)^(1/2) - 1)^2 - (560*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) +
 1)^(1/2) - 1)^6 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2)
 - 1)^8 - (4368*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
 + (8008*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (114
40*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + (12870*((x
^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (11440*((x^(1/2)
 - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (8008*((x^(1/2) - 1)^(
1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (4368*((x^(1/2) - 1)^(1/2) -
1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^24)
/((x^(1/2) + 1)^(1/2) - 1)^24 - (560*((x^(1/2) - 1)^(1/2) - 1i)^26)/((x^(1/
2) + 1)^(1/2) - 1)^26 + (120*((x^(1/2) - 1)^(1/2) - 1i)^28)/((x^(1/2) + 1)^
(1/2) - 1)^28 - (16*((x^(1/2) - 1)^(1/2) - 1i)^30)/((x^(1/2) + 1)^(1/2) - 1
)^30 + ((x^(1/2) - 1)^(1/2) - 1i)^32/((x^(1/2) + 1)^(1/2) - 1)^32 + 1) - (5
*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/16
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

## 3.641 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\,dx$

**Optimal**. Leaf size=104

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{1}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi** [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {280, 323, 330, 52}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{1}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 - (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))
/3 - ArcCosh[Sqrt[x]]/8
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\,dx = \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} - \frac{1}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} - \frac{1}{8}\int \frac{\sqrt{\phantom{x}}}{\sqrt{-1+\sqrt{x}}}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1}$$

**Mathematica [A]** time = 0.06, size = 99, normalized size = 0.95

$$\frac{\sqrt{\sqrt{x}+1}\,\sqrt{x}\left(8x^{5/2}-2x^{3/2}-8x^2+2x-3\sqrt{x}+3\right)+6\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{24\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 - 3*Sqrt[x] + 2*x - 2*x^(3/2) - 8*x^2 + 8*x^(
5/2)) + 6*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(24*Sqrt[-1
+ Sqrt[x]])
```

**IntegrateAlgebraic [A]** time = 1.30, size = 176, normalized size = 1.69

$$\frac{\left(\frac{3(\sqrt{x}-1)^5}{(\sqrt{x}+1)^5}+\frac{47(\sqrt{x}-1)^4}{(\sqrt{x}+1)^4}+\frac{78(\sqrt{x}-1)^3}{(\sqrt{x}+1)^3}+\frac{78(\sqrt{x}-1)^2}{(\sqrt{x}+1)^2}+\frac{47(\sqrt{x}-1)}{\sqrt{x}+1}+3\right)\sqrt{\sqrt{x}-1}}{12\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}-1\right)^6\sqrt{\sqrt{x}+1}}-\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] ((3 + (3*(-1 + Sqrt[x])^5)/(1 + Sqrt[x])^5 + (47*(-1 + Sqrt[x])^4)/(1 + Sqr
t[x])^4 + (78*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (78*(-1 + Sqrt[x])^2)/(1
+ Sqrt[x])^2 + (47*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]])/(12*(
-1 + (-1 + Sqrt[x])/(1 + Sqrt[x]))^6*Sqrt[1 + Sqrt[x]]) - ArcTanh[Sqrt[-1 +
 Sqrt[x]]/Sqrt[1 + Sqrt[x]]]/4
```

**fricas [A]** time = 0.41, size = 57, normalized size = 0.55

$$\frac{1}{24}\left(8\,x^2-2\,x-3\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{1}{16}\,\log\left(2\,\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*l
og(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac** [A] time = 0.22, size = 127, normalized size = 1.22

$$\frac{1}{120}\left(\left(2\left(\left(4\left(5\sqrt{x}-26\right)\left(\sqrt{x}+1\right)+321\right)\left(\sqrt{x}+1\right)-451\right)\left(\sqrt{x}+1\right)+745\right)\left(\sqrt{x}+1\right)-405\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{60}\left(\left(2\left(3\left(4\sqrt{x}-17\right)\left(\sqrt{x}+1\right)+133\right)\left(\sqrt{x}+1\right)-295\right)\left(\sqrt{x}+1\right)+195\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*((2*((4*(5*sqrt(x) - 26)*(sqrt(x) + 1) + 321)*(sqrt(x) + 1) - 451)*(s
qrt(x) + 1) + 745)*(sqrt(x) + 1) - 405)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
 + 1/60*((2*(3*(4*sqrt(x) - 17)*(sqrt(x) + 1) + 133)*(sqrt(x) + 1) - 295)*(
sqrt(x) + 1) + 195)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/4*log(sqrt(sqrt
(x) + 1) - sqrt(sqrt(x) - 1))
```

**maple** [A] time = 0.05, size = 65, normalized size = 0.62

$$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-8\sqrt{x-1}\,x^{\frac{5}{2}}+2\sqrt{x-1}\,x^{\frac{3}{2}}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)+3\sqrt{x-1}\,\sqrt{x}\right)}{24\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-8*(x-1)^(1/2)*x^(5/2)+2*(x-1)^(
1/2)*x^(3/2)+3*(x-1)^(1/2)*x^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

**maxima** [A] time = 0.49, size = 47, normalized size = 0.45

$$\frac{1}{3}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{1}{4}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{8}\,\sqrt{x-1}\,\sqrt{x}-\frac{1}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(x - 1)^(3/2)*x^(3/2) + 1/4*(x - 1)^(3/2)*sqrt(x) + 1/8*sqrt(x - 1)*sqr
t(x) - 1/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

**mupad** [B] time = 31.39, size = 632, normalized size = 6.08

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-i}{\sqrt{\sqrt{x}+1}-1}\right)}{2}-\frac{\frac{35\left(\sqrt{\sqrt{x}-1}-i\right)^{3}}{6\left(\sqrt{\sqrt{x}+1}-1\right)^{3}}+\frac{757\left(\sqrt{\sqrt{x}-1}-i\right)^{5}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{5}}+\frac{7339\left(\sqrt{\sqrt{x}-1}-i\right)^{7}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{7}}+\frac{41929\left(\sqrt{\sqrt{x}-1}-i\right)^{9}}{3\left(\sqrt{\sqrt{x}+1}-1\right)^{9}}+\frac{25661\left(\sqrt{\sqrt{x}-1}-i\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{25661\left(\sqrt{\sqrt{x}-1}-i\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}+\frac{41929\left(\sqrt{\sqrt{x}-1}-i\right)^{15}}{3\left(\sqrt{\sqrt{x}+1}-1\right)^{15}}+\frac{7339\left(\sqrt{\sqrt{x}-1}-i\right)^{17}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{17}}+\frac{757\left(\sqrt{\sqrt{x}-1}-i\right)^{19}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{19}}+\frac{35\left(\sqrt{\sqrt{x}-1}-i\right)^{21}}{6\left(\sqrt{\sqrt{x}+1}-1\right)^{21}}-\frac{\left(\sqrt{\sqrt{x}-1}-i\right)^{23}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{23}}-\frac{\sqrt{\sqrt{x}-1}-i}{2\left(\sqrt{\sqrt{x}+1}-1\right)}}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-i\right)^{4}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{4}}-\frac{220\left(\sqrt{\sqrt{x}-1}-i\right)^{6}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{6}}+\frac{495\left(\sqrt{\sqrt{x}-1}-i\right)^{8}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{8}}-\frac{792\left(\sqrt{\sqrt{x}-1}-i\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-i\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\sqrt{x}-1}-i\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+\frac{495\left(\sqrt{\sqrt{x}-1}-i\right)^{16}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{16}}-\frac{220\left(\sqrt{\sqrt{x}-1}-i\right)^{18}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{18}}+\frac{66\left(\sqrt{\sqrt{x}-1}-i\right)^{20}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{20}}-\frac{12\left(\sqrt{\sqrt{x}-1}-i\right)^{22}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{22}}+\frac{\left(\sqrt{\sqrt{x}-1}-i\right)^{24}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{24}}-\frac{12\left(\sqrt{\sqrt{x}-1}-i\right)^{2}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] - atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1))/2 - ((35*((x^
(1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (757*((x^(1/2)
- 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) + (7339*((x^(1/2) - 1)^
(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (41929*((x^(1/2) - 1)^(1/2
) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25661*((x^(1/2) - 1)^(1/2) -
1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25661*((x^(1/2) - 1)^(1/2) - 1i)^13
)/((x^(1/2) + 1)^(1/2) - 1)^13 + (41929*((x^(1/2) - 1)^(1/2) - 1i)^15)/(3*(
(x^(1/2) + 1)^(1/2) - 1)^15) + (7339*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*((x^
(1/2) + 1)^(1/2) - 1)^17) + (757*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(1/2
) + 1)^(1/2) - 1)^19) + (35*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2) + 1
)^(1/2) - 1)^21) - ((x^(1/2) - 1)^(1/2) - 1i)^23/(2*((x^(1/2) + 1)^(1/2) -
1)^23) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1/2) - 1)))/((66*((x
```

```
^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((x^(1/2) - 1)
^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) - 1)^(1/2) - 1
i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^
(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1
)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2)
- 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14
+ (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (220*(
(x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (66*((x^(1/2)
- 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/2) - 1)^(1/2
) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2) - 1i)^24/((
x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

## 3.642 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx$

**Optimal**. Leaf size=73

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi** [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {280, 323, 330, 52}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/2 - ArcCosh[Sqrt[x]]/4
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx = \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{8}\int \frac{1}{\sqrt{-1+\sqrt{x}}\ \cdots}$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{4}\,\text{Subst}\Bigg(\int \frac{\cdots}{\sqrt{-1\cdots}}$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

**Mathematica** [A] time = 0.08, size = 87, normalized size = 1.19

$$\frac{\sqrt{\sqrt{x}+1}\,\sqrt{x}\left(2x^{3/2}-2x-\sqrt{x}+1\right)+2\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{4\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 - Sqrt[x] - 2*x + 2*x^(3/2)) + 2*Sqrt[1 - Sqr
t[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(4*Sqrt[-1 + Sqrt[x]])
```

**IntegrateAlgebraic** [B] time = 1.48, size = 404, normalized size = 5.53

$$\frac{-4\sqrt{\sqrt{x}+1}\left(-1136x^{7/2}-1096x^{5/2}+17296x^{3/2}-4752x^3+7240x^2+55360x+28224\sqrt{x}-18816\right)-4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(-194x^{7/2}-6079x^{5/2}+6992x^{3/2}-3120x^3-104x^2+38632x+74488\sqrt{x}+32592\right)+\sqrt{3}\left(-4\sqrt{\sqrt{x}-1}\left(656x^{7/2}+3832x^{5/2}-10928x^{3/2}+3408x^3-1192x^2-41472x-52416\sqrt{x}-18816\right)-4\left(1800x^{7/2}-1148x^{5/2}-23268x^{3/2}+112x^4+3416x^3-6678x^2-41440x-10872\sqrt{x}+10864\right)\right)}{24960x^{3/2}+\sqrt{3}\sqrt{\sqrt{x}+1}\left(-5248x^{3/2}-22016x-11264\sqrt{x}+7168\right)+\sqrt{\sqrt{x}-1}\left(9088x^{3/2}+\sqrt{3}\sqrt{\sqrt{x}+1}\left(-896x^{3/2}-14400x-28672\sqrt{x}-12416\right)+47104x+60416\sqrt{x}+21504\right)+1552x^2+49408x+13312\sqrt{x}-12416}+\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}-1}{\sqrt{3}-\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (-4*Sqrt[1 + Sqrt[x]]*(-18816 + 28224*Sqrt[x] + 55360*x + 17296*x^(3/2) + 7
240*x^2 - 1096*x^(5/2) - 4752*x^3 - 1136*x^(7/2)) - 4*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*(32592 + 74488*Sqrt[x] + 38632*x + 6992*x^(3/2) - 104*x^2 -
 6079*x^(5/2) - 3120*x^3 - 194*x^(7/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]]*(-
18816 - 52416*Sqrt[x] - 41472*x - 10928*x^(3/2) - 1192*x^2 + 3832*x^(5/2) +
 3408*x^3 + 656*x^(7/2)) - 4*(10864 - 10872*Sqrt[x] - 41440*x - 23268*x^(3/
2) - 6678*x^2 - 1148*x^(5/2) + 3416*x^3 + 1800*x^(7/2) + 112*x^4)))/(-12416
 + 13312*Sqrt[x] + 49408*x + 24960*x^(3/2) + 1552*x^2 + Sqrt[3]*Sqrt[1 + Sq
rt[x]]*(7168 - 11264*Sqrt[x] - 22016*x - 5248*x^(3/2)) + Sqrt[-1 + Sqrt[x]]
*(21504 + 60416*Sqrt[x] + 47104*x + 9088*x^(3/2) + Sqrt[3]*Sqrt[1 + Sqrt[x]
]*(-12416 - 28672*Sqrt[x] - 14400*x - 896*x^(3/2)))) + ArcTanh[(-1 + Sqrt[-
1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

**fricas** [A] time = 0.40, size = 52, normalized size = 0.71

$$\frac{1}{4}(2x-1)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{1}{8}\log\left(2\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac [B]** time = 0.19, size = 92, normalized size = 1.26

$$\frac{1}{12}\left(\left(2\left(3\sqrt{x}-10\right)\left(\sqrt{x}+1\right)+43\right)\left(\sqrt{x}+1\right)-39\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{3}\left(\left(2\sqrt{x}-5\right)\left(\sqrt{x}+1\right)+9\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*((2*(3*sqrt(x) - 10)*(sqrt(x) + 1) + 43)*(sqrt(x) + 1) - 39)*sqrt(sqrt
(x) + 1)*sqrt(sqrt(x) - 1) + 1/3*((2*sqrt(x) - 5)*(sqrt(x) + 1) + 9)*sqrt(s
qrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))
```

**maple [A]** time = 0.05, size = 52, normalized size = 0.71

$$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-2\sqrt{x-1}\,x^{\frac{3}{2}}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)+\sqrt{x-1}\,\sqrt{x}\right)}{4\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-2*(x-1)^(1/2)*x^(3/2)+(x-1)^(1/2
)*x^(1/2)+ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

**maxima [A]** time = 0.58, size = 37, normalized size = 0.51

$$\frac{1}{2}\left(x-1\right)^{\frac{3}{2}}\sqrt{x}+\frac{1}{4}\sqrt{x-1}\,\sqrt{x}-\frac{1}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(x - 1)^(3/2)*sqrt(x) + 1/4*sqrt(x - 1)*sqrt(x) - 1/4*log(2*sqrt(x - 1)
 + 2*sqrt(x))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

## 3.643 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

**Optimal**. Leaf size=37

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {280, 330, 52}

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\
&= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 72, normalized size = 1.95

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(\sqrt{x}-1\right)+2\sqrt{1-\sqrt{x}}\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] ((-1 + Sqrt[x])*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt
[1 - Sqrt[x]]/Sqrt[2]])/Sqrt[-1 + Sqrt[x]]
```

**IntegrateAlgebraic [B]** time = 1.17, size = 268, normalized size = 7.24

$$\frac{-4\sqrt{\sqrt{x}+1}\left(-20x^{3/2}-4x+96\sqrt{x}+48\right)-4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(-7x^{3/2}-28x+10\sqrt{x}+84\right)+\sqrt{3}\left(-4\sqrt{\sqrt{x}-1}\left(12x^{3/2}+20x-32\sqrt{x}-48\right)-4\left(14x^{3/2}+4x^2-18x-70\sqrt{x}-28\right)\right)}{\sqrt{3}\sqrt{\sqrt{x}+1}\left(-48\sqrt{x}-32\right)+\sqrt{\sqrt{x}-1}\left(\sqrt{3}\sqrt{\sqrt{x}+1}\left(-16\sqrt{x}-56\right)+80\sqrt{x}+96\right)+28x+112\sqrt{x}+56}+4\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}-1}{\sqrt{3}-\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] (-4*Sqrt[1 + Sqrt[x]]*(48 + 96*Sqrt[x] - 4*x - 20*x^(3/2)) - 4*Sqrt[-1 + Sq
rt[x]]*Sqrt[1 + Sqrt[x]]*(84 + 10*Sqrt[x] - 28*x - 7*x^(3/2)) + Sqrt[3]*(-4
*Sqrt[-1 + Sqrt[x]]*(-48 - 32*Sqrt[x] + 20*x + 12*x^(3/2)) - 4*(-28 - 70*Sq
rt[x] - 18*x + 14*x^(3/2) + 4*x^2)))/(56 + Sqrt[3]*(-32 - 48*Sqrt[x])*Sqrt[
1 + Sqrt[x]] + Sqrt[-1 + Sqrt[x]]*(96 + Sqrt[3]*(-56 - 16*Sqrt[x])*Sqrt[1 +
 Sqrt[x]] + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) + 4*ArcTanh[(-1 + Sqrt[-1 + S
qrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

**fricas [A]** time = 0.41, size = 46, normalized size = 1.24

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac [B]** time = 0.18, size = 57, normalized size = 1.54

$$\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\left(\sqrt{x}-2\right)+2\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*(sqrt(x) - 2) + 2*sqrt(sqrt(x) + 1)*sqr
t(sqrt(x) - 1) + 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

**maple [B]** time = 0.05, size = 72, normalized size = 1.95

$$-\frac{\sqrt{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}}+\sqrt{\sqrt{x}-1}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2),x)
```

```
[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(3/2)-(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)-((x
^(1/2)-1)*(x^(1/2)+1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^(1/2)
+(x-1)^(1/2))
```

**maxima [A]** time = 0.49, size = 26, normalized size = 0.70

$$\sqrt{x-1}\sqrt{x} - \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))
```

**mupad [B]** time = 5.07, size = 41, normalized size = 1.11

$$\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1} - \ln\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1} + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2),x)
```

```
[Out] x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) - log((x^(1/2) - 1)^(1/2)*(
x^(1/2) + 1)^(1/2) + x^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/sqrt(x), x)
```

## 3.644 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

**Optimal**. Leaf size=67

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\sqrt{\sqrt{x}-1}\,\sqrt{x}\,\sqrt{\sqrt{x}+1}+2\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {327, 280, 330, 52}

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\sqrt{\sqrt{x}-1}\,\sqrt{x}\,\sqrt{\sqrt{x}+1}+2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2
*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(
a1*a2*c^(2*n)*(m + 1)), Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p
, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IG
tQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{\ }}}$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{\ }}\right.$$

$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\cosh^{-1}\left(\sqrt{x}\right)$$

**Mathematica [A]** time = 0.08, size = 74, normalized size = 1.10

$$\frac{2\left(-\frac{\sqrt{\sqrt{x}+1}\left(\sqrt{x}-1\right)}{\sqrt{x}} - 2\sqrt{1-\sqrt{x}}\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)\right)}{\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-(((-1 + Sqrt[x])*Sqrt[1 + Sqrt[x]])/Sqrt[x]) - 2*Sqrt[1 - Sqrt[x]]*Arc
Sin[Sqrt[1 - Sqrt[x]]/Sqrt[2]]))/Sqrt[-1 + Sqrt[x]]
```

**IntegrateAlgebraic [B]** time = 1.13, size = 184, normalized size = 2.75

$$\frac{\left(\sqrt{\sqrt{x}-1}-1\right)\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)\left(\sqrt{\sqrt{x}-1}+\sqrt{3}\sqrt{\sqrt{x}+1}-\sqrt{x}-2\right)}{\left(\sqrt{3}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\sqrt{\sqrt{x}-1}+2\sqrt{3}\sqrt{\sqrt{x}+1}-2\sqrt{x}-3\right)\sqrt{x}} - 8\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}-1}{\sqrt{3}-\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((-3 - 2*Sqrt[-1 + Sqrt[x]]
+ 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
] - 2*Sqrt[x])*Sqrt[x]) - 8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sq
rt[1 + Sqrt[x]])]
```

**fricas [A]** time = 0.40, size = 55, normalized size = 0.82

$$-\frac{x\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)+2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="fricas
")
```

```
[Out] -(x*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1) + 2*sqrt(x
)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x)/x
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac")
[Out] Timed out
```

**maple [A]** time = 0.05, size = 47, normalized size = 0.70

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)-\sqrt{x-1}\right)}{\sqrt{x-1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(3/2),x)
[Out] 2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(ln(x^(1/2)+(x-1)^(1/2))*x^(1/2)-(x-1
)^(1/2))/(x-1)^(1/2)/x^(1/2)
```

**maxima [A]** time = 1.28, size = 27, normalized size = 0.40

$$-\frac{2\sqrt{x-1}}{\sqrt{x}}+2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxima
")
[Out] -2*sqrt(x - 1)/sqrt(x) + 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

**mupad [B]** time = 6.25, size = 129, normalized size = 1.93

$$8\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)-\frac{\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{2\left(\sqrt{\sqrt{x}+1}-1\right)^2}+\frac{1}{2}}{\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}}-\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{2\left(\sqrt{\sqrt{x}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(3/2),x)
[Out] 8*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) - ((5*((x^(1/
2) - 1)^(1/2) - 1i)^2)/(2*((x^(1/2) + 1)^(1/2) - 1)^2) + 1/2)/((((x^(1/2) -
1)^(1/2) - 1i)^3/((x^(1/2) + 1)^(1/2) - 1)^3 + ((x^(1/2) - 1)^(1/2) - 1i)/(
(x^(1/2) + 1)^(1/2) - 1)) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1
/2) - 1))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

## 3.645 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

**Optimal**. Leaf size=31

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

**Rubi [A]** time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {265}

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

**Mathematica [A]** time = 0.02, size = 31, normalized size = 1.00

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

**IntegrateAlgebraic [B]** time = 2.38, size = 512, normalized size = 16.52

$$\frac{\left(\frac{\left(\sqrt{\sqrt{x}-1}-1\right)^2}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^2}+1\right)\left(\frac{9\left(\sqrt{\sqrt{x}-1}-1\right)^8}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^8}+\frac{42\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^7}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^7}+\frac{256\left(\sqrt{\sqrt{x}-1}-1\right)^6}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^6}+\frac{294\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^5}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^5}+\frac{638\left(\sqrt{\sqrt{x}-1}-1\right)^4}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^4}+\frac{294\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^3}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^3}+\frac{256\left(\sqrt{\sqrt{x}-1}-1\right)^2}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^2}+\frac{42\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)}{\sqrt{3}-\sqrt{\sqrt{x}+1}}+9\right)\left(\frac{1}{384}-\frac{\sqrt{\sqrt{x}-1}}{384}\right)\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-\sqrt{3}\right)\left(-2x-2\sqrt{\sqrt{x}-1}\,\sqrt{x}+\sqrt{3}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+2\sqrt{3}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-3\sqrt{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(9 + (9*
(-1 + Sqrt[-1 + Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (42*Sqrt[3]*
(-1 + Sqrt[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (256*(-1 + S
```

```
qrt[-1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (294*Sqrt[3]*(-1 +
Sqrt[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (638*(-1 + Sqrt[-1
 + Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (294*Sqrt[3]*(-1 + Sqrt[-
1 + Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (256*(-1 + Sqrt[-1 + Sqr
t[x]])^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (42*Sqrt[3]*(-1 + Sqrt[-1 + Sqr
t[x]]))/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/384 - Sqrt[-1 + Sqrt[x]]/384)*(Sq
rt[3] - Sqrt[1 + Sqrt[x]])^12)/((-Sqrt[3] + Sqrt[1 + Sqrt[x]])*(-3*Sqrt[x]
- 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + Sqrt
[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - 2*x)^3)
```

**fricas [A]** time = 0.42, size = 30, normalized size = 0.97

$$\frac{2\left((x-1)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+x^2\right)}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="fricas
")
```

```
[Out] 2/3*((x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x^2)/x^2
```

**giac [B]** time = 0.23, size = 48, normalized size = 1.55

$$\frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^8+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

**maple [A]** time = 0.05, size = 23, normalized size = 0.74

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(x-1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(5/2),x)
```

```
[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)/x^(3/2)
```

**maxima [A]** time = 1.32, size = 10, normalized size = 0.32

$$\frac{2\,(x-1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxima
")
```

```
[Out] 2/3*(x - 1)^(3/2)/x^(3/2)
```

**mupad [B]** time = 5.26, size = 31, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{3}-\frac{2\,\sqrt{\sqrt{x}+1}}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(5/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/3 - (2*(x^(1/2) + 1)^(1/2))
/3))/x^(3/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

## 3.646 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$

**Optimal**. Leaf size=63

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

**Rubi [A]** time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx&=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{2}{5}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx\\&=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 36, normalized size = 0.57

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}(2x+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(3 + 2*x))/(15*x^(5/2))
```

**IntegrateAlgebraic [B]** time = 10.09, size = 836, normalized size = 13.27

$$\frac{\left(\frac{\left(\sqrt{\sqrt{x}-1}-1\right)^{2}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{2}}+1\right)\left(\frac{45\left(\sqrt{\sqrt{x}-1}-1\right)^{16}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{16}}+\frac{300\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{15}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{15}}+\frac{2960\left(\sqrt{\sqrt{x}-1}-1\right)^{14}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{14}}+\frac{6620\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{13}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{13}}+\frac{34252\left(\sqrt{\sqrt{x}-1}-1\right)^{12}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{12}}+\frac{46980\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{11}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{11}}+\frac{152688\left(\sqrt{\sqrt{x}-1}-1\right)^{10}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{10}}+\frac{129140\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{9}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{9}}+\frac{254222\left(\sqrt{\sqrt{x}-1}-1\right)^{8}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{8}}+\frac{129140\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{7}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{7}}+\frac{152688\left(\sqrt{\sqrt{x}-1}-1\right)^{6}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{6}}+\frac{46980\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{5}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{5}}+\frac{34252\left(\sqrt{\sqrt{x}-1}-1\right)^{4}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{4}}+\frac{6620\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{3}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{3}}+\frac{2960\left(\sqrt{\sqrt{x}-1}-1\right)^{2}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{2}}+\frac{300\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)}{\sqrt{3}-\sqrt{\sqrt{x}+1}}+45\right)\left(\frac{1}{122880}-\frac{\sqrt{\sqrt{x}-1}}{122880}\right)\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{20}}{\left(\sqrt{\sqrt{x}+1}-\sqrt{3}\right)\left(-2x-2\sqrt{\sqrt{x}-1}\sqrt{x}+\sqrt{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+2\sqrt{3}\sqrt{\sqrt{x}+1}\sqrt{x}-3\sqrt{x}\right)^{5}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(45 + (4
5*(-1 + Sqrt[-1 + Sqrt[x]])^16)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^16 + (300*Sqr
t[3]*(-1 + Sqrt[-1 + Sqrt[x]])^15)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^15 + (2960
*(-1 + Sqrt[-1 + Sqrt[x]])^14)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^14 + (6620*Sqr
t[3]*(-1 + Sqrt[-1 + Sqrt[x]])^13)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^13 + (3425
2*(-1 + Sqrt[-1 + Sqrt[x]])^12)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^12 + (46980*S
qrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^11)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^11 + (15
2688*(-1 + Sqrt[-1 + Sqrt[x]])^10)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^10 + (1291
40*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^9)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^9 + (
254222*(-1 + Sqrt[-1 + Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (1291
40*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (
152688*(-1 + Sqrt[-1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (4698
0*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (3
4252*(-1 + Sqrt[-1 + Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (6620*S
qrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (2960
*(-1 + Sqrt[-1 + Sqrt[x]])^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (300*Sqrt[3
]*(-1 + Sqrt[-1 + Sqrt[x]]))/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/122880 - Sqr
t[-1 + Sqrt[x]]/122880)*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^20)/((-Sqrt[3] + Sqrt
[1 + Sqrt[x]])*(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[
1 + Sqrt[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]
 - 2*x)^5)
```

**fricas [A]** time = 0.41, size = 37, normalized size = 0.59

$$\frac{2\left(2x^{3}+\left(2x^{2}+x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/
x^3
```

**giac [B]** time = 0.28, size = 90, normalized size = 1.43

$$\frac{128\left(15\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}-20\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+80\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+64\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) +
1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5
```

**maple** [A] time = 0.05, size = 28, normalized size = 0.44

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(x-1)(2x+3)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(7/2),x)
[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(2*x+3)/x^(5/2)
```

**maxima** [A] time = 1.31, size = 21, normalized size = 0.33

$$\frac{4\,(x-1)^{\frac{3}{2}}}{15\,x^{\frac{3}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxima
")
[Out] 4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)
```

**mupad** [B] time = 5.07, size = 43, normalized size = 0.68

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{15}-\frac{2\,\sqrt{\sqrt{x}+1}}{5}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{15}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(7/2),x)
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/15 - (2*(x^(1/2) + 1)^(1/2)
)/5 + (4*x^2*(x^(1/2) + 1)^(1/2))/15))/x^(5/2)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(7/2), x)
```

## 3.647 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} dx$

**Optimal**. Leaf size=94

$$\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2)) + (8*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(35*x^(5/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}} dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}} dx \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{8}{35}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{}}{x^{5/2}} \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\right.}{105x^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 41, normalized size = 0.44

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}\left(8x^2+12x+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7/
2))
```

**IntegrateAlgebraic [B]** time = 36.64, size = 1160, normalized size = 12.34

$$\frac{\left(\frac{\left(\sqrt{\sqrt{x}-1}-1\right)^{2}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{2}}+1\right)\left(\frac{315\left(\sqrt{\sqrt{x}-1}-1\right)^{24}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{24}}+\frac{2730\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{23}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{23}}+\frac{37520\left(\sqrt{\sqrt{x}-1}-1\right)^{22}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{22}}+\frac{123550\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{21}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{21}}+\frac{965902\left(\sqrt{\sqrt{x}-1}-1\right)^{20}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{20}}+\frac{2042838\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{19}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{19}}+\frac{10643536\left(\sqrt{\sqrt{x}-1}-1\right)^{18}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{18}}+\frac{15378258\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{17}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{17}}+\frac{56109493\left(\sqrt{\sqrt{x}-1}-1\right)^{16}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{16}}+\frac{57833188\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{15}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{15}}+\frac{151811360\left(\sqrt{\sqrt{x}-1}-1\right)^{14}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{14}}+\frac{112621740\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{13}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{13}}+\frac{212123652\left(\sqrt{\sqrt{x}-1}-1\right)^{12}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{12}}+\frac{112621740\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{11}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{11}}+\frac{151811360\left(\sqrt{\sqrt{x}-1}-1\right)^{10}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{10}}+\frac{57833188\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{9}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{9}}+\frac{56109493\left(\sqrt{\sqrt{x}-1}-1\right)^{8}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{8}}+\frac{15378258\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{7}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{7}}+\frac{10643536\left(\sqrt{\sqrt{x}-1}-1\right)^{6}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{6}}+\frac{2042838\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{5}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{5}}+\frac{965902\left(\sqrt{\sqrt{x}-1}-1\right)^{4}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{4}}+\frac{123550\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^{3}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{3}}+\frac{37520\left(\sqrt{\sqrt{x}-1}-1\right)^{2}}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{2}}+\frac{2730\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)}{\sqrt{3}-\sqrt{\sqrt{x}+1}}+315\right)\left(\frac{1}{55050240}-\frac{\sqrt{\sqrt{x}-1}}{55050240}\right)\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{28}}{\left(\sqrt{\sqrt{x}+1}-\sqrt{3}\right)\left(-2x-2\sqrt{\sqrt{x}-1}\sqrt{x}+\sqrt{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+2\sqrt{3}\sqrt{\sqrt{x}+1}\sqrt{x}-3\sqrt{x}\right)^{7}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(315 + (
315*(-1 + Sqrt[-1 + Sqrt[x]])^24)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^24 + (2730*
Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^23)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^23 + (3
7520*(-1 + Sqrt[-1 + Sqrt[x]])^22)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^22 + (1235
50*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^21)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^21 +
 (965902*(-1 + Sqrt[-1 + Sqrt[x]])^20)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^20 + (
2042838*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^19)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])
^19 + (10643536*(-1 + Sqrt[-1 + Sqrt[x]])^18)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])
^18 + (15378258*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^17)/(Sqrt[3] - Sqrt[1 + S
qrt[x]])^17 + (56109493*(-1 + Sqrt[-1 + Sqrt[x]])^16)/(Sqrt[3] - Sqrt[1 + S
qrt[x]])^16 + (57833188*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^15)/(Sqrt[3] - Sq
rt[1 + Sqrt[x]])^15 + (151811360*(-1 + Sqrt[-1 + Sqrt[x]])^14)/(Sqrt[3] - S
qrt[1 + Sqrt[x]])^14 + (112621740*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^13)/(Sq
rt[3] - Sqrt[1 + Sqrt[x]])^13 + (212123652*(-1 + Sqrt[-1 + Sqrt[x]])^12)/(S
qrt[3] - Sqrt[1 + Sqrt[x]])^12 + (112621740*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]
])^11)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^11 + (151811360*(-1 + Sqrt[-1 + Sqrt[x
]])^10)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^10 + (57833188*Sqrt[3]*(-1 + Sqrt[-1
+ Sqrt[x]])^9)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^9 + (56109493*(-1 + Sqrt[-1 +
Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (15378258*Sqrt[3]*(-1 + Sqrt
[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (10643536*(-1 + Sqrt[-
1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (2042838*Sqrt[3]*(-1 + S
qrt[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (965902*(-1 + Sqrt[
-1 + Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (123550*Sqrt[3]*(-1 + S
qrt[-1 + Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (37520*(-1 + Sqrt[-
1 + Sqrt[x]])^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (2730*Sqrt[3]*(-1 + Sqrt
[-1 + Sqrt[x]]))/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/55050240 - Sqrt[-1 + Sqr
t[x]]/55050240)*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^28)/((-Sqrt[3] + Sqrt[1 + Sqr
t[x]])*(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 + Sqrt
[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - 2*x)^
7)
```

**fricas [A]** time = 0.40, size = 44, normalized size = 0.47

$$\frac{2\left(8x^{4}+\left(8x^{3}+4x^{2}+3x-15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{105x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sq
rt(x) - 1))/x^4
```

**giac** [A] time = 0.29, size = 111, normalized size = 1.18

$$\frac{4096\left(35\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}-70\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+168\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+224\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+128\right)}{105\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x)
+ 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^
8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^7
```

**maple** [A] time = 0.05, size = 33, normalized size = 0.35

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(x-1)\left(8x^2+12x+15\right)}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(9/2),x)
```

```
[Out] 2/105*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(8*x^2+12*x+15)/x^(7/2)
```

**maxima** [A] time = 1.55, size = 31, normalized size = 0.33

$$\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{3}{2}}}+\frac{8\,(x-1)^{\frac{3}{2}}}{35\,x^{\frac{5}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="maxima
")
```

```
[Out] 16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3/
2)/x^(7/2)
```

**mupad** [B] time = 5.04, size = 55, normalized size = 0.59

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{35}-\frac{2\,\sqrt{\sqrt{x}+1}}{7}+\frac{8\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{105}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(9/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/35 - (2*(x^(1/2) + 1)^(1/2)
)/7 + (8*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/105))/
x^(7/2)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

## 3.648 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$

**Optimal**. Leaf size=125

$$\frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}}+\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}}+\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

**Rubi [A]** time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}}+\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}}+\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(9*x^(9/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(21*x^(7/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(5/2)) + (32*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)
)/(315*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{2}{3}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{8}{21}\int \frac{\sqrt{-1+\sqrt{x}}}{x^{7}}\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}(}{105x^{5/2}}\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}(}{105x^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 0.37

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}\left(16x^3+24x^2+30x+35\right)}{315x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))/
(315*x^(9/2))
```

**IntegrateAlgebraic [B]** time = 111.25, size = 1484, normalized size = 11.87

result too large to display

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(945 + (
945*(-1 + Sqrt[-1 + Sqrt[x]])^32)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^32 + (10080
*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^31)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^31 + (
178080*(-1 + Sqrt[-1 + Sqrt[x]])^30)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^30 + (77
1120*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^29)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^29
 + (7981176*(-1 + Sqrt[-1 + Sqrt[x]])^28)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^28
+ (22539888*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^27)/(Sqrt[3] - Sqrt[1 + Sqrt[
x]])^27 + (159375840*(-1 + Sqrt[-1 + Sqrt[x]])^26)/(Sqrt[3] - Sqrt[1 + Sqrt
[x]])^26 + (319049280*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^25)/(Sqrt[3] - Sqrt
[1 + Sqrt[x]])^25 + (1649481052*(-1 + Sqrt[-1 + Sqrt[x]])^24)/(Sqrt[3] - Sq
rt[1 + Sqrt[x]])^24 + (2469292416*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^23)/(Sq
rt[3] - Sqrt[1 + Sqrt[x]])^23 + (9691565216*(-1 + Sqrt[-1 + Sqrt[x]])^22)/(
Sqrt[3] - Sqrt[1 + Sqrt[x]])^22 + (11128795920*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt
[x]])^21)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^21 + (33755226952*(-1 + Sqrt[-1 + S
qrt[x]])^20)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^20 + (30109229136*Sqrt[3]*(-1 +
Sqrt[-1 + Sqrt[x]])^19)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^19 + (71150262752*(-1
 + Sqrt[-1 + Sqrt[x]])^18)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^18 + (49500527328*
Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^17)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^17 + (9
1239315046*(-1 + Sqrt[-1 + Sqrt[x]])^16)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^16 +
 (49500527328*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^15)/(Sqrt[3] - Sqrt[1 + Sqr
t[x]])^15 + (71150262752*(-1 + Sqrt[-1 + Sqrt[x]])^14)/(Sqrt[3] - Sqrt[1 +
Sqrt[x]])^14 + (30109229136*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])^13)/(Sqrt[3]
- Sqrt[1 + Sqrt[x]])^13 + (33755226952*(-1 + Sqrt[-1 + Sqrt[x]])^12)/(Sqrt[
3] - Sqrt[1 + Sqrt[x]])^12 + (11128795920*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])
^11)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^11 + (9691565216*(-1 + Sqrt[-1 + Sqrt[x]
])^10)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^10 + (2469292416*Sqrt[3]*(-1 + Sqrt[-1
 + Sqrt[x]])^9)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^9 + (1649481052*(-1 + Sqrt[-1
 + Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (319049280*Sqrt[3]*(-1 +
Sqrt[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (159375840*(-1 + S
qrt[-1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (22539888*Sqrt[3]*(
-1 + Sqrt[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (7981176*(-1
+ Sqrt[-1 + Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (771120*Sqrt[3]*
(-1 + Sqrt[-1 + Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (178080*(-1
+ Sqrt[-1 + Sqrt[x]])^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (10080*Sqrt[3]*(
-1 + Sqrt[-1 + Sqrt[x]]))/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/10569646080 - S
qrt[-1 + Sqrt[x]]/10569646080)*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^36)/((-Sqrt[3]
 + Sqrt[1 + Sqrt[x]])*(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3
]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*
Sqrt[x] - 2*x)^9)
```

**fricas [A]** time = 0.40, size = 49, normalized size = 0.39

$$\frac{2\left(16\,x^{5}+\left(16\,x^{4}+8\,x^{3}+6\,x^{2}+5\,x-35\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{315\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="frica
s")
```

```
[Out] 2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1))/x^5
```

**giac** [A] time = 0.43, size = 132, normalized size = 1.06

$$\frac{16384\left(315\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{20}-756\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}+1344\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+2304\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+2304\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+1024\right)}{315\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac"
)
```

```
[Out] 16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt(
x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqrt(x
) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^4 + 4)^9
```

**maple** [A] time = 0.06, size = 38, normalized size = 0.30

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(x-1)\left(16x^{3}+24x^{2}+30x+35\right)}{315x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(11/2),x)
```

```
[Out] 2/315*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(16*x^3+24*x^2+30*x+35)/x^(
9/2)
```

**maxima** [A] time = 1.22, size = 41, normalized size = 0.33

$$\frac{32\,(x-1)^{\frac{3}{2}}}{315\,x^{\frac{3}{2}}}+\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{5}{2}}}+\frac{4\,(x-1)^{\frac{3}{2}}}{21\,x^{\frac{7}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] 32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)^
(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)
```

**mupad** [B] time = 5.02, size = 67, normalized size = 0.54

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{63}-\frac{2\,\sqrt{\sqrt{x}+1}}{9}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{315}+\frac{32\,x^4\,\sqrt{\sqrt{x}+1}}{315}\right)}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(11/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/63 - (2*(x^(1/2) + 1)^(1/2)
)/9 + (4*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/315 +
(32*x^4*(x^(1/2) + 1)^(1/2))/315))/x^(9/2)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)
[Out] Timed out
```

## 3.649 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$

**Optimal**. Leaf size=104

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}+\frac{5}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{5}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}+\frac{5}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{5}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 + (5*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/
2))/3 + (5*ArcCosh[Sqrt[x]])/8
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{5}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= \frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1 \ldots}}$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{\ldots}}$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{\ldots}}$$

$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{\ldots}}$$

**Mathematica [A]** time = 0.05, size = 67, normalized size = 0.64

$$\frac{1}{24}\left(\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}\,\left(8x^2+10x+15\right)+30\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2) + 30*ArcT
anh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/24
```

**IntegrateAlgebraic [A]** time = 1.42, size = 176, normalized size = 1.69

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{33\left(\sqrt{x}-1\right)^5}{\left(\sqrt{x}+1\right)^5}+\frac{5\left(\sqrt{x}-1\right)^4}{\left(\sqrt{x}+1\right)^4}+\frac{90\left(\sqrt{x}-1\right)^3}{\left(\sqrt{x}+1\right)^3}+\frac{90\left(\sqrt{x}-1\right)^2}{\left(\sqrt{x}+1\right)^2}+\frac{5\left(\sqrt{x}-1\right)}{\sqrt{x}+1}+33\right)}{12\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}-1\right)^6\sqrt{\sqrt{x}+1}}+\frac{5}{4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] ((33 + (33*(-1 + Sqrt[x])^5)/(1 + Sqrt[x])^5 + (5*(-1 + Sqrt[x])^4)/(1 + Sq
rt[x])^4 + (90*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (90*(-1 + Sqrt[x])^2)/(1
 + Sqrt[x])^2 + (5*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]])/(12*(
-1 + (-1 + Sqrt[x])/(1 + Sqrt[x]))^6*Sqrt[1 + Sqrt[x]]) + (5*ArcTanh[Sqrt[-
1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]])/4
```

**fricas [A]** time = 0.41, size = 57, normalized size = 0.55

$$\frac{1}{24}\left(8\,x^2+10\,x+15\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-\frac{5}{16}\,\log\left(2\,\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/16
*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac [A]** time = 0.23, size = 76, normalized size = 0.73

$$\frac{1}{24}\left(\left(2\left(\left(4\left(\sqrt{x}+1\right)\left(\sqrt{x}-4\right)+45\right)\left(\sqrt{x}+1\right)-55\right)\left(\sqrt{x}+1\right)+85\right)\left(\sqrt{x}+1\right)-33\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{5}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*((4*(sqrt(x) + 1)*(sqrt(x) - 4) + 45)*(sqrt(x) + 1) - 55)*(sqrt(x)
 + 1) + 85)*(sqrt(x) + 1) - 33)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/4*l
og(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

**maple [A]** time = 0.06, size = 65, normalized size = 0.62

$$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8\sqrt{x-1}\,x^{\frac{5}{2}}+10\sqrt{x-1}\,x^{\frac{3}{2}}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)+15\sqrt{x-1}\,\sqrt{x}\right)}{24\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*(x-1)^(1/2)*x^(5/2)+10*(x-1)^(1
/2)*x^(3/2)+15*(x-1)^(1/2)*x^(1/2)+15*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

**maxima [A]** time = 0.61, size = 47, normalized size = 0.45

$$\frac{1}{3}\sqrt{x-1}\,x^{\frac{5}{2}}+\frac{5}{12}\sqrt{x-1}\,x^{\frac{3}{2}}+\frac{5}{8}\sqrt{x-1}\,\sqrt{x}+\frac{5}{8}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(x
) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

**mupad [B]** time = 27.09, size = 632, normalized size = 6.08

$$\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2}-\frac{-\frac{175\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{311\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{8361\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{42259\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25295\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{25295\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}+\frac{42259\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{15}}{3\left(\sqrt{\sqrt{x}+1}-1\right)^{15}}+\frac{8361\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{17}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{17}}+\frac{311\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{19}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{19}}-\frac{175\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{21}}{6\left(\sqrt{\sqrt{x}+1}-1\right)^{21}}+\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{23}}{2\left(\sqrt{\sqrt{x}+1}-1\right)^{23}}+\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)}{2\left(\sqrt{\sqrt{x}+1}-1\right)}}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{16}}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{18}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{18}}+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{20}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{20}}-\frac{12\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{22}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{22}}+\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{24}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{24}}-\frac{12\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\sqrt{x}+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] (5*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/2 - ((311*(
(x^(1/2) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) - (175*((x^(1/
2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (8361*((x^(1/2) -
1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (42259*((x^(1/2) - 1)^(
1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25295*((x^(1/2) - 1)^(1/2)
 - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25295*((x^(1/2) - 1)^(1/2) - 1i)
^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (42259*((x^(1/2) - 1)^(1/2) - 1i)^15)/(
3*((x^(1/2) + 1)^(1/2) - 1)^15) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*(
(x^(1/2) + 1)^(1/2) - 1)^17) + (311*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(
1/2) + 1)^(1/2) - 1)^19) - (175*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2)
 + 1)^(1/2) - 1)^21) + (5*((x^(1/2) - 1)^(1/2) - 1i)^23)/(2*((x^(1/2) + 1)^
(1/2) - 1)^23) + (5*((x^(1/2) - 1)^(1/2) - 1i))/(2*((x^(1/2) + 1)^(1/2) - 1
)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((
x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) -
```

```
1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) -
 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/(
(x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2)
 + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1
/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)
^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (6
6*((x^(1/2) - 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/
2) - 1)^(1/2) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2)
 - 1i)^24/((x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

## 3.650 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$

**Optimal**. Leaf size=73

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{3}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi** [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{3}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*x^(3/2))/2 + (3*ArcCosh[Sqrt[x]])/4
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx &= \frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx \\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{8}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1 \ldots}} \\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{4}\text{Subst}\left(\int \frac{\ldots}{\sqrt{-1+\ldots}}\right. \\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 62, normalized size = 0.85

$$\frac{1}{4}\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}(2x+3)+6\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x) + 6*ArcTanh[Sqrt[(-
1 + Sqrt[x])/(1 + Sqrt[x])]])/4
```

**IntegrateAlgebraic** [A] time = 1.32, size = 136, normalized size = 1.86

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{5(\sqrt{x}-1)^3}{(\sqrt{x}+1)^3}+\frac{3(\sqrt{x}-1)^2}{(\sqrt{x}+1)^2}+\frac{3(\sqrt{x}-1)}{\sqrt{x}+1}+5\right)}{2\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}-1\right)^4\sqrt{\sqrt{x}+1}}+\frac{3}{2}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] ((5 + (5*(-1 + Sqrt[x])^3)/(1 + Sqrt[x])^3 + (3*(-1 + Sqrt[x])^2)/(1 + Sqrt
[x])^2 + (3*(-1 + Sqrt[x]))/(1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]])/(2*(-1 + (-1
 + Sqrt[x])/(1 + Sqrt[x]))^4*Sqrt[1 + Sqrt[x]]) + (3*ArcTanh[Sqrt[-1 + Sqrt
[x]]/Sqrt[1 + Sqrt[x]]])/2
```

**fricas** [A] time = 0.40, size = 52, normalized size = 0.71

$$\frac{1}{4}(2x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{3}{8}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac** [A] time = 0.23, size = 59, normalized size = 0.81

$$\frac{1}{4}\left(\left(2\left(\sqrt{x}+1\right)\left(\sqrt{x}-2\right)+9\right)\left(\sqrt{x}+1\right)-5\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{3}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((2*(sqrt(x) + 1)*(sqrt(x) - 2) + 9)*(sqrt(x) + 1) - 5)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1) - 3/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

**maple** [A] time = 0.06, size = 55, normalized size = 0.75

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(2\sqrt{x-1}\,x^{\frac{3}{2}}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)+3\sqrt{x-1}\,\sqrt{x}\right)}{4\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

[Out] 1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*(x-1)^(1/2)*x^(3/2)+3*(x-1)^(1/2
)*x^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)

**maxima [A]** time = 0.46, size = 37, normalized size = 0.51

$$\frac{1}{2}\sqrt{x-1}\,x^{\frac{3}{2}}+\frac{3}{4}\sqrt{x-1}\,\sqrt{x}+\frac{3}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")

[Out] 1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1) +
 2*sqrt(x))

**mupad [B]** time = 18.76, size = 429, normalized size = 5.88

$$3\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)+\frac{\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}-\frac{3\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{15}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{15}}-\frac{3\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)}{\sqrt{\sqrt{x}+1}-1}}{1+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{70\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{8\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{16}}-\frac{8\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{\left(\sqrt{\sqrt{x}+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)

[Out] 3*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) + ((23*((x^(1
/2) - 1)^(1/2) - 1i)^3)/((x^(1/2) + 1)^(1/2) - 1)^3 + (333*((x^(1/2) - 1)^(
1/2) - 1i)^5)/((x^(1/2) + 1)^(1/2) - 1)^5 + (671*((x^(1/2) - 1)^(1/2) - 1i)
^7)/((x^(1/2) + 1)^(1/2) - 1)^7 + (671*((x^(1/2) - 1)^(1/2) - 1i)^9)/((x^(1
/2) + 1)^(1/2) - 1)^9 + (333*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2) + 1)^
(1/2) - 1)^11 + (23*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1
)^13 - (3*((x^(1/2) - 1)^(1/2) - 1i)^15)/((x^(1/2) + 1)^(1/2) - 1)^15 - (3*
((x^(1/2) - 1)^(1/2) - 1i))/((x^(1/2) + 1)^(1/2) - 1))/((28*((x^(1/2) - 1)^
(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (8*((x^(1/2) - 1)^(1/2) - 1i)^
2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (56*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2
) + 1)^(1/2) - 1)^6 + (70*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2
) - 1)^8 - (56*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
+ (28*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (8*((x^
(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + ((x^(1/2) - 1)^(1
/2) - 1i)^16/((x^(1/2) + 1)^(1/2) - 1)^16 + 1)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)

[Out] Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)

## 3.651 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$

**Optimal**. Leaf size=35

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {323, 330, 52}

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx &= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx\\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 55, normalized size = 1.57

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+2\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcTanh[Sqrt[-1 + Sqrt[x]]
/Sqrt[1 + Sqrt[x]]]
```

**IntegrateAlgebraic** [B] time = 1.27, size = 268, normalized size = 7.66

$$\frac{-4\sqrt{\sqrt{x}+1}\left(-20x^{3/2}-4x+96\sqrt{x}+48\right)-4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(-7x^{3/2}-28x+10\sqrt{x}+84\right)+\sqrt{3}\left(-4\sqrt{\sqrt{x}-1}\left(12x^{3/2}+20x-32\sqrt{x}-48\right)-4\left(14x^{3/2}+4x^2-18x-70\sqrt{x}-28\right)\right)}{\sqrt{3}\sqrt{\sqrt{x}+1}\left(-48\sqrt{x}-32\right)+\sqrt{\sqrt{x}-1}\left(\sqrt{3}\sqrt{\sqrt{x}+1}\left(-16\sqrt{x}-56\right)+80\sqrt{x}+96\right)+28x+112\sqrt{x}+56}-4\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}-1}{\sqrt{3}-\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (-4*Sqrt[1 + Sqrt[x]]*(48 + 96*Sqrt[x] - 4*x - 20*x^(3/2)) - 4*Sqrt[-1 + Sq
rt[x]]*Sqrt[1 + Sqrt[x]]*(84 + 10*Sqrt[x] - 28*x - 7*x^(3/2)) + Sqrt[3]*(-4
*Sqrt[-1 + Sqrt[x]]*(-48 - 32*Sqrt[x] + 20*x + 12*x^(3/2)) - 4*(-28 - 70*Sq
rt[x] - 18*x + 14*x^(3/2) + 4*x^2)))/(56 + Sqrt[3]*(-32 - 48*Sqrt[x])*Sqrt[
1 + Sqrt[x]] + Sqrt[-1 + Sqrt[x]]*(96 + Sqrt[3]*(-56 - 16*Sqrt[x])*Sqrt[1 +
 Sqrt[x]] + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) - 4*ArcTanh[(-1 + Sqrt[-1 + S
qrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

**fricas** [A] time = 0.41, size = 46, normalized size = 1.31

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac** [A] time = 0.22, size = 39, normalized size = 1.11

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log(sqrt(sqrt(x) + 1) - sqr
t(sqrt(x) - 1))
```

**maple** [A] time = 0.06, size = 41, normalized size = 1.17

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(\ln\left(\sqrt{x}+\sqrt{x-1}\right)+\sqrt{x-1}\sqrt{x}\right)}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*((x-1)^(1/2)*x^(1/2)+ln(x^(1/2)+(x-1)^(
1/2)))/(x-1)^(1/2)
```

**maxima** [A] time = 0.45, size = 24, normalized size = 0.69

$$\sqrt{x-1}\,\sqrt{x}+\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)
```

**sympy** [C] time = 27.91, size = 83, normalized size = 2.37

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix} \middle| {\frac{1}{x}} \right)}{2\pi^{\frac{3}{2}}}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix} \middle| {\frac{e^{2i\pi}}{x}} \right)}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0),
()), 1/x)/(2*pi**(3/2)) - I*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()),
((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*pi)/x)/(2*pi**(3/2))
```

## 3.652 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx$

**Optimal**. Leaf size=8

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {330, 52}

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 2*ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x, \sqrt{x}\right)\\ &= 2\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

**Mathematica [B]** time = 0.01, size = 26, normalized size = 3.25

$$4\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 4*ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]]
```

**IntegrateAlgebraic [B]** time = 1.02, size = 38, normalized size = 4.75

$$-8\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}-1}{\sqrt{3}-\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] -8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

**fricas [B]** time = 0.41, size = 27, normalized size = 3.38

$$-\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] -log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

**giac [B]** time = 0.20, size = 20, normalized size = 2.50

$$-4\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] -4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

**maple [B]** time = 0.05, size = 40, normalized size = 5.00

$$\frac{2\sqrt{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2*((x^(1/2)-1)*(x^(1/2)+1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^
(1/2)+(x-1)^(1/2))
```

**maxima [B]** time = 0.65, size = 16, normalized size = 2.00

$$2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

**mupad [B]** time = 5.29, size = 6, normalized size = 0.75

$$2\,\mathrm{acosh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] 2*acosh(x^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

## 3.653 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx$

**Optimal**. Leaf size=29

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

**Rubi [A]** time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {265}

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

**Mathematica [A]** time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

**IntegrateAlgebraic [B]** time = 1.14, size = 188, normalized size = 6.48

$$\frac{\left(\frac{\left(\sqrt{\sqrt{x}-1}-1\right)^3}{2\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^3}+\frac{\sqrt{\sqrt{x}-1}-1}{2\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)}\right)\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^4}{-2x-2\sqrt{\sqrt{x}-1}\,\sqrt{x}+\sqrt{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+2\sqrt{3}\sqrt{\sqrt{x}+1}\,\sqrt{x}-3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (((-1 + Sqrt[-1 + Sqrt[x]])^3/(2*(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3) + (-1 + S
qrt[-1 + Sqrt[x]])/(2*(Sqrt[3] - Sqrt[1 + Sqrt[x]])))*(Sqrt[3] - Sqrt[1 + S
qrt[x]])^4)/(-3*Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 +
 Sqrt[x]]*Sqrt[x] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] -
2*x)
```

**fricas [A]** time = 0.42, size = 25, normalized size = 0.86

$$\frac{2\left(\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x
```

**giac [A]** time = 0.22, size = 25, normalized size = 0.86

$$\frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)
```

**maple [A]** time = 0.06, size = 20, normalized size = 0.69

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2)
```

**maxima [A]** time = 1.21, size = 10, normalized size = 0.34

$$\frac{2\sqrt{x-1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*sqrt(x - 1)/sqrt(x)
```

**mupad [B]** time = 5.56, size = 19, normalized size = 0.66

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] (2*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

## 3.654 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx$

**Optimal**. Leaf size=63

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

**Rubi [A]** time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx&=\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{2}{3}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx\\&=\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{4\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3\sqrt{x}}\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 36, normalized size = 0.57

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(2x+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(1 + 2*x))/(3*x^(3/2))
```

**IntegrateAlgebraic [B]** time = 2.75, size = 512, normalized size = 8.13

$$\frac{\left(\frac{\left(\sqrt{\sqrt{x}-1}-1\right)^2}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^2}+1\right)\left(\frac{3\left(\sqrt{\sqrt{x}-1}-1\right)^8}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^8}+\frac{6\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^7}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^7}+\frac{32\left(\sqrt{\sqrt{x}-1}-1\right)^6}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^6}+\frac{42\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^5}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^5}+\frac{106\left(\sqrt{\sqrt{x}-1}-1\right)^4}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^4}+\frac{42\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)^3}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^3}+\frac{32\left(\sqrt{\sqrt{x}-1}-1\right)^2}{\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^2}+\frac{6\sqrt{3}\left(\sqrt{\sqrt{x}-1}-1\right)}{\sqrt{3}-\sqrt{\sqrt{x}+1}}+3\right)\left(\frac{1}{384}-\frac{\sqrt{\sqrt{x}-1}}{384}\right)\left(\sqrt{3}-\sqrt{\sqrt{x}+1}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-\sqrt{3}\right)\left(-2x-2\sqrt{\sqrt{x}-1}\sqrt{x}+\sqrt{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+2\sqrt{3}\sqrt{\sqrt{x}+1}\sqrt{x}-3\sqrt{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] ((1 + (-1 + Sqrt[-1 + Sqrt[x]])^2/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2)*(3 + (3*
(-1 + Sqrt[-1 + Sqrt[x]])^8)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^8 + (6*Sqrt[3]*(
-1 + Sqrt[-1 + Sqrt[x]])^7)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^7 + (32*(-1 + Sqr
t[-1 + Sqrt[x]])^6)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^6 + (42*Sqrt[3]*(-1 + Sqr
t[-1 + Sqrt[x]])^5)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^5 + (106*(-1 + Sqrt[-1 +
Sqrt[x]])^4)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^4 + (42*Sqrt[3]*(-1 + Sqrt[-1 +
Sqrt[x]])^3)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^3 + (32*(-1 + Sqrt[-1 + Sqrt[x]]
)^2)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])^2 + (6*Sqrt[3]*(-1 + Sqrt[-1 + Sqrt[x]])
)/(Sqrt[3] - Sqrt[1 + Sqrt[x]]))*(1/384 - Sqrt[-1 + Sqrt[x]]/384)*(Sqrt[3]
- Sqrt[1 + Sqrt[x]])^12)/((-Sqrt[3] + Sqrt[1 + Sqrt[x]])*(-3*Sqrt[x] - 2*Sq
rt[-1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + Sqrt[3]*Sq
rt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - 2*x)^3)
```

**fricas [A]** time = 0.40, size = 34, normalized size = 0.54

$$\frac{2\left((2x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x^2\right)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2
```

**giac [A]** time = 0.22, size = 48, normalized size = 0.76

$$\frac{128\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

**maple [A]** time = 0.06, size = 25, normalized size = 0.40

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(2x+1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*x+1)/x^(3/2)
```

**maxima** [A] time = 1.20, size = 21, normalized size = 0.33

$$\frac{4\sqrt{x-1}}{3\sqrt{x}} + \frac{2\sqrt{x-1}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 4/3*sqrt(x - 1)/sqrt(x) + 2/3*sqrt(x - 1)/x^(3/2)
```

**mupad** [B] time = 5.50, size = 33, normalized size = 0.52

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{4x}{3}+\frac{2\sqrt{x}}{3}+\frac{4x^{3/2}}{3}+\frac{2}{3}\right)}{x^{3/2}\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((4*x)/3 + (2*x^(1/2))/3 + (4*x^(3/2))/3 + 2/3))/(x^(3
/2)*(x^(1/2) + 1)^(1/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

## 3.655 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}}\,dx$

**Optimal**. Leaf size=94

$$\frac{8\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

**Rubi** [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{8\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(5*x^(5/2)) + (8*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(15*x^(3/2)) + (16*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
])/(15*Sqrt[x])
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}}\,dx &= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{4}{5}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx\\ &= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{8}{15}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{}}}\\ &= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{16\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15\sqrt{x}}\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 41, normalized size = 0.44

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^2+4x+3\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(3 + 4*x + 8*x^2))/(15*x^(5/2))
```

**IntegrateAlgebraic** [A] time = 1.37, size = 127, normalized size = 1.35

$$\frac{4\left(\frac{15\left(\sqrt{x}-1\right)^4}{\left(\sqrt{x}+1\right)^4}+\frac{20\left(\sqrt{x}-1\right)^3}{\left(\sqrt{x}+1\right)^3}+\frac{58\left(\sqrt{x}-1\right)^2}{\left(\sqrt{x}+1\right)^2}+\frac{20\left(\sqrt{x}-1\right)}{\sqrt{x}+1}+15\right)\sqrt{\sqrt{x}-1}}{15\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}+1\right)^5\sqrt{\sqrt{x}+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (4*(15 + (15*(-1 + Sqrt[x])^4)/(1 + Sqrt[x])^4 + (20*(-1 + Sqrt[x])^3)/(1 +
 Sqrt[x])^3 + (58*(-1 + Sqrt[x])^2)/(1 + Sqrt[x])^2 + (20*(-1 + Sqrt[x]))/(
1 + Sqrt[x]))*Sqrt[-1 + Sqrt[x]])/(15*(1 + (-1 + Sqrt[x])/(1 + Sqrt[x]))^5*
Sqrt[1 + Sqrt[x]])
```

**fricas** [A] time = 0.41, size = 39, normalized size = 0.41

$$\frac{2\left(8\,x^3+\left(8\,x^2+4\,x+3\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
)/x^3
```

**giac** [A] time = 0.26, size = 69, normalized size = 0.73

$$\frac{4096\left(5\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^8+10\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+8\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
4)^5
```

**maple** [A] time = 0.06, size = 30, normalized size = 0.32

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^2+4x+3\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^2+4*x+3)/x^(5/2)
```

**maxima [A]** time = 1.30, size = 31, normalized size = 0.33

$$\frac{16\sqrt{x-1}}{15\sqrt{x}}+\frac{8\sqrt{x-1}}{15\,x^{\frac{3}{2}}}+\frac{2\sqrt{x-1}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 16/15*sqrt(x - 1)/sqrt(x) + 8/15*sqrt(x - 1)/x^(3/2) + 2/5*sqrt(x - 1)/x^(5
/2)
```

**mupad [B]** time = 5.66, size = 43, normalized size = 0.46

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{8\,x}{15}+\frac{16\,x^2}{15}+\frac{2\,\sqrt{x}}{5}+\frac{8\,x^{3/2}}{15}+\frac{16\,x^{5/2}}{15}+\frac{2}{5}\right)}{x^{5/2}\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((8*x)/15 + (16*x^2)/15 + (2*x^(1/2))/5 + (8*x^(3/2))/
15 + (16*x^(5/2))/15 + 2/5))/(x^(5/2)*(x^(1/2) + 1)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

## 3.656 $\int \frac{1+x^6}{x(1-x^6)} \, dx$

**Optimal**. Leaf size=15

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

**Rubi [A]** time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 72}

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^6}{x\left(1-x^6\right)} \, dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1+x}{(1-x)x} \, dx,x,x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right) dx,x,x^6\right)\\ &= \log(x)-\frac{1}{3}\log\left(1-x^6\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 15, normalized size = 1.00

$$\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{x\left(1-x^6\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^6)/(x*(1 - x^6)),x]
[Out] IntegrateAlgebraic[(1 + x^6)/(x*(1 - x^6)), x]
```

**fricas [A]** time = 0.40, size = 11, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^6-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="fricas")
[Out] -1/3*log(x^6 - 1) + log(x)
```

**giac [A]** time = 0.19, size = 16, normalized size = 1.07

$$\frac{1}{6}\log\left(x^6\right)-\frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="giac")
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

**maple [B]** time = 0.05, size = 36, normalized size = 2.40

$$\ln(x)-\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}-\frac{\ln\left(x^2-x+1\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x/(-x^6+1),x)
[Out] -1/3*ln(x-1)-1/3*ln(x^2+x+1)-1/3*ln(x+1)+ln(x)-1/3*ln(x^2-x+1)
```

**maxima [A]** time = 0.46, size = 15, normalized size = 1.00

$$-\frac{1}{3}\log\left(x^6-1\right)+\frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")
[Out] -1/3*log(x^6 - 1) + 1/6*log(x^6)
```

**mupad [B]** time = 0.09, size = 11, normalized size = 0.73

$$\ln(x)-\frac{\ln\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/(x*(x^6 - 1)),x)
[Out] log(x) - log(x^6 - 1)/3
```

**sympy [A]** time = 0.12, size = 10, normalized size = 0.67

$$\log(x)-\frac{\log\left(x^6-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x/(-x**6+1),x)
```

```
[Out] log(x) - log(x**6 - 1)/3
```

## 3.657

$$\int (ex)^m (a+bx^n)^p \left(a(1+m)+b(1+m+n+np)x^n\right) dx$$

**Optimal**. Leaf size=22

$$\frac{(ex)^{m+1}(a+bx^n)^{p+1}}{e}$$

**Rubi** [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {449}

$$\frac{(ex)^{m+1}(a+bx^n)^{p+1}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m (a+bx^n)^p \left(a(1+m)+b(1+m+n+np)x^n\right) dx = \frac{(ex)^{1+m}(a+bx^n)^{1+p}}{e}$$

**Mathematica** [C] time = 0.17, size = 110, normalized size = 5.00

$$x(ex)^m (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(\frac{bx^n(m+np+n+1)\,{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1}+a\,{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n};-\frac{bx^n}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] (x*(e*x)^m*(a + b*x^n)^p*(a*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n,
 -((b*x^n)/a)] + (b*(1 + m + n + n*p)*x^n*Hypergeometric2F1[(1 + m + n)/n,
-p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n)))/(1 + (b*x^n)/a)^p
```

**IntegrateAlgebraic** [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (ex)^m (a+bx^n)^p \left(a(1+m)+b(1+m+n+np)x^n\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x
^n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n +
 n*p)*x^n), x]
```

**fricas [A]** time = 0.41, size = 40, normalized size = 1.82

$$\left(bxx^{n}e^{\left(m\log(e)+m\log(x)\right)} + axe^{\left(m\log(e)+m\log(x)\right)}\right)(bx^{n} + a)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="fri
cas")
```

```
[Out] (b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)))*(b*x^n + a)
^p
```

**giac [A]** time = 0.29, size = 38, normalized size = 1.73

$$(bx^{n} + a)^{p}bxx^{m}x^{n}e^{m} + (bx^{n} + a)^{p}axx^{m}e^{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="gia
c")
```

```
[Out] (b*x^n + a)^p*b*x*x^m*x^n*e^m + (b*x^n + a)^p*a*x*x^m*e^m
```

**maple [F]** time = 0.67, size = 0, normalized size = 0.00

$$\int \left(\left(np + m + n + 1\right) b\, x^{n} + (m + 1)\, a\right) (ex)^{m} \left(b\, x^{n} + a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^n+a)^p*(a*(m+1)+b*(n*p+m+n+1)*x^n),x)
```

```
[Out] int((e*x)^m*(b*x^n+a)^p*(a*(m+1)+b*(n*p+m+n+1)*x^n),x)
```

**maxima [A]** time = 0.95, size = 36, normalized size = 1.64

$$\left(ae^{m}xx^{m} + be^{m}xe^{\left(m\log(x)+n\log(x)\right)}\right)(bx^{n} + a)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="max
ima")
```

```
[Out] (a*e^m*x*x^m + b*e^m*x*e^(m*log(x) + n*log(x)))*(b*x^n + a)^p
```

**mupad [B]** time = 4.94, size = 31, normalized size = 1.41

$$\left(a\,x\,{\left(e\,x\right)}^m+b\,x^{n+1}\,{\left(e\,x\right)}^m\right)\,{\left(a+b\,x^n\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a*(m + 1) + b*x^n*(m + n + n*p + 1))*(a + b*x^n)^p,x)
```

```
[Out] (a*x*(e*x)^m + b*x^(n + 1)*(e*x)^m)*(a + b*x^n)^p
```

**sympy [B]** time = 8.43, size = 39, normalized size = 1.77

$$ae^{m}xx^{m}\left(a + bx^{n}\right)^{p} + be^{m}xx^{m}x^{n}\left(a + bx^{n}\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(n*p+m+n+1)*x**n),x)
```

```
[Out] a*e**m*x*x**m*(a + b*x**n)**p + b*e**m*x*x**m*x**n*(a + b*x**n)**p
```

## 3.658 $\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx$

**Optimal**. Leaf size=63

$$-\frac{b\log(a+bx^n)}{an(bc-ad)}+\frac{d\log(c+dx^n)}{cn(bc-ad)}+\frac{\log(x)}{ac}$$

**Rubi [A]** time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log(a+bx^n)}{an(bc-ad)}+\frac{d\log(c+dx^n)}{cn(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)*n) + (d*Log[c + d*x^n])/(c
*(b*c - a*d)*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= \frac{\log(x)}{ac}-\frac{b\log(a+bx^n)}{a(bc-ad)n}+\frac{d\log(c+dx^n)}{c(bc-ad)n}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 56, normalized size = 0.89

$$\frac{-bc\log(a+bx^n)+ad\log(c+dx^n)-adn\log(x)+bcn\log(x)}{abc^2n-a^2cdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*n*Log[x] - a*d*n*Log[x] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a*
b*c^2*n - a^2*c*d*n)
```

**IntegrateAlgebraic** [A] time = 0.07, size = 67, normalized size = 1.06

$$\frac{b\log\left(a+bx^n\right)}{an(ad-bc)}+\frac{d\log\left(c+dx^n\right)}{cn(bc-ad)}+\frac{\log\left(x^n\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] Log[x^n]/(a*c*n) + (b*Log[a + b*x^n])/(a*(-(b*c) + a*d)*n) + (d*Log[c + d*x
^n])/(c*(b*c - a*d)*n)
```

**fricas** [A] time = 0.45, size = 58, normalized size = 0.92

$$-\frac{bc\log\left(bx^n+a\right)-ad\log\left(dx^n+c\right)-(bc-ad)n\log(x)}{\left(abc^2-a^2cd\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^2
 - a^2*c*d)*n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx^n+a)(dx^n+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)
```

**maple** [A] time = 0.10, size = 69, normalized size = 1.10

$$\frac{b\ln\left(b\,x^n+a\right)}{(ad-bc)\,an}-\frac{d\ln\left(d\,x^n+c\right)}{(ad-bc)\,cn}+\frac{\ln\left(x^n\right)}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] -1/n/c*d/(a*d-b*c)*ln(d*x^n+c)+1/n/a*b/(a*d-b*c)*ln(b*x^n+a)+1/n/c/a*ln(x^n
)
```

**maxima** [A] time = 0.57, size = 69, normalized size = 1.10

$$-\frac{b\log\left(\frac{bx^n+a}{b}\right)}{abcn-a^2dn}+\frac{d\log\left(\frac{dx^n+c}{d}\right)}{bc^2n-acdn}+\frac{\log(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n -
 a*c*d*n) + log(x)/(a*c)
```

**mupad** [B] time = 5.72, size = 162, normalized size = 2.57

$$\frac{b\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{d\,x\,\left(a^2\,d\,n-a\,b\,c\,n\right)}\right)}{a^2\,d\,n-a\,b\,c\,n}+\frac{d\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{b\,x\,\left(b\,c^2\,n-a\,c\,d\,n\right)}\right)}{b\,c^2\,n-a\,c\,d\,n}+\frac{\ln(x)\,\left(n-1\right)}{a\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] (b*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(d*x*(a^2*d*n - a*b*
c*n))))/(a^2*d*n - a*b*c*n) + (d*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b
*c*n*x^n)/(b*x*(b*c^2*n - a*c*d*n))))/(b*c^2*n - a*c*d*n) + (log(x)*(n - 1)
)/(a*c*n)
```

**sympy [A]** time = 4.02, size = 332, normalized size = 5.27

$$\begin{cases} \frac{\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an}}{c} & \text{for } d = 0 \\ \frac{-\frac{x^{-n}}{cn} + \frac{d\log\left(x^{-n}+\frac{d}{c}\right)}{c^2 n}}{b} & \text{for } a = 0 \\ \frac{cn\log(x)}{ac^2n+acdnx^n} - \frac{c\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n} + \frac{c}{ac^2n+acdnx^n} + \frac{dnx^n\log(x)}{ac^2n+acdnx^n} - \frac{dx^n\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n} & \text{for } b = \frac{ad}{c} \\ \frac{-\frac{x^{-n}}{an} + \frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2 n}}{d} & \text{for } c = 0 \\ \frac{\log(x)}{(a+b)(c+d)} & \text{for } n = 0 \\ \frac{\frac{\log(x)}{c} - \frac{\log\left(\frac{c}{d}+x^n\right)}{cn}}{a} & \text{for } b = 0 \\ \frac{adn\log(x)}{a^2cdn-abc^2n} - \frac{ad\log\left(\frac{c}{d}+x^n\right)}{a^2cdn-abc^2n} - \frac{bcn\log(x)}{a^2cdn-abc^2n} + \frac{bc\log\left(\frac{a}{b}+x^n\right)}{a^2cdn-abc^2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((log(x)/a - log(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-x**(-n)/(c*n
) + d*log(x**(-n) + d/c)/(c**2*n))/b, Eq(a, 0)), (c*n*log(x)/(a*c**2*n + a*
c*d*n*x**n) - c*log(c/d + x**n)/(a*c**2*n + a*c*d*n*x**n) + c/(a*c**2*n + a
*c*d*n*x**n) + d*n*x**n*log(x)/(a*c**2*n + a*c*d*n*x**n) - d*x**n*log(c/d +
 x**n)/(a*c**2*n + a*c*d*n*x**n), Eq(b, a*d/c)), ((-x**(-n)/(a*n) + b*log(x
**(-n) + b/a)/(a**2*n))/d, Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(n, 0)),
 ((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), (a*d*n*log(x)/(a**2*c*d*
n - a*b*c**2*n) - a*d*log(c/d + x**n)/(a**2*c*d*n - a*b*c**2*n) - b*c*n*log
(x)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a/b + x**n)/(a**2*c*d*n - a*b*c**2*
n), True))
```

## 3.659 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx$

**Optimal**. Leaf size=101

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{cn(bc-ad)^2}+\frac{b}{an(bc-ad)(a+bx^n)}$$

**Rubi [A]** time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{cn(bc-ad)^2}+\frac{b}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*(a + b*x^n)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^n])/(a^2*(b*c - a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx&=\frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}\,dx,x,x^n\right)}{n}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{a^2cx}+\frac{b^2}{a(-bc+ad)(a+bx)^2}+\frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)}-\frac{d^3}{c(bc-ad)^2(c+dx)}\right)dx,x,x^n\right)}{n}\\&=\frac{b}{a(bc-ad)n(a+bx^n)}+\frac{\log(x)}{a^2c}-\frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2n}-\frac{d^2\log(c+dx^n)}{c(bc-ad)^2n}\end{aligned}$$

**Mathematica [A]** time = 0.33, size = 97, normalized size = 0.96

$$\frac{-\frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2}+\frac{n\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{c(bc-ad)^2}+\frac{b}{a(bc-ad)(a+bx^n)}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b/(a*(b*c - a*d)*(a + b*x^n)) + (n*Log[x])/(a^2*c) - (b*(b*c - 2*a*d)*Log[
a + b*x^n])/(a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2))/n
```

**IntegrateAlgebraic [A]** time = 0.13, size = 109, normalized size = 1.08

$$\frac{\left(2abd-b^2c\right)\log\left(a+bx^n\right)}{a^2n(ad-bc)^2}+\frac{\log\left(x^n\right)}{a^2cn}-\frac{d^2\log\left(c+dx^n\right)}{cn(bc-ad)^2}-\frac{b}{an(ad-bc)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(b/(a*(-(b*c) + a*d)*n*(a + b*x^n))) + Log[x^n]/(a^2*c*n) + ((-(b^2*c) + 2
*a*b*d)*Log[a + b*x^n])/(a^2*(-(b*c) + a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*
(b*c - a*d)^2*n)
```

**fricas [B]** time = 0.47, size = 224, normalized size = 2.22

$$\frac{ab^2c^2-a^2bcd+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)nx^n\log(x)+\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)n\log(x)-\left(ab^2c^2-2\,a^2bcd+\left(b^3c^2-2\,ab^2cd\right)x^n\right)\log\left(bx^n+a\right)-\left(a^2bd^2x^n+a^3d^2\right)\log\left(dx^n+c\right)}{\left(a^2b^3c^3-2\,a^3b^2c^2d+a^4bcd^2\right)nx^n+\left(a^3b^2c^3-2\,a^4bc^2d+a^5cd^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x) +
 (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d +
(b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*log
(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^3*b^
2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^n+a\right)^2\left(dx^n+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)
```

**maple [A]** time = 0.10, size = 131, normalized size = 1.30

$$\frac{2bd\ln\left(b\,x^n+a\right)}{\left(ad-bc\right)^2an}-\frac{b^2c\ln\left(b\,x^n+a\right)}{\left(ad-bc\right)^2a^2n}-\frac{d^2\ln\left(d\,x^n+c\right)}{\left(ad-bc\right)^2cn}-\frac{b}{\left(ad-bc\right)\left(b\,x^n+a\right)an}+\frac{\ln\left(x^n\right)}{a^2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] -1/n/(a*d-b*c)^2/c*d^2*ln(d*x^n+c)-1/n/a*b/(a*d-b*c)/(b*x^n+a)+2/n*b/(a*d-b
*c)^2/a*ln(b*x^n+a)*d-1/n*b^2/(a*d-b*c)^2/a^2*ln(b*x^n+a)*c+1/n/c/a^2*ln(x^
n)
```

**maxima [A]** time = 0.67, size = 151, normalized size = 1.50

$$-\frac{d^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n-2\,abc^2dn+a^2cd^2n}-\frac{\left(b^2c-2\,abd\right)\log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n}+\frac{b}{a^2bcn-a^3dn+\left(ab^2cn-a^2bdn\right)x^n}+\frac{\log(x)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c
- 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n) +
 b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a+b\,x^n\right)^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(1/(x*(a + b*x^n)^2*(c + d*x^n)), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed
```

## 3.660 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$

**Optimal**. Leaf size=130

$$\frac{bx^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{b^3x^{4n}}{4dn}$$

**Rubi** [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{bx^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}+\frac{b^3x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)^3*x^n)/(d^4*n)) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*
n))/(2*d^3*n) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2*n) + (b^3*x^(4*n))/(4*d*
n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}\left(a+bx^n\right)^3}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{c+dx}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^3}{d^4}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x}{d^3}-\frac{b^2(bc-3ad)x^2}{d^2}+\frac{b^3x^3}{d}+\frac{c(bc-ad)^3}{d^4(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{(bc-ad)^3x^n}{d^4n}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^{2n}}{2d^3n}-\frac{b^2(bc-3ad)x^{3n}}{3d^2n}+\frac{b^3x^{4n}}{4dn}+\frac{c(bc-ad)}{}\end{aligned}$$

**Mathematica** [A] time = 0.35, size = 115, normalized size = 0.88

$$\frac{6bd^2x^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)-4b^2d^3x^{3n}(bc-3ad)+12dx^n(ad-bc)^3+12c(bc-ad)^3\log\left(c+dx^n\right)+3b^3d^4x^{4n}}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (12*d*(-(b*c) + a*d)^3*x^n + 6*b*d^2*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2
*n) - 4*b^2*d^3*(b*c - 3*a*d)*x^(3*n) + 3*b^3*d^4*x^(4*n) + 12*c*(b*c - a*d
)^3*Log[c + d*x^n])/(12*d^5*n)
```

**IntegrateAlgebraic [A]** time = 0.11, size = 155, normalized size = 1.19

$$\frac{x^n\left(12a^3d^3-36a^2bcd^2+18a^2bd^3x^n+36ab^2c^2d-18ab^2cd^2x^n+12ab^2d^3x^{2n}-12b^3c^3+6b^3c^2dx^n-4b^3cd^2x^{2n}+3b^3d^3x^{3n}\right)}{12d^4n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (x^n*(-12*b^3*c^3 + 36*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 12*a^3*d^3 + 6*b^3*c^
2*d*x^n - 18*a*b^2*c*d^2*x^n + 18*a^2*b*d^3*x^n - 4*b^3*c*d^2*x^(2*n) + 12*
a*b^2*d^3*x^(2*n) + 3*b^3*d^3*x^(3*n)))/(12*d^4*n) + (c*(b*c - a*d)^3*Log[c
 + d*x^n])/(d^5*n)
```

**fricas [A]** time = 0.45, size = 177, normalized size = 1.36

$$\frac{3\,b^3d^4x^{4\,n}-4\left(b^3cd^3-3\,ab^2d^4\right)x^{3\,n}+6\left(b^3c^2d^2-3\,ab^2cd^3+3\,a^2bd^4\right)x^{2\,n}-12\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)x^n+12\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)\log\left(dx^n+c\right)}{12\,d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^3*d^4*x^(4*n) - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^(3*n) + 6*(b^3*c^2*
d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^(2*n) - 12*(b^3*c^3*d - 3*a*b^2*c^2*d^
2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^
2*d^2 - a^3*c*d^3)*log(d*x^n + c))/(d^5*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^n+a\right)^3 x^{2\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)
```

**maple [B]** time = 0.08, size = 284, normalized size = 2.18

$$-\frac{a^3c\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^2n}+\frac{a^3\mathrm{e}^{n\ln(x)}}{dn}+\frac{3a^2b\,c^2\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^3n}-\frac{3a^2bc\,\mathrm{e}^{n\ln(x)}}{d^2n}+\frac{3a^2b\,\mathrm{e}^{2n\ln(x)}}{2dn}-\frac{3a\,b^2c^3\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^4n}+\frac{3a\,b^2c^2\mathrm{e}^{n\ln(x)}}{d^3n}-\frac{3a\,b^2c\,\mathrm{e}^{2n\ln(x)}}{2d^2n}+\frac{a\,b^2\mathrm{e}^{3n\ln(x)}}{dn}+\frac{b^3c^4\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^5n}-\frac{b^3c^3\mathrm{e}^{n\ln(x)}}{d^4n}+\frac{b^3c^2\mathrm{e}^{2n\ln(x)}}{2d^3n}-\frac{b^3c\,\mathrm{e}^{3n\ln(x)}}{3d^2n}+\frac{b^3\mathrm{e}^{4n\ln(x)}}{4dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a^3-3/d^2/n*exp(n*ln(x))*a^2*b*c+3/d^3/n*exp(n*ln(x))*a*
b^2*c^2-1/d^4/n*exp(n*ln(x))*b^3*c^3+1/4*b^3/d/n*exp(n*ln(x))^4+3/2*b/d/n*e
xp(n*ln(x))^2*a^2-3/2*b^2/d^2/n*exp(n*ln(x))^2*a*c+1/2*b^3/d^3/n*exp(n*ln(x
))^2*c^2+b^2/d/n*exp(n*ln(x))^3*a-1/3*b^3/d^2/n*exp(n*ln(x))^3*c-c/d^2/n*ln
(d*exp(n*ln(x))+c)*a^3+3*c^2/d^3/n*ln(d*exp(n*ln(x))+c)*a^2*b-3*c^3/d^4/n*l
n(d*exp(n*ln(x))+c)*a*b^2+c^4/d^5/n*ln(d*exp(n*ln(x))+c)*b^3
```

**maxima [A]** time = 0.53, size = 231, normalized size = 1.78

$$a^3\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)+\frac{1}{12}\,b^3\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)-\frac{1}{2}\,ab^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+\frac{3}{2}\,a^2b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^3*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*log((d*x^
n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) - 12
*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x
^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*log((d*x^n
+ c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

**mupad** [F]     time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}\,{\left(a+b\,x^n\right)}^3}{c+d\,x^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

**sympy** [A]     time = 135.87, size = 320, normalized size = 2.46

$$\begin{cases} \frac{\left(a+b\right)^{3} \log{\left(x \right)}}{c} & \text{for}\: d = 0 \wedge n = 0 \\ \frac{\frac{a^{3} x^{2 n}}{2 n} + \frac{a^{2} b x^{3 n}}{n} + \frac{3 a b^{2} x^{4 n}}{4 n} + \frac{b^{3} x^{5 n}}{5 n}}{c} & \text{for}\: d = 0 \\ \frac{\left(a+b\right)^{3} \log{\left(x \right)}}{c+d} & \text{for}\: n = 0 \\ - \frac{a^{3} c \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{2} n} + \frac{a^{3} x^{n}}{d n} + \frac{3 a^{2} b c^{2} \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{3} n} - \frac{3 a^{2} b c x^{n}}{d^{2} n} + \frac{3 a^{2} b x^{2 n}}{2 d n} - \frac{3 a b^{2} c^{3} \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{4} n} + \frac{3 a b^{2} c^{2} x^{n}}{d^{3} n} - \frac{3 a b^{2} c x^{2 n}}{2 d^{2} n} + \frac{a b^{2} x^{3 n}}{d n} + \frac{b^{3} c^{4} \log{\left(\frac{c}{d} + x^{n} \right)}}{d^{5} n} - \frac{b^{3} c^{3} x^{n}}{d^{4} n} + \frac{b^{3} c^{2} x^{2 n}}{2 d^{3} n} - \frac{b^{3} c x^{3 n}}{3 d^{2} n} + \frac{b^{3} x^{4 n}}{4 d n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x**(2*n)/(2*n)
 + a**2*b*x**(3*n)/n + 3*a*b**2*x**(4*n)/(4*n) + b**3*x**(5*n)/(5*n))/c, Eq
(d, 0)), ((a + b)**3*log(x)/(c + d), Eq(n, 0)), (-a**3*c*log(c/d + x**n)/(d
**2*n) + a**3*x**n/(d*n) + 3*a**2*b*c**2*log(c/d + x**n)/(d**3*n) - 3*a**2*
b*c*x**n/(d**2*n) + 3*a**2*b*x**(2*n)/(2*d*n) - 3*a*b**2*c**3*log(c/d + x**
n)/(d**4*n) + 3*a*b**2*c**2*x**n/(d**3*n) - 3*a*b**2*c*x**(2*n)/(2*d**2*n)
+ a*b**2*x**(3*n)/(d*n) + b**3*c**4*log(c/d + x**n)/(d**5*n) - b**3*c**3*x*
*n/(d**4*n) + b**3*c**2*x**(2*n)/(2*d**3*n) - b**3*c*x**(3*n)/(3*d**2*n) +
b**3*x**(4*n)/(4*d*n), True))
```

## 3.661 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$

**Optimal**. Leaf size=90

$$-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}+\frac{x^n(bc-ad)^2}{d^3n}-\frac{bx^{2n}(bc-2ad)}{2d^2n}+\frac{b^2x^{3n}}{3dn}$$

**Rubi** [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{x^n(bc-ad)^2}{d^3n}-\frac{bx^{2n}(bc-2ad)}{2d^2n}-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}+\frac{b^2x^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] ((b*c - a*d)^2*x^n)/(d^3*n) - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2*n) + (b^2*x^
(3*n))/(3*d*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3}-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^2}{d}-\frac{c(bc-ad)^2}{d^3(c+dx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{(bc-ad)^2x^n}{d^3n}-\frac{b(bc-2ad)x^{2n}}{2d^2n}+\frac{b^2x^{3n}}{3dn}-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}\end{aligned}$$

**Mathematica** [A] time = 0.14, size = 82, normalized size = 0.91

$$\frac{-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4}+\frac{x^n(bc-ad)^2}{d^3}-\frac{bx^{2n}(bc-2ad)}{2d^2}+\frac{b^2x^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (((b*c - a*d)^2*x^n)/d^3 - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2) + (b^2*x^(3*n)
)/(3*d) - (c*(b*c - a*d)^2*Log[c + d*x^n])/d^4)/n
```

**IntegrateAlgebraic [A]** time = 0.08, size = 97, normalized size = 1.08

$$\frac{x^n\left(6a^2d^2-12abcd+6abd^2x^n+6b^2c^2-3b^2cdx^n+2b^2d^2x^{2n}\right)}{6d^3n}-\frac{c(bc-ad)^2\log\left(c+dx^n\right)}{d^4n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (x^n*(6*b^2*c^2 - 12*a*b*c*d + 6*a^2*d^2 - 3*b^2*c*d*x^n + 6*a*b*d^2*x^n +
2*b^2*d^2*x^(2*n)))/(6*d^3*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)
```

**fricas [A]** time = 0.44, size = 108, normalized size = 1.20

$$\frac{2\,b^2d^3x^{3\,n}-3\left(b^2cd^2-2\,abd^3\right)x^{2\,n}+6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^n-6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(dx^n+c\right)}{6\,d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x
^n + c))/(d^4*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^n+a\right)^2x^{2\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(2*n - 1)/(d*x^n + c), x)
```

**maple [A]** time = 0.08, size = 173, normalized size = 1.92

$$-\frac{a^2c\ln\left(d\,e^{n\ln(x)}+c\right)}{d^2n}+\frac{a^2e^{n\ln(x)}}{dn}+\frac{2ab\,c^2\ln\left(d\,e^{n\ln(x)}+c\right)}{d^3n}-\frac{2abc\,e^{n\ln(x)}}{d^2n}+\frac{ab\,e^{2n\ln(x)}}{dn}-\frac{b^2c^3\ln\left(d\,e^{n\ln(x)}+c\right)}{d^4n}+\frac{b^2c^2e^{n\ln(x)}}{d^3n}-\frac{b^2c\,e^{2n\ln(x)}}{2d^2n}+\frac{b^2e^{3n\ln(x)}}{3dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a^2-2/d^2/n*exp(n*ln(x))*a*b*c+1/d^3/n*exp(n*ln(x))*b^2*
c^2+1/3*b^2/d/n*exp(n*ln(x))^3+b/d/n*exp(n*ln(x))^2*a-1/2*b^2/d^2/n*exp(n*l
n(x))^2*c-c/d^2/n*ln(d*exp(n*ln(x))+c)*a^2+2*c^2/d^3/n*ln(d*exp(n*ln(x))+c)
*a*b-c^3/d^4/n*ln(d*exp(n*ln(x))+c)*b^2
```

**maxima [A]** time = 0.70, size = 150, normalized size = 1.67

$$a^2\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)-\frac{1}{6}\,b^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+ab\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) - 1/6*b^2*(6*c^3*log((d*x^n
+ c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + a*
b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}\left(a+b\,x^n\right)^2}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

**sympy [A]** time = 63.51, size = 202, normalized size = 2.24

$$\begin{cases} \frac{(a+b)^2\log(x)}{c} & \text{for}\ d=0 \wedge n=0 \\ \frac{\frac{a^2x^{2n}}{2n}+\frac{2abx^{3n}}{3n}+\frac{b^2x^{4n}}{4n}}{c} & \text{for}\ d=0 \\ \frac{(a+b)^2\log(x)}{c+d} & \text{for}\ n=0 \\ -\frac{a^2c\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{a^2x^n}{dn}+\frac{2abc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{2abcx^n}{d^2n}+\frac{abx^{2n}}{dn}-\frac{b^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{b^2c^2x^n}{d^3n}-\frac{b^2cx^{2n}}{2d^2n}+\frac{b^2x^{3n}}{3dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x**(2*n)/(2*n)
 + 2*a*b*x**(3*n)/(3*n) + b**2*x**(4*n)/(4*n))/c, Eq(d, 0)), ((a + b)**2*lo
g(x)/(c + d), Eq(n, 0)), (-a**2*c*log(c/d + x**n)/(d**2*n) + a**2*x**n/(d*n
) + 2*a*b*c**2*log(c/d + x**n)/(d**3*n) - 2*a*b*c*x**n/(d**2*n) + a*b*x**(2
*n)/(d*n) - b**2*c**3*log(c/d + x**n)/(d**4*n) + b**2*c**2*x**n/(d**3*n) -
b**2*c*x**(2*n)/(2*d**2*n) + b**2*x**(3*n)/(3*d*n), True))
```

## 3.662 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx$

**Optimal**. Leaf size=60

$$\frac{c(bc-ad)\log(c+dx^n)}{d^3n}-\frac{x^n(bc-ad)}{d^2n}+\frac{bx^{2n}}{2dn}$$

**Rubi [A]** time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{x^n(bc-ad)}{d^2n}+\frac{c(bc-ad)\log(c+dx^n)}{d^3n}+\frac{bx^{2n}}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)*x^n)/(d^2*n)) + (b*x^(2*n))/(2*d*n) + (c*(b*c - a*d)*Log[c +
 d*x^n])/(d^3*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{(bc-ad)x^n}{d^2n}+\frac{bx^{2n}}{2dn}+\frac{c(bc-ad)\log(c+dx^n)}{d^3n}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 50, normalized size = 0.83

$$\frac{dx^n(2ad-2bc+bdx^n)+2c(bc-ad)\log(c+dx^n)}{2d^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3*
n)
```

**IntegrateAlgebraic** [A] time = 0.05, size = 55, normalized size = 0.92

$$\frac{\left(bc^2-acd\right)\log\left(c+dx^n\right)}{d^3n}+\frac{x^n\left(2ad-2bc+bdx^n\right)}{2d^2n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (x^n*(-2*b*c + 2*a*d + b*d*x^n))/(2*d^2*n) + ((b*c^2 - a*c*d)*Log[c + d*x^n
])/(d^3*n)
```

**fricas** [A] time = 0.43, size = 56, normalized size = 0.93

$$\frac{bd^2x^{2n}-2\left(bcd-ad^2\right)x^n+2\left(bc^2-acd\right)\log\left(dx^n+c\right)}{2\,d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n +
c))/(d^3*n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx^n+a)x^{2n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(2*n - 1)/(d*x^n + c), x)
```

**maple** [A] time = 0.07, size = 87, normalized size = 1.45

$$-\frac{ac\ln\left(d\,e^{n\ln(x)}+c\right)}{d^2n}+\frac{a\,e^{n\ln(x)}}{dn}+\frac{b\,c^2\ln\left(d\,e^{n\ln(x)}+c\right)}{d^3n}-\frac{bc\,e^{n\ln(x)}}{d^2n}+\frac{b\,e^{2n\ln(x)}}{2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a-1/d^2/n*exp(n*ln(x))*b*c+1/2*b/d/n*exp(n*ln(x))^2-c/d^
2/n*ln(d*exp(n*ln(x))+c)*a+c^2/d^3/n*ln(d*exp(n*ln(x))+c)*b
```

**maxima** [A] time = 0.70, size = 83, normalized size = 1.38

$$a\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)+\frac{1}{2}\,b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/2*b*(2*c^2*log((d*x^n + c)
/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^{2\,n-1}\,\left(a+b\,x^n\right)}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

**sympy** [A] time = 26.97, size = 105, normalized size = 1.75

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{\frac{ax^{2n}}{2n} + \frac{bx^{3n}}{3n}}{c} & \text{for } d = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n = 0 \\ -\frac{ac\log\left(\frac{c}{d}+x^n\right)}{d^2n} + \frac{ax^n}{dn} + \frac{bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{bcx^n}{d^2n} + \frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x**(2*n)/(2*n) + b*x
**(3*n)/(3*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 0)), (-a*c*log(
c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d + x**n)/(d**3*n) - b*c
*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))
```

## 3.663 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx$

**Optimal**. Leaf size=54

$$\frac{c\log(c+dx^n)}{dn(bc-ad)}-\frac{a\log(a+bx^n)}{bn(bc-ad)}$$

**Rubi** [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 72}

$$\frac{c\log(c+dx^n)}{dn(bc-ad)}-\frac{a\log(a+bx^n)}{bn(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a\log(a+bx^n)}{b(bc-ad)n}+\frac{c\log(c+dx^n)}{d(bc-ad)n}\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 44, normalized size = 0.81

$$-\frac{ad\log(a+bx^n)-bc\log(c+dx^n)}{b^2cdn-abd^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*d*Log[a + b*x^n] - b*c*Log[c + d*x^n])/(b^2*c*d*n - a*b*d^2*n))
```

**IntegrateAlgebraic** [A] time = 0.06, size = 55, normalized size = 1.02

$$-\frac{a\log(a+bx^n)}{bn(bc-ad)}-\frac{c\log(c+dx^n)}{dn(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) - (c*Log[c + d*x^n])/(d*(-(b*c) + a
*d)*n)
```

**fricas** [A] time = 0.44, size = 45, normalized size = 0.83

$$-\frac{ad\log\left(bx^{n}+a\right)-bc\log\left(dx^{n}+c\right)}{\left(b^{2}cd-abd^{2}\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a*d*log(b*x^n + a) - b*c*log(d*x^n + c))/((b^2*c*d - a*b*d^2)*n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2n-1}}{(bx^{n}+a)(dx^{n}+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

**maple** [A] time = 0.07, size = 59, normalized size = 1.09

$$\frac{a\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{(ad-bc)\,bn}-\frac{c\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{(ad-bc)\,dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] a/(a*d-b*c)/b/n*ln(b*exp(n*ln(x))+a)-c/d/n/(a*d-b*c)*ln(d*exp(n*ln(x))+c)
```

**maxima** [A] time = 0.59, size = 60, normalized size = 1.11

$$-\frac{a\log\left(\frac{bx^{n}+a}{b}\right)}{b^{2}cn-abdn}+\frac{c\log\left(\frac{dx^{n}+c}{d}\right)}{bcdn-ad^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n -
 a*d^2*n)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^{2n-1}}{\left(a+b\,x^{n}\right)\,\left(c+d\,x^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

**sympy** [**F(-2)**]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.664 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)} dx$

**Optimal**. Leaf size=75

$$\frac{a}{bn(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{n(bc-ad)^2}-\frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

**Rubi [A]** time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{a}{bn(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{n(bc-ad)^2}-\frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)} dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2}+\frac{bc}{(bc-ad)^2(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)}\right) dx, x, x^n\right)}{n}\\ &= \frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 58, normalized size = 0.77

$$\frac{\frac{a(bc-ad)}{b(a+bx^n)}+c\log(a+bx^n)-c\log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] ((a*(b*c - a*d))/(b*(a + b*x^n)) + c*Log[a + b*x^n] - c*Log[c + d*x^n])/((b
*c - a*d)^2*n)
```

**IntegrateAlgebraic [A]** time = 0.09, size = 75, normalized size = 1.00

$$\frac{a}{bn(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{n(bc-ad)^2}-\frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

**fricas [A]** time = 0.45, size = 120, normalized size = 1.60

$$\frac{abc-a^2d+\left(b^2cx^n+abc\right)\log(bx^n+a)-\left(b^2cx^n+abc\right)\log(dx^n+c)}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)nx^n+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*log(b*x^n + a) - (b^2*c*x^n + a*b*c)*l
og(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2 -
2*a^2*b^2*c*d + a^3*b*d^2)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2\,n-1}}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

**maple [A]** time = 0.09, size = 109, normalized size = 1.45

$$\frac{c\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{\left(a^2d^2-2abcd+b^2c^2\right)n}-\frac{c\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{\left(a^2d^2-2abcd+b^2c^2\right)n}+\frac{\mathrm{e}^{n\ln(x)}}{(ad-bc)\left(b\,\mathrm{e}^{n\ln(x)}+a\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] 1/(a*d-b*c)/n*exp(n*ln(x))/(b*exp(n*ln(x))+a)+c/n/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(b*exp(n*ln(x))+a)-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(d*exp(n*ln(x))+c
)
```

**maxima [A]** time = 0.52, size = 121, normalized size = 1.61

$$\frac{c\log\left(\frac{bx^n+a}{b}\right)}{b^2c^2n-2\,abcdn+a^2d^2n}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2n-2\,abcdn+a^2d^2n}+\frac{a}{ab^2cn-a^2bdn+\left(b^3cn-ab^2dn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c*log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*log((d*x^n +
 c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n + (
b^3*c*n - a*b^2*d*n)*x^n)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.665 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$

**Optimal**. Leaf size=105

$$\frac{a}{2bn(bc-ad)\left(a+bx^n\right)^2}-\frac{c}{n(bc-ad)^2\left(a+bx^n\right)}-\frac{cd\log\left(a+bx^n\right)}{n(bc-ad)^3}+\frac{cd\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

**Rubi [A]** time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{a}{2bn(bc-ad)\left(a+bx^n\right)^2}-\frac{c}{n(bc-ad)^2\left(a+bx^n\right)}-\frac{cd\log\left(a+bx^n\right)}{n(bc-ad)^3}+\frac{cd\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]

[Out] a/(2*b*(b*c - a*d)*n*(a + b*x^n)^2) - c/((b*c - a*d)^2*n*(a + b*x^n)) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n
)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{\left(a+bx^n\right)^3\left(c+dx^n\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^3(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^3}+\frac{bc}{(bc-ad)^2(a+bx)^2}-\frac{bcd}{(bc-ad)^3(a+bx)}+\frac{cd^2}{(bc-ad)^3(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= \frac{a}{2b(bc-ad)n\left(a+bx^n\right)^2}-\frac{c}{(bc-ad)^2n\left(a+bx^n\right)}-\frac{cd\log\left(a+bx^n\right)}{(bc-ad)^3n}+\frac{cd\log\left(c+dx^n\right)}{(bc-ad)^3n}\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 97, normalized size = 0.92

$$\frac{\frac{a}{2b(bc-ad)(a+bx^n)^2}-\frac{c}{(bc-ad)^2(a+bx^n)}-\frac{cd\log(a+bx^n)}{(bc-ad)^3}+\frac{cd\log(c+dx^n)}{(bc-ad)^3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (a/(2*b*(b*c - a*d)*(a + b*x^n)^2) - c/((b*c - a*d)^2*(a + b*x^n)) - (c*d*L
og[a + b*x^n])/(b*c - a*d)^3 + (c*d*Log[c + d*x^n])/(b*c - a*d)^3)/n
```

**IntegrateAlgebraic** [A] time = 0.12, size = 100, normalized size = 0.95

$$\frac{-a^2d-abc-2b^2cx^n}{2bn(bc-ad)^2\left(a+bx^n\right)^2}-\frac{cd\log\left(a+bx^n\right)}{n(bc-ad)^3}+\frac{cd\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (-(a*b*c) - a^2*d - 2*b^2*c*x^n)/(2*b*(b*c - a*d)^2*n*(a + b*x^n)^2) - (c*d
*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n)
```

**fricas** [B] time = 0.46, size = 267, normalized size = 2.54

$$-\frac{ab^2c^2-a^3d^2+2\left(b^3c^2-ab^2cd\right)x^n+2\left(b^3cdx^{2n}+2ab^2cdx^n+a^2bcd\right)\log\left(bx^n+a\right)-2\left(b^3cdx^{2n}+2ab^2cdx^n+a^2bcd\right)\log\left(dx^n+c\right)}{2\left(\left(b^6c^3-3ab^5c^2d+3a^2b^4cd^2-a^3b^3d^3\right)nx^{2n}+2\left(ab^5c^3-3a^2b^4c^2d+3a^3b^3cd^2-a^4b^2d^3\right)nx^n+\left(a^2b^4c^3-3a^3b^3c^2d+3a^4b^2cd^2-a^5bd^3\right)n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*n
) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*a*
b^2*c*d*x^n + a^2*b*c*d)*log(d*x^n + c))/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*
b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^
2*c*d^2 - a^5*b*d^3)*n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\left(bx^n+a\right)^3\left(dx^n+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

**maple** [A] time = 0.11, size = 203, normalized size = 1.93

$$\frac{cd\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)n}-\frac{cd\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)n}+\frac{-\frac{bc\,\mathrm{e}^{n\ln(x)}}{\left(a^2d^2-2abcd+b^2c^2\right)n}+\frac{\left(-abd-b^2c\right)a}{2\left(a^2d^2-2abcd+b^2c^2\right)b^2n}}{\left(b\,\mathrm{e}^{n\ln(x)}+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] (-b*c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/2*a*(-a*b*d-b^2*c)/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(b*exp(n*ln(x))+a)^2+c*d/n/(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*exp(n*ln(x))+a)-c*d/n/(a^3*d^3-3*a^2*b*c*d
^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*exp(n*ln(x))+c)
```

**maxima** [B] time = 0.66, size = 243, normalized size = 2.31

$$-\frac{cd\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3ab^2c^2dn+3a^2bcd^2n-a^3d^3n}+\frac{cd\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3ab^2c^2dn+3a^2bcd^2n-a^3d^3n}-\frac{2b^2cx^n+abc+a^2d}{2\left(a^2b^3c^2n-2a^3b^2cdn+a^4bd^2n+\left(b^5c^2n-2ab^4cdn+a^2b^3d^2n\right)x^{2n}+2\left(ab^4c^2n-2a^2b^3cdn+a^3b^2d^2n\right)x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^
3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*
d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n - 2*a
^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n)*x^
(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a+b\,x^n\right)}^3\,\left(c+d\,x^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.666 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$

**Optimal**. Leaf size=158

$$\frac{bx^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)}{3d^3n}-\frac{b^2x^{4n}(bc-3ad)}{4d^2n}-\frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}+\frac{cx^n(bc-ad)^3}{d^5n}-\frac{x^{2n}(bc-ad)^3}{2d^4n}+\frac{b^3x^{5n}}{5dn}$$

**Rubi [A]** time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$\frac{bx^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)}{3d^3n}-\frac{b^2x^{4n}(bc-3ad)}{4d^2n}-\frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}+\frac{cx^n(bc-ad)^3}{d^5n}-\frac{x^{2n}(bc-ad)^3}{2d^4n}+\frac{b^3x^{5n}}{5dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)^3*x^n)/(d^5*n) - ((b*c - a*d)^3*x^(2*n))/(2*d^4*n) + (b*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3*n) - (b^2*(b*c - 3*a*d)*x^(4*
n))/(4*d^2*n) + (b^3*x^(5*n))/(5*d*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/
(d^6*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}\left(a+bx^n\right)^3}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^3}{c+dx}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)^3}{d^5}+\frac{(-bc+ad)^3x}{d^4}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^2}{d^3}-\frac{b^2(bc-3ad)x^3}{d^2}+\frac{b^3x^4}{d}-\frac{c^2(bc-ad)^3}{d^5(c+dx)}\right)d\right.}{n}\\ &= \frac{c(bc-ad)^3x^n}{d^5n}-\frac{(bc-ad)^3x^{2n}}{2d^4n}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^{3n}}{3d^3n}-\frac{b^2(bc-3ad)x^{4n}}{4d^2n}+\frac{b^3x}{5a}\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 138, normalized size = 0.87

$$\frac{20bd^3x^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)-15b^2d^4x^{4n}(bc-3ad)-60c^2(bc-ad)^3\log\left(c+dx^n\right)+30d^2x^{2n}(ad-bc)^3+60cdx^n(bc-ad)^3+12b^3d^5x^{5n}}{60d^6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (60*c*d*(b*c - a*d)^3*x^n + 30*d^2*(-(b*c) + a*d)^3*x^(2*n) + 20*b*d^3*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n) - 15*b^2*d^4*(b*c - 3*a*d)*x^(4*n) +
12*b^3*d^5*x^(5*n) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)
```

**IntegrateAlgebraic [A]** time = 0.13, size = 220, normalized size = 1.39

$$\frac{x^n\left(-60a^3cd^3+30a^3d^4x^n+180a^2bc^2d^2-90a^2bcd^3x^n+60a^2bd^4x^{2n}-180ab^2c^3d+90ab^2c^2d^2x^n-60ab^2cd^3x^{2n}+45ab^2d^4x^{3n}+60b^3c^4-30b^3c^3dx^n+20b^3c^2d^2x^{2n}-15b^3cd^3x^{3n}+12b^3d^4x^{4n}\right)}{60d^5n}-\frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (x^n*(60*b^3*c^4 - 180*a*b^2*c^3*d + 180*a^2*b*c^2*d^2 - 60*a^3*c*d^3 - 30*
b^3*c^3*d*x^n + 90*a*b^2*c^2*d^2*x^n - 90*a^2*b*c*d^3*x^n + 30*a^3*d^4*x^n
+ 20*b^3*c^2*d^2*x^(2*n) - 60*a*b^2*c*d^3*x^(2*n) + 60*a^2*b*d^4*x^(2*n) -
15*b^3*c*d^3*x^(3*n) + 45*a*b^2*d^4*x^(3*n) + 12*b^3*d^4*x^(4*n)))/(60*d^5*
n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/(d^6*n)
```

**fricas [A]** time = 0.49, size = 230, normalized size = 1.46

$$\frac{12\,b^3d^5x^{5\,n}-15\left(b^3cd^4-3\,ab^2d^5\right)x^{4\,n}+20\left(b^3c^2d^3-3\,ab^2cd^4+3\,a^2bd^5\right)x^{3\,n}-30\left(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5\right)x^{2\,n}+60\left(b^3c^4d-3\,ab^2c^3d^2+3\,a^2bc^2d^3-a^3cd^4\right)x^n-60\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\log\left(dx^n+c\right)}{60\,d^6n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^3*d^5*x^(5*n) - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^(4*n) + 20*(b^3*c
^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^(3*n) - 30*(b^3*c^3*d^2 - 3*a*b^2*c
^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^(2*n) + 60*(b^3*c^4*d - 3*a*b^2*c^3*d^2
 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b
*c^3*d^2 - a^3*c^2*d^3)*log(d*x^n + c))/(d^6*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^n+a\right)^3x^{3\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)
```

**maple [B]** time = 0.07, size = 342, normalized size = 2.16

$$\frac{a^3c^2\ln\left(x^n+\frac{c}{d}\right)}{d^3n}-\frac{a^3c\,x^n}{d^2n}+\frac{a^3x^{2n}}{2dn}-\frac{3a^2b\,c^3\ln\left(x^n+\frac{c}{d}\right)}{d^4n}+\frac{3a^2b\,c^2x^n}{d^3n}-\frac{3a^2bc\,x^{2n}}{2d^2n}+\frac{a^2b\,x^{3n}}{dn}+\frac{3a\,b^2c^4\ln\left(x^n+\frac{c}{d}\right)}{d^5n}-\frac{3a\,b^2c^3x^n}{d^4n}+\frac{3a\,b^2c^2x^{2n}}{2d^3n}-\frac{a\,b^2c\,x^{3n}}{d^2n}+\frac{3a\,b^2x^{4n}}{4dn}-\frac{b^3c^5\ln\left(x^n+\frac{c}{d}\right)}{d^6n}+\frac{b^3c^4x^n}{d^5n}-\frac{b^3c^3x^{2n}}{2d^4n}+\frac{b^3c^2x^{3n}}{3d^3n}-\frac{b^3c\,x^{4n}}{4d^2n}+\frac{b^3x^{5n}}{5dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] 1/5*b^3/d/n*(x^n)^5+3/4*b^2/d/n*(x^n)^4*a-1/4*b^3/d^2/n*(x^n)^4*c+b/d/n*(x^
n)^3*a^2-b^2/d^2/n*(x^n)^3*a*c+1/3*b^3/d^3/n*(x^n)^3*c^2+1/2/d/n*(x^n)^2*a^
3-3/2/d^2/n*(x^n)^2*a^2*b*c+3/2/d^3/n*(x^n)^2*a*b^2*c^2-1/2/d^4/n*(x^n)^2*b
^3*c^3-c/d^2/n*x^n*a^3+3*c^2/d^3/n*x^n*a^2*b-3*c^3/d^4/n*x^n*a*b^2+c^4/d^5/
n*x^n*b^3+c^2/d^3/n*ln(x^n+c/d)*a^3-3*c^3/d^4/n*ln(x^n+c/d)*a^2*b+3*c^4/d^5
/n*ln(x^n+c/d)*a*b^2-c^5/d^6/n*ln(x^n+c/d)*b^3
```

**maxima [A]** time = 0.66, size = 286, normalized size = 1.81

$$-\frac{1}{60}\,b^3\left(\frac{60\,c^5\log\left(\frac{dx^n+c}{d}\right)}{d^6n}-\frac{12\,d^4x^{5\,n}-15\,cd^3x^{4\,n}+20\,c^2d^2x^{3\,n}-30\,c^3dx^{2\,n}+60\,c^4x^n}{d^5n}\right)+\frac{1}{4}\,ab^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)-\frac{1}{2}\,a^2b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+\frac{1}{2}\,a^3\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/60*b^3*(60*c^5*log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^(5*n) - 15*c*d^3*x
^(4*n) + 20*c^2*d^2*x^(3*n) - 30*c^3*d*x^(2*n) + 60*c^4*x^n)/(d^5*n)) + 1/4
*a*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n
) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^3*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

```
[Out] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Timed out
```

## 3.667 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n} dx$

**Optimal**. Leaf size=118

$$\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

**Rubi [A]** time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] -((c*(b*c - a*d)^2*x^n)/(d^4*n)) + ((b*c - a*d)^2*x^(2*n))/(2*d^3*n) - (b*(
b*c - 2*a*d)*x^(3*n))/(3*d^2*n) + (b^2*x^(4*n))/(4*d*n) + (c^2*(b*c - a*d)^
2*Log[c + d*x^n])/(d^5*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n} dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{c+dx} dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^4}+\frac{(-bc+ad)^2x}{d^3}-\frac{b(bc-2ad)x^2}{d^2}+\frac{b^2x^3}{d}+\frac{c^2(bc-ad)^2}{d^4(c+dx)}\right) dx, x, x^n\right)}{n}\\ &= -\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log(c+d}{d^5n}\end{aligned}$$

**Mathematica [A]** time = 0.27, size = 103, normalized size = 0.87

$$\frac{12c^2(bc-ad)^2\log(c+dx^n)-4bd^3x^{3n}(bc-2ad)+6d^2x^{2n}(bc-ad)^2-12cdx^n(bc-ad)^2+3b^2d^4x^{4n}}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (-12*c*d*(b*c - a*d)^2*x^n + 6*d^2*(b*c - a*d)^2*x^(2*n) - 4*b*d^3*(b*c - 2
*a*d)*x^(3*n) + 3*b^2*d^4*x^(4*n) + 12*c^2*(b*c - a*d)^2*Log[c + d*x^n])/(1
2*d^5*n)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 141, normalized size = 1.19

$$\frac{x^n\left(-12a^2cd^2+6a^2d^3x^n+24abc^2d-12abcd^2x^n+8abd^3x^{2n}-12b^2c^3+6b^2c^2dx^n-4b^2cd^2x^{2n}+3b^2d^3x^{3n}\right)}{12d^4n}+\frac{c^2(bc-ad)^2\log\left(c+dx^n\right)}{d^5n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (x^n*(-12*b^2*c^3 + 24*a*b*c^2*d - 12*a^2*c*d^2 + 6*b^2*c^2*d*x^n - 12*a*b*
c*d^2*x^n + 6*a^2*d^3*x^n - 4*b^2*c*d^2*x^(2*n) + 8*a*b*d^3*x^(2*n) + 3*b^2
*d^3*x^(3*n)))/(12*d^4*n) + (c^2*(b*c - a*d)^2*Log[c + d*x^n])/(d^5*n)
```

**fricas [A]** time = 0.48, size = 146, normalized size = 1.24

$$\frac{3\,b^2d^4x^{4\,n}-4\left(b^2cd^3-2\,abd^4\right)x^{3\,n}+6\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^{2\,n}-12\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^n+12\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\log\left(dx^n+c\right)}{12\,d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^n+a\right)^2x^{3\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)
```

**maple [B]** time = 0.07, size = 236, normalized size = 2.00

$$\frac{a^2c^2\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^3n}-\frac{a^2c\,\mathrm{e}^{n\ln(x)}}{d^2n}+\frac{a^2\mathrm{e}^{2n\ln(x)}}{2dn}-\frac{2ab\,c^3\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^4n}+\frac{2ab\,c^2\mathrm{e}^{n\ln(x)}}{d^3n}-\frac{abc\,\mathrm{e}^{2n\ln(x)}}{d^2n}+\frac{2ab\,\mathrm{e}^{3n\ln(x)}}{3dn}+\frac{b^2c^4\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^5n}-\frac{b^2c^3\mathrm{e}^{n\ln(x)}}{d^4n}+\frac{b^2c^2\mathrm{e}^{2n\ln(x)}}{2d^3n}-\frac{b^2c\,\mathrm{e}^{3n\ln(x)}}{3d^2n}+\frac{b^2\mathrm{e}^{4n\ln(x)}}{4dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] 1/4*b^2/d/n*exp(n*ln(x))^4+1/2/d/n*exp(n*ln(x))^2*a^2-1/d^2/n*exp(n*ln(x))^
2*a*b*c+1/2/d^3/n*exp(n*ln(x))^2*b^2*c^2+2/3*b/d/n*exp(n*ln(x))^3*a-1/3*b^2
/d^2/n*exp(n*ln(x))^3*c-c/d^2/n*exp(n*ln(x))*a^2+2*c^2/d^3/n*exp(n*ln(x))*a
*b-c^3/d^4/n*exp(n*ln(x))*b^2+c^2/d^3/n*ln(d*exp(n*ln(x))+c)*a^2-2*c^3/d^4/
n*ln(d*exp(n*ln(x))+c)*a*b+c^4/d^5/n*ln(d*exp(n*ln(x))+c)*b^2
```

**maxima [A]** time = 0.54, size = 192, normalized size = 1.63

$$\frac{1}{12}\,b^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)-\frac{1}{3}\,ab\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+\frac{1}{2}\,a^2\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] 1/12*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3
*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/3*a*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^2*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}\,(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed
```

## 3.668 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n} \, dx$

**Optimal**. Leaf size=86

$$-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

**Rubi [A]** time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)*x^n)/(d^3*n) - ((b*c - a*d)*x^(2*n))/(2*d^2*n) + (b*x^(3*n))
/(3*d*n) - (c^2*(b*c - a*d)*Log[c + d*x^n])/(d^4*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n} \, dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{c+dx} \, dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^3}+\frac{(-bc+ad)x}{d^2}+\frac{bx^2}{d}-\frac{c^2(bc-ad)}{d^3(c+dx)}\right) dx, x, x^n\right)}{n}\\ &= \frac{c(bc-ad)x^n}{d^3n}-\frac{(bc-ad)x^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn}-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 78, normalized size = 0.91

$$\frac{-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4}+\frac{cx^n(bc-ad)}{d^3}-\frac{x^{2n}(bc-ad)}{2d^2}+\frac{bx^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] ((c*(b*c - a*d)*x^n)/d^3 - ((b*c - a*d)*x^(2*n))/(2*d^2) + (b*x^(3*n))/(3*d
) - (c^2*(b*c - a*d)*Log[c + d*x^n])/d^4)/n
```

**IntegrateAlgebraic [A]** time = 0.07, size = 82, normalized size = 0.95

$$\frac{x^n\left(-6acd+3ad^2x^n+6bc^2-3bcdx^n+2bd^2x^{2n}\right)}{6d^3n}+\frac{\left(ac^2d-bc^3\right)\log\left(c+dx^n\right)}{d^4n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (x^n*(6*b*c^2 - 6*a*c*d - 3*b*c*d*x^n + 3*a*d^2*x^n + 2*b*d^2*x^(2*n)))/(6*
d^3*n) + ((-(b*c^3) + a*c^2*d)*Log[c + d*x^n])/(d^4*n)
```

**fricas [A]** time = 0.43, size = 82, normalized size = 0.95

$$\frac{2\,bd^3x^{3\,n}-3\left(bcd^2-ad^3\right)x^{2\,n}+6\left(bc^2d-acd^2\right)x^n-6\left(bc^3-ac^2d\right)\log\left(dx^n+c\right)}{6\,d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)*
x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx^n+a)x^{3\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(3*n - 1)/(d*x^n + c), x)
```

**maple [A]** time = 0.07, size = 125, normalized size = 1.45

$$\frac{a\,c^2\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^3n}-\frac{ac\,\mathrm{e}^{n\ln(x)}}{d^2n}+\frac{a\,\mathrm{e}^{2n\ln(x)}}{2dn}-\frac{b\,c^3\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^4n}+\frac{b\,c^2\mathrm{e}^{n\ln(x)}}{d^3n}-\frac{bc\,\mathrm{e}^{2n\ln(x)}}{2d^2n}+\frac{b\,\mathrm{e}^{3n\ln(x)}}{3dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] 1/3*b/d/n*exp(n*ln(x))^3+1/2/d/n*exp(n*ln(x))^2*a-1/2/d^2/n*exp(n*ln(x))^2*
b*c-c/d^2/n*exp(n*ln(x))*a+c^2/d^3/n*exp(n*ln(x))*b+c^2/d^3/n*ln(d*exp(n*ln
(x))+c)*a-c^3/d^4/n*ln(d*exp(n*ln(x))+c)*b
```

**maxima [A]** time = 0.53, size = 112, normalized size = 1.30

$$-\frac{1}{6}\,b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+\frac{1}{2}\,a\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/6*b*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) +
 6*c^2*x^n)/(d^3*n)) + 1/2*a*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n)
 - 2*c*x^n)/(d^2*n))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}\,\left(a+b\,x^n\right)}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

```
[Out] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

**sympy [A]** time = 57.94, size = 139, normalized size = 1.62

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d=0 \wedge n=0 \\ \frac{\frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=0 \\ \frac{ac^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{acx^n}{d^2n} + \frac{ax^{2n}}{2dn} - \frac{bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{bc^2x^n}{d^3n} - \frac{bcx^{2n}}{2d^2n} + \frac{bx^{3n}}{3dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x**(3*n)/(3*n) + b*x
**(4*n)/(4*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 0)), (a*c**2*lo
g(c/d + x**n)/(d**3*n) - a*c*x**n/(d**2*n) + a*x**(2*n)/(2*d*n) - b*c**3*lo
g(c/d + x**n)/(d**4*n) + b*c**2*x**n/(d**3*n) - b*c*x**(2*n)/(2*d**2*n) + b
*x**(3*n)/(3*d*n), True))
```

## 3.669 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$

**Optimal**. Leaf size=71

$$\frac{a^2\log(a+bx^n)}{b^2n(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

**Rubi** [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 72}

$$\frac{a^2\log(a+bx^n)}{b^2n(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) - (c^2*Log[c + d*x^n
])/(d^2*(b*c - a*d)*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= \frac{x^n}{bdn}+\frac{a^2\log(a+bx^n)}{b^2(bc-ad)n}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)n}\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 66, normalized size = 0.93

$$\frac{\frac{a^2\log(a+bx^n)}{b^2(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)}+\frac{x^n}{bd}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x^n/(b*d) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^n])/
(d^2*(b*c - a*d)))/n
```

**IntegrateAlgebraic [A]** time = 0.07, size = 70, normalized size = 0.99

$$\frac{a^2\log\left(a+bx^n\right)}{b^2n(bc-ad)}+\frac{c^2\log\left(c+dx^n\right)}{d^2n(ad-bc)}+\frac{x^n}{bdn}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) + (c^2*Log[c + d*x^n
])/(d^2*(-(b*c) + a*d)*n)
```

**fricas [A]** time = 0.46, size = 74, normalized size = 1.04

$$\frac{a^2d^2\log\left(bx^n+a\right)-b^2c^2\log\left(dx^n+c\right)+\left(b^2cd-abd^2\right)x^n}{\left(b^3cd^2-ab^2d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n)
/((b^3*c*d^2 - a*b^2*d^3)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3\,n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

**maple [A]** time = 0.10, size = 78, normalized size = 1.10

$$-\frac{a^2\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{(ad-bc)\,b^2n}+\frac{c^2\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{(ad-bc)\,d^2n}+\frac{\mathrm{e}^{n\ln(x)}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] 1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(d*exp(n*ln(x))+c)-a^2/(a*d-b*c)
/b^2/n*ln(b*exp(n*ln(x))+a)
```

**maxima [A]** time = 0.63, size = 81, normalized size = 1.14

$$\frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3cn-ab^2dn}-\frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{bcd^2n-ad^3n}+\frac{x^n}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c*
d^2*n - a*d^3*n) + x^n/(b*d*n)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3\,n-1}}{\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.670 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$

**Optimal**. Leaf size=95

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

**Rubi [A]** time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)}+\frac{c^2}{(bc-ad)^2(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^2}{b^2(bc-ad)n(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n}+\frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 90, normalized size = 0.95

$$\frac{-\frac{a^2}{b^2(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{d(bc-ad)^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (-(a^2/(b^2*(b*c - a*d)*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(b
^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2))/n
```

**IntegrateAlgebraic [A]** time = 0.11, size = 95, normalized size = 1.00

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)}+\frac{\left(a^2d-2abc\right)\log\left(a+bx^n\right)}{b^2n(bc-ad)^2}+\frac{c^2\log\left(c+dx^n\right)}{dn(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) + ((-2*a*b*c + a^2*d)*Log[a + b*x^n]
)/(b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(-(b*c) + a*d)^2*n)
```

**fricas [A]** time = 0.45, size = 166, normalized size = 1.75

$$-\frac{a^2bcd-a^3d^2+\left(2\,a^2bcd-a^3d^2+\left(2\,ab^2cd-a^2bd^2\right)x^n\right)\log\left(bx^n+a\right)-\left(b^3c^2x^n+ab^2c^2\right)\log\left(dx^n+c\right)}{\left(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3\right)nx^n+\left(ab^4c^2d-2\,a^2b^3cd^2+a^3b^2d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)*
x^n)*log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*log(d*x^n + c))/((b^5*c^2*d
 - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^
3*b^2*d^3)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\left(bx^n+a\right)^2\left(dx^n+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

**maple [A]** time = 0.08, size = 163, normalized size = 1.72

$$\frac{a^2d\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{\left(a^2d^2-2abcd+b^2c^2\right)b^2n}-\frac{2ac\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{\left(a^2d^2-2abcd+b^2c^2\right)bn}+\frac{c^2\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{\left(a^2d^2-2abcd+b^2c^2\right)dn}+\frac{a^2}{(ad-bc)\left(b\,\mathrm{e}^{n\ln(x)}+a\right)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] a^2/(a*d-b*c)/b^2/n/(b*exp(n*ln(x))+a)+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*
ln(d*exp(n*ln(x))+c)+a^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*ln(b*exp(n*ln(x)
)+a)*d-2*a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/n*ln(b*exp(n*ln(x))+a)*c
```

**maxima [A]** time = 0.58, size = 147, normalized size = 1.55

$$\frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2dn-2\,abcd^2n+a^2d^3n}-\frac{a^2}{ab^3cn-a^2b^2dn+\left(b^4cn-ab^3dn\right)x^n}-\frac{\left(2\,abc-a^2d\right)\log\left(\frac{bx^n+a}{b}\right)}{b^4c^2n-2\,ab^3cdn+a^2b^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*b
^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*log((
b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

**sympy** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed
```

## 3.671 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$

**Optimal**. Leaf size=120

$$-\frac{a^2}{2b^2n(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{n(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{n(bc-ad)^3}$$

**Rubi** [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$-\frac{a^2}{2b^2n(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{n(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] -a^2/(2*b^2*(b*c - a*d)*n*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a*
d)^2*n*(a + b*x^n)) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c +
 d*x^n])/((b*c - a*d)^3*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2}+\frac{bc^2}{(bc-ad)^3(a+bx)}-\frac{c^2d}{(bc-ad)^3(c+dx)}\right) dx, x, x^n\right)}{n}\\ &= -\frac{a^2}{2b^2(bc-ad)n(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2n(a+bx^n)}+\frac{c^2\log(a+bx^n)}{(bc-ad)^3n}-\frac{c^2\log(\ldots)}{(bc-\ldots)}\end{aligned}$$

**Mathematica** [A] time = 0.27, size = 112, normalized size = 0.93

$$\frac{-\frac{a^2}{2b^2(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{(bc-ad)^3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (-1/2*a^2/(b^2*(b*c - a*d)*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a
*d)^2*(a + b*x^n)) + (c^2*Log[a + b*x^n])/(b*c - a*d)^3 - (c^2*Log[c + d*x^
n])/(b*c - a*d)^3)/n
```

**IntegrateAlgebraic [A]** time = 0.15, size = 110, normalized size = 0.92

$$-\frac{a\left(a^2d-3abc+2abdx^n-4b^2cx^n\right)}{2b^2n(bc-ad)^2\left(a+bx^n\right)^2}+\frac{c^2\log\left(a+bx^n\right)}{n(bc-ad)^3}-\frac{c^2\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] -1/2*(a*(-3*a*b*c + a^2*d - 4*b^2*c*x^n + 2*a*b*d*x^n))/(b^2*(b*c - a*d)^2*
n*(a + b*x^n)^2) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c + d*
x^n])/((b*c - a*d)^3*n)
```

**fricas [B]** time = 0.45, size = 301, normalized size = 2.51

$$\frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\left(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2\right)x^n+2\left(b^4c^2x^{2\,n}+2\,ab^3c^2x^n+a^2b^2c^2\right)\log\left(bx^n+a\right)-2\left(b^4c^2x^{2\,n}+2\,ab^3c^2x^n+a^2b^2c^2\right)\log\left(dx^n+c\right)}{2\left(\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)nx^{2\,n}+2\left(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3\right)nx^n+\left(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3\right)n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log
(b*x^n + a) - 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log(d*x^n
 + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^(2*n)
 + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x^n +
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3\,n-1}}{\left(bx^n+a\right)^3\left(dx^n+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

**maple [A]** time = 0.12, size = 214, normalized size = 1.78

$$-\frac{c^2\ln\left(b\,e^{n\ln(x)}+a\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)n}+\frac{c^2\ln\left(d\,e^{n\ln(x)}+c\right)}{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)n}+\frac{\frac{(-ad+2bc)a\,e^{n\ln(x)}}{\left(a^2d^2-2abcd+b^2c^2\right)bn}+\frac{(-ad+3bc)a^2}{2\left(a^2d^2-2abcd+b^2c^2\right)b^2n}}{\left(b\,e^{n\ln(x)}+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] ((-a*d+2*b*c)*a/n/b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*exp(n*ln(x))+1/2*a^2*(-a*d+
3*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(b*exp(n*ln(x))+a)^2+c^2/n/(a^3*d
^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*exp(n*ln(x))+c)-c^2/n/(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*exp(n*ln(x))+a)
```

**maxima [B]** time = 0.68, size = 262, normalized size = 2.18

$$\frac{c^2\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n}-\frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n}+\frac{3\,a^2bc-a^3d+2\left(2\,ab^2c-a^2bd\right)x^n}{2\left(a^2b^4c^2n-2\,a^3b^3cdn+a^4b^2d^2n+\left(b^6c^2n-2\,ab^5cdn+a^2b^4d^2n\right)x^{2\,n}+2\left(ab^5c^2n-2\,a^2b^4cdn+a^3b^3d^2n\right)x^n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3
*d^3*n) - c^2*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d
^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n)/(
a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*c*d*
n + a^2*b^4*d^2*n)*x^(2*n) + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^3*d^2
*n)*x^n)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{{\left(a+b\,x^n\right)}^3\,\left(c+d\,x^n\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.672 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

**Optimal**. Leaf size=14

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (x^14*(b + c*x)^14)/14
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}x^{14}(b+cx)^{14}$$

**Mathematica [B]** time = 0.01, size = 172, normalized size = 12.29

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] IntegrateAlgebraic[x^13*(b + c*x)^13*(b + 2*c*x), x]
```

**fricas [B]** time = 0.35, size = 154, normalized size = 11.00

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8+143x^{19}c^5b^9+\frac{143}{2}x^{18}c^4b^{10}+26x^{17}c^3b^{11}+\frac{13}{2}x^{16}c^2b^{12}+x^{15}cb^{13}+\frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

**giac [A]** time = 0.15, size = 13, normalized size = 0.93

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

**maple [B]** time = 0.04, size = 155, normalized size = 11.07

$$\frac{1}{14}c^{14}x^{28}+b\,c^{13}x^{27}+\frac{13}{2}b^{2}c^{12}x^{26}+26b^{3}c^{11}x^{25}+\frac{143}{2}b^{4}c^{10}x^{24}+143b^{5}c^{9}x^{23}+\frac{429}{2}b^{6}c^{8}x^{22}+\frac{1716}{7}b^{7}c^{7}x^{21}+\frac{429}{2}b^{8}c^{6}x^{20}+143b^{9}c^{5}x^{19}+\frac{143}{2}b^{10}c^{4}x^{18}+26b^{11}c^{3}x^{17}+\frac{13}{2}b^{12}c^{2}x^{16}+b^{13}c\,x^{15}+\frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(c*x+b)^13*(2*c*x+b),x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

**maxima [B]** time = 0.57, size = 154, normalized size = 11.00

$$\frac{1}{14}c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}b^{2}c^{12}x^{26}+26\,b^{3}c^{11}x^{25}+\frac{143}{2}b^{4}c^{10}x^{24}+143\,b^{5}c^{9}x^{23}+\frac{429}{2}b^{6}c^{8}x^{22}+\frac{1716}{7}b^{7}c^{7}x^{21}+\frac{429}{2}b^{8}c^{6}x^{20}+143\,b^{9}c^{5}x^{19}+\frac{143}{2}b^{10}c^{4}x^{18}+26\,b^{11}c^{3}x^{17}+\frac{13}{2}b^{12}c^{2}x^{16}+b^{13}cx^{15}+\frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

**mupad [B]** time = 0.16, size = 154, normalized size = 11.00

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b + c*x)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

**sympy [B]** time = 0.12, size = 175, normalized size = 12.50

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^{2}x^{16}}{2}+26b^{11}c^{3}x^{17}+\frac{143b^{10}c^{4}x^{18}}{2}+143b^{9}c^{5}x^{19}+\frac{429b^{8}c^{6}x^{20}}{2}+\frac{1716b^{7}c^{7}x^{21}}{7}+\frac{429b^{6}c^{8}x^{22}}{2}+143b^{5}c^{9}x^{23}+\frac{143b^{4}c^{10}x^{24}}{2}+26b^{3}c^{11}x^{25}+\frac{13b^{2}c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(c*x+b)**13*(2*c*x+b),x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

## 3.673 $\int x^{27} \left(b+cx^2\right)^{13} \left(b+2cx^2\right) \, dx$

**Optimal**. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

**Rubi [A]** time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)\\ &= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14} \end{aligned}$$

**Mathematica [B]** time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^27*(b + c*x^2)^13*(b + 2*c*x^2), x]
```

**fricas [B]** time = 0.36, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b^8 + \frac{143}{2}x^{38}c^5b^9 + \frac{143}{4}x^{36}c^4b^{10} + 13x^{34}c^3b^{11} + \frac{13}{4}x^{32}c^2b^{12} + \frac{1}{2}x^{30}cb^{13} + \frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

**giac [B]** time = 0.19, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

**maple [B]** time = 0.04, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

**maxima [B]** time = 0.73, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

**mupad [B]** time = 4.93, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^27*(b + c*x^2)^13*(b + 2*c*x^2),x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

**sympy [B]** time = 0.12, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^{2}x^{32}}{4}+13b^{11}c^{3}x^{34}+\frac{143b^{10}c^{4}x^{36}}{4}+\frac{143b^{9}c^{5}x^{38}}{2}+\frac{429b^{8}c^{6}x^{40}}{4}+\frac{858b^{7}c^{7}x^{42}}{7}+\frac{429b^{6}c^{8}x^{44}}{4}+\frac{143b^{5}c^{9}x^{46}}{2}+\frac{143b^{4}c^{10}x^{48}}{4}+13b^{3}c^{11}x^{50}+\frac{13b^{2}c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

## 3.674 $\int x^{41} \left(b+cx^3\right)^{13} \left(b+2cx^3\right) dx$

**Optimal**. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

**Rubi [A]** time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^3\right)\\ &= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}\end{aligned}$$

**Mathematica [B]** time = 0.02, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] IntegrateAlgebraic[x^41*(b + c*x^3)^13*(b + 2*c*x^3), x]
```

**fricas [B]** time = 0.36, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14}+\frac{1}{3}x^{81}c^{13}b+\frac{13}{6}x^{78}c^{12}b^{2}+\frac{26}{3}x^{75}c^{11}b^{3}+\frac{143}{6}x^{72}c^{10}b^{4}+\frac{143}{3}x^{69}c^{9}b^{5}+\frac{143}{2}x^{66}c^{8}b^{6}+\frac{572}{7}x^{63}c^{7}b^{7}+\frac{143}{2}x^{60}c^{6}b^{8}+\frac{143}{3}x^{57}c^{5}b^{9}+\frac{143}{6}x^{54}c^{4}b^{10}+\frac{26}{3}x^{51}c^{3}b^{11}+\frac{13}{6}x^{48}c^{2}b^{12}+\frac{1}{3}x^{45}cb^{13}+\frac{1}{42}x^{42}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

**giac [B]** time = 0.16, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

**maple [B]** time = 0.04, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}b\,c^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}c\,x^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

**maxima [B]** time = 0.62, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

**mupad [B]** time = 4.77, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^41*(b + c*x^3)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

**sympy** [B] time = 0.12, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^{2}x^{48}}{6} + \frac{26b^{11}c^{3}x^{51}}{3} + \frac{143b^{10}c^{4}x^{54}}{6} + \frac{143b^{9}c^{5}x^{57}}{3} + \frac{143b^{8}c^{6}x^{60}}{2} + \frac{572b^{7}c^{7}x^{63}}{7} + \frac{143b^{6}c^{8}x^{66}}{2} + \frac{143b^{5}c^{9}x^{69}}{3} + \frac{143b^{4}c^{10}x^{72}}{6} + \frac{26b^{3}c^{11}x^{75}}{3} + \frac{13b^{2}c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**41*(c*x**3+b)**13*(2*c*x**3+b),x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

## 3.675 $\int x^{-1+14n}\left(b+cx^{n}\right)^{13}\left(b+2cx^{n}\right)\,dx$

**Optimal**. Leaf size=21

$$\frac{x^{14n}\left(b+cx^{n}\right)^{14}}{14n}$$

**Rubi** [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {446, 74}

$$\frac{x^{14n}\left(b+cx^{n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+14n}\left(b+cx^{n}\right)^{13}\left(b+2cx^{n}\right)\,dx &= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^{n}\right)}{n}\\ &= \frac{x^{14n}\left(b+cx^{n}\right)^{14}}{14n}\end{aligned}$$

**Mathematica** [A] time = 0.19, size = 21, normalized size = 1.00

$$\frac{x^{14n}\left(b+cx^{n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

**IntegrateAlgebraic** [A] time = 0.04, size = 21, normalized size = 1.00

$$\frac{x^{14n}\left(b+cx^{n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

**fricas [B]** time = 0.46, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14\,bc^{13}x^{27n} + 91\,b^{2}c^{12}x^{26n} + 364\,b^{3}c^{11}x^{25n} + 1001\,b^{4}c^{10}x^{24n} + 2002\,b^{5}c^{9}x^{23n} + 3003\,b^{6}c^{8}x^{22n} + 3432\,b^{7}c^{7}x^{21n} + 3003\,b^{8}c^{6}x^{20n} + 2002\,b^{9}c^{5}x^{19n} + 1001\,b^{10}c^{4}x^{18n} + 364\,b^{11}c^{3}x^{17n} + 91\,b^{12}c^{2}x^{16n} + 14\,b^{13}cx^{15n} + b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

**giac [B]** time = 0.57, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14\,bc^{13}x^{27n} + 91\,b^{2}c^{12}x^{26n} + 364\,b^{3}c^{11}x^{25n} + 1001\,b^{4}c^{10}x^{24n} + 2002\,b^{5}c^{9}x^{23n} + 3003\,b^{6}c^{8}x^{22n} + 3432\,b^{7}c^{7}x^{21n} + 3003\,b^{8}c^{6}x^{20n} + 2002\,b^{9}c^{5}x^{19n} + 1001\,b^{10}c^{4}x^{18n} + 364\,b^{11}c^{3}x^{17n} + 91\,b^{12}c^{2}x^{16n} + 14\,b^{13}cx^{15n} + b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

**maple [B]** time = 0.08, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n} + \frac{b^{13}c\,x^{15n}}{n} + \frac{13b^{12}c^{2}x^{16n}}{2n} + \frac{26b^{11}c^{3}x^{17n}}{n} + \frac{143b^{10}c^{4}x^{18n}}{2n} + \frac{143b^{9}c^{5}x^{19n}}{n} + \frac{429b^{8}c^{6}x^{20n}}{2n} + \frac{1716b^{7}c^{7}x^{21n}}{7n} + \frac{429b^{6}c^{8}x^{22n}}{2n} + \frac{143b^{5}c^{9}x^{23n}}{n} + \frac{143b^{4}c^{10}x^{24n}}{2n} + \frac{26b^{3}c^{11}x^{25n}}{n} + \frac{13b^{2}c^{12}x^{26n}}{2n} + \frac{b\,c^{13}x^{27n}}{n} + \frac{c^{14}x^{28n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

**maxima [B]** time = 0.64, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28n}}{14\,n} + \frac{bc^{13}x^{27n}}{n} + \frac{13\,b^{2}c^{12}x^{26n}}{2\,n} + \frac{26\,b^{3}c^{11}x^{25n}}{n} + \frac{143\,b^{4}c^{10}x^{24n}}{2\,n} + \frac{143\,b^{5}c^{9}x^{23n}}{n} + \frac{429\,b^{6}c^{8}x^{22n}}{2\,n} + \frac{1716\,b^{7}c^{7}x^{21n}}{7\,n} + \frac{429\,b^{8}c^{6}x^{20n}}{2\,n} + \frac{143\,b^{9}c^{5}x^{19n}}{n} + \frac{143\,b^{10}c^{4}x^{18n}}{2\,n} + \frac{26\,b^{11}c^{3}x^{17n}}{n} + \frac{13\,b^{12}c^{2}x^{16n}}{2\,n} + \frac{b^{13}cx^{15n}}{n} + \frac{b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

**mupad [B]** time = 5.21, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n} + \frac{c^{14}\,x^{28\,n}}{14\,n} + \frac{13\,b^{12}\,c^{2}\,x^{16\,n}}{2\,n} + \frac{26\,b^{11}\,c^{3}\,x^{17\,n}}{n} + \frac{143\,b^{10}\,c^{4}\,x^{18\,n}}{2\,n} + \frac{143\,b^{9}\,c^{5}\,x^{19\,n}}{n} + \frac{429\,b^{8}\,c^{6}\,x^{20\,n}}{2\,n} + \frac{1716\,b^{7}\,c^{7}\,x^{21\,n}}{7\,n} + \frac{429\,b^{6}\,c^{8}\,x^{22\,n}}{2\,n} + \frac{143\,b^{5}\,c^{9}\,x^{23\,n}}{n} + \frac{143\,b^{4}\,c^{10}\,x^{24\,n}}{2\,n} + \frac{26\,b^{3}\,c^{11}\,x^{25\,n}}{n} + \frac{13\,b^{2}\,c^{12}\,x^{26\,n}}{2\,n} + \frac{b^{13}\,c\,x^{15\,n}}{n} + \frac{b\,c^{13}\,x^{27\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(14*n - 1)*(b + c*x^n)^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+14*n)*(b+c*x**n)**13*(b+2*c*x**n),x)
```

```
[Out] Timed out
```

## 3.676 $\int x^{-1+m} (a + bx^n)^{-1+p} \left(am + b(m + np)x^n\right) dx$

**Optimal**. Leaf size=13

$$x^m (a + bx^n)^p$$

**Rubi [A]** time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {449}

$$x^m (a + bx^n)^p$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] x^m*(a + b*x^n)^p
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+m} (a + bx^n)^{-1+p} \left(am + b(m + np)x^n\right) dx = x^m (a + bx^n)^p$$

**Mathematica [C]** time = 0.18, size = 107, normalized size = 8.23

$$\frac{x^m (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(bx^n(m + np)\,{}_2F_1\left(\frac{m+n}{n}, 1 - p; \frac{m}{n} + 2; -\frac{bx^n}{a}\right) + a(m + n)\,{}_2F_1\left(\frac{m}{n}, 1 - p; \frac{m+n}{n}; -\frac{bx^n}{a}\right)\right)}{a(m + n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] (x^m*(a + b*x^n)^p*(a*(m + n)*Hypergeometric2F1[m/n, 1 - p, (m + n)/n, -((b
*x^n)/a)] + b*(m + n*p)*x^n*Hypergeometric2F1[(m + n)/n, 1 - p, 2 + m/n, -(
(b*x^n)/a)]))/(a*(m + n)*(1 + (b*x^n)/a)^p)
```

**IntegrateAlgebraic [F]** time = 0.10, size = 0, normalized size = 0.00

$$\int x^{-1+m} (a + bx^n)^{-1+p} \left(am + b(m + np)x^n\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),
x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p
)*x^n), x]
```

**fricas [B]** time = 0.43, size = 32, normalized size = 2.46

$$\left(bxx^{m-1}x^n + axx^{m-1}\right)(bx^n + a)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="frica
s")
```

```
[Out] (b*x*x^(m - 1)*x^n + a*x*x^(m - 1))*(b*x^n + a)^(p - 1)
```

**giac [B]** time = 0.28, size = 70, normalized size = 5.38

$$bxx^{n}e^{\left(p\log(bx^{n}+a)+m\log(x)-\log(bx^{n}+a)-\log(x)\right)}+axe^{\left(p\log(bx^{n}+a)+m\log(x)-\log(bx^{n}+a)-\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="giac"
)
```

```
[Out] b*x*x^n*e^(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x)) + a*x*e^(
p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x))
```

**maple [F]** time = 0.68, size = 0, normalized size = 0.00

$$\int\left(am+\left(np+m\right)b\,x^{n}\right)x^{m-1}\left(b\,x^{n}+a\right)^{p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-1)*(b*x^n+a)^(p-1)*(a*m+b*(n*p+m)*x^n),x)
```

```
[Out] int(x^(m-1)*(b*x^n+a)^(p-1)*(a*m+b*(n*p+m)*x^n),x)
```

**maxima [A]** time = 0.84, size = 16, normalized size = 1.23

$$e^{\left(p\log(bx^{n}+a)+m\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="maxim
a")
```

```
[Out] e^(p*log(b*x^n + a) + m*log(x))
```

**mupad [B]** time = 4.81, size = 25, normalized size = 1.92

$$\left(a\,x^{m}+b\,x^{m+n}\right)\,\left(a+b\,x^{n}\right)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*(a*m + b*x^n*(m + n*p))*(a + b*x^n)^(p - 1),x)
```

```
[Out] (a*x^m + b*x^(m + n))*(a + b*x^n)^(p - 1)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a+b*x**n)**(-1+p)*(a*m+b*(n*p+m)*x**n),x)
```

```
[Out] Timed out
```

## 3.677 $\int \frac{b+2cx}{x(b+cx)}\,dx$

**Optimal**. Leaf size=8

$$\log(x(b+cx))$$

**Rubi** [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {72}

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(x*(b + c*x)),x]
[Out] Log[x] + Log[b + c*x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx}{x(b+cx)}\,dx &= \int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx\\ &= \log(x)+\log(b+cx)\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 9, normalized size = 1.12

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(x*(b + c*x)),x]
[Out] Log[x] + Log[b + c*x]
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{x(b+cx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(x*(b + c*x)),x]
[Out] IntegrateAlgebraic[(b + 2*c*x)/(x*(b + c*x)), x]
```

**fricas** [A] time = 0.40, size = 10, normalized size = 1.25

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")
[Out] log(c*x^2 + b*x)
```

**giac** [A] time = 0.15, size = 11, normalized size = 1.38

$$\log(|cx + b|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")
[Out] log(abs(c*x + b)) + log(abs(x))
```

**maple** [A] time = 0.04, size = 9, normalized size = 1.12

$$\ln((cx + b)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x/(c*x+b),x)
[Out] ln(x*(c*x+b))
```

**maxima** [A] time = 0.48, size = 9, normalized size = 1.12

$$\log(cx + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")
[Out] log(c*x + b) + log(x)
```

**mupad** [B] time = 4.66, size = 8, normalized size = 1.00

$$\ln(x\ (b + c\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(x*(b + c*x)),x)
[Out] log(x*(b + c*x))
```

**sympy** [A] time = 0.12, size = 8, normalized size = 1.00

$$\log\left(bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x)
[Out] log(b*x + c*x**2)
```

## 3.678 $\int \frac{b+2cx^2}{x\left(b+cx^2\right)} \, dx$

**Optimal**. Leaf size=15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+2cx^2}{x\left(b+cx^2\right)} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right) dx, x, x^2\right)\\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(x*(b + c*x^2)), x]
```

**fricas [A]** time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

**giac [A]** time = 0.15, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

**maple [A]** time = 0.05, size = 14, normalized size = 0.93

$$\ln(x)+\frac{\ln\left(c\,x^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x/(c*x^2+b),x)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

**maxima [A]** time = 0.49, size = 17, normalized size = 1.13

$$\frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

**mupad [B]** time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x*(b + c*x^2)),x)
[Out] log(b + c*x^2)/2 + log(x)
```

**sympy [A]** time = 0.18, size = 12, normalized size = 0.80

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x/(c*x**2+b),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

## 3.679 $\int \frac{b+2cx^3}{x(b+cx^3)}\,dx$

**Optimal**. Leaf size=15

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

**Rubi** [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^3\right)\\ &= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

**IntegrateAlgebraic** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(x*(b + c*x^3)), x]
```

**fricas [A]** time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

**giac [A]** time = 0.16, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

**maple [A]** time = 0.05, size = 14, normalized size = 0.93

$$\ln(x)+\frac{\ln\left(c\,x^3+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x/(c*x^3+b),x)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

**maxima [A]** time = 0.55, size = 17, normalized size = 1.13

$$\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

**mupad [B]** time = 4.63, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^3+b\right)}{3}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x*(b + c*x^3)),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

**sympy [A]** time = 0.19, size = 12, normalized size = 0.80

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x/(c*x**3+b),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

## 3.680 $\int \frac{b+2cx^n}{x(b+cx^n)} dx$

**Optimal**. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

**Rubi [A]** time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^n}{x(b+cx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^n\right)}{n}\\ &= \log(x)+\frac{\log(b+cx^n)}{n}\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

**IntegrateAlgebraic [A]** time = 0.03, size = 24, normalized size = 1.60

$$\frac{\log(bn+cnx^n)}{n}+\frac{\log(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
[Out] Log[x^n]/n + Log[b*n + c*n*x^n]/n
```

**fricas [A]** time = 0.44, size = 17, normalized size = 1.13

$$\frac{n\log(x) + \log\left(cx^n + b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="fricas")
[Out] (n*log(x) + log(c*x^n + b))/n
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,cx^n + b}{(cx^n + b)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/((c*x^n + b)*x), x)
```

**maple [A]** time = 0.05, size = 17, normalized size = 1.13

$$\frac{\ln\left(\left(c\,x^n + b\right)x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/x/(b+c*x^n),x)
[Out] 1/n*ln(x^n*(b+c*x^n))
```

**maxima [B]** time = 0.49, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

**mupad [B]** time = 4.72, size = 15, normalized size = 1.00

$$\ln(x) + \frac{\ln\left(b + c\,x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(x*(b + c*x^n)),x)
[Out] log(x) + log(b + c*x^n)/n
```

**sympy** [A] time = 0.62, size = 29, normalized size = 1.93

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \log(x) & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/x/(b+c*x**n),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))
```

## 3.681 $\int \frac{b+2cx}{x^8(b+cx)^8} \, dx$

**Optimal**. Leaf size=14

$$-\frac{1}{7x^7(b+cx)^7}$$

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {74}

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{b+2cx}{x^8(b+cx)^8} \, dx = -\frac{1}{7x^7(b+cx)^7}$$

**Mathematica [A]** time = 0.02, size = 14, normalized size = 1.00

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{x^8(b+cx)^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)/(x^8*(b + c*x)^8), x]
```

**fricas [B]** time = 0.42, size = 81, normalized size = 5.79

$$-\frac{1}{7\left(c^7x^{14} + 7\,bc^6x^{13} + 21\,b^2c^5x^{12} + 35\,b^3c^4x^{11} + 35\,b^4c^3x^{10} + 21\,b^5c^2x^9 + 7\,b^6cx^8 + b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

**giac [A]** time = 0.17, size = 13, normalized size = 0.93

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

**maple [B]** time = 0.06, size = 177, normalized size = 12.64

$$\frac{c^7}{7\left(cx+b\right)^7 b^7}+\frac{c^7}{\left(cx+b\right)^6 b^8}+\frac{4c^7}{\left(cx+b\right)^5 b^9}+\frac{12c^7}{\left(cx+b\right)^4 b^{10}}+\frac{30c^7}{\left(cx+b\right)^3 b^{11}}+\frac{66c^7}{\left(cx+b\right)^2 b^{12}}+\frac{132c^7}{\left(cx+b\right) b^{13}}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}-\frac{1}{7b^7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x^8/(c*x+b)^8,x)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7-1/7
/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b
^9*c^2/x^5+1/b^8*c/x^6
```

**maxima [B]** time = 0.61, size = 81, normalized size = 5.79

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

**mupad [B]** time = 7.09, size = 12, normalized size = 0.86

$$-\frac{1}{7\,x^7\,\left(b+c\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(x^8*(b + c*x)^8),x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

**sympy [B]** time = 0.89, size = 87, normalized size = 6.21

$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x**8/(c*x+b)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

## 3.682 $\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}\,dx$

**Optimal**. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

**Rubi [A]** time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)\\ &= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8), x]
```

**fricas [B]** time = 0.41, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

**giac [A]** time = 0.15, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^{4}+bx^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

**maple [B]** time = 0.06, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{2}+b\right)^{7}c}-\frac{b^{5}}{\left(c\,x^{2}+b\right)^{6}c}-\frac{4b^{4}}{\left(c\,x^{2}+b\right)^{5}c}-\frac{12b^{3}}{\left(c\,x^{2}+b\right)^{4}c}-\frac{30b^{2}}{\left(c\,x^{2}+b\right)^{3}c}-\frac{66b}{\left(c\,x^{2}+b\right)^{2}c}-\frac{132}{\left(c\,x^{2}+b\right)c}\right)c^{8}}{2b^{13}}-\frac{66c^{6}}{b^{13}x^{2}}+\frac{33c^{5}}{b^{12}x^{4}}-\frac{15c^{4}}{b^{11}x^{6}}+\frac{6c^{3}}{b^{10}x^{8}}-\frac{2c^{2}}{b^{9}x^{10}}+\frac{c}{2b^{8}x^{12}}-\frac{1}{14b^{7}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x)
```

```
[Out] -1/2*c^8/b^13*(-4*b^4/c/(c*x^2+b)^5-66*b/c/(c*x^2+b)^2-1/7*b^6/c/(c*x^2+b)^
7-b^5/c/(c*x^2+b)^6-12*b^3/c/(c*x^2+b)^4-132/c/(c*x^2+b)-30*b^2/c/(c*x^2+b)
^3)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

**maxima [B]** time = 0.64, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

**mupad [B]** time = 2.32, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\left(c\,x^{2}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

**sympy [B]** time = 1.36, size = 87, normalized size = 5.44

$$-\frac{1}{14b^7x^{14} + 98b^6cx^{16} + 294b^5c^2x^{18} + 490b^4c^3x^{20} + 490b^3c^4x^{22} + 294b^2c^5x^{24} + 98bc^6x^{26} + 14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

## 3.683 $\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$

**Optimal**. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

**Rubi [A]** time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)\\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

**IntegrateAlgebraic [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8), x]
```

**fricas [B]** time = 0.39, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

**giac [A]** time = 0.17, size = 15, normalized size = 0.94

$$-\frac{1}{21\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

**maple [B]** time = 0.06, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{3}+b\right)^{7}c}-\frac{b^{5}}{\left(c\,x^{3}+b\right)^{6}c}-\frac{4b^{4}}{\left(c\,x^{3}+b\right)^{5}c}-\frac{12b^{3}}{\left(c\,x^{3}+b\right)^{4}c}-\frac{30b^{2}}{\left(c\,x^{3}+b\right)^{3}c}-\frac{66b}{\left(c\,x^{3}+b\right)^{2}c}-\frac{132}{\left(c\,x^{3}+b\right)c}\right)c^{8}}{3b^{13}}-\frac{44c^{6}}{b^{13}x^{3}}+\frac{22c^{5}}{b^{12}x^{6}}-\frac{10c^{4}}{b^{11}x^{9}}+\frac{4c^{3}}{b^{10}x^{12}}-\frac{4c^{2}}{3b^{9}x^{15}}+\frac{c}{3b^{8}x^{18}}-\frac{1}{21b^{7}x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-4*b^4/c/(c*x^3+b)^5-66*b/c/(c*x^3+b)^2-1/7*b^6/c/(c*x^3+b)^
7-b^5/c/(c*x^3+b)^6-12*b^3/c/(c*x^3+b)^4-132/c/(c*x^3+b)-30*b^2/c/(c*x^3+b)
^3)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

**maxima [B]** time = 0.61, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

**mupad [B]** time = 9.98, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,{\left(c\,x^3+b\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

**sympy [B]** time = 1.91, size = 87, normalized size = 5.44

$$-\frac{1}{21b^7x^{21} + 147b^6cx^{24} + 441b^5c^2x^{27} + 735b^4c^3x^{30} + 735b^3c^4x^{33} + 441b^2c^5x^{36} + 147bc^6x^{39} + 21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

## 3.684 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$

**Optimal**. Leaf size=21

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {446, 74}

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-7n}\,(b+2cx^n)}{(b+cx^n)^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-7n}}{7n\,(b+cx^n)^7}\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

**IntegrateAlgebraic [A]** time = 0.10, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

**fricas [B]** time = 0.48, size = 105, normalized size = 5.00

$$-\frac{1}{7\left(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+21\,b^{2}c^{5}nx^{12\,n}+35\,b^{3}c^{4}nx^{11\,n}+35\,b^{4}c^{3}nx^{10\,n}+21\,b^{5}c^{2}nx^{9\,n}+7\,b^{6}cnx^{8\,n}+b^{7}nx^{7\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,cx^{n}+b)x^{-7\,n-1}}{(cx^{n}+b)^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)
```

**maple [B]** time = 0.10, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^{7}n}+\frac{c\,x^{-6n}}{b^{8}n}-\frac{4c^{2}x^{-5n}}{b^{9}n}+\frac{12c^{3}x^{-4n}}{b^{10}n}-\frac{30c^{4}x^{-3n}}{b^{11}n}+\frac{66c^{5}x^{-2n}}{b^{12}n}+\frac{\left(9009b^{5}c\,x^{n}+20020b^{4}c^{2}x^{2n}+24024b^{3}c^{3}x^{3n}+16380b^{2}c^{4}x^{4n}+6006b\,c^{5}x^{5n}+924c^{6}x^{6n}+1716b^{6}\right)c^{7}}{7\,(c\,x^{n}+b)^{7}\,b^{13}n}-\frac{132c^{6}x^{-n}}{b^{13}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7*n)*(b+2*c*x^n)/(c*x^n+b)^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(c*x^n+b)^7
```

**maxima [B]** time = 0.79, size = 612, normalized size = 29.14

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^{2}c^{11}x^{11\,n}+9579570\,b^{3}c^{10}x^{10\,n}+8270262\,b^{4}c^{9}x^{9\,n}+4018014\,b^{5}c^{8}x^{8\,n}+934362\,b^{6}c^{7}x^{7\,n}+45045\,b^{7}c^{6}x^{6\,n}-5005\,b^{8}c^{5}x^{5\,n}+1001\,b^{9}c^{4}x^{4\,n}-273\,b^{10}c^{3}x^{3\,n}+91\,b^{11}c^{2}x^{2\,n}-35\,b^{12}cx^{n}+15\,b^{13}}{b^{14}c^{7}nx^{14\,n}+7\,b^{15}c^{6}nx^{13\,n}+21\,b^{16}c^{5}nx^{12\,n}+35\,b^{17}c^{4}nx^{11\,n}+35\,b^{18}c^{3}nx^{10\,n}+21\,b^{19}c^{2}nx^{9\,n}+7\,b^{20}cnx^{8\,n}+b^{21}nx^{7\,n}}+\frac{360360\,c^{7}\log(x)}{b^{15}}-\frac{360360\,c^{7}\log\left(\frac{cx^{n}+b}{c}\right)}{b^{15}n}\right)+\frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^{2}c^{10}x^{10\,n}+9579570\,b^{3}c^{9}x^{9\,n}+8270262\,b^{4}c^{8}x^{8\,n}+4018014\,b^{5}c^{7}x^{7\,n}+934362\,b^{6}c^{6}x^{6\,n}+45045\,b^{7}c^{5}x^{5\,n}-5005\,b^{8}c^{4}x^{4\,n}+1001\,b^{9}c^{3}x^{3\,n}-273\,b^{10}c^{2}x^{2\,n}+91\,b^{11}cx^{n}-35\,b^{12}}{b^{13}c^{7}nx^{13\,n}+7\,b^{14}c^{6}nx^{12\,n}+21\,b^{15}c^{5}nx^{11\,n}+35\,b^{16}c^{4}nx^{10\,n}+35\,b^{17}c^{3}nx^{9\,n}+21\,b^{18}c^{2}nx^{8\,n}+7\,b^{19}cnx^{7\,n}+b^{20}nx^{6\,n}}+\frac{360360\,c^{6}\log(x)}{b^{14}}-\frac{360360\,c^{6}\log\left(\frac{cx^{n}+b}{c}\right)}{b^{14}n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
```

```
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

**mupad [B]** time = 4.99, size = 105, normalized size = 5.00

$$-\frac{1}{7\,x^{7\,n}\,\left(b^7\,n+c^7\,n\,x^{7\,n}+7\,b^6\,c\,n\,x^n+7\,b\,c^6\,n\,x^{6\,n}+21\,b^5\,c^2\,n\,x^{2\,n}+35\,b^4\,c^3\,n\,x^{3\,n}+35\,b^3\,c^4\,n\,x^{4\,n}+21\,b^2\,c^5\,n\,x^{5\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(x^(7*n + 1)*(b + c*x^n)^8),x)
```

```
[Out] -1/(7*x^(7*n)*(b^7*n + c^7*n*x^(7*n) + 7*b^6*c*n*x^n + 7*b*c^6*n*x^(6*n) +
21*b^5*c^2*n*x^(2*n) + 35*b^4*c^3*n*x^(3*n) + 35*b^3*c^4*n*x^(4*n) + 21*b^2
*c^5*n*x^(5*n)))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

```
[Out] Timed out
```

## 3.685 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx$

**Optimal**. Leaf size=52

$$-\frac{1}{24}\left(x^{16}+1\right)^{3/2}-\frac{\sqrt{x^{16}+1}}{8}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

**Rubi [A]** time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 206}

$$-\frac{1}{24}\left(x^{16}+1\right)^{3/2}-\frac{\sqrt{x^{16}+1}}{8}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] -Sqrt[1 + x^16]/8 - (1 + x^16)^(3/2)/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(
4*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx &= \frac{1}{16}\text{Subst}\left(\int \frac{x\sqrt{1+x}}{1-x}\,dx, x, x^{16}\right)\\
&= -\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{16}\text{Subst}\left(\int \frac{\sqrt{1+x}}{1-x}\,dx, x, x^{16}\right)\\
&= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx, x, x^{16}\right)\\
&= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x^{16}}\right)\\
&= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 44, normalized size = 0.85

$$\frac{1}{24}\left(3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)-\sqrt{x^{16}+1}\left(x^{16}+4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
[Out] (-(Sqrt[1 + x^16]*(4 + x^16)) + 3*Sqrt[2]*ArcTanh[Sqrt[1 + x^16]/Sqrt[2]])/
24
```

**IntegrateAlgebraic [A]** time = 0.07, size = 46, normalized size = 0.88

$$\frac{1}{24}\sqrt{x^{16}+1}\left(-x^{16}-4\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
[Out] ((-4 - x^16)*Sqrt[1 + x^16])/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(4*Sqrt[2
])
```

**fricas [A]** time = 0.42, size = 46, normalized size = 0.88

$$-\frac{1}{24}\left(x^{16}+4\right)\sqrt{x^{16}+1}+\frac{1}{16}\sqrt{2}\log\left(\frac{x^{16}+2\sqrt{2}\sqrt{x^{16}+1}+3}{x^{16}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="fricas")
[Out] -1/24*(x^16 + 4)*sqrt(x^16 + 1) + 1/16*sqrt(2)*log((x^16 + 2*sqrt(2)*sqrt(x
^16 + 1) + 3)/(x^16 - 1))
```

**giac [A]** time = 0.15, size = 56, normalized size = 1.08

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^{16}+1}\right|}{2\left(\sqrt{2}+\sqrt{x^{16}+1}\right)}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="giac")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^16
+ 1))/(sqrt(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

**maple [C]** time = 1.15, size = 86, normalized size = 1.65

$$\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{x^{16}\,\mathrm{RootOf}\left(\_Z^2-2\right)+3\,\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{x^{16}+1}}{(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)\left(x^8+1\right)}\right)}{16}-\frac{\left(x^{16}+4\right)\sqrt{x^{16}+1}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^31*(x^16+1)^(1/2)/(-x^16+1),x)
```

```
[Out] -1/24*(x^16+4)*(x^16+1)^(1/2)+1/16*RootOf(_Z^2-2)*ln(-(RootOf(_Z^2-2)*x^16+
4*(x^16+1)^(1/2)+3*RootOf(_Z^2-2))/(x-1)/(x+1)/(x^2+1)/(x^4+1)/(x^8+1))
```

**maxima [A]** time = 1.25, size = 53, normalized size = 1.02

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(x^16 + 1))/(sqrt
(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

**mupad [B]** time = 4.82, size = 37, normalized size = 0.71

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^{16}+1}}{2}\right)}{8}-\frac{\sqrt{x^{16}+1}}{8}-\frac{\left(x^{16}+1\right)^{3/2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^31*(x^16 + 1)^(1/2))/(x^16 - 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(x^16 + 1)^(1/2))/2))/8 - (x^16 + 1)^(1/2)/8 - (x^1
6 + 1)^(3/2)/24
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)
```

```
[Out] Timed out
```

## 3.686 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx$

**Optimal**. Leaf size=93

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

**Rubi** [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 105, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a
] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/Sq
rt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\right) - d\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\\
&= -\left((2c)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\right) - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\
&= \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{b}\\
&= \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}
\end{aligned}
$$

**Mathematica [A]** time = 0.68, size = 142, normalized size = 1.53

$$
\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\,x\sqrt{c+\frac{d}{x}}\,\sqrt{bc-ad}\,\sqrt{\frac{b(cx+d)}{x(bc-ad)}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{bcx+bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d/x]*x*Sqrt[(b*(d + c*x))/((b*c - a*d)
*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*d + b*c*x) + (2*S
qrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a]
```

**IntegrateAlgebraic [B]** time = 2.67, size = 442, normalized size = 4.75

$$
\frac{c\sqrt{\frac{a}{c}}\sqrt{ax+b}\sqrt{c+\frac{d}{x}}\left(\sqrt{ax+b}-\sqrt{\frac{a}{c}}\sqrt{cx+d}\right)\left(-2c\sqrt{\frac{a}{c}}\sqrt{ax+b}\sqrt{cx+d}+a(2cx+d)+bc\right)\left(-\frac{2c\sqrt{\frac{a}{c}}\log\left(\sqrt{\frac{a(cx+d)}{c}-\frac{ad}{c}+b}-\sqrt{\frac{a}{c}}\sqrt{cx+d}\right)}{a}-\frac{2\sqrt{c}\sqrt{d}\sqrt{\frac{a}{c}}\tanh^{-1}\left(-\frac{\sqrt{a}(cx+d)}{\sqrt{b}\sqrt{c}\sqrt{d}}+\frac{\sqrt{c}\sqrt{\frac{a}{c}}\sqrt{cx+d}\sqrt{\frac{a(cx+d)}{c}-\frac{ad}{c}+b}}{\sqrt{a}\sqrt{b}\sqrt{d}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}\right)}{\sqrt{a}\sqrt{b}}\right)}{\sqrt{a+\frac{b}{x}}\sqrt{cx+d}\left(a\left(4c^2x\sqrt{\frac{a}{c}}\sqrt{ax+b}+3cd\sqrt{\frac{a}{c}}\sqrt{ax+b}-4acx\sqrt{cx+d}-ad\sqrt{cx+d}\right)+bc\left(c\sqrt{\frac{a}{c}}\sqrt{ax+b}-3a\sqrt{cx+d}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (Sqrt[a/c]*c*Sqrt[c + d/x]*Sqrt[b + a*x]*(Sqrt[b + a*x] - Sqrt[a/c]*Sqrt[d
+ c*x])*(b*c - 2*Sqrt[a/c]*c*Sqrt[b + a*x]*Sqrt[d + c*x] + a*(d + 2*c*x))*(
(-2*Sqrt[a/c]*Sqrt[c]*Sqrt[d]*ArcTanh[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]) -
 (Sqrt[a]*(d + c*x))/(Sqrt[b]*Sqrt[c]*Sqrt[d]) + (Sqrt[a/c]*Sqrt[c]*Sqrt[d
+ c*x]*Sqrt[b - (a*d)/c + (a*(d + c*x))/c])/(Sqrt[a]*Sqrt[b]*Sqrt[d])])/(Sq
rt[a]*Sqrt[b]) - (2*Sqrt[a/c]*c*Log[-(Sqrt[a/c]*Sqrt[d + c*x]) + Sqrt[b - (
a*d)/c + (a*(d + c*x))/c]])/a))/(Sqrt[a + b/x]*Sqrt[d + c*x]*(b*c*(Sqrt[a/c
]*c*Sqrt[b + a*x] - 3*a*Sqrt[d + c*x]) + a*(3*Sqrt[a/c]*c*d*Sqrt[b + a*x] +
 4*Sqrt[a/c]*c^2*x*Sqrt[b + a*x] - a*d*Sqrt[d + c*x] - 4*a*c*x*Sqrt[d + c*x
])))
```

**fricas [B]** time = 0.71, size = 757, normalized size = 8.14

$$\left[\frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8a^2c^2x^2-b^2c^2-6abcd-a^2d^2-4\left(2a^2cx^2+\left(abc+a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-8\left(abc^2+a^2cd\right)x\right)+\frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8b^2d^2+\left(b^2c^2+6abcd+a^2d^2\right)x^2-4\left(2b^2dx+\left(b^2c+abd\right)x^2\right)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\left(b^2cd+abd^2\right)x}{x^2}\right),\ -\sqrt{-\frac{c}{a}}\arctan\left(\frac{2ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2acx+bc+ad}\right)+\frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8b^2d^2+\left(b^2c^2+6abcd+a^2d^2\right)x^2-4\left(2b^2dx+\left(b^2c+abd\right)x^2\right)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\left(b^2cd+abd^2\right)x}{x^2}\right),\ \sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(2bdx+\left(bc+ad\right)x^2\right)\sqrt{-\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\left(acdx^2+bd^2+\left(bcd+ad^2\right)x\right)}\right)+\frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8a^2c^2x^2-b^2c^2-6abcd-a^2d^2-4\left(2a^2cx^2+\left(abc+a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-8\left(abc^2+a^2cd\right)x\right),\ -\sqrt{-\frac{c}{a}}\arctan\left(\frac{2ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2acx+bc+ad}\right)+\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(2bdx+\left(bc+ad\right)x^2\right)\sqrt{-\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\left(acdx^2+bd^2+\left(bcd+ad^2\right)x\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^
2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)
- 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt(
(a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), -sqrt(-c/a)
*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
 + a*d)) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt(
(c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), sqrt(-d/b)*arctan(1/2*(2*b*d*
x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*
x^2 + b*d^2 + (b*c*d + a*d^2)*x)) + 1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*
c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x), -sqrt(-c/a)*
arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
+ a*d)) + sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt
((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x))]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \frac{d}{x}}}{\sqrt{a + \frac{b}{x}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

**maple [B]** time = 0.10, size = 143, normalized size = 1.54

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\,\left(-\sqrt{bd}\,c\ln\left(\frac{2acx+ad+bc+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}}{2\sqrt{ac}}\right)+\sqrt{ac}\,d\ln\left(\frac{adx+bcx+2bd+2\sqrt{bd}\,\sqrt{(cx+d)(ax+b)}}{x}\right)\right)x}{\sqrt{(cx+d)\,(ax+b)}\,\sqrt{bd}\,\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x)
```

```
[Out] -((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((c*x
+d)*(a*x+b))^(1/2)+2*b*d)/x)*d*(a*c)^(1/2)-ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+
b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*c*(b*d)^(1/2))/((c*x+d)*(a*x+b)
)^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \frac{d}{x}}}{\sqrt{a + \frac{b}{x}}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c + \frac{d}{x}}}{x\, \sqrt{a + \frac{b}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)),x)
```

```
[Out] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + \frac{d}{x}}}{x \sqrt{a + \frac{b}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)/x/(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

## 3.687 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=252

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}-\frac{5(bc-ad)^2(ad+7bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n}+\frac{5(bc-ad)(ad+7bc)(a}{96bd^3}$$

**Rubi [A]** time = 0.23, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}-\frac{(ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n}+\frac{5(bc-ad)(ad+7bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n}-\frac{5(bc-ad)^2(ad+7bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n}+\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b*d^4*
n) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b*
d^3*n) - ((7*b*c + a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b*d^2*n) + (
(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d*n) + (5*(b*c - a*d)^3*(7*b*c + a*
d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(3/2
)*d^(9/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}\left(a+bx^n\right)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bdn} - \frac{(7bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{8bdn}$$

$$= -\frac{(7bc+ad)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{(5(bc-ad)(7bc+ad))}{4\ldots}$$

$$= \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a\ldots}{\ldots}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

**Mathematica [A]** time = 1.09, size = 223, normalized size = 0.88

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\left(c+dx^n\right)\left(15a^3d^3+a^2bd^2\left(118dx^n-191c\right)+ab^2d\left(265c^2-172cdx^n+136d^2x^{2n}\right)+b^3\left(-105c^3+70c^2dx^n-56cd^2x^{2n}+48d^3x^{3n}\right)\right)+15(ad+7bc)(bc-ad)^{7/2}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{192b^2d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*c^
3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^(7/
2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])
```

**IntegrateAlgebraic [F]** time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}\left(a+bx^n\right)^{5/2}}{\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

[Out] Defer[IntegrateAlgebraic][(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],
 x]

**fricas** [A]    time = 0.54, size = 607, normalized size = 2.41

$$\left[-\frac{15 \left(7 b^{4} c^{4} - 20 a b^{3} c^{3} d + 18 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} - a^{4} d^{4}\right) \sqrt{b d} \log\left(8 b^{2} d^{2} x^{2 n} + b^{2} c^{2} + 6 a b c d + a^{2} d^{2} - 4 \left(2 \sqrt{b d} b d x^{n} + (b c + a d) \sqrt{b d}\right) \sqrt{b x^{n} + a} \sqrt{d x^{n} + c} + 8 \left(b^{2} c d + a b d^{2}\right) x^{n}\right) - 4 \left(48 b^{4} d^{4} x^{3 n} - 105 b^{4} c^{3} d + 265 a b^{3} c^{2} d^{2} - 191 a^{2} b^{2} c d^{3} + 15 a^{3} b d^{4} - 8 \left(7 b^{4} c d^{3} - 17 a b^{3} d^{4}\right) x^{2 n} + 2 \left(35 b^{4} c^{2} d^{2} - 86 a b^{3} c d^{3} + 59 a^{2} b^{2} d^{4}\right) x^{n}\right) \sqrt{b x^{n} + a} \sqrt{d x^{n} + c}}{768 b^{2} d^{5} n}, -\frac{15 \left(7 b^{4} c^{4} - 20 a b^{3} c^{3} d + 18 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} - a^{4} d^{4}\right) \sqrt{-b d} \arctan\left(\frac{\left(2 \sqrt{-b d} b d x^{n} + (b c + a d) \sqrt{-b d}\right) \sqrt{b x^{n} + a} \sqrt{d x^{n} + c}}{2 \left(b^{2} d^{2} x^{2 n} + a b c d + \left(b^{2} c d + a b d^{2}\right) x^{n}\right)}\right) - 2 \left(48 b^{4} d^{4} x^{3 n} - 105 b^{4} c^{3} d + 265 a b^{3} c^{2} d^{2} - 191 a^{2} b^{2} c d^{3} + 15 a^{3} b d^{4} - 8 \left(7 b^{4} c d^{3} - 17 a b^{3} d^{4}\right) x^{2 n} + 2 \left(35 b^{4} c^{2} d^{2} - 86 a b^{3} c d^{3} + 59 a^{2} b^{2} d^{4}\right) x^{n}\right) \sqrt{b x^{n} + a} \sqrt{d x^{n} + c}}{384 b^{2} d^{5} n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] [-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^
2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*
x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3
*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3
- 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d
^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7*b^4*c^
4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*
d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*
d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2
*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c
))/(b^2*d^5*n)]

**giac** [F]    time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")

[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)

**maple** [F]    time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{5}{2}}\, x^{2n-1}}{\sqrt{d\,x^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)

[Out] int(x^(2*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)

**maxima** [F]    time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)

**mupad** [F]    time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2n-1}\,(a + b\,x^n)^{5/2}}{\sqrt{c + d\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

## 3.688 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=199

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}+\frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n}-\frac{(ad+5bc)\,(a+bx^n)^{3/2}\sqrt{c}}{12bd^2n}$$

**Rubi [A]** time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 80, 50, 63, 217, 206}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}-\frac{(ad+5bc)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}+\frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n}+\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b*d^3*n) - (
(5*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b*d^2*n) + ((a + b*x^n
)^(5/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(3/2)*d^(7/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{(5bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{6bdn}$$

$$= -\frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} + \frac{((bc-ad)(5bc+ad))\,\text{S}}{8b}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

**Mathematica [A]** time = 0.67, size = 178, normalized size = 0.89

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\left(3a^2d^2+2abd\,(7dx^n-11c)+b^2\left(15c^2-10cdx^n+8d^2x^{2n}\right)\right)-3(bc-ad)^{5/2}(ad+5bc)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{24b^2d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^
n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*b*
c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n]
)/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])
```

**IntegrateAlgebraic [F]** time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],
 x]
```

**fricas** [A] time = 0.50, size = 469, normalized size = 2.36

$$\left[\frac{3\left(5b^3c^3 - 9ab^2c^2d + 3a^2bcd^2 + a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^{2n} + b^2c^2 + 6abcd + a^2d^2 - 4\left(2\sqrt{bd}bdx^n + (bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c} + 8\left(b^2cd + abd^2\right)x^n\right) + 4\left(8b^3d^3x^{2n} + 15b^3c^2d - 22ab^2cd^2 + 3a^2bd^3 - 2\left(5b^3cd^2 - 7ab^2d^3\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{96b^2d^4n}, \frac{3\left(5b^3c^3 - 9ab^2c^2d + 3a^2bcd^2 + a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{\left(2\sqrt{-bd}bdx^n + (bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n} + abcd + \left(b^2cd + abd^2\right)x^n\right)}\right) + 2\left(8b^3d^3x^{2n} + 15b^3c^2d - 22ab^2cd^2 + 3a^2bd^3 - 2\left(5b^3cd^2 - 7ab^2d^3\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{48b^2d^4n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^
n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a
*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2
*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)
)/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3
)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqr
t(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^
2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d
^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^2*d^4*n)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

**maple** [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{3}{2}}\, x^{2n-1}}{\sqrt{d\,x^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(2*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}\,(a + b\,x^n)^{3/2}}{\sqrt{c + d\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

## 3.689 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=146

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

**Rubi [A]** time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b*d^2*n) + ((a + b*x^n)
^(3/2)*Sqrt[c + d*x^n])/(2*b*d*n) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqr
t[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(3/2)*d^(5/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} - \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}}{8bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}}{4}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}}{4b^2d^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{(bc-ad)(3bc+ad)\tanh^{-1}}{4b^{3/2}d^{5/2}n}$$

**Mathematica [A]** time = 0.51, size = 141, normalized size = 0.97

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\,(ad-3bc+2bdx^n)+(ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c -
a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])
```

**IntegrateAlgebraic [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x
]
```

**fricas [A]** time = 0.46, size = 359, normalized size = 2.46

$$\left[-\frac{\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{bd}\,\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2-4\left(2\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)-4\left(2b^2d^2x^n-3b^2cd+abd^2\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{16b^2d^3n}, -\frac{\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)-2\left(2b^2d^2x^n-3b^2cd+abd^2\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{8b^2d^3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(
b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(2*b
^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3
*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*sqrt(
-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^
2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n - 3*
b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}\,x^{2\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

**maple [F]** time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^{n}+a}\;x^{2n-1}}{\sqrt{d\,x^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(2*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}\,x^{2\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}\sqrt{a + bx^{n}}}{\sqrt{c + dx^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

## 3.690 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}} dx$

**Optimal**. Leaf size=89

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

**Rubi** [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d*n) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2dn}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2dn}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

**Mathematica** [A] time = 0.32, size = 123, normalized size = 1.38

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n) - \sqrt{bc-ad}\,(ad+bc)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*
d]])/(b^2*d^(3/2)*n*Sqrt[c + d*x^n])
```

**IntegrateAlgebraic** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x
]
```

**fricas** [A] time = 0.52, size = 281, normalized size = 3.16

$$\left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}\,bd + (bc+ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2n} + b^2c^2 + 6\,abcd + a^2d^2 - 4\left(2\sqrt{bd}\,bdx^n + (bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c} + 8\left(b^2cd + abd^2\right)x^n\right)}{4\,b^2d^2n}, \frac{2\sqrt{bx^n+a}\sqrt{dx^n+c}\,bd + (bc+ad)\sqrt{-bd}\,\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)}{2\,b^2d^2n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (
b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^
2)*x^n))/(b^2*d^2*n), 1/2*(2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a
*d)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*s
```

```
qrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*
d^2)*x^n)))/(b^2*d^2*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

**maple [F]** time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{b\,x^{n}+a}\,\sqrt{d\,x^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
[Out] int(x^(2*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{a+bx^{n}}\,\sqrt{c+dx^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(2*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

## 3.691 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}+\frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

**Rubi [A]** time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}+\frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{bn}$$

$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2n}$$

$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2n}$$

$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}$$

**Mathematica [A]** time = 0.90, size = 122, normalized size = 1.34

$$\frac{2\left(\frac{ab(c+dx^n)}{(bc-ad)\sqrt{a+bx^n}} + \frac{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}\right)}{b^2n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*((a*b*(c + d*x^n))/((b*c - a*d)*Sqrt[a + b*x^n]) + (Sqrt[b*c - a*d]*Sqrt
[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c -
a*d]])/Sqrt[d]))/(b^2*n*Sqrt[c + d*x^n])
```

**IntegrateAlgebraic [F]** time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),
 x]
```

**fricas [B]** time = 0.60, size = 408, normalized size = 4.48

$$\left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}\,abd + \left(\left(b^2c-abd\right)\sqrt{bd}\,x^n + \left(abc-a^2d\right)\sqrt{bd}\right)\log\left(8\,b^2d^2x^{2n} + b^2c^2 + 6\,abcd + a^2d^2 + 4\left(2\sqrt{bd}\,bdx^n + (bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c} + 8\left(b^2cd+abd^2\right)x^n\right)}{2\left(\left(b^4cd-ab^3d^2\right)nx^n + \left(ab^3cd-a^2b^2d^2\right)n\right)}, \frac{2\sqrt{bx^n+a}\sqrt{dx^n+c}\,abd - \left(\left(b^2c-abd\right)\sqrt{-bd}\,x^n + \left(abc-a^2d\right)\sqrt{-bd}\right)\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)}{\left(b^4cd-ab^3d^2\right)nx^n + \left(ab^3cd-a^2b^2d^2\right)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)*
x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n +
```

```
a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)*n*x
^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d
 - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arctan(1/2
*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n
 + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^4*c*d - a
*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

**maple [F]** time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(b\,x^n+a)^{\frac{3}{2}}\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
[Out] int(x^(2*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a+b\,x^n)^{3/2}\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

## 3.692 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=95

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

**Rubi** [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 78, 37}

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*n*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*n*Sqrt[a + b*x^n])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^n\right)}{n}\\
&= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{(3bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^n\right)}{3b(bc-ad)n}\\
&= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 57, normalized size = 0.60

$$\frac{2\sqrt{c+dx^n}\,(-2ac+adx^n-3bcx^n)}{3n(bc-ad)^2\,(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a +
b*x^n)^(3/2))
```

**IntegrateAlgebraic [F]** time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),
 x]
```

**fricas [A]** time = 0.74, size = 135, normalized size = 1.42

$$-\frac{2\,(2\,ac+(3\,bc-ad)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{3\,\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)nx^{2\,n}+2\,\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)nx^n+\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*a*c + (3*b*c - a*d)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)/((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

**maple [F]** time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(b\,x^n+a)^{\frac{5}{2}}\,\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(2*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a+b\,x^n\right)}^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

## 3.693 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=358

$$\frac{(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{128b^2d^5n}-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\,\sqrt{c+}}{192b^2d^4n}$$

**Rubi [A]** time = 0.40, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n}-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n}+\frac{(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n}-\frac{(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{128b^{5/2}d^{11/2}n}-\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n}+\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
 + d*x^n])/(128*b^2*d^5*n) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(192*b^2*d^4*n) + ((63*b^2*c^2 + 14
*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(240*b^2*d^3*n) -
(3*(3*b*c + a*d)*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(40*b^2*d^2*n) + (x^n*(
a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d*n) - ((b*c - a*d)^3*(63*b^2*c^2 +
14*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c +
 d*x^n])])/(128*b^(5/2)*d^(11/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}\left(a+bx^n\right)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{5bdn}$$

$$= -\frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\left(63b^2c^2+14ab\right.}{}$$

$$= \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+}}{40b^2d^2n}$$

$$= -\frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{\left(63b^2c^2+14abcd\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

**Mathematica [A]** time = 2.02, size = 274, normalized size = 0.77

$$\frac{\sqrt{c+dx^n}\left(\frac{5(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\left(-\frac{16d^3\left(a+bx^n\right)^3}{15(ad-bc)^3}-\frac{4d^2\left(a+bx^n\right)^2}{3(bc-ad)^2}-\frac{2d\left(a+bx^n\right)}{ad-bc}-\frac{2\sqrt{d}\sqrt{a+bx^n}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{4bd^5}-\frac{24(ad+3bc)\left(a+bx^n\right)^4}{bd}+64x^n\left(a+bx^n\right)^4\right)}{320bdn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*x
^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a +
b*x^n))/(-(b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d^3*
(a + b*x^n)^3)/(15*(-(b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(
Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]))))/(4*b*d^5)))/(320*b*d*n*Sqrt[a + b*x^n])
```

**IntegrateAlgebraic [F]** time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}\left(a+bx^n\right)^{5/2}}{\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],
 x]
```

**fricas [A]** time = 0.58, size = 771, normalized size = 2.15

$$\left[-\frac{15\left(63b^5c^5-175ab^4c^4d+150a^2b^3c^3d^2-30a^3b^2c^2d^3-5a^4bcd^4-3a^5d^5\right)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4\left(2\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)-4\left(384b^5d^5x^{4n}+945b^5c^4d-2310ab^4c^3d^2+1564a^2b^3c^2d^3-90a^3b^2cd^4-45a^4bd^5-144\left(3b^5cd^4-7ab^4d^5\right)x^{3n}+8\left(63b^5c^2d^3-148ab^4cd^4+93a^2b^3d^5\right)x^{2n}-2\left(315b^5c^3d^2-749ab^4c^2d^3+481a^2b^3cd^4-15a^3b^2d^5\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{7680b^3d^6n}, \frac{15\left(63b^5c^5-175ab^4c^4d+150a^2b^3c^3d^2-30a^3b^2c^2d^3-5a^4bcd^4-3a^5d^5\right)\sqrt{-bd}\arctan\left(\frac{\left(2\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)+2\left(384b^5d^5x^{4n}+945b^5c^4d-2310ab^4c^3d^2+1564a^2b^3c^2d^3-90a^3b^2cd^4-45a^4bd^5-144\left(3b^5cd^4-7ab^4d^5\right)x^{3n}+8\left(63b^5c^2d^3-148ab^4cd^4+93a^2b^3d^5\right)x^{2n}-2\left(315b^5c^3d^2-749ab^4c^2d^3+481a^2b^3cd^4-15a^3b^2d^5\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{3840b^3d^6n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b
^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(384*b
^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3
- 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n)
 + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b
^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*s
qrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^5 - 175*a
*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a
^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d
))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d +
a*b*d^2)*x^n)) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^
2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d
^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^
3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4
 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^n+a\right)^{\frac{5}{2}}x^{3n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

**maple [F]** time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{n}+a)^{\frac{5}{2}}\,x^{3n-1}}{\sqrt{d\,x^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
[Out] int(x^(3*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^{n}+a)^{\frac{5}{2}}x^{3\,n-1}}{\sqrt{dx^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3\,n-1}\,(a+b\,x^{n})^{5/2}}{\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

## 3.694 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=291

$$-\frac{(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64b^2d^4n}+\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n}+\frac{(b}{}$$

**Rubi** [A] time = 0.32, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n}-\frac{(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64b^2d^4n}+\frac{(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{9/2}n}-\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n}+\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
+ d*x^n])/(64*b^2*d^4*n) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^
n)^(3/2)*Sqrt[c + d*x^n])/(96*b^2*d^3*n) - ((7*b*c + 3*a*d)*(a + b*x^n)^(5/
2)*Sqrt[c + d*x^n])/(24*b^2*d^2*n) + (x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n]
)/(4*b*d*n) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(5/2)*d^(9/2)*n
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn}$$

$$= -\frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\left(35b^2c^2+10ab\ldots\right.}{\ldots}$$

$$= \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+\ldots}}{24b^2d^2n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+\ldots\right.}{\ldots}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+\ldots\right.}{\ldots}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+\ldots\right.}{\ldots}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+\ldots\right.}{\ldots}$$

**Mathematica [A]** time = 1.01, size = 241, normalized size = 0.83

$$\frac{3(bc-ad)^{5/2}\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\left(9a^3d^3+3a^2bd^2(5c-2dx^n)-ab^2d\left(145c^2-92cdx^n+72d^2x^{2n}\right)+b^3\left(105c^3-70c^2dx^n+56cd^2x^{2n}-48d^3x^{3n}\right)\right)}{192b^3d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*
d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 - 7
```

```
0*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)*(3
5*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSi
nh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^3*d^(9/2)*n*Sqrt[c +
d*x^n])
```

**IntegrateAlgebraic [F]** time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}\left(a+bx^n\right)^{3/2}}{\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],
 x]
```

**fricas [A]** time = 0.54, size = 607, normalized size = 2.09

$$\left[\frac{3\left(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)+4\left(48\,b^4d^4x^{3\,n}-105\,b^4c^3d+145\,ab^3c^2d^2-15\,a^2b^2cd^3-9\,a^3bd^4-8\left(7\,b^4cd^3-9\,ab^3d^4\right)x^{2\,n}+2\left(35\,b^4c^2d^2-46\,ab^3cd^3+3\,a^2b^2d^4\right)x^n\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{768\,b^3d^5n},\ -\frac{3\left(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{2\left(b^2d^2x^{2\,n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)-2\left(48\,b^4d^4x^{3\,n}-105\,b^4c^3d+145\,ab^3c^2d^2-15\,a^2b^2cd^3-9\,a^3bd^4-8\left(7\,b^4cd^3-9\,ab^3d^4\right)x^{2\,n}+2\left(35\,b^4c^2d^2-46\,ab^3cd^3+3\,a^2b^2d^4\right)x^n\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{384\,b^3d^5n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d
^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d
*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^
3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 -
 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)
*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^4*c^4 -
 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d
)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a
)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^
3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2
 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^5*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^n+a\right)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

**maple [F]** time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^n+a\right)^{\frac{3}{2}}\,x^{3n-1}}{\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{3\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3\,n-1}\,(a + b\,x^n)^{3/2}}{\sqrt{c + d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

## 3.695 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=221

$$\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}-\frac{(3ad+5bc)\,(}{}$$

**Rubi [A]** time = 0.25, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}-\frac{(3ad+5bc)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n}+\frac{x^n\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b^2*
d^3*n) - ((5*b*c + 3*a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b^2*d^2*n)
 + (x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(8*b^(5/2)*d^(7/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{3bdn} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{3bdn}$$

$$= -\frac{(5bc+3ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{3bdn} + \frac{\left(5b^2c^2+2abcd+\right.}{}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + $$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + $$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + $$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + $$

**Mathematica [A]** time = 0.62, size = 191, normalized size = 0.86

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\left(c+dx^n\right)\left(-3a^2d^2+2abd\left(dx^n-2c\right)+b^2\left(15c^2-10cdx^n+8d^2x^{2n}\right)\right)-3(bc-ad)^{3/2}\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*x^n)
 + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(3/2)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c + d*x^n])
```

**IntegrateAlgebraic [F]**  time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n], x
]
```

**fricas [A]**  time = 0.51, size = 471, normalized size = 2.13

$$\left[-\frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4\left(2\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)-4\left(8b^3d^3x^{2n}+15b^3c^2d-4ab^2cd^2-3a^2bd^3-2\left(5b^3cd^2-ab^2d^3\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{96b^3d^4n}, \frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)+2\left(8b^3d^3x^{2n}+15b^3c^2d-4ab^2cd^2-3a^2bd^3-2\left(5b^3cd^2-ab^2d^3\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{48b^3d^4n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n
 + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*
b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b
*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt
(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^
n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n
)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*
(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^4*n)
]
```

**giac [F]**  time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}\,x^{3n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

**maple [F]**  time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^n+a}\;x^{3n-1}}{\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]**  time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}\,x^{3n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(3*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

## 3.696 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=150

$$-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}-\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

**Rubi [A]** time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 90, 80, 63, 217, 206}

$$-\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b^2*d^2*n) + (x^n*Sqrt[
a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d*n) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(5/2)*d^(5/2
)*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{4b^{5/2}}$$

**Mathematica [A]** time = 0.46, size = 157, normalized size = 1.05

$$\frac{\sqrt{bc-ad}\left(3a^2d^2+2abcd+3b^2c^2\right)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)(-3ad-3bc+2bdx^n)}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt[
b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c -
a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)*n*
Sqrt[c + d*x^n])
```

**IntegrateAlgebraic [F]** time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x
]
```

**fricas [A]** time = 0.50, size = 361, normalized size = 2.41

$$\left[\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4\left(2\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)+4\left(2b^2d^2x^n-3b^2cd-3abd^2\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{16b^3d^3n}, -\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\sqrt{-bd}\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)-2\left(2b^2d^2x^n-3b^2cd-3abd^2\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{8b^3d^3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(2*
b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*
d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*
sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c
)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n
 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple [F]** time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{b\,x^{n}+a}\,\sqrt{d\,x^{n}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{a+bx^{n}}\,\sqrt{c+dx^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(3*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

## 3.697 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=133

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

**Rubi** [A] time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 89, 80, 63, 217, 206}

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (Sqrt[a + b*
x^n]*Sqrt[c + d*x^n])/(b^2*d*n) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*d^(3/2)*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{b^2(bc-ad)n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.}{b^3dn}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x,\right.}{b^3dn}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}
\end{aligned}
$$

**Mathematica [A]** time = 0.52, size = 185, normalized size = 1.39

$$
\frac{\sqrt{bc-ad}\left(-3a^2d^2+2abcd+b^2c^2\right)\sqrt{a+bx^n}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\left(c+dx^n\right)\left(-3a^2d+ab\left(c-dx^n\right)+b^2cx^n\right)}{b^3d^{3/2}n(ad-bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-(b*Sqrt[d]*(c + d*x^n)*(-3*a^2*d + b^2*c*x^n + a*b*(c - d*x^n))) + Sqrt[b
*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(b^3
*d^(3/2)*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])
```

**IntegrateAlgebraic [F]** time = 0.31, size = 0, normalized size = 0.00

$$
\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),
 x]
```

**fricas [B]** time = 0.57, size = 540, normalized size = 4.06

$$\left[\frac{4\left(ab^2cd-3a^2bd^2+\left(b^3cd-ab^2d^2\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+\left(\left(b^3c^2+2ab^2cd-3a^2bd^2\right)\sqrt{bd}\,x^n+\left(ab^2c^2+2a^2bcd-3a^3d^2\right)\sqrt{bd}\right)\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2-4\left(2\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)}{4\left(\left(b^5cd^2-ab^4d^3\right)nx^n+\left(ab^4cd^2-a^2b^3d^3\right)n\right)}, \frac{2\left(ab^2cd-3a^2bd^2+\left(b^3cd-ab^2d^2\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+\left(\left(b^3c^2+2ab^2cd-3a^2bd^2\right)\sqrt{-bd}\,x^n+\left(ab^2c^2+2a^2bcd-3a^3d^2\right)\sqrt{-bd}\right)\arctan\left(\frac{\left(2\sqrt{-bd}\,bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)}{2\left(\left(b^5cd^2-ab^4d^3\right)nx^n+\left(ab^4cd^2-a^2b^3d^3\right)n\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n +
 (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^5*c*d
^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2*c*d -
 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) +
 ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2 + 2*a^2
*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a
*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d
+ (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2
- a^2*b^3*d^3)*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

**maple [F]** time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(b\,x^n+a)^{\frac{3}{2}}\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{(a+b\,x^n)^{3/2}\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

## 3.698 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

**Optimal**. Leaf size=147

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\,(a+bx^n)^{3/2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}$$

**Rubi** [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 89, 78, 63, 217, 206}

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\,(a+bx^n)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*n*(a + b*x^n)^(3/2)) + (4*a*(3*
b*c - 2*a*d)*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)^2*n*Sqrt[a + b*x^n]) + (2*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*Sqrt
[d]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^n\right)}{n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^n\right)}{3b^2(bc-ad)n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,d\right.}{b^2n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,d\right.}{b^3n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}}+\frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\right.}{b^3n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}
\end{aligned}
$$

**Mathematica [A]** time = 0.81, size = 217, normalized size = 1.48

$$
\frac{2\sqrt{c+dx^n}\left(\frac{(3b^2c^2-a^2d^2)(a+bx^n)}{d(bc-ad)^2}+\frac{a^2}{ad-bc}-\frac{3(a+bx^n)\left(\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}-\sqrt{d}\sqrt{a+bx^n}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)\right)}{d\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{3b^2n\,(a+bx^n)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^n)
)/(d*(b*c - a*d)^2) - (3*(a + b*x^n)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/
(b*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sq
rt[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)])))/(3
*b^2*n*(a + b*x^n)^(3/2))
```

**IntegrateAlgebraic [F]** time = 0.85, size = 0, normalized size = 0.00

$$
\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),
 x]
```

**fricas [B]** time = 0.79, size = 769, normalized size = 5.23

$$\left[\frac{4\left(5a^2b^2cd-3a^3bd^2+2\left(3ab^3cd-2a^2b^2d^2\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+3\left(\left(b^4c^2-2ab^3cd+a^2b^2d^2\right)\sqrt{bd}x^{2n}+2\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)\sqrt{bd}x^n+\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)\sqrt{bd}\right)\log\left(8b^2d^2x^{2n}+b^2c^2+6abcd+a^2d^2+4\left(2\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+8\left(b^2cd+abd^2\right)x^n\right)}{6\left(\left(b^7c^2d-2ab^6cd^2+a^2b^5d^3\right)nx^{2n}+2\left(ab^6c^2d-2a^2b^5cd^2+a^3b^4d^3\right)nx^n+\left(a^2b^5c^2d-2a^3b^4cd^2+a^4b^3d^3\right)n\right)}, \frac{2\left(5a^2b^2cd-3a^3bd^2+2\left(3ab^3cd-2a^2b^2d^2\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}-3\left(\left(b^4c^2-2ab^3cd+a^2b^2d^2\right)\sqrt{-bd}x^{2n}+2\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)\sqrt{-bd}x^n+\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)\sqrt{-bd}\right)\arctan\left(\frac{\left(2\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\left(b^2d^2x^{2n}+abcd+\left(b^2cd+abd^2\right)x^n\right)}\right)}{3\left(\left(b^7c^2d-2ab^6cd^2+a^2b^5d^3\right)nx^{2n}+2\left(ab^6c^2d-2a^2b^5cd^2+a^3b^4d^3\right)nx^n+\left(a^2b^5c^2d-2a^3b^4cd^2+a^4b^3d^3\right)n\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)
*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)*x^
n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^7*
c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5
*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^
3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2
)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(
-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*arctan(1/2*(2
*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n +
c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^7*c^2*d - 2*
a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a
^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n)]
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

**maple [F]** time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(b\,x^n+a)^{\frac{5}{2}}\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

## 3.699 $\int x^p(b+cx)^p(b+2cx)\,dx$

**Optimal**. Leaf size=20

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

**Rubi** [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

```
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

**Mathematica** [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

```
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

**IntegrateAlgebraic** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^p(b+cx)^p(b+2cx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^p*(b + c*x)^p*(b + 2*c*x), x]
```

**fricas** [A] time = 0.42, size = 25, normalized size = 1.25

$$\frac{\left(cx^2+bx\right)(cx+b)^p x^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="fricas")
[Out] (c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)
```

**giac** [A] time = 0.16, size = 35, normalized size = 1.75

$$\frac{(cx+b)^p cx^2 x^p + (cx+b)^p bxx^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="giac")
[Out] ((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)
```

**maple** [A] time = 0.05, size = 21, normalized size = 1.05

$$\frac{x^{p+1}(cx+b)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(c*x+b)^p*(2*c*x+b),x)
[Out] x^(p+1)*(c*x+b)^(p+1)/(p+1)
```

**maxima** [A] time = 0.82, size = 29, normalized size = 1.45

$$\frac{\left(cx^2+bx\right)e^{\left(p\log(cx+b)+p\log(x)\right)}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")
[Out] (c*x^2 + b*x)*e^(p*log(c*x + b) + p*log(x))/(p + 1)
```

**mupad** [B] time = 4.80, size = 22, normalized size = 1.10

$$\frac{x\,x^p\,{(b+c\,x)}^p\,\left(b+c\,x\right)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(b + c*x)^p*(b + 2*c*x),x)
[Out] (x*x^p*(b + c*x)^p*(b + c*x))/(p + 1)
```

**sympy** [A] time = 3.10, size = 46, normalized size = 2.30

$$\begin{cases} \frac{bxx^p(b+cx)^p}{p+1} + \frac{cx^2x^p(b+cx)^p}{p+1} & \text{for}\: p \neq -1 \\ \log(x) + \log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(c*x+b)**p*(2*c*x+b),x)
[Out] Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

## 3.700 $\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)dx$

**Optimal**. Leaf size=27

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)dx=\frac{x^{2(1+p)}\left(b+cx^2\right)^{1+p}}{2(1+p)}$$

**Mathematica [C]** time = 0.13, size = 97, normalized size = 3.59

$$\frac{x^{2p+2}\left(b+cx^2\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2 + 2*p)*(b + c*x^2)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2
)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

**IntegrateAlgebraic [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2), x
]
```

**fricas** [A] time = 0.43, size = 32, normalized size = 1.19

$$\frac{\left(cx^{3}+bx\right)\left(cx^{2}+b\right)^{p}x^{2\,p+1}}{2\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^2 + b)^p*x^(2*p + 1)/(p + 1)
```

**giac** [B] time = 0.19, size = 52, normalized size = 1.93

$$\frac{\left(cx^{2}+b\right)^{p}cx^{3}e^{\left(2\,p\log(x)+\log(x)\right)}+\left(cx^{2}+b\right)^{p}bxe^{\left(2\,p\log(x)+\log(x)\right)}}{2\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2 + b)^p*c*x^3*e^(2*p*log(x) + log(x)) + (c*x^2 + b)^p*b*x*e^(2*p
*log(x) + log(x)))/(p + 1)
```

**maple** [A] time = 0.04, size = 26, normalized size = 0.96

$$\frac{x^{2p+2}\left(c\,x^{2}+b\right)^{p+1}}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p+1)*(c*x^2+b)^p*(2*c*x^2+b),x)
```

```
[Out] 1/2*x^(2*p+2)*(c*x^2+b)^(p+1)/(p+1)
```

**maxima** [A] time = 0.89, size = 35, normalized size = 1.30

$$\frac{\left(cx^{4}+bx^{2}\right)e^{\left(p\log\left(cx^{2}+b\right)+2\,p\log(x)\right)}}{2\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

**mupad** [B] time = 4.88, size = 47, normalized size = 1.74

$$\left(c\,x^{2}+b\right)^{p}\,\left(\frac{c\,x^{2\,p+1}\,x^{3}}{2\,p+2}+\frac{b\,x\,x^{2\,p+1}}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p + 1)*(b + c*x^2)^p*(b + 2*c*x^2),x)
```

```
[Out] (b + c*x^2)^p*((c*x^(2*p + 1)*x^3)/(2*p + 2) + (b*x*x^(2*p + 1))/(2*p + 2))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)
```

```
[Out] Timed out
```

## 3.701 $\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx$

**Optimal**. Leaf size=27

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] (x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

**Mathematica [C]** time = 0.10, size = 97, normalized size = 3.59

$$\frac{x^{3p+3}\left(b+cx^3\right)^p\left(\frac{cx^3}{b}+1\right)^{-p}\left(2c(p+1)x^3\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] (x^(3 + 3*p)*(b + c*x^3)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3
)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

**IntegrateAlgebraic [F]** time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3), x
]
```

**fricas** [A] time = 0.43, size = 32, normalized size = 1.19

$$\frac{\left(cx^4 + bx\right)\left(cx^3 + b\right)^p x^{3\,p+2}}{3\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="fricas")
[Out] 1/3*(c*x^4 + b*x)*(c*x^3 + b)^p*x^(3*p + 2)/(p + 1)
```

**giac** [B] time = 0.26, size = 56, normalized size = 2.07

$$\frac{\left(cx^3 + b\right)^p cx^4 e^{\left(3\,p\log\left(x\right)+2\,\log\left(x\right)\right)} + \left(cx^3 + b\right)^p bxe^{\left(3\,p\log\left(x\right)+2\,\log\left(x\right)\right)}}{3\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="giac")
[Out] 1/3*((c*x^3 + b)^p*c*x^4*e^(3*p*log(x) + 2*log(x)) + (c*x^3 + b)^p*b*x*e^(3
*p*log(x) + 2*log(x)))/(p + 1)
```

**maple** [A] time = 0.04, size = 26, normalized size = 0.96

$$\frac{x^{3p+3}\left(c\,x^3 + b\right)^{p+1}}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x)
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(p+1)/(p+1)
```

**maxima** [A] time = 0.69, size = 35, normalized size = 1.30

$$\frac{\left(cx^6 + bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)\right)}}{3\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

**mupad** [B] time = 4.90, size = 47, normalized size = 1.74

$$\left(c\,x^3 + b\right)^p\,\left(\frac{c\,x^{3\,p+2}\,x^4}{3\,p+3} + \frac{b\,x\,x^{3\,p+2}}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*p + 2)*(b + c*x^3)^p*(b + 2*c*x^3),x)
[Out] (b + c*x^3)^p*((c*x^(3*p + 2)*x^4)/(3*p + 3) + (b*x*x^(3*p + 2))/(3*p + 3))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)
[Out] Timed out
```

## 3.702 $\int x^{-1+n(1+p)} (b + cx^n)^p (b + 2cx^n) \, dx$

**Optimal**. Leaf size=27

$$\frac{x^{n(p+1)} (b + cx^n)^{p+1}}{n(p+1)}$$

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{n(p+1)} (b + cx^n)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] (x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+n(1+p)} (b + cx^n)^p (b + 2cx^n) \, dx = \frac{x^{n(1+p)} (b + cx^n)^{1+p}}{n(1+p)}$$

**Mathematica [C]** time = 0.16, size = 101, normalized size = 3.74

$$\frac{(b + cx^n)^p \left(\frac{cx^n}{b} + 1\right)^{-p} \left(b(p+2)x^{n(p+1)} {}_2F_1\left(-p, p+1; p+2; -\frac{cx^n}{b}\right) + 2c(p+1)x^{n(p+2)} {}_2F_1\left(-p, p+2; p+3; -\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] ((b + c*x^n)^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3 +
 p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*(1 + (c*x^n)/b)^p)
```

**IntegrateAlgebraic [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int x^{-1+n(1+p)} (b + cx^n)^p (b + 2cx^n) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n), x
]
```

**fricas [A]** time = 0.45, size = 35, normalized size = 1.30

$$\frac{(cxx^n + bx)(cx^n + b)^p x^{np+n-1}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="fricas")
[Out] (c*x*x^n + b*x)*(c*x^n + b)^p*x^(n*p + n - 1)/(n*p + n)
```

**giac [B]** time = 0.25, size = 66, normalized size = 2.44

$$\frac{(cx^n + b)^p cxx^n e^{\left(np\log(x) + n\log(x) - \log(x)\right)} + (cx^n + b)^p bxe^{\left(np\log(x) + n\log(x) - \log(x)\right)}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="giac")
[Out] ((c*x^n + b)^p*c*x*x^n*e^(n*p*log(x) + n*log(x) - log(x)) + (c*x^n + b)^p*b
*x*e^(n*p*log(x) + n*log(x) - log(x)))/(n*p + n)
```

**maple [F]** time = 0.69, size = 0, normalized size = 0.00

$$\int (2c\,x^n + b)\,x^{(p+1)n-1}\,(c\,x^n + b)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n*(p+1))*(c*x^n+b)^p*(b+2*c*x^n),x)
[Out] int(x^(-1+n*(p+1))*(c*x^n+b)^p*(b+2*c*x^n),x)
```

**maxima [A]** time = 0.96, size = 39, normalized size = 1.44

$$\frac{\left(cx^{2n} + bx^n\right)e^{\left(np\log(x) + p\log(cx^n + b)\right)}}{n\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="maxima")
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

**mupad [B]** time = 4.86, size = 54, normalized size = 2.00

$$\left(\frac{b\,x\,x^{n\,\left(p+1\right)-1}}{n\,\left(p+1\right)} + \frac{c\,x\,x^n\,x^{n\,\left(p+1\right)-1}}{n\,\left(p+1\right)}\right)\,(b + c\,x^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n*(p + 1) - 1)*(b + c*x^n)^p*(b + 2*c*x^n),x)
[Out] ((b*x*x^(n*(p + 1) - 1))/(n*(p + 1)) + (c*x*x^n*x^(n*(p + 1) - 1))/(n*(p +
1)))*(b + c*x^n)^p
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n*(1+p))*(b+c*x**n)**p*(b+2*c*x**n),x)
[Out] Timed out
```

# Chapter 4

# Appendix

## Local contents

## 4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

**Mathematica format** Mathematica_syntax_CAS_integration_elementary_version.zip

**Maple and Mupad format** Maple_syntax_CAS_integration_elementary_version.zip

**Sympy format** SYMPY_syntax_CAS_integration_elementary_version.zip

**Sage math format** SAGE_syntax_CAS_integration_elementary_version.zip

## 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```